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Time-Varying Uncertainty and the Credit Channel

Abstract

We extend the Carlstrom and Fuerst (1997) agency cost model of business cycles by including time varying uncertainty in the technology shocks that affect capital production. We first demonstrate that standard linearization methods can be used to solve the model yet second moments enter the economy's equilibrium policy functions. We then demonstrate that an increase in uncertainty causes, ceteris paribus, a fall in investment supply. A second key result is that time varying uncertainty results in countercyclical bankruptcy rates - a finding which is consistent with the data and opposite the result in Carlstrom and Fuerst. Third, we show that persistence of uncertainty affects both quantitatively and qualitatively the behavior of the economy. However, the shocks to uncertainty imply a quantitatively small role for uncertainty over the business cycle.

- *JEL Classification: E4, E5, E2*
- Keywords: *agency costs, credit channel, time-varying uncertainty.*

1 Introduction

The impact of risk on aggregate investment and lending activity, while extensively studied in theoretical models, has received little attention in quantitative macroeconomic settings. In large part, this has been due to computational methods, i.e. linearization methods, which impose certainty equivalence so that second moments play no role. We address this omission in this paper by using the credit channel model of Carlstrom and Fuerst (1997). In particular, we model time varying uncertainty as a mean preserving spread in the distribution of the technology shocks affecting capital production and explore how changes in uncertainty affect equilibrium characteristics.[1] This setting is useful for several reasons: First, the impact of uncertainty on investment via the lending channel is fairly transparent so that economic intuition is enhanced. Second, the economic environment is a variant of a typical real business cycle model so that key parameters can be calibrated to the data. Third, we demonstrate that linearization solution methods can be employed yet this does not eliminate the influence of second moments on equilibrium. That is, in solving for the linear equilibrium policy functions, the vector of state variables includes the variance of technology shocks buffeting the capital production sector.

The main results can be summarized as follows. In contrast to an aggregate technology shock which affects investment demand, we show that an increase in uncertainty results in a shift in the investment supply schedule. In particular, an increase in uncertainty will cause an increase in the price of capital and a fall in investment activity. Another important result is that time-varying uncertainty produces countercyclical bankruptcy rates. In contrast, Carlstrom and Fuerst's (1997) analysis of aggregate technology shocks produced the counterfactual prediction of procyclical bankruptcy rates.

On a less positive note, we also demonstrate that the quantitative magnitude of these effects

[1] Our choice of model and analysis of shocks to second moments is similar to that in Christiano, Motto, and Rostagno (2003) in which they examined the role that uncertainty and several other factors played in the Great Depression. Given their interest in the particular historical eposide, they did not examine in detail the role that uncertainty plays in a credit channel model.

is small relative to that of an aggregate technology shock. While this result argues against the importance of second moment effects, we think it is premature to eliminate changes in uncertainty as an important impulse mechanism to the economy. The credit channel model we examine has a sufficiently simple structure so that linearization methods can be employed to analyze second moments; it is quite possible, however, that this structure is precisely why uncertainty does not play a critical quantitative role. (But it is true that, uncertainty shocks of the order recently studied in the paper by Bloom (2005) would indeed have quantitatively meaningful effects.) We see our efforts as primarily pedagogical and argue that richer (e.g. non-linear) environments and more sophisticated numerical methods will be needed to fully explore the role of time-varying uncertainty.

2 Model

We employ the agency cost business cycle model of Carlstrom and Fuerst (1997) to address the financial intermediaries' role in the propagation of productivity shocks and extend their analysis by introducing time-varying uncertainty. Since, for the most part, the model is identical to that in Carlstrom and Fuerst, the exposition of the model will be brief with primary focus on the lending channel. A full presentation of the model is given in the appendix.

The model is a variant of a standard RBC model in which an additional production sector is added. This sector produces capital using a technology which transforms investment into capital. In a standard RBC framework, this conversion is always one-to-one; in the Carlstrom and Fuerst framework, the production technology is subject to technology shocks. (The aggregate production technology is also subject to technology shocks as is standard.) This capital production sector is owned by entrepreneurs who finance their production via loans from a risk neutral financial intermediation sector - this lending channel is characterized by a loan contract with a fixed interest rate. (Both capital production and the loans are intra-period.) If a capital producing firm realizes a low technology shock, it will declare bankruptcy and the financial intermediary will take over production; this activity is subject to monitoring costs.

The timing of events is as follows:

1. The exogenous state vector of technology shocks and uncertainty shocks, denoted $(\theta_t, \sigma_{\omega,t})$, is realized.

2. Firms hire inputs of labor and capital from households and entrepreneurs and produce output via an aggregate production function.

3. Households make their labor, consumption and savings/investment decisions. The household transfers q_t consumption goods to the banking sector for each unit of investment.

4. With the savings resources from households, the banking sector provide loans to entrepreneurs via the optimal financial contract. The contract is defined by the size of the loan (i_t) and a cutoff level of productivity for the entrepreneurs technology shock, $\bar{\omega}_t$.

5. Entrepreneurs use their net worth and loans from the banking sector as inputs into their capital-creation technology.

6. The idiosyncratic technology shock of each entrepreneur is realized. If $\omega_{j,t} \geq \bar{\omega}_t$ the entrepreneur is solvent and the loan from the bank is repaid; otherwise the entrepreneur declares bankruptcy and production is monitored by the bank at a cost of μi_t.

7. Entrpreneurs that are solvent make consumption choices; these in part determine their net worth for the next period.

A schematic of the implied flows is presented in Figure 1 and complete description of the economy is given in the appendix. We now focus on the lending contract and the role of time varying uncertainty.

2.1 Optimal Financial Contract

The optimal financial contract between entrepreneur and the Capital Mutual Fund is described by Carlstrom and Fuerst (1997). But for expository purposes as well as to explain our approach in addressing the second moment effect on equilibrium conditions, we briefly outline the model. In deriving the optimal contract, both entrepreneurs and lenders take the price of capital, q, and net worth, n, as given.

The entrepreneur has access to a stochastic technology that transforms i_t units of consumption into $\omega_t i_t$ units of capital. In Carlstrom and Fuerst (1997), the technology shock ω_t was assumed to be distributed as $i.i.d.$ with $E(\omega_t) = 1$. While we maintain the assumption of constant mean, we assume that the standard deviation is time-varying. Specifically, we assume that the standard deviation of the capital production technology shock is governed by the following AR(1) process

$$\sigma_{\omega,t} = \bar{\sigma}_{\omega}^{1-\zeta} \sigma_{\omega,t-1}^{\zeta} u_t \tag{1}$$

where $\zeta \in (0,1)$ and $u_t \sim i.i.d$ with a mean of unity.[2] The unconditional mean of the standard deviation is given by $\bar{\sigma}_{\omega}$. The realization of ω_t is privately observed by entrepreneur – banks can observe the realization at a cost of μi_t units of consumption.

The entrepreneur enters period t with one unit of labor endowment and z_t units of capital. Labor is supplied inelastically while capital is rented to firms, hence income in the period is $w_t + r_t z_t$. This income along with remaining capital determines net worth (denoted as n_t and denominated in units of consumption) at time t:

$$n_t = w_t + z_t \left(r_t + q_t \left(1 - \delta\right)\right) \tag{2}$$

With a positive net worth, the entrepreneur borrows $(i_t - n_t)$ consumption goods and agrees

[2] This autoregressive process is used so that, when the model is log- linearized, $\hat{\sigma}_{\omega,t}$ (defined as the percentage deviations from $\bar{\sigma}_{\omega}$) follows a standard, mean-zero AR(1) process.

to pay back $\left(1+r^k\right)(i_t - n_t)$ capital goods to the lender, where r^k is the interest rate on loans. Thus, the entrepreneur defaults on the loan if his realization of output is less then the re-payment, i.e.

$$\omega_t < \frac{\left(1+r^k\right)(i_t - n_t)}{i_t} \equiv \bar{\omega}_t \tag{3}$$

The optimal borrowing contract is given by the pair $(i_t, \bar{\omega}_t)$ that maximizes entrepreneur's return subject to the lender's willingness to participate (all rents go to the entrepreneur). Denoting the $c.d.f.$ and $p.d.f.$ of ω_t as $\Phi(\omega_t; \sigma_{\omega,t})$ and $\phi(\omega_t; \sigma_{\omega,t})$ respectively, the contract is determined by the solution to:[3]:

$$\max_{\{i,\bar{\omega}\}} q i_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \text{ subject to } q i_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \geq (i-n)$$

where

$$f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) = \int_{\bar{\omega}_t}^{\infty} \omega \phi\left(\omega; \sigma_{\omega,t}\right) d\omega - \left[1 - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right)\right] \bar{\omega}_t$$

which can be interpreted as the fraction of the expected net capital output received by the entrepreneur,

$$g\left(\bar{\omega}_t; \sigma_{\omega,t}\right) = \int_{-\infty}^{\bar{\omega}_t} \omega \phi\left(\omega; \sigma_{\omega,t}\right) d\omega + \left[1 - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right)\right] \bar{\omega}_t - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \mu$$

which represents the lender's fraction of expected capital output, $\Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right)$ is the bankruptcy rate.. Also note that $f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) + g\left(\bar{\omega}_t; \sigma_{\omega,t}\right) = 1 - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \mu$: the right hands side is the average amount of capital that is produced. This is split between entrepreneurs and lenders while monitoring costs reduce net capital production.

