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UNIFORM PRICING AND SOCIAL WELFARE

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Abstract. We re-examine the case for uniform pricing in a monopolistic third-degree price-discrimination setting by introducing differentiated costs. A profit-maximizing monopolist could then use price differentiation to reduce the production of the more costly goods, thereby decreasing average cost and increasing welfare. Indeed, monopolistic price differentiation can improve welfare and also aggregate consumer surplus even if, as in the benchmark linear case, total output does not increase. Accordingly, the welfare criterion based on total output fails and should be replaced by the computation of well-defined price indexes. These results possibly pave the way for a more optimistic assessment of monopolistic pricing.

Keywords: uniform pricing, third-degree price discrimination, welfare bounds, price and quantity indexes.
JEL *Classification*: D11, D42, L51.

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I. Introduction

It is well known that imposing uniform pricing on a monopolist instead of allowing third-degree price discrimination has ambiguous welfare implications. However, Schmalensee (1981), Varian (1985) and Schwartz (1990) proved that, if (marginal) costs are common, a necessary condition for monopolistic third-degree price discrimination to raise aggregate welfare is that total output increases under discriminatory pricing. Moreover, in the benchmark case of linear demand and cost functions, total output does not change if the markets served by the monopolist are the same both under uniform and differentiated pricing:[1] see Schmalensee (1981) and Tirole (1988: p. 139). Since the linear setting is usually adopted because it allows an explicit computation of the results and provides a first-order approximation to the general case, the literature on the welfare effect of allowing monopolistic price differentiation tends to be rather pessimistic:[2] see e.g. Schmalensee (1981: p. 246) and Varian (1989: pp. 622-623).

In this paper we reconsider the case for monopolistic uniform pricing by departing from the standard set-up, which assumes that there is a common cost of serving different markets. Accordingly, we are not really concerned with (third-degree) price discrimination as *narrowly* defined, since this would require that the *same* commodity be sold at different prices to different consumers: see e.g. Phlips (1981: p. 5). Nevertheless, we think our setting interesting for several reasons. First, the standard set up with equal marginal costs is a textbook idealization with little empirical content. For example, what guarantees that the marginal cost of serving geographically distant markets is the same? Second, perhaps the most appealing definition of price discrimination (attributed to George Stigler, 1987), which says that a firm price discriminates when the ratio in prices is different from the ratio in marginal costs for two "similar" (possibly identical) goods offered by the firm (see e.g. Varian, 1989: p. 598 and Stole, 2003: pp. 1-2), encompasses the case of different marginal costs (and so do other standard definitions: see e.g. Tirole, 1988: pp. 133-4 and Clerides, 2004). But since they are difficult to verify, it seems clear that, as a policy prescription, the use of uniform pricing should be tested against the possibility of *unaccounted* marginal cost differences. Moreover, in our second-best setting even the no-price-discrimination condition *à la* Stigler does not need to be optimal:[3] see below.

Third, while we should perhaps refer to them as examples of price discrimination (once the latter is properly defined: see e.g. the comments in Tirole, 1988: p. 133), there are sectors with

[1] For the sake of simplicity, we make this assumption throughout this paper. Notice that in fact price differentiation may lead to a Pareto improvement if it causes some market to open: see e.g. Schmalensee (1981), Varian (1985) and Tirole (1988: pp. 137-9).

[2] This comment only applies to the case of an otherwise unregulated monopoly: see e.g. Armstrong and Vickers (1991).

acknowledged differentiated marginal costs, such as the mail or telephone ones, in which uniform rates are imposed by public regulation (avoiding, for example, tariff discrimination between rural and urban areas), sometimes on a political basis invoking "horizontal equity" (this is the case, for instance, of the Italian residential electricity sector). In these cases, uniform pricing entails cross-subsidies whose welfare consequences have not yet been entirely understood. Fourth, the use of a uniform price may serve as a (potentially anticompetitive) strategic practice not to reveal the underlying cost structure. While this aspect is not considered here, the related additional social costs should in principle be weighed against the other welfare consequences of avoiding price differentiation, which are the subject of this paper. Finally, even if the monopolistic results we are concerned with do not necessary apply to other imperfectly competitive markets, in the literature it is sometimes suggested that they hold more generally (see e.g. the references given in Galera, 2003, p. 3). But the assumption of a common marginal cost would seem definitely too strong in the case of different firms.[4]

Not surprisingly, it turns out that in our enlarged setting the condition of an increase in output is no longer necessary to achieve an improvement in welfare through price discrimination. In particular, we are able to find many cases in which monopolistic price differentiation increases total welfare and aggregate consumer surplus even if, as in the benchmark linear case, total output does not increase. This happens if demands are close enough, as is to some extent intuitive, and also in some cases in which there is no systematic relation across markets between the demand and the cost structures (in general, a reasonable assumption). The intuition is straightforward: the economic rationale for imposing uniform pricing in the standard setting rests on the result that a *given* quantity of the *same* good should be distributed according to a common price (a result which is implied by the optimality of marginal cost pricing). But with different marginal costs no general principle can be invoked to support uniform pricing (goods are economically *different* and first-best optimality would rather call for the weaker condition of having the prices ratios equal to the marginal cost ratios),[5] which has no special properties.

Indeed, the socially efficient production of a given total amount of output (an unusual second-best problem if goods are not identical) would require that the *differences* between prices and the relative marginal costs be equal across markets. This price structure, which will never be achieved

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[3] Indeed, our results provide some theoretical foundations to an alternative price-discrimination definition based on price-cost margins: see Clerides (2004) for the empirical relevance of the distinction among different formulations.

[4] After the first version of this paper was written in 2003, we had the opportunity of reading the papers by Galera (2003) and Aguirre (2004), which show by the way of examples that even with linear demands price discrimination can be welfare improving if there is more than one firm. We believe that our results, which apply to the monopolistic case, nicely complement (and somehow generalize) the point made by those papers.

by uniform pricing if marginal costs are different (nor by imposing a no-price-discrimination condition *à la* Stigler), to some extent is more similar[6] to the one that would be chosen by an unregulated monopolist (a special case of Ramsey pricing, as is well known). In fact, in order to maximize his profit, a monopolist could use the alleged price authority to increase the price of the more costly goods, thereby decreasing *average* total cost and also increasing welfare. Indeed, he actually does that if the demand elasticities are not "perversely" correlated with the unit costs (this should be the case if demands are "close enough"). Some part of the cost reduction so achieved could then be "passed" trough prices to the consumers but, of course, the monopolist would also try to use his pricing flexibility to extract more surplus from them. Notice that this might give rise to a second-best conflict between social welfare and consumer surplus concerns. In general, welfare effects unfortunately maintain their ambiguity (as should be expected): however, we show that the welfare criterion based on total output fails in general and argue that it should be replaced by the computation of well-defined price indexes. Our results also suggest that price differentiation by an unregulated monopolist should perhaps be considered more optimistically.

The paper is organized as follows: in section II the model by Schmalensee (1981) is generalized to the case of different (constant) marginal costs, and the results by Varian (1985) are applied to it. Section III contains the main point made by this paper: by discussing the special case of equal demand elasticities, we show that price discrimination by a monopolist can be welfare and (aggregate) consumer surplus improving even if it does not increase total output. We also use the model by Schmalensee (1981) to discuss the case of monopolistic "piecemeal" price discrimination. Several results are gathered together in Proposition 1. Section IV focuses on the case of linear demands, and deals with the use of price indexes to check conditions for welfare improvements and the role of cost and demand variability. Results are gathered in Proposition 2. Section V discusses our results and concludes. More technical derivations are confined to the Appendixes.

II. The model

We refer to the model in Schmalensee (1981), which can be seen as a special case of Varian (1985). In particular, a monopolist is selling in N distinguishable markets. Let $q_i(p_i)$ be the demand function in market i (i = 1, …, N), where p_i is the price charged by the monopolist and q_i the quantity he sells. It is assumed that the relevant cost function is given by $C(\boldsymbol{q}) = \boldsymbol{c}'\boldsymbol{q} = \Sigma_i c_i q_i$, where

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[5] Notice that this is the condition underlying the broad definition of price discrimination due to Stigler and quoted above. Also notice that the unregulated monopolist would use this first-best price *structure* if demand elasticities were equal across markets.

[6] Phlips (1981: pp. 3-4) commented negatively on this similarity, but by referring to the narrowly defined case of a unique, common marginal cost.

$\boldsymbol{q}' = [q_1,q_2,\ldots,q_N]$ is the vector[7] of the monopolist's output quantities, c_i indicates the constant unit cost of output q_i and $\boldsymbol{c}' = [c_1,c_2,\ldots,c_N]$. We generalize Schmalensee's (1981) setting by assuming that c_i is possibly different from c_j, $i \neq j$, $i,j = 1, \ldots, N$. The net profit generated in market i can then be written $\pi_i(p_i) = (p_i - c_i)q_i(p_i)$: following most of the literature,[8] it is assumed that the $\pi_i(p_i)$ are smooth, strictly concave functions. The total profit function is thus given by $\boldsymbol{\Pi}(\boldsymbol{p}) = (\boldsymbol{p} - \boldsymbol{c})'\boldsymbol{q}(\boldsymbol{p}) = \Sigma_i\pi_i(p_i)$, where $\boldsymbol{p}' = [p_1,p_2,\ldots,p_N]$ is the vector of prices that the monopolist charges. It is also assumed that consumers have quasi-linear preferences. Thus, since there are no income and distributional effects, we can think in terms of a representative consumer with indirect utility function $V(\boldsymbol{p}) = v(\boldsymbol{p}) + y_0$, where y_0 is the total endowment of the *numeraire* (the Marshallian composite commodity), who consumes all the goods produced by the monopolist. By quasi-linearity, $v(\cdot)$ is convex and the demand system faced by the monopolist is given by $\boldsymbol{q}(\boldsymbol{p}) = -\boldsymbol{D_p}v(\boldsymbol{p})$, where $\boldsymbol{D_x}f(\boldsymbol{x})$ is the gradient of $f(\cdot)$. Aggregate (social) welfare (the Marshallian indicator) can then be written as $W(\boldsymbol{p}) = \boldsymbol{\Pi}(\boldsymbol{p}) + v(\boldsymbol{p})$.

