The Impact of Cost Changes on Industry Dynamics[1]

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Abstract

This paper inquires into the response of industry dynamics to increases in costs. We show that increases in marginal and fixed costs may have interesting, non-obvious effects on entry and exit. Before costs change, the model exhibits behavior that matches many industries such as manufacturing and retail: fewer but larger firms over time, and significant amounts of entry and exit. When costs rise, price rises and the market quantity supplied falls, but the amount of entry and exit may rise or fall. The most intuitive outcome from a cost increase is the *competitor neutral* case, in which entry decreases and exit increases. Two other possible cases are the *entrant favoring* case, in which entry and exit both increase, and the *incumbent favoring* case, in which entry and exit both decrease. The model places restrictions on which outcomes are possible given which costs rise (marginal or fixed). The entrant favoring case can arise only from an increase in marginal cost, which favors small entering firms relative to larger incumbents. The incumbent favoring case can come about only from an increase in fixed cost, which favors incumbents with their larger market share relative to small entrants. These restrictions allow one to infer the nature of the cost increases even when costs are not directly observed. The model can be used to examine the impacts of cost-increasing regulation or exogenous process innovation on industry dynamics.

Keywords: marginal cost, fixed cost, dynamic industry models, entry, exit, failure, market size.

JEL Codes: L11, D43

1 Introduction

This paper inquires into the response of industry dynamics—namely, entry and exit—to increases in costs. At first glance it may appear that there is not much to explore; when the cost of production in a market rises, one naturally expects that entry will decrease and exit increase as firms adjust to the new cost structure. In a static model of entry it is hard to imagine any outcome other than the equilibrium number of firms dropping as costs rise, regardless of how the model is specified. Similarly, in many dynamic models, cost increases result in less entry and more exit. However, the present work shows that in a dynamic setting some surprising outcomes can happen. In particular, a cost increase may result in more entry or less exit than before. These results do not depend on game-theoretic strategic interactions among firms, or from firms' investment decisions, two common features of models of industry dynamics in the industrial organization literature that can lead to a rich set of results. Rather, in our model entry growth and exit deterrence can follow from a relatively simple model with atomistic firms.

The present work was motivated by an empirical investigation into the effects on firms of The Americans with Disabilities Act (ADA) of 1990, the most recent major federal antidiscrimination law. The ADA seeks to prevent employment and wage discrimination against disabled workers, and to ensure the physical accessibility of businesses to disabled customers. Complying with the ADA creates costs for firms, which may alter entry and exit decisions. Compliance costs stem from provisions mandating accommodation of disabled workers and customers, and from the civil lawsuits and penalties to which the ADA exposes firms. In Prieger (2004), we conduct an empirical examination to determine if the social regulation had a measurable impact on the number of firms, the entry of new firms, and the failure rates of existing firms in the retail sector. Treating regulatory or legal changes as cost increases for firms is in accord with other empirical literature in other settings (e.g., Baicker and Chandra (2004)), which typically assumes that the only potential impacts are decreased entry and increased exit.[1] We find that the ADA is associated with a decline in the number of food stores in each market; similar results are obtained for other segments of the retail sector. The surprising empirical finding, however, is that some variables related to ADA-induced cost increases, such as disability-related labor complaints and court cases, are associated with increases in entry or decreases in exit. These unintuitive empirical results led to a deeper look into what one should expect to find when costs rise in a dynamic setting.

In the model developed here, a variant of Klepper's (1996) model, we show that increases in marginal and fixed costs may indeed have interesting and non-obvious effects on entry and exit. Before costs change, the model exhibits behavior that matches many industries such as manufacturing and retail: fewer but larger firms over time and significant amounts of entry and exit. When costs rise, price rises and the market quantity supplied falls, but the amount of entry and exit may rise or fall. Entry can rise or exit can fall because cost increases affect different cohorts heterogeneously. The oldest incumbents are the largest firms and entrants are the smallest firms in the model, and cost changes lead to composition effects in the market structure. It may happen that a few large incumbents exit and make room for a larger number of entrants, or that so many potential firms are prevented from entering that fewer incumbents exit when cost rises.

