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Risk-Sharing Externalities and Its Implications for Equity Premium in an Infinite-Horizon Economy

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Abstract This paper examines asset prices when risk-sharing externalities are incorporated into an infinite-horizon model where consumers are exposed to the endogenous income risks. It is shown that there exist multiple types of equilibria depending on the degree of market participation. Under incomplete participation, income risks cannot be fully diversified as they induce higher precautionary savings, which are conducive in turn to higher asset prices. However, the exposure to additional dividend risks can lead at the same time to a lower demand for risky assets. The aggregate effect is an increase in the equity risk premium and a decrease in the risk-free rate. Thus, the evidence suggests that the equity premium and risk-free rate puzzles can be partly explained by infinite-horizon models with incomplete market participation.

Keywords Risk-sharing externalities, endogenous income risks, incomplete market participation, asset prices
JEL classification D11, E21, E44, G12

1. Introduction

The main objective of this paper is to provide some theoretical explanation for non-market participation from some economic agents. This issue is important because by determining the rationale behind the lack of full participation, it may be possible to provide some insights on the equity premium and risk-free rate puzzles. The equity premium puzzle reflects the difficulties in explaining the difference between the observed returns on equity and risk-free rates beyond the important factors of risk aversion and aggregate consumption risk. Even upon allowing for extremely high risk-aversion, the implied risk-free rate exceeds the historical rates, which leads in turn to the risk-free rate puzzle. Mehra and Prescott (1985) examine the equity premium puzzle using the Lucas (1978) exchange economy, where the equity premium is estimated as the risk aversion parameter multiplied by the covariance between the aggregate consumption and return on equity. On the other hand, the risk-free puzzle is addressed by Weil (1989), where the risk-free rate is approximated by the sum of time preference rate and the risk aversion parameter multiplied by the expected consumption growth. The raison d'être of these puzzles lies in the evidence that the aggregate consumption risk implied by national consumption statistics is rather small, which renders the explana-

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tion of high equity premia based on the risk aversion parameter, arguably difficult. It is equally difficult to explain the observed risk-free rates based on this high degree of risk aversion. Hence, these puzzles appear to be intertwined and should be ideally addressed simultaneously.

There are several studies that attempt to provide an economic rationale for these findings.[1] Part of this literature is based on the complete Arrow-Debreu economy, where full market participation is implicitly assumed and consumers are able to diversify their idiosyncratic income shocks using Arrow-Debreu securities. The equity premium puzzle derives from the observation that if there were only insurable idiosyncratic risks, investors would be exposed to small aggregate risks. It is also difficult for models based on the representative agent and complete contingent markets to explain the behavior of asset returns. In addition, there are attempts to examine the utility function, such as the habit formation where consumers have the incentive to smooth out their consumption. Alternatively, it is possible to examine market incompleteness where risk-sharing conditions may be affected by market frictions. This implies that the aggregate consumption can be smoothed out but individual consumptions are not.

The degree of market participation is important in the explanation of consumption and asset returns. The traditional assumption that all consumers participate in fully-integrated markets is rather implausible. There is indeed strong evidence to the contrary. Incomplete market participation is evident in many countries and the proportion of non-market participants is rather significant. Only a fraction of consumers may invest in equity, while others may prefer to hold bank deposits only. Mankiw and Zeldes (1991) provided early evidence that stockholders represent only a small fraction of sample of consumers.[2] The theoretical explanation for incomplete market participation is discussed in the literature based on fixed-entry costs (Abel 2001; Vissing-Jorgensen 2002b; Weil 1992b), transaction costs and liquidity limits (Allen and Gale 1994; Williamson 1994), borrowing constraints (Constantinides et al. 2002) and model uncertainty (Cao et al. 2005). If consumers are assumed more plausibly, to incur alternative types of income risk under incomplete market participation, then they are not necessarily able to adequately diversify their income shocks. This implies that Lucas economy models should consider endogenous income risks and incomplete market participation.[3]

Thus, the raison d'être of this paper lies in providing a rationale for non-market participation, and the focus is made on risk-sharing externalities. The modelling approach is based on an infinite-horizon exchange economy. It is related to early studies by Allen and Gale (1994), who endogenize the decision on market participation, and examine the volatility of asset prices. Their evidence suggests that asset price volatility

[1] Kocherlakota (1996) offers a survey of the literature on the equity premium puzzle.

[2] The evidence provided by Mankiw and Zeldes (1991) is based on the estimation of Euler equations for stockholders and non-stockholders using data from Panel Study of Income Dynamics. The proportion of stockholders to sample consumers amounts to 27%.

[3] Incomplete market participation may be understood with respect to disparities in wealth. The relationship between wealth, market participation and income risk diversification is important, but it falls beyond the scope of the present study. Suffice it to state here, that it is possible for consumers to increase the relative level of wealth through market participation, which is conducive to higher returns on equity.

is reduced under the full participation equilibrium. Similar assumptions are made here as income risks are assumed to be diversifiable under complete market participation. In light of the critical issue raised by Allen and Gale (2001) as to how various risk-sharing mechanisms interact, this paper examines the conditions under which there exist two distinct mechanisms providing risk-sharing opportunities. One mechanism takes place through implicit asset transactions between market participants whereas the other involves banking transactions for non-market participants. It is assumed that income risks are diversifiable under a given mechanism when consumers do not consider the alternative mechanism. However, if consumers fall into two distinct groups, then income risks cannot be fully diversified. Also, the present analysis is related to Weil (1992b), who examines the effect of exogenous hand-to-mouth consumption on asset prices. It differs however from this earlier study as the focus is rather made here on the endogenization of the decision on market participation into the optimization problem. The introduction of risk-sharing externalities and beliefs about the degree of incomplete market participation allows for the examination of the effects of consumer behavior on asset prices under multiple types of equilibria. Thus, the assumptions underlying the present model are different from those adopted in previous studies.

The present model contributes to the literature with new evidence that there is potentially multiple equilibria with incomplete market participation, full market participation and strictly no market participation. Indeed, when risk-sharing externalities are considered in an infinite-horizon model, it can be shown that the existence of such equilibria depends on the degree of market participation. There is evidence that income risks cannot be fully diversified under incomplete market participation. The exposure to such risks implies higher precautionary savings and an increase in asset prices. However, the exposure to additional dividend risks can be conducive at the same time to a lower demand for risky assets. In aggregate, there is a decrease in the risk-free rate and a simultaneous increase in equity risk premium. Thus, the results suggest that the existence of risk-sharing externalities can in part, explain simultaneously the incomplete market participation, equity premium and risk-free rate puzzles. The paper also sheds light on the new concept of market participation risk premium, which exists only under equilibrium conditions of incomplete market participation.