Clearly, the unregulated monopolist would adopt a price vector $\boldsymbol{p}^*$ uniquely characterized by the FOCs $\boldsymbol{D_p\Pi}(\boldsymbol{p}^*) = \boldsymbol{0}$, where $\boldsymbol{0}$ is the relevant null vector; i.e., $i = 1, \ldots, N$:

$$\pi_i'(p_i^*) = (p_i^* - c_i)q_i'(p_i^*) + q_i(p_i^*) = 0\,, \tag{1}$$

which is just the familiar condition of marginal revenue and marginal cost equality in each market. On the contrary, if the monopolist is forced to use a uniform price he would charge all buyers p^*, which is uniquely defined by:

$$\sum_{i=1}^{N}\pi_i'(p^*) = \sum_{i=1}^{N}[(p^* - c_i)q_i'(p^*) + q_i(p^*)] = 0\,. \tag{2}$$

It is standard to adopt Joan Robinson's terminology and to call a market i "strong" if $p_i^* > p^*$: conventionally, we write that in such a case $i \in S$. Accordingly, a market i is "weak" if $p_i^* < p^*$ (and thus $i \in W$) and "intermediate" if $p_i^* = p^*$ (and thus $i \in I$). Schmalensee (1981: pp. 243-4) noted that the removal of the uniform pricing constraint can be usefully characterized as follows. Consider the problem of maximizing $\boldsymbol{\Pi}(\boldsymbol{p})$ under the linear constraint:

$$\sum_{i=1}^{N}\pi_i'(p^*)(p_i - p^*) \leq t\,, \tag{3}$$

[7] Bold characters are used to indicate vectors; conventionally, $\boldsymbol{x}$ is a column vector and $\boldsymbol{x}'$ indicates its transpose (a row vector).

[8] Without concavity of the profit (of the demand) functions, price discrimination might even move all prices in the same direction: see Nahata et alii (1990).

where t is some nonnegative real number. The solution $\boldsymbol{p}(t)$ to such a problem is uniquely defined by the FOCs $\boldsymbol{D_p}\boldsymbol{\Pi}(\boldsymbol{p}(t)) = \lambda(t)\boldsymbol{D_p}\boldsymbol{\Pi}(p^*\boldsymbol{\iota})$, where $\boldsymbol{\iota}$ is the relevant unit vector and $\lambda(t)$ is the nonnegative relevant Lagrangean multiplier (it measures the marginal increase in profit due to a marginal increase in t: i.e., $\lambda(t) = d\Pi(\boldsymbol{p}(t))/dt = \Sigma_i \pi_i'(p_i(t))p_i'(t))$.

The price system $\boldsymbol{p}(t)$ has a number of properties:[9] in particular, $\boldsymbol{p}(0) = p^*\boldsymbol{\iota}$ and $\lambda(0) = 1$ (this value depends on the normalization of the price differences implicit in (3)), and there exits a finite $\underline{t} \geq 0$ such that $\boldsymbol{p}(t) = \boldsymbol{p}^*$ and $\lambda(t) = 0$ for all $t \geq \underline{t}$. Moreover, $\lambda'(t) < 0$ and $sign\{p_i'(t)\} = sign\{\pi_i'(p_i(t))\} = sign\{\pi_i'(p^*)\}$ for all $t < \underline{t}$, $i = 1, \ldots, N$. Thus, $i \in S$ ($i \in W$) only if $\pi_i'(p^*) > 0$ ($\pi_i'(p^*) < 0$) and, unless all markets are intermediate (in which case the imposition of uniform pricing would have no effect at all), the removal of the uniform pricing constraint monotonically increases some prices and decreases others. Analogously, let us introduce the "virtual" unit cost of market i at t, $c_i(t)$, uniquely characterized by the condition:

$$(p_i(t) - c_i(t))q_i'(p_i(t)) + q_i(p_i(t)) \equiv 0. \tag{4}$$

Notice that $c_i(t)$ is obviously equal to market i's marginal revenue at $q_i(p_i(t))$: i.e., $\pi_i'(p_i(t)) = (c_i(t) - c_i)q_i'(p_i(t))$. Clearly, $i \in S$ ($i \in W$) if and only if $c_i > c_i(t)$ ($c_i < c_i(t)$), for $t < \underline{t}$ ($c_i(\underline{t}) = c_i$), $i = 1, \ldots, N$. This says that price rises (decreases) are "naturally" associated to high (low) marginal costs and low (high) marginal revenue at t. Finally, notice that $\boldsymbol{\iota}'\boldsymbol{D_p}\boldsymbol{\Pi}(\boldsymbol{p}(t)) = \Sigma_i \pi_i'(p_i(t)) = 0$ for all t (a sort of "average" equality of marginal revenues and marginal costs applies to all t), and thus, by differentiation:

$$\sum_{i=1}^{N} \pi_i''(p_i(t))p_i'(t) = \sum_{i=1}^{N} [2q_i'(p_i(t)) + (p_i(t) - c_i)q_i''(p_i(t))]p_i'(t) = 0 \tag{5}$$

(Schmalensee, 1981: p. 244, provides a graphical analysis for the case with $N = 2$ and $c_1 = c_2$; see below our Figure 3).

Very generally, Varian (1985) established the following welfare bounds for any change in (linear) prices from $\boldsymbol{p}_0$ to $\boldsymbol{p}_1$ (the result follows from the convexity of $v(\cdot)$):

$$\boldsymbol{p}_0'\boldsymbol{\Delta q} - \Delta C \geq \Delta W \geq \boldsymbol{p}_1'\boldsymbol{\Delta q} - \Delta C, \tag{6}$$

[9] Interestingly, one might think of $(\boldsymbol{p}(t),\lambda(t))$ as a dynamic system for which $-\Pi(\boldsymbol{p}(t))$ results in a Liapunov function and $(\boldsymbol{p}^*,0)$ is a globally stable equilibrium.

where $\boldsymbol{\Delta q} = \boldsymbol{q}(\boldsymbol{p}_1) - \boldsymbol{q}(\boldsymbol{p}_0)$, $\Delta C = C(\boldsymbol{q}(\boldsymbol{p}_1)) - C(\boldsymbol{q}(\boldsymbol{p}_0))$ and $\Delta W = W(\boldsymbol{p}_1) - W(\boldsymbol{p}_0)$.[10] Basically, (6) says that the price systems $\boldsymbol{p}_0$ and $\boldsymbol{p}_1$ provide useful information to assess the welfare effect of the change $\boldsymbol{\Delta q}$. In our setting, i.e. with $\boldsymbol{p}_0 = p^*\boldsymbol{\iota}$ and $\boldsymbol{p}_1 = \boldsymbol{p}^*$, (6) specializes to the linear (with respect to $\boldsymbol{\Delta q}$) expression:

$$(p^*\boldsymbol{\iota} - \boldsymbol{c})'\boldsymbol{\Delta q} \geq \Delta W \geq (\boldsymbol{p}^* - \boldsymbol{c})'\boldsymbol{\Delta q}\,, \tag{7}$$

which immediately says, as Varian (1985: p. 872) noted, that *if* marginal cost is the same across markets (i.e., $\boldsymbol{c} = c\boldsymbol{\iota}$) an increase in total output ($\boldsymbol{\iota}'\boldsymbol{\Delta q} > 0$) is a necessary condition for welfare to increase following a change from uniform pricing (notice that this does not depend on p^* and $\boldsymbol{p}^*$ being chosen by a maximizing monopolist). This result, already established by Schmalensee (1981) a few years earlier, confirmed a conjecture that dates back to the seminal works of Arthur C. Pigou (1920: chapter 16) and Joan Robinson (1933: chapter 13).[11]

In fact, it is easy to see that, if total output does not change, the change in welfare can be written $\Delta W = \Delta v + \Delta_P p - \Delta C$, where $\Delta_P p = \boldsymbol{q}(\boldsymbol{p}^*)'(\boldsymbol{p}^* - p^*\boldsymbol{\iota}) = \boldsymbol{q}(\boldsymbol{p}^*)'\boldsymbol{\Delta p}$ is the relevant Paasche price variation for the representative consumer, and $-\Delta v = v(p^*\boldsymbol{\iota}) - v(\boldsymbol{p}^*)$ is the relevant Hicksian equivalent variation (see Bertoletti, 2002). And it is a well-known result of consumer theory (see e.g. Deaton and Muellbauer, 1980: chapter 7) that the former variation is never larger than the latter (unless in the very special case of zero substitution effects, in which case they are equal). Thus, to avoid a welfare decrease following price differentiation (which causes a decrease in (gross) consumer surplus if total output does not change), we need a (strict) decrease in total cost. Notice that this result is due to the underlying preferences (and to the use of linear prices), and does not depend on the cost structure.

Following Schmalensee (1981: p. 244), by using (5) it is also easy to see that the change in total output, $Q(\boldsymbol{p}(t)) = \Sigma_i q_i(p_i(t))$, according to t is given by:

$$\begin{aligned}\frac{dQ(\boldsymbol{p}(t))}{dt} &= \sum_{i=1}^{N} q_i'(p_i(t))p_i'(t) = -\frac{1}{2}\sum_{i=1}^{N}(p_i(t)-c_i)q_i''(p_i(t))p_i'(t) \\ &= \frac{1}{2}\sum_{i=1}^{N}[\frac{q_i(p_i(t))}{q_i'(p_i(t))} - (c_i(t)-c_i)]q_i''(p_i(t))p_i'(t).\end{aligned} \tag{8}$$

This shows that total output does not change (i.e., $\boldsymbol{\iota}'\boldsymbol{\Delta q} = 0$) if the demand system is linear ($q_i'' = 0$, $i = 1, ..., N$), as Schmalensee (1981) noted for the case with a common unit cost. This result, that

[10] Notice that $\boldsymbol{p}_0'\boldsymbol{\Delta q}$ and $\boldsymbol{p}_1'\boldsymbol{\Delta q}$ are respectively the Laspeyres and Paasche quantity variations.
[11] Varian (1985: p. 875) also showed that the result generalizes to the case of (common) increasing marginal costs, and Schwartz (1990) proved that it applies to the case of (common) decreasing marginal costs as well.

again goes back to Pigou and Robinson, generalizes to the case of interdependent demands, and even to the case of *common* variable marginal costs: see Appendix A. Notice that the second term in the last expression of (8) can be signed according to the concavity of demand: i.e., it is positive if and only if demand of market i is strictly convex ($q_i'' > 0$).