The most intuitive outcome from a cost increase is the *competitor neutral* case, in which entry decreases and exit increases. Two other possible cases are the *entrant favoring* case, in which entry and exit both increase, and the *incumbent favoring* case, in which entry and exit both decrease. The model places restrictions on which outcomes are possible given which costs rise (marginal or fixed). The entrant favoring case can arise only from an increase in marginal cost, which favors small entering firms relative to larger incumbents which suffer greater inframarginal loss. The incumbent favoring case can come about only from an increase in fixed cost, which favors incumbents with their larger market share relative to small entrants. These restrictions allow one to infer the nature

[1] In Baicker and Chandra (2004) the "firm" is a physician, and the variation in legal climate is provided by regional differences in medical malpractice costs.

of the cost increases even when costs are not directly observed. For example, if one observes that entry and exit both increased after a cost-increasing event, such as a new regulation, then the model implies that marginal costs must have risen. Thus hypotheses about how regulation affects cost can be tested. The same model could be used to examine the impacts of other forms of cost-increasing regulation or exogenous process innovation on industry dynamics.

In this paper we consider the immediate, short-run impact of cost changes on entry and exit. In contrast to the emphasis in previous literature (discussed in the next section) on steady-state outcomes, we believe that in empirical settings what we observe is not instantaneous switching from one steady state to another. Therefore the theory is developed to create predictions and restrictions on how dynamics change immediately after costs change, which is merely the first step toward the new steady state.

The plan of the paper is as follows. In the next section we discuss the relevant literature. Section 3 introduces the theoretical model of firm dynamics, and section 4 examines the response to the cost changes.

2 Relation to the Literature on Entry and Exit

Interest in the entry and exit of firms has lead to a rich literature in industrial organization. Here we discuss the implications of cost changes in existing models in order to highlight how our results differ. In simple models, when costs rise, entry typically falls and exit increases. Bresnahan and Reiss (1990) provide a straightforward static entry model that illustrates the typical findings. By assuming that demand is proportional to the size of the market, Bresnahan and Reiss (1990) can define threshold market sizes S^i determining the number of entrants:

$$S^i = \frac{F_j}{(P^i - c_j)q_j^i}$$

where i indexes the number of entrants, j is the marginal entrant, F is fixed cost, P^i is the price when there are i firms, c is marginal cost, and q is demand per consumer. Clearly increases in fixed or marginal cost increase the size of the market needed to support a given number of entrants, which implies that the number of entrants decreases as costs rise. Regarding exit, consider Ghemawat and Nalebuff's (1985) declining industry game, in which two firms with capacity K_i have marginal costs of c (there are no fixed costs in the model). Firms produce at full capacity, and inverse demand as a function of total output and time is $P(K_1 + K_2, t)$, with $\partial P/\partial t < 0$. The optimal time t_i^* for firm i to exit is defined by $P(K, t_i^*) = c$, where $K = K_1 + K_2$ if the firm is the first to exit and K_i otherwise. To find the effect of an increase in marginal cost, totally differentiate to yield $(\partial P/\partial t)\, dt_i^* = dc$, whence it follows that $dt_i^*/dc < 0$. Thus when costs increase, exit increases in the short run (in the sense that exit is brought closer to the present in time).

Most simple models provide similar results from exogenous changes in cost. However, it is important to distinguish between exogenous cost shifts and costs that are under the control of the firms. For example, in the canonical entry deterrence games (e.g., Dixit (1980)) incumbents can invest to lower marginal cost in the future, which may deter entry by rivals. Thus in a cross-section of markets where such games are being played, lower marginal costs may be correlated with *less* entry. In this paper we model the impact of changes in regulation or law that affect a firm's costs, and thus focus exclusively on the case of exogenous cost shocks.