In the remainder of this paper, Section 2 introduces the economic environment and examines the consumer decision problems. Section 3 describes the conditions of multiple equilibria under incomplete market participation. Section 4 analyzes asset prices using numerical calculus. It discusses the equity premium puzzle and risk-free rate puzzle, and it examines the role of social security, which allows consumers to insure income shocks. Section 5 concludes the paper.

2. The environment

Consider an exchange economy populated with infinitely lived continuum consumers, who consume a single good. Time is indexed with $t = -1, 0, 1, \ldots$ This economy starts at period -1, and all consumers can be classified into either type-A or type-B agents depending on their unrestricted or restricted access to stock markets. Type-A agents (stockholders) hold equity while type-B agents (non-stockholders) do not. The existence of non-stockholders has been thoroughly described in the literature, with some studies providing a theoretical rationale for such consumers (e.g., Abel 2001; Allen and Gale 1994; Cao et al. 2005; Constantinides et al. 2002; Vissing-Jorgensen 2002b; Williamson 1994).

Suppose that an aggregate fraction $1-\mu$ $(\in[0,1])$ of the population falls under type-A agents. Therefore, market participants hold stocks x_{t+1} and inside bonds b_{t+1} given the asset prices p_t and q_t in period t, respectively. By definition, an inside bond implies that the net supply is equal to zero. The equity yields stochastic dividends $\widetilde{d}_{t+1}$ and capital gains $\widetilde{p}_{t+1}$ in period $t+1$. Non-market participants hold deposits $\widehat{b}_{t+1}$ given the net deposit interest rate $\overline{r}_{t+1}$.

There exists a representative firm which raises money on μ from bank borrowing and $1-\mu$ through equity financing, and allocates the funds to an investment project, which yields $\widetilde{X}$ units where $\{\widetilde{X}_t\}_{t=0}^{\infty}$ is a sequence of independent and identically distributed cash flows. This assumption is rather plausible as the extent of direct financing is function of the demand for equity from market participants. The capital structure is endogenized by the degree of market participation. Assume that the loan rate is equivalent to the deposit interest rate $\overline{r}$. Since the dividend is obtainable from residual profits, it can be expressed as follows:

$$\widetilde{d}=\frac{\widetilde{X}-\mu(1+\overline{r})}{1-\mu}$$

An increase in type-B agents induces a rise in the leverage ratio.[4]

Consumers receive the stochastic labor income $\{\widetilde{y}_{it}\}_{t=0}^{\infty}$ for $i=A,B$ of consumption good over their infinite lifetime. Future labor income risk is assumed to be dependent on the degree of incomplete market participation, $\{\widetilde{y}_{it}(\mu)\}$ for $i=A,B$. Assume that $\widetilde{y}_{it}$ is independently and identically distributed over time, and there is no aggregate uncertainty. Furthermore, let $\tilde{y}_{it}$ be subject to $\mathrm{E}_{t-1}[\widetilde{y}_{it}(\mu)]=\mathrm{E}[\widetilde{y}_{it}(\mu)]\equiv\bar{y}$ and $Var_{t-1}[\widetilde{y}_{it}(\mu)]=Var[\widetilde{y}_{it}(\mu)]=\sigma_i(\mu)$, where E denotes the expectations operator, and Var represents the variance. It is assumed that the income risk $\sigma_A(\mu)$ is increasing in $\mu\in[0,1]$, which ensures the existence of equilibrium. On the other hand, $\sigma_B(\mu)$ must be rather decreasing in μ. Agents are of only two types, which means that either type-A or type-B agents must absorb the unilateral labor-income shocks in the model without aggregate risk. Hence, increases in the number of non-market participants under these conditions, have the effect of diminishing their consumption risks. Income risks are assumed to be diversifiable when all consumers belong to the same risk-sharing group. However, if consumers fall into two distinct groups, then they cannot fully diversify income risks. In an economy with full participation, income risks are assumed to be diversifiable, that is, $\sigma_A(0)=0$ and $\sigma_B(1)=0$.[5]

[4] In the Lucas economy, all agents invest in equity and inside bond, which implies that the net supply of bonds is equal to zero. Such a model setting does not allow for the examination of leverage effects.

[5] The background theory on these assumptions is in Appendix 1 which explains the risk-sharing externalities. These assumptions are based on two empirical observations. First, the volatility of aggregate consumption is very small and this may be due to the lack of variability in both stockholders' and non-stockholders'

The assumption of common beliefs can be made with regard to stock market participation because the ratio of market participation affects in turn, the magnitude of income shocks. Indeed, within this participation game, income shocks are generated by the degree of incomplete market participation. The linkage between incomplete participation and income risks can be understood in light of the higher volatility in the consumption patterns of market participants, which are due to limited risk-sharing opportunities.

Each agent consumes the amount c_{it} of goods in period t. The preferences of these agents are defined by the following expected value of aggregate discounted utility functions:

$$U_{i0} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\gamma}}{1-\gamma} \quad \text{for } i = A, B, \ \beta \in (0,1) \text{ and } \gamma > 0, \tag{1}$$

where β and γ represent the discount factor and the risk aversion parameter, respectively.

These settings are useful in redefining the consumer's problem under incomplete market participation using the standard income-fluctuations approach. The following subsection examines type-A agent's problem of utility maximization.

