Managing sovereign credit risk in bond portfolios

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Abstract

With the recent development of the European debt crisis, traditional index bond management has been severely called into question. We focus here on the risk issues raised by the classical market-capitalization weighting scheme. We propose an approach to properly measure sovereign credit risk in a fixed-income portfolio. For that, we assume that CDS spreads follow a SABR process and we derive a sovereign credit risk measure based on CDS spreads and duration of portfolio bonds. We then consider two alternative weighting methods which are fundamental indexation and risk-based indexation. Fundamental indexation is based on GDP indexation whereas risk-based indexation uses a risk budgeting approach based on our sovereign credit risk measure. We then compare all these methods in terms of risk, diversification and performance. We show that the risk budgeting approach is the most appropriate scheme to manage sovereign credit risk in bond portfolios and gives very appealing results with respect to active management of bond portfolios.

Keywords: Sovereign credit risk, credit spread, convex risk measure, sabr model, CDS, bond indices, fundamental indexation, risk-based indexation, risk budgeting.

JEL classification: G11, C58, C60, H63.

1 Introduction

The impact of the European debt crisis may be huge. From an economic point of view, we know that economic growth, inflation and foreign exchanges are and will be highly affected for a long time. Today, there is a lot of uncertainty about the economic and fiscal policies that Europe and the rest of the world will adopt. Some risk scenarios are more plausible than others, but nobody knows what the final outcome will be. From a financial investment point

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of view, the European crisis questions investment policies of pension funds, institutionals and even retail investors. For example, there is a big issue on strategic asset allocation and long-term investment policy, because the allocation between asset classes depends on two economic pillars, namely the output and the inflation (Eychenne *et al.*, 2011). A stagnation risk scenario will then not produce the same results as an inflation risk scenario in terms of the equity/bond asset mix decision.

Another consequence of the the European debt crisis is its impact on bond portfolio management. Indeed, pension funds and institutionals are massively invested in bonds, and in particular sovereign bonds. For a long time, sovereign bonds of developed countries have been considered a safe asset. This is especially true since the creation of the eurozone and European investors have largely diversified their portfolios by investing in non-domestic bonds since 2000. The stability of the eurozone over the last decade explains this phenomenon: a European investor could prefer Italian bonds to German bonds, because they offered a better return for a risk increment which was considered to be negligible. Until recently, bond management in the eurozone was then principally explained by the search of spread. This style of active management has been encouraged by the industry of passive management with benchmarks and indexes based on debt weightings. But the eurozone crisis and the rediscovery of the sovereign credit risk leads to rethink the management of bond portfolios by giving a better place to risk management.

The paper is organized as follows. In section two, we define precisely the sovereign risk and explain why country risk is so specific with respect to other risks. In section three, we give some insight about the properties of a good sovereign credit risk measure. It allows to define a risk measure which is consistent from a theoretical point of view and which could be used in practice for bond portfolios. We then apply this risk measure to analyse market-cap weighted bond indexes. In section four, we consider how fundamental indexation and risk-based indexation could correct the drawbacks of market-capitalization indexation with regard to our sovereign credit risk measure. Section five presents a comparative analysis between the three indexation schemes (debt-capitalization, fundamental and risk-based indexations). Finally, section six offers some concluding remarks.

2 Rediscovering sovereign credit risk

In August 2011 Standard and Poor's downgraded the credit rating of US government debt from AAA for the first time. Even five years ago, such an event would have seemed very unlikely; historically, the debt of major developed countries has been considered almost free of credit risk. However, the 2008 financial crisis and the resulting global recession has played havoc with this conception. Sovereign issuers' creditworthiness is now under increasing scrutiny, especially in Europe, while bond investors now face a significant new challenge, one that's especially important for those using a passive, index-based approach.

2.1 The country risk

Sovereign default risk has a long history. Recent debt crises, such as those in 1982 (Mexico and Latin America), 1997-1998 (Russia and Asia) or 2001 (Argentina and Turkey), have been concentrated on emerging markets. Nevertheless, it's been many decades since a "developed" country defaulted. According to Reinhart and Rogoff (2009), there have been at least 250 sovereign external defaults and 68 defaults on domestic public debt since 1800.

Generally, country risk encompasses three types of risk covering financial economic crises (e.g. the 2008 subprime crisis), external debt defaults (e.g. the 1997 Russian default) and banking system (e.g. the 1990 Swedish crisis). One thing these crises have in common is that they generally occur in highly leveraged economies. Kindleberger (1939, 1978) has extensively studied these crises and observes that they show the same patterns. It starts with a period of overenthusiasm followed by a period of loss of confidence. The crisis is then triggered by credit rationing and its impact on the business cycle. Reinhart and Rogoff (2009) have called these patterns the "*syndrome of this-time-is-different*":

> *The essence of this-time-is-different syndrome is simple. It is rooted in the firmly held belief that **financial crises are things that happen to other people in other countries at other times**; crises do not happen to us, here and now. We are doing things better, we are smarter, we have learned from past mistakes. The old rules of valuation no longer apply. The current boom, unlike the many booms that preceded catastrophic collapses in the past (even in our country), is built on sound fundamentals, structural reforms, technological innovations, and good policy. Or so the story goes [...] Unfortunately, a **highly leveraged economy** can unwittingly be sitting with its back at the edge of a financial cliff for many years before chance and circumstance provoke a **crisis of confidence that pushes it off**.*

One of the problems with a sovereign crisis is that the time to recovery may be very long compared to a crisis caused by a pure financial bubble (e.g. the dot.com episode) or by corporate defaults. The main reason is that a firm or an economic sector can deleverage quickly, but not a country. Another problem is the contagion effect. Wyplosz (1999) argues that the market is not perfect and the failures of the market produce multiple equilibria (bad/good). In this context, contagion may appear naturally even if fundamentals of the economy are good. In the end, the main problem of sovereign crises is that the economic cost may be huge[1].

2.2 The eurozone crisis: a debt crisis or a Euro crisis?

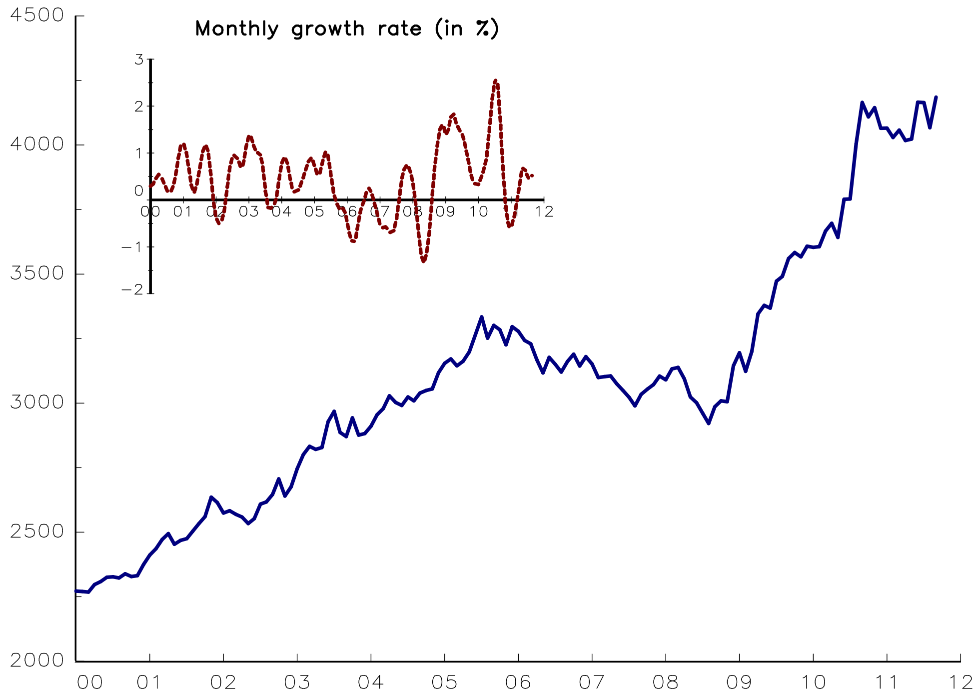
The eurozone crisis finds its roots in the US subprime crisis in 2007-2008. After the bankruptcy of Lehman Brothers in September 2008 and the fear of "*the return of depression economics*" (Krugman, 2009), European countries have largely participated to the G20 global spending plan. One of these Keynesian stimulus consequences is that European debt has dramatically increased during the period 2009-2011. In Figure 1, we have represented the evolution of the eurozone debt market value[2]. In two years, from November 2008 to November 2010, the debt market value increased by 38%. This is larger than the 33% increase which took place during the preceding nine years, between January 2000 and November 2008.

Another factor which explains the eurozone crisis is the status of Greece. In October 2009, Prime Minister George Papandreou warns that the finances of his country are in a "*state of emergency*". In April 2010, Greece sovereign debt rating is cut to BB+. In May 2010, Greece receives a first bailout from EU and IMF. However, this plan is not sufficient and a new bailout is decided in July 2011. With the case of Greece, EU has shown its lack

[1] Argentina (55% of GDP, 1980-82), Japan (10% of GDP, 1990s), Chile (41% of GDP, 1981-87), Israel (30% of GDP, 1977-83), United States (3% of GDP, 1980s), Sweden (10% of GDP, 1990-93), etc. (Crockett 1997, Wyplosz 1999).

