Copula-Based Nonlinear Models of Spatial Market Linkages *

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Abstract

An extensive empirical literature has addressed a wide array of issues pertaining to price linkages over space and across time. Empirical models of price linkages have been used to measure market power and to characterize the operation of markets that are separated by space, time, and product form. The long history of these empirical models extends from simple tests of price correlation, to conventional regression tests, to modern time series models that account for nonstationarity, nonlinearities, and threshold behavior in market linkages. This paper proposes a alternative and potentially novel approach to analyzing these same types of time series data in a nonlinear fashion. Copula-based models that consider the joint distribution of prices separated by space are developed and applied to weekly prices for important lumber products at geographically distinct markets. In particular, we consider prices taken from weekly editions of the Random Lengths publication for homogeneous OSB products.

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1 Introduction

Notions of price parity, spatial arbitrage, and price transmission characterize many basic principles and relationships in economics. At the core, markets should efficiently function so as to eliminate any potential for riskless profits through arbitrage and trade. This fundamental condition is often called the "Law of One Price" (LOP)—a concept whose nomenclature reflects the considerable confidence that economists place in its adherence. Over the years there has been considerable interest in and debate about the empirical validity of the Law of One Price (LOP), especially as it pertains to markets for tradeable goods. On one hand, economists take it as being nearly axiomatic that freely functioning markets for traded, homogeneous products should ensure that prices are efficiently linked across regional markets, the implication being that no persistent opportunities for spatial arbitrage profits exist. [1] The general implication underlying these basic concepts is that prices for homogeneous products at different geographic locations in otherwise freely functioning markets should differ by no more than transport and transactions costs, the latter including, for example, insurance, contracting fees, licensing fees, legal fees, and possibly a risk premium. On the other hand, there is substantial empirical evidence in a huge literature that finds that the adjustment lags required to restore arbitrage equilibria are often found to be far longer than would seem natural based upon any reasonable understanding of the mechanics of physical trade as it pertains to the markets in question.

A related avenue of research considers price transmission in a more general sense. Here the focus is typically on how shocks or changes in market conditions at one location or level of the market are transmitted to other locations or market levels. Price transmission models

[1] Distinctions between tests of LOP the and spatial market integration are not especially meaningful. In both cases, the economic phenomena being evaluated (spatial market arbitrage) is identical. A survey of both strands of literature can be found in Fackler and Goodwin (2001).

are often applied in considerations of vertical market linkages. For example, the extent to which raw commodity markets are impacted by changes at the retail level is an issue that has received considerable attention in the empirical literature. Although the economic phenomena being evaluated in these studies may be slightly different, the empirical tools used to evaluate such market linkages are often identical to those used in evaluating spatial market linkages.

Early empirical studies generally failed to find support in favor of LOP. Isard (1977) found rather conclusive evidence against the LOP using disaggregate data for traded goods. Isard's conclusions were subsequently confirmed for a variety of commodities in a wide array of market settings by, among others, Richardson (1978), Thursby, Johnson, and Grennes (1986), Benninga and Protopapadakis (1988), and Giovannini (1988). Goodwin, Grennes, and Wohlgenant (1990) did, however, find some support for the LOP when it was specified in terms of price expectations as opposed to observed prices. After Engle and Granger's seminal paper (1987), cointegration techniques have been used to rationalize the LOP as a long—run concept. By adopting this view of the LOP, economists were able to find more compelling evidence in favor of the LOP, including, for example, Buongiorno and Uusivuori (1992) (U.S. pulp and paper exports), Michael, Nobay, and Peel (1994) (international wheat prices), Bessler and Fuller (1993) (U.S. regional wheat markets), and Jung and Doroodian (1994) (softwood lumber markets).

The most recent literature in this area has applied smooth or discrete threshold time series models that typically consider refinements of autoregressive or vector error correction models in analyzing price relationships. The underlying motivation is that adjustments to equilibrium may not be linear, and that this nonlinearity may, in turn, be associated with hard–to–observe transactions costs associated with arbitrage. The theoretical underpinnings for transactions–costs–induced nonlinearity in the LOP have been put forward by Dumas (1992), although the basic idea dates back at least to the work of Heckscher (1916), who noted that transactions costs may define "commodity points" within which prices are not directly linked because the price differences are less than the costs of trade.

A recent example includes an analysis of manufactured lumber products (oriented strand board or OSB) in the U.S. undertaken by Holt, Prestemon, and Goodwin (2011). Their analysis applied smooth transition vector autoregression (STAR) models to consider price relationships among spatially-distinct North American markets for manufactured OSB. The application was notable in light of recent litigation that charged that OSB manufacturers had practiced discriminatory and noncompetitive pricing during the latter part of the decade. The analysis revealed that nonlinearity is an important feature of price relationships in these markets and that the price parity relationships implied by economic theory and efficient arbitrage were generally supported by the STAR models. Other empirical investigations of the role of nonlinearity as pertains to the LOP have been reported by Goodwin and Piggott (2001), Lo and Zivot (2001), Sephton (2003), Balcombe, Bailey, and Brooks (2007), and Park, Mjelde, and Bessler (2007). In general, these studies have found support for threshold effects, with the path of adjustment to equilibrium depending typically on the size if not the sign of the shock. In particular, large shocks that lead to profitable arbitrage opportunities net of transactions costs are quickly eliminated whereas smaller shocks, which may not be large enough to result in profitable arbitrage opportunities, may elicit a much smaller effect or even no adjustment at all.

This extensive literature has several common themes and generally involves the application of conventional time series models to finely sampled, nonstationary price data. In this paper, we propose an alternative and potentially novel approach to analyzing these same types of time series data in a nonlinear fashion. We develop copula-based models that consider the joint distribution of prices separated by space and apply them to weekly prices for homogeneous OSB products at geographically distinct North American markets. Although copula models have been extensively used in financial economics and risk management studies, to our knowledge, this paper is the first attempt to utilize time varying copulas in explaining spatial market linkages. [2]

[2]Patton (2006) allows for time variation in the conditional joint distribution of the returns on the Deutsche mark/U.S. dollar and Japanese Yen/U.S. dollar exchange rates by allowing the parameter(s) of a given copula to vary through time.

Our approach is a natural extension of the existing (and abundant) time-series evaluations of spatial price linkages. Our approach involves direct examination of the joint probability distribution of the key economic variables of interest. In this way, the approach is really no different than standard maximum likelihood methods applied to structural or non-structural econometric models. However, we give particular attention to the nature of the jointness or correlation between these key variables. In particular, we allow this correlation to be "state-dependent" and therefore to depend upon market conditions at any particular point in time. In this manner, our approach is analogous to the regime-switching and threshold models that are frequently applied in evaluating spatial and vertical market linkages.

The plan of our paper is as follows. The next section outlines conventional empirical approaches typically used to evaluate spatial price linkages. We then propose an alternative approach that is based upon copula models of the joint likelihood function. The third section presents an empirical application of these models to an important, regionally-traded homogeneous commodity market—the North American Oriented Strand Board (OSB) market. In particular, we consider price linkages among four regionally separate OSB markets. OSB is of interest because it has become one of the leading building materials used in the construction sector of the U.S. and in many other countries. OSB surpassed plywood as the leading engineered wood product in the mid-1990s in the U.S. The final section contains a summary of the results and conclusions.

2 Econometric Models of Spatial Price Relationships

As we have noted, a vast empirical literature has considered a wide array of empirical models of price relationships across space, time, and market form. This literature has evolved from a simple consideration of correlation coefficients and linear regression models to regime-switching, time-series models that allow for a form of "state-dependence" in characterizing price linkages. The most recent literature is usually based upon a standard autoregressive model of the form:

$$\Delta(p_t^i - p_t^j) = \alpha + \beta(p_{t-1}^i - p_{t-1}^j) \quad (1)$$

where p^i and p^j are logarithmic prices and β is a parameter that reflects the degree of market integration.[3] In particular, β represents the degree of "error-correction" that characterizes departures from price parity, which are reflected in large values of $p_{t-1}^i - p_{t-1}^j$. The "error" term, represents proportional deviations from market equilibrium. In some cases, α is taken to represent a proportional price difference that reflects transactions costs.[4]

Recent empirical evaluations of spatial price linkages have recognized that the presence of transactions costs, which are notoriously difficult to measure but nonetheless are likely to be relevant in any consideration of spatial commodity trade, may result in nonlinearities in estimates of equation 1. Two specific avenues have been adopted to account for such nonlinearities. In the first, a "threshold" parameter that reflects the presence of transactions costs, is estimated. The linkage between prices varies depending upon whether the departure from equilibrium represented by $p_{t-1}^i - p_{t-1}^j$ is large enough to evoke spatial arbitrage. In this case, a discrete break occurs between regimes where one regime may represent a case of no trade while another represents conditions of profitable trade and arbitrage. These models are typically referred to as "threshold autoregressive" (TAR) or "threshold vector error-correction" (TVEC) models.

