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INCENTIVE-COMPATIBLE AND EFFICIENT RESOURCE ALLOCATION IN LARGE ECONOMIES: AN EXACT AND LOCAL APPROACH

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ABSTRACT

The main result of this paper characterizes possibly non-symmetric strategy-proof and efficient choice functions as Perfectly Competitive. Efficiency is defined as impossibility of improvement by reallocation of commodity among finite sets of agents, and largeness of the economy is captured by a weak aggregation-condition called "local separability". Individual rationality constraints with respect to an assignment of endowments imply that the resulting allocations must be Walrasian relative to the assignment of endowments. The exact, local approach combined with a normality assumption on the domain of preferences allows the proofs to remain elementary throughout.

1. INTRODUCTION

Since Adam Smith's praises of the Invisible Hand, mainstream economics has been engaged in a love affair with the ability of idealized competitive markets to systematically yield efficient outcomes in a decentralized fashion. Decentralization has been understood as economy of information transmission and as incentive compatibility of information generation. The pioneering work of Hurwicz (1972) has made the second aspect accessible to rigorous micro-economic analysis; in particular, the question can be posed whether the privileged place of competitive markets both in theory and empirical reality can be accounted for in these terms, or whether there exist other "mechanisms" that combine efficiency with incentive compatibility.

It is well-known that in economies with a finite number of agents, price-taking is not fully incentive compatible, and that strategy-proofness and efficiency cannot coexist[1]. On the other hand, in "large" economies in which agents are infinitesimal, price-taking is incentive compatible; Walrasian equilibrium becomes genuine Perfect competition[2]. A large economy is therefore the natural setting to investigate the entire class of mechanisms that reconcile incentive compatibility and efficiency. Indeed, following Hammond (1979), the literature has obtained a number of results which characterize Walrasian allocations as the outcomes of strategy-proof and efficient mechanisms.

The present paper goes beyond the existing literature by considering general non-symmetric mechanisms satisfying an aggregation condition called "local separability". By contrast, the literature has focused on characterizing "envy-free" allocations as the equilibrium outcomes of symmetric mechanisms. Allowing for a-symmetries in

[1]Except in very special circumstances.

[2]This point has been emphasized particularly by Ostroy and Makowski.

the mechanism is important conceptually, for the basic intuition behind the privileged place attributed to the competitive mechanism – that in order to achieve efficiency in an incentive compatible way, private opportunity costs have to agree with social opportunity costs[3] – has nothing to do with symmetry of the mechanism. Furthermore, absence of symmetry may easily allow for qualitatively different incentive compatible mechanisms; this is the case in finite exchange economies, where for instance "serial dictatorships" (a la Satterthwaite-Sonnenschein (1981)) are inherently asymmetric.[4] Asymmetry is also of interest also from a mechanism-design perspective which asks whether and when lump-sum transfers (redistribution without loss of first-best efficiency) are feasible in the presence of private information. Non-symmetric mechanisms arise here when some of the criteria relevant to the evaluation of an allocation are observable; an example would be income subsidies based on observable handicaps.[5] As a consequence of our main result, it, will be shown that if resources are privately owned, incentive compatible redistribution of income will generally result in a loss of first-best efficiency, even if it is based on observable non-preference characteristics (Theorem 2 in section 5).

Using a measure-based definition of Pareto efficiency (on which an allocation is Pareto efficient if it is not possible to make a set of agents *of strictly positive measure* better off without making a set of agents of positive measure worse off), the literature has remained silent on non-symmetric mechanisms, perhaps because strategy-proofness is almost without force in this context (see observation 1 following Theorem

[3]See section 4 for more details.

[4]Barbera-Jackson (1995) also, suggest that non-symmetric mechanisms may differ substantially from symmetric ones and may be of independent interest.

[5]We note that to a limited extent, non-symmetries can be captured by conventional approaches; what is needed is that there is a finite number of observably distinct types such that the distribution of characteristics within ***each type*** satisfies the required connectedness or richness assumptions.

2); this seems counterintuitive and moreover clearly off the mark as an approximate characterization of what happens in large finite economies. A more satisfactory result can be achieved by defining an efficiency criterion which takes individual agents seriously. To this effect, the concept of "Finite Consumption Efficiency" (FCE) is introduced in section **3.** FCE requires that it must not be feasible to make a finite set of agents strictly better off by a reallocation of commodities among them that leaves the allocation of outside agents untouched.

The concept of finite consumption efficiency is akin to that of a "finite core" developed by Kaneko and Wooders (see, in particular, Kaneko and Wooders (1986) as well as Hammond, Kaneko and Wooders (1988) and of "multilateral incentive compatibility" (see Hammond (1987)), as is the underlying philosophy of taking individual agents seriously. Indeed, as Kaneko and Wooders point out, in some contexts such as assignment games, it seems hard to conceive of a meaningful alternative to a finite approach; incentive compatibility is another inherently individualistic non-cooperative concept.

The setting of an "effectively large" economy is captured by an assumption of "local separability" which requires that an agent's consumption does not depend on changes in the characteristics of a finite number of other agents. As shown in section **3,** FCE combined with local separability implies under regularity assumptions that agents are "shadow-price takers". These two conditions define thereby an "exact" approach to the study of efficient allocations in large economies that may be helpful in other contexts.

Section 4 contains the central result; of the paper, a characterization of strategy-proof, locally separable and finitely consurnption-efficient choice functions as Perfectly Competitive. The result assumes a domain of preferences with normal demand functions.[6]

[6]A prior, non-elementary result based on normal preferences has been obtained by Mas-Collel (1987).

The normality assumption gives rise to a proof whose simplicity matches the essential simplicity of the underlying economic argument; in particular, we do not need to employ the rather heavy mathematical machinery involved in standard proofs which typically rely on topological measure spaces of smooth preferences (cf. Champsaur-Laroque (1981), Mas-Colell (1985)).[7]

In proposing an exact approach to the study of large economies, we do not mean to deny the legitimacy of the measure-theoretic one. Rather, the present paper shows that an exact approach can improve on measure-theoretic ones in two ways: first, as illustrated by Theorem 2, exact finitary conditions such as FCE may have substantial extra bite. In addition, in the context of this paper an exact approach reduces the role of measure-theoretic infrastructure to a minimum (it enters here only in the definition of feasibility), and thereby purifies the economic logic of the argument. (A more detailed justification of the joint use of exact and measure-theoretic notions is offered in section 7.)

That the measure-theoretic approach is not altogether complete has been argued in the literature in a somewhat indirect way via the claim that for a continuum economy to be economically meaningful, it must be a well-defined limit of large economies, hence regular (cf. Champsaur-Laroque (1982), Makowski-Ostroy (1992)). We observe in section 6 that this line of argument, in conjunction with a measure-theoretic definition of consumption efficiency, in fact entails FCE. However, a downside of this "asymptotic approach" is that existence (for example of regular Walrasian equilibrium) becomes at best generic. To the degree one attaches intrinsic intelligibility and interest to infinite economies, the weaker exact approach seems preferable, since it does not create difficulties with existence, and since its defining conditions, local

[7]Indeed, the starting point of this paper was the attempt to explain the essence of the standard results without the use of measure-theory to my graduate-students.

separability and finite consumption efficiency, can be motivated self-sufficiently in infinite economies.