[2] We have estimated the monthly growth rate of the European debt by using a cubic spline exponential model.

Figure 1: Evolution of the eurozone debt market value (in MEUR)



Source: Datastream.

of coordination and reaction. However, Wyplosz (1999) argues that hesitations may be a factor of contagion. The later the decision, the lower the credibility and the greater this crisis.

The European crisis is not only a crisis of debt sustainability. It is also a crisis of the Euro currency. In 1961, Mundell develops the theory of optimal currency area which describes the optimal characteristics for the creation of a currency or a monetary union. To drive an economic policy and to face a crisis, there are several instruments and ajustement variables like foreign exchange rates, supply of money, prices/wages or inflation/unemployment trade-offs. If a monetary union is created, it loses the control over foreign exchange rates. In this case, an external balance deficit cannot be compensated by a depreciation of the currency and will eventually lead to higher unemployment. Because a monetary union could be viewed as a region where the exchange rate is fixed inside and flexible outside, participants of the monetary union must share some common principles as explained by Mundell:

> *If the world can be divided into regions within each of which there is factor mobility and between which there is factor immobility, then each of these regions should have a separate currency which fluctuates relative to all other currencies.*

This is the idea behind the *Maastricht Treaty* and the *Stability and Convergence Programmes* (SCP). They lay down the common rules applicable to the European countries in order to ensure the convergence of fiscal and economic policies. However, most of these common rules are not respected. In some sense, the Eurozone crisis is not only a debt crisis, but also the crisis of the Euro.

2.3 Sovereign credit risk in bond indices

Traditionally, bond indices have been constructed according to the methodology of weighting by market capitalisation. This means that each country in the index is given a weight proportional to its level of outstanding debt[3]. The simplicity of this approach and the recognition of a capitalisation-weighted index as the "market portfolio" has contributed to the success of the methodology. Yet, intuitively, it is easy to note a basic flaw in this allocation scheme, since it gives higher index weightings to the most indebted countries, regardless of their capacity to service their debt. A country facing financial hardship and trapped in a debt spiral to remain solvent would see its index weight increase until the whole mechanism collapses and an exclusion from the index occurs. Depending on the index, exclusion can be triggered by specific events, such as a downgrade or, in the worst case, a default.

The case of Greece illustrates this point. Since the beginning of the financial crisis, Greece has struggled with a high debt and refinancing burden. As the country relied more and more on borrowing, the weight of Greek debt increased in European bond indices until the point where Greece was downgraded and excluded from some of the key indices. Passive investors then had no choice but to sell their distressed bonds into a depressed market, leading to significant losses. From the perspective of efficient markets theory, such a risk could be acceptable if it were compensated by an additional return, resulting in similar risk-adjusted returns from the debt of different countries. However, this does not seem to be verified in practice, as Robert Arnott has pointed out (2010).

Another approach to bond indexing is to base weightings on fundamental data. The underlying idea is that weightings should reward those countries with high income, rather than those with high levels of debt. Toloui (2010) proposes to weight country exposures by GDP instead of by outstanding debt. This approach has the advantage of addressing some of the shortcomings of capitalisation-weighted indices:

- it allocates more to less indebted countries;
- it does not have a backward-looking bias, unlike capitalisation-weighted indices (which reflect past patterns of debt issuance);
- it does not suffer from the "buy high, sell low" problem characteristic of the capitalisation-weighted approach.

Both these weighting schemes integrate fundamental data in an attempt to allocate more to countries less likely to default, leading eventually to risk mitigation. Here fundamental factors are used as basic sovereign risk indicators. More comprehensive indicators could also be considered[4]. Brodsky *et al.* (2011) present the Blackrock Sovereign Risk Index, which is based on a wide range of fundamental factors, both quantitative and qualitative. Agencies' credit ratings are certainly the most established type of sovereign risk indicator. These ratings, although mainly driven by fundamental metrics, as shown by Cantor and Packer (1996), put a greater emphasis on discretionary analysis. Finally Gray *et al.* (2007) offer an alternative to scoring/discretionary models. They adapt Merton's seminal work (1974) on corporate credit ratings to derive a sovereign risk indicator for emerging markets. From a pragmatic standpoint, all these risk indicators have proven to be correlated significantly

[3] In the rest of the article the terms "capitalisation-based" and "debt-based" are used interchangeably to refer to such a weighting scheme.

[4] For example, Hilscher and Nosbusch (2010) consider not only the absolute level of fundamentals but also the volatility of fundamentals.

with the credit default swap (CDS) or yield spreads observed in the markets. Such indicators could therefore be deemed reliable as inputs for a weighting scheme.

Whichever approach one chooses, measuring sovereign risk is a prerequisite for anyone running a government bond portfolio. Below, we suggest a theoretical framework for such a measurement.

3 Measuring the risk of sovereign bond portfolios

The volatility of prices return, a measure traditionally used for equity risk, cannot be applied to sovereign bonds. Instead of measuring the specific risk of a sovereign country, volatility will also reflect global movements in interest rate yield curves. Even if we consider a portfolio of sovereign bonds following a single yield curve, the constituent bonds will react differently to interest rate movements, due to their differing sensitivities (duration and convexity). For instance, low duration bonds will consistently display lower volatilities than longer duration bonds, regardless of their credit risk. The same conclusion applies when price returns are used to measure bonds' co-movements through a computation of correlations. As our interest lies in discriminating among countries on the basis of their credit risk, and not on issue-specific features, we must turn to other risk measures.

In Table 1, we have reported some popular measures of country risk. The first one is the rating given by credit rating agencies like Standard and Poor's, Moody's Investors Service or Fitch Ratings. The second one corresponds to the measure computed by Euromoney whereas the third one is the opacity score developed by the Milken Institute. All these three measures are very interesting, but are difficult to use to define a risk measure. For example, the last two measures do not measure exactly the default of sovereign debt. Ratings are better for that, but there is inertia in revising these ratings. Moreover, there are some evidence that rating movements lag market-based indicators (Di Cesare, 2006). In Table 1, we have also reported the CDS spreads of several countries for September 1$^{\mathrm{st}}$, 2011 and October 4$^{\mathrm{th}}$, 2011. For example, the downgrades of Spain and Italy during the first two weeks of October seem to be anticipated by the CDS market.

3.1 Defining the risk measure

The use of a coherent convex risk measure is important. We consider a portfolio of n assets and we denote by x_i the weight of the i^{th} asset in the portfolio. Let $\mathcal{R}(x_1, \ldots, x_n)$ be the risk measure which depends on the portfolio weights $x = (x_1, \ldots, x_n)$. We say that $\mathcal{R}$ is a coherent convex risk measure if it satisfies this mathematical property:

$$\mathcal{R}(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i \cdot \frac{\partial\, \mathcal{R}(x_1, \ldots, x_n)}{\partial\, x_i}$$

This property is called the Euler decomposition of the risk measure. It means that the risk of the portfolio may be decomposed as a sum of the weights times the marginal risks. In this case, we may define exactly what the risk contribution of each asset is:

$$\mathrm{RC}_i = x_i \cdot \frac{\partial\, \mathcal{R}(x_1, \ldots, x_n)}{\partial\, x_i}$$

Working with convex risk measures simplifies the risk analysis of the portfolio, because we may attribute to each asset its contribution to the global portfolio risk.

Table 1: Some popular measures of country risk

Country	S&P Rating[1]	Euromoney Country Risk[2]		Opacity Score[3]	CDS Spread (in bp)	
		Score	Rank		01/09/11	04/10/11
Austria	AAA	84.01	13	16	123	186
Belgium	AA+	77.81	19	19	249	309
Finland	AAA	86.96	8	9	64	85
France	AAA	80.90	16	23	163	201
Germany	AAA	84.98	11	17	76	122
Greece	CCC	52.38	65	31	2,291	5,736
Ireland	BBB+	62.33	43	15	781	726
Italy	A	71.20	30	36	384	487
Netherlands	AAA	86.67	9	24	80	117
Portugal	BBB-	61.35	44	25	957	1,167
Spain	AA	66.71	36	26	376	391
Norway	AAA	93.44	1		44	52
Switzerland	AAA	90.31	3	22	58	79
Denmark	AAA	89.21	4	15	100	153
Sweden	AAA	88.74	5	14	54	66
Canada	AAA	87.17	7	20		
United Kingdom	AAA	80.22	17	18	76	102
United States	AA+	82.10	15	22	52	52
Japan	AA-	74.66	25	25	102	155

[1] Sep. 15th 2011 [2] March 2011 [3] 2009, Milken Institute

Bond prices may not be directly used for our purpose, but yields derived from them might. When compared with a reference risk-free rate at corresponding maturities, yield spreads act as an incremental return, compensating the investor for the risk associated with the issuer. Ceteris paribus, the higher the yield spread, the higher the credit risk. Unlike price returns, this measure does not depend on a bond's duration or convexity. However, it is not obvious what reference rate should be used or which maturity should be chosen. An alternative to using bond yield spreads is to use CDS spreads, which exempt us from selecting a reference rate for each issue under study[5]. CDS spreads correspond to the value investors are willing to pay to insure against an issuer's default risk. CDS spreads therefore allow us to isolate the sovereign credit risk of an issue, regardless of its currency, yield curve or maturity[6].

