WORKING PAPERS

## Fertility and Child Occupation: Theory and Evidence from Senegal

# Fertility and Child Occupation: Theory and Evidence from Senegal* 

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#### Abstract

This paper analyzes household fertility and child occupation decisions in a risky environment. Fertility decisions are made first, when only the distribution of shocks is known. When shocks are realized and fertility is fixed, parents adapt by allocating children's occupations, i.e. school, paid work and domestic chores. Fertility is decreasing with the shock probability and increasing with parental permanent income. Households facing an adverse shock make more use of child labor and send fewer children to school, unless the total number of children is small. These predictions are tested with data from the Senegalese SEHW (2003) following this two-step methodology. A Poisson model estimates the number of children with classical instruments and household-level information on shock distribution, confirming the theory's predictions on fertility. A multivariate Tobit model estimates the determinants of children occupations, including the occurrence of shocks and accounting for the endogeneity of fertility. The number of children increases (decreases) the probability of child specialization (multiple activities). Shock-related variables have an adverse effect on schooling.


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JEL classification codes: I24, I25, J13, J24, O12, O15

[^0]
## 1 Introduction

The negative relationship between family size and schooling is one of the pillars of the economics of the household literature. This finding is often explained using an argument of resources dilution: parents have limited time and money to devote to the education of their children, so that those with fewer children invest more per child. The seminal contribution in this literature is due to Becker \& Lewis (1973). Their model postulates that the shadow price of the number of children is an increasing function of child quality and that the shadow price of child quality is an increasing function of the number of children. This interdependence of shadow prices leads parents to find a tradeoff between the quantity and the quality of children. Numerous empirical studies confirm the qualityquantity tradeoff prediction. Rosenzweig \& Wolpin (1980), using Indian data, show that an exogenous increase in fertility significantly decreases the level of schooling of all children measured as the age standardized sum of the educational attainment of all children in the household. Psacharopoulos \& Patrinos (1997) show a delaying effect in schooling related to the number of children in Peruvian families; and this effect increases as the number of siblings increase. Using data from Vietnam, Anh et al. (1998) find a negative relationship between school attendance and family size, after controlling for individual and household characteristics. In the Korean case, Lee (2008) finds significant evidence of negative impact of each additional child on the monthly household expenditure for education.

However, contrary to Becker and Lewis theory and evidence from other regions, early studies in sub-Saharan Africa revealed a reverse association between family size and schooling. Gomes (1984) and Chernichovsky (1985) found that educational attainment has a positive relationship with family size respectively in urban Kenya and rural Botswana. Later studies mitigate the early findings but results remain mixed. Montgomery \& Kouame (1993) report negative link between family size and schooling in urban areas of Côte d'Ivoire but a positive one in rural areas; Lloyd \& Blanc (1996) reveal significant negative relationships in only two nations (Kenya and Namibia) out of seven sub-Saharan countries.

Why is the quality-quantity tradeoff not very consistent in Africa? According to Gomes (1984), African parents can have many children and educate a high proportion of them, as long as they can induce the advantaged children to finance their other siblings. The existence of such an intra-household coordination mechanism promotes specialization of household members in relation to their number, abilities and opportunity costs of their time.

This paper revisits the link between family size and children occupation in an African setting using evidence from Senegal. The paper investigates how children are assigned to different activities or combination of activities in relation with adult and child market performance and number of children in the household.

The contribution of this paper is twofold.
First, it provides a theoretical framework that links household composition, labor market performance and children's time allocation. There is little theoretical work that directly addresses this link in an african setting despite numerous empirical studies on occupations in Africa. Our theoretical model contains the classical ingredients of the literature, in that the household head maximizes her utility, which depends on consumption of both tradable and domestic goods and children's number and education. Furthermore,
our model captures two original aspects of the fertility/quality puzzle that, to the best of our knowledge, have been neglected in this literature. First, it has become a well documented fact that transitory income shocks play an important role in the determination of child school trajectories. ${ }^{1}$ Indeed, in a context of incomplete insurance and capital markets, shocks may induce parents to adjust the children's occupations according to their needs. Second, once shocks are realized, parents cannot adjust fertility instantaneously. This point must make shocks an important determinant of fertility decisions. Therefore, the timing of our model is the following. Fertility decisions are made first, in a risky environment, while children's occupations - namely school, paid work and domestic chores - are assigned in a second period, when shocks are realized. ${ }^{2}$

The model's main predictions are the following. Fertility is higher in households with more favorable ex-ante distribution of parental income. More precisely, fertility is decreasing in the probability of facing an adverse shock and in the magnitude of such a potential shock. Conversely, fertility increases with parental permanent income. Ex post, the number of working children is smaller among households with high parental labor productivity, high child domestic productivity and low supervision costs. The number of domestic children is positively affected by fertility, while the opposite stands for parental supply of domestic labor. The number of school children is likely to increase with fertility if parental labor income is large and/or the cost of raising children is low. The impact of fertility on schooling will also be larger if preferences for education are strong. Children occupations are affected by shocks in the following way. Households which face an adverse shock make more use of child labor than they would have if the shock had not occured. The impact of the shock on other decision variables depends on the total number of children.

The second contribution concerns the empirical analysis. We test the model's predictions in two steps.

First, a Poisson model estimates the number of children with information on the household's ex ante distribution of income and standard instruments. This approach allows us to treat the demographic composition of the household as endogenously determined. Following our theory, the number of children in the household results of household life-cycle decision concerning fertility. We control for the number of children using as instruments shocks that have affected households during the last decade and the age of the household head at his first child. While these instruments may have an impact on children's occupations, credible alternative instruments are unfortunately not available from the dataset.

Second, a multivariate Tobit model estimates the determinants of children occupa-

[^1]tions bringing in the occurrence of shocks. This estimation accounts for the endogeneity of fertility by introducing the residuals of the first estimation following Smith \& Blundell (1986) two-steps procedure. In the multivariate Tobit model, the dependent variables are the numbers of children within a household involved in each occupation (home production, market work, or schooling) or without one. This approach is innovative since most studies, except Orbeta (2005), consider individual occupational status to estimate the determinants of household decisions of children's occupation. ${ }^{1}$ Individual outcomes implicitly assume the independence of the decision for each child within the same household, whereas coordination within the household invalidates this assumption. It is obvious that the number of children present in the household as well as the occupation of each child influence parents' decisions on the activities of the other siblings. Using the number of children in different activities allows having dependent variables closer to reality and taking account the issue of correlations among children within the same household and overcoming the problem of independence of the decision for each child.

## 2 Theoretical framework

### 2.1 The model

The household head makes her decision about fertility in a risky environment, when the distribution of outcomes is known but not its realization. The allocation of children's activities - namely school, paid work and domestic chores - takes place later, when shocks are realized. This setting captures the idea that depending on the shocks they face, parents cannot adjust fertility instantaneously but can allocate the time of their children accordingly. Formally, parent labor productivity $w_{p}$ can be high with probability $p$ ( $w_{p}=$ $w$ ), or low with probability $1-p$. In this case, a negative shock $\Delta$ occurs and $w_{p}=w-\Delta .^{3}$ The timing is the following: 1) The household head decides on her number of children N. 2) Nature determines whether a negative shock has taken place (i.e. parental labor productivity $w_{p}$ is realized). 3) The household head allocates the occupations of each of her $N$ children and her own time.