The plan of the paper is as follows. Section 2 presents the choice-functional framework of the paper, with particular emphasis on the notion of a "neighborhood of economies" ("local domain"). Section **3** introduces the methodologically distinctive assumptions of the present paper, local separability and finite consumption efficiency. The main result of the paper, a characterization of possibly non-symmetric Perfectly Competitive choice functions, is demonstrated in section 4. As shown in section 5, in the presence of individual rationality constraints the result implies strong restrictions on allocations; these implications are not obtainable with a measure-theoretic definition of efficiency.

The last two sections provide further support for the exact approach. Section 6 discusses the difference between exact and measure-theoretic efficiency concepts; in particular, it is argued that FCE is sound also from an asymptotic point of view which views infinite economies as limiting descriptions of large finite economies. Section 7 deals with conceptual issues concerning the definition of feasibility and efficiency in large economies; in particular, they explain how FCE is consistent with a measure-theoretic definition of feasibility.

2. FRAMEWORK AND NOTATION

In the following, a "social choice" approach to resource allocation in (large) private good economies is undertaken. Individual agents $\mathbf{i} \in \mathbf{I}$ are described by their preferences over allocations $f = (f_i)_{i \in I}$,which, in a private goods context, can be identified with preferences over commodity bundles $x \in \mathbf{R}^{\ell}_{+}$. $\mathbf{I}$ will assumed to be infinite unless otherwise specified. Resource scarcity is described by the requirement that allocations have to be feasible, i.e., that $f \in F \subset \mathbf{R}_+^{\ell \times I}$, where F is endowed with appropriate structure to reflect the private goods environment. A hallmark of the social choice approach is the absence of individual endowments in the description of the economy. Information about these may be added – for instance, in order to impose individual rationality constraints, – but doing so is optional.

This is an interestingly different perspective on competitive "Walrasian" allocations, in that many of our standard conceptions about the central role of competitive allocations are derived from an environment in which private property rights are already well-specified. These intuitions have found their deepest and most developed expression in the "core-equivalence Theorems" for large economies. By contrast, an axiomatization of competitive allocatians in the more abstract social choice framework provides eo ipso an account of the role of claims to private property implicit in the definition of a Walrasian allocation.

An agent i is described by a **preference relation** $\succ$ on $\mathbf{R}_+^\ell$ $(= \{x \in \mathbf{R}_+^\ell | x \geq 0))$. Throughout, preferences $\succ$ are assumed to be *asymmetric*, monotone ($x > y$ implies $x \succ y$) and such that the sets $\{y | y \succ x)$ are convex and open in $\mathbf{R}_+^\ell$ for every $x \in \mathbf{R}_+^\ell$. They are also assumed to lead to well-defined choices from budget sets, i.e., denoting $\phi(p, m; \succ) := \{x \in \mathbf{R}_+^\ell | \; p \cdot x \leq m, \text{ and for no } y \in \mathbf{R}_+^\ell \text{ such that } p \cdot y \leq m : \; y \succ x\}$, it is assumed that $\phi(p, m; \succ)$ is non-empty whenever $p \gg 0$ and $m > 0$. Throughout, prices are normalized to have Euclidean norm 1; i.e. $p \in S_+^\ell = \{x \in \mathbf{R}_+^\ell | \|x\| = 1, x \geq 0)$, and often $p \in S_{++}^\ell = \{x \in \mathbf{R}_+^\ell | \|x\| = 1, x \gg 0)$.

Let $\mathcal{P}$ denote a class of such relations. An **allocation** is a mapping $f : \mathbf{I} \rightarrow \mathbf{R}_+^\ell$ (and may be viewed as an element of $\mathbf{R}_+^{\ell \times I}$). An **economy** is a mapping $\mathcal{E} : \mathbf{I} \rightarrow \mathcal{P}$.

In this paper, the feasible set F is held fixed and plays only a subordinate role. An economy can thus be identified with its preference profile. We will consider domains $\mathcal{D}$ of economies that are closed under changes of the preferences of single agents.

Definition 1 $\underline{\mathcal{E}' \sim \mathcal{E}}$ if $\mathcal{E}$ and $\mathcal{E}'$ *differ* in at most a finite number of individuals, *i.e.*, if $\#\{i \in I | \ \mathcal{E}(i) \neq \mathcal{E}'(i)\} < \infty$. $\mathcal{D} \subseteq \mathcal{P}^I$ is **locally closed** if $\mathcal{E} \in \mathcal{D}$ and $\mathcal{E}' \sim \mathcal{E}$ imply $\mathcal{E}' \in \mathcal{D}$. *A* locally closed $\mathcal{D}$ is a **local domain** if $\mathcal{E}, \mathcal{E}' \in \mathcal{D}$ imply $\mathcal{E} \sim \mathcal{E}'$.

Local domains ("neighborhoods of economies") are the equivalence classes $[\mathcal{E}]$ defined by $\sim$; sometimes, to highlight the role of $\mathcal{P}$ in defining this equivalence class, we will also write [E;$\mathcal{P}$]. Note that $[\mathcal{E}] = \mathcal{P}^I$ if and only if **I** is finite. If **I** is infinite, economies in the same local domain are only microscopically different, and $\sim$ can be interpreted as relation of macroscopic *equivalence.*

A choice function C maps economies to allocations, $C : \mathcal{D} \rightarrow \mathbf{R}_+^{\ell \times I}$. C is assumed to be a proper function, that is: to be non-empty and single-valued. Properties of allocations such as efficiency and individual rationality are understood to determine analogous properties of choice function:; economy by economy.

In the following, choice functions defined on local domains can be viewed as the proper object of study. We will argue in section **3** that the notion of Perfect Competition is naturally defined in those terms; moreover, all properties used in its characterization will be local, concerning either one economy at a time (efficiency and individual rationality properties) or changes in the preferences of a finite number of agents (strategy-proofness in section 4 and "local separability" in section **3).** Choice functions defined on local domains might well turn out to be the natural object of study also in other future investigations of axiomatic resource allocation.

3. AGENTS AS SHADOW-PRICE TAKERS

In a large private goods economy, it is natural and standard to assume that an agent's consumption depends only on his characteristics and the macroscopic features of the economy (see, for instance, Harnmond (1979) and Dubey-Mas Colell-Shubik

(1980)). A weak version of this is to require that any agent's consumption be independent of any variation in the characteristics of a finite number of individuals, as in the following

Axiom 1 (Local Separability) *For all* $\mathcal{E}, \mathcal{E}' \in \mathcal{D}$, $\mathcal{E}(i) = \mathcal{E}'(i)$ *and* $\mathcal{E} \sim \mathcal{E}'$ *imply* $C(\mathcal{E})(i) = C(\mathcal{E}')(i)$.

Note that on a local domain, C is locally separable if and only if $\mathcal{E}(i) = \mathcal{E}'(i)$ implies $C(\mathcal{E})(i) = C(\mathcal{E}')(i)$. In this case, the choice function can be represented as a collection of individual choice functions $(C^i)_{i \in I}$, with $C^i : \mathcal{P} \to \mathbf{R}^{\ell}_{+}$.

Local separability helps ensure that a single agent's characteristics have no influence on shadow prices, that is: no effect on the relative scarcity of different goods. Moreover, in a strategic context, local separability of the choice function is the natural result of local separability of the underlying mechanism (analogously defined). This is analyzed in more detail in Nehring (1998) which also shows that Nash behavior in locally separable mechanisms induces *strategy-proof* choice functions.