There are several theoretical studies of industry dynamics. Three prominent models with atomistic firms are Jovanovic (1982), Hopenhayn (1992), and Klepper (1996).[2] Hopenhayn (1992) is the only one of these studies that investigates the effect of cost changes on entry and exit, and focuses on the limiting distribution instead of the short-run impact we consider. Hopenhayn (1992) finds that as entry costs rise, entry and exit both decrease (the incumbent favoring outcome, in our nomenclature). However, this is not a surprising outcome, given that entry costs are not paid by incumbents. In our model described in the next section, there are no entrant-specific costs and fixed costs are paid by all firms each period. When the fixed cost analogous to ours rises in Hopenhayn's

[2] Petrakis and Roy (1999) and Roy and Kamihigashi (2004) also explore models of industry dynamics with atomistic firms.

(1992) model, exit increases in the steady state and incumbent favoring is not possible as it is in our model. The model in section 3 is based on Klepper (1996),[3] who did not focus on exogenous cost shocks. Our theoretical model simplifies Klepper (1996) by abstracting away from endogenous innovation for the sake of more readily exploring how cost changes affect entry and exit. The model we adopt is more convenient to work with than the complex dynamical system in Jovanovic (1982) and admits non-steady state analysis more easily than does the model in Hopenhayn (1992).

Ericson and Pakes (1995) extend the literature on industry dynamics to allow imperfect competition, with extensions and variants of dynamic models with strategic interactions among firms provided by Peretto (1996), Amir and Lambson (2003), and Benkard (2004). By not adopting a game-theoretic model, we are able to more easily characterize how entry and exit change in response to structural cost changes. Furthermore, that we find a rich set of possible outcomes in our model even without incorporating strategic interactions highlights that the unexpected outcomes do not depend on imperfect competition. However, the assumption of atomistic firms makes our model less suited to study of industries with few firms such as telecommunications or automobile manufacturing. We note that models of imperfect but symmetric competition such as Amir and Lambson (2003) cannot lead to the "unexpected" outcomes we find in our model, because the less-obvious outcomes depend heavily on composition effects (such as large incumbents being replaced by smaller entrants).[4] Furthermore, models such as that of Ericson and Pakes (1995) typically focus on steady-state solutions due to the complexity of the non-steady-state dynamics, and we are also interested in the transient response of industry to cost shocks.

Other studies model entry and exit in dynamic models with imperfectly competitive markets but without strategic interactions among firms. Asplund and Nocke (2003) construct a model to reflect the stylized fact from empirical studies that entry and exit rates are positively correlated

[3] See also Klepper (2002).

[4] Note however that in models with endogenous cost-reducing R&D many outcomes are possible even when competition is symmetric. For example, in Peretto (1996) there are a multiplicity of equilibria, and when exogenous fixed cost rises it is possible that the number of firms rises in some equilbria while falling in others.

across industries. In accord with their goal, they find that when fixed costs rise, entry and exit rates decrease (the incumbent favoring outcome). Our model shows that even in a simpler model with atomistic firms the incumbent favoring outcome is possible but is not the only potential outcome. Some of the recent international trade literature uses models in this vein as well. One such paper that performs comparative statics with respect to (trade) regulation-induced cost changes is Melitz (2003), who assumes monopolistic competition among firms. Melitz (2003) shows that when trade reform lowers the variable cost of trade or the fixed cost of entry in export markets, the least productive firms exit the domestic market and the most productive firms enter the export markets. These composition effects are akin to some of our results, although our results are all in the context of a single market and do not rely on reallocation of firms between markets.

In addition to the theoretical studies, there are several empirical studies of entry using structural econometric models, mostly in static or two-period settings (Bresnahan and Reiss, 1987; Bresnahan and Reiss, 1990; Berry, 1992; Mazzeo, 2002; Seim, 2004). Given the static nature of these models, the impact of cost changes on entry and exit can not be separated from the impact on the number of firms in the market. Although Pakes, Ostrovsky and Berry (2004) present a fully dynamic structural econometric model, their focus is quite different than the present work. Pakes et al. (2004) incorporate strategic interactions and asymmetric information among firms, and are more concerned with developing a suitable econometric model than in comparative statics or dynamics. Among the numerous reduced-form empirical studies of industry dynamics (see Caves (1998) for a survey), of most interest here may be Manjón-Antolin (2004), which emphasizes the importance of firm size in the determination of entry and exit rates, which is related to the importance of composition effects that drive the results of our model.