2.1 Market participants

Type-A consumers decide to participate in the equity market given the endowment with asset x_0. They hold equity x_{t+1} and bond b_{t+1} in period t, as they are associated with the following dynamic optimization problem:

$$\max \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_{At}^{1-\gamma}}{1-\gamma} \right]$$

subject to the sequence of budget constraints given by

$$c_{At} + p_t x_{t+1} + q_t b_{t+1} = w_{At}, \tag{2}$$

$$w_{At+1} = \widetilde{y}^{A}_{t+1} + (\widetilde{d}_{t+1} + \widetilde{p}_{t+1}) x_{t+1} + b_{t+1}, \tag{3}$$

where w_{At} denotes the wealth in period t.

consumptions. Alternatively, while each agent's consumption may exhibit a degree of volatility, consumption risks are offset on aggregate. Second, the empirical evidence from Attanasio et al. (2002), Brav et al. (2002) and Vissing-Jorgensen (2002a) supports the second possibility. The covariance between market participants' consumption and stock returns is found to be substantially large while that between non-market participants and stock returns is small. This means that full-integrated consumption exhibits less fluctuation, and that variations in market participants' consumptions can be offset by changes in non-market participants. However, it should be noted also that Brav et al. (2002) suggest that part of the volatility of consumption reported in the Consumers Expenditure Survey might be due to measurement errors.

2.2 Non-market participants

If the value function of type-B is larger than that of type-A $U_{B0} \geq U_{A0}$ given the prevailing beliefs about the degree of incomplete market participation, the decisions made with respect of current income in terms of consumption and deposit can be characterized by:

$$\max \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_{Bt}^{1-\gamma}}{1-\gamma} \right]$$

$$\text{s.t.} \quad c_{Bt} + \widehat{b}_{t+1} = y_{Bt} + (1+\bar{r}_t)\widehat{b}_t \tag{4}$$

Under these conditions, type-B agents have no incentive to participate into the equity market. In equilibrium, non-stockholders provide one unit of deposit and consume $y_{Bt} + \bar{r}_t$.

2.3 Equilibrium asset prices

The Euler equations are given by:

$$p_t c_{At}^{-\gamma} = \beta \mathrm{E}_t[(\widetilde{p}_{t+1} + \widetilde{d}_{t+1})\widetilde{c}_{At+1}^{-\gamma}], \tag{5}$$

$$q_t c_{At}^{-\gamma} = \beta \mathrm{E}_t[\widetilde{c}_{At+1}^{-\gamma}] \tag{6}$$

Therefore, asset prices can be expressed as follows:

$$p_t = \beta \mathrm{E}_t \left[(\widetilde{p}_{t+1} + \widetilde{d}_{t+1}) \cdot \left(\frac{\widetilde{c}_{At+1}}{c_{At}} \right)^{-\gamma} \right],$$

$$q_t = \beta \mathrm{E}_t \left[\left(\frac{\widetilde{c}_{At+1}}{c_{At}} \right)^{-\gamma} \right]$$

It is assumed that per capita equity and deposit are supplied to the extent of one unit. Given the assumption of inside bond, the consumption of type-A agents and type-B agents in period t can be expressed as

$$c_{At}^{*} = y_{At} + d_t, \tag{7}$$

$$c_{Bt}^{*} = y_{Bt} + \bar{r}_t. \tag{8}$$

The gross risk-free rate R_{t+1}^F is defined as:[6]

$$R_{t+1}^F \equiv \frac{1}{q_t} = \frac{(y_{At} + d_t)^{-\gamma}}{\beta \mathrm{E}_t \left[\left(\widetilde{y}_{At+1}(\mu) + \widetilde{d}_{t+1} \right)^{-\gamma} \right]} \tag{9}$$

[6] The deposit interest rate can be expressed as $1 + \bar{r}_{t+1} = \mathrm{E}_t c_{Bt+1}^{\gamma} / (\beta c_{Bt}^{\gamma})$. To derive the theoretical equity premium, the following reference is made only to the bond interest rate for market participants.

Also, the expected gross return on equity can be determined as:

$$\begin{aligned} \mathrm{E}_t\left[\widetilde{R}_{t+1}\right] &\equiv \frac{\mathrm{E}_t\left[\widetilde{p}_{t+1}+\widetilde{d}_{t+1}\right]}{p_t} \\ &= \frac{\mathrm{E}_t\left[\widetilde{p}_{t+1}+\widetilde{d}_{t+1}\right](y_{At}+d_t)^{-\gamma}}{\beta \mathrm{E}_t\left[(\widetilde{p}_{t+1}+\widetilde{d}_{t+1})\left(\widetilde{y}_{At+1}(\mu)+\widetilde{d}_{t+1}\right)^{-\gamma}\right]} \end{aligned} \tag{10}$$

In addition, it is possible to define the expected equity premium π as:

$$\begin{aligned} \pi_{t+1} &\equiv \frac{\mathrm{E}_t\left[\widetilde{R}_{t+1}\right]}{R^F_{t+1}} \\ &= \frac{\mathrm{E}_t\left[\widetilde{p}_{t+1}+\widetilde{d}_{t+1}\right]\mathrm{E}_t\left[\left(\widetilde{y}_{At+1}(\mu)+\widetilde{d}_{t+1}\right)^{-\gamma}\right]}{\mathrm{E}_t\left[(\widetilde{p}_{t+1}+\widetilde{d}_{t+1})\left(\widetilde{y}_{At+1}(\mu)+\widetilde{d}_{t+1}\right)^{-\gamma}\right]} \end{aligned} \tag{11}$$

Consequently, equilibrium asset prices are represented by the behavior of type-A consumers. Given an increase in consumption risk, there is a tendency for market participants to raise their precautionary savings, which in turn augments the demand for both risky and risk-free assets. However, because of risk aversion due to higher consumption risk under CRRA preference, the price of risky assets decreases, thereby driving excess returns on equity higher. Thus, it is important to examine whether there exist rational non-market participants or not. The following section addresses the welfare properties and multiple types of equilibria.

3. Multiple equilibria

Using equations (1) and (7), the lifetime indirect utility function V^*_{A0} of market participants is characterized by:

$$V^*_{A0} = \frac{\mathrm{E}[\widetilde{y}_A(\mu)+\widetilde{d}]^{1-\gamma}}{(1-\gamma)(1-\beta)} \tag{12}$$

In the same way, from equations (1) and (8), the lifetime indirect utility function V^*_{B0} of non-market participants can be expressed as

$$V^*_{B0} = \frac{\mathrm{E}[\widetilde{y}_B(\mu)+\overline{r}]^{1-\gamma}}{(1-\gamma)(1-\beta)}. \tag{13}$$

It is clear that when $\mathrm{E}[\widetilde{y}_B(\mu)+\overline{r}]^{1-\gamma}/(1-\gamma) \geq \mathrm{E}[\widetilde{y}_A(\mu)+\widetilde{d}]^{1-\gamma}/(1-\gamma)$, it is optimal for consumers to be non-stockholders. The individual decision is based on the relationship between V^*_{B0} and V^*_{A0}, and on whether consumers participate in the stock market. Therefore, non-stockholders believe that market participants would be able to absorb income risks. It is not optimal for these consumers to participate in the stock market if the expected return on equity cannot compensate for risks involved with market participation. For simplicity, it is assumed that the stochastic income ($\tilde{y}_t$) and dividend ($\tilde{d}_t$) are binomially distributed.