Alternatives to this simple model permit the switching between regimes to occur at a gradual and smooth pace. The speed and degree of adjustment is implied by parameters of a "transition" function. A number of different specifications of such "smooth transition autoregressive " (STAR) models have been developed in the literature. Such models essentially nest the TAR versions such that they permit a more flexible evaluation of price linkages. The behavior underlying spatial price linkages is likely to be discrete—representing the two

[3] See, for example, Taylor (2001), who applies regime-switching, time-series models of this form to empirical tests of purchasing power parity—an aggregate version of the LOP.

[4] A specification that is often referred to as an "iceberg" model, reflecting the fact that the value of the commodity melts away via a lower price as it is shipped.

states of trade/no-trade. However, in that empirical evaluations of such models almost always involves some degree of aggregation, the patterns of adjustment may be of a more smooth nature and therefore may favor the STAR-type models.

These models have provided considerable flexibility in modeling spatial and vertical price linkages. The results of allowing for such flexibility and accounting for unobservable transactions costs have generally provided much greater support for the concept of market integration and efficiency. However, in empirical practice, they often suffer from complications resulting from parameters that may be unidentified under certain null hypotheses and a resulting need to rely upon non-standard inferential techniques.

Our approach involves a simple extension or re-characterization of the fundamental relationship expressed in equation 1. We make use of the widely-recognized correspondence between β in equation 1 and the standard, linear Pearson correlation coefficient:

$$\hat{\beta} = \hat{\rho}\frac{\hat{\sigma_y}}{\hat{\sigma_x}} \tag{2}$$

where y and x correspond to the random variables $\Delta(p_t^i - p_t^j)$ and $p_{t-1}^i - p_{t-1}^j$, ρ is the Pearson correlation coefficient, and σ_p represents the standard deviation of random variable p. The "error-correction" relationship that characterizes the linkage between markets i and j is represented in the sample correlation coefficient ρ. To the extent that β realizes regime switching, the coefficient ρ will also reflect switching. To the extent that such switching is dependent upon market conditions (i.e., as reflected in the price differential), the correlation coefficient ρ may exhibit state dependence.

The empirical approach adopted here involves considering the joint distribution function of $\Delta(p_t^i - p_t^j)$ and $p_{t-1}^i - p_{t-1}^j$. We make use of a widely-recognized, fundamental result known as Sklar's (1959) Theorem, which implies that any joint probability function can be represented in terms of the marginal densities and a function known as a "copula." In particular, Sklar's Theorem implies that, for any continuous $p-$variate cumulative probability function F, a unique copula function $C(\cdot)$ exists for which

$$F(x_1, x_2, ..., x_p) = C(F_1(x_1), ..., F_p(x_p); \xi), \tag{3}$$

where $F_i(\cdot)$ are marginal distributions and ξ is a set of parameters that measures dependence.

2.1 Copulas

Copula models have recently realized widespread application in empirical models of joint probability distributions.[5] The models essentially use a "copula" function to tie together two marginal probability functions that may (or may not) be related to one another. Much of the work on copulas has been motivated by their applicability to the issues in risk management, insurance and financial economics (see among others; Cherubini et al. (2004), Rodriguez (2003), Hu (2006), Patton (2006), Jondeau and Rockinger (2006)). In agricultural economics literature, copula models have been used extensively in the design and rating of crop revenue insurance contracts, where the inverse correlation of prices and yields plays an important role in pricing revenue risk.

Copulas are functions that join or *couple* multivariate distributions to their one–dimensional marginal distribution functions. A $p-$dimensional copula, $C(u_1, u_2, \ldots, u_p)$, is a multivariate distribution function in the unit hypercube $[0,1]^p$ with uniform $U(0,1)$ marginal distributions. As long as the marginal distributions are continuous, there is a unique copula associated to the joint distribution, F, that can be obtained as,

$$C(u_1, u_2, \ldots u_p) = F(F_1^{-1}(u_1), \ldots, F_p^{-1}(u_p)) \tag{4}$$

On the other hand, given a $p-$dimensional copula, $C(u_1, \ldots u_p)$, and p univariate distributions, $F_1(x_1), \ldots, F_p(x_p)$, the function 3 is a p-variate distribution function with margins $F_1, \ldots F_p$ whose corresponding density function written as

$$f(x_1, x_2, \ldots x_p) = c(F_1(x_1), \ldots, F_p(x_p)) \prod_{i=1}^{p} f_i(x_i) \tag{5}$$

Provided that it exists, the density function of the copula, c, can be derived using 4 and marginal density functions, f_i:

[5]For details on construction and properties of copulas, see among others Nelsen (1999, 2006) and Joe (1997).

$$c(u_1, u_2, \dots u_p) = \frac{f(F_1^{-1}(u_1), \dots, F_p^{-1}(u_p))}{\prod_{i=1}^{p} f_i(F_i^{-1}(u_i))}$$

There is a large number of parametric families of copulas in the literature (see, for example, Nelsen (2006)). Two of the most commonly used copula families are elliptical copulas and Archimedean copulas. Gaussian and t-copulas are elliptical whereas the Clayton and Gumbel are among Archimedean copulas.

2.1.1 Elliptical Copulas

Gaussian Copula:

The Gaussian (or normal) copula, which is obtained from the multivariate normal distribution with correlation matrix, R, is the most basic copula and it is written as

$$C_R^{Ga}(u_1, u_2, \dots u_p) = \int_{-\infty}^{\Phi^{-1}(u_1)} \dots \int_{-\infty}^{\Phi^{-1}(u_p)} \frac{1}{\sqrt{2\pi^p(1-|R|)}} \times exp\left\{\frac{-\mathbf{u}'R^{-1}\mathbf{u}}{2}\right\} d\mathbf{u} \qquad (6)$$

where $\mathbf{u} = (u_1, \dots u_p)$ and Φ^{-1} is the inverse of the cumulative distribution function of the univariate standard normal distribution.

t - copula:

The Gaussian copula assumes that there is no dependence in the tails of the distribution. Therefore, it is often more useful to consider the $t-$copula, which can be obtained from the multivariate $t-$distribution with ν degrees of freedom and correlation matrix, R:

$$C_{\nu,R}^{t}(u_1, u_2, \dots u_p) = \int_{-\infty}^{t_\nu^{-1}(u_1)} \dots \int_{-\infty}^{t_\nu^{-1}(u_p)} \frac{\Gamma(\frac{\nu+p}{2})(1+\frac{\mathbf{u}'R^{-1}\mathbf{u}}{\nu})^{-\frac{\nu+p}{2}}}{\Gamma(\frac{\nu}{2})\sqrt{(\pi\nu)^p|R|}} d\mathbf{u} \qquad (7)$$

where $t_\nu^{-1}(u_1)$ denotes the inverse of the distribution function of the standard univariate t-distribution with ν degrees of freedom. Note that the Gaussian copula is a special case of the $t-$copula where ν goes to infinity. The properties of $t-$copula were studied by Embrechts et al. (2002), Fang et al. (2002), and Demarta and McNeil (2005). The $t-$copula model has received much attention recently, particularly in the context of modeling multivariate financial data (e.g., daily relative or logarithmic price changes on a number of stocks). One reason for the success of the $t-$copula is its ability to capture the phenomenon of dependent extreme values. The dependence in elliptical distributions is essentially determined by covariances (see Embrechts et al. (2002) and Glasserman (2004) for discussions on using $t-$distributions for applications in risk management).

2.1.2 Archimedean Copulas

Let function $\phi : [0,1] \rightarrow [0,\infty)$ be a strict *Archimedean copula generator* function and suppose its inverse ϕ^{-1} is monotonic on $[0,\infty)$. A strict generator is a decreasing function $\phi : [0,1] \rightarrow [0,\infty)$ that satisfies $\phi(0) = \infty$ and $\phi(1) = 0$. An Archimedean copula is defined as follows:

$$C(u_1, u_2, \ldots, u_p) = \phi^{-1}\Big(\phi(u_1) + \cdots + \phi(u_p)\Big)$$

We use the following Archimedean copulas: Clayton and Gumbel copulas.[6]

Clayton Copula:

Let the generator function $\phi(u) = \theta^{-1}\left(u^{-\theta} - 1\right)$. A Clayton copula is defined as

$$C_\theta^C(u_1, u_2, \ldots u_p) = \left[\sum_{i=1}^{p} u_i^{-\theta} - p + 1\right]^{-1/\theta} \tag{8}$$

with $\theta > 0$.

