Seasonal cycles in the housing market

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2012

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Version: January 18, 2012

Abstract: The housing market exhibits a puzzling yet repetitive seasonal boom and bust cycle where prices *and* trade volume rise in summers and fall in winters. This paper presents a search model that analytically generates the observed deterministic cycle.

Keywords: housing, search, thin and thick markets, seasonality
JEL: D39, D49, D83

1. INTRODUCTION

Every year the residential real estate market goes through a boom and bust cycle. In summers prices tend to rise, trade speeds up and the trade volume goes up. In winters the opposite happens; prices fall, it takes longer to sell and the trade volume falls. What is puzzling, the cycle is highly predictable and repetitive, which seemingly defies the no-arbitrage condition; hence difficult to explain with the standard durable asset models.[1]

This paper presents a search model, based on two pillars, addressing this puzzle. First, the market is decentralized and transactions occur via search and matching. It takes time to locate and investigate a house and not every house is a good match for every customer. Second, the search market is 'thick' in summers and 'thin' in winters, which means that better matches are formed in summers.[2]

In this setup we analytically show that the market systematically fluctuates between 'hot' and 'cold' periods as observed in reality. In a thick market higher quality matches are formed, so buyers are willing to pay more. This is why prices rise in summers. In addition, agents cannot transfer the additional value across seasons, so they have strong incentives to trade while the market is still thick, which is why trade speeds up in summers. The short time to sale in summers signals that prices could be even higher. Indeed we show that if winter never came—that is if the market remained thick throughout—then the equilibrium price would be higher. With winter in sight, sellers do not raise prices sufficiently as they want to take advantage of the summer market while it lasts.

It is also easy to understand why agents prefer to trade immediately—assuming a suitable match is found—rather than waiting for the 'favorite' season because there is no guarantee to find a better match in the next season. In equilibrium sellers are strictly better off trading

[1] Ngai and Tenreyro [7] empirically document this systematic boom and bust cycle in the UK (for every year between 1983-2007) and in the US (for every year between 1991-2007). They, too, present a setup generating deterministic cycles, but the results are based on quantitative simulations; not analytic. A related paper is Krainer [6]. However in his setup if the persistence parameter is set $\lambda = 0$ so that seasons alternate deterministically then one obtains the wrong cycle; the market is cold in the summer and hot in the winter. See also Ferraris and Watanabe [2].

[2] Goodman [4] finds that the construction of new houses and sales of existing properties are about twice as high in summers as in winters. Home buyers, too, tend to move in summers. He finds that about 1/3 of all moves occur in June, July and August, roughly twice the proportion occurring in December, January and February. While the reasons for moving in the summer are clear for households with school-age children or newlyweds, remaining households have nearly identical seasonal patterns. The seasonality in household moves clearly coincides with the seasonality in construction and home sales, which indicates that the market is thick in summers and thin in winters.

immediately; buyers, on the other hand, are left indifferent between purchasing and walking away.

2. MODEL

The economy consists of a continuum of risk neutral buyers and sellers. Each seller has a house from which he derives no utility and each buyer seeks to purchase one. Buyers receive periodic housing services starting the period after the purchase and continuing forever. Time is discrete and infinite and deterministically alternates between two seasons, summer and winter; if the current season is summer then the next one is winter, and so on. Agents meet with some time invariant probability $\alpha \in (0,1)$ and after inspecting the house, a buyer realizes his valuation $v \in [0,1]$, a random draw from a cdf $F(\cdot)$.[3] Two buyers may differ in their valuations for the same house, which captures the notion that buyers have idiosyncratic tastes and preferences. To generate seasonality we assume that the search market is thick in the summer and thin in the winter. The following assumption captures this notion.