3.2 Modelling the dynamics of the spreads

Let $S_i(t)$ be the spread of the i^{th} country. We assume that it follows the following diffusion SABR process (Hagan *et al.*, 2002):

$$\mathrm{d}S_i(t) = \sigma_i^S \cdot S_i(t)^{\beta_i} \cdot \mathrm{d}W_i(t)$$

Using historical observations of spreads we can calibrate the parameters σ_i^S and β_i by the log-likelihood function. Here we consider the CDS spreads of the eleven European countries belonging to the Citigroup European Government Bond Index (EGBI) since January 2008. We obtain values for β_i close to one[7]. For simplicity's sake we will assume that the spreads follow a lognormal diffusion in the rest of the article. This implies that spreads vary proportionally to their absolute levels. This assumption seems reasonable in view of CDS spread data over the last few years. Moreover the credit risk component of bond volatility is thus proportional to its spread[8]. It can be estimated as the product of its duration D_i, the spread volatility σ_i^S and the spread level $S_i(t)$:

$$\sigma_i^B = D_i \cdot \sigma_i^S \cdot S_i(t)$$

Finally we model state-dependence, defining Γ as the correlation matrix of the Brownian motions $W_i(t)$. Therefore, Γ can be estimated by computing the empirical correlation matrix of the relative variations of historical spreads. The credit risk of a portfolio of n sovereign bonds is then defined as the volatility of the CDS basket which would perfectly hedge the sovereign risk of the portfolio:

$$\mathcal{R}(x) = \sqrt{x^\top \Sigma x}$$

where $\Sigma_{i,j} = \Gamma_{i,j} \cdot \sigma_i^B \cdot \sigma_j^B$ is the the covariance between the bond of country i and the bond of country j and x is the vector of country weights.

[5] One drawback of this measure is that the CDS market is less liquid than the bonds market. Perhaps a better measure may be the asset swap spread, but it implies some calibration issues. For the purpose of this paper, the use of CDS spreads is sufficient to illustrate our methodology.

[6] This approach is frequently used by recent studies on this subject (Longstaff *et al.*, 2007, Ejsing and Lemke, 2011).

[7] Results are given in Appendix A.1. We obtain an average value of 0.957 if we consider the 11 countries.

[8] Let $B_i(t, D_i)$ be a zero-coupon risky bond of maturity (or duration) D_i. We have:

$$\mathrm{d}\ln B_i(t, D_i) = -D_i \cdot \mathrm{d}R(t) - D_i \cdot \mathrm{d}S_i(t)$$

with $R(t)$ the "risk-free" interest rate.

3.3 Some stylized facts

In Tables 2, 3 and 4, we report the estimated values of the σ_i^S and $\Gamma_{i,j}$ parameters for the 11 member countries of the euro for different dates. In September 2011, we notice that the spread volatilities σ_i^S seem to fluctuate around 60%, ranging from 53% to 70% and cross-correlations tend to cluster around 65%. We also remark that Finland, which is the country with the lowest level of debt, relative to the size of its economy, is the least correlated with the other markets. At the opposite end of the scale, Portugal, Italy, Greece and Spain, four of the most heavily indebted European countries, exhibit a high level of correlation between each other. This could confirm the intuition that contagion would be more likely to spread among such indebted countries, whereas countries such as Finland would be less susceptible to the default of other Eurozone countries. Comparing these estimates to those from three years earlier, we obtain different results for volatility and correlation. In July 2008, the spread volatility fluctuated between 58% and 107%, with an average of 77%. This is due to the log-normality model property of the spread process: the smaller the spreads, the larger the (relative) volatility. As for the correlations, we notice a sharp increase. The average correlation rose from 38% in January 2008 to 62% in September 2011. In Figure 2, we have reported the evolution of the Γ matrix. We observe some interesting facts. First, the default of Lehman Brothers has shifted the average correlation. It is particularly obvious for the safe countries like Finland, Germany and Netherlands. Second, we also observe that the correlation between GIIPS reaches its maximum in October 2010, and decreases since this date. Third, the correlation between safe countries (Finland, Germany and Netherlands) has increased recently illustrating a period of flight-to-quality.

Figure 2: Evolution of the Γ matrix

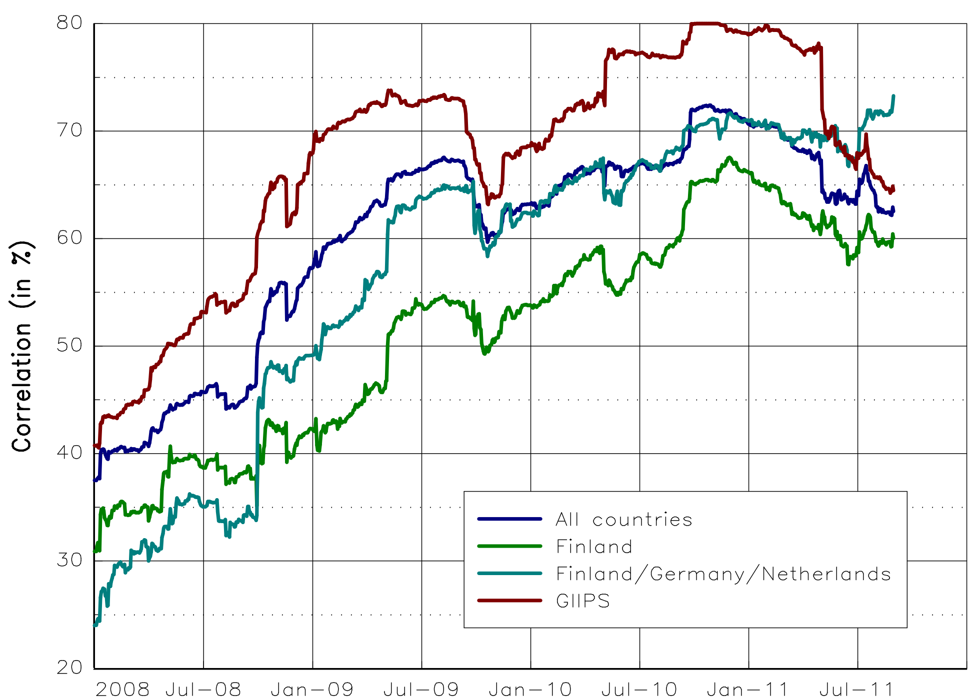


Table 2: Statistics of σ_i^S

Country	July-08	July-09	July-10	July-11	September-11
Austria	66.9%	105.8%	77.8%	45.9%	54.4%
Belgium	73.3%	88.9%	78.0%	57.2%	65.8%
Finland	106.8%	97.9%	75.2%	49.8%	55.7%
France	78.2%	91.8%	97.7%	57.3%	63.9%
Germany	81.3%	98.5%	74.9%	48.8%	55.7%
Greece	67.5%	65.8%	94.7%	49.4%	56.8%
Ireland	80.3%	99.8%	73.4%	51.7%	53.5%
Italy	58.8%	69.5%	89.3%	61.5%	69.3%
Netherlands	95.2%	103.7%	68.6%	47.3%	54.9%
Portugal	66.9%	72.0%	116.6%	53.5%	58.3%
Spain	72.4%	76.8%	90.2%	58.9%	64.6%

Table 3: Estimated Γ matrix (January 2008)

Country	AT	BE	FI	FR	DE	GR	IE	IT	NL	PT	ES
Austria	100%										
Belgium	54%	100%									
Finland	27%	34%	100%								
France	55%	60%	42%	100%							
Germany	51%	43%	23%	50%	100%						
Greece	42%	38%	29%	47%	43%	100%					
Ireland	33%	35%	41%	37%	27%	22%	100%				
Italy	47%	50%	37%	60%	47%	61%	35%	100%			
Netherlands	22%	38%	32%	29%	17%	4%	29%	22%	100%		
Portugal	38%	48%	23%	45%	45%	58%	29%	58%	12%	100%	
Spain	35%	49%	22%	33%	33%	35%	28%	41%	30%	41%	100%

Table 4: Estimated Γ matrix (September 2011)