III. The case of equal elasticities

The two results concerning total output illustrated in the previous section are at the basis of the skeptical view of monopolistic differentiated pricing that we mentioned in section I. That view is rather widespread in spite of being in striking contrast with the related literature on price regulation, which tends to suggest that to allow a monopolist some price flexibility is welfare enhancing: see e.g. Armstrong and Vickers (1991) and Vickers (1997). In fact, we believe that a negative assessment of price differentiation is generally unwarranted, because the welfare criterion based on total output generally fails. Consider the following illustrative examples. Suppose that demands have the same elasticity at the uniform price p^*, i.e., consider a case somehow "dual" with respect to the standard setting. Admittedly, as it turns out, this is a favorable situation for allowing price differentiation, but it should be borne in mind that it would be difficult to defend the use of uniform pricing if demand were very different. Moreover, intuition (and results below) suggests that the real point is the correlation across markets among the demand and the cost differences: why should we think of any specific correlation in the general case? Accordingly, we start with the no-correlation simplest case.

First, also suppose that demands are linear. Then it must be the case that the imposition of uniform pricing does not change total output. In the 2-goods case the situation is depicted in Figure 1, where $\Delta Q = 0$ indicates the locus of prices which corresponds to the same total output $Q(p^*\boldsymbol{\iota})$, and $V = v(p^*\boldsymbol{\iota})$ is the relevant consumer surplus indifference curve. The vector $\boldsymbol{q}(p^*\boldsymbol{\iota})$ is orthogonal to the plane $\Delta Q = 0$ due to the assumption of equal demand elasticities. In fact, the no total-output change condition can be written: $-\Sigma_i(q_i(p^*)\varepsilon_i(p^*)/p^*)dp_i = 0$, where $\varepsilon_i(p_i) = -q_i'(p_i)p_i/q_i(p_i)$ is the demand elasticity in market i. Notice that any differentiated price vector actually chosen by a profit-maximizing monopolist would *increase* consumer surplus, and thus social welfare by a trivial "revealed preference" result.

The previous argument depends on another well-known result of consumer theory (again see e.g. Deaton and Muellbauer, 1980: chapter 7): i.e., that the Laspeyres price variation $\Delta_L p = \boldsymbol{q}(p^*\boldsymbol{\iota})'(\boldsymbol{p}^* - p^*\boldsymbol{\iota}) = \boldsymbol{q}(p^*\boldsymbol{\iota})'\boldsymbol{\Delta p}$ is never larger than the Hicksian equivalent variation (the price locus $\Delta_L p = 0$ just describes the hyperplane tangent to $V = v(p^*\boldsymbol{\iota})$ at $p^*\boldsymbol{\iota}$). Thus, a nonpositive Laspeyres price variation is a sufficient condition for a consumer surplus (and then welfare) increase. In our example of equal elasticities and linear demands, the Laspeyres price variation from uniform pricing is given by $\Sigma_i q_i(p^*)(p_i^* - p^*) = - p^*\varepsilon(p^*)^{-1}\Sigma_i q_i'(p^*)\Delta p_i$ (where $\varepsilon(p^*)$ is the common value of the demand elasticity), which is nonpositive if:

$$\sum_{i=1}^{N} q_i'(p^*)\Delta p_i = \sum_{i=1}^{N} \Delta q_i = \Delta Q \geq 0, \tag{9}$$

where $\Delta Q = Q(\boldsymbol{p}^*) - Q(p^*\boldsymbol{\iota})$.

Now suppose that demands are (strictly) concave: in that case, depicted by Figure 2, the constant total output locus will have a concave graph. In this situation (concave demands with equal elasticities at $p^*\boldsymbol{\iota}$) any price differentiation by the monopolist that does not decrease total output would clearly realize a *sufficient* condition for a welfare improvement. Formally, in (9): $q_i'(p^*)\Delta p_i \geq \Delta q_i$. Indeed, note that, if demands are concave, $p^*\boldsymbol{\iota}$ *minimizes* $v(\boldsymbol{p})$ over the set $\{\boldsymbol{p} \mid Q(\boldsymbol{p}) \geq Q(p^*\boldsymbol{\iota})\}$. This property of the uniform pricing (which does not depend on the cost structure) might come as a surprise, but it is just due to the familiar substitution effect. Suppose that, starting from uniform pricing, prices are differentiated just in such a way that to an increase $dq_i > 0$ in a market i there corresponds a decrease $dq_j = - dq_i < 0$ in another market j. The first-order impact on the representative consumer's utility due to this changes is given by $dV = - q_i(p)dp_i - q_j(p)dp_j = p(dq_i/\varepsilon_i(p) + dq_j/\varepsilon_j(p)) = pdq_i(1/\varepsilon_i(p) - 1/\varepsilon_j(p))$, where p is the uniform price. The previous expression is of course null if demand elasticities are equal at the uniform price, but the overall impact will be positive because consumers trade the monopolist's commodities off against the other goods (in our quasi-linear preferences setting, substituting them with and for the *numeraire*). That is, in the case of both markets the first-order term overstates the true impact on consumer surplus (this is just a verbal restatement of the convexity of $v(\cdot)$). Notice that the first-order impact of a price change on the consumer surplus depends on the demand elasticity and the related quantity change.

Finally, suppose that demands are isoelastic, i.e., $q_i(p_i) = k_i p^{-\varepsilon_i}$, with $k_i > 0$ and $\varepsilon_i > 1$. It is easy to see that, under the assumption of equal demand elasticities ($\varepsilon_i = \varepsilon$, $i = 1, ..., N$), $\Delta_L p = 0$: see Appendix B. Accordingly, in such a case monopolistic price differentiation increases total output, welfare and aggregate consumer surplus. More generally, even in the case of (possibly strictly) convex demand functions, a welfare improvement is achieved if the price change from the uniform price structure is "small" and output does not decrease. To see it formally, consider the change in welfare associated with $t < \underline{t}$:

$$
\begin{aligned}
\frac{dW(\boldsymbol{p}(t))}{dt} &= \sum_{i=1}^{N}[(c_i(t)-c_i)q_i'(p_i(t))p_i'(t)-q_i(p_i(t))p_i'(t)] \\
&= \sum_{i=1}^{N}(p_i(t)-c_i)q_i'(p_i(t))p_i'(t) \qquad (10) \\
&= \sum_{i=1}^{N}p_i(t)\frac{dq_i(p_i(t))}{dt}-\frac{dC(\boldsymbol{q}(\boldsymbol{p}(t)))}{dt}.
\end{aligned}
$$

The first term in the square bracket in (10) reflects the change in profit in market i and is always positive, while the second is the correspondent change in consumer surplus, which is positive only if $i \in W$. Since for large enough t it will be $p_i(t) > c_i$ for all i,[12] the second expression in (10) makes it clear that in general it is not possible to sign the overall welfare impact of the increase in t between 0 and $\underline{t}$. But note that welfare is increasing at t if, geometrically, the projection of $d\boldsymbol{q}(\boldsymbol{p}(t))/dt$ on $(\boldsymbol{p}(t) - \boldsymbol{c})$ points in the same direction as the latter vector (i.e., if the inner product $(\boldsymbol{p}(t) - \boldsymbol{c})'d\boldsymbol{q}(\boldsymbol{p}(t))/dt$ is positive). Since the monopolist will choose the price change $d\boldsymbol{p}(t)/dt$ which achieves the maximum (available) value for $(\boldsymbol{c}(t) - \boldsymbol{c})'d\boldsymbol{q}(\boldsymbol{p}(t))/dt$, this may point towards a "wrong" direction only if $(\boldsymbol{p}(t) - \boldsymbol{c}(t))'d\boldsymbol{q}(\boldsymbol{p}(t))/dt = -\Sigma_i q_i(p_i(t))p_i'(t) < 0$. That is, only if price rises are associated with the largest demands (so that consumer surplus decreases).

The third expression in (10) is just a continuous version of (7). It implies:[13]

$$
\frac{dW(\boldsymbol{p}(0))}{dt} = p^* \frac{dQ(\boldsymbol{p}(0))}{dt} - \sum_{i=1}^{N} c_i \frac{dq_i(p_i(0))}{dt}, \qquad (11)
$$

which means that a *piecemeal policy* allowing the introduction of (small) price differentiation would cause a welfare improvement by reducing the *average* total cost if it does not also reduce total output, except in the "perverse" case in which quantity rises are on average associated with large marginal costs. Notice that $c_i(0) = p^*/m_i(p^*)$, where $m_i(p_i) = \varepsilon_i(p_i)/(\varepsilon_i(p_i) - 1)$ is the monopolistic mark up in market i. Thus, if demands have the same elasticity, in spite of their possibly being convex, $d\boldsymbol{q}(\boldsymbol{p}(0))/dt$ and $(\boldsymbol{p}(0) - \boldsymbol{c}(0)) = (p^* - c(0))\boldsymbol{\iota}$ (where $c_i(0) = c(0)$, $i = 1, ..., N$) are at worst orthogonal if $dQ(\boldsymbol{p}(0))/dt \geq 0$. In particular, in such a case, by using:

$$
\frac{dC(\boldsymbol{q}(\boldsymbol{p}(t)))}{dt} = \sum_{i=1}^{N} c_i(t) \frac{dq_i(p_i(t))}{dt} - \lambda(t), \qquad (12)
$$

and $\lambda(0) = 1$, we get:

[12] Note that $(p_i(t) - c_i)$ may even be negative in some markets for some t, but in such a case the correspondent element of the welfare change would certainly be positive.
[13] Notice that $dW(\boldsymbol{p}(0))/dt = 0$ if the demand system is linear and marginal cost is the same across markets.

$$\frac{dW(\boldsymbol{p}(0))}{dt} = \frac{p^*}{\varepsilon(p^*)} \frac{dQ(\boldsymbol{p}(0))}{dt} + 1 . \tag{13}$$

Thus, if demands have the same elasticity, a profit-maximizing monopolistic "piecemeal policy" of price differentiation would indeed generally (whatever the concavity of demand) be welfare improving if it does not decreases total output. Notice that the first term on the right-hand side of (13) is just $dv(\boldsymbol{p}(t))/dt$, which says that, for a small price differentiation, (aggregate) consumer surplus increases if and only if total output increases.