Gumbel Copula:

Let the generator function $\phi(u) = (-\log u)^\theta$. A Gumbel copula is defined as

$$C_\theta^{Gu}(u_1, u_2, \ldots, u_p) = \exp\left\{-\left[\sum_{i=1}^{p}(-\log u_i)^\theta\right]^{1/\theta}\right\} \tag{9}$$

with $\theta > 1$.

[6] We also consider rotated versions of each of these copula functions. A copula is rotated by using $1 - u_i^x$ in place of u_i^x, where u_i^x is the quantile corresponding to the marginal distribution for x at observation i.

2.1.3 Dependence

Various dependence measures between two random variables, X_1 and X_2, depend only on their copula function. Kendall's tau is a very useful alternative to the linear correlation coefficient and it is defined as

$$\tau = 4\int_0^1\int_0^1 C(u_1,u_2), c(u_1,u_2)du_1du_2 - 1$$

Kendall's tau has the same form for a bivariate Gaussian copula and a bivariate $t-$copula with correlation coefficient ρ:

$$\tau = \frac{2}{\pi}\text{arcsin}\rho$$

Another useful dependence measures between two variables are the coefficients of upper tail dependence, λ_u, and lower tail dependence, λ_l, which are defined as

$$\lambda_u = \lim_{q\to 1} P(X_2 > F_{X_2}^{-1}(q)|X_1 > F_{X_1}^{-1}(q)) \tag{10}$$

$$\lambda_l = \lim_{q\to 0} P(X_2 \leq F_{X_2}^{-1}(q)|X_1 \leq F_{X_1}^{-1}(q)) \tag{11}$$

λ_u and λ_l can be expressed as a function of copula as follows.

$$\lambda_u = \lim_{q\to 1}\frac{1-2q+C(q,q)}{1-q} \tag{12}$$

$$\lambda_l = \lim_{q\to 0}\frac{C(q,q)}{q} \tag{13}$$

The Gaussian copula is characterized by zero tail dependence. The $t-$copula exhibits tail dependence which is determined by,

$$\lambda_u = \lambda_l = 2t_{\upsilon+1}\left(\frac{-\sqrt{\upsilon+1}\sqrt{1-\rho}}{\sqrt{1+\rho}}\right)$$

where $t_{\upsilon+1}$ denotes the cumulative distribution function of the standard univariate Student–t distribution with $\upsilon+1$ degrees of freedom. The Clayton copula exhibits greater dependence in the negative tail than in the positive; and the Gumbel copula is exhibits greater dependence in the positive tail than in the negative.

3 Empirical Application

Assume there is a homogeneous commodity traded in two regional markets represented, respectively, by location indices i and j. The regional market prices for the goods are denoted by P_i and P_j. The per–unit revenue to arbitragers selling in region j is therefore $(1-\kappa)P_j$, where κ denotes the per–unit loss in value for the commodity due to transactions (transport) costs, $0 < \kappa < 1$. In general, the greater the distance between locations i and j, the closer is κ to one. A simple model of spatial price relationships that incorporates the effects of transaction costs (and possibly other frictions), then, can be written as

$$1/(1-\kappa) \geq P_i/P_j \geq (1-\kappa) \tag{14}$$

or, after taking natural logaritms and denoting $p_i = ln(P_i)$ and $p_j = ln(P_j)$,

$$-ln(1-\kappa) \geq (p_i - p_j) \geq ln(1-\kappa) \tag{15}$$

The implication from 15 is there is a band, $[-ln(1-\kappa), ln(1-\kappa)]$, within which no profitable arbitrage activity will occur; arbitrage is, however, profitable when log price differences, $p_i - p_j$, fall outside of the limits of the band. Over time we would expect that log price differences within the limits of the band would follow something very close to a unit root process, likely without drift. But, log price differences that fall outside of the limits of the band should be mean reverting. The relation in 15 implies a transactions cost band, which has often been assumed in the literature (see., e.g., Balcombe, Bailey, and Brooks, 2007, or Goodwin and Piggott, 2001), and which typically yields an empirical model consistent with the threshold autoregressive models described above (see, for example, Goodwin and Piggott (2001)). As noted, these models typically find that market price adjustments to shocks to parity condition tend to be faster or more apparent when the shocks are large. Threshold models typically allow the speed or degree of adjustment to vary in accordance with the size of the disequilibrium implied in parity relationships.

The copula approach offers a way of representing the multivariate distribution in terms of its (possibly) dependent marginals. This may be accomplished by following a variety of estimation approaches, including conventional joint maximum likelihood estimation or by following a two-stage statistical procedure that separately estimates the marginal distributions and the copula function. In this analysis, we chose nonparametric (empirical) *c.d.f.* functions so as to allow for maximum flexibility. That said, properties of individual marginal distributions may be of interest in their own right. Such properties can be discerned the applying maximum likelihood or method of moments estimation techniques in conjunction with or preceding ML estimation of the joint distribution (i.e., by way of the copula).

In the case of the joint probability distribution among spatially linked price pairs, we fit a copula model between $\Delta(p_t^i - p_t^j)$ and $(p_{t-1}^i - p_{t-1}^j)$.The particular choice of the copula function determines the nature of correlation. Standard linear correlation generally implies a constant correlation coefficient. In contrast, different functional relationships between random variables, including those that vary across the marginals, can be achieved with copula functions. In particular, the parametric form of the copula can, in some cases, permit considerably flexibility in how adjustments may differ as price differences become larger or smaller. In particular, in our application, we consider six different copula models (Gaussian, t, Clayton, rotated Clayton, Gumbel and rotated Gumbel) which allow for varying degrees of tail (or state) dependence as the degrees of freedom parameter changes. In a t-copula, for example, a smaller degrees of freedom parameter (which we denote as ν) will imply a greater degree of tail dependence. Conversely, as the degrees of freedom parameter increases, the t-copula approaches a Gaussian copula and tail dependence therefore approaches zero.

As an alternative to the standard copula model, we also consider a second and more deliberate approach to allowing for state-dependence in the joint distribution of regional prices. We accomplish this by allowing one or more parameters of the copula function to vary as market conditions change. We adopt an approach that is very similar to that applied in standard nonlinear threshold models of prices in that we allow lagged price differences (analogous to an "error correction" term) to directly impact the parameters of copulas that characterizes the relationship between price pairs in the regional markets. For Gaussian and t-copulas, we allow the off-diagonal element of the correlation matrix R in equations 6 and

7 above to be a function of market conditions, reflected in the lagged price differentials, as follows:

$$\rho_t = \alpha_0 + \alpha_1 \varphi(p^i_{t-1} - p^j_{t-1}) \tag{16}$$

where $(p^i_{t-1} - p^j_{t-1})$, $i, j = 1, \ldots, 4$ is the log price differences at time $t - 1$ and φ is the empirical *c.d.f.* function which implies various *states* of regional price differences.[7] The degrees of freedom parameter in the t-copula is also allowed to vary in the following manner:

$$\nu_t = \beta_0 + \beta_1 \varphi(p^i_{t-1} - p^j_{t-1}) \tag{17}$$

Finally, we allow the shape parameter, θ, in the Clayton and Gumbel copula to vary with time using the following functional relationship for the parameter θ.

$$\theta_t = \gamma_0 + \gamma_1 \varphi(p^i_{t-1} - p^j_{t-1}) \tag{18}$$

In accordance with the conventional "error correction" behavior anticipated in spatially integrated markets, we expect to see the parameters α_1, β_1 and γ_1 to be statistically significant. In the case of asymmetric copulas, a parametric structure that only allows tail dependence in one direction may be implied. This is justified if trade tends to mostly be unidirectional, as is typical in most regional market relationships. In such cases, depending on the direction of trade flows and which price is usually higher, the sign of these parameters could be negative or positive. This reflects the fact that an increase in the higher price or a decrease in the lower price will trigger a tighter relationship between the two prices (in first-differenced form) in the subsequent period. This assumes that markets display a relatively stable basis relationship, such that one price is generally above another (a characteristic that exists in most regional markets, where one market us usually "upstream" and another is "downstream"). We estimate the parameters of the copula models using maximum likelihood estimation.

[7] We considered a number of such "forcing variables"—variables that force the change in correlation or dependence. In the end, the empirical *c.d.f.* of the price differential, which represents a normalized measure of the size of deviations from parity, yielded the best results from among the alternatives considered. The optimal choice of a forcing variable remains a topic of ongoing research.