- **Assumption 1.** *The cdf in the summer, F_s, likelihood ratio dominates the one in the winter, F_w; that is $f_s(v)/f_w(v)$ is increasing. In addition, the "iso-probability curve" $\Phi(v) \equiv F_w^{-1} \circ F_s(v) : [0,1] \to [0,1]$ is increasing and strictly convex.*[4]

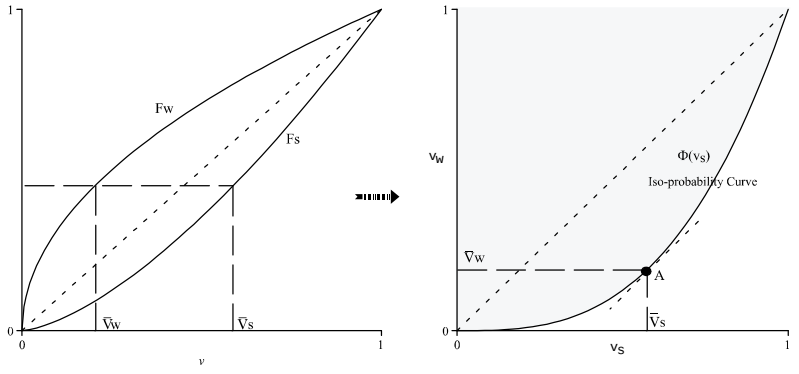

Figure 1 – Obtaining the Iso-Probability Curve from the cdfs.

Likelihood ratio dominance implies first order stochastic dominance (FOSD), $F_s(v) < F_w(v)$, as well as hazard rate dominance, $h_s(v) < h_w(v)$, where $h = f/S$. Given a threshold valuation v, the expression $1 - F(v)$ is the probability that the buyer encounters a house better than v. FOSD implies that in summers it is easier to find a suitable match; alternatively, controlling for the probability of sale, it says in summers higher quality matches are formed. This roughly captures the thin-thick argument. However FOSD alone is not enough to obtain what we are after; other assumptions are also needed.

The curve Φ is obtained by drawing horizontal lines across the cdfs and tracing combinations of v_s and v_w satisfying $F_s(v_s) = F_w(v_w)$; see Figure 1. Any point inside Φ (shaded area) has $S_s > S_w$ saying that buyers are more likely to purchase in the summer. For clarity, the points on the border of Φ are denoted with capital letters. The slope of Φ equals to $f_s(V_s)/f_w(V_w)$. Strict convexity ensures that there exists a unique point $A = (\overline{V}_s, \overline{V}_w) \in (0,1)^2$ on Φ such that $f_s(\overline{V}_s) = f_w(\overline{V}_w)$. Furthermore (i) $f_s(V_s) > f_w(V_w)$ for $V_s > \overline{V}_s$ and $V_w > \overline{V}_w$; (ii)

[3] This assumption is adopted by Jovanovic [5], Wolinsky [8], Krainer [6] among others.

[4] The strict convexity of Φ is added as an extra assumption, but under certain circumstances the likelihood ratio dominance is a sufficient condition for it; for instance if $f_s' \geq 0$ and $f_w' \leq 0$ with one inequality strict.

$f_s(V_s) < f_w(V_w)$ for $V_s < \overline{V}_s$ and $V_w < \overline{V}_w$.[5]

The next assumption is needed mainly for technical purposes (second order condition and the uniqueness of the equilibrium).

- **Assumption 2.** *The survival function* $S_x = 1 - F_x$ *is log-concave, that is* $f_x^2(v) + f_x'(v) S_x(v) > 0$, $\forall v$ *and* $x = s, w$.[6]

The trading mechanism is price posting and prices are indexed by the season.[7] If agents trade at p_x then the seller receives payoff p_x; the buyer starts receiving dividends v; both agents leave the search market and are replaced by clones (this is needed to maintain stationarity). Agents who do not trade receive zero and continue to the next round to play the same game.