Table 1. Distribution of $\tilde{y}_t$ and $\tilde{d}_t$

Dividend	Type-A	Type-B	
$\bar{d}+\varepsilon$	$\bar{y}+\lambda\mu$	$\bar{y}-\lambda(1-\mu)$	with probability 0.5
$\bar{d}-\varepsilon$	$\bar{y}-\lambda\mu$	$\bar{y}+\lambda(1-\mu)$	with probability 0.5

Table 1 describes the binomial distribution of $\tilde{y}_t$ and $\tilde{d}_t$, where $\overline{d}$ and λ represent the expected dividend and the magnitude of income shocks which take positive values.[7] Suppose that $\bar{y}-(1-\lambda)\mu$ takes positive values in order to avoid cases of zero-consumption for non-market participants. This simplification is consistent with the previous assumptions about the endogeneity of income risk. Indeed, if the proportion of non-stockholders decreases, then the income shocks of market participants decrease. In particular, under a full participation economy, income shocks can be, by assumption, absorbed through implicit asset trading. In addition, the assumption that income $\bar{y}$ is constant over time implies that the aggregate labor income does not fluctuate. This is supported by evidence that the aggregate risk reflected by national consumption statistics is small (Mehra and Prescott 1985). Attanasio et al. (2002), Brav et al. (2002) and Vissing-Jorgensen (2002a) also report that the covariance of consumption growth with stock returns based on information from the Consumer Expenditure Survey, is much lower for non-stockholders than for stockholders.

Before characterizing the equilibrium conditions for this game, it is useful to introduce the following restrictions:

$$\frac{0.5(\bar{y}+\bar{d}+\varepsilon)^{1-\gamma}+0.5(\bar{y}+\bar{d}-\varepsilon)^{1-\gamma}}{1-\gamma} > \frac{0.5(\bar{y}+\bar{r}-\lambda)^{1-\gamma}+0.5(\bar{y}+\bar{r}+\lambda)^{1-\gamma}}{1-\gamma} \tag{P1}$$

$$\frac{0.5(\bar{y}+\lambda+\bar{d}+\varepsilon)^{1-\gamma}+0.5(\bar{y}-\lambda+\bar{d}-\varepsilon)^{1-\gamma}}{1-\gamma} < \frac{(\bar{y}+\bar{r})^{1-\gamma}}{1-\gamma} \tag{P2}$$

If (P1) and (P2) are satisfied, then the indirect utility of stockholders (non-stockholders) dominates that of non-stockholders (stockholders) if $\mu=0$ (1). It is now possible to introduce the following proposition, which indicates that the participation game has multiple equilibria.

Proposition 1 (Multiple Equilibria)**.** *If conditions (P1) and (P2) are satisfied, there*

[7] It is assumed that income shocks λ take only positive values, but even under negative values, it can be shown that dividend fluctuations can offset income shocks.

exists a unique fixed point μ^* *such that:*

$$\begin{aligned} V_{A0}^* &> V_{B0}^*, \text{ if } 0 \leq \mu < \mu^* \\ V_{A0}^* &= V_{B0}^*, \text{ if } \mu = \mu^* \\ V_{A0}^* &< V_{B0}^*, \text{ if } \mu^* < \mu \leq 1 \end{aligned}$$

Proof. See Appendix 2.

It is noted first that the indirect utility function of stockholders (non-stockholders) is monotone decreasing (increasing) in μ.[8] This implies that the indirect lifetime utility functions of each type of consumers meet at the fixed point μ^*. When either (P1) or (P2) is not satisfied, there exists a unique full-participation equilibrium. Note that the restriction of non-negativity imposed on dividends ($\bar{d} \geq \varepsilon$) and deposit interest rate $\bar{r} = 0$ necessarily lead to condition (P1) because the upper bound on V_{B0}^* is smaller than the lower bound on V_{A0}^* even when $\lambda \simeq 0$:

$$\frac{0.5(\bar{y}+2\bar{d})^{1-\gamma}+0.5\bar{y}^{1-\gamma}}{1-\gamma} > \frac{\bar{y}^{1-\gamma}}{1-\gamma}$$

In addition, (P2) is more likely to be satisfied when λ takes large values. The most risky environment $\lambda \simeq \bar{y}$ and $\varepsilon \simeq \bar{d}$ ensures that (P1) and (P2) are satisfied.

The remainder of the section examines the equilibrium conditions under incomplete market participation. The following relation can be obtained when $\mu = \mu^*$:

$$\begin{aligned} &\frac{0.5(\bar{y}+\bar{d}+\varepsilon+\lambda\mu^*)^{1-\gamma}+0.5(\bar{y}+\bar{d}-\varepsilon-\lambda\mu^*)^{1-\gamma}}{(1-\gamma)(1-\beta)} \\ &= \frac{0.5[\bar{y}+\bar{r}-\lambda(1-\mu^*)]^{1-\gamma}+0.5[\bar{y}+\bar{r}+\lambda(1-\mu^*)]^{1-\gamma}}{(1-\gamma)(1-\beta)} \end{aligned} \tag{14}$$

The indirect utilities of stockholders and non-stockholders evaluated at the two consumption states are equivalent. This is illustrated in Figure 1 where a fixed point μ^* involving the payoffs from market participation decision is considered.

It is useful to introduce at this point of the analysis the concept of certainty equivalent of the stochastic consumption $CE(c_i^*)$. In equilibrium under incomplete market participation, the certainty equivalent of stockholders is equal to that of non-stockholders from equation (14), i.e. $CE(c_A^*) = CE(c_B^*)$. Furthermore, $E(c_A^*) - CE$ and $E(c_B^*) - CE$ can be interpreted as the stockholders' and non-stockholders' risk premia, respectively. The market participation risk premium (MPRP) can thus be defined as $E(c_A^*) - E(c_B^*)$. The level of expected returns is function of the exposure to market participation risks, which are inclusive of uninsured income risks as well as dividend risks.