The household head maximizes her utility, which depends on consumption of both tradable and domestic goods and children's education. The head is endowed with one unit of productive time. A share $(1-d)$ of this time is to be allocated to paid work, which has a productivity per unit of time of $w_{p}$, and a share $d$ to domestic good production. The household head's productivity per unit of domestic labor is normalized to 1. Raising a child irrespective of his/her activity costs $\theta$. Let us now turn to child activities. Our data shows that children are mostly assigned to a single activity. ${ }^{4}$ This may be justified by the existence of scale economies, which should particularly be the case in schooling, where partial attendance is very unlikely to provide significant results. Similarly, learning by doing may lead working children to specialize in this occupation. This observation leads us to depart from the standard interpretation of models of fertility and child qual-

[^2]ity/occupation where children are treated homogeneously and where the household head's decision variables are the shares of time allocated to an average representative child. Instead, we interpret occupations in our model as the number of children allocated to each activity, with the constraint that the sum of children of each type equals total fertility decided in the first period:
$$
n_{l}+n_{d}+n_{e}=N
$$
where $n_{l}, n_{d}$ and $n_{e}$ are respectively the numbers of working, domestic and school children. Working children earn a wage rate of $w_{c} \leq w_{p}$. However, supervision costs are required for child labor. These costs are captured by parameters $s_{l}$ and are convex in the number of children. ${ }^{5}$ The household head's tradable good consumption equals
\[

$$
\begin{equation*}
C=(1-d) w_{p}+n_{l} w_{c}-s_{l} n_{l}^{2}-N \theta . \tag{1}
\end{equation*}
$$

\]

Domestic children allocate their whole unit of time to produce $\alpha \leq 1$ units of the domestic good. ${ }^{6}$ The household head's domestic good consumption equals

$$
\begin{equation*}
D=d+\alpha n_{d} . \tag{2}
\end{equation*}
$$

Finally, despite the potential need to make their children work, parents are altrusistic towards their children. They care about child quality, i.e. education, since the latter determines children's future income and welfare. As in previous models of endogeneous fertility, the utility derived from child quality is equal to the product between the number of children and their quality, with the particular feature here that children may be reach different qualities according to their occupation:

$$
\begin{align*}
E & =n_{e} Q+\left(n_{d}+n_{l}\right) q  \tag{3}\\
& =N Q-\left(n_{d}+n_{l}\right)(Q-q),
\end{align*}
$$

where $Q$ and $q$ represent the quality attached to respectively educated and uneducated children. There are two interpretations to the functionnal form of $E$. The first one is in line with the concept of a child compensation mechanism of Gomes (1984): if such a mechanism exists, then parents should only care about the total human capital generated, the returns to which are redistributed among children. Even in the absence of such a mechanism, our formulation remains valid if one considers parents as utilitarist with respect to their children's education.

The household head's preferences are assumed separable in each of these three goods. For simplicity and empirical testability, preferences are assumed Cobb-Douglas, so that the household head's utility is

$$
H=\gamma \ln (C)+\delta \ln (D)+\epsilon \ln (E)
$$

Given these preferences, we further assume without loss of generality that $\gamma+\delta+\epsilon=1$. Let us now turn to the resolution of the model.

We proceed by backward induction, and start at stage 3, where both fertility and the state of the world are realized and the household head must assign the number of children

[^3]to each occupation and allocate her own time. For $N$ and $w_{p}$ given, the head's objective at this stage is to maximize $H$ with respect to $d, n_{d}$ and $n_{l}$, subject to (1) to (3). First order conditions with respect to respectively $d, n_{d}$ and $n_{l}$ are
\[

$$
\begin{align*}
\frac{\gamma w_{p}}{C} & =\frac{\delta}{D}  \tag{FOC}\\
\frac{\delta \alpha}{D} & =\frac{\epsilon}{N \mu-\left(n_{d}+n_{l}\right)}  \tag{d}\\
\frac{\gamma\left(w_{c}-2 s_{l} n_{l}\right)}{C} & =\frac{\epsilon}{N \mu-\left(n_{d}+n_{l}\right)}, \tag{l}
\end{align*}
$$
\]

where $\mu \equiv \frac{Q}{Q-q}$. Let us briefly comment on $\mu$. If $q>0$, that is, despite the fact that some children may be uneducated, those are still considered as "goods" by their parents, then $\mu>1$. If on the contrary $q \leq 0$, that is, parents suffer so much from having uneducated children that those may be interpreted as "bads" (the presence of such children decreases $E)$, then $\mu \in[0 ; 1]$.

Combining all three conditions, one obtains the optimal number of working children,

$$
\begin{align*}
n_{l}^{*} & =\frac{w_{c}-\alpha w_{p}}{2 s_{l}}  \tag{l}\\
& =\frac{\alpha}{2 s_{l}}\left(\frac{w_{c}}{\alpha}-w_{p}\right) .
\end{align*}
$$

The existence of working children requires child wage $w_{c}$ to be larger than the threshold $\alpha w_{p}$. While $w_{c}$ can be interpreted as the limit when $n_{l}$ tends to zero of child labor net marginal productivity $\left(\lim _{n_{l} \rightarrow 0} \frac{\partial\left(n_{l} w_{c}-s_{l} n_{l}^{2}\right)}{\partial n_{l}}\right)$, the threshold $\alpha w_{p}$ is nothing but the the $n_{l}^{*}$ th child's net marginal productivity $\left.\frac{\partial\left(n_{l} w_{c}-s_{l} n_{l}^{2}\right)}{\partial n_{l}}\right|_{n_{l}=n_{l}^{*}}$. An alternative interpretation of the condition for the existence of child labor is that child labor productivity (relative to child domestic productivity) $\frac{w_{c}}{\alpha}$ is larger than adult relative productivity $\frac{w_{p}}{1}$. Two cases can be distiguished here. First, when adult labor productivity is low, the number of working children will be strictly positive. However, when adult labor productivity is high, it may be that $w_{c}-\alpha w_{p}<0$, in which case there might be no child labor at the optimum: $n_{l}^{*}=0$. We focus here on the case where child labor is interior in both states of the world. The case where child labor is at a corner in the good state of the world does not add anything to the analysis.

Using $n_{l}^{*}=\frac{w_{c}-\alpha w_{p}}{2 s_{l}} \geq 0$, we are left with a two-equation, two-unknown system, which provides the following optimal decisions:

$$
\begin{align*}
& d^{*}=\frac{1-\gamma}{w_{p}}\left(w_{p}-N \theta+\frac{w_{c}^{2}-\alpha^{2} w_{p}^{2}}{4 s_{l}}\right)-\gamma \alpha\left(\mu N-\frac{w_{c}-\alpha w_{p}}{2 s_{l}}\right)  \tag{*}\\
& n_{d}^{*}=-\frac{\epsilon}{\alpha w_{p}}\left(w_{p}-N \theta+\frac{w_{c}^{2}-\alpha^{2} w_{p}^{2}}{4 s_{l}}\right)+(1-\epsilon)\left(\mu N-\frac{w_{c}-\alpha w_{p}}{2 s_{l}}\right) \tag{d}
\end{align*}
$$

At equilibrium, the levels of the utility function's three arguments are:

$$
\begin{aligned}
C^{*} & =\gamma Y, \\
D^{*} & =\frac{\delta}{w_{p}} Y, \\
E^{*} & =\frac{\epsilon(Q-q)}{\alpha w_{p}} Y,
\end{aligned}
$$

where

$$
\begin{equation*}
Y=w_{p}+N\left(\mu \alpha w_{p}-\theta\right)+\frac{\left(w_{c}-\alpha w_{p}\right)^{2}}{4 s_{l}} \tag{Y}
\end{equation*}
$$

Consumption of each good depends on its respective coefficient and on relative prices, and on total household normalized income $Y$. At the optimum, the "returns" to each type of child are equalized at $\mu \alpha w_{p}-\theta$. While education and domestic chores have constant returns, child labor returns are decreasing due to supervision costs, so that the latter provides a surplus of $\frac{\left(w_{c}-\alpha w_{p}\right)^{2}}{4 s_{l}} .^{7}$ In order to make relevant the distinction between the two states of the world (absence and occurance of an adverse shock $\Delta$ ) and to obtain an interior solution for $N$ in period 1, we make the following assumption.

Assumption 0: $\mu \alpha(w-\Delta)-\theta<0<\mu \alpha w-\theta$ or equivalently $\Delta>w-\frac{\theta}{\alpha}\left(1-\frac{q}{Q}\right)>$ 0.

Assumption 0 implies that when the household faces an adverse shock, the negative impact this shock exerts on child marginal contribution $\mu \alpha(w-\Delta)$ makes $Y$ decrease with the number of children (and therefore on the head's utility) since this contribution does not cover child cost $\theta$. On the contrary, children in a household which does not face an adverse shock provide at the optimum high contributions which cover their cost.