3 The Model

Here we construct a model to investigate the response of industry dynamics to increases in costs. In each period $t = 1, 2, \ldots$, there is a continuum of atomistic potential entrant firms indexed by their fixed cost $F \in [\underline{F}, \bar{F}] \equiv \mathcal{F}, 0 < \underline{F} < \bar{F}$. The fixed costs are paid each period, and are avoidable if a firm decides to exit (or not enter) the market. The fixed costs may represent the costs of business licenses, complying with local regulations, or lumpy investments that fully depreciate each period. The mass of firms with fixed cost less than or equal to F is $\mu(F)$, where measure μ is a non-negative monotonically increasing, differentiable, and bounded function on $\mathcal{F}$. To simplify dynamics it is assumed that there is a mass point of most-efficient firms: $\mu(\underline{F}) > 0$. Firms have no costs if they do not enter: outside opportunities are normalized to zero. The firms have constant returns to scale technology resulting in constant marginal cost c.

Changes in regulation or business law, or adverse technology shocks can unexpectedly increase the firm's marginal or fixed costs. Changes in marginal cost increase c directly; changes in fixed cost increase F by $\phi \geq 0$. A concrete example, explored in a companion paper (Prieger, 2004), is the impact of the Americans with Disabilities Act on firm's cost. As a form of social regulation, the ADA requires firms to alter their actions, compelling them to hire, accommodate, or make their premises accessible to the disabled. Assuming the firms were profit-maximizing before the regulation, the ADA can be modeled at the most basic level as a cost increase for firms.

Firm i entering in period k has output in period $t \geq k$ denoted $q(i)_t^k$ (or $q(i)_t$; cohort superscripts are dropped when the cohort is not of interest). The total market quantity produced at t is $Q_t = \sum_{k=1}^{t} \int_{\mathcal{F}_t^k} q(i)_t^k d\mu$, where $\mathcal{F}_t^k$ is the subset of $\mathcal{F}$ containing the cohort of firms that entered in period k that are still in the market at time t. Consumers view firms' products as homogeneous. Market demand at time t, $f_t(p)$, is a function of the current market price only, where f_t is continuous and downward-sloping in price, and increases (for given p) over time. If a firm stays in the market it keeps all previous customers and attracts a share of new buyers (and those whose previous supplier exited) in proportion to last period's "market share" $s_{t-1} \equiv q(i)_{t-1}/Q_{t-1}$.[5] This advantage of incumbency may be due to consumer inertia, or to stocks of advertising or brand-recognition "goodwill" not explicitly modeled. The firm can also sell more product in amount $\Delta q(i)_t$ by incurring marketing cost $m(\Delta q)$, where $m(0) = m'(0) = 0$, $m'(\Delta q) > 0$, $m''(\Delta q) > 0$ for all $\Delta q \geq 0$, with m' unbounded above. Thus firm i at time t sells

$$q(i)_t = s_{t-1}Q_t + \Delta q(i)_t \tag{1}$$

where the first term is zero for entrants, who have no previous production. It will be shown below that price declines and Q_t increases over time in equilibrium, and therefore $q(i)_t$ increases over time for any firm staying in the market. Many of the propositions in the paper rely on the fact that the oldest incumbents produce the most output; the model is not suited to industries in which large-scale entry is seen, where entrants leap-frog past incumbents.