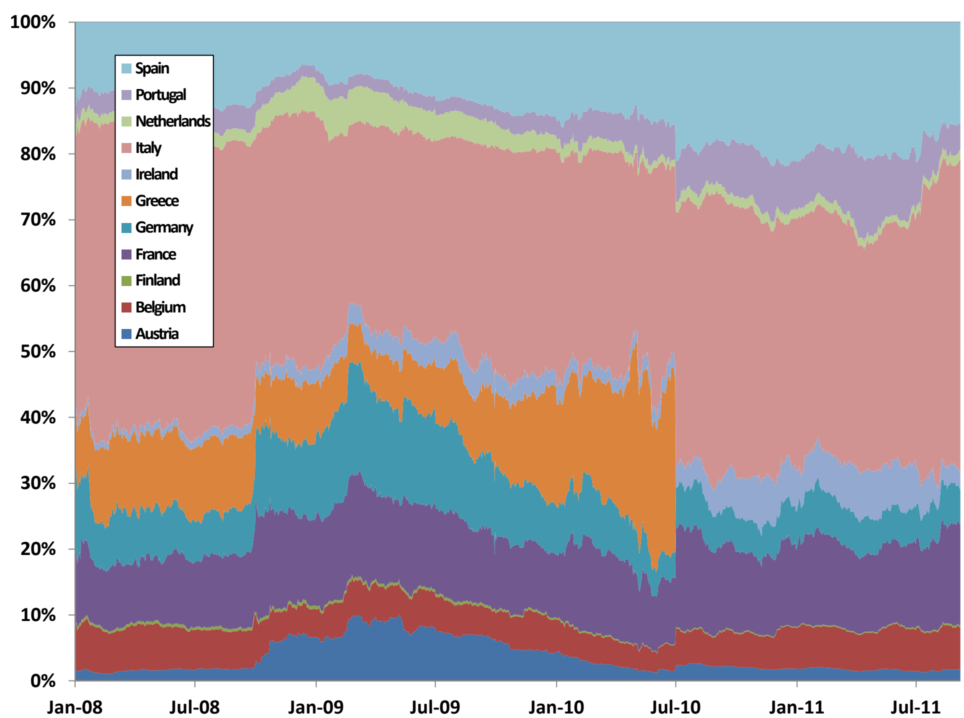
Country	AT	BE	FI	FR	DE	GR	IE	IT	NL	PT	ES
Austria	100%										
Belgium	75%	100%									
Finland	68%	64%	100%								
France	72%	70%	61%	100%							
Germany	77%	72%	65%	72%	100%						
Greece	38%	48%	35%	37%	37%	100%					
Ireland	58%	62%	55%	56%	59%	54%	100%				
Italy	71%	85%	66%	67%	69%	48%	68%	100%			
Netherlands	69%	67%	72%	67%	78%	45%	56%	68%	100%		
Portugal	55%	66%	50%	55%	57%	54%	80%	73%	52%	100%	
Spain	67%	76%	56%	60%	61%	49%	69%	80%	56%	71%	100%

3.4 Application to the EGBI portfolio

Once these parameters are estimated, we can deduce for any sovereign bond portfolio the credit risk contribution of each country's debt. Using the previous formula to define $\mathcal{R}(x)$, we are able to compute the marginal credit-linked volatility and risk contribution, country by country[9]. The marginal risk expresses the impact of a small change in a country's index weight on the credit-linked volatility, whereas the risk contribution reflects the portion of the overall credit-linked volatility that a particular country is accountable for.

Let us consider the Citigroup EGBI index. Figure 3 shows the evolution of the credit risk contribution (RC) of each country since the beginning of 2008. We notice the increase of Greece's risk contribution since 2008. On April 27$^{\text{th}}$ 2010, the country's sovereign debt rating was cut to BB+ by Standard & Poor's. Therefore Greece lost its investment grade status and exited the EGBI index at its next rebalancing date. The risk contribution of Greece reached a maximum of 28.5% on April 30$^{\text{th}}$, 2010, while at the end of June 2010, just before its exit from the index, it remained very high, at 26%. Since July 2010, we also notice an increase in the credit risk of Portugal, Ireland, Italy and Spain.

Figure 3: Evolution of risk contributions for the EGBI index



In Table 5 we report the weights and risk contributions for the constituents of the EGBI on some rebalancing dates. In July 2010 (after the exclusion of Greece) the Portugal, Ireland, Italy and Spain account for 35% of the index weights, but 70% of the sovereign risk. In July 2011, these figures remain approximately the same[10]. The risk contribution of these four

[9] See Appendix A.2 for details.
[10] The weights decrease by 1% whereas the risk contributions increase by 3%.

Table 5: Weights and risk contributions of the EGBI index

Country	July-08		July-09		July-10		July-11		September-11	
	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC
Austria	4.1%	1.7%	3.6%	7.7%	4.1%	2.3%	4.3%	1.5%	4.3%	1.8%
Belgium	6.2%	6.1%	6.5%	5.1%	6.3%	5.7%	6.4%	6.5%	6.4%	6.7%
Finland	1.2%	0.4%	1.3%	0.5%	1.3%	0.2%	1.6%	0.2%	1.6%	0.3%
France	20.5%	9.8%	20.4%	13.2%	22.2%	15.1%	23.1%	13.3%	23.1%	15.5%
Germany	24.4%	6.1%	22.3%	13.0%	22.9%	6.0%	22.1%	5.3%	22.1%	5.5%
Greece	4.9%	11.4%	5.4%	8.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Ireland	1.0%	1.3%	1.5%	4.3%	2.1%	3.3%	1.4%	5.4%	1.4%	2.7%
Italy	22.1%	45.2%	22.4%	29.5%	23.4%	38.7%	23.1%	38.5%	23.1%	46.3%
Netherlands	5.3%	1.7%	5.3%	4.1%	6.1%	1.6%	6.2%	1.2%	6.2%	1.5%
Portugal	2.4%	3.9%	2.3%	2.3%	2.1%	6.3%	1.6%	6.6%	1.6%	4.4%
Spain	7.8%	12.4%	9.1%	11.8%	9.6%	20.9%	10.3%	21.3%	10.3%	15.5%
Sovereign Risk	0.70%		2.59%		6.12%		4.02%		8.12%	

countries is therefore very high, although smaller than the 83% reached during the peak of May 2011. By contrast, in February 2009 their collective risk contribution was only 46%[11].

4 Implications for the management of bond portfolios

Capitalisation-weighted indices presenting clear drawbacks, we propose an alternative approach to indexation, based on sovereign risk. Using the framework described previously, it is possible to decompose the risk of a sovereign bond portfolio into individual country contributions. Given the portfolio weights, we can find the risk contributions. Risk budgeting is basically the same process in reverse. Given a set of defined risk contributions, risk budgeting provides a portfolio allocation. The problem with traditional indices is that investors may face a high level of risk concentration. To avoid that, investors can start by defining a set of risk budgets that they find appropriate and then derive the corresponding weights. Even if the risk budgets are fixed, the portfolio weights still vary over time as the risk associated with each country fluctuates. This way, when an individual country's situation deteriorates, this is reflected in its spread and thereby its risk, leading eventually to a reduction of its weight. Risk budgeting thus allows investors to control the distribution of risk over time. However, it does not address the problem of how to choose the right risk budgets.

4.1 Fundamental indexation

The GDP weighting approach can help us to define the risk budgets. Toloui (2010) writes:

> "*An alternative to market-cap weighting is to weight country exposures in global bond indexes by GDP instead of market capitalisation. [. . .] In contrast to market-cap weighting, GDP weighting does not reward countries with high levels of debt issuance: Countries with higher debt-to-GDP levels generally have a lower representation in a GDP-weighted index versus a market- cap-weighted index.*"

[11] The decomposition was 5.8% for Greece, 3.0% for Ireland, 27.1% for Italy, 1.9% for Portugal and 8.3% for Spain.

Table 6: Weights and risk contributions of the GDP indexation

Country	July-08		July-09		July-10		July-11		September-11	
	GDP	RC	GDP	RC	GDP	RC	GDP	RC	GDP	RC
Austria	3.1%	1.4%	3.1%	7.0%	3.1%	1.7%	3.2%	1.0%	3.2%	1.3%
Belgium	3.8%	4.0%	3.8%	3.2%	3.9%	3.3%	4.0%	3.5%	4.0%	4.0%
Finland	2.0%	0.8%	1.9%	0.7%	2.0%	0.3%	2.1%	0.3%	2.1%	0.3%
France	21.2%	11.2%	21.5%	14.9%	21.4%	13.4%	21.5%	10.6%	21.5%	13.8%
Germany	27.4%	7.6%	27.2%	17.0%	27.7%	6.7%	27.9%	5.8%	27.9%	6.7%
Greece	2.6%	6.2%	2.7%	4.4%	2.6%	15.7%	2.4%	19.8%	2.4%	13.8%
Ireland	2.0%	3.0%	1.9%	5.6%	1.8%	2.6%	1.7%	5.9%	1.7%	3.3%
Italy	17.4%	37.5%	17.3%	23.5%	17.2%	25.8%	17.0%	23.9%	17.0%	32.6%
Netherlands	6.5%	2.5%	6.5%	5.3%	6.5%	1.6%	6.6%	1.2%	6.6%	1.6%
Portugal	1.9%	3.3%	1.9%	2.0%	1.9%	5.3%	1.9%	6.7%	1.9%	5.1%
Spain	12.0%	22.6%	12.0%	16.5%	11.8%	23.7%	11.8%	21.4%	11.8%	17.5%
Sovereign Risk	0.64%		2.47%		6.59%		4.56%		8.26%	

In our opinion, the GDP weighting approach is a first step towards avoiding the "buy high, sell low" problem of capitalisation weighting. A direct extension of this approach is to consider the GDP-RB (risk budgeting) weighting approach. In this case, the weights are computed such that the risk contribution of each country is proportional to its GDP. Using GDP as the basis for a risk budget allows us to consider GDP weighting schemes as a special case of GDP-RB weighting approach (in other words, GDP weighting implies that all countries present the same risk levels and correlations).