3.1 Data

For the empirical analyses, we consider regional North American markets for a prominent traded commodity—oriented strand board (OSB). OSB is a manufactured wood product that was first introduced in 1978 (the forerunner to oriented strand board was waferboard).[8] The Structural Board Association (SBA) reports that in 1980 OSB panel production in North America was 751 million square feet (on a $3/8^{th}$'s inch basis), but that by as early as 2005 this number had grown to 25 billion square feet. The SBA also reports that by 2000 OSB production exceeded that of plywood, and that by 2006 OSB production enjoyed a sixty–percent market share among all panel products in North America. OSB now accounts for the largest share of the overall panel wood products market.

Spatial linkages in this market are of particular interest because it is a good that is widely traded across considerable distances within the North American continent. Consumption is widespread and spatially dispersed while production tends to be concentrated in particular regions such as the U.S. South and Eastern Canada. Depletion of old–growth timber stocks that traditionally served as a source for panel wood products brought about tremendous growth in the use of engineered wood products such as OSB. A burgeoning housing market (and its more recent contraction) have brought about a number of significant shocks to this rapidly expanding industry. Construction market responses to large hurricanes such as Andrew in 1992 and Katrina in 2005 are another source of OSB market price volatility that merits clearer understanding for better quantifying the economic impacts of these catastrophic events. These and related factors underscore the importance of understanding and quantitatively measuring linkages among regional OSB markets.

The data set consists of OSB in four regional North American markets. Specifically, the regions examined are: (1) Eastern Canada (production deriving from plants in Ontario and Quebec); (2) North Central (production deriving from plants in Wisconsin, Michigan, and

[8] OSB is engineered by using waterproof and heat cured resins and waxes, and consists of rectangular shaped wood strands that are arranged in oriented layers. OSB is produced in long, continuous mats which are then cut into panels of varying sizes. In this regard OSB is similar to plywood, although OSB is generally considered to have more uniformity than plywood and is, moreover, cheaper to produce.

Minnesota);(3) Southeast (production deriving from plants in Georgia, Alabama, Mississippi, South Carolina, and Tennessee); and (4) Southwest (production deriving from plants in Texas, Louisiana, Arkansas, and Oklahoma). The result is there are six pairwise spatial price relationships that may be examined. The price data are for panels of $7/16^{th}$'s inch oriented strand board, and are expressed in U.S. dollars per thousand square feet. All price data are observed on a weekly basis and were obtained from the industry source Random Lengths.[9] The data span the period from February 3, 1995 through August 20, 2010, which yields 812 weekly observations. The basic unit of analysis used throughout the analysis is the natural logarithm of the price ratio, that is, $ln(P_t^i/P_t^j)$, where i and j indicates regional location (i.e., $i, j = 1, \ldots, 4$) and a subscripted t is a time index such that $t = 1, \ldots, T$, where $T = 812$.

3.2 Results

Our initial empirical analysis begins with a consideration of the relationship between the first-difference of the price differential ($\Delta(p_t^i - p_t^j$) and the lagged price differential ($p_{t-1}^i - p_{t-1}^j$). Because certain Archimedean copula functional relationships are only able to accommodate positive correlation, we utilize the negative value of the price differential (or $p_{t-1}^j - p_{t-1}^i$) as the right hand side regressor. Figure 1 illustrates the sample data for each of the six market pair combinations. The anticipated positive correlation is apparent for each market pair, suggesting adherence to the conditions required for spatial market integration. We applied OLS estimation techniques to the standard error-correction specification presented in equation 1 above (again, with the sign of the lagged price differential switched). Parameter estimates and summary statistics are presented in Table 1. The results indicate a reasonably strong degree of integration among the regional OSB markets. In fact, the half-lives of deviations from equilibrium conditions implied by these estimates are very similar to those

[9] *Random Lengths* is an independent, privately owned price reporting service, providing information on commonly produced and consumed wood products in the U.S., Canada, and other countries since 1944. Reported open–market sales prices are based on hundreds of weekly telephone interviews with product buyers and sellers. These interviews are with producers, wholesalers, distributors, secondary manufacturers, buying groups, treaters, and some large retailers. The regional OSB price data used are FOB mill price averages.

presented by Goodwin et al. (2011) in an application of STAR models to a similar set of OSB data.[10]

Marginal cumulative distributions for $(\Delta(p_t^i - p_t^j)$ and $(p_{t-1}^i - p_{t-1}^j)$ were represented using nonparametric, empirical cdf's. Again, this approach affords us maximum flexibility in evaluating the functional relationships underlying market linkages. Standard maximum likelihood estimation techniques were used to fit the six different copula models described above to the resulting data. Table 2 presents ML estimates of the copula parameters and summary statistics. In particular, values of the log-likelihood functions and of the AIC model fitting criterion are presented for each copula/market-pair combination. Likewise, measures of tail dependence, as described above, are also presented in the table. The correlation, degrees of freedom (in the case of the T copula), and shape parameters are all highly statistically significant. Recall that a t distribution converges to a Gaussian distribution as the degrees of freedom increases. In four of the six cases, the degrees of freedom parameters for the T copula are less than 30, which indicates greater platykurtosis than would be suggested by a Gaussian copula.

Tail dependence is an important indicator of how the relationship between the variables of interest (price differentials) behaves under extreme events. Recall that, by construction, the Gaussian copula has zero tail dependence. The T copula allows for positive dependence but imposes symmetry in dependence in the upper and lower tails of the distributions. The Clayton and rotated Gumbel copulas allow for lower tail dependence but impose zero upper tail dependence while the opposite is true for the rotated Clayton and the Gumbel copulas.

Selection among the alternative copula models can be guided through a consideration of the log likelihood function values and the AIC criteria. The rotated Clayton copula is supported in three of the six cases (Eastern Canada and the Southeast US, the North Central and Southeast US, and the North Central and Southwest US). Price comparisons for East Canada and the North Central and Southwest US markets favor Gaussian and T copulas, though in the latter case, the high degrees of freedom for the T copula estimates indicates a

[10]Deviation half-lives represent the weeks required to eliminate one-half of the deviation from equilibrium and are given by $ln(0.5)/ln(1-\beta)$.

relationship very similar to that of the Gaussian, with no tail dependence. Estimates of the asymmetric Archimedean copulas (variants of the Clayton and Gumbel copulas) all indicate strong tail dependence. Interpretation of tail dependence in cases where such dependence is only allowed in one tail can be aided by a consideration of the typical basis relationships among markets. In particular, to the extent that one market tends to export to another (i.e., a case of upstream/downstream market relationships), we generally expect to see price differences tending to be either positive or negative, but not both. This reflects the presence of transactions costs which are a component of basis price differences. This asymmetry in commodity flows is a relatively common feature in most basic commodity markets, including manufactured wood products. Figure 2 presents nonparametric densities for the price differentials for all six pairs of markets. In five of six cases, definite patterns of basis, reflecting a relationship where one market price is generally above another, are indicated. This suggests that the asymmetric tail dependence associated with the Clayton and Gumbel copulas (and their rotated versions) may be appropriate.[11]

Outside of examining tail dependence measures, the easiest way to characterize the market integration relationships among the market pairs is to consider the joint pdf functions implied by the copula estimates. To this end, we simulated the joint distributions implied by the copula estimates that were favored by the log-likelihood and AIC values. We assumed standard normal marginals for the differenced and lagged price differentials.[12] Contours of the resulting joint densities are presented for the favored copula functions in Figure 3. The densities illustrate patterns of tail dependence, where linkages between markets is stronger for larger deviations from price parity. In particular, estimates of the rotated Clayton and Gumbel copulas in panels (d), (e), and (f) of Figure 3 illustrate tighter correlation in the tails, corresponding to stronger price adjustments when price differences are higher. That

[11]Ongoing work is considering mixtures of copulas in order to permit greater flexibility in representing asymmetric tail dependence.

[12]Although the sample of 812 observations allows accurate estimation of the copula parameters, it is relatively thin for the purposes of illustrating the joint distribution. Instead, we utilize a much denser grid of values generated from a standard normal.

said, the patterns are subtle and may reflect the fact that each copula function is relatively restrictive in terms of the extent of tail dependence permitted.

We next considered allowing greater flexibility in the market relationships represented by the copula function estimates by allowing shape, correlation, and degrees of freedom parameters to vary according to the distribution of the price differential. In particular, we allowed parameters to vary with the empirical quantile of the price differential. Again, standard maximum likelihood estimation techniques were used to obtain parameter estimates.[13] The resulting parameter estimates are presented in Table 3. It is important to note that the standard copula functions presented in Table 2 are nested within the specifications presented in Table 3. This allows standard likelihood ratio tests of the parameters and alternative specifications.

The state-dependent versions of the copula models provide substantial improvements in fit over the standard versions in many (but not all) cases. Likelihood function values favor the augmented T copula in four of six cases. The expanded Gumbel copula is favored in two cases by the AIC and the augmented rotated Clayton and Gumbel copulas receive support in two cases each. It is relevant to note that the parameter estimates corresponding to the state dependence effect (i.e., the coefficients on the empirical cdf values of the lagged price differences) are frequently statistically significant, even in cases where the copula is not favored over alternatives by the log-likelihood and AIC values.