A firm's profit at t is:

$$\pi(i)_t = q(i)_t\,(p_t - c) - m(\Delta q(i)_t) - \phi - F \tag{2}$$

where before any costs change ϕ is zero. Firms are atomistic price takers. Following Klepper (1996), firms can project the current period's market-clearing price, but are myopic in that they base entry, exit, and production decisions only on current period's profits (2). Furthermore, firms do not anticipate the cost change. Given an expectation of the market-clearing price, each firm decides by how much to expand output by choosing the optimal $\Delta q(i)^*_t$ should the firm decide to be in the market. Given $\Delta q(i)^*_t$, if $\pi(i)^*_t > 0$ a firm decides to stay in the market, if an incumbent, or to enter, if not where $\pi(i)^*_t$ is the maximized profit given expected price p_t. Firms will not enter (or will exit) if $\pi(i)^*_t < 0$, and are indifferent if $\pi(i)^*_t = 0$,

The equilibrium price p^*_t is determined by supply equaling demand: $Q^*_t = f_t(p^*_t)$, where Q^*_t is the equilibrium market supply under the optimal entry, exit, and output expansion decisions.[6] It

[5] With a continuum of firms, the actual market share of a single firm is zero. The important things about the quantity $q(i)_{t-1}/Q_{t-1}$ is that it is well-defined and that it integrates to one under measure μ.

[6] Since μ is differentiable, it is continuous, and an equilibrium price exists. If μ is discontinuous (for example,

is assumed that f_1 is large enough and costs are small enough so that p_1^* exists. A final assumption of the model, to simplify the dynamics, is:

Assumption 1 *$\pi(i)_t^*$ is increasing in p_t in an interval around p_{t-1}^*, with p_t^* lying within this interval.*

To maximize profit, a firm staying in the market chooses the optimal increase in output in accordance with the first-order condition resulting from maximization of (2),

$$m'(\Delta q(i)_t^*) = (p_t - c) \tag{3}$$

which implicitly defines $\Delta q(i)_t^*$. The assumptions on m assure that $\Delta q(i)_t^* > 0$ whenever price is above marginal cost, as it must be in equilibrium. A description of the equilibrium in this model, in the absence of any cost change, is in the following proposition. Most of the results and many of the proofs for Proposition 1 are similar to those in Klepper (1996) and are in the appendix.

Proposition 1 (Basic Results) *Let $V(i)_t = q(i)_t\,(p_t - c) - m(\Delta q(i)_t)$ be variable profit (exclusive of fixed cost ϕ and F), and asterisks denote optimal quantities and sets. Holding c and ϕ fixed, an equilibrium price sequence $\{p_t^*\}$ exists, and the following hold in equilibrium:*

1. *$p_t^* < p_{t-1}^*$ and $Q_t^* > Q_{t-1}^*$. Prices fall and output expands over time.*

2. *$q(i)_t^{k*} > q(i)_{t-1}^{k*}$ and $s(i)_t^{k*} > s(i)_{t-1}^{k*}$ $\forall i \in \mathcal{F}_t^{k*}, t, k \leq t$. Firms grow over time or exit.*

3. *$\Delta q(i)_t^{k*} = \Delta q_t^*$ $\forall$ $i \in \mathcal{F}_t^{k*}, t, k \leq t$. Output expansion is the same for all firms in the market in period t.*

4. *$q(i)_t^{k*} = q_t^{k*}$ and $V(i)_t^{k*} = V_t^{k*}$ $\forall$ $i \in \mathcal{F}_t^{k*}, t, k \leq t$. Output and variable profits are the same for all firms of the same cohort in the market in period t.*

because of discrete firms), excess demand may not equal zero in equilibrium because a marginal increase in price may flip the market from excess demand to excess supply upon entry of a single firm.

5. *There exists an* F_t^k *such that* $\mathcal{F}_t^{k*} = [\underline{F}, F_t^k]\ \forall t, k \leq t$. *For firms of a cohort, only those with fixed costs below a threshold value remain in the market in period* t.

6. $\Delta q_t^* < \Delta q_s^*$, $V_t^{t*} < V_s^{s*}$, *and* $F_t^t < F_s^s$ *for* $t > s$. *The output, variable profit, and threshold fixed cost of entrants fall over time.*

7. *Exit occurs each period.*

8. *Entry decreases each period, and eventually ceases in finite time.*

9. *The number of firms in the market may increase at first, but eventually declines monotonically.*[7]

The "number of firms" in the last point is to be read as "the measure of $\bigcup_{k=1}^{t} \mathcal{F}_t^k$ with respect to μ." Regarding Proposition 1.8, the measure of entering firms declines to zero as $t \to \infty$ in all cases. If firms with the minimum fixed cost have non-zero measure, then there is a date t after which no more firms enter (see the appendix).