2.1. Buyers

Let Ω_x denote a buyer's value of search in season $x = s, w$.

$$\Omega_x = \alpha \int_0^1 \max\left\{\tfrac{v}{1-\beta} - p_x, \beta\Omega_{\widetilde{x}}\right\} dF_x(v) + (1-\alpha)\beta\Omega_{\widetilde{x}}$$

With probability α he meets a seller. If he purchases he gets $v/(1-\beta) - p_x$.[8] If he walks away he obtains $\beta\Omega_{\widetilde{x}}$ which is the discounted value of search in the next season. With probability $1-\alpha$ he does not encounter a seller and moves on to the next season. Manipulation yields $\Omega_x = \theta\tau_x + \beta\theta\tau_{\widetilde{x}}$, where $\theta = \alpha/\{(1-\beta)^2(1+\beta)\}$ and

$$\tau_x = \int_0^1 \max\{v - (1-\beta)(p_x + \beta\Omega_{\widetilde{x}}), 0\} dF_x(v).$$

For any given price p_x there is a threshold *reservation value* v_x satisfying

$$v_x = (1-\beta)(p_x + \beta\Omega_{\widetilde{x}}), \tag{1}$$

which is where buyers are indifferent between purchasing and searching. So the decision is simple: purchase if $v \geq v_x$ and keep searching otherwise. Obviously not all meetings result in trade; for trade to occur the house must turn out to be a good match, which happens with probability $S_x(v_x)$. Inserting (1) into τ_x and using integration by parts one gets

$$\Omega_x = \theta \int_{v_x}^1 S_x(v)\, dv + \beta\theta \int_{v_{\widetilde{x}}}^1 S_{\widetilde{x}}(v)\, dv.$$

Substituting Ω_x into the indifference condition (1) one gets the *indifference curves* in the summer and the winter; see Figure 2. The upward slope means that the higher the price p_x

[5] To see this note that $dF_w^{-1} \circ F_s(V_s)/dV_s = f_s(V_s)/f_w(V_w)$. The function is convex and lies underneath the 45^0 line cutting it at the origin and at $(1,1)$. The claim follows from the Intermediate Value Theorem. Note that the strict convexity of Φ ensures that $\overline{V}_s$ and $\overline{V}_w$ are strictly between zero and one.

[6] Log-concavity of the survival function is a standard and mild assumption; many well known distributions including Uniform, Normal, Exponential, χ^2 distributions are log concave—see Bagnoli and Bergstrom [1] for details.

[7] In reality buyers typically attempt to negotiate discounts off the posted price; see for instance Gill and Thanassoulis [3], however due to space constraints we assume that the transaction takes place at the posted price.

[8] The buyer receives v starting the period after the purchase and continuing forever; the present value of the stream equals to $v/(1-\beta)$.

the higher the associated v_x and therefore the smaller the chance of a trade.

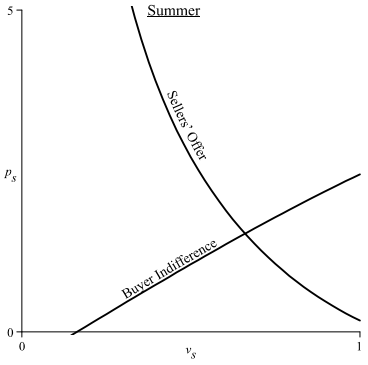

Figure 2a

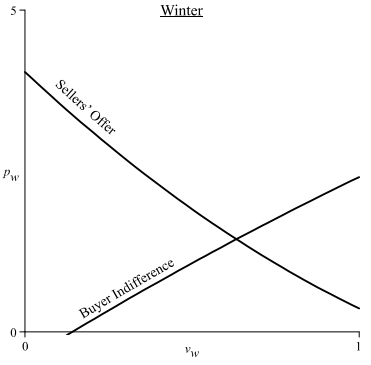

Figure 2b

2.2. Sellers

A seller quotes p_x in season x taking as given the indifference condition (1). His value of search equals to

$$\Pi_x = \alpha S_x(v_x) \max\{p_x, \beta\Pi_{\widetilde{x}}\} + \{1 - \alpha S_x(v_x)\}\beta\Pi_{\widetilde{x}}.$$