The necessary conditions for the optimal contract problem are

$$\frac{\partial(.)}{\partial \bar{\omega}} : q i f'\left(\bar{\omega}\right) = -\lambda q i \frac{\partial g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{\partial \bar{\omega}}$$

[3] The notation $\Phi(\omega; \sigma_{\omega,t})$ is used to denote that the distribution function is time-varying as determined by the realization of the random variable, $\sigma_{\omega,t}$. For expositional purposes, we suppress the time notation on the price of capital and net worth since these are treated as parameters in this section.

where λ_t is the shadow price of capital. Using the definitions of $f(\bar{\omega}_t;\sigma_{\omega,t})$ and $g(\bar{\omega}_t;\sigma_{\omega,t})$, this can be rewritten as:

$$1-\frac{1}{\lambda_t}=\frac{\phi(\bar{\omega}_t;\sigma_{\omega,t})}{1-\Phi(\bar{\omega}_t;\sigma_{\omega,t})}\mu \tag{4}$$

As shown by eq.(4), the shadow price of capital is an increasing function of the relevant Inverse Mill's ratio (interpreted as the conditional probability of bankruptcy) and the agency costs. If the product of these terms equals zero, then the shadow price equals the cost of capital production, i.e. $\lambda_t = 1$.

The second necessary condition is:

$$\frac{\partial(.)}{\partial i_t}: qf(\bar{\omega}_t;\sigma_{\omega,t}) = -\lambda_t\left[1-qg(\bar{\omega}_t;\sigma_{\omega,t})\right]$$

Solving for q using the first order conditions, we have

$$\begin{aligned} q^{-1} &= \left[\left(f(\bar{\omega}_t;\sigma_{\omega,t})+g(\bar{\omega}_t;\sigma_{\omega,t})\right)+\frac{\phi(\bar{\omega}_t;\sigma_{\omega,t})\,\mu f(\bar{\omega}_t;\sigma_{\omega,t})}{\frac{\partial f(\bar{\omega}_t;\sigma_{\omega,t})}{\partial\bar{\omega}}}\right] \\ &= \left[1-\Phi(\bar{\omega}_t;\sigma_{\omega,t})\,\mu+\frac{\phi(\bar{\omega}_t;\sigma_{\omega,t})\,\mu f(\bar{\omega}_t;\sigma_{\omega,t})}{\frac{\partial f(\bar{\omega}_t;\sigma_{\omega,t})}{\partial\bar{\omega}}}\right] \\ &\equiv \left[1-D(\bar{\omega}_t,\sigma_{\omega,t})\right]=F(\bar{\omega}_t,\sigma_{\omega,t}) \end{aligned} \tag{5}$$

where $D(\bar{\omega}_t;\sigma_{\omega,t})$ can be thought of as the total default costs.

It is straightforward to show that equation (5) defines an implicit function $\bar{\omega}(q,\sigma_{\omega,t})$ that is increasing in q. Also note that, in equilibrium, the price of capital, q, differs from unity due to the presence of the credit market frictions. (Note that $\frac{\partial f(\bar{\omega}_t;\sigma_{\omega,t})}{\partial\bar{\omega}}=\Phi(\bar{\omega}_t;\sigma_{\omega,t})-1<0$.)

The incentive compatibility constraint implies

$$i_t=\frac{1}{\left(1-qg(\bar{\omega}_t;\sigma_{\omega,t})\right)}n \tag{6}$$

Equation (6) implies that investment is linear in net worth and defines a function that represents the amount of consumption goods placed in to the capital technology: $i(q, n, \sigma_{\omega,t})$. The fact that the function is linear implies that the aggregate investment function is well defined.

The effect of an increase in uncertainty on investment in this model can be understood by first turning to eq. (5). Under the assumption that the price of capital is unchanged, this implies that the costs of default, represented in the function $D(\bar{\omega}_t, \sigma_{\omega,t})$, must also be unchanged. With a mean-preserving spread in the distribution for ω_t, this implies that $\bar{\omega}_t$ will fall. As a consequence, the lenders' expected capital return, $g(\bar{\omega}_t; \sigma_{\omega,t})$, will also fall since, as shown in the appendix, $g(\bar{\omega}_t; \sigma_{\omega,t}) \approx \bar{\omega}_t$. Given the incentive compatibility constraint,

$$qi_t g(\bar{\omega}_t; \sigma_{\omega,t}) = (i_t - n)$$

the fall in the left-hand side induces a fall in i_t. This relationship is shown numerically (using the parameter values described in the next section) in Figure 2.

The effects of the two technology shocks: the aggregate technology shock, θ_t, and the uncertainty shock, $\sigma_{\omega,t}$, on the capital market can be summarized graphically as shown in Figure 3. While not analyzed explicitly here, an aggregate technology shock shifts the location of the capital demand curve as both the income effect and, if shocks are positively autocorrelated, the substitution effect of higher expected marginal productivity of capital causes the demand curve to shift outward for a positive technology shock. This will, ceteris paribus, cause the price of capital to increase; note this explains the procyclical bankruptcy rates in Carlstrom and Fuerst (1997) given that $\partial\bar{\omega}/\partial q > 0$ as mentioned previously. In contrast, an increase in uncertainty causes the investment supply function to shift leftward resulting in a higher price of capital but smaller quantity of investment. These partial equilibrium results are not overturned in the general equilibrium setup.

2.2 Equilibrium

Equilibrium in the economy is represented by market clearing in four markets: the labor markets for households and entrepreneurs and the goods markets for consumption and capital. Letting (H_t, H_t^e) denote the aggregate labor supply of, respectively households and entrepreneurs, we have

$$H_t = (1 - \eta)\, l_t \tag{7}$$

where l_t denotes labor supply of households and η denotes the fraction of entrepreneurs in the economy.

$$H_t^e = \eta \tag{8}$$

Goods market equilibrium is represented by

$$C_t + I_t = Y_t \tag{9}$$

where $C_t = (1 - \eta)\, c_t + \eta c_t^e$ and $I_t = \eta i_t$. (Note upper case variables denotes aggregate quantities while lower case denote per-capita quantities.)

The law of motion of aggregate capital is given by:

$$K_{t+1} = (1 - \delta)\, K_t + I_t \left[1 - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \mu\right] \tag{10}$$

A competitive equilibrium is defined by the decision rules for (aggregate capital, entrepreneurs capital, household labor, entrepreneur's labor, the price of capital, entrepreneur's net worth, investment, the cutoff productivity level, household consumption, and entrepreneur's consumption) given by the vector: $\{K_{t+1}, Z_{t+1}, H_t, H_t^e, q_t, n_t, i_t, \bar{\omega}_t, c_t, c_t^e\}$ where these decision rules are stationary functions of $\{K_t, Z_t, \theta_t, \sigma_{\omega,t}\}$ and satisfy the following equations[4]

$$\begin{aligned}
\nu c_t &= \alpha_H \frac{Y_t}{H_t} && (11)\\
\frac{q_t}{c_t} &= \beta E_t \left\{ \frac{1}{c_{t+1}} \left(q_{t+1} (1-\delta) + \alpha_K \frac{Y_{t+1}}{K_{t+1}} \right) \right\} && (12)\\
q_t &= \left\{ 1 - \Phi(\bar{\omega}; \sigma_{\omega,t}) \mu + \frac{\phi(\bar{\omega}; \sigma_{\omega,t}) \mu f(\bar{\omega}; \sigma_{\omega,t})}{f'(\bar{\omega}_t)} \right\}^{-1} && (13)\\
i_t &= \frac{1}{(1 - q_t g(\bar{\omega}; \sigma_{\omega,t}))} n_t && (14)\\
q_t &= \beta \gamma E_t \left\{ \left(q_{t+1} (1-\delta) + \alpha_K \frac{Y_{t+1}}{K_{t+1}} \right) \left(\frac{q_{t+1} f(\bar{\omega}; \sigma_{\omega,t})}{(1 - q_{t+1} g(\bar{\omega}; \sigma_{\omega,t}))} \right) \right\} && (15)\\
n_t &= \alpha_{H^e} \frac{Y_t}{H_t^e} + Z_t \left(q_t (1-\delta) + \alpha_K \frac{Y_t}{K_t} \right) && (16)\\
Z_{t+1} &= \eta n_t \left\{ \frac{f(\bar{\omega}; \sigma_{\omega,t})}{1 - q_t g(\bar{\omega}; \sigma_{\omega,t})} \right\} - \eta \frac{c_t^e}{q_t} && (17)\\
\theta_{t+1} &= \theta_t^{\rho} \xi_{t+1} \text{ where } \xi_t \sim i.i.d. \text{ with } E(\xi_t) = 1 && (18)\\
\sigma_{\omega,t+1} &= \bar{\sigma}_{\omega}^{1-\zeta} \sigma_{\omega,t}^{\zeta} u_{t+1} \text{ where } u_t \sim i.i.d. \text{ with } E(u_t) = 1 && (19)
\end{aligned}$$

The first equation represents the labor-leisure choice for households while the second equation is the necessary condition associated with household's savings decision. The third and fourth equation are from the optimal lending contract while the fifth equation is the necessary condition associated with entrepreneur's savings decision. The sixth equation is the determination of net worth while the seventh gives the evolution of entrepreneur's capital. (The evolution of aggregate capital is given in eq. (10)). The final two equations represent the laws of motion for the aggregate technology and uncertainty shock respectively.