In the context of choice functions defined on local domains, merely "approximate" criteria of efficiency ψ are inadequate; a property ψ is *approximate* if f satisfies ψ in $\mathcal{E}$ whenever f satisfies ψ in some —-equivalent $\mathcal{E}'$. Clearly, all measure-theoretic efficiency concepts are approximate.

Approximate concepts are inadequate, since by definition they are irresponsive to changes of preferences within [*I*]. Hence, any locally constant choice function that yields an "approximately efficient" allocation in some economy $\mathcal{E}$ yields approximately efficient allocations throughout the local domain $[\mathcal{E}]$. Moreover, intuitively obvious Pareto improvements *are* missed by these concepts, such as the possibility that two agents improve by exchanging their commodity bundles. In their stead, we propose the following exact, finitary one.

Axiom 2 (Finite Consumption Efficiency) The allocation f satisfies FCE if for no $g \in \mathbf{R}_+^{\ell \times I}$ and no finite set $J \subset I$ *such* that $g_i = f_i$ for all $i \in I \setminus J$, $\sum_{i \in J} g_i = \sum_{i \in J} f_i$, and $g_i \succ_i f_i$ *for* all $i \in J$.

In words: For f to satisfy FCE, it must not be feasible to make a finite set of agents strictly better off by a reallocation of commodities among them that leaves the allocation of outside agents untouched.[8]

Remark. Under the maintained assumptions on preferences, FCE can be shown[9] to be equivalent to the existence of a price vector $p \in S_{++}^{\ell}$ supporting every agent's consumption. Thus, FCE can be viewed as equivalent to consumption efficiency for reallocations among groups of agents of *arbitrary* size; in particular, it implies the absence of improving reallocations among groups of traders with positive measure (i.e. μCE which is formally defined in section 5). For further extensive discussion of the status and content of FCE, see sections 6 and 7.

It seems natural to expect that FCE combined with local separability implies "shadow-price *taking*", i.e., independence of shadow prices from variations in any agent's preferences. This property will be crucial for the subsequent substantive results.

Definition 2 C is **uniformly supported** on $[\mathcal{E}, \mathcal{P}]$ by $p \in S_+^{\ell}$ if, for all $i \in I$ *and all* $\succ \in \mathcal{P}$, $C(\succ, \mathcal{E}_{-i})(i) \in \phi(p, p \cdot C(\succ, \mathcal{E}_{-i})(i); \succ)$.

The following example shows that shadow-price taking , i.e., uniform supportedness is not implied by local separability by itself.

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[8]FCE can also be interpreted as a renegotiation-proofness condition, weakening Gale (1980) and Harnmond's (1987) "multilateral incentive-compatibility" condition.

[9]With the help of Tychonoff's theorem.

Example 1 *Let* $\mathrm{P} = \{\succ^{\alpha}\}_{\alpha\in(0,1)}$, $\succ^{\alpha}$ *being representable by a utility function* U^{α} *with* $U^{a}(x_1, x_2) = \max(x_1 - 1, 0)^{\alpha} \max(x_2 - 1, 0)^{1-\alpha} + \max(\min(x_1, x_2), 1)$.

Consider any local domain $\mathcal{D} \subseteq \mathcal{P}^{I}$.

Define C *by* $C^{i^*}(\succ) = (2,2)\ \ \forall\ \succ\in\mathcal{P}$, *and* $C^{i}(\succ) = (1,1)\ \ \forall\ \succ\in \mathrm{P},\ i \# i*$.

C **is** *locally separable and FCE. However, each* $\mathcal{E} \in \mathcal{D}$ *has a unique supporting price* $\frac{(\alpha^*,1-\alpha^*)}{\|(\alpha^*,1-\alpha^*)\|}$ *determined by* $\mathcal{E}(i^*) = \succ^{\alpha^*}$.

Some regularity conditions are needed to obtain the desired conclusion. The first will play a crucial role in the proof of the main result of the paper. It is essentially a geometric version of standard smoothness (C2) conditions. For the purpose of immediate interest, proposition 1, though, the condition is clearly stronger than necessary.

Define the "radius of curvature" of the upper contour set of $\succ$ at $x \in \mathbf{R}^{\ell}_{++}$ by

$$\rho(x,\succ) := \sup\{r \mid \exists p \in S^{\ell}_{++} : \{z \mid z \succ r\} \supseteq B^{r}(rp + x) + \mathbf{R}^{\ell}_{+}\},$$

with $\sup\emptyset = 0$ by convention.

Axiom 3 (Uniform Smoothness)

For all $y > 0$, $\inf\left\{\rho(x,\succ) \mid \succ\in\mathcal{P}, x \in \mathbf{R}^{\ell}_{++} : \|x\| \geq \gamma\right\} > 0$.

Remark. The clause "$\|x\| \geq \gamma$" has been inserted to allow for arbitrarily small curvature radii near the origin; those come about easily, as for instance in the case of Cobb-Douglas preferences.

To exploit the smoothness of preferences, a regularity assumption on the choice function is also needed.

Definition 3 A *choice function* C *is* **weakly interior** *if, for every economy* $\mathrm{E} \in \mathcal{D}$ *there are at least two agents* $i_1, i_2 \in I$ *and two preference relations* $\succ_{i_1}, \succ_{i_2} \in \mathcal{P}$ *such that* $C(\succ_{i_k}, \mathcal{E}_{-i_k})(i_k) \gg 0$ *for* $k = 1, 2$.

Proposition 1 *If a locally separable, weakly interior choice function* C *on a uniformly smooth domain* $\mathcal{D}$ *satisfies FCE, then* C *is uniformly supported on every local subdomain* $[\mathcal{E}] \subseteq \mathcal{D}$.

Proof. W.l.o.g., $\mathcal{D}$ is a local domain. Under the assumptions on C and $\mathcal{D}$, there exist at least two agents $i_1, i_2 \in \mathbf{I}$ and preference relations $\succ_{i_1}, \succ_{i_2} \in \mathbf{P}$ such that $C^{i_k}(\succ_{i_k})$ is uniquely supported by p_k, k $= 1, 2$. **By** FCE, $p_1 = p_2 =:$ p. For any $j \in I$ and $\succ \in \mathbf{P}$, there exists $\mathcal{E} \in \mathcal{D}$ such that $\mathcal{E}(j) = \succ$ and $\mathcal{E}(i_k) = \succ_{i_k}$, for $k = 1$ or **k** $=$ **2.** Assume, w.l.o.g. that $\mathcal{E}(i_1) = \succ_{i_1}$. In view of the openness of preferences, the convexity of both $\succ$ and $\succ_{i_1}$ and the unique supportedness of $C^{i_1}(\succ_{i_1})$ by p, it follows from FCE via a standard separation argument that p supports $C^j(\succ)$ as well. ■

4. A CHARACTERIZATION OF PERFECTLY COMPETITIVE CHOICE FUNCTIONS

It is a major advantage of infinite models of economic resource allocation that they endow the notion of "price taking" with real rather than merely approximate meaning. To emphasize the difference to the finite case, such allocations and choice functions are sometimes referred to as "Perfectly Competitive" rather than merely "Walrasian", a usage which we will follow.[10] Perfect Competition as effective price taking is naturally expressed in *local choice-functional* terms: the value of an agent's consumption (his shadow income) evaluated at constant shadow-prices remains constant as *his preferences change.* This is captured by the following definition.