Moreover, it is not difficult to see that, under the stronger assumption of *equal demands*, total output increases only if demand is strictly convex. In fact, by evaluating (8) at $t = 0$, it follows that, in the case of equal demands:

$$\begin{aligned}\frac{dQ(\boldsymbol{p}(0))}{dt} &= \frac{1}{2}\frac{q''(p^*)}{q'(p^*)}\left[\frac{q(p^*)}{q'(p^*)}\frac{dQ(\boldsymbol{p}(0))}{dt} - \lambda(0)\right] \\ &= -\frac{q''(p^*)q'(p^*)}{2q'(p^*)^2 - q(p^*)q''(p^*)}.\end{aligned} \tag{14}$$

(14) says that, if demands are equal across markets, when allowing a small amount of price differentiation total output decreases if demand is strictly concave ($q'' < 0$) and increases if demand is strictly convex.[14] In turn, (14) implies that:

$$\frac{dW(\boldsymbol{p}(0))}{dt} = \frac{2q'(p^*)^2}{2q'(p^*)^2 - q(p^*)q''(p^*)}, \tag{15}$$

which says that, if demands are equal across markets, a piecemeal policy introducing small price differentiation always increases welfare in our setting (see footnote 14). Of course, if marginal costs are all equal as well, as in the standard set up, the unregulated monopolist adopts $\boldsymbol{p}^* = p^* \boldsymbol{\iota}$, and the removal of the uniform pricing constraint has neither a positive nor negative effect.

We summarize our results in the following Proposition.

PROPOSITION 1.

I. *If under uniform pricing the demand elasticities are equal across markets, monopolistic price differentiation does increase both welfare and aggregate consumer surplus:*

1) *if demands are linear;*
2) *if demands are isoelastic;*

[14] This is true under demand convexity if $2\varepsilon(p^*) < \eta(p^*)$, where $\eta(p^*) = - p^* q''(p^*)/q'(p^*)$, but such a condition must be satisfied if the profit function has to be concave. Notice that the previous condition is always satisfied if demands are isoelastic.

3) *if demands are concave and total output does not decrease.*

II. *If only small price differentiations (a so-called piecemeal policy) are allowed to the monopolist:*

4) *welfare always increases under equal demands (if demand is strictly convex total output increases, raising consumer surplus as well);*
5) *welfare and (weakly) consumer surplus increase if total output does not decrease under equal demand elasticities.*

In fact, a profit-maximizing monopolist freed by the uniform pricing constraint should be willing to use this freedom to decreases the *average* unit cost, even when leaving total output unchanged (as happens in the benchmark linear case). And this could then be welfare improving, as suggested by (7) in the case of discrete price variations. However, if elasticities at the uniform price are different, he might be willing to increase prices in markets which are more costly to serve (if demand elasticities are positively correlated to marginal cost), and the directions of cost and output variations become difficult to establish even for a small price differentiation. Moreover, consumer surplus might decrease more rapidly than profit increases, thus reducing total welfare. This tends to happen if prices rise in markets where quantities are larger, which may well be the case since in those markets the demand elasticities tend to be smaller. Will in fact some of the previous results apply to the general case of unequal demand functions?

IV. The linear case

Following Varian (1985: pp. 873-4), we can notice that, by (1), the right-hand side of (7) can be written: $\Sigma_i c_i \Delta q_i/(\varepsilon_i(p_i^*) - 1)$, where $\Delta q_i = q_i(p_i^*) - q_i(p^*)$ (note that, if $c_i/(\varepsilon_i(p_i^*) - 1) = (p_i^* - c_i) = \rho$, $i = 1, ..., N$, then a weak increase in total output is *sufficient* for monopolistic price differentiation to produce a welfare improvement). Moreover, for concave demand functions, a sufficient condition for the right-hand side of (7) being non negative is that the Paasche price variation $\Delta_P p$ is non positive.[15] However, while they have different information requirements (the latter being considerably less demanding, since it does not involve unit costs), $\Sigma_i c_i \Delta q_i/(\varepsilon_i(p_i^*) - 1) \geq 0$ and $\Delta_P p \leq 0$ are *equivalent*, *sufficient* conditions for a welfare improvement under monopolistic pricing if the demand system is linear. We also know from the discussion in section II that, under linear demands, a *necessary* condition for avoiding a welfare decrease due to price differentiation is that total cost

[15] Notice that this does not guarantee that consumer surplus does not decrease: again see e.g. Deaton and Muellbauer (1980: chapter 7).

does decrease. In fact, in the case of a linear demand system the welfare bounds in (7) become: $-\Delta C \geq \Delta W \geq -\Delta_P p$.[16]

Since $\Delta_L p \leq 0$ is always a *sufficient* condition for a consumer surplus (and then a welfare) improvement, it thus turns out that under linear demands we can simply replace the invalid output criterion by checking the sign of the price variations $\Delta_L p$ and $\Delta_P p$. Negative value for those variations are indeed sufficient conditions respectively for a consumer surplus or just for a welfare improvement. Note that these conditions are valid also for the case of strictly concave demands, and that their verification does not require cost knowledge.[17] Of course, in principle one can also directly compute the welfare variations, but in general it is difficult to draw unambiguous conclusions (we present some relevant statistics in Appendix C). Notice that it is easily obtained that $p^* = (Cov\{c_i,b_i\} + \underline{a} + \underline{bc})/(2\underline{b})$ and $p_i^* = (a_i + b_i c_i)/(2b_i)$, where $q_i(p_i) = a_i - b_i p_i$ ($a_i,b_i > 0$, $a_i/b_i > c_i$, $i = 1, ..., N$), $Cov\{c_i,b_i\} = (\Sigma_i c_i b_i)/N - \underline{cb}$ is the covariance between c_i and b_i across markets, and $\underline{a} = (\Sigma_i a_i)/N$, $\underline{b} = (\Sigma_i b_i)/N$ and $\underline{c} = (\Sigma_i c_i)/N$ are respectively the average value of a_i, b_i and c_i. Also notice that the previous expressions imply that the average value of $-b_i \Delta p_i = \Delta q_i$ is null, as we already knew. Finally, note that $(p_i^* - c_i) = a_i/(2b_i) - c_i/2$.

The simplest (unambiguous) case arises if a_i/b_i is the same across markets (notice that this restricts demand variability only at a single point, perhaps very far from the quantity-price couples observed in the market).[18] In that case $(p_i^* - p_j^*) = (c_i - c_j)/2$ and thus the only reason for monopolistic price differentiation is to reflect the marginal cost differences (in the standard setting with a marginal cost common across markets, one gets $p^* = p_i^*$ and allowing price differentiation has no effect at all).[19] In our setting, one should then expect that relaxing the constraint of uniform pricing would increase welfare. Indeed, allowing the monopolist price flexibility once again increases both welfare and consumers surplus. To see why, consider that in the linear case:

$$\Delta W = \frac{1}{2}[\Delta\Pi + \sum_{i=1}^{N}(\frac{a_i}{b_i} - c_i)\Delta q_i], \tag{16}$$

where $\boldsymbol{\Delta\Pi} = \Pi(\boldsymbol{p}^*) - \Pi(p^*\boldsymbol{\iota})$, and then:

[16] There seems to be a misprint in Varian (1985: p. 875) concerning the expression of the welfare change when the demand system is linear.

[17] It is worth noting that the actual computation of these price variations is common practice in the industries regulated by price caps: see e.g. Armstrong and Vickers (1991).

[18] The vertical intercept parameter of a linear demand function is sometimes identified with the "reservation price" (willingness to pay) of the "richest" consumer: see for example Tirole (1988: pp. 143-4). But in our setting the assumption of quasi-linear preferences and no income effect does preclude such an interpretation.

[19] Notice that this is nevertheless a case of price discrimination according to the Stigler's broad definition we quoted in section I.

$$\Delta v = \frac{\sum_{i=1}^{N}(\frac{a_i}{b_i} - c_i)\Delta q_i - \Delta \Pi}{2}. \tag{17}$$

Thus, the condition $\Sigma_i(a_i/b_i - c_i)\Delta q_i > 0$ is sufficient for a welfare improvement and necessary for a consumer surplus improvement. Moreover, that condition is equivalent to $\Delta C < 0$ if $a_i/b_i = \kappa$, $i = 1, ..., N$: i.e., if all demands have the same vertical intercept, a decrease in total cost is a necessary and sufficient condition for a welfare improvement. In such a case (see Appendix C for details):

$$\Delta W = \frac{3N}{8}[\sum_{i=1}^{N}\frac{(c_i b_i)^2}{Nb_i} - \frac{1}{\underline{b}}\left(\frac{\sum_{i=1}^{N}\mathrm{c}_i b_i}{N}\right)^2] = 3\Delta v, \tag{18}$$

which, since the function $g(x,y) = x^2/y$ is convex on $\Re^2_+$, is necessarily non negative by the Jensen's inequality (notice that the right-hand side of (18) becomes $(3bN/8)Var\{c_i\}$ if the demand functions are just equal across markets).