The model exhibits behavior that matches many industries (manufacturing and retail, for example): fewer but larger firms over time, with significant amounts of entry and exit.[8] The most efficient firms (those with the lowest fixed cost) remain in the market and expand their share, at the expense of higher-cost firms that are forced to exit. Industry dynamics for a typical set of parameter values are depicted in Figure 1 for the initial 10 periods.[9] The number of firms in the top panel declines over time, in accord with Proposition 1.9. In this example the number of firms begin to decline immediately, although Proposition 1.9 allows the possibility that the industry can grow at first if demand increases enough. The other panels of Figure 1 decompose the firm count

[7] The "number of firms" is to be read as "the measure of $\bigcup_{k=1}^{t} \mathcal{F}_t^k$ with respect to μ."

[8] Proposition 1.8 may require additional explanation. The measure of entering firms declines to zero as $t \to \infty$ in all cases. The second part of Prop. 1.8 means that if firms with the minimum fixed cost have non-zero measure, then there is a date t after which no more firms enter. The proof in the appendix makes clear the distinction.

[9] In this and following figures, demand is $Q_t = 1000t - p$, adjustment cost is $m(\Delta q) = 100(\Delta q)^3$, F is distributed uniformly on $[0.01, 10.0]$, the measure of potential entrants each period is 10,000, and $c = 1$.

into its determinants, entry and exit. Entry ceases entirely in period 13 (not shown in the figure), as suggested by Proposition 1.8.

4 The Impact of Cost Changes

With the basic behavior of industry dynamics in the model established, we can now examine the impact of an unanticipated increase in cost. In the period cost increases, it is assumed that the firms know that costs have changed before they make their entry, exit, and output decisions.

Proposition 2 (Impact of Cost Increases) *In the period t in which cost increases, the following hold, compared to the same period were cost not to increase:*

1. *p_t^* rises and Q_t^* falls.*
2. *The number of entering firms can increase or decrease.*
3. *The number of incumbent firms can increase or decrease.*

The first point follows naturally from the fact that while cost rises for all firms, the demand function is unchanged (see appendix for proof). The second and third results may be shown numerically. In these statements "number of firms" is to be read as "the measure of $\mathcal{F}_t^k$ with respect to μ" for the relevant $\mathcal{F}_t^k$. Numerical examples showing that entry and exit can each move in either direction are shown in Figures 2–5, which are discussed below. Given that market quantity falls, when the number of firms increases it must be that each firm produces less or smaller entrants replace larger incumbents (a composition effect),[10] or both. If enough new firms enter, as may happen if μ has a large mass of entrants close to the threshold entry level before the cost change, then the number of firms in the market can even rise. In static models such as Bresnahan and Reiss (1987), in which there is no distinction between entrants and incumbents, the number of firms can only decrease when costs rise.

[10]Entrants always produce less than do incumbents, by Propositions 1.2 and 1.6.

Thus, this relatively simple model generates interesting, varied, and non-obvious responses to the cost changes. The possibilities for entry and exit are listed in Table 1. The most intuitive case is the *competitor neutral* case, in which entry decreases and exit increases in response to the cost changes. When entry increases, it can be shown that the scale of entry Δq_t also increases (Lemmas 5 and 6 in the appendix). Thus, since market quantity in total falls, entry can increase only at the expense of the number of incumbents, the quantity each incumbent produces, or both. Industry dynamics for an example of the competitor neutral case resulting from an increase in fixed cost is depicted in Figure 2.[11] In the example shown in the figure entry drops immediately to zero when cost increases, although in general this need not be the case.