The utility level under the optimal allocation is

$$
\begin{aligned}
H^{*}\left(w_{p}, N\right) & =\gamma \ln \left(C^{*}\right)+\delta \ln \left(D^{*}\right)+\epsilon \ln \left(E^{*}\right) \\
& =K\left(w_{p}\right)+\ln \left(Y\left(w_{p}, N\right)\right),
\end{aligned}
$$

where $K\left(w_{p}\right)=\gamma \ln (\gamma)+\delta \ln \left(\frac{\delta}{w_{p}}\right)+\epsilon \ln \left(\frac{\epsilon(Q-q)}{\alpha w_{p}}\right)$. Let us now turn to the fertility decision in period 1 , before shocks are realized. The objective is to maximize $E H^{*}=p H^{*}(w, N)+$ $(1-p) H^{*}(w-\Delta, N)$ with respect to $N$, which provides the following first order condition:

$$
\begin{aligned}
\frac{\partial E H^{*}}{\partial N}= & p \frac{\mu \alpha w-\theta}{\left(w+N(\mu \alpha w-\theta)+\frac{\left(w_{c}-\alpha w\right)^{2}}{4 s_{l}}\right)} \\
& +(1-p) \frac{\mu \alpha(w-\Delta)-\theta}{\left((w-\Delta)+N(\mu \alpha(w-\Delta)-\theta)+\frac{\left(w_{c}-\alpha(w-\Delta)\right)^{2}}{4 s_{l}}\right)} \\
= & 0
\end{aligned}
$$

[^4]The level of $N$ that maximizes expected utility is therefore: ${ }^{8}$

$$
N^{*}=p \frac{\left((w-\Delta)+\frac{\left(w_{c}-\alpha(w-\Delta)\right)^{2}}{4 s_{l}}\right)}{(\theta-\alpha \mu(w-\Delta))}-(1-p) \frac{\left(w+\frac{\left(w_{c}-\alpha w\right)^{2}}{4 s_{l}}\right)}{(\alpha \mu w-\theta)} .
$$

### 2.2 Comparative Statics

We start the comparative statics analysis by studying the first stage decision. More precisely, we study first the impact of the various parameters of the distribution of adult income - namely $p, \Delta$ and $w$ - on fertility. Isolating $p$, one can rewrite $N^{*}$ as

$$
\begin{aligned}
N^{*}= & p\left(\frac{\left((w-\Delta)+\frac{\left(w_{c}-\alpha(w-\Delta)\right)^{2}}{4 s_{l}}\right)}{(\theta-\alpha \mu(w-\Delta))}+\frac{\left(w+\frac{\left(w_{c}-\alpha w\right)^{2}}{4 s_{l}}\right)}{(\alpha \mu w-\theta)}\right) \\
& -\frac{\left(w+\frac{\left(w_{c}-\alpha w\right)^{2}}{4 s_{l}}\right)}{(\alpha \mu w-\theta)}
\end{aligned}
$$

A higher $p$ decreases the optimal number of children for two reasons. On the one hand, the potential decrease in $Y$ due to fertility when an adverse shock occurs is less likely and on ther other hand, the potential increase in $Y$ due to fertility when an adverse shock does not occur is is more likely. Note that there exist values of shock probability such that fertility is at a corner (equal to zero). ${ }^{9}$ Let us study the impact of the adverse shock's magnitude on fertility.

$$
\begin{aligned}
\frac{\partial N^{*}}{\partial \Delta} & =-\frac{p}{(\theta-\alpha \mu(w-\Delta))^{2}}\left(\alpha \mu\left(\frac{\left(w_{c}^{2}-\alpha^{2}(w-\Delta)^{2}\right)}{4 s_{l}}-\frac{\left(w_{c}-\alpha(w-\Delta)\right)}{2 s_{l}} \frac{\theta}{\mu}\right)+\theta\right) \\
& =-\frac{p}{(\theta-\alpha \mu(w-\Delta))^{2}}\left(\alpha \mu\left(n_{l}^{*} w_{c}-n_{l}^{* 2} s_{l}-n_{l}^{*} \frac{\theta}{\mu}\right)+\theta\right)
\end{aligned}
$$

This derivative will be negative under the sufficient condition that in the presence of an adverse shock, the income generated by the optimal number of working children, $n_{l}^{*} w_{c}-n_{l}^{* 2} s_{l}$, is superior to its quality-adjusted cost $n_{l}^{*} \frac{\theta}{\mu}{ }^{10}$ Because of the convexity of supervision costs, this very natural condition imposes an upper bound on the number of

$$
\begin{aligned}
& { }^{8} \text { The second order condition is valid: } \\
& \qquad \begin{aligned}
\frac{\partial^{2} H_{*}}{\partial N^{2}} & =-p \frac{(\theta-\alpha \mu w)^{2}}{Y(w, N)^{2}}-(1-p) \frac{(\theta-\alpha \mu(w-\Delta))^{2}}{Y(w-\Delta, N)^{2}} \\
& <0
\end{aligned}
\end{aligned}
$$

${ }^{9}$ Indeed, fertility is at a corner if the probability of the good state of the world $p$ is smaller than $\left.p_{0} \in\right] 0 ; 1[$ where

$$
p_{0} \equiv \frac{w+\frac{\left(w_{c}-\alpha w\right)^{2}}{4 s_{l}}}{\left(\frac{(\alpha \mu w-\theta)}{(\theta-\alpha \mu(w-\Delta))}\left((w-\Delta)+\frac{\left(w_{c}-\alpha(w-\Delta)\right)^{2}}{4 s_{l}}\right)+\left(w+\frac{\left(w_{c}-\alpha w\right)^{2}}{4 s_{l}}\right)\right)} .
$$

[^5]working children. This upper bound, $\frac{w_{c}-\theta}{s_{l}}$, is larger than $n_{l}^{*}$ if and only if the condition stated in Assumption1 is satisfied.

Assumption 1: $0<\theta-\mu \alpha(w-\Delta)<\mu w_{c}-\theta$.
Assumption 1 entails two components. First, it states that $w_{c}>\frac{\theta}{\mu}$, which is a rather minor assumption: the marginal productivity of child labor when $n_{l}$ tends to zero (i.e. the highest marginal productivity level) is likely to be higher that the cost of raising a child. Second, Assumption 1 relates to Assumption 0 regarding the domain of children's marginal net contribution in the bad state of the world, $\theta-\alpha \mu(w-\Delta)$. While assumption 0 imposes an upper bound on the latter (i.e. 0), Assumption 1 imposes a lower bound, namely $-\left(\mu w_{c}-\theta\right)$. This upper bound means that the marginal cost of the last working child (net of his/her contribution) in the bad state of the world cannot exceed the net marginal benefit of child labor when $n_{l}$ tends to zero.

Finally, we study the impact of adult labor productivity $w$ on fertility:

$$
\begin{aligned}
\frac{\partial N}{\partial w}= & \frac{p}{(\theta-\alpha \mu(w-\Delta))^{2}}\left(\frac{\left(\alpha\left(w_{c}-\alpha(w-\Delta)\right)\left(\left(\mu w_{c}-\theta\right)-(\theta-\alpha \mu(w-\Delta))\right)\right)}{4 s_{l}}+\theta\right) \\
& +\frac{1-p}{(\theta-w \alpha \mu)^{2}}\left(\frac{\alpha\left(w_{c}-w \alpha\right)\left(\mu w_{c}-\theta+w \alpha \mu-\theta\right)}{4 s_{l}}+\theta\right) .
\end{aligned}
$$

Assumptions 0 and 1 ensure that both the first and second terms are positive. We summarize the first part of this comparative statics analysis by summarizing its results in Proposition 1:

Proposition 1. In the first stage, the household head's decision on the number of children is negatively affected by the probability of facing an adverse shock. Under the sufficient but not necessary condition stated in Assumption 1, this decision is also negatively affected by the magnitude of such a potential shock and positively affected by the head's labor productivity $w$.

The second part of this comparative statics exercise is to study what are the impacts of the first two stages - fertility decisions and the occurence of shocks - on $n_{l}, n_{d}, n_{e}$ and $d$. We start by presenting the impact of fertility decisions in Proposition 2.

Proposition 2. In the third stage, the number of domestic children is positively affected by fertility, while the opposite stands for parental supply of domestic labor. The number of school children is likely to increase with fertility if parental labor income is large and/or the cost of raising children is low. The impact of fertility on schooling will also be larger if the utility derived from having an (un)educated child is high (low).

While the effects of fertility on $n_{l}, n_{d}$ and $d$ are straightforward, we present here the effect of $N$ on $n_{e}$ :

$$
\frac{\partial n_{e}}{\partial N}=\epsilon \frac{\alpha w_{p}-\theta}{\alpha w_{p}}+(1-\epsilon)(1-\mu)
$$

The effects of all parameters cited in Proposition 2 are self-explanatory, except the welfare derived from having children. The welfare attached to an uneducated child, $q$, has a positive impact on $\mu$. Also, if $q$ equals 0 , or the utility derived from having an educated child $(Q)$ tends to infinity, $\mu$ tends to unity, so that $\frac{\partial n_{e}}{\partial N}=\epsilon \frac{\alpha w_{p}-\theta}{\alpha w_{p}}$. This quantity is, under

Assumption 0, positive in the good state of the world, and negative if an adverse shock occurs.