In order to consider the distributional properties that underlie price linkages among the market pairs, we again simulated the implied joint pdf functions. We chose to present examples for each pair that were either given support by the model fitting criteria or that had highly significant state-dependent parameter coefficients. Figure 4 presents the resulting joint pdf contours. A different picture of the price linkages emerges from the augmented models. In particular, very strong patterns of tail dependence, corresponding to large deviations from equilibrium among the prices, are revealed. In some cases, the correlation is stronger

[13] We also used a simulated annealing stochastic search algorithm to obtain starting values for standard quasi-Newton optimization procedures.

for positive deviations while in others correlation appears much stronger for negative price differences. Again, this reflects the basis patterns illustrated in Figure 2.

In accordance with existing research, the results indicate that market adjustments are generally larger in response to large price differences which reflect more substantial disequilibrium conditions (and therefore bigger arbitrage opportunities). The implications are very similar to those provided in other estimation approaches that allow for nonlinearities. In particular, regime switching and threshold models generally imply that price linkages and adjustment patterns are stronger and quicker when deviations from equilibrium are large. This reflects the presence of transactions costs and Heckscher's "commodity points."

4 Summary and Concluding Remarks

We evaluated the adherence to the economic conditions typically required for efficiently linked markets by considering the degree and nature of correlation implied by copula models of joint distributions of spatially related prices. To allow and model nonlinear behavior that might be caused by transactions costs, we adopted specific classes of copula functions that allow for "state-dependent" correlation, where the state is defined by the degree of market disequilibrium represented by spatial price differences at any point in time. We find that transactions costs bands are implied by certain nonlinear patterns of correlation. In addition, we consider more flexible copula models that allow parameters of the joint distributions to vary according to the "state" of market disequilibrium. We find that such models provide even stronger evidence of nonlinearities in market linkages.

One weakness of the copula approach is that it is usually difficult to select a specific parametric copula. We highlight alternative model fitting criteria that may be of value in comparing alternative copula models. Such an approach is, however, hindered by the fact that such comparisons do not necessarily comprise formal specification tests. Further, model fitting criteria may not be fully comparable across different copula families. Ongoing research is considering more formal approaches to specification testing, including tests based upon standard Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling specification tests. In such an approach, the dimensionality of the problem is reduced by comparing joint cdf values to the empirical cdf. This approach to testing offers promise in allowing for more formal model comparisons.

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Table 1. OLS Estimates of Autoregressive Error-Correction Price Parity Model[a]

$$\Delta(p_t^i - p_t^j) = \alpha - \beta(p_{t-1}^i - p_{t-1}^j)$$

Parameter	Estimate	Standard Error	t-Ratio	Deviation Half-Life	R^2
Eastern Canada and NC US					
α	−0.0066	0.0013	−5.26		0.0555
β	0.1136	0.0165	6.89	5.75	
Eastern Canada and SE US					
α	−0.0019	0.0012	−1.60		0.0217
β	0.0467	0.0110	4.24	14.51	
Eastern Canada and SW US					
α	−0.0045	0.0013	−3.40		0.0391
β	0.0815	0.0142	5.74	8.15	
Eastern Canada and SW US					
α	0.0008	0.0010	0.82		0.0264
β	0.0534	0.0114	4.68	12.62	
NC US and SE US					
α	0.0002	0.0010	0.23		0.0576
β	0.1148	0.0163	7.03	5.68	
SE US and SW US					
α	−0.0007	0.0007	−0.94		0.0279
β	0.0563	0.0117	4.81	11.97	

[a] An asterisk indicates statistical significance at the $\alpha = .10$ or smaller level. Deviation half-lives represent the weeks required to eliminate one-half of the deviation from equilibrium and are given by $ln(0.5)/ln(1-\beta)$.

Table 2. Copula Parameter Estimates (With Empirical Marginals)[a]

$C(\Delta(p_t^i - p_t^j), (p_{t-1}^i - p_{t-1}^j))$

Copula	Parameter	Estimate	Standard Error	t Ratio	Log Likelihood	AIC	Lower Tail Dependence	Upper Tail Dependence
East Canada and North Central US								
Gaussian	ρ	0.2321	0.0337	6.90*	21.8316	−41.6632†	0.0000	0.0000
T	ρ	0.2265	0.0337	6.72*	22.4752†	−40.9503	0.0007	0.0007
	ν	22.7499	5.1802	4.39*				
Clayton	θ	0.2330	0.0494	4.72*	14.9767	−27.9534	0.0511	0.0000
Rotated Clayton	θ	0.2724	0.0486	5.61*	19.9210	−37.8420	0.0000	0.0785
Gumbel	θ	1.1504	0.0281	40.96*	21.1173	−40.2345	0.0000	0.1733
Rotated Gumbel	θ	1.1391	0.0281	40.47*	18.9214	−35.8428	0.1623	0.0000
East Canada and Southeast US								
Gaussian	ρ	0.1530	0.0345	4.43*	9.3252	−16.6505	0.0000	0.0000
T	ρ	0.1510	0.0346	4.37*	9.4024	−14.8049	0.0000	0.0000
	ν	96.9999	1.0641	91.16*				
Clayton	θ	0.1328	0.0455	2.92*	5.6982	−9.3964	0.0054	0.0000
Rotated Clayton	θ	0.1742	0.0408	4.27*	9.4390†	−16.8780†	0.0000	0.0187
Gumbel	θ	1.0859	0.0237	45.78*	8.0347	−14.0693	0.0000	0.1067
Rotated Gumbel	θ	1.0774	0.0241	44.70*	7.7772	−13.5544	0.0972	0.0000
East Canada and Southwest US								
Gaussian	ρ	0.1968	0.0340	5.79*	15.5670	−29.1340†	0.0000	0.0000
T	ρ	0.1949	0.0342	5.70*	15.7462†	−27.4924	0.0000	0.0000
	ν	46.9979	23.8265	1.97*				
Clayton	θ	0.2086	0.0454	4.59*	12.4103	−22.8206	0.0360	0.0000
Rotated Clayton	θ	0.2034	0.0462	4.40*	11.6107	−21.2214	0.0000	0.0331
Gumbel	θ	1.1177	0.0270	41.42*	12.5964	−23.1927	0.0000	0.1408
Rotated Gumbel	θ	1.1195	0.0266	42.07*	13.2852	−24.5703	0.1426	0.0000

[a] An asterisk indicates statistical significance at the $\alpha = .10$ or smaller level. Maximum likelihood and minimum AIC values are identified by "†".

Table 2. (continued)[a]

Copula	Parameter	Estimate	Standard Error	t Ratio	Log Likelihood	AIC	Lower Tail Dependence	Upper Tail Dependence
North Central US and Southeast US								
Gaussian	ρ	0.1859	0.0347	5.36*	13.8513	−25.7027	0.0000	0.0000
T	ρ	0.1807	0.0348	5.18*	14.4285	−24.8569	0.0004	0.0004
	ν	23.3336	0.5798	40.25*				
Clayton	θ	0.0940	0.0462	2.03*	2.6494	−3.2988	0.0000	0.0006
Rotated Clayton	θ	0.2949	0.0478	6.17*	24.2816†	−46.5631†	0.0953	0.0000
Gumbel	θ	1.1403	0.0254	44.85*	21.1548	−40.3096	0.0000	0.1635
Rotated Gumbel	θ	1.0753	0.0272	39.57*	5.9797	−9.9594	0.0948	0.0000
North Central US and Southwest US								
Gaussian	ρ	0.2436	0.0346	7.04*	24.1257	−46.2515	0.0000	0.0000
T	ρ	0.2360	0.0358	6.59*	27.1568	−50.3136	0.0321	0.0321
	ν	9.0134	3.8569	2.34*				
Clayton	θ	0.2323	0.0487	4.77*	14.1956	−26.3911	0.0506	0.0000
Rotated Clayton	θ	0.3210	0.0540	5.94*	26.4397†	−50.8794†	0.0000	0.1154
Gumbel	θ	1.1792	0.0299	39.50*	29.3536	−56.7073	0.0000	0.2000
Rotated Gumbel	θ	1.1504	0.0304	37.82*	18.2574	−34.5148	0.1733	0.0000
Southeast US and Southwest US								
Gaussian	ρ	0.2052	0.0357	5.75*	16.9584	−31.9168	0.0000	0.0000
T	ρ	0.2027	0.0365	5.55*	19.5611	−35.1221	0.0192	0.0192
	ν	10.2438	4.9235	2.08*				
Clayton	θ	0.2543	0.0529	4.81*	17.1453	−32.2907	0.0655	0.0000
Rotated Clayton	θ	0.2000	0.0452	4.43*	11.0723	−20.1446	0.0000	0.0313
Gumbel	θ	1.1226	0.0287	39.14*	12.6694	−23.3388	0.0000	0.1459
Rotated Gumbel	θ	1.1453	0.0287	39.97*	19.3011†	−36.6022†	0.1683	0.0000

[a] An asterisk indicates statistical significance at the $\alpha = .10$ or smaller level. Maximum likelihood and minimum AIC values are identified by "†".