It is clear from Figure 2 that if the consumption of stockholders becomes more volatile, then a higher MPRP is required. In equilibrium, the MPRP can be shown to

[8] This fixed point can be regarded as a sunspot equilibrium, which can be achieved when consumers have self-fulfilling expectations μ^*. Reference on sunspot models can be also made to earlier studies by Azariadis (1981), Azariadis and Guesnerie (1986) and Cass and Shell (1983), inter alias.

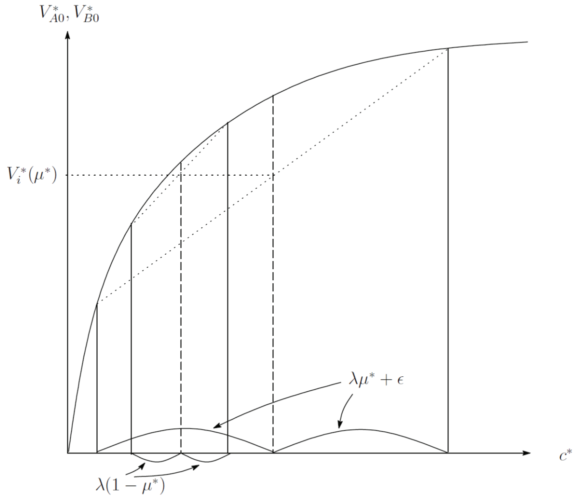

Figure 1. Risk aversion and equilibrium incomplete market participation

be equal to $\overline{d} - \overline{r}$. Also, if consumers are more risk averse and dividends are fixed, then the equilibrium conditions can only be sustained through adjustments in the degree of incomplete market participation (μ^* decreases).

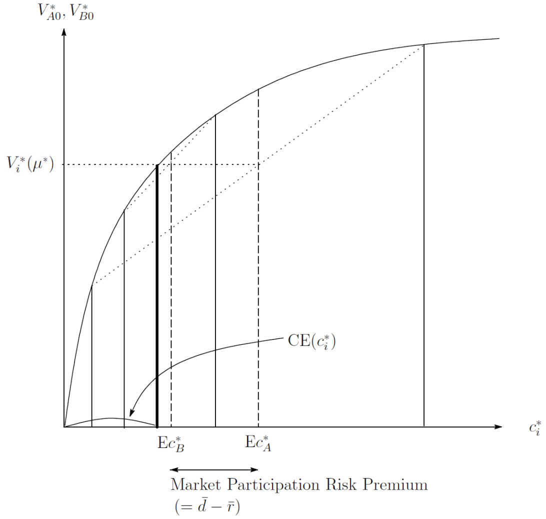

Figure 2. Certainty equivalent and market participation risk premium

4. Numerical analysis

The equilibria asset prices can be also characterized with simple numerical calculus. The CRRA preferences, labor income shocks, deposit interest rate and equity can be described by the set of parameters (β, γ, $\bar{y}$, λ, $\bar{r}$, $\mathrm{E}[\widetilde{R}]$, ε).

It is possible to derive the degree of incomplete market participation (μ^*), equity premium and risk-free rate. The equilibrium incomplete market participation (μ^*) can be obtained by fixing (β, $\bar{y}$, λ, $\bar{r}$, $\mathrm{E}[\widetilde{R}]$, ε) and altering the risk aversion parameter $\gamma \in (0, 10]$. Finally, in order to examine the equilibrium risk-free rate and equity premium, the set of (γ, μ^*) and fixed values (β, $\bar{y}$, λ, $\bar{r}$, $\mathrm{E}[\widetilde{R}]$, ε) are substituted into equations (9) and (11).

4.1 Equilibrium incomplete market participation

For the purposes of benchmark calibration, the time period is set to one year, the discount factor at $\beta = 0.9$, the coefficient of relative risk aversion at $\gamma = 1$, and the deposit interest rate at $\bar{r} = 0.75\%$. The levels of equity risk and labor income shocks need to be determined however. Suppose that $\mathrm{E}[\widetilde{R}] = 1.07$, $\varepsilon = 0.16$, $\bar{y} = 1$, and $\lambda = 0.3$, which implies that the likelihood of a 15 percent change in return on equity, and a 30 percent change in labor income, amounts to 50 percent.[9]

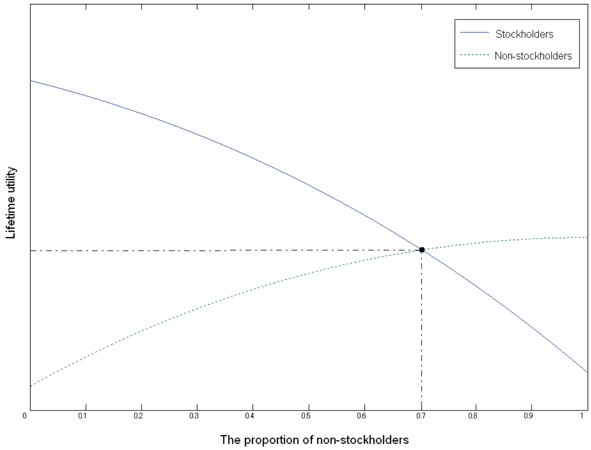

Figure 3. Multiple equilibria and welfare

[9] This level of volatility on equity returns is consistent with the results from Campbell (2000). There are however, no precise estimates of the level of individual income risks. Hence, the present analysis is based on various levels of income risk. When the expected labor income is high relative to expected equity returns, the lifetime indirect utility of market participants dominates that for non-stockholders irrespective of the ratio of non-market participation μ.

Figure 3 describes the effect of non-market participants on the value function of each agent, and presents the multiple equilibria $\{E1, E2, E3\}$. If μ is smaller than 70 percent, then the value function of market participants is larger than that for non-market participants. This implies that under such conditions, consumers decide to participate also in asset markets. Therefore, the fraction of non-stockholders is equal to zero, which reflects a full participation economy. When μ is larger than 0.7 however, there is no demand for equity and bonds as all consumers decide not to participate in asset markets. In the case where $\mu = 0.7$, the equilibrium E1 is achieved, with a positive fraction of rational non-market participants.