Let us now consider the eleven national sub-indices of the Citigroup EGBI index. Each sub-index can be seen as a proxy of the relevant country's outstanding debt and will be treated as if it were a bond. A GDP weighting approach consists in fixing at each rebalancing date the weight of the i^{th} country in the portfolio as equal to the share of its GDP in the total GDP of the Eurozone:

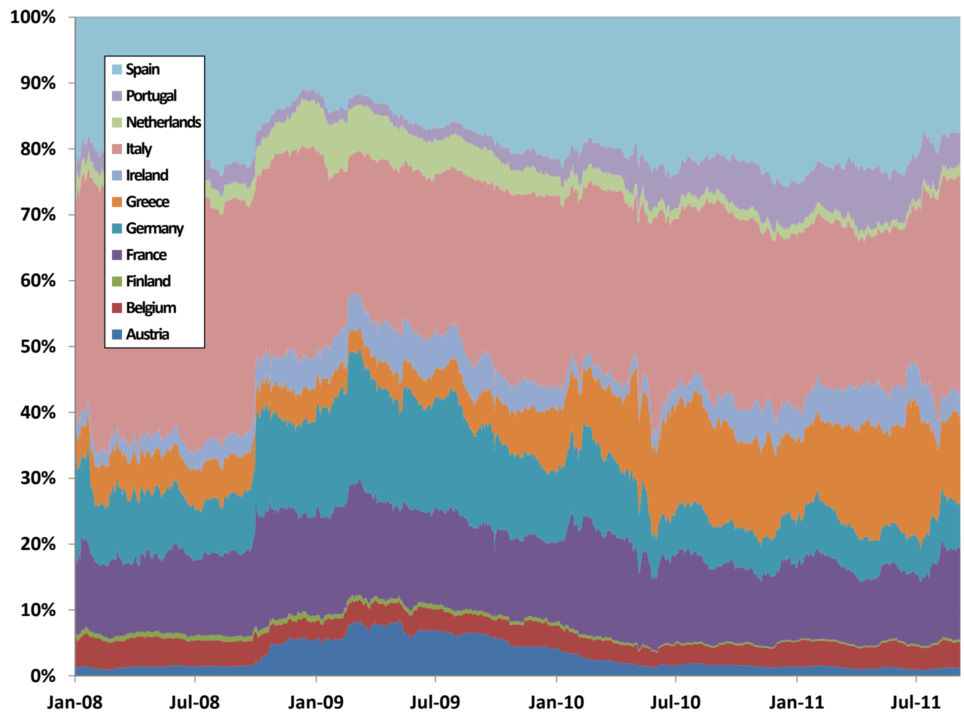
$$x_i = \frac{\text{GDP}_i}{\sum_{j=1}^{11} \text{GDP}_j}$$

In Table 6 and Figure 4 we use a GDP weighting scheme[12]. Under GDP-based indexation Greece has a smaller index allocation than under capitalisation weighting. For example, in July 2008, the weight of Greece was 4.91% by market capitalisation (Table 5) but only 2.61% under GDP-based indexation (Table 6). We observe the same phenomenon for Italy and Portugal. Nevertheless, we notice that this type of indexation increases the weight of Spain or Ireland. This is because the debt to GDP ratio is lower than average for these countries. We remark that GDP indexation produces a more balanced portfolio in terms of risk than debt-based indexation, but that risk is still concentrated in a few countries.

Another disappointing result is that GDP indexation produces sovereign credit risk measures similar to those of DEBT indexation. For example, the sovereign risk measure is equal to 8.26% for the GDP indexation in September 2011 which is close to the 8.12% obtained with the EGBI portfolio.

[12]The GDP column may also be viewed as the weights of the bond portfolio.

Figure 4: Evolution of risk contributions for the GDP indexation



4.2 Risk-based indexation

Risk-based indexation is more difficult to define for bond portfolios than for equity portfolios. For the later, a lot of allocation methods exist and are adapted to define a risk-based portfolio[13]. These methods, however, are not suited for bond portfolios[14] except for the risk budgeting techniques that prove appropriate.

4.2.1 An introduction to risk budgeting allocation methods

The risk budgeting method is explained in Meucci (2005) and Sherer (2007). This technique makes it possible to control the risk allocated to each constituents of a portfolio. Let $x = (x_1, \ldots, x_n)$ be the portfolio weights. If we use a coherent convex risk measure $\mathcal{R}(x_1, \ldots, x_n)$, we have :

$$\mathcal{R}(x_1, \ldots, x_n) = \sum_{i=1}^{n} \mathrm{RC}_i (x_1, \ldots, x_n)$$

where $\mathrm{RC}_i (x_1, \ldots, x_n)$ is the risk contribution of the i^{th} asset :

$$\mathrm{RC}_i (x_1, \ldots, x_n) = x_i \cdot \frac{\partial \mathcal{R}(x_1, \ldots, x_n)}{\partial x_i}$$

[13] We can cite for example the equally-weighted portfolio (DeMiguel *et al.*, 2009), the minimum variance portfolio (Clarke *et al.*, 2006), the erc portfolio (Maillard *et al.*, 2010) or the mdp/msr portfolio (Choueifaty and Coignard, 2008 and Martellini, 2008).

[14] Moreover, some of these methods face some concentration problems (Roncalli, 2011). For example, if we consider the minimum variance portfolio, we will obtained a very concentrated portfolio in Finland and Germany.

Table 7: Comparison of traditional and risk budgeting approaches

Traditional approach

Asset	Weight	Marginal Risk	Risk Contribution	
			Absolute	Relative
1	**60.00%**	18.80%	11.28%	66.67%
2	**20.00%**	23.94%	4.79%	28.30%
3	**20.00%**	4.26%	0.85%	5.03%
Volatility			16.92%	

Risk budgeting approach

Asset	Weight	Marginal Risk	Risk Contribution	
			Absolute	Relative
1	48.50%	17.69%	8.58%	**60.00%**
2	13.17%	21.71%	2.86%	**20.00%**
3	38.32%	7.46%	2.86%	**20.00%**
Volatility			14.30%	

To define a risk budgeting (or RB) portfolio, we consider a set of given risk budgets $\{\mathrm{RB}_1, \ldots, \mathrm{RB}_n\}$. Then we have to solve the following non-linear system of n equations[15]:

$$\begin{cases} \mathrm{RC}_1(x_1, \ldots, x_n) = \mathrm{RB}_1 \\ \vdots \\ \mathrm{RC}_n(x_1, \ldots, x_n) = \mathrm{RB}_n \end{cases}$$

Remark 1 *The philosophy of the risk budgeting approach is then the opposite of the traditional approach. In the traditional approach, the weights of the portfolio are given and we determine the risk contributions according to these weights. Conversely, in the risk budgeting approach, the risk contributions of the portfolio are given and we determine the weights according to these risk contributions. To illustrate these differences, we consider three assets. The volatilities are respectively* 20%, 30% *and* 15%. *The correlations are* 60% *between the first asset and the second asset and* 10% *between the first two assets and the third asset. We assume that the risk measure is the volatility of the portfolio. In Table 7, we report the volatility decomposition of the portfolio with weights* 60%, 20% *and* 20%. *We notice that the third asset has a weight contribution of* 20% *but a risk contribution of only* 5%. *If we now consider a portfolio such that the risk budgets are* 60%, 20% *and* 20%, *the weights are* 48.5%, 13.2% *and* 38.3%. *This means that we must set the third asset weight to* 38.3% *if we want it to account for* 20% *of the portfolio risk.*

4.2.2 The GDP-RB indexation

For the GDP-RB approach, we have to estimate the vector of weights such that the risk contribution of the i^{th} country is equal to the part of its GDP in the total GDP of the Eurozone:

$$\mathrm{RC}_i = \frac{\mathrm{GDP}_i}{\sum_{j=1}^{11} \mathrm{GDP}_j}$$

The solution is shown in Figure 5 and in Table 8. By construction, the risk contributions of GDP-RB indexation correspond to the weights of GDP indexation. For example, in July 2008, Austria had a 3.1% weight in the GDP indexed portfolio, the same as its 3.1%

[15] The numerical solution may be obtained by using non-linear optimisation procedures like the sequential quadratic programming (SQP) algorithm.