Table 3. State-Varying Copula Parameter Estimates (With Empirical Marginals)[a]

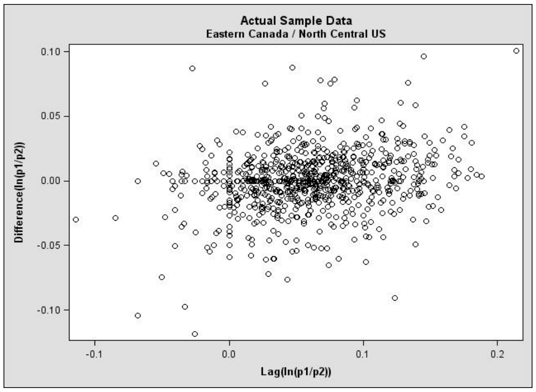
Copula	Parameter	Estimate	Standard Error	t Ratio	Log Likelihood	AIC
East Canada and North Central US						
Gaussian	α_0	0.4722	0.1160	4.07*	21.8477	−39.6955
	α_1	−0.0306	0.1752	−0.17		
T	β_0	3.1118	0.2334	13.33*	22.4884†	−36.9769
	β_1	−0.0627	0.1775	−0.35		
	α_0	0.4598	0.1155	3.98*		
	α_1	−0.0294	0.1771	−0.17		
Clayton	γ_0	−1.6088	0.3156	−5.10*	15.2875	−26.5751
	γ_1	0.4299	0.5653	0.76		
Rotated Clayton	γ_0	−1.5461	0.2948	−5.25*	20.8664	−37.7327
	γ_1	0.6750	0.5094	1.33		
Gumbel	γ_0	−1.4855	0.2844	−5.22*	22.2578	−40.5155†
	γ_1	−0.7259	0.4846	−1.50		
Rotated Gumbel	γ_0	−1.6248	0.3179	−5.11*	19.6744	−35.3487
	γ_1	−0.6186	0.5182	−1.19		
East Canada and Southeast US						
Gaussian	α_0	0.1614	0.1213	1.33	10.4785	−16.9569†
	α_1	0.2605	0.1724	1.51		
T	β_0	1.7962	0.0046	393.17*	12.2880†	−16.5759
	β_1	5.9694	0.0279	213.92*		
	α_0	0.1136	0.1255	0.91		
	α_1	0.3149	0.1759	1.79*		
Clayton	γ_0	−3.0667	2.2884	−1.34	7.9397	−11.8794
	γ_1	2.1981	2.8932	0.76		
Rotated Clayton	γ_0	−1.7492	0.2849	−6.14*	9.4391	−14.8781
	γ_1	0.0051	0.5748	0.01		
Gumbel	γ_0	−2.3758	0.5101	−4.66*	8.0505	−12.1010
	γ_1	−0.1272	0.6718	−0.19		
Rotated Gumbel	γ_0	−1.6544	0.3429	−4.83*	10.0751	−16.1502
	γ_1	−1.6406	0.9143	−1.79*		
East Canada and Southwest US						
Gaussian	α_0	0.3817	0.1142	3.34*	15.5670	−27.1340†
	α_1	0.0005	0.1675	0.00		
T	β_0	2.8031	8.7312	0.32	16.0060†	−24.0119
	β_1	1.7504	0.8010	2.19*		
	α_0	0.3816	0.1248	3.06*		
	α_1	−0.0052	0.1695	−0.03		
Clayton	γ_0	−1.7806	0.3317	−5.37*	12.9265	−21.8530
	γ_1	0.5979	0.5495	1.09		
Rotated Clayton	γ_0	−1.8161	0.3244	−5.60*	12.2014	−20.4028
	γ_1	0.6317	0.5720	1.10		
Gumbel	γ_0	−1.6868	0.3298	−5.11*	13.6599	−23.3198
	γ_1	−0.8052	0.5349	−1.51		
Rotated Gumbel	γ_0	−1.7798	0.3265	−5.45*	13.8704	−23.7409
	γ_1	−0.6024	0.5319	−1.13		

[a] An asterisk indicates statistical significance at the $\alpha = .10$ or smaller level. Maximum likelihood and minimum AIC values are identified by "†".

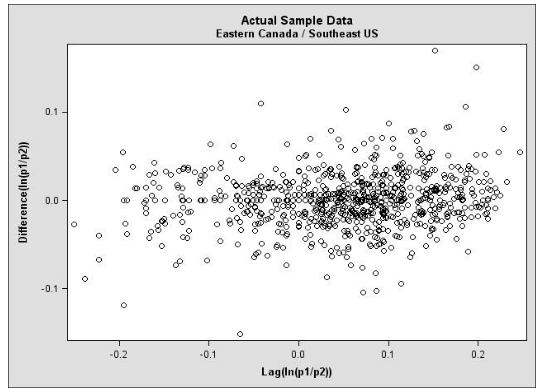
Table 3. (continued)[a]

Copula	Parameter	Estimate	Standard Error	t Ratio	Log Likelihood	AIC
North Central US and Southeast US						
Gaussian	α_0	0.1235	0.1144	1.08	17.2462	−30.4924
	α_1	0.4496	0.1767	2.54*		
T	β_0	4.9419	0.7371	6.70*	18.6964	−29.3928
	β_1	−2.9548	0.7400	−3.99*		
	α_0	0.1194	0.1144	1.04		
	α_1	0.4479	0.1795	2.50*		
Clayton	γ_0	−15.4014	6.0202	−2.56*	12.1436	−20.2873
	γ_1	15.8854	6.3794	2.49*		
Rotated Clayton	γ_0	−1.0931	0.2066	−5.29*	24.6527†	−45.3055†
	γ_1	−0.4065	0.4634	−0.88		
Gumbel	γ_0	−2.4396	0.4616	−5.29*	22.0515	−40.1030
	γ_1	0.7301	0.5664	1.29		
Rotated Gumbel	γ_0	−0.4673	0.3334	−1.40	16.8480	−29.6960
	γ_1	−12.0656	4.8408	−2.49*		
North Central US and Southwest US						
Gaussian	α_0	0.3899	0.1137	3.43*	24.7267	−45.4534
	α_1	0.1814	0.1710	1.06		
T	β_0	2.4357	0.5523	4.41*	27.6791	−47.3581
	β_1	−0.6625	0.6551	−1.01		
	α_0	0.3910	0.1174	3.33*		
	α_1	0.1411	0.1816	0.78		
Clayton	γ_0	−1.8396	0.3520	−5.23*	15.9671	−27.9342
	γ_1	0.9918	0.5320	1.86*		
Rotated Clayton	γ_0	−1.1999	0.2164	−5.54*	26.5556	−49.1113
	γ_1	0.1915	0.4032	0.48		
Gumbel	γ_0	−1.6045	0.2810	−5.71*	29.4723†	−54.9447†
	γ_1	−0.1922	0.3840	−0.50		
Rotated Gumbel	γ_0	−1.3222	0.2614	−5.06*	20.9626	−37.9253
	γ_1	−1.0827	0.4962	−2.18*		
Southeast US and Southwest US						
Gaussian	α_0	0.5267	0.1201	4.39*	18.0378	−32.0756
	α_1	−0.2445	0.1719	−1.42		
T	β_0	1.6808	0.5253	3.20*	21.1449†	−34.2898
	β_1	1.1726	0.5989	1.96*		
	α_0	0.5129	0.1268	4.04*		
	α_1	−0.2336	0.1792	−1.30		
Clayton	γ_0	−1.2689	0.2319	−5.47*	17.3373	−30.6746
	γ_1	−0.3083	0.5377	−0.57		
Rotated Clayton	γ_0	−2.0613	0.3757	−5.49*	13.2789	−22.5577
	γ_1	1.1977	0.5587	2.14*		
Gumbel	γ_0	−1.3420	0.2777	−4.83*	16.3131	−28.6261
	γ_1	−1.4288	0.5729	−2.49*		
Rotated Gumbel	γ_0	−2.0178	0.3677	−5.49*	19.3503	−34.7007†
	γ_1	0.1479	0.4620	0.32		

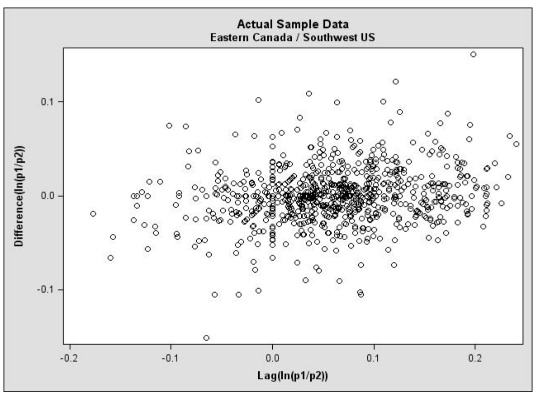
[a] An asterisk indicates statistical significance at the $\alpha = .10$ or smaller level. Maximum likelihood and minimum AIC values are identified by "†".