[10]**This distinction has been emphasized especially in the work of Ostroy and Makowski.**

Definition 4 $C : \mathcal{D} \to \mathbf{R}_+^{\ell \times I}$ *is* **Perfectly Competitive** *if, for all local subdomains* $\mathcal{D}' \subset \mathcal{D}$, *there exists a price vector* $p \in S_+^{\ell}$ *and an income assignment* $m : I \to \mathbf{R}_+$ *such that* $C(\mathcal{E})(i) \in \phi(p, m(i); (\mathcal{E}(i))$ *for all* $\mathcal{E} \in \mathcal{D}'$ *and* $i \in I$.

Roughly put, the definition requires that all agents maximize preferences subject to a given income and common prices which they do not influence. Note that it permits an agents' income to depend on macroscopic features of the economy in arbitrary ways.

Note also that on a domain of smooth preferences with single-valued demand functions, any FCE allocation f in an economy $\mathcal{E}$ uniquely extends to a Perfectly Competitive choice function C which is furthermore locally separable. As a result, existence of Perfectly Competitive choice functions on a local domain consistent with a given allocation of property rights (cf. section 5) is ensured by the existence of competitive equilibria in one of its economies.

While common supporting prices originate in efficiency conditions, it is natural to look for an axiomatic basis for the givenness of an agent's income in incentive compatibility properties, in particular strategy-proofness which amounts to requiring that an agent's consumption maximize his preferences subject to **a** given budget constraint (of arbitrary shape); the task of the mathematical argument is to show that under appropriate assumptions, the budget constraint must be linear.

Axiom 4 (Strategy-proofness)

For no $\mathcal{E} \in \mathcal{V}$, $i \in I$ *and no* $\succ' \in \mathcal{P}$: $\quad C(\succ', \mathcal{E}_{-i})(i) \succ C(\mathcal{E})(i)$

Remark. While in finite settings strategy-proofness characterizes implementability in dominant strategy equilibrium, in infinite settings it is of much broader applicability. In the latter, strategy-proofness is entailed by Nash equilibrium behavior in "locally separable mechanisms"[11], and by Bayesian Nash equilibria with independent types. For symmetric mechanisms, these claims are straightforward and well-known; they are shown to generalize to non-symmetric mechanisms in Nehring (1998).

Are strategy-proof and efficient choice functions Perfectly Competitive as indicated? Intuition suggests that if strategy-proof choice functions are to systematically lead to efficient allocations, private incentives have to be aligned with social opportunity costs. Since in a large private goods economy social opportunity costs can be measured in terms of shadow prices that are independent of any particular agent's characteristics, such an alignment occurs if and only if the value of an agent's consumption evaluated at these prices is constant, i.e., if the choice function is Perfectly Competitive.

The intuition also suggests that the key to the desired characterization result is sufficient richness of the domain $\mathcal{P}$ of preferences an agent might have; this is exactly what Theorem 1 below delivers. In agreement with the above intuition, however, neither symmetry nor the distribution of preferences in the actual economy matter. As described, the present situation parallels that of the characterization of Groves mechanisms, both in terms of the intuition commonly presented and in terms of the formal structure of the characterization (see Green and Laffont (1977) and Holmstrom (1979)) which also makes assumptions on the domain of preferences only. By contrast, the literature on the subject (Hammond (1979), Champsaur-Laroque (1981) and others) is restricted to characterizing the allocations resulting from symmetric mechanisms, and hinges on strong assumptions on the distribution of preferences in the economy. The only exception to the latter seems to be Makowski and Ostroy (1992)[12].

[11] Defined analogously to locally separable choice-functions

[12] See also section 6.

The relevant richness condition is a connectedness assumption, a simplified version of standard ones. For its statement, we need to define a family of metrics on the domain of preferences; let

$$d^{(p,\overline{m})}(\succ,\succ') := \sup_{m\leq \overline{m}} \|\phi(p,m;\succ) - \phi(p,m;\succ')\|.$$

Axiom 5 (Lipschitz Connectedness) *For all* $\succ,\succ'\in P$ *and all* $p,\overline{m}$ *there exists a mapping* $h:[0,1]\rightarrow\mathcal{P}$ *and* $L<\infty$ *such that i)* $h(0)=\succ$, *ii)* $h(1)=\succ'$, *and iii)* $d^{(p,\overline{m})}(h(t),h(t'))\leq L\cdot\mid t-t'\mid$ *for all* $t,t'\in[0,1]$.

In contrast to most of the literature, we will finally assume that preferences generate *normal* demand behavior. This simplifies the proof significantly.

Axiom 6 (Normality) *For all* $\succ\in\mathcal{P}$, $p\in S^{\ell}_{++}$, *and all* $x,y\in\mathbf{R}^{\ell}_{+}$:
$x\in\phi(p,p\cdot x,\succ)$, $y\in\phi(p,p\cdot y,\succ)$ *and* $p\cdot y\geq(\leq)p\cdot x$ *imply* $y\geq(\leq)x$.

Note that normality implies single-valuedness of the demand functions $\phi(..,\succ)$. To avoid technicalities associated with boundary consumptions, we will assume choice functions to be interior.

Definition 5 C *is* **interior** *if, for all* $\mathcal{E}\in\mathcal{D}$ *and all* $i\in I$, $\quad C(\mathcal{E})(i)\gg 0$.

Theorem 1 *Consider a locally closed domain of economies* $\mathcal{D}\subseteq\mathcal{P}^I$ *such that* $\mathcal{P}$ *is normal, uniformly smooth and Lipschitz connected. An interior choice function defined on such* $\mathcal{D}$ *is Perfectly Competitive if and only if it is locally separable, FCE and stmtegy-proof.*

Proof.

Necessity of FCE and strategy-proofness is clear; that of local separability follows from the single-valuedness of demand functions entailed by the definition of normality.

For sufficiency, note first that from the nature of the claim, it is without loss of generality to assume $\mathcal{D}$ to be a local domain, i.e. to be of the form [I]By Proposition 1, the assumptions of the theorem ensure the existence of a uniformly supporting price vector $p^* \in S^{\ell}_{+}$, hence by the monotonicity of preferences in fact $p^* \in S^{\ell}_{++}$. Using the notation of section 2, it thus needs to be shown that for all $i \in \mathbf{I}$, the agents' shadow income $p^* \cdot C^i(\succ)$ is constant as a function of $\succ \in \mathbf{P}$. Thus, fix some $i \in \mathbf{I}$.

The following mathematical fact plays the role of the envelope theorem in standard proofs; in very rough terms, it can be viewed as a non-infinitesimal version thereof. It is Illustrated in figure 1.

Lemma 1 *Given $p \in S^{\ell}_{++}$ and $r > 0$, there exist $\varepsilon > 0$ and $K < \infty$ such that for all $w \in \mathbf{R}^{\ell}$ and $y \in \mathbf{R}^{\ell}_{+}$,*

i) $p \cdot w = 0$,

ii) $\|w\| \leq \varepsilon$ *and*

iii) $y + w \notin (B^r(rp) + \mathrm{R}^{\ell}_{+})$

imply $\|y\| \leq K\|w\|^2$.

Proof. See appendix. □

Figure 1 about here

The key to the proof is the following lemma which shows that preferences close to each other receive similar shadow incomes; the rest is mathematics.

Let $m(\succ) := p^* . C^i(\succ)$ for $\succ \in \mathbf{P}$.