Let us finish our comparative statics analysis by studying the role of adverse shocks. Firstly, it is straightforward to show that the number of working children is larger if a shock occurs than otherwise:

$$
\begin{aligned}
n_{l}(w-\Delta)-n_{l}(w) & =\frac{\alpha \Delta}{2 s_{l}} \\
& >0
\end{aligned}
$$

A negative shock on adult labor productivity makes child labor more attractive. Larger shocks, or lower supervision costs, or larger child domestic productivity strengthen this result. Also note that, unlike the other comparative statics we are going to study below, the impact of the shock on child labor does not depend on the total number of children. We now study the effect of the shock on the number of educated children.

$$
n_{e}(w-\Delta)-n_{e}(w)=-\frac{\epsilon \Delta}{\alpha w(w-\Delta)}\left(N \theta-\frac{w_{c}^{2}-\alpha^{2} w(w-\Delta)}{4 s_{l}}\right)
$$

Loosely speaking, the number of school children will be lower in the event of a shock if the total cost of children $N \theta$ is larger than the "average" income generated by child labor (net of supervision costs) $\frac{w_{c}^{2}-\alpha^{2} w(w-\Delta)}{4 s_{l}}$. More precisely, $n_{e}$ will be lower in the event of a shock if and only if $N$ is larger than the threshold $N_{e} \equiv \frac{w_{c}^{2}-\alpha^{2} w(w-\Delta)}{4 s_{l} \theta}$. Let us now substitute $N^{*}$ for $N$ in order to obtain a condition that only depends on parameters. Then, the third ratio in the previous equation, $N \theta-\frac{w_{c}^{2}-\alpha^{2} w(w-\Delta)}{4 s_{l}}$ equals

$$
\frac{w(\theta-\alpha \mu(w-\Delta))-p \theta \Delta}{(\theta-w \alpha \mu)(\theta-w \alpha \mu+\Delta \alpha \mu)}\left(\frac{\alpha\left(\left(w_{c}-\alpha w\right)\left(\frac{\mu\left(w_{c}+\alpha w\right)}{2}-\theta\right)+\Delta \alpha(w \alpha \mu-\theta)\right)}{2 s_{l}}+\theta\right)
$$

Except for the first numerator, $w(\theta-\alpha \mu(w-\Delta))-p \theta \Delta$, all terms of the right hand side are positive. The latter is positive if and only if $p$ is smaller than $p_{e} \equiv \frac{w(\theta-\alpha \mu(w-\Delta))}{\theta \Delta} \in$ $] 0 ; 1\left[.^{11}\right.$ This is in line with our previous interpretation (i.e. that $N$ should be small enough) since we know from Proposition 1 that fertility is increasing with $p$. If this is the case, the effect of facing an adverse shock on the number of educated children is negative:

$$
\left.n_{e}(w-\Delta)-n_{e}(w)<0 \Leftrightarrow p<p_{e} \equiv \frac{w(\theta-\alpha \mu(w-\Delta))}{\theta \Delta} \in\right] 0 ; 1[
$$

It will show clearer to interpret this result after the shock's effects on all variables have been studied. Let us thus proceed with $n_{d}$. Since $N$ is fixed at this stage, the effect of a shock on the number of domestic children is simply the opposite of the sum of the effects on $n_{d}$ and $n_{e}$.

$$
n_{d}(w-\Delta)-n_{d}(w)=\frac{\epsilon \Delta}{\alpha w(w-\Delta)}\left(N \theta-\frac{w_{c}^{2}+\left(\frac{2}{\epsilon}-1\right) \alpha^{2} w(w-\Delta)}{4 s_{l}}\right)
$$

The number of domestic children is likely to be higher in case of a shock if the total

[^6]number of children is larger than $N_{d} \equiv \frac{w_{c}^{2}+\left(\frac{2}{\epsilon}-1\right) \alpha^{2} w(w-\Delta)}{4 s_{l} \theta}>N_{e}$. Therefore, if $N>N_{d}$, a shock results in a larger $n_{d}$ and a lower $n_{e}$. If $N \in\left[N_{e} ; N_{d}[\right.$, a shock still results in a lower $n_{e}$, but results in a lower $n_{d}$ as well. Finally, if $N<N_{e}$, a shock results in lower $n_{d}$ and even a larger $n_{e}$. Note that unlike the comparative statics on $n_{l}$ and $n_{e}$, the sign of the shock here depends on a preference parameter: the smaller the descending altruism parameter $\epsilon$, the more likely will there be a decrease in $n_{d}$ if a shock occurs. Parents who care relatively more about their children's education are more likely to substitute child labor for domestic labor. The impact of the shock on the adult allocation of time, which we study now, also depends on a preference parameter, i.e. $\gamma$ :
$$
d(w-\Delta)-d(w)=\frac{\Delta}{w(w-\Delta)}\left(\frac{\alpha^{2} w(w-\Delta)}{2 s_{l}}-(1-\gamma)\left(N \theta-\frac{w_{c}^{2}-\alpha^{2} w(w-\Delta)}{4 s_{l}}\right)\right) .
$$

A decrease in adult labor productivity has two types of effects on the allocation of adult time. On the one hand, this shock creates a negative income effect, which may be overcome by increasing adult paid labor supply. On the other hand, this shock affects relative productivities both between agents and between activities. As we have seen above, this substitution effect translates into an increase in child labor. Also, adult domestic work becomes relatively more productive, which may lead to an increase in $d$. The sign of the net effect on $d$ depends mainly on two elements. First, it depends negatively on $N$. More precisely, under the sufficient but not necessary condition that $N<N_{e}$, the total cost of children has a small impact on $C$, the negative income effect is thus dominated and the household head will decrease her paid labor supply in order to maintain a sufficient level of the consumption good. This condition is however not necessary, and larger values of $N$ may be compatible with an increase in adult domestic work. This is more likely to be the case if the preference parameter for domestic good consumption $\gamma$ is high. However, it can be shown that for $N \geq N_{d}$, the occurence of the shock leads to a decrease in adult domestics labor. ${ }^{12}$

We summarize all the effects of an adverse shock on the stage 3 decision variables in Proposition 3.

[^7]Proposition 3. If an adverse shock occurs in stage 2, the household head will allocate more children to paid labor than she would have in the absence of such a shock. This increase in child paid labor does not depend on the total number of children, unlike the other variables of interest for which the following structure applies:

- If the total number of children is sufficiently small $\left(N<N_{e}\right)$, the shock induces the household head to increase her supply of domestic labor and to decrease the number of domestic children. This decrease in child domestic labor more than compensates the increase in child paid labor. Consequently, the number of school children increases.
- If the total number of children is intermediate ( $N \in\left[N_{e} ; N_{d}[\right.$ ), the shock induces the household head to compensate the increase in child paid labor by decreases in both school and domestic labor. The variation in adult domestic labor supply depends positively on preferences towards domestic good consumption.
- If the total number of children is large ( $N \geq N_{d}$ ), the shock induces the household head to decrease her supply of domestic labor and to increase the number of domestic children. Consequently, both increases in child paid and domestic labor are to be compensated by a decrease in the number of school children.


## 3 Data and main variables

Our empirical analysis is based on the data set from the Senegalese Survey on Education and Household Wellbeing. This survey was carried out in 2003 by the DPS (Department of Planning and Statistics) of the Republic of Senegal in collaboration with the Ministry of Education (Senegal), Cornell University (USA), CREA (University of Dakar) and INRA (France). The survey comprises three series of questionnaires: household-, community(village and city) and school-level. The household questionnaire consists of ten different books collecting information about household and individual characteristics, education, health, dwelling, migration, employment, transfers, expenses and assets. The community questionnaire collects information on the characteristics (infrastructures, availability of school facilities, social and economic organization, economy, etc) of the villages and cities where the surveyed populations live. The school component of the survey contains information on school facilities such as the setting characteristics, the number of teacher, the teachers' level, etc.

The survey includes 1,811 households containing 19,137 individuals. After exclusion for age, sex, and other inconsistencies, our estimation sample consists of 1,801 households comprising 19,017 individuals. Our population study covers all children aged 6-18 years.