[4] A more thorough presentation of the equilibrium conditions are presented in the Appendix.

3 Equilibrium Characteristics

3.1 Steady-state analysis

While our focus is primarily on the cyclical behavior of the economy, an examination of the steady-state properties of the economy is useful for two reasons. First, by studying the interaction between uncertainty (i.e. the variance of the technology shock affecting the capital production sector) and the steady-state, the intuition for how time-varying uncertainty affects the cyclical characteristics of the economy is improved. Second, it is important to point out that changes in the second moment of technology shocks affect the level of the economy - most notably consumption and output. That is, since the cyclical analysis presented in the next section is characterized in terms of deviations from steady-state, the impact of changes in uncertainty on the *level* of economic activity is lost.[5]

For this analysis, we use, to a large extent, the parameters employed in Carlstrom and Fuerst's (1997) analysis. Specifically, the following parameter values are used:

Table 1: Parameter Values

β	α	δ	μ
0.99	0.36	0.02	0.25

Agents discount factor, the depreciation rate and capital's share are fairly standard in RBC analysis. The remaining parameter, μ, represents the monitoring costs associated with bankruptcy. This value, as noted by Carlstrom and Fuerst (1997) is relatively prudent given estimates of bankruptcy costs (which range from 20% (Altman (1984) to 36% (Alderson and Betker (1995) of firm assets).

The remaining parameters, (σ, γ), determine the steady-state bankruptcy rate (which we denote as br and is expressed in percentage terms) and the risk premium (denoted rp) associated

[5] This statement is in reference to Lucas's analysis of the cost of business cycles (Lucas (1987) in which the trend and cycle are treated as distinct. In contrast, our analysis demonstrates that the cyclical behavior of the economy has implications for the level of the steady-state. If one were using an endogenous growth model, cyclical behavior may well have implications for the trend.

with bank loans.[6] To examine the role of uncertainty on the steady-state behavior of the economy, we hold the bankruptcy rate constant to that studied in Carlstrom and Fuerst and increase the standard deviation by slightly less than 50%; the implied values for γ and the risk premium are given in Table 2:[7]

Table 2: Parameter Values

Economy	$\boldsymbol{\sigma}$	rp	$\boldsymbol{\gamma}$
Economy I ($C\&F$)	0.207	1.87%	0.9474
Economy II	0.30	2.42%	0.954

The effect of greater uncertainy in the capital production sector is seen in Table 3. (All values in Table 3 are percentage changes relative to the Carlstrom and Fuerst economy.) Consistent with the partial equilibrium analysis presented earlier, a mean-preserving spread in entrepreneur's technology shock causes the price of capital to increase and steady-state capital to fall. This also implies a decrease in consumption, a slight increase in steady-state labor, and a fall in steady-state output.

[6] The fraction of entrepreneurs in the economy, η, is not a critical parameter for the behavior of the economy. As Carlstrom and Fuerst note, it is simply a normalization. Aggregate consumption in the model is indeed a weighted average of household and entrepreneurial consumption but the weights are determined by the steady-state level of per-capita consumption for these groups. This is endogenously determined - but not by η. This is demonstrated at the end of the Appendix.

[7] As discussed in Carlstrom and Fuerst, a bankruptcy rate of 0.974% (per quarter) and an annual risk premium of 187 basis points are broadly consistent with U.S. data.

Table 3: Steady-State Effects of Greater Uncertainty

(comparison to Carlstrom & Fuerst Economy)

variable	Economy II
c	-0.19
k	-0.51
h	0.04
y	-0.16
q	0.35
z	28.4
n	28.7

3.2 Cyclical Behavior

As described in Section 2, eqs. (11) through (19) determine the equilibrium properties of the economy. To analyze the cyclical properties of the economy, we linearize (i.e. take a first-order Taylor series expansion) of these equations around the steady-state values and express all terms as percentage deviations from steady-state values. This numerical approximation method is standard in quantitative macroeconomics. What is not standard in this model is that the second moment of technology shocks hitting the capital production sector will influence equilibrium behavior and, therefore, the equilibrium policy rules. That is, linearizing the equilibrium conditions around the steady-state typically imposes certainty equivalence so that variances do not matter. In this model, however, the variance of the technology shock can be treated as an additional state variable through its role in determining lending activities and, in particular, the nature of the lending contract.[8] Linearizing the system of equilibrium conditions does not eliminate that role in this economy and, hence, we think that this is an attractive feature of the model.

While the previous section analyzed the steady-state behavior of four different economies, in

[8] Specifically, ω_t is assumed to be log normally distributed. Hence, the linear approximation to the equations describing the financial contract (eqs. (24) and (25)) will include the second moment of ω_t.

this section we employ the same parameters as in the Carlstrom and Fuerst model (Economy I in the previous section). We depart from Carlstrom and Fuerst by relaxing the *i.i.d.* assumption for the capital sector technology shock. This is reflected in the law of motion for the standard deviation of the technology shock which is given in eq. (19); for convenience this is rewritten below:

$$\sigma_{\omega,t+1} = \bar{\sigma}_{\omega}^{1-\zeta} \sigma_{\omega,t}^{\zeta} u_{t+1}$$

As in Carlstrom and Fuerst, the standard deviation of the technology shock ω_t is, on average, equal to 0.207. That is, we set $\bar{\sigma}_{\omega} = 0.207$. We then examine two different economies characterized by the persistence in uncertainty, i.e. the parameter ζ. In the low persistence economy, we set $\zeta = 0.05$ while in the moderate persistence economy we set $\zeta = 0.90$. The behavior of these two economies is analyzed by examining the impulse response functions of several key variables to a 1% innovation in σ_{ω}. These are presented in Figures 4-6.

We first turn to aggregate output and household consumption and investment. With greater uncertainty, the bankruptcy rate increases in the economy (this is verified in Figure 5), which implies that agency costs increase. The rate of return on investment for the economy therefore falls. Households, in response, reduce investment and increase consumption and leisure. The latter response causes output to fall. Note that the consumption and leisure response is increasing in the degree of persistence. This is not the case, however, for investment - this is due to the increase in the price of capital (see Figure 5) and reflects the behavior of entrepreneurs. This behavior is understood after first examining the lending channel.

The increase in uncertainty affects, predictably, all three key variables in the lending channel: the price of capital, the risk premium associated with loans and the bankruptcy rate. As already mentioned, the bankruptcy rate increases and, in the high persistence economy, this increased rate of bankruptcy lasts for several quarters. This result implies that the bankruptcy rate is countercyclical in this economy; in contrast, in the analysis by Carlstrom and Fuerst the bankruptcy rate was, counterfactually, procyclical.[9] Their focus was on the effects of innovation to the aggregate technology shock and, because of the assumed persistence in this shock, is driven by the change in the first moment of the aggregate production shock. Our analysis demonstrates that second moment effects may play a significant role in these correlations over the business cycle. Further research, both empirical and theoretical, in this area would be fruitful . Returning to the model, the increased bankruptcy rate implies that the price of capital is greater and this increase lasts longer in the high persistence economy. The same is true for the risk premium on loans.

Figure 6 reports the consumption and net worth of entrepreneurs in the economies. In contrast to all other variables, persistence has a dramatic qualitative effect on entrepreneurs' behavior. With low persistence, entrepreneurs exploit the high price of capital to increase consumption: the lack persistence provides no incentive to increase investment. Since the price of capital quickly returns to its steady-state values, the increased consumption erodes entrepreneurs' net worth. To restore net worth to its steady-state value, consumption falls temporarily. The behavior in the high persistence economy is quite different: the price of capital is high and forecast to stay high so investment increases dramatically. Initially, the investment is financed by lower consumption, but as entrepreneurs net worth increases (due to greater capital and a higher price of capital) consumption also increases. This endogenous response by entrepreneurs is why, in the high persistence economy, the initial fall in aggregate investment is not as great in the high persistence economy.