Table 2. Risk aversion parameters and equilibrium incomplete market participation

β	γ	$\bar{y}$	$E[\widetilde{R}]$	ε	λ	μ^*
0.9	1	1	1.07	0.16	0.3	0.70
	2					0.48
	3					0.41
	4					0.38
	5					0.36
	6					0.35
	7					0.35
	8					0.34
	9					0.34
	10					0.34

These results are briefly presented in Table 2 where the first row refers to the baseline calibration, whereas the remaining lines allow for the examination of the relationship between risk aversion and equilibrium incomplete market participation. In the benchmark case, the fraction of market participants in equilibrium (E1) represents only 30 percent, which is similar to the evidence from Mankiw and Zeldes (1991). An increase in risk aversion has the effect of driving down the degree of incomplete market participation at equilibrium. In light of this analytical evidence on the relationship between risk aversion and incomplete market participation, the following section attempts to explain the equity premium puzzle and the risk-free rate puzzle.

4.2 Equity premium and risk-free rate

The analysis has shown that there is a potential for multiple equilibria, including equilibrium under incomplete market participation (E1), under full participation (E2), and under no market participation (E3). The equity premium and risk-free rate can be characterized only for the equilibrium conditions under incomplete or full market participation.

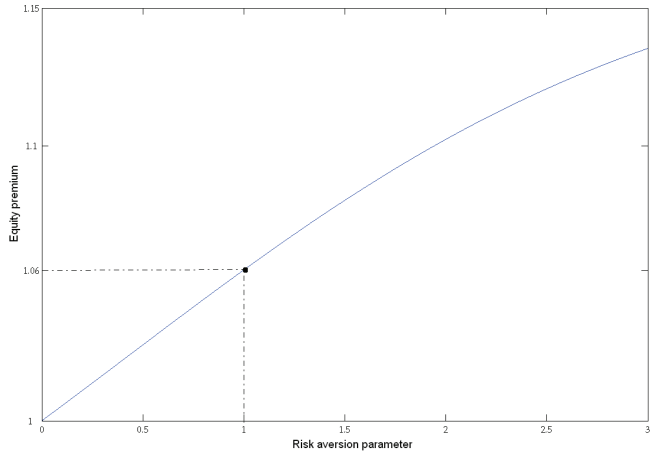

Figure 4. Equity premium under incomplete market participation

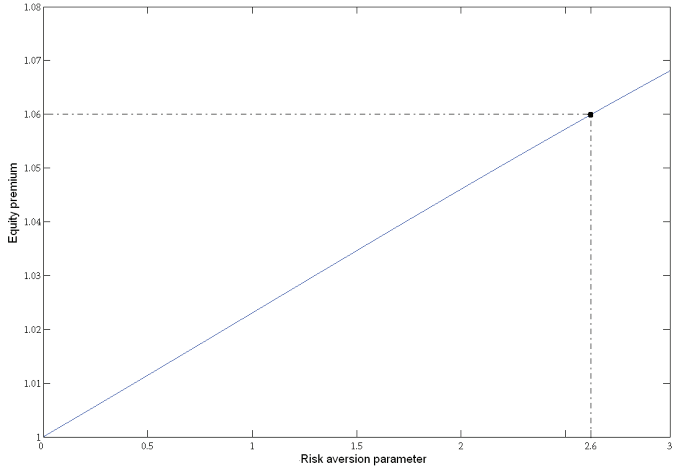

Figure 5. Equity premium under full participation

Figure 4 displays the calibrated gross equity premia, where the parameters μ^* and γ are assumed to be equal to 0.7 and 1, respectively. This setting allows for the replication of the historical average equity premium of 6%, which is documented in the literature. Figure 5 displays the equity premium in the case of equilibrium under full participation. It is required that the coefficient of relative risk aversion amounts to 2.6 in order to achieve the equity premium value of 6% reported in the literature.

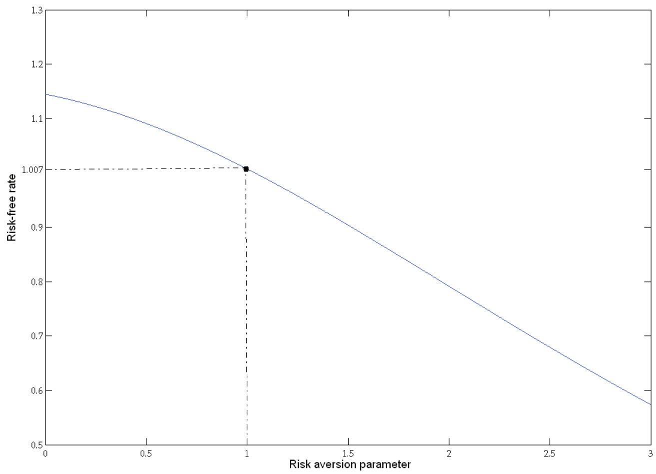

Figure 6. Risk-free rate under incomplete market participation

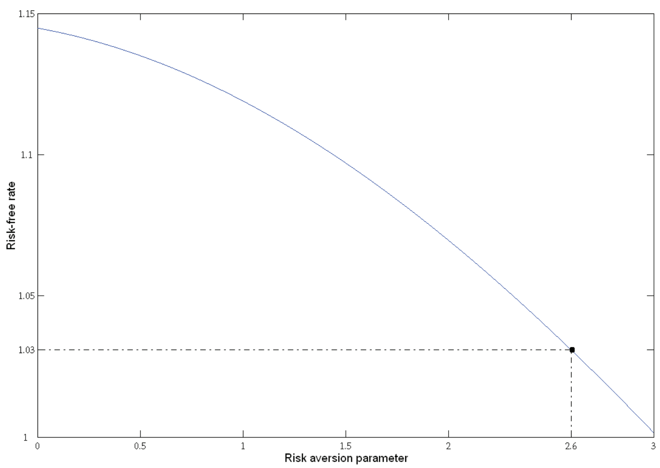

Figure 7. Risk-free rate under full participation

The calibrated risk-free rate represents the average of the two values obtained under different states. The risk-free rate, shown in Figure 6, falls from 1.13 to 0.58 as the coefficient of relative risk aversion rises from 0 to 3. In equilibrium, the net risk-free rate is about 0.75 percent, which is consistent with the estimated values in the empirical literature.[10] However, the risk-free rate puzzle does not seem to be completely

[10] The value of γ is set to 1, which is consistent with the equity premium and risk-free rate documented in the literature.

solved since the risk-free rate in Figure 7 is about 3 percent which is relatively high by historical standards. Thus, the present model can partly explain the equity premium and risk-free rate puzzles under incomplete market participation.