Lemma 2 *There exist $\varepsilon^* > 0$ and $K^* < \infty$ such that for any $\succ, \succ' \in P$ satisfying $d^{(p^*, m(\succ))}(\succ, \succ') \leq \varepsilon^*$:*

$|m(\succ') - m(\succ)| \leq K^* \left[d^{(p^*, m(\succ))}(\succ, \succ')\right]^2$.

Proof. Take any $\succ' \in \mathcal{P}$, and set $\gamma^* := \min_{k<\ell} C^i_k(\succ') > 0$ (by interiority). Note that in fact for any $\succ \in \mathbf{P}$, $\|C^i(\succ)\| \geq \gamma^*$ by strategy-proofness and the monotonicity of preferences. By the interiority of the choice function and uniform smoothness, there exists $r^* > 0$ such that

$$\{z \mid z \succ C^i(\succ)\} \supseteq B^{r^*}(r^*p^* + C^i(\succ)) + \mathbf{R}^\ell_+ \text{ for all } \succ \in \mathbf{P}. \tag{1}$$

For $p = p^*$ and $r = r^*$, fix some $\varepsilon =: \varepsilon^*$ and $K =: K^*$ from lemma *1*.

Consider $\succ, \succ' \in \mathcal{P}$ such that $d^{(p^*,m(\succ))}(\succ, \succ') \leq \varepsilon^*$.

Let $x := C^i(\succ)$, $w := \phi(p^*, m(\succ); \succ') - x$, and $y := C^i(\succ') - \phi(p^*, m(\succ); \succ')$; thus $C^i(\succ') = x + w + y$.

By strategy-proofness and (1),

$C^i(\succ') = x + w + y \notin B^{r^*}(x + r^*p^*) + \mathbf{R}^\ell_+$, which implies

$$w + y \notin B^{r^*}(r^*p^*) + \mathbf{R}^\ell_+ \quad . \tag{2}$$

Similarly, strategy-proofness and (1) yields $C^i(\succ) = x \notin B^{r^*}(x + w + y + r^*p^*) + \mathbf{R}^\ell_+$, which implies

$$-w - y \notin B^{r^*}(r^*p^*) + \mathbf{R}^\ell_+ \ . \tag{3}$$

By the normality of $\succ'$, $y \geq 0$ or $y \leq 0$. If $y \geq 0$, application of lemma 1 to *(2)* yields $\|y\| \leq d^{(p^*,m(\succ))}(\succ, \succ')^2 \cdot K^*$. If $y \leq 0$, application of lemma 1 to *(3)* yields the same inequality. Since also $|p^* \cdot C^i(\succ') - p^* \cdot C^i(\succ)| = |p^* \cdot y| \leq \|y\|$, the claim follows. □

Fix now $\succ^*$, and let $M = p^* \cdot C^i(\succ^*) (\varepsilon^*)^2 \cdot K^*$.

Take $\succ^{**} \in \mathcal{P}$ such that $d^{(p^*,M)}(\succ^*, \succ^{**}) \leq \varepsilon^*$. By Lipschitz connectedness, there exist $L < \infty$ and a mapping $h : [0,1] \to \mathcal{P}$ such that i) $h(0) = \succ^*$, ii) $h(1) = \succ^{**}$, and iii) $d^{(p^*,M)}(h(t), h(t')) \leq L \cdot |t - t'|$ for all $t, t' \in [0,1]$.

For any natural number $n > L/\varepsilon^*$,

$$|m(\succ') - m(\succ)| \leq \sum_{j=1,\ldots,n} \left| m\left(h(\frac{j}{n})\right) - m\left(h(\frac{j-1}{n})\right)\right| \leq nK^*(\frac{L}{n})^2$$

by lemma 2.

Since this inequality holds for arbitrary sufficiently large n, it follows that in fact $m(\succ^{**}) = m(\succ^*)$ for any $\succ^*, \succ^{**} \in \mathcal{P}$ within ε^* of each other.

Finally, since Lipschitz Connectedness implies ordinary path-connectedness, this conclusion extends to arbitrary $\succ^*, \succ^{**} \in \mathcal{P}$. ■

Remark 1. Compared to typical results in the literature such as that of Champsaur-Laroque (1981), Theorem 1 is stronger in not assuming transitivity and completeness of preferences. It is weaker, on the other hand, in assuming normality of the induced demand function. Mas-Colell (1987) also assumes normality, and replaces connectedness by a global richness assumption on preferences.

Remark 2. The main advantage of using normality lies in the resulting simplification of the proof. The present proof is elementary, while those in the literature rely on rather heavy mathematical machinery such as topological spaces of preferences and measures thereon.

More substantively, its *non-infinitesimal* approach suggests the robustness of the result to various infinitary assumptions. A particularly interesting type of result can be obtained by translating Theorem 1 into a result about Pareto efficient and envy-free allocations, in which case $\mathcal{P}$ is to be interpreted as the set of preferences of some agent in the actual economy $\mathcal{E}$. The proof of Theorem 1 allows one to show for *finite* economies that if the set of actual preferences is "almost connected", then the allocation must be "almost equal-income" Walrasian. A rough outline of the key steps of such a proof along the lines of Theorem 1 are given now .

To obtain *some* bound at all on the difference between the supporting incomes of any two agents, it must be possible to connect their consumptions by a path of consumption bundles of other agents whose longest link does not exceed a bound ε^{**} in length; the value of the bound is determined via lemma 1 by a lower bound on the curvature radius of any agent's preferences.

If this condition is satisfied, the difference between the supporting incomes of any two agents can be bounded by a "connectedness distance" of the allocation multiplied by a proportionality factor K^{**}; the relevant *connectedness distance* between their consumption bundles is defined as the shortest "path length" between the bundles, the path length in terms being given by the sum of the *squared* Euclidean distances of adjacent consumption bundles. As in the proof of Theorem 1, bounds on the distances between consumption bundles can be replaced by bounds on the distances between preferences.

5. PROPERTY RIGHTS IN AN EXCHANGE ECONOMY

It is clear and has been observed above that any finitely consumption-efficient allocation can be extended to a Perfectly Competitive choice function; thus, strategy-proofness alone fails to impose additional restrictions on efficient resource allocation in any given economy. This may change quite dramatically, however, if additional conditions are imposed. As an example, consider participation constraints based on property rights.

Let $w : \mathbf{I} \to \mathbf{R}^{\ell}_{+}$ denote an allocation of property rights, with ω_i representing the commodity bundle agent i is entitled to, i's endowment.

Axiom 7 f is individually rational with *respect to* w, if, *for no* $i \in I$, $\omega_i \succ_i f_i$.

Individual rationality has maximal bite if the domain of preferences is "comprehensive" rather than merely connected.[13]

Axiom 8 **A** *domain of preferences P is* **comprehensive** *if it contains a subdomain Q that is normal, uniformly smooth, Lipschitx convected and has the property that for every* $x \in \mathbf{R}^{\ell}_{++}$ *and* $p \in S^{\ell}_{++}$, *there exists* $\succ \in Q$ *such that* $\{z \in \mathbf{R}^{\ell}_{+} \mid z \succ x\} \subseteq \{z \in \mathbf{R}^{\ell}_{+} \mid p \cdot z > p \cdot x\}$.

Comprehensiveness requires of Q that it contain, for every strictly positive consumption bundle x and every strictly positive price vector p, a preference relation + that has p as its "gradient of preference".