The dataset contains information on time use for household chores (cooking, cleaning, caring younger siblings, old-people or sick persons, fetching water or firewood, etc) and homework for all children aged 6-18 years. However, it does not include information on time use for class attendance or for market work. Therefore, we use binary variables to capture child participation to each occupation. We consider three main occupations: household chores, market work, and schooling. We consider that a child is involve in household chores if he/she reports spending more than two hours per day performing domestic tasks. Note that, for consistency reason, our sample excludes all cases reporting spending more than seventy hours per week doing household chores. We define child labor as any productive activity (paid or non-paid) performed within (in the family farm or the
family business) or outside the household. Finally, a child is considered at school if he/she reports attending a class in a formal (public or private) or a community school at least 3 days a week. Classes range from first year of primary school to the last one of senior secondary school.

We provide in Table 1 information about the proportions of children involved in all categories, including combinations of occupations and absence of occupation. In the sample, most children are specialized in one activity. The first category of specialization is 'going to school'. For this category, we do not notice any gender gap between boys and girls. However, the sample reveals a clear decrease in school enrollment as children age. Of the children not attending school in the sample, most fall in the category 'domestic chores only'. In this category, a clear gender pattern appears, as children grow old; among children of age 15 and above, 30 percent of the girls are specialized in doing domestic chores versus 23 percent for boys. Few proportions of children combine two or three activities. The main type of activity combination is the association school-market work (around 5 percent of the children). Conversely, a sizeable proportion of children report not being involved in any activity (around 10 percent of the children within the household). This is not unexpected since most survey data show that a substantial number of children neither attend school nor work at home or in the labor market (Cigno \& Rosati (2005)). These children are idle partly because of under-reporting of child labor market and domestic chores. However, Deb \& Rosati (2002) show that these children are clearly different from the other children who go to school, perform domestic chores and/or work.

Table No 1: Proportion of children in different activities (average per household, \%)

|  | Age categories and gender |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Activities | $6-18$ |  | $6-10$ |  | $11-14$ | $15-18$ |  |  |
|  | Boys | Girls | Boys | Girls | Boys | Girls | Boys | Girls |
|  | 37.3 | 37.9 | 38.5 | 39.6 | 40.0 | 39.6 | 34.0 | 34.5 |
| Domestic chores only | 22.0 | 25.1 | 21.7 | 21.6 | 20.6 | 23.4 | 23.5 | 30.1 |
| Market work only | 15.9 | 13.7 | 14.8 | 14.4 | 14.3 | 13.2 | 18.2 | 13.7 |
| School and domestic chores | 2.9 | 3.4 | 3.5 | 3.1 | 3.1 | 4.4 | 2.3 | 2.6 |
| Market work and domestic chores | 2.9 | 3.6 | 3.1 | 3.1 | 3.2 | 3.2 | 2.5 | 4.6 |
| Market work and school | 5.2 | 4.6 | 5.7 | 5.9 | 5.6 | 4.6 | 4.5 | 3.2 |
| All activities | 0.9 | 1.0 | 1.0 | 1.0 | 0.8 | 1.0 | 0.8 | 1.0 |
| No occupation | 12.9 | 10.7 | 11.7 | 11.2 | 12.5 | 10.7 | 14.2 | 10.3 |

The explanatory variables used in the estimation consist in demographic and socioeconomic characteristics of the household and neighborhood variables. The householdlevel variables include:

- Number of children aged from 0 to 20 years old;
- The head of household current age and his 'age at the birth of his first child'; we introduce these age variables in order to control for cohort effects in fertility and for non-completed fertility. We use 'head of household age at his first child' as an instrument to estimate the number of children. We assume that this variable directly affects the number of children in the household but does not affect children's time-use.
- The gender-ratio within the household.

The socio-economic variables include:

- Adult literacy rate within the household: we capture it using the proportions of adult males and females who completed junior secondary school. These variables allow us to capture the earning capacity of adults living in the households as well as preferences for education.
- Household asset index to measure the possessions of the household. We use this index as a proxy of ex ante parental income and wealth. Unfortunalety, the survey did not include questions on expenditures or income. We generate this index using multiple correspondence analysis (MCA). We use binary indicators on household level assets (the presence or absence of TV, fridge, radio-tape, fan, and furniture), and categorical indicators on two variables (types of cooking and lighting energies). We excluded any productive asset to avoid correlation with children's activities.
- Household land possession: this variable captures the surface of land for agriculture owned by the household. We compute this variable using the surface of land measured in ares ( 100 squares meters).
- Business ownership: a dummy variable indicates whether owns a non-agricultural business or not.

The last set of variables we use regards the household neighborhood. This comprises:

- Distance to source of water,
- Number of primary school and the number of secondary school, so as to control for access to schooling facilities within the neighborhood,
- Average level of schooling fees in primary and secondary school, so as to control for the local education policy,
- Shocks: this is the most important element in our analysis, we account for shocks that each household has ever experienced. These shocks include crop shocks, business lost, and unemployment. We split the shocks in two categories. The first comprises old shocks viz shocks that have happened at least five years ago while the second category includes shocks that occurred one or two years before the survey. Using these 'old shocks', we generate for each household the probability of facing adverse shocks. Afterward, we use this probability as an instrument to estimate the number of children that adults in the household have decided to have (just to remind that this decision takes place in the first period). The second category of shocks (recent shocks) will be used to estimate parents' decision of children's time-use.

Summary statistics for the variables used are presented in Table 2.

| Variables | Mean | Std. Dev. |
| :---: | :---: | :---: |
| Children per household and category |  |  |
| School only | 2.03 | 1.97 |
| Domestic chores only | 1.22 | 1.34 |
| market work only | 0.79 | 1.14 |
| School and domestic chores | 0.17 | 0.61 |
| Market work and domestic chore | 0.17 | 0.49 |
| Market work and school | 0.26 | 0.76 |
| All activities | 0.06 | 0.39 |
| No oocupation | 0.65 | 1.06 |
| Children 6-18 per household | 5.36 | 2.93 |
| Children 0-20 per household | 6.41 | 3.57 |
| Number female adults | 2.93 | 1.69 |
| Number of male adults | 2.11 | 1.54 |
| Household size | 13.03 | 6.04 |
| Female adult literacy (\%) | 19.71 | 52.44 |
| Male adult literacy (\%) | 34.59 | 68.62 |
| Age of head of household | 54.10 | 12.48 |
| Head of household age at first child (male) | 34.60 | 8.59 |
| Head of household age at first child (female) | 24.65 | 6.00 |
| Agricultural land (surface in ares) | 531.15 | 734.52 |
| Number of primary school per district | 1.75 | 0.91 |
| Number of secondary school per district | 0.68 | 0.77 |

## 4 Estimation strategy

First, a Poisson model is used to estimate the number of children with information on the household's ex ante distribution of income and standard instruments. The ex ante distribution is captured with proxies of parental permanent income and probability of facing a shock. Permanent income is based on the wealth index and parental education. The probability of shocks is obtained with the fitted values of a probit model that estimates the occurence of shocks on the whole set of household, neighborhood and area characteristics, except family size and related variables. We control for the number of children using as instruments shocks that have affected households during the last decade and the age of the household head at his first child. While these instruments may have an impact on children's occupations, credible alternative instruments are unfortunately not available from the dataset. Preliminary results suggest that the predictions of Proposition 1 are validated by the data (see other files).

Second, a multivariate Tobit model estimates the determinants of children occupations including the occurence of shocks. Three categories of activities are considered for children's occupation: school attendance, income-generating work (market work), and housework. The literature usually assumes these activities as mutually exclusive. The reality, however, is that children may use their time in different combinations of activities. This paper estimates children's occupation considering also combined activities. To estimate our model, let $I_{i j}^{*}$ be the proportion of children doing activity $i$ in household $j$;
$i=1, \ldots m$, $m$ being the number of activities.The reduced form equations representing activities are assumed to take the form:

$$
\begin{gather*}
I_{i j}^{*}=\alpha n_{j}+X_{j}^{\prime} \beta+\epsilon_{i j}  \tag{4}\\
\text { where }: n_{j}=Z_{j} \gamma+X_{j}^{\prime} \varphi+\mu_{j}
\end{gather*}
$$

where $n_{j}$ captures the number of children of schooling age in household $j, X$ is a vector of explanatory variables and $\epsilon_{i j}$ an error term assumed to be distributed with mean 0 and variance $\sigma_{i j}^{2}$. $Z_{j}$ corresponds to the instruments controling for the endogeneity of $n_{j}$ and $\mu_{j}$ is the error term of the equation representing the number of children in household $j$.