A further analysis of the equilibrium characteristics of the high persistence economy is presented in Table 4 in which a few, key second moments are reported. For comparison, the moments implied by the model when subject to total factor productivity shocks (θ_t) or information shocks (σ_ω) are given along with the corresponding moments from the US data. Note that, while time varying uncertainty induces greater volatility in labor, investment, and the capital stock, the dis-

[9] In the Carlstrom and Fuerst (1997) model, a technology shock increases output and the demand for capital. The resulting increase in the price of capital implies greater lending activity and, hence, an increase in the bankruptcy rate (and risk premia). Here, greater uncertainty results in greater bankruptcy rates even though investment falls; since labor is also reduced, this produces countercyclical bankruptcy rates and risk premia.

crepancy between the moments from the artificial economy and the actual data are not that much different than arising from a standard RBC model subject to productivity shocks. This behavior stands in stark contrast to the financial intermediation model of Cooper and Ejarque (2000) in which labor and investment were countercyclical and capital stock volatility was over 5 times greater than GDP volatility.[10] Their analysis did not present an explicit model of the financial intermediation sector and our analysis suggests that the endogenous response of this sector to shocks is important and leads to improved performance of the model. The model does imply negative correlation between consumption and investment hence we reach the same conclusion as Cooper and Ejarque (1997): shocks to uncertainty can not be the dominant shock in the economy since this correlation is counterfactual to business cycle behavior. This observation does not, in our opinion, rule out uncertainty as playing a role in business cycle behavior - it simply can not be the sole or dominant factor. A second important feature seen in Table 4 is the quantitatively small role that second moment shocks have on the economy; as seen in the first column, a 1% innovation to the aggregate technology shock produces volatility in GDP over 60 times larger than that from a comparable shock to the conditional standard deviation. This is addressed in the next section.

[10] Cooper and Ejarque (2000) analyze two versions of their model: one in which financial intermediation plays a role in financing both undepreciated and new capital and another in which only new capital (i.e. investment) uses financial intermediaries. The countercyclical behavior of labor and investment is seen in the first version; however, both models exhibit high volatility of the capital stock.

Table 4: Business Cycle Characteristics[11]

shocks	σ_y	Volatility relative to y: c	h	i	k	Correlation with y: c	h	i	k
θ	0.046	0.63	0.59	2.72	0.87	0.84	0.81	0.91	0.65
σ_ω	0.0007	0.64	1.43	5.14	1.03	-0.54	0.93	0.97	0.36
US data[12]	1.71	0.49	0.86	3.15	0.36	0.76	0.86	0.90	-0.08

4 Conclusion

The effect of uncertainty as characterized by second moment effects has been largely ignored in quantitative macroeconomics due to the numerical approximation methods typically employed during the computational exercise. The analysis presented here uses standard solution methods (i.e. linearizing around the steady-state) but exploits features of the Carlstrom and Fuerst (1997) agency cost model of business cycles so that time varying uncertainty can be analyzed. While development of more general solution methods that capture second moments effects is encouraged, we think that the intuitive nature of this model and its standard solution method make it an attractive environment to study the effects of time-varying uncertainty.

Our primary findings fall into four broad categories. First, we demonstrate that uncertainty affects the level of the steady-state of the economy so that welfare analysis of uncertainty that focus entirely on the variability of output (or consumption) will understate the true costs of uncertainty. Second, we demonstrate that time varying uncertainty results in countercyclical bankruptcy rates - a finding which is consistent with the data and opposite the result in Carlstrom and Fuerst. Third, we show that persistence of uncertainty effects both quantitatively and qualitatively the behavior of the economy. Quantitatively, however, the impact of an increase is significantly less than that of an aggregate technology shock. We conclude that further research is needed in

[11] For this comparative analysis, the standard deviation of the innovation to both shocks was assumed to be 0.007. This figure is typical for total factor productivity shocks but whether this is a good figure for shocks to the second moments is an open question. We also assumed that both shocks exhibit high persistence with an autocorrelation of 0.95 for θ_t and 0.90 for σ_ω.

[12] The US figures are from Kydland and Prescott (1990).

(at least) two dimensions: the characterization of uncertainty shocks (i.e., second moments or rare catastrophic events) and the development of richer theoretical models which introduce more non-linearities in the equations defining equilibrium.

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5 Appendix:

5.1 The Lending Channel: Approximation analysis

To find a simple analytical formula for investment in the partial equilibrium model described in the text, is convenient to assume to use the substitution $\overline{\omega} = \exp(\omega_1)$ in order to use the normal rather than lognormal distribution for the technology shock ω_t. Using this permits equations (5) and (6) to be expressed in the form

$$\frac{i}{n} = (1 - qg_1(\omega_1, \sigma))^{-1}, \tag{20}$$

$$\frac{1 - q^{-1}}{\mu} = \text{const} = \Phi_1(\omega_1, \sigma) + \exp(-\omega_1)\phi_1(\omega_1, \sigma)\frac{f_1(\omega_1, \sigma)}{1 - \Phi_1(\omega_1, \sigma)} \tag{21}$$

where $f_1(\omega_1, \sigma) = f(\overline{\omega}, \sigma)$, $g_1(\omega_1, \sigma) = g(\overline{\omega}, \sigma)$ and so forth.

We need to find a simple approximation for the equations above. To do that we will use the asymptotic expansion on large parameter $|\omega_1/\sigma| \gg 1$. Evaluated at steady-state levels, the numerical value of $\omega_1/\sigma \approx -2.4$ and so can be considered as "large" here since its square appears as an argument of exponent function. Then we have the following representations of terms in (20),(21) (note that the mean of ω_1 has been shifted by $\sigma^2/2$ in order to maintain a mean-preserving spread):

$$\begin{aligned}
\Phi_1(\omega_1, \sigma) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\omega_1/\sigma+\sigma/2} \exp(-\frac{x^2}{2})dx \simeq \frac{1}{\sqrt{2\pi}|\frac{\omega_1}{\sigma} + \frac{\sigma}{2}|}\exp[-\frac{1}{2}(\frac{\omega_1}{\sigma} + \frac{\sigma}{2})^2], \\
f_1(\omega_1, \sigma) &= 1 - \exp(\omega_1)[1 - \Phi_1(\omega_1, \sigma)] - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\omega_1/\sigma} \exp[\sigma x - \frac{1}{2}(x + \frac{\sigma}{2})^2]dx \\
&\simeq 1 - \exp(\omega_1), \\
g_1(\omega_1, \sigma) &= 1 - \mu\Phi_1(\omega_1, \sigma) - f_1(\omega_1, \sigma) \simeq \exp(\omega_1)
\end{aligned} \tag{22}$$

The asymptotic expansion of $\Phi_1(\omega_1, \sigma)$ uses the following chain of exact and approximate relations:

$$\int_{-\infty}^{-X} \exp(-\frac{x^2}{2})dx = \frac{1}{X}\int_0^{\infty} \exp\left[-\frac{1}{2}\left(-X - \frac{y}{X}\right)^2\right] dy =$$

$$\frac{1}{x}\exp(-\frac{X^2}{2})\int_0^{\infty} \exp\left(-y - \frac{y^2}{2X^2}\right) dy \simeq \frac{1}{x}\exp(-\frac{X^2}{2})\int_0^{\infty} \exp\left(-y\right) dy = \frac{1}{x}\exp(-\frac{X^2}{2})$$

Here we assume $-X$ to be a large negative number and perform variable substitution $x = -X - y/X$. Note that neglecting the term $-\frac{y^2}{2X^2}$ in the exponent under the integral produces the zero-order term of an asymptotic series. (For the detailed theory of asymptotic series and its applications see Olver (1997).) The approximation for $f_1(\omega_1, \sigma)$ and $g_1(\omega_1, \sigma)$ uses the smallness of $\Phi_1(\omega_1, \sigma)$, which is equal to the bankruptcy rate $br \simeq 0.00974$. The last integral term in the expression for $f_1(\omega_1, \sigma)$ differs from $\Phi_1(\omega_1, \sigma)$ by the factor $\exp(\sigma x)$under the integral, which is smaller than 1, because the range of integration is negative. So that term is less than $\Phi_1(\omega_1, \sigma)$ and can be also neglected.