The bite of individual rationality constraints is further enhanced if all resources are privately owned. This is easiest expressed using some measure-theoretic formalism (in a rather loose way; the technical details are entirely standard and omitted).

Let μ denote a non-atomic measure on the space of agents. Private ownership of resources is expressed by the following feasibility condition.

Axiom 9 **f** *is* **feasible** *with respect to w if* $\int f dp \leq \int w dp$.

Under these assumptions, individual rationality constraints imply that agents are effectively entitled to the full *value* of their endowments.

Definition 6 **f** *is* **Walrasian** *with respect to w if, for some* $p \in S^{\ell}_{+}$ *and* $\mu-a.e.$ $i \in I$, $f_i \in \phi(p, p \cdot \omega_i, \mathcal{E}(i))$.

Theorem 2 *Consider a choice function C on a locally closed domain* $\mathcal{D}$ *such that* $\mathcal{P}$ *is comprehensive and such that C restricted to* $\mathcal{D} \cap \mathcal{Q}^I$ *is interior. If C is locally separable, FCE, strategy-proof and individually rational as well as privately feasible with respect to the allocation of property rights* w, *then for all* $\mathcal{E} \in \mathcal{D}$, $C(\mathcal{E})$ *is Walrasian with respect to w.*

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[13] Mas-Colell (1987) uses a condition with similar flavor

Proof. W.l.o.g., assume **D** to be a local domain. Begin by considering the restriction of C to $\mathcal{D} \cap \mathcal{Q}^I$ denoted by $\overline{C}$. By Theorem 1, $\overline{C}$ is Perfectly Competitive for some price vector p^* and income assignment m^*.Let $H_{p^*,w} := \{z \in \mathbf{R}^\ell_+ \mid \mathbf{p^*}. z = w)$.

By comprehensiveness and interiority, for every $i \in \mathbf{I}$, $\{\overline{C}^i(\succ) \mid \succ \in \mathcal{Q}\} = H_{p^*,m^*(i)} \cap \mathbf{R}^\ell_{++}$.

Hence by individual rationality and monotonicity of preferences (as well as uniform smoothness to deal with non-interior ω_i), $m^*(i) \geq p^* \cdot \omega_i$, for every $i \in \mathbf{I}$.

By private feasibility, this implies $m^*(i) = p^* \cdot \omega_i$ for μ-a.e. $i \in \mathbf{I}$.

The claim of the theorem follows from showing that

$$\{C^i(\succ) \mid \succ \in \mathcal{P}\} \subseteq H_{p^*,m^*(i)}.$$

To verify this, note that by monotonicity of preferences and strategy-proofness, $\{C^i(t) \mid \succ \in \mathsf{P}) \cap \mathbf{R}^\ell_{++} \subseteq H_{p^*,m^*(i)}$, as well as $\{C^i(\succ) \mid \succ \in \mathsf{P}) \subseteq \{z \in \mathbf{R}^\ell_+ \mid \mathbf{p^*} \cdot z \geq w)$.

Finally, $\{C^i(\succ) \mid \succ \in \mathcal{P}\} \setminus \mathbf{R}^\ell_{++} \subseteq \{z \in \mathbf{R}^\ell_+ \mid \mathbf{p^*}. z \geq w)$, by monotonicity of preferences, strategy-proofness and uniform smoothness. ■

Theorem 2 does not appear to have an equivalent in the literature. This may be due to the fact that if FCE is replaced by a conventional measure-theoretic (hence approximate) efficiency criterion such as μCE (about to be defined), strategy-proofness has no force at all.

Axiom 10 *An allocation f satisfies μ**CE** if, for no g such that $\int g \, d\mu = \int f \, d\mu$: $\mu(g_i \succ_i f_i) > 0$ and $\mu(g_i \succ_i f_i$ or $g_i = f_i) = 1$.*

Observation 1 *Suppose **f** is an allocation in $\mathcal{E}$ that is μCE and individually rational with respect to w. Let P be any domain of preferences containing $\{\mathcal{E}(i) \mid i \in I)$. Then there exists a choice function C on $[\mathcal{E}, \mathcal{P}]$ that is locally separable and strategy-proof as well as μCE and individually rational with respect to* w *on $[\mathcal{E}, \mathcal{P}]$.*

Proof. Define C^i by $C^i(\succ) = \begin{cases} \omega_i & \text{if } \omega_i \succ f_i \\ f_i & \text{otherwise} \end{cases}$

It is easily verified that the resulting C has all properties claimed for it. □

6. FINITE VERSUS MEASURE-THEORETIC EFFICIENCY

In terms of assumptions, besides local separability, the principal departure of this paper from the literature has been the replacement of a measure-theoretic concept of efficiency by a finite one. This move leads to results that do not depend on the particular mathematization of infinity chosen, and which can be proved by elementary means. Beyond these "intended consequences, Theorem 2 shows that the obtainable results can be much stronger. The following remarks describe and try to account for this difference.

First, it should be noted that, viewed from within an infinite model, the difference between the two efficiency concepts is substantive, not merely technical: FCE corresponds to an exact, μCE to an approximate version of consumption efficiency. This is seen more clearly if μCE is reformulated – under our monotonicity assumptions equivalently – as the following condition:

$$\text{For no } g \in \mathbf{R}_+^{\ell \times I} : \int g d\mu < \int f d\mu \quad \text{and} \quad g_i \succ_i f_i \text{ for all } i \in I.$$

Thus, an allocation satisfies μCE if it is not possible to reallocate consumption bundles among agents in a way that leaves them as well off as before while saving an amount of resources that is significant in the aggregate. As an "approximate" efficiency concept, μCE allows for the possibility of saving a finite total amount of resources by finite reallocations.

The distinction between an exact and an approximate notion has a parallel from an asymptotic point of view. **A** sequence of allocations can be defined as "asymptotically *efficient*" if the per capita amount of resources that can be saved by some improving :reallocation shrinks to zero as the economy becomes large. It can be shown that (appropriately defined) limits of sequences of consumption efficient allocations are FCE in the limit economy, whereas the limits of sequences of asymptotically efficient allocations are μCE.[14]

It might thus seem that in a strategic context, μCE is the concept of primary interest, since the non-cooperative outcomes of "asymptotically locally separable mechanisms" (such as finite strategic market games) are typically at best asymptotically efficient. For at least two reasons, such a conclusion seems mistaken.

First, when C is appropriately continuous in a sense that will be made precise below, μCE in fact implies FCE. Since such continuity is arguably essential for the meaningfulness of an infinite model as a limiting representation of large finite models, the asymptotic point of view lends further support to FCE.

Secondly, in the important special case of quasi-linear preference domains, full consumption efficiency[15] is achievable in finite economies, even in dominant strategy equilibrium. For this case, our FCE based results establish the unity and qualitative continuity between finite and infinite economies – with the difference, of course, that full Pareto efficiency is attainable only in the latter.[16]

To define the concept of continuity relevant to the first point, it is notationally easiest to redefine economies as triples $(I, \mathcal{E}, \mu)$ and allow sets and measures of agents to vary.

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[14] Cf. Nehring (1997).

[15] In the sense of FCE; in finite quasi-linear economies, it is well-known that full Pareto-efficiency cannot be achieved due to the unavoidability of some surplus of the transferable commodity.

[16] For symmetric mechanisms, this point has been made before by Makowski-Ostroy (1992).