The endogeneity of $n_{j}$ is addressed by adapting the two-step procedure proposed by Smith \& Blundell (1986). The first step, where $n_{j}$ is regressed on the set of exogenous explanatory variables $X_{j}$, and the set of instruments $Z_{j}$ has been described above. At the second stage, the residuals $\widehat{\mu}$ from the first stage are included as an additional regressor in each equation of activity as follows:

$$
\begin{equation*}
I_{i j}^{*}=\alpha n_{j}+\tau \widehat{\mu}+X_{j}^{\prime} \beta+\epsilon_{i j} \tag{5}
\end{equation*}
$$

Note that our dependent variable corresponds to the proportion of children in a given activity or combination of activities. Therefore, the system of equations in (1) implies that for the $i t h$ equation, the single dependent variable $I_{i j}^{*}$ is observed with non-negative values. Next, consider that the $i t h$ equation, $I_{i j}$ is determined as follows:

$$
\begin{equation*}
I_{i j}=\max \left(I_{i j}^{*}, 0\right) \tag{6}
\end{equation*}
$$

Equation (6) indicates that the proportion of children doing activity $i$ possesses mixed discrete-continuous distribution. For each activity, we observe either the discrete outcome $I_{i j}^{*}=0$ or the continuous outcome $I_{i j}^{*}>0$. The discrete outcome occurs when households choose for their children the specialization in activity $k,(k \neq i)$. Hence, for the equation $I_{i j}$ non-negligible proportions of its values may be identically zero. Equations (5) and (6) combined constitute a Tobit or censored model, where the dependent variable $I_{i j}$ is censored at zero. When there is no correlation between decisions to be allocated to each occupation, equations (5) and (6) can be specified and estimated using a single equation Tobit model. However, when decisions are determined by the same process, a single equation approach would fail to capture the interactions effects across equations representing activities. In that case, equations (5) and (6) should be considered as a multivariate Tobit model.

The multivariate Tobit model allows accommodating cross-equation restrictions using joint estimation of the equations. In what follows, we use this approach to investigate the determinants of children's activities. As far as the authors know, this is the first paper using this estimation strategy to study children's occupation. The multivariate Tobit system has been mainly used to estimate systems of demand equations where some consumers choose not to buy several the goods or systems of input demand and supply equations where firms choose not to produce several of the outputs or not to use several of the inputs.

Considering equations (5) and (6) as a multivariate Tobit model, we can rewrite the
system as follows:

$$
\begin{align*}
\mathbf{I}_{j}^{*} & =\mathbf{f}\left(\mathbf{n}_{j}, \mathbf{X}_{j}, \widehat{\boldsymbol{\mu}}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\tau}\right)+\epsilon_{j}  \tag{7}\\
I_{i j} & =\max \left(I_{i j}^{*}, 0\right) \tag{8}
\end{align*}
$$

where $\mathbf{I}_{j}^{*}=\left(I_{1 j}^{*}, \ldots, I_{m j}^{*}\right)^{\prime}, \epsilon_{j}=\left(\epsilon_{1 j}, \ldots, \epsilon_{m j}\right)^{\prime} \mathcal{N}(0, \Sigma)\left(\epsilon_{j}\right.$ is distributed multivariate normal with mean zero and $(m \times m)$ covariance matrix $\Sigma)$, and $\epsilon_{j}$ are iid across households. ${ }^{13}$

In the system above, there are $m$ equations of activities and a sample data of $N$ observations $(j=1, \ldots ., N)$.The parameter estimates in equations (7) and (8) can be obtained by maximizing the likelihood function of the sample. Since there are $m$ dependent variables, there would be $2^{m}$ possible combinations of observations at their censoring points. Let $C^{r}$ be a censoring regime with values equal to zero for the censored equations and one for the non-censored equations, $r=1,2, \ldots, 2^{m}$.

The sample likelihood function which accounts for all censoring regimes of all observations is:

$$
\begin{equation*}
L=\prod_{j=1}^{N} L_{j}^{C^{r}}\left(I_{j}, \Sigma\right) \tag{9}
\end{equation*}
$$

where $L_{j}^{C^{r}}$ gives the likelihood of the case when the $j t h$ observation falls into regime $r$ (for further details see Barslund (2007)).

Evaluating the likelihood function in (9) may be intractable, especially when the number of dependent variables, $m$, is large. In other words, it may be difficult to calculate the expression (9) since it involves high dimensional integrals. The solution consists therefore of simulating rather than calculating these integrals using probability simualtion methods. These methods are based on the fact that the integral of interest represents the probability of an event in the popualtion.

There are different probability simulations methods, but the Maximum Simulated Likelihood (MSL) appears to outperform all others. The only problem is that it is computationally time consuming. The maximum simulated likelihood is a conceptually simple extension of the maximum likelihood estimation. However, instead of gathering the loglikelihood through analytical or numerical methods, the log-likelihood is simulated and then maximized to obtain maximum simulated estimators of the model parameters.

In our estimations we use this method exploiting the mvtobit STATA procedure developed by Barslund (2007).

## 5 Results

As indicated above, we first estimate the parameters underlying the number of children using Poisson, afterward we investigate the determinants of children's occupations. We begin by analyzing the estimates from the 'number of children' equation. Next, we describe the estimates of the determinants of children's occupations.

[^8]
### 5.1 Number of children

Table 3 reports the Poisson estimates for the Number of Children 0-20 years old. Results likely support the predictions of Proposition 1. Except for crops, the higher the exposure to shocks, fewer is the number of children in the household. However, contrary to a priori expectations, the exposure to crops shock is not statistically significant. The negative sign of the business and unemployment probability shocks suggests that non-agricultural households account for the potential shocks they face when deciding of the number of children to have. In contrast, shocks are less likely source of concern in agricultural households' decisions of the number of children to have.

Results also indicate progressive effect of wealth on the number of children; richer households seem to have more children. The estimates for the wealth quintiles are $11 \%$, $12 \%, 13 \%, 14 \%$ for the second to the fifth quintile, respectively. Conversely, large landholdings increase the number of children in the household.

We also find that households with more educated women have fewer children. However, the coefficient on the number of the literated adult male is not statistically significant.

Table No 3: Poisson estimates for Number of Children (0-20)

| Explanatory variables | Coefficients | Std. Error |
| :--- | :--- | :---: |
| Male literacy | -0.049 | $(0.034)$ |
| Female literacy | $-0.212^{* * *}$ | $(0.059)$ |
| Wealth index, 2nd quintile | $0.108^{* *}$ | $(0.042)$ |
| Wealth index, 3rd quintile | $0.121^{* * *}$ | $(0.039)$ |
| Wealth index, 4th quintile | $0.128^{* * *}$ | $(0.034)$ |
| Wealth index, 5th quintile | $0.141^{* * *}$ | $(0.033)$ |
| Business shock probability | $-0.773^{* *}$ | $(0.325)$ |
| Unemployment probability | $-0.594^{* *}$ | $(0.278)$ |
| Crops'shocks probability | 0.029 | $(0.115)$ |
| Agricultural land surface | $0.011^{* * *}$ | $(0.002)$ |
| Land surface x Crops' shocks | 0.006 | $(0.009)$ |
| Head of household age at first child | $-0.009^{* * *}$ | $(0.001)$ |
| Head of household, male | $0.251^{* * *}$ | $(0.035)$ |
| Constant | $1.886^{* * *}$ | $(0.055)$ |
| Log_pseudolikelihood |  | -3664.110 |
| Chi2 |  | 269.3723 |
| $*$ p<0.1, $* *$ p $<0.05, * * *$ p $<0.01$ |  |  |

In addition, where the head of household starts tardy his childbearing, the number of children is less important.

### 5.2 Children Occupations

Tables 4 and 5 present the results from the multivariate Tobit regressions. The model specification allows for the existence of covariance effects acrosss equations representing differents occupations. We investigate the existence of these interactions effects using a Wald test of the restrictions that ( $\rho_{21}=\rho_{31}=\rho_{41}=\rho_{32}=\rho_{42}=\rho_{43}=0$ ). Results in both
tables show that the Wald test of the restrictions that all $\rho_{j k}=0$ yields a test statistic of 438.913 and 461.326 (respectively) which is asymptotically distributed as chi-square with six degrees of freedom $(p=0.000)$. This indicates strong evidence of significant contemporaneous cross-effect between the differents time-use options for children within a household. Therefore, we conclude that the system-wide estimator provides an efficiency gain over univariate specifications of the tobit models. We calculate the standard errors using a robust covariance estimation procedure. Note that our estimation approach is computationally very demanding. Therefore, for sake of efficiency, we restrict our estimations to single activities (chores, work, school, or idle), using combinations of activities as a reference.