Substituting (22) into (20) and (21) we arrive to the relations:

$$\frac{i}{n} = (1 - q\exp(\omega_1))^{-1}, \tag{23}$$

$$\frac{1-q^{-1}}{\mu} \simeq \frac{\exp\left(-\frac{1}{2}\left(\frac{\omega_1}{\sigma} + \frac{\sigma}{2}\right)^2\right)}{\sqrt{2\pi}}\left(\frac{1}{\left|\frac{\omega_1}{\sigma} + \frac{\sigma}{2}\right|} + \frac{\exp(-\omega_1) - 1}{\sigma}\right) \tag{24}$$

Neglecting the small terms $\sigma^2/4$ and $\left|\frac{\omega_1}{\sigma} + \frac{\sigma}{2}\right|^{-1}$ in (24) we can rewrite it in the form:

$$\frac{\sigma}{\omega_1}\exp\left(\frac{1}{2}(\frac{\omega_1}{\sigma})^2\right)=\frac{\mu}{\sqrt{2\pi}\,(1-q^{-1})}\exp(-\frac{\omega_1}{2})\frac{\exp(-\omega_1)-1}{\omega_1}$$

Taking a log of it we obtain:

$$\frac{\omega_1}{\sigma}=-\sqrt{2\left(\ln w\left(\omega_1\right)+\ln\left|\frac{\omega_1}{\sigma}\right|\right)}, \tag{25}$$

where

$$w\left(\omega_1\right)=\frac{\mu}{\sqrt{2\pi}\,(1-q^{-1})}\exp(-\frac{\omega_1}{2})\frac{\exp(-\omega_1)-1}{\omega_1}$$

The asymptotic solution of (25) can be obtained using logarithmic precision. For that we can assume $\omega_1 \simeq \omega_s(\omega_s$ is the constant steady-state value) in the expression for $w\left(\omega_1\right)$ and iterate (25) one time:

$$\omega_1 \simeq -c_\sigma \sigma,$$

where the constant $c_\sigma=\sqrt{2\left(\ln w\left(\omega_s\right)+\ln\sqrt{2\ln w\left(\omega_s\right)}\right)}$. Substituting the last formula into (23) we obtain the final relation:

$$\frac{i}{n}\simeq\left(1-q\exp(-c_\sigma\sigma)\right)^{-1}$$

Figure 2 graphs this relationship along with the exact relationship determined via numerical methods (all parameter values are those in Economy I). As can be seen, the approximation is quite good.

5.2 Model Description

5.2.1 Households

The representative household is infinitely lived and has expected utility over consumption c_t and leisure $1 - l_t$ with functional form given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) + \nu(1 - l_t)\right] \tag{26}$$

where E_0 denotes the conditional expectation operator on time zero information, $\beta \in (0, 1)$, $\nu > 0$, and l_t is time t labor. The household supplies labor, l_t, and rents its accumulated capital stock, k_t, to firms at the market clearing real wage, w_t, and rental rate r_t, respectively, thus earning a total income of $w_t l_t + r_t k_t$. The household then purchases consumption good from firms at price of one (i.e. consumption is the numeraire), and purchases new capital, i_t, at a price of q_t. Consequently, the household's budget constraint is

$$w_t l_t + r_t k_t \geq c_t + q_t i_t \tag{27}$$

The law of motion for households' capital stock is standard:

$$k_{t+1} = (1 - \delta) k_t + i_t \tag{28}$$

where $\delta \in (0, 1)$ is the depreciation rate on capital.

The necessary conditions associated with the maximization problem include the standard labor-leisure condition and the intertemporal efficiency condition associated with investment. Given the functional form for preferences, these are:

$$\nu c_t = w_t \tag{29}$$

$$\frac{q_t}{c_t} = \beta E_t \left(\frac{q_{t+1}(1-\delta) + r_{t+1}}{c_{t+1}} \right) \tag{30}$$

5.2.2 Firms

The economy's output is produced by firms using Cobb-Douglas technology[13]

$$Y_t = \theta_t K_t^{\alpha_K} H_t^{\alpha_H} \left(H_t^e\right)^{\alpha_{H^e}} \tag{31}$$

where Y_t represents the aggregate output, θ_t denotes the aggregate technology shock, K_t denotes the aggregate capital stock, H_t denotes the aggregate household labor supply, H_t^e denotes the aggregate supply of entrepreneurial labor, and $\alpha_K + \alpha_H + \alpha_{H^e} = 1$.[14]

The profit maximizing representative firm's first order conditions are given by the factor market's condition that wage and rental rates are equal to their respective marginal productivities:

$$w_t = \alpha_H \frac{Y_t}{H_t} \tag{32}$$

$$r_t = \alpha_K \frac{Y_t}{K_t} \tag{33}$$

$$w_t^e = \alpha_{H^e} \frac{Y_t}{H_t^e} \tag{34}$$

where w_t^e denotes the wage rate for entrepreneurial labor.

5.2.3 Entrepreneurs

A risk neutral representative entrepreneur's course of action is as follows. To finance his project at period t, he borrows resources from the Capital Mutual Fund according to an optimal financial contract. The entire borrowed resources, along with his total net worth at period t, are then

[13] Note that we denote aggregate variables with upper case while lower case represents per-capita values. Prices are also lower case.

[14] As in Carlstrom and Fuerst, we assume that the entrepreneur's labor share is small, in particular, $\alpha_{H^e} = 0.0001$. The inclusion of entrepreneurs' labor into the aggregate production function serves as a technical device so that entrepreneurs' net worth is always positive, even when insolvent.

invested into his capital creation project. If the representative entrepreneur is solvent after observing his own technology shock, he then makes his consumption decision; otherwise, he declares bankruptcy and production is monitored (at a cost) by the Capital Mutual Fund.

5.3 Entrepreneur's Consumption Choice

To rule out self-financing by the entrepreneur (i.e. which would eliminate the presence of agency costs), it is assumed that the entrepreneur discounts the future at a faster rate than the household. This is represented by following expected utility function:

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_t^e \tag{35}$$

where c_t^e denotes entrepreneur's consumption at date t, and $\gamma \in (0,1)$. This new parameter, γ, will be chosen so that it offsets the steady-state internal rate of return to entrepreneurs' investment.

At the end of the period, the entrepreneur finances consumption out of the returns from the investment project implying that the law of motion for the entrepreneur's capital stock is:

$$z_{t+1} = n_t \left\{ \frac{f(\bar{\omega}; \sigma_{\omega,t})}{1 - q_t g(\bar{\omega}; \sigma_{\omega,t})} \right\} - \frac{c_t^e}{q_t} \tag{36}$$

Note that the expected return to internal fund is $\frac{q_t f(\bar{\omega};\sigma_{\omega,t}) i_t}{n_t}$; that is, the net worth of size n_t is leveraged into a project of size i_t, entrepreneurs keep the share of the capital produced and capital is priced at q_t consumption goods. Since these are intra-period loans, the opportunity cost is 1.[15]

Consequently, the representative entrepreneur maximizes his expected utility function in equation (35) over consumption and capital subject to the law of motion for capital, equation (36),

[15] As noted above, we require in steady-state $1 = \gamma \frac{q_t f(\bar{\omega}_t)}{(1 - q_t g(\bar{\omega}_t))}$.

and the definition of net worth given in equation (2). The resulting Euler equation is as follows:

$$q_t = \beta\gamma E_t \left\{ \left(q_{t+1}(1-\delta) + r_{t+1}\right) \left(\frac{q_{t+1} f(\bar{\omega}; \sigma_{\omega,t})}{(1 - q_{t+1} g(\bar{\omega}; \sigma_{\omega,t}))} \right) \right\}$$

5.4 Financial Intermediaries

The Capital Mutual Funds (CMFs) act as risk-neutral financial intermediaries who earn no profit and produce neither consumption nor capital goods. There is a clear role for the CMF in this economy since, through pooling, all aggregate uncertainty of capital production can be eliminated. The CMF receives capital from three sources: entrepreneurs sell undepreciated capital in advance of the loan, after the loan, the CMF receives the newly created capital through loan repayment and through monitoring of insolvent firms, and, finally, those entrepreneur's that are still solvent, sell some of their capital to the CMF to finance current period consumption. This capital is then sold at the price of q_t units of consumption to households for their investment plans.

5.5 Steady-state conditions in the Carlstrom and Fuerst Agency Cost Model

We first present the equilibrium conditions and express these in scaled (by the fraction of entrepreneurs in the economy) terms. Then the equations are analyzed for steady-state implications. As in the text, upper case variables denote aggregate wide while lower case represent household variables. Preferences and technology are:

$$\begin{aligned} U(\tilde{c}, 1-l) &= \ln \tilde{c} + \nu(1-l) \\ Y &= \theta K^{\alpha} \left[(1-\eta) l\right]^{1-\alpha-\phi} \eta^{\phi} \end{aligned}$$

Where η denotes the fraction of entrepreneurs in the economy and θ is the technology shock. Note that aggregate household labor is $L = (1-\eta) l$ while entrepreneurs inelastically supply one unit of labor. We assume that the share of entrepreneur's labor is approximately zero so that the production function is simply

$$Y = \theta K^{\alpha} \left[(1-\eta)\, l \right]^{1-\alpha}$$

This assumption implies that entrepreneurs receive no wage income (see eq. (9) in C&F.