To fully grasp the previous result, consider the case of a piecemeal policy of price differentiation with linear, possibly different, demands. We already know that total output does not change along the path from 0 to $\underline{t}$ (see (8)), which implies that $\Sigma_i b_i p_i'(t) = 0$. Moreover, (4) implies that $c_i(t) = 2p_i(t) – a_i/b_i$: accordingly, marginal revenues (demand elasticities) are equal at $t = 0$, i.e., $c_i(0) = c(0)$, $i = 1, ..., N$, if all demands have the same vertical intercept. We can immediately deduce from our analysis in section II that in such a case welfare increases and consumer surplus does not change following the introduction of small price differentiation. Note that, more generally, we get from (11) and (12):

$$\frac{dW(\boldsymbol{p}(0))}{dt} = -\sum_{i=1}^{N} c_i \frac{dq_i(p_i(0))}{dt} = \sum_{i=1}^{N}(a_i / b_i)\frac{dq_i(p_i(0))}{dt} + 1, \tag{19}$$

which says that a necessary and sufficient condition for a welfare improvement is a negative correlation between c_i and $dq_i(p_i(0))/dt$, while a positive correlation between a_i/b_i (and thus $c_i(0)$) and $dq_i(p_i(0))/dt$ would be necessary and sufficient for a consumer surplus improvement. Note that while the former condition appears quite natural, the latter seems to require some form of correlation between the demand parameters. Moreover, what happens if marginal cost is the same across markets is that the increase in profit is exactly matched by the decrease in consumer surplus, so that there is no welfare change (in the case of a piecemeal policy). If, on the contrary, a_i/b_i is the same across markets, then there is no consumer surplus change and the increase in welfare is just due to cost reduction.

Figure 3 illustrates the situation with *equal marginal costs* for the 2-markets case. Without loss of generality, the price in the strong market is indicated on the horizontal axis. Points u, d, f indicate respectively the position of uniform pricing, unconstrained monopolistic pricing and the first best. We show three iso-profit loci (surrounding d) indicated by Π, and three iso-welfare loci (surrounding f) indicated by W. The portion ud of the plane $\Delta Q = 0$ is the locus of the $\boldsymbol{p}(t)$ price vectors.[20] The line fd is instead the locus of the Ramsey price vectors (tangency points between the iso-profit and iso-welfare curves). Notice that at u the relevant iso-welfare curve is tangent to the $\Delta Q = 0$ plane (since the slope of the former is given by $[- b_s(p_s - c_s)]/[b_w(p_w - c_w)]$): accordingly, u dominates any other point of that locus with respect to social welfare (since $a_s/b_s > a_w/b_w$, it is the case that at u the plane $\Delta Q = 0$ is steeper than the relevant representative consumer indifference curve (not shown for the sake of simplicity), whose slope is given by $- q_s(p_s)/q_w(p_w)$). Figure 4 illustrates instead the situation with *equal demand elasticities* at p^* ($a_s/b_s = a_w/b_w$). We show three iso-welfare loci and, for simplicity, a single iso-profit locus. Notice that the $\Delta Q = 0$ plane is steeper than the relevant welfare locus at u, while they are tangent at σ (thus $p_s - p_w = c_s - c_w$ at that point).[21] Since $p_s^* - p_w^* = (c_s - c_w)/2$, it must be that point d lies between u and σ. Accordingly, it necessarily welfare-dominates u.

Note that a very special case arises if $a_i/b_i - c_i = 2\rho$, $i = 1, ..., N$. In such a case (16) and (17) imply that $\Delta W = \Delta\pi/2 = - \Delta v > 0$, and of course $\Delta C < 0$, unless there is no market variability at all and $\boldsymbol{p}^* = p^*\boldsymbol{\iota}$ (notice that (19) implies $dC(\boldsymbol{q}(\boldsymbol{p}(0))/dt = - ½ = dv(\boldsymbol{p}(0)/dt = - dW(\boldsymbol{p}(0)/dt)$. The reason is simple: in this case the Ramsey price vector $\boldsymbol{p}^*$ satisfies the second-best conditions $(p_i^* - c_i) = \rho$, $i = 1, ..., N$, we mentioned in section I, and thus $\boldsymbol{p}^*$ *maximizes* $W(\boldsymbol{p})$ over the set $\{\boldsymbol{p} \mid Q(\boldsymbol{p}) = Q(p^*\boldsymbol{\iota})\}$. But, at the same time, $\boldsymbol{p}^*$ *minimizes* $v(\boldsymbol{p})$ over the previous set, since it equalizes q_i/q_i' across markets. The situation is illustrated in Figure 5 for the two-markets case (notice that points d and σ coincide). These results neatly illustrate the potential conflict between welfare versus consumer surplus concerns in our second-best setting. However, they require a good deal of cross demand and cost parameter correlation, which in general is hardly plausible.

Two other cases can be analyzed in more depth. Suppose that $a_i = a$, $i = 1, ..., N$: i.e., all demands have the same horizontal intercept. In such a case we get:

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[20] Notice that ud is the locus of tangencies of the relevant iso-profit curves with 45° lines of the form $p_s - p_w = T$ (this follows from (3)).

[21] Also notice that in Figure 3 point σ would coincide with u.

$$\Delta C = \frac{aN}{2\underline{b}} Cov\{c_i, b_i\} + \frac{N}{2}[\frac{1}{\underline{b}} \left(\frac{\sum_{i=1}^{N} c_i b_i}{N} \right)^2 - \sum_{i=1}^{N} \frac{(c_i b_i)^2}{N b_i}]. \tag{20}$$

Thus, a non positive covariance of demand slope and marginal cost would be sufficient to get a cost decrease by price differentiation (if all demands have the same horizontal intercept but marginal costs differ). Notice that a negative covariance means that at p^* (at $t = 0$) higher marginal costs tend to be associated with smaller marginal revenues, so that average total cost should indeed be expected to decrease under price discrimination. The problem is that, under a negative covariance, price rises would also be associated with larger demands, so harming the representative consumer (as is suggested by (17) and (19)). Actually, this is the way monopolistic price discrimination operates if marginal costs are equal across markets. In fact, even assuming no cross correlation between cost and demand parameters (why should there be any in the general case?), the sign of the welfare (and also of the consumer surplus) change stays ambiguous (see (C6) in Appendix C). Clearly, in the two-markets case, a *necessary* condition for price differentiation to cause a welfare increase is that $c_s > c_w$ (market i is strong if and only if $a_i/b_i - a_j/b_j > c_j - c_i$, $i \neq j$, $i,j = 1,2$). It is easy to see that a *sufficient* condition for a welfare improvement is then $c_s - c_w > a_s/b_s - a_w/b_w$; i.e., $(c_s - c_w)/a > 1/b_s - 1/b_w$, if the horizontal intercept is common across markets (things are in such a case very much as in Figure 4). Indeed, $a_s/b_s < a_w/b_w$ (i.e., $b_w < b_s$ in the case of a common demand horizontal intercept) is a *sufficient* condition for monopolistic price differentiation to deliver even an aggregate consumer surplus increase (in fact, in such a case the no-total-output-change locus is steeper than the relevant representative consumer indifference curve at u). However, Figure 6 shows a "perverse" case in which $c_s < c_w$ and (any degree of) monopolistic price differentiation decreases social welfare with respect to the case of uniform pricing (notice that, in such a case, necessarily $a_s/b_s > a_w/b_w$ and thus price differentiation decreases aggregate consumer surplus).[22]

Now suppose that $b_i = b$, $i = 1, \ldots, N$: in such a case:

$$\Delta C = -\frac{N}{2}[Cov\{a_i, c_i\} + bVar\{c_i\}], \tag{21}$$

[22] Notice that Figures 4 and 6 can be thought of as the results of "perturbations" of the market fundamentals (the demand and cost parameters) with respect to the case of demand (elasticity) and marginal cost equal across markets. In the latter case the iso-profit loci would be centered on the uniform price, points u, d and σ would coincide (at that point the relevant indifference curve of the representative consumer would be tangent to the plane $\Delta Q = 0$, as in Figure 1) and of course the uniform pricing rule would have no hold.

where $Cov\{a_i,c_i\} = (\Sigma_i a_i c_i)/N - \underline{a}\ \underline{b}$ is the covariance between a_i and c_i across markets. Notice that a nonnegative covariance of demand intercept and marginal cost is a sufficient condition to get a cost decrease (if all demands have the same slope and costs differ). Indeed, in this case a positive covariance means that at p^* higher marginal costs tend to be associated to smaller marginal revenue, so that, once again, average total cost should be expected to decrease. But again a welfare improvement cannot be guaranteed, because even if a_i and c_i were uncorrelated, prices would rise in the markets with the largest intercept values, and this would have a negative first-order effect on consumer surplus.

In fact, a simple computation shows that (see Appendix C for details) that, with equal demand slope:

$$\Delta W = \frac{N}{8b}[4b^2 Var\{c_i\} - Var\{a_i - bc_i\}], \qquad (22)$$

and

$$\Delta v = \frac{N}{8b}[Var\{a_i - bc_i\} - 4Var\{a_i\}]. \qquad (23)$$

(22) and (23) confirm that a welfare improvement cannot be guaranteed, since it depends on the relative variability of a_i and c_i and on their correlation. In particular, notice in (22) that if the quantity demanded at the competitive equilibrium level is the same across markets, then ΔW achieves its maximum (positive) value. Of course this can only happen (with identical slope) if marginal costs are "perfectly" correlated with the intercept parameters (indeed, it requires $a_i = b(c_i + 2\rho)$, $i = 1, ..., N$: see above), and then Δv gets its minimum (negative) value, as shown by (23). However, assuming $Cov\{a_i,c_i\} = 0$, it can also be shown that $Var\{c_i\} > Var\{a_i\}/(3b^2)$ becomes a necessary and sufficient condition for a welfare improvement due to price differentiation, while $Var\{c_i\} > 3Var\{a_i\}/b^2$ becomes a necessary and sufficient condition for a consumer surplus increase (see (C7) in Appendix C). It is easy to see that, in the two-markets case, if the slope is common across markets, $c_s - c_w > (1/b)(a_s - a_w)$ is a *sufficient* condition for price differentiation to cause a welfare increase (it must also be the case that $c_s - c_w > (1/b)(a_w - a_s)$: then things are once again very much as in Figure 4; in addition, if $a_w > a_s$ monopolistic price differentiation also raises aggregate consumer surplus).

The main results of this section are summarized in Proposition 2.

PROPOSITION 2.