Definition 7 $(I',\mathcal{E}',\mu')$ is a J-perturbation of $(I,\mathcal{E},\mu)$ if $I' \supseteq I$ and $I' \setminus I$ can partitioned into sets $\{F_j\}_{j\in J}$ with the following properties:

i) $\mathcal{E}'(i) = \begin{cases} \mathcal{E}(i) \text{ if } i \in I \\ \mathcal{E}(j) \text{ if } i \in F_j \end{cases}$,

ii) $\mu'(H \cap I) = \mu(H \cap I)$ for all H, $\mu'(F_j) > 0$ for all $j \in J$, and

iii) $i \in F_j \Rightarrow C(I',\mathcal{E}',\mu')(i) = C(I',\mathcal{E}',\mu')(j)$.

$(I,\mathcal{E},\mu)$ is a point of continuity of C if, for every finite subset J of I, there exists a sequence of J-perturbations $(I^k,\mathcal{E}^k,\mu^k)$ such that $\mu^k(F_j)$ converges to zero for all $j \in J$ and such that $C(j)(I^k,\mathcal{E}^k,\mu^k)$ converges to $C(j)(I,\mathcal{E},\mu)$ for all $j \in J$.[17]

Observation 2 If C satisfies μCE at $(I,\mathcal{E},\mu)$ and at all *sufficiently* small J-perturbations of $(I,\mathcal{E},\mu)$, for all finite $J \subset I$, and if $(I,\mathcal{E},\mu)$ *is* a point of continuity of μ, $C(I,\mathcal{E},\mu)$ satisfies in fact FCE.

Proof. For any J-perturbation, μCE implies the absence of improving reallocations among the agents in J. By the openness of preferences and continuity of C at μ, this absence carries over to the limit. This implies FCE since J can be chosen arbitrarily. ■

7. DEFINING FEASIBILITY AND EFFICIENCY IN LARGE ECONOMIES

It is generally understood that in large (non-atomic continuum) economies, one must define and interpret notions of feasibility and efficiency with care. In particular, it is clear that the following two definitions jointly rule out the existence of any efficient allocation.

[17] A similar assumption of continuity has been made by Makowski and Ostroy (1992).

Condition 1 **(Standard Feasibility)** $\int f d\mu \leq \int \omega d\mu$.

Condition 2 **(Naive Efficiency)** *For no feasible* g, $g_i \succ_i f_i$ *whenever* $g_i \neq f_i$.

(Again, we will concentrate on exchange economies; the first condition is merely a restatement of feasibility given allocation of endowments *w.)*

The standard move to rescue existence has been to require that the set of improving agents have positive measure. While in most applications this approach has delivered sound results, this is much less clear in the present context (as pointed out in section *5).* Moreover, from a conceptual point of view, the standard approach restricts meaningful trade to infinite sets of agents, which, at least for notions of ***consumption*** efficiency (and, analogously, for notions of blocking by coalitions in core settings), seems economically unattractive: however large the economy, the most plausible coalitions to trade/block are ones that are small in ***absolute*** size, indeed of small finite cardinality such as two. (A more radical interpretation of continuum economies holds that all meaningful statements in the continuum require an "up to measure zero" clause; however, on such an interpretation, even the notion of strategy-proofness becomes problematic or at least loses transparency.)

In the following, we will take a closer look at the interpretation of measure-theoretic definitions of feasibility and argue that, when properly interpreted, they are naturally complemented by exact finitary ones (which, in the present paper, turn out to do essentially all the work).

The first step in the argument is to recognize that the measure-theoretic definition of feasibility, condition 1, does not have the same obvious "physical" interpretation for a continuum economy that it has for a finite economy. Indeed, it is conceptually quite problematic when interpreted "naively"; it contradicts our intuitive notions of feasibility and scarcity for it entails that given any feasible allocation ***f,*** there is another feasible allocation g that yields greater consumption for some agent without *any* reduction in the consumption of others. In a related vein, Kaneko and Wooders (1986) emphasize that Standard Feasibility should be viewed as an "idealization" *derive* it by a limiting argument .

A more satisfactory interpretation of the standard definition is obtained by reading it "coarsely". Intuitively, in an economy with an infinite number of agents, it is clearly possible to increase the consumption of a particular agent by decreasing the consumption of other agents by *arbitrarily small* positive amounts. A legitimate limit version of this is to allow the consumption of a particular agent to increase by a positive non-infinitesimal amount while the consumption of other agents goes down by *infinitesimal* but still *non-zero* amounts. Such reallocations could be modelled explicitly in an economy in which agents' consumptions take values in the "non-standard reals". One can view the approach taken in this paper as finessing the move to non-standard analysis by specifying agents' allocations only coarsely in terms of their *real part* omitting their infinitesimal part. Thus, on a Coarse Interpretation, Naive Efficiency is clearly inappropriate even if Standard Feasibility is assumed, and there is no need to rule it out artificially by denying the meaningfulness of distinguishing between two allocations that coincide almost everywhere.

On the other hand, on a Coarse Interpretation, the standard definition of Pareto efficiency is still clearly *necessary* for intuitive Pareto efficiency. That is, one will still want to impose the following condition which can be viewed as a *partial* definition of Pareto efficiency for continuum economies:

Condition 3 *If f is Pareto efficient, then there is no feasible g such that $g_i \succ_i f_i$ for all $i \in I$.*

(Under monotonicity of preferences, this is equivalent to requiring the absence of allocations that improve a set of agents of positive measure and leave all other agents' consumptions unchanged.) The condition is valid even under a Coarse Interpretation since the strict preferences does not depend on knowledge of the infinitesimal parts of agents' consumption. But the Coarse Interpretation has the non-standard implication that *in geneml* it is illegitimate to postulate that $g_i \geq f_i$ implies $g_i \succsim f_i$; note that it is precisely the absence of this implication which eliminates the problem of Naive Efficiency.

Condition **3** is not self-evidently *sufficient* for intuitive Pareto efficiency; in particular, as we have argued in section **3,** it fails to capture the possibility of improving reallocations among a finite number of agents. In other words, one should also require the following complementary condition.

Condition *4* ***Iff*** *is Pareto efficient, then f is FCE.*

In the context of a Coarse Interpretation, the special beauty of considering reallocations among finite subsets of agents is that, in this special case, one can legitimately identify equality of the real parts of consumption (of the unaffected agents) "$g_i = f_i$" with equality of their fully-specified consumption, and hence can legitimately infer their indifference between the allocations.

Thus, the Coarse Interpretation yields a conceptually coherent justification of Finite Consumption Efficiency as a component condition of full Pareto efficiency that avoids Naive Efficiency and its paradoxes. It allows one to refer to allocations exactly, using measure-theoretic mathematics when appropriate without having to give up the notion of a well-defined agent.

APPENDIX

Proof of Lemma 1:

Consider 6 : $\mathbf{R} \times \mathbf{R} \times S^\ell \times S^\ell_+ \to \mathbf{R}$ defined by

$$\delta(\alpha, \beta, \widehat{w}, \widehat{y}) := \|rp - \alpha\widehat{w} - \beta\widehat{y}\|^2.$$

6 is C^∞ with $\delta(0,0,\widehat{w},\widehat{y}) = r^2$, $\frac{\partial\delta}{\partial\alpha}\mid_{(0,0,\widehat{w},\widehat{y})} = -2rp\cdot\widehat{w} = 0$ and $\frac{\partial\delta}{\partial\beta}\mid_{(0,0,\widehat{w},\widehat{y})} = -2rp\cdot\widehat{y} < 0$ for all $\widehat{w}, \widehat{y}$.