There are nine equilibrium conditions:

The resource constraint

$$(1-\eta)\, \tilde{c}_t + \eta c_t^e + \eta i_t = Y_t = \theta_t K_t^{\alpha} \left[(1-\eta)\, l_t \right]^{1-\alpha} \tag{37}$$

Let $c = \frac{(1-\eta)\tilde{c}}{\eta}$, $h = \frac{(1-\eta)}{\eta} l$, and $k_t = \frac{K_t}{\eta}$ then eq(37) can be written as:

$$c_t + c_t^e + i_t = \theta_t k_t^{\alpha} h_t^{1-\alpha} \tag{38}$$

Household's intratemporal efficiency condition

$$\tilde{c}_t = \frac{(1-\alpha)}{\nu} K_t^{\alpha} \left[(1-\eta)\, l_t \right]^{-\alpha}$$

Defining $\nu_0 = \frac{\eta}{1-\eta}\nu$, this can be expressed as:

$$\nu_0 c_t = (1-\alpha)\, k_t^{\alpha} h_t^{-\alpha} \tag{39}$$

Law of motion of aggregate capital stock

$$K_{t+1} = (1-\delta)\, K_t + \eta i_t \left[1 - \Phi\left(\bar{\omega};\sigma_{\omega,t}\right)\mu\right]$$

Dividing by η yields the scaled version:

$$k_{t+1} = (1-\delta)\, k_t + i_t \left[1 - \Phi\left(\bar{\omega};\sigma_{\omega,t}\right)\mu\right] \tag{40}$$

Household's intertemporal efficiency condition

$$q_t \frac{1}{\tilde{c}_t} = \beta E_t \left\{ \frac{1}{\tilde{c}_{t+1}} \left[q_{t+1}(1-\delta) + \theta_{t+1}\alpha K_{t+1}^{\alpha-1} \left[(1-\eta)\, l_{t+1}\right]^{1-\alpha} \right] \right\}$$

Dividing both sides by $\frac{1-\eta}{\eta}$ and scaling the inputs by ηyields:

$$q_t \frac{1}{c_t} = \beta E_t \left\{ \frac{1}{c_{t+1}} \left[q_{t+1}(1-\delta) + \theta_{t+1}\alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} \right] \right\} \tag{41}$$

The conditions from the financial contract are already in scaled form:

Contract efficiency condition

$$q_t = \frac{1}{1 - \Phi\left(\bar{\omega};\sigma_{\omega,t}\right)\mu + \phi\left(\bar{\omega};\sigma_{\omega,t}\right)\mu \frac{f\left(\bar{\omega};\sigma_{\omega,t}\right)}{f'\left(\bar{\omega}_t\right)}} \tag{42}$$

Contract incentive compatibility constraint

$$\frac{i_t}{n_t} = \frac{1}{1 - q_t g\left(\bar{\omega};\sigma_{\omega,t}\right)} \tag{43}$$

Where n_t is entrepreneur's net worth.

Determination of net worth

$$\eta n_t = Z_t \left[q_t (1-\delta) + \theta_t K_t^{\alpha-1} \left[(1-\eta) l_t\right]^{1-\alpha} \right]$$

or, in scaled terms:

$$n_t = z_t \left[q_t (1-\delta) + \theta_t k_t^{\alpha-1} h_t^{1-\alpha} \right] \tag{44}$$

Note that z_t denotes (scaled) entrepreneur's capital.

Law of motion of entrepreneur's capital

$$Z_{t+1} = \eta n_t \left\{ \frac{f(\bar{\omega}; \sigma_{\omega,t})}{1 - q_t g(\bar{\omega}; \sigma_{\omega,t})} \right\} - \eta \frac{c_t^e}{q_t}$$

Or, dividing by η

$$z_{t+1} = n_t \left\{ \frac{f(\bar{\omega}; \sigma_{\omega,t})}{1 - q_t g(\bar{\omega}; \sigma_{\omega,t})} \right\} - \frac{c_t^e}{q_t} \tag{45}$$

Entrepreneur's intertemporal efficiency condition

$$q_t = \gamma \beta E_t \left\{ \left[q_{t+1} (1-\delta) + \theta_{t+1} \alpha K_{t+1}^{\alpha-1} \left[(1-\eta) l_{t+1}\right]^{1-\alpha} \right] \left(\frac{q_{t+1} f(\bar{\omega}; \sigma_{\omega,t})}{1 - q_{t+1} g(\bar{\omega}; \sigma_{\omega,t})} \right) \right\}$$

Or, in scaled terms:

$$q_t = \gamma \beta E_t \left\{ \left[q_{t+1} (1-\delta) + \theta_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} \right] \left(\frac{q_{t+1} f(\bar{\omega}; \sigma_{\omega,t})}{1 - q_{t+1} g(\bar{\omega}; \sigma_{\omega,t})} \right) \right\} \tag{46}$$

5.6 Definition of Steady-state

Steady-state is defined by time-invariant quantities:

$$c_t = \hat{c}, c_t^e = \hat{c}^e, k_t = \hat{k}, \bar{\omega}_t = \hat{\omega}, h_t = \hat{h}, q_t = \hat{q}, z_t = \hat{z}, n_t = \hat{n}, i_t = \hat{\imath}$$

So there are nine unknowns. While we have nine equilibrium conditions, the two intertemporal efficiency conditions become identical in steady-state since C&F impose the condition that the internal rate of return to entrepreneur is offset by their additional discount factor:

$$\gamma \left(\frac{\hat{q} f(\hat{\omega})}{1 - \hat{q} g(\hat{\omega})} \right) = 1 \tag{47}$$

This results in an indeterminacy - but there is a block recursiveness of the model due to the calibration exercise. In particular, we demonstrate that the risk premium and bankruptcy rate determine $(\hat{\omega}, \sigma)$ - these in turn determine the steady-state price of capital. From eq.(41)we have:

$$\hat{q} = \frac{\alpha\beta}{1 - \beta(1-\delta)} \hat{k}^{\alpha-1} \hat{h}^{1-\alpha} = \frac{\alpha\beta}{1 - \beta(1-\delta)} \frac{\hat{y}}{\hat{k}} \tag{48}$$

From eq.(39)we have:

$$\hat{h} = \frac{1-\alpha}{\nu_0} \frac{\hat{k}^{\alpha} \hat{h}^{1-\alpha}}{\hat{c}} = \frac{1-\alpha}{\nu_0} \frac{\hat{y}}{\hat{c}} \tag{49}$$

From eq.(40)we have:

$$\hat{k} = \frac{1 - \Phi(\hat{\omega}) \mu}{\delta} \hat{\imath} \tag{50}$$

Note that these three equations are normally (i.e. in a typical RBC framework) used to find steady-state $\left(\hat{k}, \hat{h}, \hat{c}\right)$ - because $\hat{q} = 1$. Here since the price of capital is endogenous, we have four unknowns. From eq. (44)and eq. (41)we have

$$\hat{n} = \hat{z}\left(\hat{q}\left(1-\delta\right)+\alpha\frac{\hat{y}}{\hat{k}}\right) = \hat{z}\frac{\hat{q}}{\beta} \tag{51}$$

From eq. (45)and the restriction on the entrepreneur's additional discount factor (eq. (47)), we have

$$\hat{z} = \hat{n}\frac{1}{\hat{q}\gamma} - \frac{\hat{c}^e}{\hat{q}} \tag{52}$$

Combining eqs. (51)and (52) yields:

$$\frac{\hat{c}^e}{\hat{n}} = \frac{1}{\gamma} - \beta \tag{53}$$

We have the two conditions from the financial contract

$$\hat{q} = \frac{1}{1-\Phi\left(\hat{\omega}\right)\mu+\phi\left(\hat{\omega}\right)\mu\frac{f\left(\hat{\omega}\right)}{f'\left(\hat{\omega}\right)}} \tag{54}$$

And

$$\hat{\imath} = \frac{1}{1-\hat{q}\left(1-\Phi\left(\hat{\omega}\right)\mu-f\left(\hat{\omega}\right)\right)}\hat{n} \tag{55}$$

Finally, we have the resource constraint:

$$\hat{c}+\hat{c}^e+\hat{\imath} = \hat{k}^{\alpha}\hat{h}^{1-\alpha} \tag{56}$$

The eight equations $(48), (49), (50), (51), (52), (54), (55), (56)$ are insufficient to find the nine unknowns. However, the risk premium, denoted as ζ, is defined by the following

$$\hat{q}\hat{\omega}\frac{\hat{\imath}}{\hat{\imath}-\hat{n}} = \zeta \tag{57}$$

But we also know (from eq.(55) that

$$\frac{\hat{n}}{\hat{\imath}} = 1 - \hat{q}g(\hat{\omega})$$

Rearranging eq.(57) yields:

$$\frac{\hat{q}\hat{\omega}}{\zeta} = 1 - \frac{\hat{n}}{\hat{\imath}}$$

substituting from the previous expression yields

$$\hat{\omega} = \zeta g(\hat{\omega}) \tag{58}$$

Let br = bankruptcy rate – this observable also provides another condition on the distribution. That is, we require:

$$\Phi(\hat{\omega}) = br \tag{59}$$

The two equations eq.(58) and eq. (59) can be solved for the two unknowns - $(\hat{\omega}, \sigma)$. By varying the bankruptcy rate and the risk premium, we can determine different levels of uncertainty $(\sigma)$and the cutoff point $(\hat{\omega})$.