If demands are linear, monopolistic price differentiation:

1) *has ambiguous welfare effects in the general case, and a total output increase is* not *a necessary condition for a welfare*

improvement: however, negative Laspeyres and Paasche price variations provide sufficient conditions respectively for an aggregate consumer surplus or just a welfare improvement;

2) *increases both consumer surplus and welfare if demands have the same vertical intercept;*
3) *with a null cost and demand parameter covariance, it decreases total cost if demands have the same horizontal intercept or the same slope*, *and in the latter case also raises welfare* (or *even consumer surplus) if marginal cost variability is large enough with respect to demand variability.*

The lesson of Proposition 2 is that the traditional output criterion does not apply even to the linear demand system case unless one can be sure that marginal costs are equal. On the contrary, in such a case one can use the *Paasche* price index to check whether sufficient conditions for a welfare improvement are satisfied (this actually requires just demand concavity), or the *Laspeyres* price index to see if even an aggregate consumer surplus can be guaranteed (this more severe condition is indeed totally general). Moreover, information on the relative variability (and correlation) among demand and cost conditions appear crucial to judge the welfare impact of imposing monopolistic uniform pricing. It may even happen that the Ramsey pricing freely chosen by the profit-maximizing monopolist is the (second-best) welfare optimum.

V. Discussion and conclusions

In this paper we have re-considered the case for imposing uniform pricing on a monopolist in a third-degree price discrimination setting with (possibly) differentiated marginal costs. As should be expected, we find that such an imposition has ambiguous welfare results. However, in the case of equal demand elasticities at the uniform price (a case somehow dual to the one traditionally considered by the literature), it tends to be socially better to allow some (perhaps mild) form of price differentiation (also an aggregate consumer surplus increase can be guaranteed if total output does not decrease). The main reason is that the price flexibility granted to the monopolist produces in such circumstances socially valuable average cost reductions. Moreover, from the social point of view there is nothing special in a uniform price (we have shown that it can be the "worst" pricing option), unless it is coupled with a common marginal cost. Similar results apply to the benchmark case of differentiated linear demands, if they have the same vertical intercept or, under null demand and cost parameter covariance (a seemingly reasonable assumption in general) and equal demand slope, if the variability of marginal cost is large enough with respect to demand variability. We conclude that the welfare criterion based on total output to allow price differentiation is *very* fragile. On the contrary, sufficient robust conditions for achieving a consumer surplus improvement, or just a welfare increase (assuming demand concavity), can be checked by using the relevant (informatively undemanding) Laspeyres and Paasche price indexes.

The previous results come as no surprise. Phlips (1981: p. 1) observed more than twenty years ago that, due to the presence of some market power: "generally, discriminatory prices will be required for an optimal allocation of resources in real life situation". In fact, it is well known that in a second-best environment the imposition of an additional constraint may improve efficiency, but this is not always the case. It turns out that, even if output does not decrease, we cannot guarantee a welfare improvement by imposing a uniform pricing constraint on a monopolist, unless we are sure that the commodity is *exactly* the same. And apart from that case, the use of uniform pricing is just the addition of a further constraint to a second-best setting. The crucial element is then the assumption of a *single* commodity. Several authors noticed this difficulty (see e.g. Phlips, 1981: pp. 5-9 and Varian, 1989: p. 599), and it should be clear that "similar" commodities are not the same commodity if they have different marginal costs. Moreover, by now many years of "new" regulation theory have taught us that we cannot assume that cost differences are easily disclosed, not to say incorporated in a regulation practice. Indeed, one message of the recent price regulation literature, from which this paper draws inspiration, stresses the (at least potential) social usefulness of allowing some price flexibility to a (regulated) monopolist (see e.g. Vickers, 1997).

This does not mean that we disregard the contributions of the by now classic works by Schmalensee (1981), Varian (1985) and Schwartz (1990), or, needless to say, of the original intuitions of Pigou (1920) and Robinson (1933). In particular, we have used the model by Schmalensee (1981), and ideas and results of all the authors quoted above, to argue that, as a policy prescription, uniform pricing is not robust, even when corroborated by the requirement that total output should not increase by allowing price differentiation. Nowadays, it is widely accepted that possible social gains in the form of cost reductions should not be dismissed, and that the computation of well-defined Laspeyres or Paasche price indexes should replace the measurement of total output changes to serve consumer surplus or social welfare concerns (indeed, we owe these results to standard consumer theory, as first shown by Varian, 1985).

It is also worth emphasizing that our results do not sustain a policy of complete monopolistic price behavior liberalization. On the one hand, we have just argued that imposing price uniformity on an otherwise unregulated monopolist is generally not a promising strategy. On the other hand, the monopolistic strategic (anti-competitive and anti-regulative) tactics are probably too complex to be described by a simple model, and we did not even attempted to do so here (thus our conclusions do not necessarily conflict with the rules of the Robinson-Patman Act). Nor have we considered the use of uniform pricing in "competitive" environments, a topic only very recently investigated by the economic literature (see e.g. Stole, 2003, Galera, 2003 and Aguirre, 2004). In the latter case, in particular, it is for example well known that the European Authorities have reasons to endorse, to some extent, a policy of uniform pricing that relate to the making of the European common market and are not based on welfare considerations (even when they do not explicitly run counter to them: see e.g. Cabral, 2000: paragraph 10.5). And a similar comment would apply to the political constraints (mentioned in section I) based on “horizontal equity” concerns (even though the latter may be weakened by our result that price differentiation can also increase aggregate consumer surplus).

Finally, we also stress that we have assumed throughout the paper that the monopolist possesses all the demand and cost information needed to maximize profit, very much as in the “Ramsey- Boiteux” tradition (also see Schwartz, 1990: p. 1262). If this were not the case, it seems to us that it would be even more difficult to sustain the use of a uniform pricing constraint: why should the policymaker be assumed to know more than the monopolist? In particular, if the differentiated pricing structure just reflects the cost knowledge of the monopolist, who is unable to get reliable demand (elasticity) information (as in the case of the so-called “Allais doctrine”), it is hard to imagine any economically sound reasons to defend the use of a uniform price.

Appendix A

We wish to show that even if the demands are not independent (i.e., $\partial q_i/\partial p_j$ is possibly non zero for some $i \neq j$, $i,j = 1, \ldots, N$), relaxing the constraint of a uniform pricing does not change monopolistic total output if the demand system is linear and marginal costs are either constant or common. We will assume that the correspondent FOC conditions do characterize the solution of the relevant profit maximization programs. Equations (1) become:

$$\frac{\partial \Pi(\boldsymbol{p}^*)}{\partial p_i} = q_i(\boldsymbol{p}^*) + \sum_{j=1}^{N} (p_j^* - \frac{\partial C(\boldsymbol{q}(\boldsymbol{p}^*))}{\partial q_j}) \frac{\partial q_j(\boldsymbol{p}^*)}{\partial p_i} = 0\,, \tag{A1}$$

and by adding up

$$\sum_{i=1}^{N} \frac{\partial \Pi(\boldsymbol{p}^*)}{\partial p_i} = \sum_{i=1}^{N} q_i(\boldsymbol{p}^*) + \sum_{i=1}^{N} \sum_{j=1}^{N} (p_j^* - \frac{\partial C(\boldsymbol{q}(\boldsymbol{p}^*))}{\partial q_j}) \frac{\partial q_j(\boldsymbol{p}^*)}{\partial p_i} = 0\,; \tag{A2}$$

i.e., by using linearity of the demand system

$$2Q(\boldsymbol{p}^*) - Q(\mathbf{0}) - \sum_{i=1}^{N} \sum_{j=1}^{N} q_{ji} \frac{\partial C(\boldsymbol{q}(\boldsymbol{p}^*))}{\partial q_j} = 0\,, \tag{A3}$$

where $q_{ji} = \partial q_j/\partial p_i$, $i,j = 1, ..., N$. Similarly, (2) becomes:

$$\frac{d\Pi(p^*\boldsymbol{\iota})}{dp^*} = \sum_{i=1}^{N}[(p^* - \frac{\partial C(\boldsymbol{q}(p^*\boldsymbol{\iota}))}{\partial q_i})\sum_{j=1}^{N}\frac{\partial q_i(p^*\boldsymbol{\iota})}{\partial p_j} + q_i(p^*\boldsymbol{\iota})] = 0\,, \tag{A4}$$

i.e.,

$$2Q(p^*\boldsymbol{\iota}) - Q(\mathbf{0}) - \sum_{i=1}^{N}\sum_{j=1}^{N} q_{ij}\frac{\partial C(\boldsymbol{q}(p^*\boldsymbol{\iota}))}{\partial q_j} = 0 \tag{A5}$$

by linearity. Finally, notice that, since $q_{ji} = q_{ij}$,[23] the left-hand side of (A3) and (A5) are identical functions with respect to Q if either marginal costs $\partial C(\boldsymbol{q})/\partial q_j$ are constant or they are equal and only depend on total output Q. QED.

Appendix B

Assuming isoelastic demands, equations (1) become:

$$\pi_i'(p_i^*) = -\varepsilon_i k_i (p_i^* - c_i)(p_i^*)^{-\varepsilon_i - 1} + k_i (p_i^*)^{-\varepsilon_i} = 0\,, \tag{B1}$$

which imply $p_i^* = (\varepsilon_i c_i)/(\varepsilon_i - 1)$. Similarly, equation (2) becomes:

$$\sum_{i=1}^{N}\pi_i'(p^*) = \sum_{i=1}^{N}(p^*)^{-\varepsilon_i}[-\varepsilon_i k_i (1 - \frac{c_i}{p^*}) + k_i] = 0\,, \tag{B2}$$

which implies, assuming $\varepsilon_i = \varepsilon$ ($i = 1, ..., N$), $p^* = (\varepsilon Cov\{c_i,k_i\}/\underline{k} + \varepsilon\underline{c})/(\varepsilon - 1)$, where $Cov\{c_i,k_i\} = (\Sigma_i c_i k_i)/N - \underline{ck}$ is the covariance between c_i and k_i across markets, and $\underline{k} = (\Sigma_i k_i)/N$ is the average value of k_i. Since $\Delta p_i = \varepsilon[c_i - \underline{c} - Cov\{c_i,k_i\}/\underline{k}]/(\varepsilon - 1)$, it follows that:

$$\begin{aligned} \Delta_L p &= \sum_{i=1}^{N} q_i(p^*)\Delta p_i \\ &= \left[\frac{\varepsilon}{\varepsilon - 1}\right]^{1-\varepsilon}\left[\frac{Cov\{c_i,k_i\}}{\underline{k}} + \underline{c}\right]^{-\varepsilon}\left[\sum_{i=1}^{N}(k_i c_i) - N\underline{kc} - NCov\{c_i,k_i\}\right] = 0. \end{aligned} \tag{B3}$$

[23] The symmetry property $q_{ji} = q_{ij}$ follows from the quasi linearity of consumers' preferences: also see Layson (1998).