Figure 5: Evolution of weights of the GDP-RB indexation

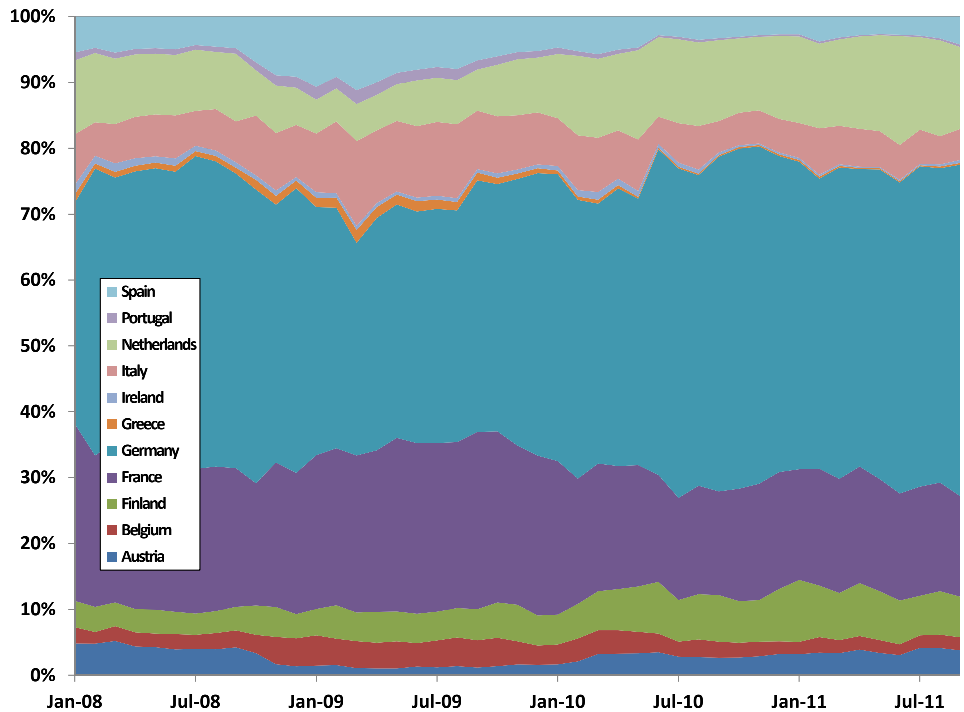
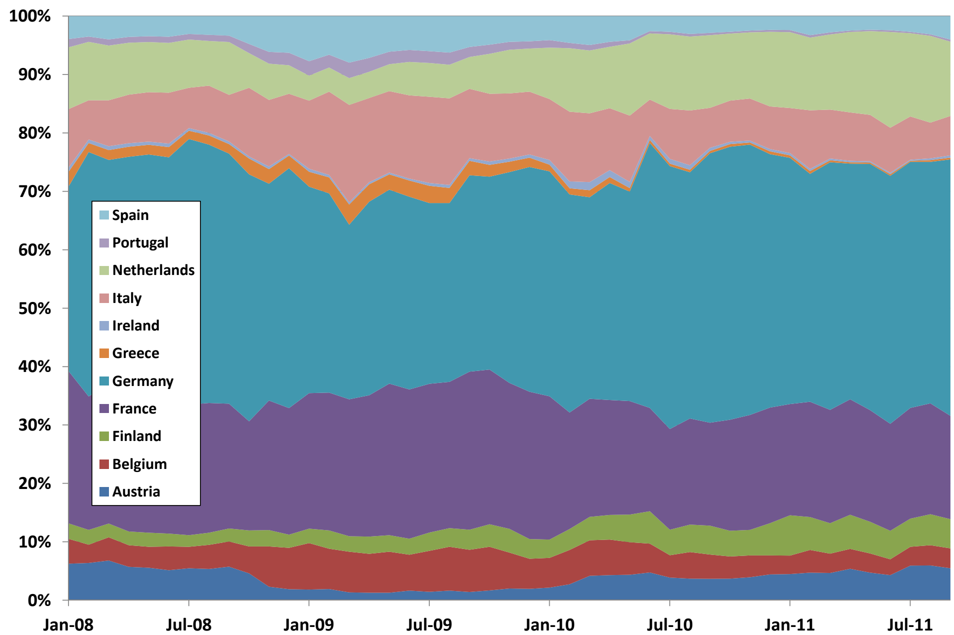


Table 8: Weights and risk contributions of the GDP-RB indexation

Country	July-08		July-09		July-10		July-11		September-11	
	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights
Austria	3.1%	3.9%	3.1%	1.2%	3.1%	2.9%	3.2%	4.2%	3.2%	3.8%
Belgium	3.8%	2.1%	3.8%	4.1%	3.9%	2.2%	4.0%	1.9%	4.0%	2.0%
Finland	2.0%	3.2%	1.9%	4.4%	2.0%	6.3%	2.1%	6.0%	2.1%	6.1%
France	21.2%	22.0%	21.5%	25.6%	21.4%	15.5%	21.5%	16.5%	21.5%	15.3%
Germany	27.4%	47.8%	27.2%	35.5%	27.7%	50.0%	27.9%	48.7%	27.9%	50.2%
Greece	2.6%	0.7%	2.7%	1.4%	2.6%	0.2%	2.4%	0.2%	2.4%	0.3%
Ireland	2.0%	0.8%	1.9%	0.6%	1.8%	0.6%	1.7%	0.2%	1.7%	0.5%
Italy	17.4%	5.3%	17.3%	11.2%	17.2%	6.0%	17.0%	5.2%	17.0%	4.7%
Netherlands	6.5%	9.2%	6.5%	6.7%	6.5%	12.8%	6.6%	14.0%	6.6%	12.5%
Portugal	1.9%	0.7%	1.9%	1.6%	1.9%	0.4%	1.9%	0.2%	1.9%	0.4%
Spain	12.0%	4.2%	12.0%	7.7%	11.8%	3.1%	11.8%	2.9%	11.8%	4.2%
Sovereign Risk	0.39%		2.10%		3.25%		1.91%		4.13%	

Figure 6: Evolution of weights of the DEBT-RB indexation



risk contribution in the portfolio based on GDP-RB indexation. In Table 8, we clearly see the impact of the sovereign credit risk of individual countries. For example, the weight of Germany is lower under GDP indexation than under GDP-RB indexation. The same phenomenon appears for other "safe" countries like Finland or the Netherlands. We also notice that the portfolio structure varies a lot across time, depending on the sovereign risk priced in by the market. The GDP-RB portfolio is further away from the GDP portfolio in periods of risk aversion, such as July 2011, whereas it is closer when sovereign risk concerns are relatively low, as in July 2009. Moreover, we notice that the GDP-RB indexation has a lower sovereign risk measure compared to DEBT or GDP indexations. For example, at the 1st September 2011, it is equal to 4.13% for the GDP-RB indexation whereas it is equal to 8.12% and 8.26% for the DEBT or GDP indexations.

4.2.3 The DEBT-RB indexation

This methodology may be applied to other metrics than GDP. For example, if we consider the amount of debt (DEBT), the portfolio is built such that:

$$\mathrm{RC}_i = \frac{\mathrm{DEBT}_i}{\sum_{j=1}^{11} \mathrm{DEBT}_j}$$

This case is illustrated in Figure 6 and Table 9. One interesting property of the risk budgeting approach is that when sovereign risk is the same for all countries, the solution corresponds to the weights of the traditional benchmark. In the case of the DEBT-RB approach, assuming equal risks gives us the weights of the capitalisation-weighting scheme (i.e. the DEBT-weighting scheme).

Table 9: Weights and risk contributions of the DEBT-RB indexation

Country	July-08		July-09		July-10		July-11		September-11	
	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights
Austria	4.1%	5.4%	3.6%	1.4%	3.9%	3.9%	4.2%	5.9%	4.2%	5.5%
Belgium	6.2%	3.6%	6.5%	7.1%	6.0%	3.8%	6.2%	3.2%	6.2%	3.4%
Finland	1.2%	2.0%	1.3%	3.1%	1.2%	4.4%	1.5%	4.8%	1.5%	5.0%
France	20.5%	22.4%	20.4%	25.5%	21.2%	17.2%	22.5%	18.9%	22.5%	17.7%
Germany	24.4%	45.8%	22.3%	30.9%	21.9%	45.0%	21.5%	42.1%	21.5%	43.9%
Greece	4.9%	1.4%	5.4%	2.9%	4.3%	0.4%	2.6%	0.2%	2.5%	0.3%
Ireland	1.0%	0.4%	1.5%	0.5%	2.0%	0.8%	1.4%	0.2%	1.4%	0.4%
Italy	22.1%	6.9%	22.4%	14.8%	22.4%	8.5%	22.5%	7.4%	22.5%	6.7%
Netherlands	5.3%	8.2%	5.3%	5.8%	5.9%	12.8%	6.0%	14.3%	6.0%	12.7%
Portugal	2.4%	0.9%	2.3%	2.0%	2.0%	0.4%	1.6%	0.2%	1.6%	0.4%
Spain	7.8%	3.0%	9.1%	6.0%	9.2%	2.7%	10.1%	2.7%	10.1%	4.0%
Sovereign Risk	0.41%		2.19%		3.63%		2.10%		4.57%	

5 Comparison of the indexing schemes

We may now simulate the four indexing approaches: DEBT-weighting, GDP-weighting, GDP-based risk budgeting and DEBT-based risk budgeting[16]. In what follows, we compare all these weighting schemes in terms of performance, risk and portfolio characteristics.