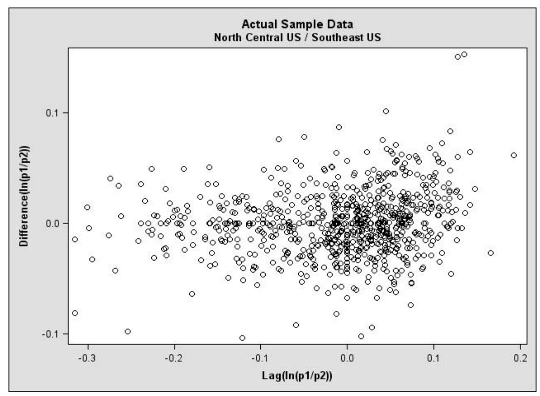
(a) Eastern Canada and NC US



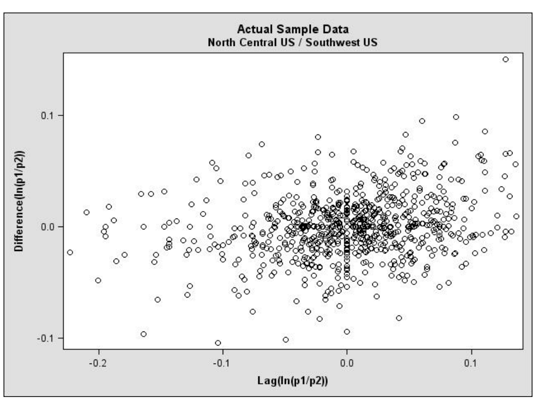
(b) Eastern Canada and SE US



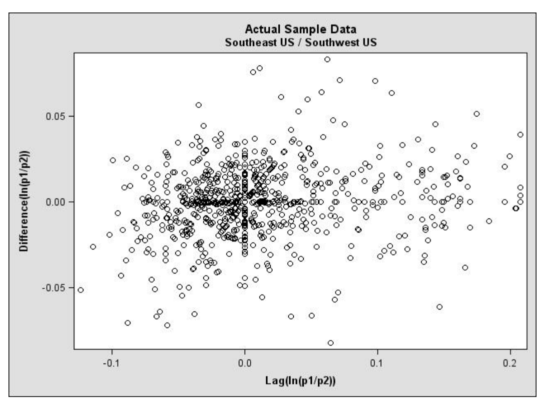
(c) Eastern Canada and SW US



(d) NC US and SE US

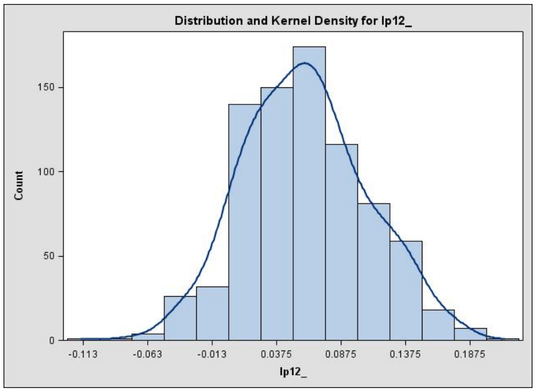


(e) NC US and SW US

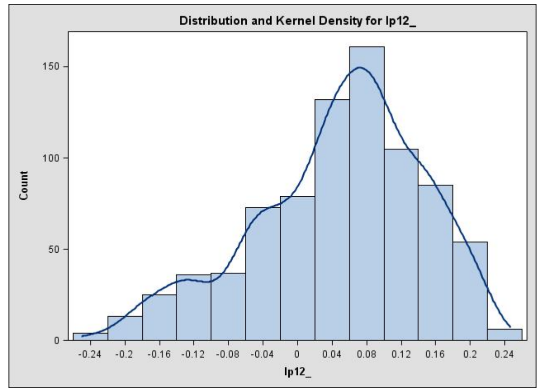


(f) SE US and SW US

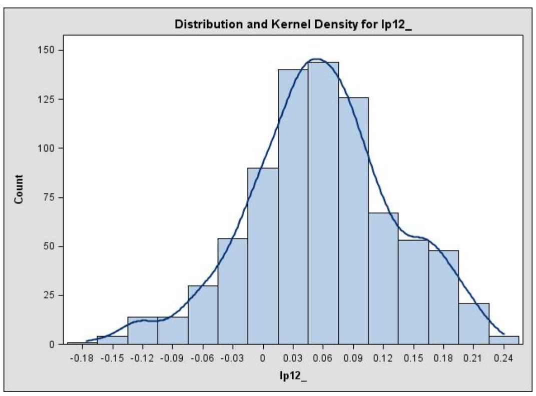
Figure 1: Regional OSB Prices Sample Data $(\Delta(p_t^i - p_t^j), (p_{t-1}^i - p_{t-1}^j))$