Appendix C

In the case of a linear demand system, direct computation shows that the following equalities hold:

$$\Delta C = \frac{N}{2\underline{b}}[Cov\{b_i, c_i\}(\underline{a} + 2\underline{b}\underline{c} + Cov\{b_i, c_i\}) - \underline{b}Cov\{a_i, c_i\} + \underline{b}(\underline{b}\underline{c}^2 - \sum_{i=1}^{N} \frac{b_i c_i^{\,2}}{N})], \tag{C1}$$

$$\Delta_L p = \frac{N}{2\underline{b}}[\underline{b}Cov\{a_i, c_i\} - \underline{a}Cov\{b_i, c_i\} + \underline{b}(\sum_{i=1}^{N} \frac{a_i^2}{Nb_i} - \frac{\underline{a}^2}{\underline{b}})], \tag{C2}$$

$$\Delta_P p = \frac{N}{4\underline{b}}[(Cov\{b_i, c_i\})^2 + 2\underline{b}\underline{c}Cov\{b_i, c_i\} + \underline{b}(\sum_{i=1}^{N} \frac{a_i^2}{Nb_i} - \frac{\underline{a}^2}{\underline{b}}) + \underline{b}(\underline{b}\underline{c}^2 - \sum_{i=1}^{N} \frac{b_i c_i^{\,2}}{N})]. \tag{C3}$$

Notice that the last term in the square bracket in (C2) (the third term in the square bracket in (C3))[24] is nonnegative by Jensen's inequality. Also notice that, if the demand parameters are not correlated with the marginal costs (perhaps the interesting case), we get $\Delta C \leq 0$ and $\Delta_L p \geq 0$, while $\Delta_P p$ cannot be signed (the last term in the square bracket in (C1) and (C3) is then certainly non positive). And remember that, by a general property, $-\Delta_P p \geq \Delta v \geq -\Delta_L p$. A little bit more can be said if demand parameters, in addition to not being correlated to marginal costs, are also uncorrelated between them. In particular, if (without demand functions and marginal costs being all equal) $a_i = a$ ($i = 1, ..., N$), we get $\Delta C = (N/2)[\underline{b}\underline{c}^2 - (\Sigma_i b_i c_i^2/N)] < 0$, $\Delta_L p = (a^2/2)[(\Sigma_i 1/b_i) - N/\underline{b}] > 0$ and $\Delta_P p = (N/4)\{a^2[\Sigma_i (Nb_i)^{-1} - 1/\underline{b}] + \underline{b}\underline{c}^2 - (\Sigma_i b_i c_i^2/N)\}$; while, if $b_i = b$ ($i = 1, ..., N$), we get $\Delta C = -(bN/2)Var\{c_i\} < 0$, $\Delta_L p = (N/(2b))Var\{a_i\} > 0$ and $\Delta_P p = (N/(4b))[Var\{a_i\} - b^2 Var\{c_i\}]$, which neatly illustrate the role of the relative variance of demand and cost parameters. Note that, in the latter case, $b^2 Var\{c_i\} > Var\{a_i\}$ is a sufficient condition for price differentiation to deliver a welfare improvement.

Moreover, if (without assuming away any cross demand and cost parameter correlation) a_i/b_i is the same across markets, then we get $\Delta_P p = \Delta C/2 = (N/(4\underline{b}))[(\Sigma_i b_i c_i/N)^2 - \underline{b}(\Sigma_i b_i c_i^2/N)] \leq 0$ and $\Delta_L p = 0$, which implies that consumer surplus (and thus welfare) increases if the monopolist actually differentiates prices (i.e., if marginal costs are different). In the special case in which the demand functions are equal (i.e., $a_i = a$ and $b_i = b$, $i = 1, ..., N$: see section III) it turns out that $\Delta C = -(bN/2)Var\{c_i\} = 2\Delta_P p \leq 0$: notice that this means that half of the cost decrease is "passed" to the representative consumer in the form of a revenue reduction. Also notice that, if $c_i = c$, $i = 1, ..., N$

[24] This term is clearly related to the variability of a_i/b_i.

(the standard set up in the literature), $\Delta C = 0$ (and thus welfare decreases if the monopolist does differentiate prices), and $\Delta_P p = \Delta_L p/2 = 1/4[(\Sigma_i a_i^2/b_i) - N(\underline{a}^2/\underline{b})] \geq 0$.

In addition, one can compute that:

$$
\begin{aligned}
\Delta W &= \sum_{i=1}^{N} \frac{[(p_i^* - c_i) + (p^* - c_i)]\Delta q_i}{2} \\
&= \frac{1}{2}\sum_{i=1}^{N}[(p^* - c_i)(q_i(c) - q_i(p^*)) - (p_i^* - c_i)(q_i(c) - q_i(p_i^*))] \\
&= \frac{-1}{2}\sum_{i=1}^{N} b_i \Delta p_i [p_i^* + p^* - 2c_i] = \frac{-1}{2}\sum_{i=1}^{N} b_i \Delta p_i [p_i^* - 2c_i] = \frac{1}{4}\sum_{i=1}^{N} b_i \Delta p_i [3c_i - \frac{a_i}{b_i}] \qquad \text{(C4)} \\
&= \frac{N}{8\underline{b}}[\underline{b}(\frac{\underline{a}^2}{\underline{b}} - \sum_{i=1}^{N}\frac{a_i^2}{Nb_i}) - 2\underline{a}Cov\{b_i, c_i\} + 2\underline{b}Cov\{a_i, c_i\} + 3\underline{b}(\sum_{i=1}^{N}\frac{(c_i b_i)^2}{Nb_i} - \frac{1}{\underline{b}}\left(\frac{\sum_{i=1}^{N} c_i b_i}{N}\right)^2)],
\end{aligned}
$$

and, finally:

$$
\begin{aligned}
\Delta v &= \frac{1}{2}\sum_{i=1}^{N}[(\frac{a_i}{b_i} - p_i^*)q_i(p_i^*) - (\frac{a_i}{b_i} - p^*)q_i(p^*)] \\
&= \frac{1}{2}\sum_{i=1}^{N} b_i \Delta p_i [p_i^* + p^* - \frac{2a_i}{b_i}] = \frac{1}{2}\sum_{i=1}^{N} b_i \Delta p_i [p_i^* - \frac{2a_i}{b_i}] = \frac{1}{4}\sum_{i=1}^{N} b_i \Delta p_i [c_i - \frac{3a_i}{b_i}] \qquad \text{(C5)} \\
&= \frac{N}{8}[3(\frac{\underline{a}^2}{\underline{b}} - \sum_{i=1}^{N}\frac{a_i^2}{Nb_i}) + \frac{2\underline{a}}{\underline{b}}Cov\{b_i, c_i\} - 2Cov\{a_i, c_i\} + (\sum_{i=1}^{N}\frac{(c_i b_i)^2}{Nb_i} - \frac{1}{\underline{b}}\left(\frac{\sum_{i=1}^{N} c_i b_i}{N}\right)^2)].
\end{aligned}
$$

Notice that the first terms in the square brackets of the last expressions in (C4) and (C5) are nonpositive, while the last terms are nonnegative. Also note that, as anticipated above, if a_i/b_i is the same across markets, allowing monopolistic price differentiation delivers a consumer surplus and welfare improvement: $\Delta W = (3N/8)[\Sigma_i(c_i b_i)^2/Nb_i) - (1/\underline{b})(\Sigma_i(c_i b_i)/N)^2] = 3\Delta v = 3/2\ \boldsymbol{\Delta\Pi} \geq 0$ ($\Delta W = (3bN/8)Var\{c_i\}$ if demand functions are just equal across markets: see section III). If, on the contrary, marginal costs are equal across markets, monopolistic price differentiation (discrimination) causes a welfare decrease, as is well known: $\Delta W = (N/8)[(\underline{a}^2/\underline{b}) - \Sigma_i(a_i^2/Nb_i)] = \Delta v/3 = -\boldsymbol{\Delta\Pi}/2 \leq 0$.

More generally, however, even with a null cross demand and cost parameter correlation, (C4) and (C5) have ambiguous signs. In particular, in such a case they reduce to:

$$\Delta W = \frac{N}{8\underline{b}}[a^2(1-\sum_{i=1}^{N}\frac{b}{Nb_i}) + 3\underline{b}(\sum_{i=1}^{N}\frac{c_i{}^2 b_i}{N} - \underline{c}^2\underline{b})],$$
$$\Delta v = \frac{N}{8\underline{b}}[3a^2(1-\sum_{i=1}^{N}\frac{b}{Nb_i}) + \underline{b}(\sum_{i=1}^{N}\frac{c_i{}^2 b_i}{N} - \underline{c}^2\underline{b})], \quad \text{(C6)}$$

and

$$\Delta W = \frac{N}{8b}[-Var\{a_i\} + 3b^2 Var\{c_i\}],$$
$$\Delta v = \frac{N}{8b}[-3Var\{a_i\} + b^2 Var\{c_i\}], \quad \text{(C7)}$$

respectively if $a_i = a$ or $b_i = b$ ($i = 1, ..., N$). Note that, in the latter case, (C7) says that $Var\{c_i\} > Var\{a_i\}/(3b^2)$ is a necessary and sufficient condition for a welfare improvement due to price differentiation, while $Var\{c_i\} > 3Var\{a_i\}/b^2$ is a necessary and sufficient condition for a consumer surplus increase.