With probability $\alpha S_x(v_x)$ he meets a buyer who agrees to purchase; so the seller obtains p_x.The condition $\max\{p_x, \beta\Pi_{\widetilde{x}}\}$ guarantees that the seller is willing to trade today rather than waiting for the next season. With the complementary probability trade does not materialize, so he moves to the next season. Conjecturing $p_x > \beta\Pi_{\widetilde{x}}$ (to be verified) the seller solves

$$\max_{p_x} \Pi_x \text{ subject to (1)}$$

taking Ω_x and $\Omega_{\widetilde{x}}$ as given. The FOC is given by

$$p_x - \beta\Pi_{\widetilde{x}} = \frac{S_x(v_x)}{f_x(v_x)(1-\beta)}.$$

If S_x is log concave then the second order condition holds; hence the FOC yields a maximum.[9]Straightforward algebra yields the profit maximizing prices P_s and P_w that a seller ought to post (*offer curves*):

$$P_x = \frac{S_x(v_x)}{f_x(v_x)(1-\beta)} + \frac{\theta S^2_{\widetilde{x}}(v_{\widetilde{x}})}{f_{\widetilde{x}}(v_{\widetilde{x}})} + \frac{\beta\theta S^2_x(v_x)}{f_x(v_x)}. \tag{2}$$

Note that the FOC implies that $P_x(v_x, v_{\widetilde{x}}) > \beta\Pi_{\widetilde{x}}$ since the expression S_x/f_x is positive for all v_x. This verifies the earlier conjecture and ensures that the seller is willing to trade immediately rather than waiting.

LEMMA 1. *Differentiating (2) yields* $dP_x/dv_x < dP_{\widetilde{x}}/dv_x < 0$, *for* $x = s, w$.

The Lemma says that offer curves are downward sloping (Figure 2), which reflects the fact that sellers face a trade-off between revenue and liquidity. For instance, in panel 2*a* for low values of v_s the probability of a sale is high, so sellers can afford to post high; however as v_s rises, liquidity concerns start to kick in and prices fall. The relationship $|dP_x/dv_x| > |dP_{\widetilde{x}}/dv_x|$ says that a change in the current season's v_x affects the current season's price a lot more than the other season's price. These inequalities will be useful later on.

[9] The proof is skipped due to the space constraint.

2.3. Equilibrium

Simultaneous intersections of the offer and indifference curves determine the equilibrium. More formally, a steady-state symmetric equilibrium is characterized by the pairs $\mathbf{v}^* = (v_s^*, v_w^*)$ and $\mathbf{p}^* = (p_s^*, p_w^*)$ satisfying indifference (1) and profit maximization (2).

The equilibrium exits and it is unique. The proof amounts to showing that there exists a unique pair $\mathbf{v}^*$ satisfying

$$\Delta_x(v_s, v_w) = P_x(v_s, v_w) + \beta\Omega_{\widetilde{x}}(v_s, v_w) - \tfrac{v_x}{1-\beta} = 0, \text{ for } x = s, w.$$

In equilibrium the difference function $D \equiv \Delta_s - \Delta_w$ must equal to zero as well. It is given by

$$D(v_s, v_w) = \underbrace{P_s - P_w}_{T_1(v_s, v_w)} + \underbrace{\beta(\Omega_w - \Omega_s) - \frac{v_s - v_w}{1-\beta}}_{T_2(v_s, v_w)}. \tag{3}$$

For expositional purposes we will focus on Δ_w and D (instead of Δ_w and Δ_s). Closed form expressions for T_1 and T_2 are provided in the appendix, see (4) and (5).

LEMMA 2. *The function Δ_w decreases in both arguments v_s and v_w while its locus, l_Δ, is downward sloping wrt v_s. Functions T_1 and D decrease in v_s and increase in v_w. Their locuses, Γ and l_D respectively, slope upwards wrt v_s.*

The equilibrium $\mathbf{v}^*$ lies at the intersection of l_Δ and l_D. The Lemma says that one curve slopes downward while the other slopes upward (see Figure 3). The only task that remains is to verify that $l_D < l_\Delta$ when $v_s = 0$ and $l_D > l_\Delta$ when $v_s = 1$. This would guarantee that they intersect once in the $(0,1)^2$ space, proving the equilibrium exists and is unique. This is a straightforward but a somewhat lengthy process; given the space constraint we skip this step and move on to discuss hot and cold markets.

3. HOT AND COLD MARKETS

Propositions 1 and 2, main results of the paper, analytically show that the market is hot in the summer (high price *and* high trade volume) and cold in the winter (the opposite). We first carry out the technical analysis and then offer some intuition.