Note that the price of capital in steady-state, is a function of $(\hat{\omega}, \sigma)$ as determined by eq. (54). The other preference parameter, γ is then determined by eq. (47). Once this is determined, the remaining unknowns: $\left(\hat{c}, \hat{c}^e, \hat{h}, \hat{\imath}, \hat{k}, \hat{z}, \hat{n}\right)$ are determined by eqs. $(48), (49), (50), (51), (53), (55), (56)$.

Finally, we note that the parameter η does not play a role in the characteristics of equilibrium and, in particular, the behavior of aggregate consumption. This can be seen by first defining aggregate consumption:

$$(1-\eta)\tilde{c}_t + \eta c_t^e = C_t^A$$

Dividing by η and using the earlier definitions:

$$c_t + c_t^e = c_t^A \tag{60}$$

Since the policy rules for household and entrepreneurial consumption are defined as the percentage deviations from steady-state, aggregate consumption will be similarly defined (and note that since $c_t^A = \frac{1}{\eta}C_t^A$, percentage deviations of aggregate consumption and scaled aggregate consumption are identical). Using an asterisk to denote percentage deviations from steady-state, we have:

$$\frac{\hat{c}}{\hat{c}+\hat{c}^e}c_t^* + \frac{\hat{c}^e}{\hat{c}+\hat{c}^e}c_t^{e*} = c_t^{A*} \tag{61}$$

It is this equation that is used to analyze the cyclical properties of aggregate consumption.

Figure 1: Flow of Funds in Credit Channel Model
(Entrepreneur labor input and income is not shown)

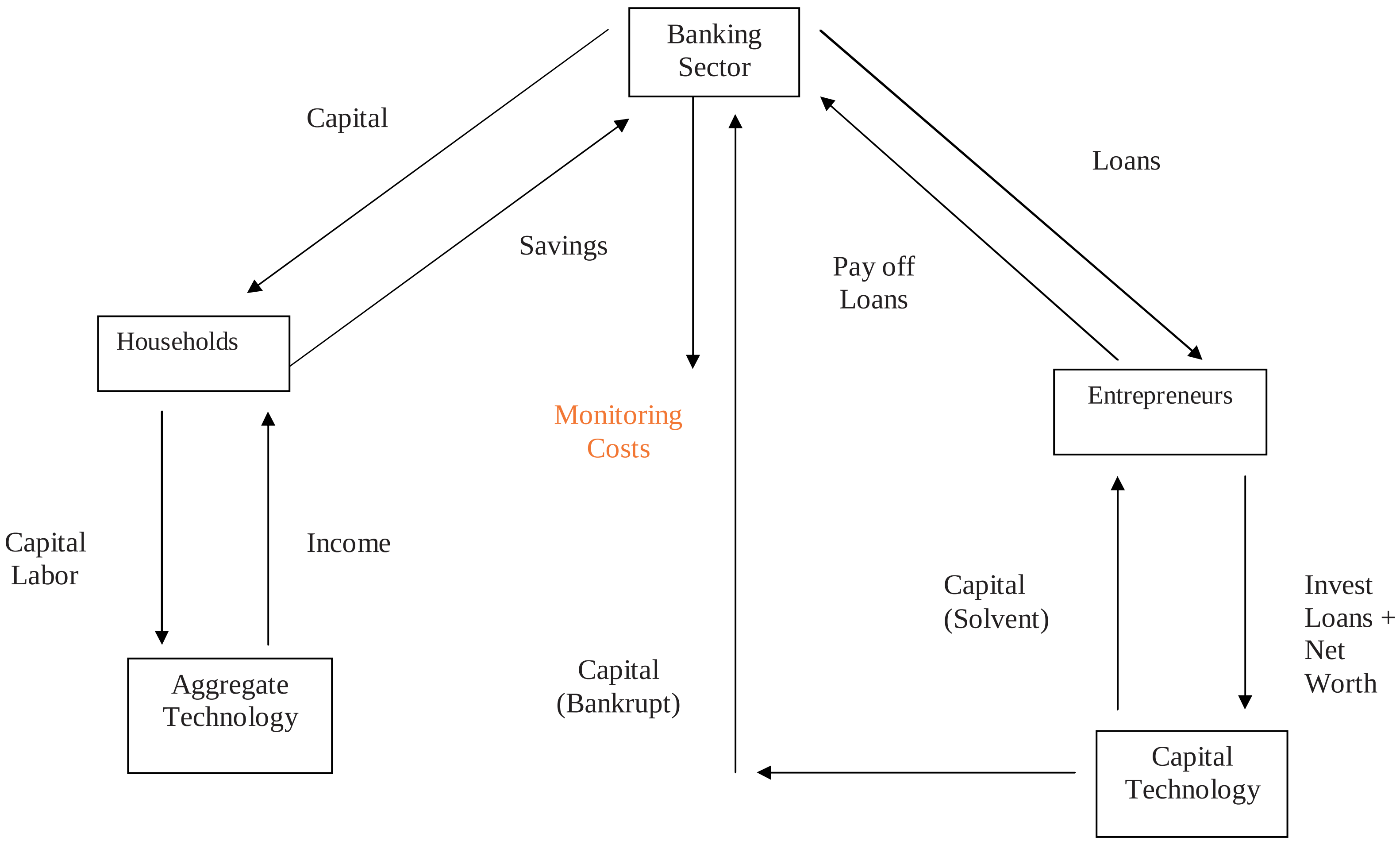

Figure 2: Exact and Approximate behavior of (i/n) as a function of σ

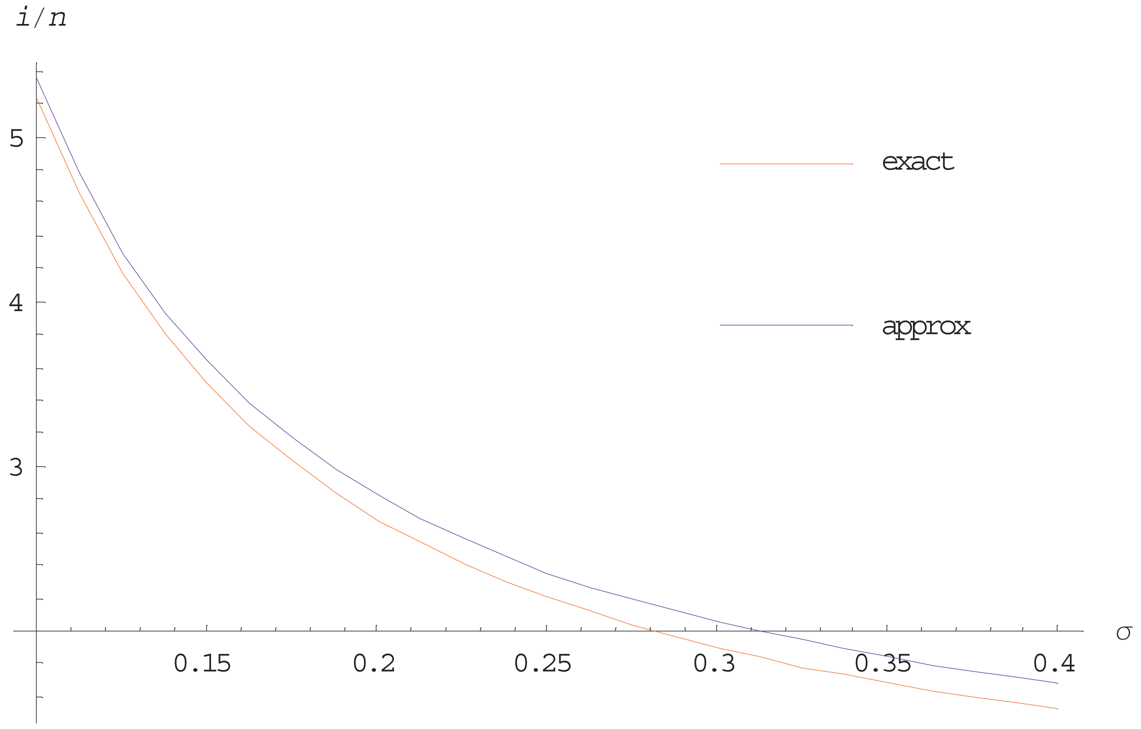

Figure 3: Technology and Uncertainty Shocks:
Effects on Investment Demand and Supply