Table 4 reports estimates of the augmented multivariate Tobit model for children's time-use following the approach of Smith \& Blundell (1986). Results clearly show the positive effect on children specialization that arises from the presence of large number of children within a household. Household with more children have larger proportion of children going exclusively to school and more children performing domestic chores or working for pay. These results are coherent with Gomes' findings and the predictions of our theoretical model.

In contrast, findings are seemingly not supporting our theoretical predictions regarding the effects of adverse shocks on children's occupations. Except for schooling, the coefficients of shocks-related variables are not statistically significant in any of the occupation equation. However, shocks' impact on schooling activity appears consistent with the model's prediction. The coefficient of the interaction term (shock x Number of children) in the equation for schooling is statistically significant and negative. Household with large number of children are likely to reduce the number of children attending school to cope with adverse shocks.

The other results are consistent with most of the results from the literature. Household with more educated adults male have more children attending school and fewer working children. It is however noteworthy that the coefficient on the female adult literacy is not significant in the schooling equation. Other studies support the contrary. The existence of school facilities in household's neighborhood displays significant and negative impact on paid work activities for children. In contrast, it favors school attendance.

In the augmented multivariate Tobit model, we account for the endogeneity of the number of children 0-20 years oldusing Smith-Blundell procedure that is analogous to the Rivers \& Vuong (1988) method for binary model. We estimate the number of children using as intruments shocks that have affected households during the last decade and the age of the household head at his first child.

To test for the potential endogeneity of the number of children, the multivariate Tobit model is also estimated by excluding the augmented residuals in all equations. This is done in order to investigate the extent of bias introduced in the estimates when potentially endogenous variables are included in the regressions. Results are given in Table 5. It shows indicate that there are no significant changes in any of the estimated coefficients. Thus, we can conclude that accounting for the endogeneity of the number of children is trivial.

Table No 4: Augmented multivariate Tobit estimates of number of children per category of occupation

| Explanatory variables | Chores |  | Work |  | School |  | No activity |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coeff. | Std. Err. | Coeff. | Std. Err. | Coeff. | Std. Err. | Coeff. | Std. Err. |
| Number of children $0-20$ years old | . $20^{* * *}$ | . 03 | $.13^{* * *}$ | . 03 | $.35^{* * *}$ | . 02 | . $17^{* * *}$ | . 03 |
| Fitted residuals | $-.00^{* * *}$ | . 00 | .01* | . 01 | -. 00 | . 00 | -. 00 | . 00 |
| Shocks | -. 03 | . 22 | -. 12 | . 28 | . 63 ** | . 25 | -. 48 | . 29 |
| Number of children $\times$ Shocks | -. 04 | . 03 | . 01 | . 03 | $-.09 * * *$ | . 03 | . 03 | . 04 |
| Fitted residual x Shocks | -. 00 | 00 | -. 01 | . 01 | . 00 | . 00 | -. 00 | . 00 |
| Male literacy | . 17 | . 13 | $-1.04^{* * *}$ | . 20 | . 49 *** | . 14 | . $45^{* *}$ | . 18 |
| Female literacy | . 31 | . 21 | -. 72 ** | . 30 | . 17 | . 21 | . 51 * | . 29 |
| Wealth index | . $23^{* * *}$ | . 08 | -. 02 | . 09 | . 12 | . 09 | . 14 | . 10 |
| Agricultural land surface | $-.03^{* *}$ | . 01 | . 01 | . 01 | -. 01 | . 01 | -. 01 | . 01 |
| Head of household, male | . 10 | . 12 | . $25^{*}$ | . 15 | -. 01 | . 14 | -. 17 | . 18 |
| Average age of children aged 0-20 | . 02 | . 02 | .05* | . 03 | $-.21^{* * *}$ | . 02 | -. 00 | . 03 |
| Girls ratio among 0-20 | . $711^{* * *}$ | . 14 | $-.75{ }^{* * *}$ | . 16 | . 03 | . 16 | $-.77^{* * *}$ | . 20 |
| Proportion of female among adults | . 26 | . 23 | -. 05 | . 26 | -. 19 | . 26 | -. 11 | . 34 |
| Multilevel schools | -. 13 | . 09 | . 02 | . 11 | . 26 ** | . 11 | -. 05 | . 12 |
| Number of primary school per district | . 03 | . 05 | $-.20^{* * *}$ | . 07 | -. 01 | . 05 | -. 02 | . 07 |
| Secondary schools per district | -. 04 | . 07 | $-.24^{* * *}$ | . 08 | . $16^{* *}$ | . 07 | . 11 | . 08 |
| Constant | $-1.41^{* * *}$ | . 46 | -. 59 | . 51 | $1.72{ }^{* * *}$ | . 47 | $-1.22^{* *}$ | . 60 |
| Number of observations | 1378 |  |  |  |  |  |  |  |
| Log_pseudolikelihood | -7233.79 |  |  |  |  |  |  |  |
| Wald Chi2(64) | 7452.15 |  |  |  |  |  |  |  |
| Likelihood ratio test of rho $21=$ rho31 $=$ rho $41=$ rho $32=$ rho42 $=$ rho $43=0: \operatorname{chi} 2(6)=438.913$ Prob $>\operatorname{chi} 2=0.0000$ |  |  |  |  |  |  |  |  |

* $\mathrm{p}<0.1,{ }^{* *} \mathrm{p}<0.05,{ }^{* * *} \mathrm{p}<0.01$

Table No 5: Multivariate Tobit estimates of number of children per category of occupation

| Explanatory variables | Chores |  | Work |  | School |  | No activity |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coeff. | Std. Err. | Coeff. | Std. Err. | Coeff. | Std. Err. | Coeff. | Std. Err. |
| Number of children $0-20$ years old | . 20 *** | . 02 | . $12^{* * *}$ | . 02 | . $35^{* * *}$ | . 02 | . $17^{* * *}$ | . 03 |
| Shocks | -. 06 | . 21 | . 09 | . 23 | . $57 * *$ | . 00 | -. 46 | . 28 |
| Number of children $\times$ Shocks | -. 04 | . 03 | . 02 | . 03 | $-.09 * * *$ | . 04 | . 03 | . 04 |
| Male literacy | . 18 | . 13 | $-1.03^{* * *}$ | . 18 | . $48^{* * *}$ | . 13 | . $48^{* * *}$ | . 19 |
| Female literacy | . 30 | . 21 | -. $64^{* *}$ | . 29 | . 22 | . 22 | . 44 | . 29 |
| Wealth index | $.23^{* * *}$ | . 08 | -. 02 | 09 | . 13 | . 09 | . 17 | . 11 |
| Agricultural land surface | -. 01 | . 01 | -. 01 | . 01 | -. 01 | . 01 | . 01 | . 01 |
| Head of household, male | . 10 | . 12 | . 22 | . 14 | -. 08 | . 14 | -. 15 | . 17 |
| Average age of children aged 0-20 | . 03 | . 02 | .05** | . 02 | $-.22^{* * *}$ | . 02 | -. 01 | . 03 |
| Girls ratio among 0-20 | . 73 *** | . 14 | $-.77^{* * *}$ | . 16 | -. 06 | . 15 | -. 69 *** | . 19 |
| Proportion of female among adults | . 12 | . 24 | . 04 | . 24 | -. 07 | . 24 | -. 10 | . 32 |
| Multilevel schools | -. 11 | . 09 | . 03 | . 10 | . $27^{* *}$ | . 10 | -. 08 | . 12 |
| Number of primary school per district | . 02 | . 05 | $-.19^{* * *}$ | . 06 | -. 01 | . 05 | -. 05 | . 07 |
| Secondary schools per district | -. 04 | . 06 | $-.25^{* * *}$ | . 07 | .19** | . 07 | . 11 | . 08 |
| Constant | -1.40 *** | . 45 | -.95* | . 48 | $1.84^{* * *}$ | . 47 | -1.16* | . 59 |
| Number of observations | 1462 |  |  |  |  |  |  |  |
| Log_pseudolikelihood | -7656.31 |  |  |  |  |  |  |  |
| Chi2(56) | 1734.09 |  |  |  |  |  |  |  |

Likelihood ratio test of rho21 $=$ rho $31=$ rho $41=\operatorname{rho} 32=$ rho $42=$ rho43 $=0: \operatorname{chi} 2(6)=461.326 \operatorname{Prob}>\operatorname{chi} 2=0.0000$

* $\mathrm{p}<0.1,{ }^{* *} \mathrm{p}<0.05,{ }^{* * *} \mathrm{p}<0.01$


## 6 Conclusion

This paper formulates and estimates a model of household fertility and child occupation. Fertility decisions are made first, in a risky environment, while the allocation of children's occupations - namely school, paid work and domestic chores - and of parental time takes place in a second period, when shocks are realized and fertility cannot be adjusted. In line with our data, children are specialized in a single occupation, which leads to child discrimination. Fertility is higher in households with more favorable ex-ante distribution of parental income. The number of domestic children is positively affected by fertility, while the opposite stands for parental supply of domestic labor. The number of school children is likely to increase with fertility if parental labor income is large and/or the cost of raising children is low. The impact of fertility on schooling will also be larger if preferences for education are strong. Adverse shocks lead to an increase in the number of working children. Large families tend to adjust to shocks by increasing domestic labor and decreasing schooling, while the opposite stands for small families, in which case parents increase their supply of domestic labor. We test the model's predictions in two steps using data from the 2003 survey Senegalese Survey on Education and Household Wellbeing. A Poisson model estimates the number of children with information on the household's ex ante distribution of income and standard instruments. A multivariate Tobit model estimates the determinants of children occupations including the occurence of shocks and accounting for the endogeneity of fertility.