3.1. Analysis

PROPOSITION 1. *The equilibrium price in the summer is higher than the one in the winter, i.e. $p_s^* > p_w^*$.*

The term T_1 is the price difference function and its locus Γ is the *iso-price* curve. Lemma 2 says that Γ is upward sloping; see Figure 3. Note that any point lying to the left of Γ has $P_s > P_w$, which is what we are after.[10] Below we show that the entire locus of $D = 0$, which of course contains the equilibrium point $\mathbf{v}^*$, lies to the left of Γ; hence the proposition. These arguments are made precise below.

Proof. Fix some v_w and let v_s' and v_s'' satisfy $T_1(v_s', v_w) = 0$ and $D(v_s'', v_w) = 0$. Note that v_s' must exceed v_w. To see why substitute $v_s = v_w = v$ into the expression for T_1, given in the appendix by (4), to obtain

$$T_1(v, v) = \tfrac{1+\beta-\alpha S_s(v)}{h_s(v)} - \tfrac{1+\beta-\alpha S_w(v)}{h_w(v)} > 0,$$

[10] To see this fix some $(\widehat{v}_w, \widehat{v}_s)$ on Γ and note that $T_1(v_s, \widehat{v}_w) > 0 \Leftrightarrow P_s(v_s, \widehat{v}_w) > P_w(v_s, \widehat{v}_w)$ for any $v_s < \widehat{v}_s$ since $\partial T_1/\partial v_s < 0$. Similarly $T_1(\widehat{v}_s, v_w) > 0$ for any $v_w > \widehat{v}_w$ since $\partial T_1/\partial v_w > 0$.

which is positive because $h_s(v) < h_w(v)$ and $S_s(v) > S_w(v)$ for all v. The former relationship is the hazard rate dominance and the latter is the FOSD. In the proof of Lemma 2 we have $\partial T_1/\partial v_s < 0$. So, if $T_1(v'_s, v_w) = 0$ then $v'_s > v_w$. This, in turn, means that $T_2(v'_s, v_w) < 0$. The expression for T_2, given in the appendix by (5), is indeed negative because $v'_s > v_w$ and FOSD.

Since $T_1(v'_s, v_w) = 0$ and $T_2(v'_s, v_w) < 0$, their sum $D(v'_s, v_w)$ is negative. Lemma 2 says that D decreases in v_s. This means that if $D(v''_s, v_w) = 0$ then $v''_s < v'_s$ and therefore

$$T_1(v''_s, v_w) > 0 \Leftrightarrow P_s(v''_s, v_w) > P_w(v''_s, v_w).$$

To see why $p^*_s > p^*_w$ substitute v^*_w for v_w. The arguments above imply that $T_1(v^*_s, v^*_w) > 0 \Leftrightarrow p^*_s > p^*_w$ because $D(v^*_s, v^*_w) = 0$. This completes the proof. ∎

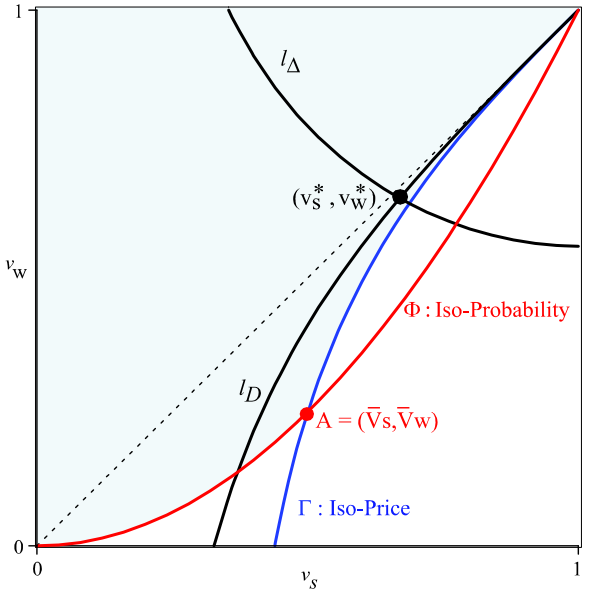

Fig 3: Locuses, Iso-Probability and Iso-Price

The next proposition establishes that trade is more likely in the summer.