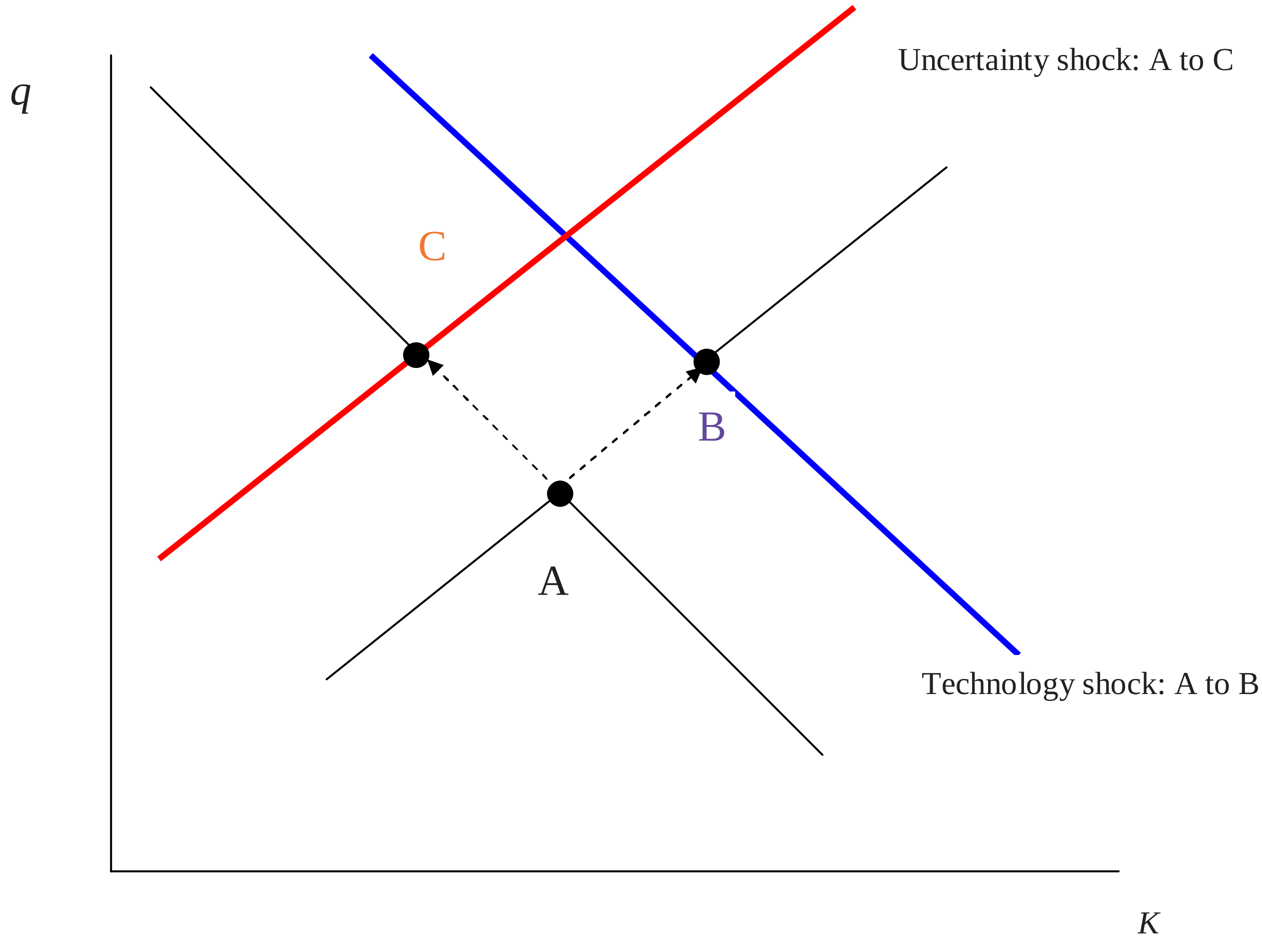

Figure 4: Response of Output, Consumption, and Investment
Low and High Persistence Economies

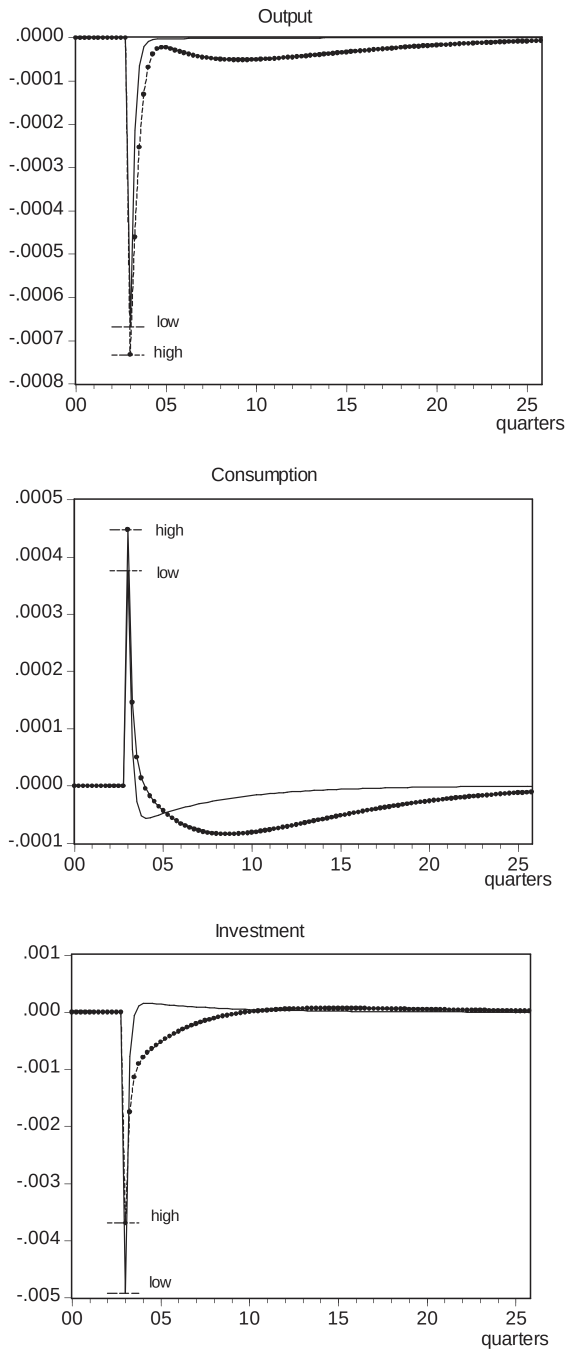

Figure 5: Response of Price of Capital, Risk Premia, and Bankruptcy Rate
Low and High Persistence Economies

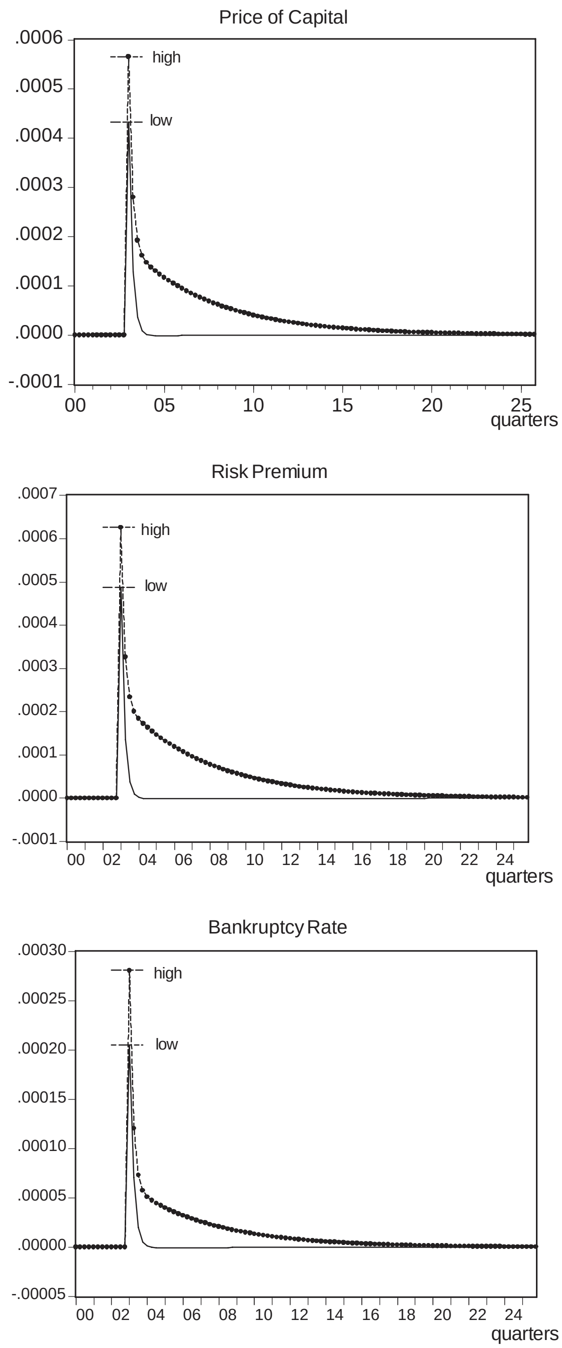

Figure 6: Response of Entrepreneur's Consumption and Net Worth
Low and High Persistence Economies