By the implicit function theorem, for any $\widehat{w}$, $\widehat{y}$, there exists a closed $\varepsilon(\widehat{w},\widehat{y})$-ball around $(0,\widehat{w},\widehat{y})$, $B^{\varepsilon(\widehat{w},\widehat{y})}(0,\widehat{w},\widehat{y})$, and a unique function $h^{(\widehat{w},\widehat{y})} : B^{\varepsilon(\widehat{w},\widehat{y})}(0,\widehat{w},\widehat{y}) \to \mathbf{R}$ such that $\delta(\alpha, h^{(\widehat{w},\widehat{y})}(\alpha,\widetilde{w},\widetilde{y}),\widetilde{w},\widetilde{y}) = r^2$ for all $a, \widetilde{w}, \widetilde{y} \in B^{\varepsilon(\widehat{w},\widehat{y})}(0,\widehat{w},\widehat{y})$.

By the connectedness and compactness of the sets S^ℓ and S^ℓ_+, it follows that there exists in fact $\varepsilon > 0$ and a continuous function $h : [-\varepsilon, +\varepsilon] \times S^\ell \times S^\ell_+ \to \mathbf{R}$ such that, for all $(\alpha,\widetilde{w},\widetilde{y}) \in [-\varepsilon,+\varepsilon] \times S^\ell \times S^\ell_+$,

i) $\delta(\alpha, h(\alpha,\widetilde{w},\widetilde{y}),\widetilde{w},\widetilde{y}) = r^2$,

ii) $h(0,\widetilde{w},\widetilde{y}) = 0$,

iii) h is C^∞ in a with $\frac{\partial h}{\partial\alpha}\mid_{(0,\widetilde{w},\widetilde{y})} = 0$.

Hence by ii) and iii), for $K := \max\left\{\left|\frac{\partial^2 h}{\partial\alpha^2}\right|_{(\alpha,\widetilde{w},\widetilde{y})} \mid (\alpha,\widetilde{w},\widetilde{y}) \in [-\varepsilon,+\varepsilon] \times S^\ell \times S^\ell_+\right\}$, one has

$$h(\alpha,\widetilde{w},\widetilde{y}) \leq \alpha^2 K \text{ for all } (\alpha,\widetilde{w},\widetilde{y}) \in [-\varepsilon,+\varepsilon] \times S^\ell \times S^\ell_+. \tag{4}$$

Consider now any w and y satisfying the assumptions of lemma 1. If $y = 0$, there is nothing to prove; if $w = 0$, the claim is immediate; assume thus $w \neq 0$ and $y \neq 0$. By the definition of h, for $a \leq \varepsilon$, $\beta \geq 0$, $\widetilde{w} \in S^\ell$, $\widetilde{y} \in S^\ell_+$, $\alpha\widetilde{w} + \beta\widetilde{y} \notin (B^r(rp) + \mathbf{R}^\ell_+)$ implies $h(\alpha,\widetilde{w},\widetilde{y}) \geq \beta$. Combining this with equation (4) applied to $\alpha = \|w\|$, $\widetilde{w} = \frac{w}{\|w\|}$, $\beta = \|y\|$, $\widetilde{y} = \frac{y}{\|y\|}$ yields $\|y\| \leq h\left(\|w\|, \frac{w}{\|w\|}, \frac{y}{\|y\|}\right) \leq \|w\|^2 \cdot K$. □

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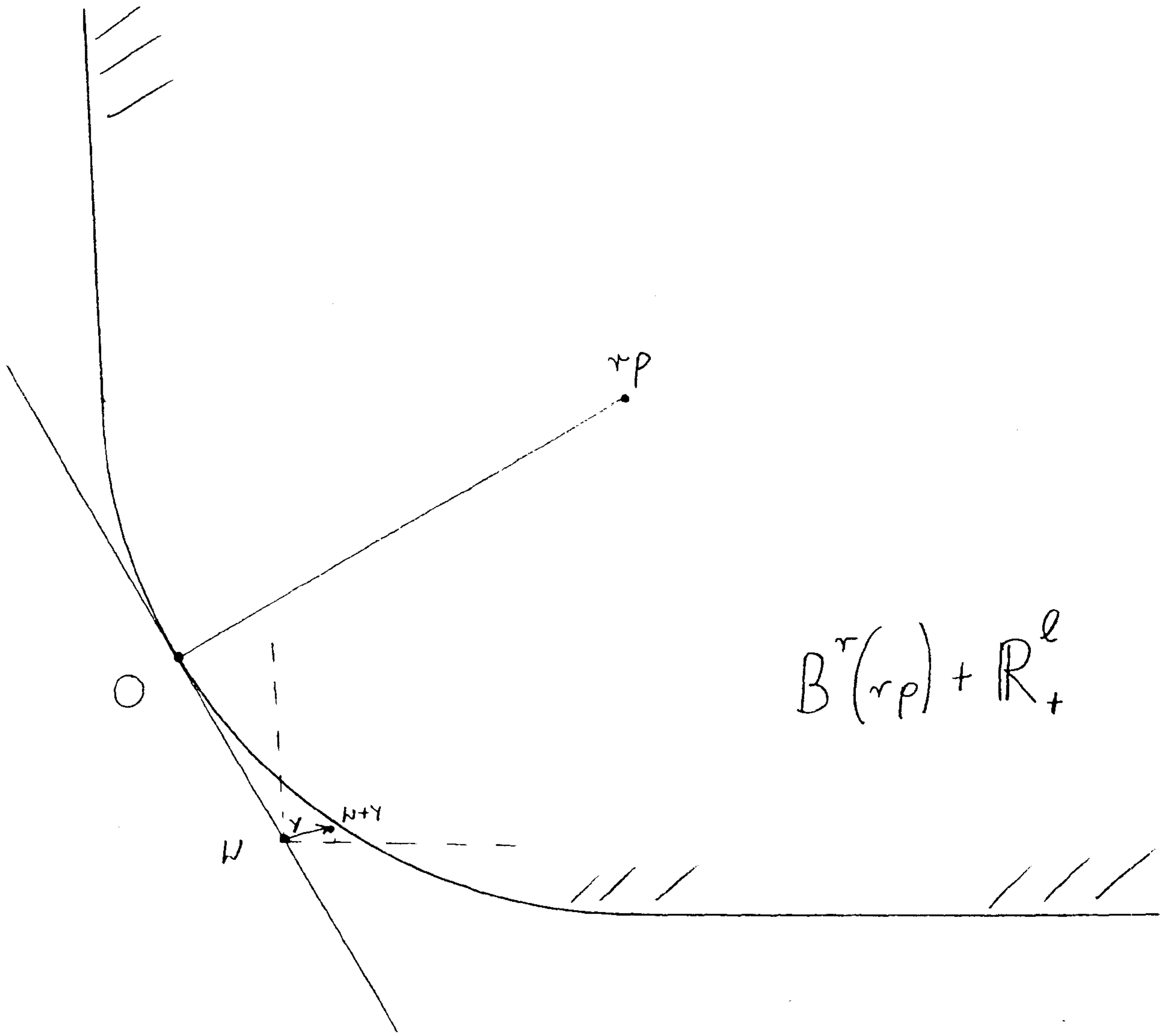

Figure 1.