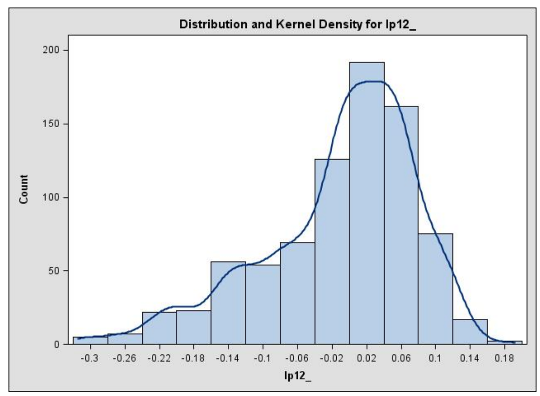
(a) Eastern Canada and NC US



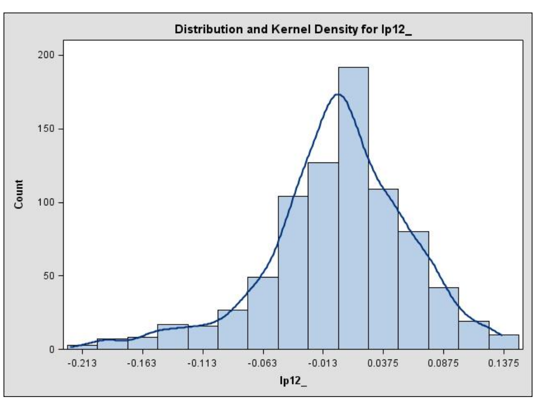
(b) Eastern Canada and SE US



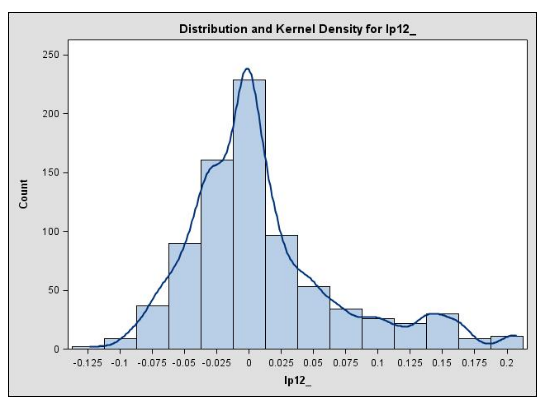
(c) Eastern Canada and SW US



(d) NC US and SE US

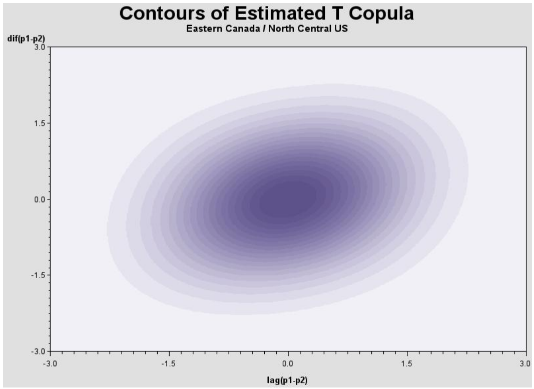


(e) NC US and SW US

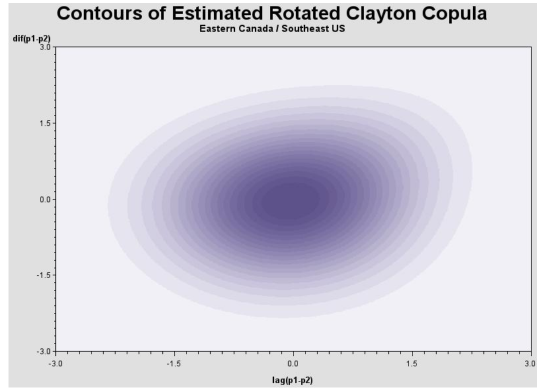


(f) SE US and SW US

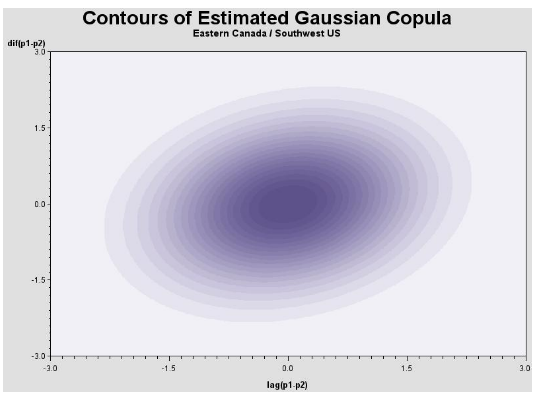
Figure 2: Distribution of Lagged Price Differentials $(p^i_{t-1} - p^j_{t-1})$



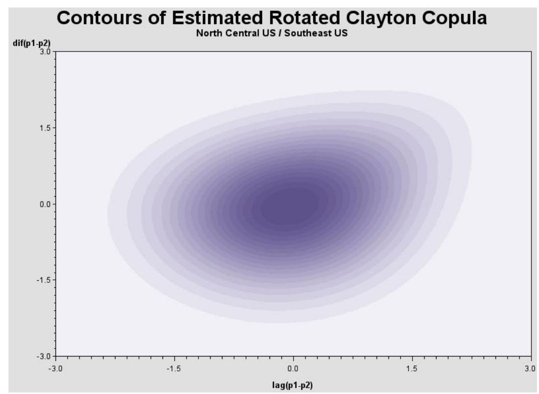
(a) Eastern Canada and NC US



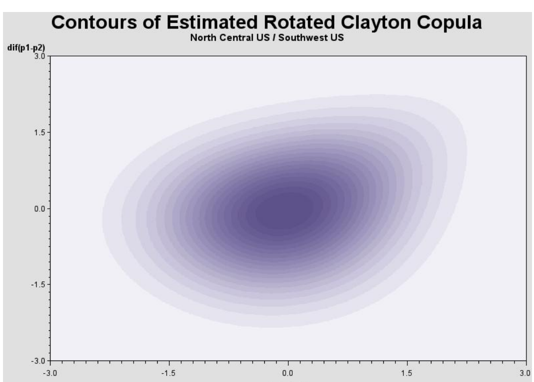
(b) Eastern Canada and SE US



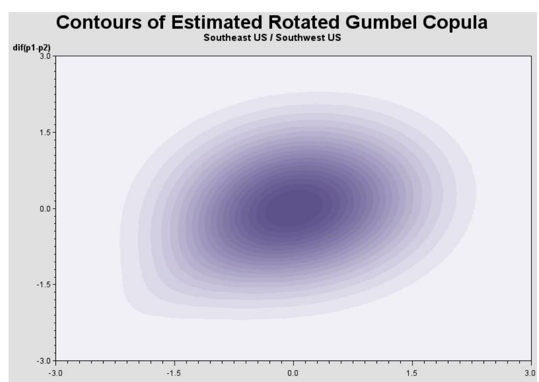
(c) Eastern Canada and SW US



(d) NC US and SE US

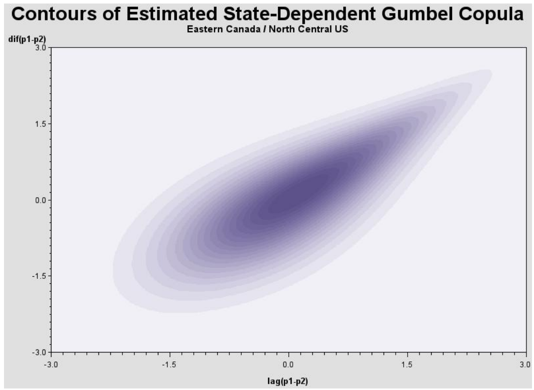


(e) NC US and SW US

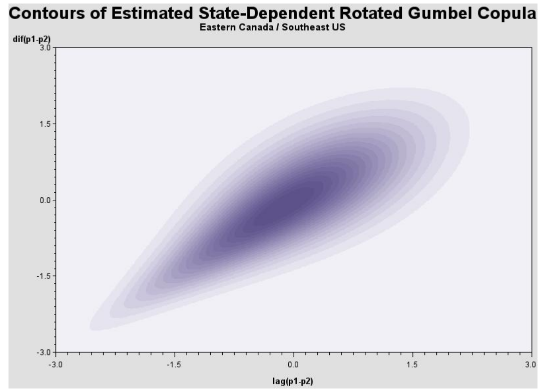


(f) SE US and SW US

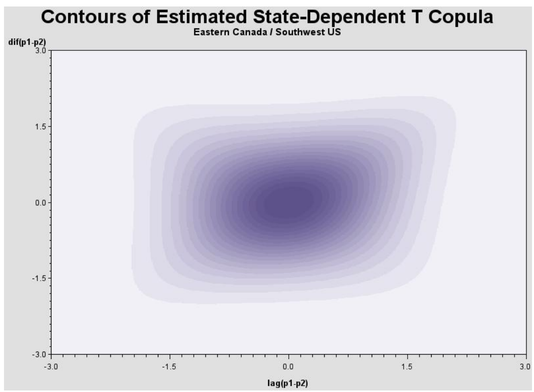
Figure 3: Contours of Estimated Copula Joint Probability Functions (With Standard Normal Marginals)



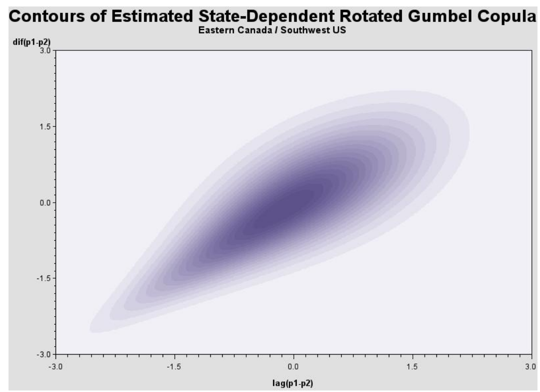
(a) Eastern Canada and NC US



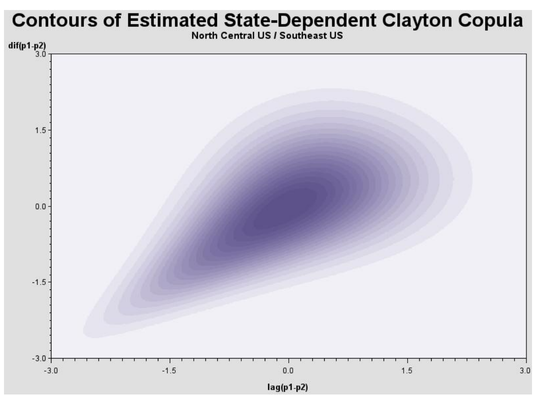
(b) Eastern Canada and SE US



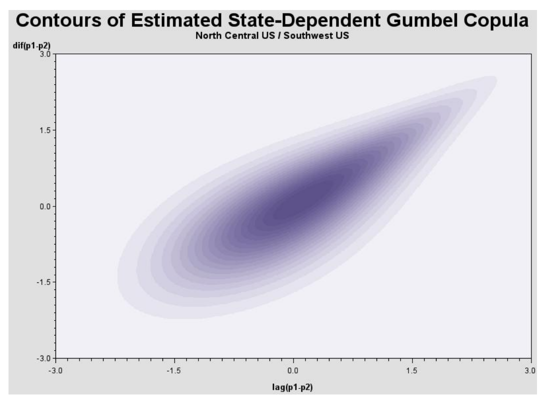
(c) Eastern Canada and SW US



(d) NC US and SE US



(e) NC US and SW US



(f) SE US and SW US

Figure 4: Contours of Estimated State-Dependent Copula Joint Probability Functions (With Standard Normal Marginals)