4.3 The role of social security and self-insurance

The modeling approach by Mankiw (1986) and Weil (1992, 1994) based on finite-lived agents in incomplete markets, suggests that the idiosyncratic income fluctuations can partly explain both the equity premium and risk-free rate puzzles. In the multi-period setting where shocks are not persistent, Levine and Zama (2002) show that market incompleteness can be partially circumvented using self-insurance, where agents can significantly hedge against the effects of idiosyncratic risk by mutually borrowing and lending assets among each other. Constantinides and Duffie (1996) show also that incomplete consumption insurance can explain the risk premium puzzle when individual endowment shocks are persistent. However, the empirical evidence lends little support to the proposition of highly persistent idiosyncratic shocks, according to Heaton and Lucas (1986). Constantinides et al. (2002) investigate the effects of limited market participation generated by borrowing constraints on asset prices in an overlapping generations economy. The existence of borrowing constraints on young agents has prohibitive effects on self-insurance.[11]

In contrast to the existing literature, the present study shows that even under the infinite horizon setting, it is possible to explain the equity premium puzzle and risk-free rate puzzle, under the condition of incomplete market participation. The evidence suggests that risk-sharing externalities may have disruptive effects on self-insurance. In particular, if the income fluctuations of market participants are correlated, it becomes more difficult to make recourse to safe assets for insurance purposes. It is explicitly assumed that market participants have access to bond markets whereas non-stockholders rely on deposits. Hence, the existence of multiple safe assets has prohibitive effects on self-insurance even if borrowing constraints do not exist. However, if the government provides alternative solutions by substituting social security τ_t for the safe assets b_t and $\widehat{b}_t$, then such income transfers can create risk-sharing opportunities between market participants and non-market participants. Indeed, the income transfers from market participants with positive income shocks to non-market participants with negative income shocks can whittle down the effects of income shocks. Thus, the development of social security programs can contribute toward the elimination of incomplete market participation and the explanation of the equity premium puzzle in the Lucas economy.

5. Conclusion

This study examined the effects of risk-sharing externalities on asset prices using the infinite-horizon model under endogenous labor income risks. In the equilibrium with incomplete market participation, this model can partly explain both the equity premium

[11] The literature also suggests that uninsured income risks promote precautionary savings. Examples of uninsured income risk models include Aiyagari (1994), Angeletos and Calvet (2006), Devereaux and Smith (1994), Huggett (1993), Mankiw (1986), and Weil (1992a, 1994), inter alia.

puzzle and risk-free rate puzzle. However, it cannot explain the risk-free rate puzzle under full participation equilibrium. This failure is attenuated by the fact that equilibrium under full participation is less likely given the prevailing constraints on market participation.

On the other hand, equilibrium with incomplete market participation is unstable because small changes in any parameter values can be conducive to departures from equilibrium. The equity and bond returns can indeed vary depending on the underlying conditions of incomplete or full participation. These results open new avenues for theoretical and empirical research on the relationship between the degree of incomplete market participation and the significance of equity risk premium and risk-free rate puzzles.

The equilibrium with incomplete market participation is worse than full-participation equilibria in terms of social welfare. However, Ohno (2009) investigates the equilibrium growth rate of capital stock and social welfare when risk-sharing externalities are incorporated into the infinite-horizon production economy. The spillover effects on production technologies lead to the under-accumulation of capital stocks. Under equilibrium incomplete market participation, the endowment risks cannot be fully diversified as they induce precautionary savings, which have the potential to increase the equilibrium growth rates and improve social welfare. This approach has the potential of shedding light on important economic issues such as the role of financial intermediaries, financial market integration, international risk-sharing puzzle, inter alia.

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Appendix 1: Risk-sharing externalities

The following example provides an intuitive explanation of risk-sharing externalities and considers two types of jobs and infinite number of agents. Each consumer decides to belong to one of two risk-sharing groups denoted by $i = A, B$. Let us consider two states ($s = 1, 2$) in two period economy. The probabilities of each state are equal to 0.5. Assume that each consumer receives the following stochastic endowments which are associated with different jobs j ($j = 1, 2$) and the ratio of each job j to consumers is assumed to be 0.5:

	Job 1	Job 2
State 1	$\bar{e}+\varepsilon$	$\bar{e}-\varepsilon$
State 2	$\bar{e}-\varepsilon$	$\bar{e}+\varepsilon$

The average endowment and endowment shocks are denoted by $\bar{e}$ and ε, respectively. Without loss of generality, it can be assumed that endowment shocks are positive and smaller than average endowments in order to eschew cases of negative consumption. Under state 1 for instance, while consumers with job 1 receive $\bar{e}+\varepsilon$ units of consumption good while those with job 2 obtain $\bar{e}-\varepsilon$ units only. Individual consumers consider participation into group i given the following utility function:

$$U(c^i_{j0}, c^i_{js}) \equiv \frac{(c^i_{j0})^{1-\gamma}}{1-\gamma} + \sum_{s=1}^{2} 0.5 \frac{(c^i_{js})^{1-\gamma}}{1-\gamma}$$

Within each group, complete Arrow-Debreu securities are available.[12] Let q^i_s be the price of a unit of consumption in date 1 state s within group i. Then, the budget constraints are given by:

$$\begin{aligned} c^i_{j0} + \sum_{s=1}^{2} q^i_s a^i_{js} &= e_0, \\ c^i_{js} &= a^i_{js} + e_{js} \end{aligned}$$

The first-order conditions can be expressed as follows:

$$q^i_s = 0.5 \left(\frac{c^i_{js}}{c^i_{j0}} \right)^{-\gamma}$$

This implies that the marginal rate of substitution for any individual within a particular group is equal to the Arrow-Debreu asset price. These asset prices differ across groups, but they are the same for all consumers within a given group irrespective of variations

[12] The Arrow-Debreu security is a security that pays one unit of numeraire if a specified state is realized and zero otherwise.

in endowment risk associated with different jobs. Therefore, the following relation can be obtained:

$$\frac{c^i_{js}}{c^i_{j0}} = \frac{c^i_{j's}}{c^i_{j'0}} \text{ for } j \neq j' \tag{A1}$$