PROPOSITION 2. *The equilibrium probability of sale in the summer is higher than the one in the winter, i.e.* $S_s(v^*_s) > S_w(v^*_w)$.

Proof. The arguments below are best understood with the aid of Figure 3. The objective is to show that the equilibrium $\mathbf{v}^*$ lies inside the iso-probability curve Φ.

Step 1. We show that point $A = (\overline{V}_s, \overline{V}_s)$ on Φ lies below l_D as well as l_Δ. To start, note that the iso-price curve Γ intersects with Φ at point A. To see why recall that at A we have $f_s(\overline{V}_s) = f_w(\overline{V}_w)$ and $S_s(\overline{V}_s) = S_w(\overline{V}_w)$; therefore $h_s(\overline{V}_s) = h_w(\overline{V}_w)$. Substituting this equalities into (4) yields $T_1(\overline{V}_s, \overline{V}_w) = 0$, which means that A lies on Γ. Since the entire Γ curve lies below l_D, so is point A.

Now we show that A lies below l_Δ. Substitute $(\overline{V}_s, \overline{V}_s)$ into Δ_w and simplify to obtain

$$\Delta_w(\overline{V}_s, \overline{V}_w) = \underbrace{\frac{1}{1-\beta}\left\{\frac{1-\beta+\alpha S_w(\overline{V}_w)}{h_w(\overline{V}_w)(1-\beta)} - \overline{V}_w\right\}}_{H(\overline{V}_w)} + \beta\theta\left\{\int_{\overline{V}_s}^1 S_s(v)\,dv + \beta\int_{\overline{V}_w}^1 S_w(v)\,dv\right\}.$$

We want to show that $\Delta_w\left(\overline{V}_s,\overline{V}_w\right)>0$. The second term is positive but the term $H\left(\overline{V}_w\right)$, which decreases in $\overline{V}_w$, is negative when $\overline{V}_w=1$, so the sign of the sum may be ambiguous. Recall that $\overline{V}_w$ itself is a function of the cdfs satisfying $F_s\left(\overline{V}_s\right)=F_w\left(\overline{V}_w\right)$ and $f_s\left(\overline{V}_s\right)=f_w\left(\overline{V}_w\right)$; it is not a free variable. The strict convexity of Φ ensures that $\overline{V}_w<1$. So fix some $\overline{V}_w<1$ and note that there exists a unique $\overline{\beta}\left(\overline{V}_w\right)<1$ such that $H\left(\overline{V}_w\right)\geq 0$ for all $\beta\geq\overline{\beta}$ i.e. if agents are patient enough. This sufficient condition ensures that $\Delta_w\left(\overline{V}_s,\overline{V}_w\right)>0$.[11] Recall that Δ_w falls both in v_w and v_s whereas its locus is downward sloping in the v_s-v_d space. This means that if $\overline{V}_s$ is kept constant then $\Delta_w\left(\overline{V}_s,v_w\right)=0$ for some point v_w greater than $\overline{V}_w$. Alternatively if $\overline{V}_w$ is held constant then $\Delta_w\left(v_s,\overline{V}_w\right)=0$ again at a point v_s greater than $\overline{V}_s$. Since l_Δ is downward sloping the arguments imply that point A lies below the locus.

Step 2. The curve l_Δ is downward sloping whereas l_D is upward sloping and point A lies beneath both. This means that their intersection point $\mathbf{v}^*$ must lie above point A, that is $v_w^*>\overline{V}_w$; so the region below $\overline{V}_w$ can be dismissed as it cannot contain the equilibrium. Now we will show that along the border of Φ that lies above A the function D is negative, that is $D\left(V_sV_w\right)$ for all $\left(V_s,V_w\right)>A=\left(\overline{V}_s,\overline{V}_w\right)$.