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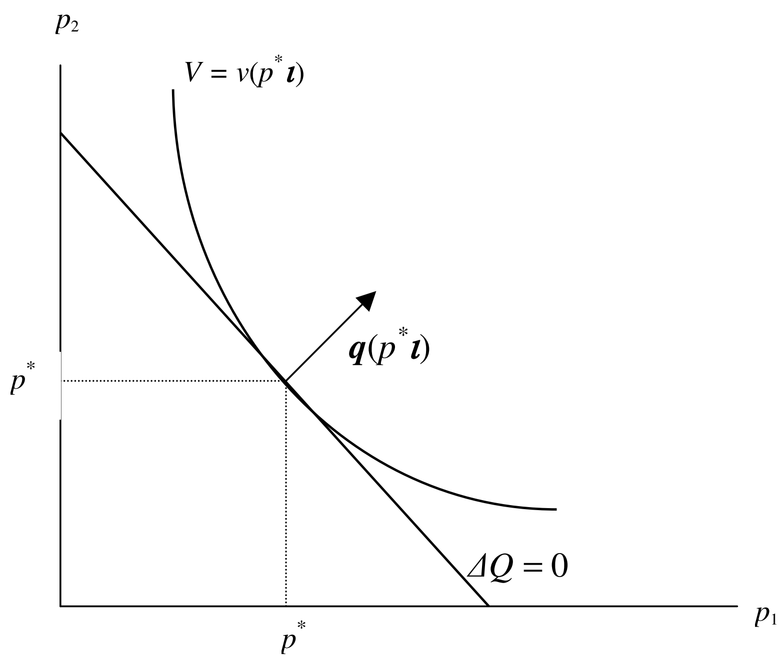

Figure 1: Uniform pricing with linear demands, equal demand *elasticites*

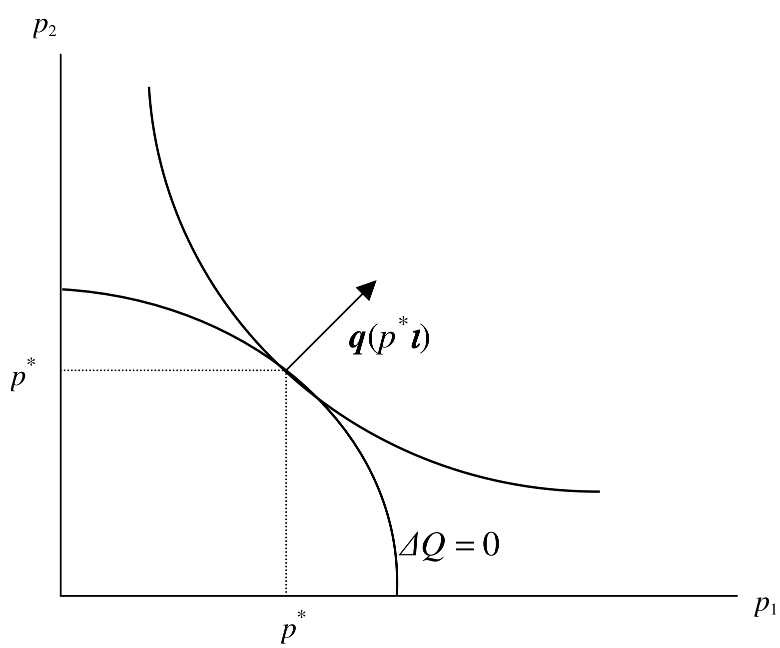

Figure 2: Uniform pricing with concave demands, equal demand *elasticites*

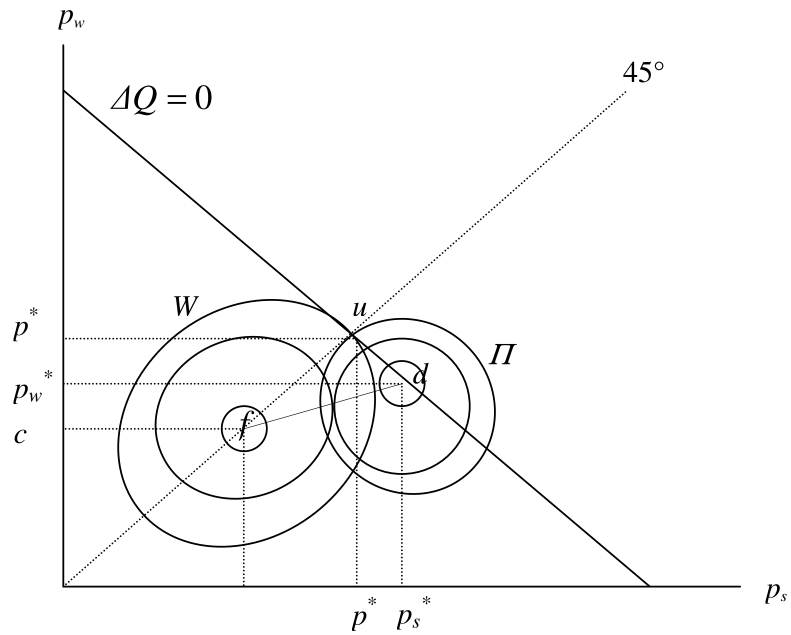

Figure 3: Monopolistic Price differentiation with equal marginal costs

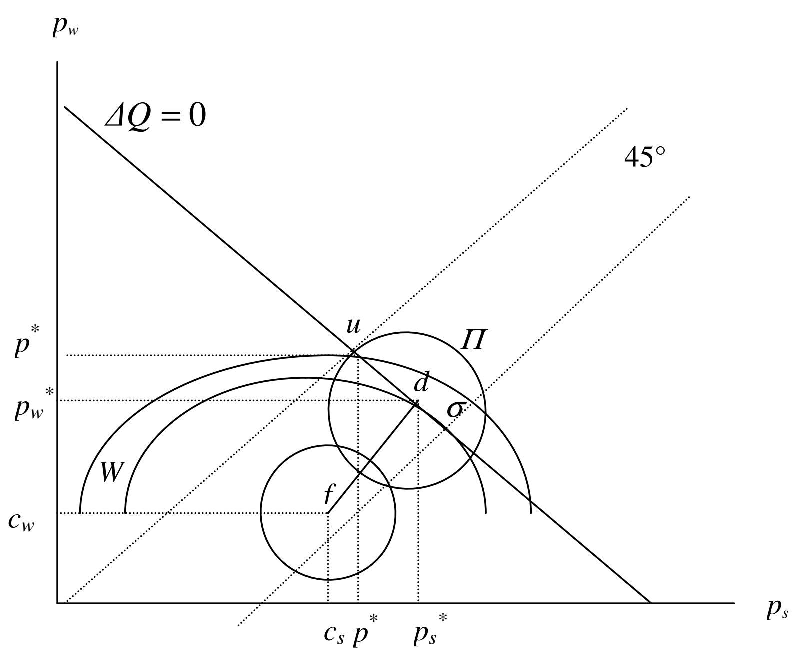

Figure 4: Monopolistic price differentiation with equal demand elasticities at p^*

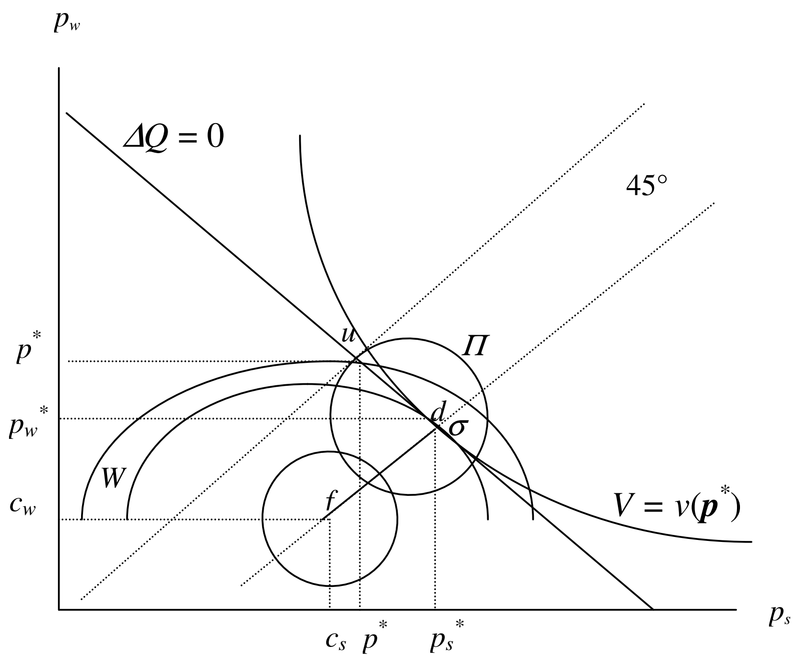

Figure 5: Efficient (second-best) monopolistic price differentiation

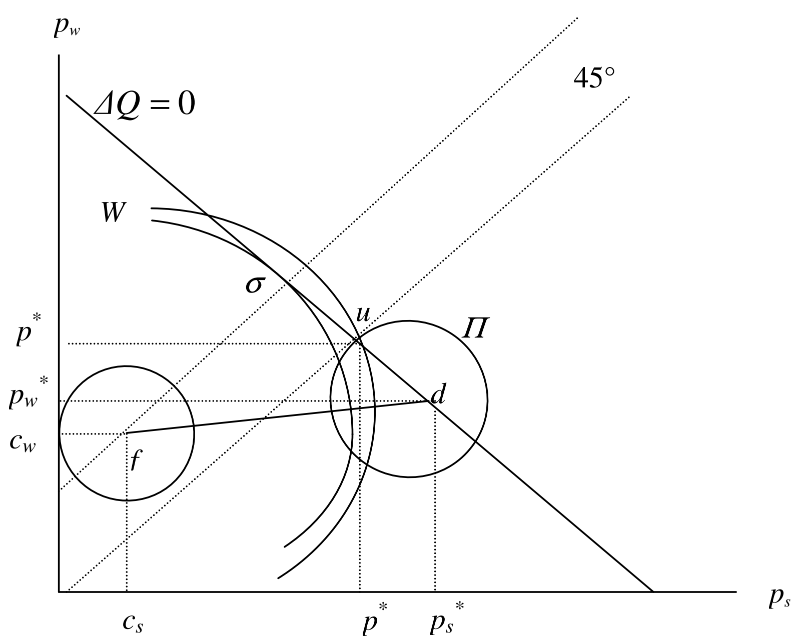

Figure 6: Socially inefficient monopolistic price differentiation