At the same way, it can be shown that

$$\frac{c^i_{js'}}{c^i_{js}} = \frac{c^i_{j's'}}{c^i_{j's}} \text{ for } j \neq j'. \tag{A2}$$

Thus for a given group, individual consumption patterns are similar. This implies that shocks to consumption across individuals within the same group are perfectly correlated. Given the proportion $\kappa_j \in [0, 0.5]$ of group-A consumers with job j, the equilibrium conditions require that:

$$\sum_j \kappa_j c^A_{js} = \sum_j \kappa_j e_{js} \text{ and } \sum_j (0.5 - \kappa_j) c^B_{js} = \sum_j (0.5 - \kappa_j) e_{js} \tag{A3}$$

Again, it is noted that individuals are classified into group A or B depending on their respective beliefs about the participation ratio κ_j. If an agent makes the prediction that $U(c^A_{j0}, c^A_{js}|\kappa_j) \geq U(c^B_{j0}, c^B_{js}|\kappa_j)$, then participation into group A is implied. In the same way, the conviction that $U(c^B_{j0}, c^B_{js}|\kappa_j) \geq U(c^A_{j0}, c^A_{js}|\kappa_j)$ is conducive to participation into group B. These relations can be regarded as individual rationality conditions.

If consumers with different types of jobs $j = 1$ and $j = 2$ participate in group A at equal rates $\kappa_1 = \kappa_2$, then it is possible to smooth their individual consumption $c^A_{js} = \bar{e}$. This is consistent with a fully-integrated Arrow-Debreu economy. However, when consumers fall into two subgroups with different participation ratios $\kappa_1 \neq \kappa_2$, then it is difficult to fully diversify away the endowment risks, which are also conducive to the exposure to consumption risks. This implies that the income variance $\sigma_A(\mu)$ is not necessarily monotone increasing in μ. However, this assumption is required to guarantee the existence of equilibria.

Indeed, even though there is no aggregate shocks to the economy, neither group can offset the individual endowment shocks. The degree of market participation can be determined under conditions of sunspot equilibria. If consumers have the same self-fulfilling expectations κ_j, then any fixed point satisfing equations (A1), (A2) and (A3) as well as the individual rationality conditions defined above, becomes an equilibrium point. Since the case of unequal participation ratios $\kappa_1 \neq \kappa_2$ is not dominant, consumers have an incentive to move into another risk-sharing group. However, transfers between groups is restricted by the assumption of one-time participation game, in which the decision of individual agents to participate into a given group is also independent of the decisions achieved by others. Thus, given the assumption that individual decisions are irreversible, equilibrium exist even under non-dominant participation conditions $\kappa_1 \neq \kappa_2$.

Appendix 2

Lemma 1. *V_{A0}^* is monotone decreasing and V_{B0}^* is monotone increasing in μ.*

Proof. The first derivative of the value function for stockholders with respect to the ratio of incomplete market participation $\partial V_{A0}^*/\partial \mu$ can be expressed as

$$\frac{\lambda}{2(1-\beta)}\left[(\bar{y}+\lambda\mu+\bar{d}+\varepsilon)^{-\gamma}-(\bar{y}-\lambda\mu+\bar{d}-\varepsilon)^{-\gamma}\right].$$

The difference in marginal utilities between state 1 and 2 $(\bar{y}+\lambda\mu+\bar{d}+\varepsilon)^{-\gamma}-(\bar{y}-\lambda\mu+\bar{d}-\varepsilon)^{-\gamma}$ is negative because $x^{-\gamma}$ is decreasing in x when $\gamma>0$, and $\bar{y}+\lambda\mu+\bar{d}+\varepsilon>\bar{y}-\lambda\mu+\bar{d}-\varepsilon(>0)$, which implies that $\partial V_{A0}^*/\partial\mu<0$. In the same way, the first derivative of the value function for non-market participants $\partial V_{B0}^*/\partial\mu$ can be expressed as

$$\frac{\lambda}{2(1-\beta)}\left\{[\bar{y}+\bar{r}-\lambda(1-\mu)]^{-\gamma}-[\bar{y}+\bar{r}+\lambda(1-\mu)]^{-\gamma}\right\}.$$

Then, $[\bar{y}+\bar{r}-\lambda(1-\mu)]^{-\gamma}-[\bar{y}+\bar{r}+\lambda(1-\mu)]^{-\gamma}>0$ implying that $\partial V_{B0}^*/\partial\mu>0$. □

Proof of Proposition 1. It is possible to demonstrate that when (P1) is satisfied, then the lifetime utility of stockholders dominates that of non-stockholders under full market participation $\mu=0$. Under these conditions, the respective utility functions can be expressed as follows:

$$\begin{aligned}
V_{A0}^*(0) &= \frac{0.5(\bar{y}+\bar{d}+\varepsilon)^{1-\gamma}+0.5(\bar{y}+\bar{d}-\varepsilon)^{1-\gamma}}{(1-\gamma)(1-\beta)},\\
V_{B0}^*(0) &= \frac{0.5(\bar{y}+\bar{r}-\lambda)^{1-\gamma}+0.5(\bar{y}+\bar{r}+\lambda)^{1-\gamma}}{(1-\gamma)(1-\beta)}
\end{aligned}$$

Similarly, the indirect utility of non-stockholders dominates that of stockholders under no market participation $\mu=1$ when (P2) is satisfied. This implies the respective utility functions

$$\begin{aligned}
V_{A0}^*(1) &= \frac{0.5(\bar{y}+\lambda+\bar{d}+\varepsilon)^{1-\gamma}+0.5(\bar{y}-\lambda+\bar{d}-\varepsilon)^{1-\gamma}}{(1-\gamma)(1-\beta)},\\
V_{B0}^*(1) &= \frac{(\bar{y}+\bar{r})^{1-\gamma}}{(1-\gamma)(1-\beta)}.
\end{aligned}$$

To demonstrate the existence of the unique fixed point μ^*, the inequalities $V_{A0}^*(0)>V_{B0}^*(0)$ and $V_{B0}^*(1)>V_{A0}^*(1)$ must be satisfied because V_{A0}^* is monotone decreasing and V_{B0}^* is monotone increasing in μ, as shown by the lemma above. □