5.1 In terms of performance

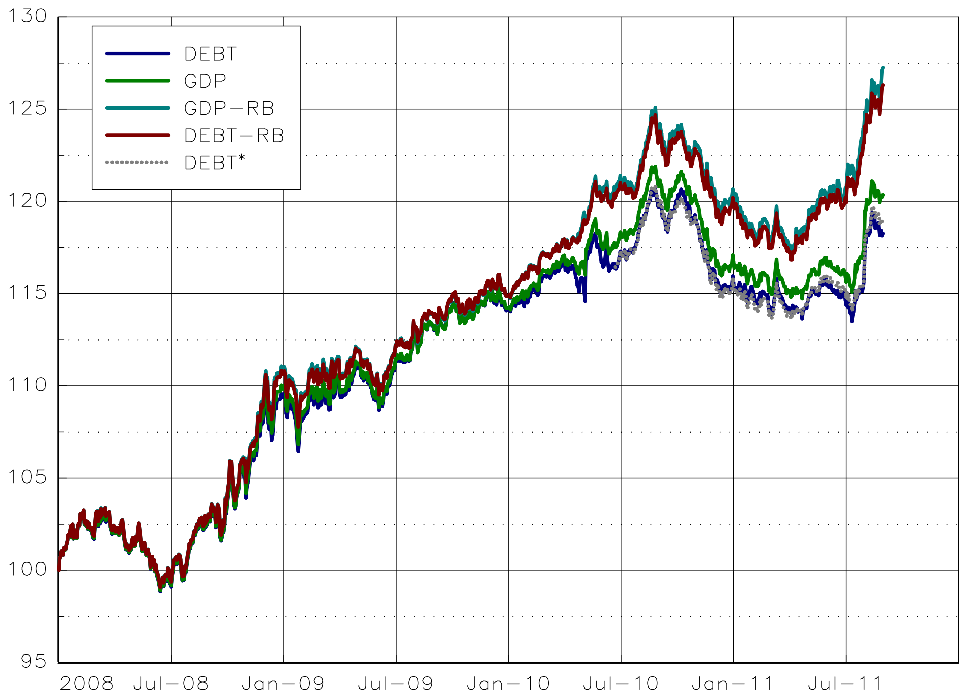
The results are given in Figure 7 and relevant statistics are shown in Table 10. We notice that alternative index approaches to capitalisation weighting post better performances. We notice that risk-based indexations yield the best returns but generate higher tracking errors. Of course, these results depend on the study period and may be explained by the economic situation since 2008.

Table 10: Main statistics (01/01/2008 – 06/09/2011)

	DEBT	GDP	GDP-RB	DEBT-RB	DEBT*	DEBT*-RB
Average Return	4.7%	5.2%	6.8%	6.5%	4.8%	6.6%
Volatility	4.4%	4.3%	4.4%	4.4%	4.4%	4.4%
Sharpe	1.05	1.19	1.53	1.49	1.10	1.50
Tracking Error		0.6%	2.2%	2.0%	0.3%	2.0%
Information Ratio		0.84	0.91	0.90	0.55	0.91
Correlation		99.2%	87.4%	89.7%	99.8%	89.6%
Beta		97.1%	87.5%	88.8%	98.9%	88.7%

[16] In the DEBT* case, we have excluded Greece since July 2010 in order to be coherent with the construction of the EGBI portfolio.

Figure 7: Simulation of the performance of the 4 indexing schemes



5.2 In terms of risk

Our purpose here is not to say that alternative indexations are better than capitalisation weighting. Having said that, the fact that only alternative methodologies can integrate a sovereign risk dimension is an undeniable asset[17]. In the risk budgeting approaches, it is easy to see how it is implemented and what benefits can be expected. Evolution of the sovereign risk measure is reported in Figure 8 for the 4 indexation schemes. As expected, we find that risk-based indexations present lower sovereign risk. If we compare the risk contribution of GIIPS countries, we observe a difference between risk-based indexation on the one hand, and capitalisation and fundamental indexation on the other hand (see Figure 9).

5.3 In terms of portfolio characteristics

In Table 11, we report the weights and risk contributions and some other statistics like yield-to-maturity, duration and average spread as at the beginning of September 2011. We also compute the concentration measure given by the Gini index. This statistic takes the value 1 in case of perfect concentration and 0 if no concentration exists. The Gini coefficient is applied both to weights and to risk contributions. When considering weights, the less concentrated portfolio (with a Gini index equal to 0) corresponds to the equally weighted portfolio, whereas the less concentrated portfolio in terms of risk corresponds to the equal risk contributions (ERC) portfolio (Maillard *et al.*, 2010). In both cases, the most concentrated

[17] In some sense, GDP, GDP-RB and DEBT-RB may be viewed as the equivalent for bonds of the existing schemes of fundamental and risk-based indexation for equities (Demey *et al.*, 2010).

Figure 8: Evolution of the risk measure for the 4 indexing schemes

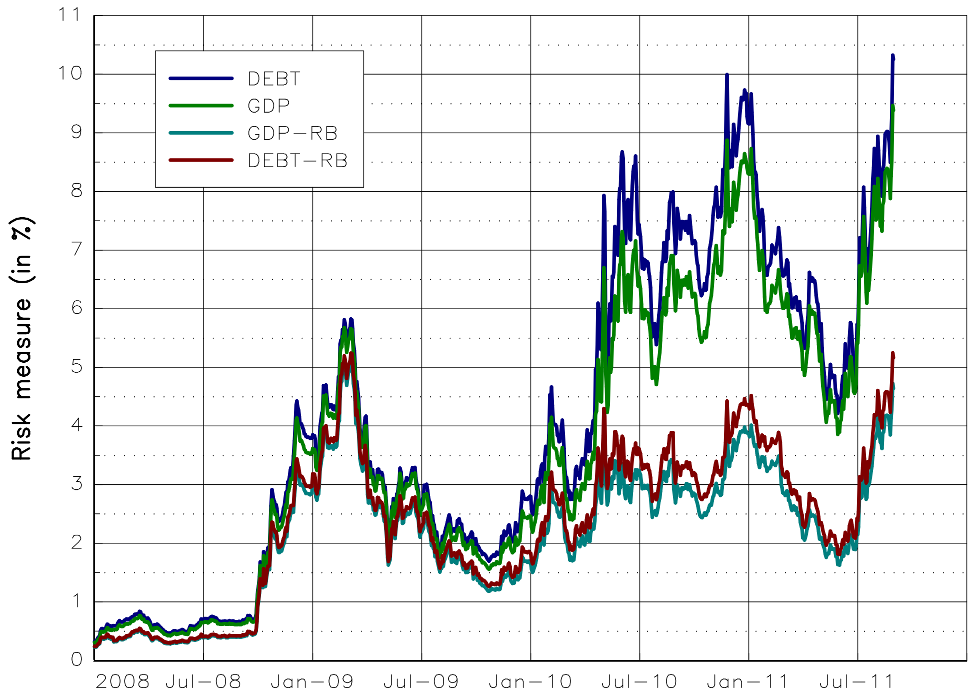
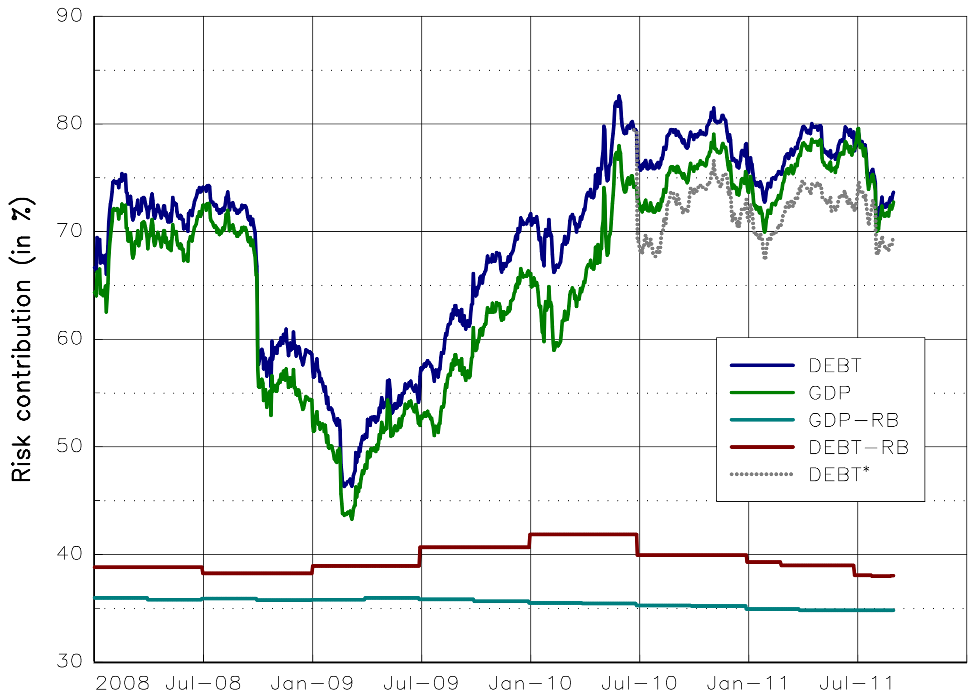