Recall that $f_s\left(V_s\right)>f_w\left(V_w\right)$ for all $\left(V_s,V_w\right)>A$ (from Assumption 1). Along such points the expressions T_1 is negative. To see why substitute $\left(V_s,V_w\right)$ into (4) use the fact that $S_s\left(V_s\right)=S_w\left(V_w\right)$ to obtain

$$T_1\left(V_s,V_w\right)=\left\{1+\beta-\alpha S_s\left(V_s\right)\right\}S_s\left(V_s\right)\left\{\tfrac{1}{f_s(V_s)}-\tfrac{1}{f_w(V_w)}\right\}<0,$$

which is negative since $f_s\left(V_s\right)>f_w\left(V_w\right)$. Similarly focusing on (5) it is easy to see that $T_2\left(V_s,V_w\right)<0$ because $V_s>V_w$ and FOSD. It follows that $D\left(V_s,V_w\right)<0$ because $D=T_1+T_2$.

Since $D<0$ along the border of Φ the equilibrium point must be inside Φ. To see why fix V_w and note that $D=0$ for some $v_s<V_s$ since D decreases in v_s. Alternatively fix V_s and note the $D=0$ for some $v_w>V_w$. Finally, since $\left(v_s^*,v_w^*\right)$ lies inside Φ we have $S_s\left(v_s^*\right)>S_w\left(v_w^*\right)$. This completes the proof. ∎

3.2. Discussion

The fact that $S_s\left(v_s^*\right)>S_w\left(v_w^*\right)$ implies that in summers the expected time on the market is short and the trade volume is high. In addition since $p_s^*>p_w^*$ we have a "hot market" in the summer (high prices, quick sales, high volume) and a "cold market" in the winter (low prices, slow sales, low volume).

In the thick market better matches are formed, which is why prices go up in the summer. In addition, agents have no means of transferring the extra value across seasons, so they have strong incentives to trade while the market is still thick; hence trade speeds up in the summer. The short sale duration in summers indicates that prices could be even higher. Indeed, with the winter in sight and afraid of not being able to take advantage of the thick summer market, sellers do not raise prices as much as they want. The simulation below confirms this insight.[12]

	Price	Probability of Sale
Summer	6.22	0.39
Winter	6.10	0.22
If winter never came	6.31	0.36

[11] Alternatively one can put some structure behind the cdfs to pin down $\overline{V}_w$ and then show that H is positive. For instance fix $F_s\left(v_s\right)=v_s^Q$ and $F_w\left(v_w\right)=v_w^q$, where $Q>q>0$ are positive real numbers. This family of distributions captures a continuum of concave and convex combinations of cdfs and covers the entire $[0,1]^2$ spectrum, yet it is simple enough to characterize $\overline{V}_w$. Basic algebra reveals that $\overline{V}_w=(q/Q)^{\frac{Q}{Q-q}}$, which decreases in Q and increases in q. Furthermore $\lim_{Q\to q}\overline{V}_w=e^{-1}$ whereas $\lim_{Q\to\infty}\overline{V}_w=0$. Notice that

$$\lim_{Q\to q}H\left(\overline{V}_w\right)=\tfrac{(1+\alpha-\beta)(e-2)/e+\alpha/e^2}{1-\beta}>0,$$

which means that $H\left(\overline{V}_w\right)$ is positive for all $Q>q$; hence $\Delta_w\left(\overline{V}_s,\overline{V}_w\right)>0$ for all $Q>q$.

[12] Based on $F_s=v^2$, $F_w=v$, $\alpha=0.5$ and $\beta=0.9$.

If winter never came ($F_w = F_s$) i.e. if the market remained thick throughout the entire year, then the equilibrium price would be even higher.

4. CONCLUSION

We have presented a search model that generates a deterministic boom and bust cycle. Although the discussion so far revolved around the housing market, the model is applicable to other search and matching settings, such as the used car market or, to some extent, the labor market, that go through similar seasonal cycles.