The Poisson estimates indicate that the data likely support our predictions with respect to the effect on fertility of the probability of facing shocks. Except for crops shock, the higher the exposure to shocks, fewer is the number of children in the household. The negative sign of the business and unemployment probability shocks suggests that nonagricultural households account for the potential shocks they face when deciding of the number of children to have. In contrast, shocks are less likely source of concern in agricultural households' decisions of the number of children to have. Our results also show a progressive effect of wealth on the number of children; richer households seem to have more children.

Conversely, estimates from the augmented multivariate Tobit model clearly show that children's specialization probability is positively related to the number of children within a household. Household with more children have larger proportion of children going exclusively to school and more children performing domestic chores or working for pay. These results are coherent with Gomes' findings and the structure of our theoretical model. Our theoretical predictions regarding the effects of adverse shocks on children's occupations are partially supported. The coefficients of shocks-related variables are statistically significant in the schooling equation. Household with large number of children are likely to reduce the number of children attending school to cope with adverse shocks.

## References

Anh, T. S., Knodel, J. E., Lam, D., \& Friedman, J. (1998). Family size and children's education in vietnam. Demography, 35, 57-70.

Barslund, M. (2007). Estimation of Tobit Type Censored Demand Systems: A Comparison of Estimators. Discussion Papers 07-16, University of Copenhagen. Department of Economics.

Becker, G. \& Lewis, G. (1973). On the interaction between the quantity and quality of children. Journal of Political Economy, 81, 279-88.

Björkman, M. (2006). Income Shocks and Gender Gaps in Education: Evidence from Uganda. Seminar Papers 744, Stockholm University, Institute for International Economic Studies.

Chernichovsky, D. (1985). Socioeconomic and demographic aspects of school enrollment and attendance in rural botswana. Economic Development and Cultural Change, 33(2), 319-32.

Cigno, A. \& Rosati, F. C. (2005). The Economics of Child Labour. Number 9780199264452 in OUP Catalogue. Oxford University Press.

Deb, P. \& Rosati, F. (2002). Determinants of Child Labor and School Attendance: The Role of Household Unobservables. Hunter College Department of Economics Working Papers 02/9, Hunter College: Department of Economics.

Dehejia, R. H., Beegle, K., \& Gatti, R. (2003). Child labor, income shocks, and access to credit. Policy Research Working Paper Series 3075, The World Bank.

Gomes, M. (1984). Family size and educational attainment in kenya. Population and Development Review, 10, 647-60.

Gubert, F. \& Robilliard, A.-S. (2007). Risk and schooling decisions in rural madagascar: A panel data-analysis. Journal of African Economies, (pp. ejm010).

Jacoby, H. G. \& Skoufias, E. (1997). Risk, financial markets, and human capital in a developing country. Review of Economic Studies, 64(3), 311-35.

Lee, J. (2008). Sibling size and investment in children's education: an asian instrument. Journal of Population Economics, 21(4), 855-875.

Lloyd, C. \& Blanc, A. (1996). Child's schooling in sub-saharan africa: The role of fathers, mothers, and others. Population and Development Review, 22, 265-98.

Montgomery, M. \& Kouame, A. (1993). Fertility and Schooling in Côte d'Ivoire: Is There a Tradeoff? Technical Report 11, The World Bank.

Orbeta, A. J. C. (2005). Number of Children and their Education in Philippine Households. Discussion Papers DP 2005-21, Philippine Institute for Development Studies.

Psacharopoulos, G. \& Patrinos, H. A. (1997). Family size, schooling and child labor in peru - an empirical analysis. Journal of Population Economics, 10(4), 387-405.

Rivers, D. \& Vuong, Q. H. (1988). Limited information estimators and exogeneity tests for simultaneous probit models. Journal of Econometrics, 39(3), 347-366.

Rosenzweig, M. R. \& Wolpin, K. I. (1980). Testing the quantity-quality fertility model: The use of twins as a natural experiment. Econometrica, 48(1), 227-40.

Smith, R. \& Blundell, R. (1986). An exogeneity test for a simultaneous equation tobit model with an application to labor supply. Econometrica, 54(3), 679-685.

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[^1]:    ${ }^{1}$ Björkman (2006) finds that income shocks have large negative and highly significant effects on female enrollment in primary schools and the effect grows stronger for older girls in Uganda. Using panel data from Tanzania, Dehejia et al. (2003) find that households respond to transitory income shocks by increasing child labor, but that the extent to which child labor is used as a buffer is lower when households have access to credit. Using panel data from Madagascar, Gubert \& Robilliard (2007) find that transitory income shocks have a significant impact on the probability of leaving school. Jacoby \& Skoufias (1997) study responses to aggregate and idiosyncratic, as well as to anticipated and unanticipated, income shocks in India. They find that seasonal fluctuations in school attendance are a form of self-insurance but one which does not result in a substantial loss of human capital on average.
    ${ }^{2}$ We distinguish market work from domestic chores instead of looking only at the trade-off between child labor and schooling since grouping work occupations (paid and domestic labor) can lead to false results because, as our model shows, the determinants of these two occupations differ. Also, parental time is to be allocated between paid work and domestic good production in the model.

[^2]:    ${ }^{1}$ However, Orbeta (2005) focuses only on children attending school and does not take into account other activities.
    ${ }^{3}$ One could equivalently write $w$ as $w_{h}$ and $w-\Delta$ as $w_{l}$, but the notation we use allows a more interesting comparative statics exercise, in which one can study the effect of the base wage $w$, and that of shocks $\Delta$ separately.
    ${ }^{4}$ Less than $15 \%$ of the sample are engaged in multiple activities.

[^3]:    ${ }^{5}$ A broader interpretation of decreasing returns to child labor may be used as well.
    ${ }^{6} \mathrm{We}$ assume that domestic work can be directly observed and does not involve any supervision costs, which is well admitted in the literature.

[^4]:    ${ }^{7}$ This surplus is simply the difference between total child labor net production and the optimal number of working children times $\alpha w_{p}$, the marginal productivity of the $n_{l}^{*}$-th working child:

    $$
    n_{l}^{*} w_{c}-s_{l} n_{l}^{* 2}-n_{l}^{*} \alpha w_{p}=\frac{\left(w_{c}-\alpha w_{p}\right)^{2}}{4 s_{l}}
    $$

[^5]:    ${ }^{10}$ If $q>0$, that is, both educated and uneducated children are "goods", then $\mu>1$, making the condition even weaker than requiring the child labor income to be larger than its actual costs.

[^6]:    ${ }^{11}$ To see that $p_{e}<1$, note that the former inequality is equivalent to $-(w \alpha \mu-\theta)(w-\Delta)<0$.

[^7]:    ${ }^{12}$ The effect of the shock on $d$ for $N=N_{d}$ equals

    $$
    \begin{aligned}
    (d(w-\Delta)-d(w))_{N=N_{d}} & =-\alpha^{2} \frac{w(w-\Delta)}{2 s_{l}} \frac{(1-(\gamma+\epsilon))}{\epsilon} \\
    & <0 .
    \end{aligned}
    $$

[^8]:    ${ }^{13}$ If $\Sigma$ is allowed varying across households, this means that equations 6 and 7 correspond to the specification of a system of heteroskedastic Tobit equations