Figure 9: Evolution of the GIIPS risk contribution



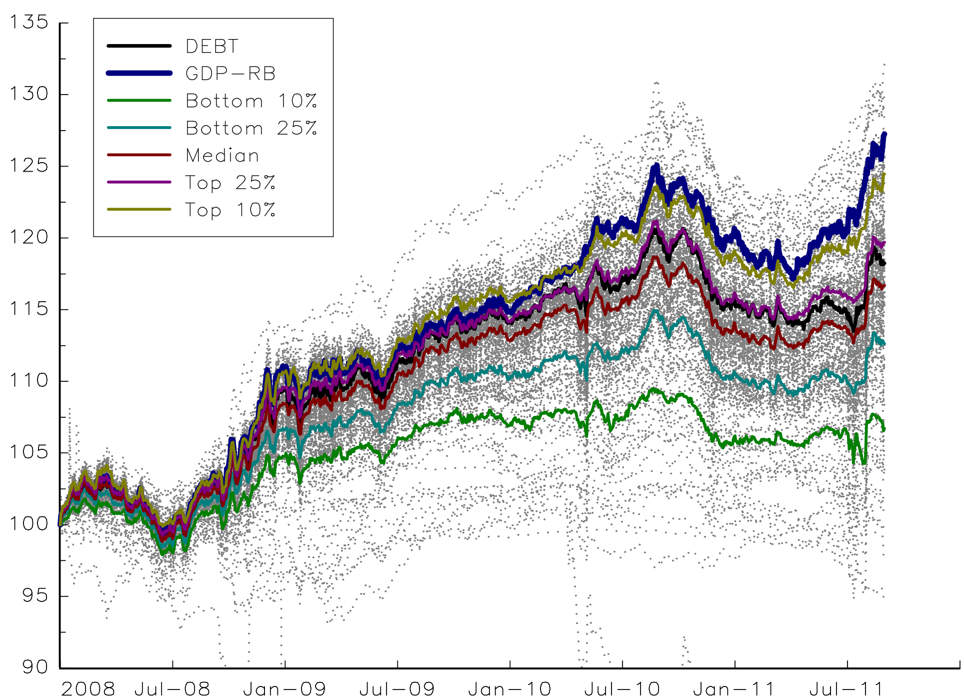
portfolio corresponds to a portfolio invested in only one country's debt. We observe in Table 11 that risk-based index approaches may lead to more concentrated portfolios in terms of weights. However the concentration in terms of risk is decreased when compared to debt-based (capitalisation) weighting. One of the main advantages of the risk budgeting approach comes from its dynamic allocation process, one which adapts to prevailing sovereign risk. By integrating the sovereign risk dimension in the investment process, indices based on risk budgeting also fit the definition of risk-based portfolios (Demey *et al.*, 2010).

5.4 With respect to active management

In Figure 10, we report the NAV of the 218 funds available in the Morningstar database under the "Bond Euro Government" category. We also report the different performance quantiles (10%, 25%, 50%, 75% and 90%) of these funds. We observe a high dispersion of performance between the top 10% and the bottom 10%.

Many studies agree that on average mutual funds underperform passive benchmarks by a statistically and economically significant margin[18]. It is particularly true for the equity asset class, but it is less verified for the bond asset class. In our example, if we accept the academic rule that the average performance of active management (represented by the quantile 50%)) is equal to the performance of the benchmark (represented by the DEBT indexation) minus the fees, we obtain an implied fees ratio equal to 36 bps per year. This academic rule seems to be verified. It is interesting to notice that risk-based indexations favorably compare with the top 10% best funds of active management.

Figure 10: Comparison of bond indexes with active management



[18]See Hereil *et al.* (2010) for a survey.

Table 11: Characteristics of the portfolios (September 2011)

Country	DEBT		GDP		GDP-RB		DEBT-RB		DEBT*		DEBT*-GDP	
	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC	Weights	RC
Austria	4.2%	1.5%	3.2%	1.3%	3.8%	3.2%	5.5%	4.2%	4.3%	1.8%	5.5%	4.3%
Belgium	6.2%	5.8%	4.0%	4.0%	2.0%	4.0%	3.4%	6.2%	6.4%	6.7%	3.4%	6.4%
Finland	1.5%	0.2%	2.1%	0.3%	6.1%	2.1%	5.0%	1.5%	1.6%	0.3%	5.0%	1.6%
France	22.5%	13.2%	21.5%	13.8%	15.3%	21.5%	17.7%	22.5%	23.1%	15.5%	17.7%	23.1%
Germany	21.5%	4.7%	27.9%	6.7%	50.2%	27.9%	43.9%	21.5%	22.1%	5.5%	43.9%	22.1%
Greece	2.5%	13.0%	2.4%	13.8%	0.3%	2.4%	0.3%	2.5%	0.0%	0.0%	0.0%	0.0%
Ireland	1.4%	2.4%	1.7%	3.3%	0.5%	1.7%	0.4%	1.4%	1.4%	2.7%	0.4%	1.4%
Italy	22.5%	40.2%	17.0%	32.6%	4.7%	17.0%	6.7%	22.5%	23.1%	46.3%	6.8%	23.1%
Netherlands	6.0%	1.3%	6.6%	1.6%	12.5%	6.6%	12.7%	6.0%	6.2%	1.5%	12.8%	6.2%
Portugal	1.6%	3.9%	1.9%	5.1%	0.4%	1.9%	0.4%	1.6%	1.6%	4.4%	0.4%	1.6%
Spain	10.1%	13.6%	11.8%	17.5%	4.2%	11.8%	4.0%	10.1%	10.3%	15.5%	4.0%	10.3%
Yield-to-Maturity	3.88%		3.83%		2.86%		2.92%		3.51%		2.88%	
Duration	6.11		6.07		6.03		6.06		6.13		6.06	
Average Spread	3.26%		3.08%		1.50%		1.63%		2.68%		1.56%	
Implied Rating	A		A		AA+		AA+		A+		AA+	
Weight Concentration	48.8%		50.7%		65.4%		61.2%		51.9%		61.6%	
Risk Concentration	56.6%		52.6%		50.7%		48.8%		64.0%		51.9%	
Risk Measure	8.97%		8.26%		4.13%		4.57%		8.12%		4.48%	

6 Conclusion

In this article, we present a useful framework to analyze the sovereign risk of a bond portfolio. By defining the credit risk of a bond portfolio as the volatility of the CDS basket which would perfectly hedge the sovereign risk of the portfolio, we can compute the sovereign risk contribution of each country and monitor it across time. Using this decomposition, we show that indexation based on risk budgeting techniques may be an efficient alternative to the traditional capitalisation-based approach to indexation. We also show that fundamental indexation is not enough and must be supplemented by risk-based methods.

We focus here on portfolios of sovereign bonds and on credit risk. However, this approach may be easily extended to take into account other risks (e.g. interest rates or foreign exchange) or to be applied to other asset classes (e.g. corporate or high yield bonds). In this last case, bond indexes based on risk budgeting techniques present an interesting feature from a Solvency II point of view because they will need less capital requirements compared to traditional bond indexes.

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A Appendix

A.1 Maximum likelihood of the diffusion process for modeling spreads

We assume that we observe spreads at some known dates $t_0, \ldots, t_m$. Let $S_{i,j}$ be the observed spread for the i^{th} country at date t_j. The log-likelihood function for the i^{th} country is given by the following formula:

$$\begin{aligned} \ell &= -\frac{m}{2}\ln 2\pi - m\ln\sigma_i^S - \frac{1}{2}\sum_{j=1}^{m}\ln\left(t_j - t_{j-1}\right) - \\ & \quad \beta_i \sum_{j=1}^{m}\ln S_{i,j-1} - \frac{1}{2}\sum_{j=1}^{m}\frac{\left(S_{i,j} - S_{i,j-1}\right)^2}{\left(\sigma_i^S S_{i,j-1}^{\beta_i}\right)^2} \end{aligned}$$

We estimate the coefficient β_i by maximizing the concentrated log-likelihood function. Figure 10 shows the maximum likelihood estimation of β_i for the period January 2008-August 2011.

Table 12: Results for the period January 2008 – August 2011

Country	AT	BE	FI	FR	DE	GR	IE	IT	NL	PT	ES	Average
estimate	0.996	1.017	0.816	0.786	0.899	1.070	0.836	1.157	0.793	1.013	1.148	0.957
std-dev.	1.10%	2.00%	1.60%	1.60%	2.00%	1.10%	0.70%	1.70%	0.90%	1.10%	2.10%	1.45%

A.2 Risk decomposition of the volatility risk measure

We consider a portfolio $x = (x_1, \ldots, x_n)$ of n assets. Let Σ be the covariance matrix of assets. The volatility of the portfolio is then equal to:

$$\sigma(x) = \sqrt{x^\top \Sigma x}$$

We deduce that the marginal volatility is:

$$\frac{\partial\,\sigma(x)}{\partial\,x} = \frac{\Sigma x}{\sqrt{x^\top \Sigma x}}$$

We define the risk contribution as the product of the weight times the marginal volatility:

$$\begin{aligned} \mathrm{RC}_i &= x_i \cdot \frac{\partial\,\sigma(x)}{\partial\,x_i} \\ &= x_i \cdot \frac{(\Sigma x)_i}{\sqrt{x^\top \Sigma x}} \end{aligned}$$

We verify that the sum of risk contributions is exactly equal to the volatility risk measure:

$$\begin{aligned} \sum_{i=1}^{m}\mathrm{RC}_i &= \sum_{i=1}^{m} x_i \cdot \frac{(\Sigma x)_i}{\sqrt{x^\top \Sigma x}} \\ &= x^\top \frac{\Sigma x}{\sqrt{x^\top \Sigma x}} \\ &= \sqrt{x^\top \Sigma x} \\ &= \sigma(x) \end{aligned}$$