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5. APPENDIX

Proof of Lemma 1. Observe that

$$\frac{dP_x}{dv_x} = -\frac{M_x(v_x)}{1-\beta} - \beta\theta S_x(v_x)\{1 + M_x(v_x)\} < 0 \quad \text{and} \quad \frac{dP_x}{dv_{\widetilde{x}}} = -\theta S_{\widetilde{x}}(v_{\widetilde{x}})\{1 + M_{\widetilde{x}}(v_{\widetilde{x}})\} < 0,$$

where $M_x(v_x) = 1 + f_x'(v_x) S_x(v_x)/f_x^2(v_x)$, which is positive because of log concavity; thus both derivatives are negative. Furthermore

$$\frac{d\{P_x - P_{\widetilde{x}}\}}{dv_x} = \frac{1}{1-\beta^2} \times \{-M_x\{1 + \beta - \alpha S_x\} + \alpha S_x\} < 0,$$

which is negative because M_x is positive. ■

Proof of Lemma 2. Let $l_\Delta(v_s) = \{v_w : \Delta_w(v_s, v_w) = 0\}$ be the locus of $\Delta_w = 0$. Its slope wrt v_s is given by (Implicit Function Theorem)

$$\frac{dl_\Delta}{dv_s} = -\frac{\partial \Delta_w}{\partial v_s} \Big/ \frac{\partial \Delta_w}{\partial v_w} < 0.$$

To see why the expression is negative note that

$$\frac{\partial \Delta_w}{\partial v_s} = \frac{\partial P_w}{\partial v_s} + \beta \frac{\partial \Omega_s}{\partial v_s} < 0,$$

which is negative because $\partial P_w/\partial v_s < 0$ (Lemma 1) and $\partial \Omega_s/\partial v_s = -\theta S_s(v_s) < 0$. Similarly one can show that $\partial \Delta_w/\partial v_w < 0$; hence $dl_\Delta/dv_s < 0$.

Now turn to the difference function D. Recall that $D = T_1 + T_2$, where T_1 and T_2 are defined in (3). Substitute for P_s, P_w, Ω_s and Ω_w and simplify to obtain

$$T_1(v_s, v_w) = \frac{1}{1-\beta} \times \left\{ \frac{1}{h_s(v_s)} - \frac{1}{h_w(v_w)} + \frac{\alpha}{1-\beta} \left[\frac{S_w(v_w)}{h_w(v_w)} - \frac{S_s(v_s)}{h_s(v_s)} \right] \right\} \quad \text{and} \tag{4}$$

$$T_2(v_s, v_w) = \frac{\alpha\beta}{1-\beta^2} \times \left\{ \int_{v_s}^1 F_s(v)\, dv - \int_{v_w}^1 F_w(v)\, dv \right\} - \frac{1+\beta(1-\alpha)}{1-\beta^2}(v_s - v_w). \tag{5}$$

Let $l_D(v_s) = \{v_w : D(v_s, v_w) = 0\}$ be the locus of $D = 0$. Its slope wrt v_s is given by

$$\frac{dl_D}{dv_s} = -\frac{\partial\{T_1 + T_2\}}{\partial v_s} \Big/ \frac{\partial\{T_1 + T_2\}}{\partial v_w} > 0.$$

The expression is positive because

$$\frac{\partial T_1}{\partial v_s} = \frac{\partial P_s - P_w}{\partial v_s} < 0 \quad \text{and} \quad \frac{\partial T_1}{\partial v_w} = \frac{\partial P_s - P_w}{\partial v_w} > 0 \quad \text{(Lemma 1)},$$

$$\frac{\partial T_2}{\partial v_s} = -\frac{1}{1-\beta} \times \left\{ 1 - \frac{\beta\alpha S_s(v_s)}{1+\beta} \right\} < 0 \quad \text{and} \quad \frac{\partial T_2}{\partial v_w} = \frac{1}{1-\beta} \times \left\{ 1 - \frac{\beta\alpha S_w(v_w)}{1+\beta} \right\} > 0.$$

Note that in the second line the expressions inside the curly brackets are positive for all parameter values. Using the same technique one can show that the locus of T_1, given by $\Gamma(v_s) = \{v_w : P_s(v_s, v_w) = P_w(v_s, v_w)\}$ is also downward sloping since $\partial T_1/\partial v_s < 0$ and $\partial T_1/\partial v_w > 0$. ■