When entry declines and the resulting lessening of competitive pressure allows more incumbents to stay in the market, so that exit also declines, we have the *incumbent favoring* case. Industry dynamics for an example of the incumbent favoring case resulting from an increase in fixed cost is depicted in Figure 3.[12]

Finally, we term the case in which entry increases and the number of incumbents falls *entrant favoring*. An entrant favoring case resulting from an increase in marginal cost is depicted in Figure 4.[13] The bottom two panels show that both entry and exit jump up in the period of the cost change. However, note that all changes in Table 1 are with reference to the same period in the baseline in which no costs changes, not with reference to changes in entry and exit over time. To illustrate the distinction, Figure 5 shows another entrant favoring case.[14] Unlike the case in Figure 4, entry and exit are lower in the period of the cost change (period 4) than the previous period. The relevant comparison, however, is to what entry and exit would have been in period 4 had costs remained unchanged. The implication for empirical application of the model is that a difference-in-differences approach (or other methods that control for secular trends) may be more appropriate to determine

[11] In this example fixed cost increases by 1.0 in period 4.

[12] In this example fixed cost increases by 0.41 in period 2.

[13] In this example marginal cost increases by 2000 in period 2. A large increase is required to separate the curves clearly in the figure.

[14] In this example marginal cost increases by 1300 in period 2.

which of the cases in Table 1 applies than a simpler before-and-after model. The latter approach applied to the data in Figure 5 would mistakenly classify this case as incumbent favoring.[15]

Examining when the various cases occur allows us to link more easily observed outcomes such as entry and exit with the less easily observed changes in cost. To simplify analysis we introduce two additional assumptions.

Assumption 2 *Fixed cost F is uniformly distributed on $\mathcal{F}$.*

Assumption 3 *At the equilibrium price $\varepsilon_t^*(p_t^* - c)/p_t^* < 1$, where $\varepsilon_t \equiv -f_t'(p_t)p_t/Q_t$, the elasticity of demand.*

Assumption 2 simplifies comparison across cohorts of the effects of the cost increase, as will be described below.[16] Assumption 3 is related to but stronger than Assumption 1. In particular, Assumption 1 is equivalent to the weaker condition that $\varepsilon_t^*(p_t^* - c)/p_t^* < 1 + \Delta q_t^*/(\Delta q_t^* + q_t^{1*})$ (see Lemma 7 in the appendix). The last term on the right side shrinks over time, and in simulations Assumption 3 is typically satisfied after a few periods, if not immediately, as the relative markup shrinks. If demand is inelastic then Assumption 3 holds in all periods. With these assumptions, we can characterize the impacts that the changes in cost have on entry and exit.

Proposition 3 (Restrictions on Observed Outcomes) *Using the definitions from Table 1, the following hold:*

1. *When marginal cost increases, the incumbent favoring case is not possible.*

2. *When fixed cost increases, under Assumptions 2 and 3 the entrant favoring case is not possible.*

[15] To complete the set of logical possibilities we should include the case in which entry increases, exit decreases, and the number of firms increases. We ignore this case. It cannot happen when fixed cost rises (the proof of Proposition 3.2 shows that entry must decrease). We have not been able to rule it out theoretically when marginal cost rises: it may be possible that a small number of large incumbents exits to make room for a larger number of smaller incumbents. However, we cannot generate it numerically.

[16] Technically, Assumption 2 violates the maintained assumption that μ has a mass point at $\underline{F}$. However, the mass point is needed only to simplify the proof of Propositions 1.8-9. Also, Assumption 2 can be changed to include the mass point at $\underline{F}$ and no results would change.

The proof of the theorem is in the appendix, but the insight is presented here. For any cohort k, the change in the number of firms in the market N_t^{k*} due to a small increase in cost is

$$\frac{dN_t^{k*}}{db} = \frac{d}{db}\int_{\mathcal{F}_t^k} d\mu = \frac{dF_t^k}{db}\mu'(F_t^k) \tag{4}$$

where $b = c$ or ϕ, and the second equality follows from Proposition 1.5 and the application of Leibnitz' rule to a Reimann-Stieltjes integral. The marginal entrant (when $k = t$) or exiting firm (when $k < t$) in a cohort satisfies $\pi(F_t^k)_t^* = 0$, thus defining F_t^k as the break-even level of fixed cost:

$$F_t^k = V_t^{k*} - \phi \tag{5}$$

where V is as defined in Proposition 1.4. Expression (4) for the change in the number of firms can then be interpreted as the change in the threshold F_t^k weighted by the density of firms at that point, $\mu'(F_t^k)$. Substituting (5) for F_t^k into (4) gives

$$\frac{dN_t^{k*}}{dc} = \left[\frac{dV_t^{k*}}{dp}\frac{dp_t^*}{dc} - q_t^k\right]\mu'(F_t^k) \tag{6}$$
$$\frac{dN_t^{k*}}{d\phi} = \left[\frac{dV_t^{k*}}{dp}\frac{dp_t^*}{d\phi} - 1\right]\mu'(F_t^k) \tag{7}$$

for marginal cost and fixed cost changes, respectively. Under Assumption 2, $\mu'(F_t^k)$ in (6)–(7) is the same for all k, which simplifies cross-cohort comparison. The second term in the brackets ($-q_t^k$ or -1) is the direct effect of the cost change on the number of firms. The negative direct effect reflects that because production is more expensive when costs rise, the threshold fixed cost decreases and fewer firms remain in the market (or enter). The first term in the brackets in (6)–(7) is the (positive) price effect of the cost change. The cost increase leads to an increase in equilibrium price (from Proposition 2.1), which increases profit (because of Assumption 1). With higher profit, more firms remain in the market (or enter). The direct and price effects thus move in opposite directions, and either can predominate, leading to various cases to consider.

Figures 6–9 depict the magnitudes of the direct and price effects for various possible cases. To understand the intuition and the figures, note that the first term in the price effect is

$$\frac{dV_t^{k*}}{dp} = \frac{d}{dp}\left[\left(s_{t-1}^k Q_t^* + \Delta q_t^*\right)(p_t^* - c)\right] = s_{t-1}^k (p_t^* - c)\frac{dQ_t^*}{dp} + q_t^{k*} \tag{8}$$

By the envelope theorem, there are no effects through changes in the optimized Δq_t^*. For the cohort entering at t, the first term on the rightmost side of (8) is zero because entrants have no pre-existing share. Consider increases in marginal cost for the moment. The sign of (6) for entrants is determined by $q_t^{k*}\,(dp_t^*/dc - 1)$. If dp_t^*/dc, the equilibrium pass-through of a marginal cost increase, is less than one, then we have the case shown in Figure 6. Here the direct effect dominates the price effect at cohort age zero and entry declines. Note the intercepts are marked using the fact that for entrants, $q_t^t = \Delta q_t^t$. For older cohorts, both effects are larger because q_t^{k*} rises with cohort age $t - k$ (an implication of Propositions 1.2 and 1.6).[17] However, the difference between the direct and price effects grows over time, due to the increasing share s_{t-1}^k in (8) as cohorts age.[18] Thus the price effect curve never crosses the direct effect curve, the bracketed term in equation (6) is negative, cohort size falls for each k, entry declines and exit increases, and we have the competitive neutral case.

The argument above began with the assumption that a marginal cost increase results in less than full pass through to consumers. However, the opposite is also possible in this model: it may be that $dp_t^*/dc > 1$. When so, the direct and price effects are as in Figure 7. The price effect is smaller than the magnitude of the direct effect for at least the entering cohort on the left of the graph, implying that entry increases. Given the shape of the curves and the fact that exit increases for at least the oldest cohort (proved in the appendix), it must be that the curves cross. This results in the entrant favoring case. In the entrant favoring case an increase in marginal cost hurts the profit of incumbents more than of entrants because existing firms have a larger automatic market share,

[17] If Assumption 2 is abandoned, then large differences in $\mu'(F_t^k)$ across k may affect the slopes of the curves in Figures 6–9.

[18] That share rises (conditional on not exiting) as a cohort ages follows from parts 2 and 6 of Proposition 1.

and thus suffer greater inframarginal loss. The greater exit of incumbents results in a price that exceeds the marginal cost increase, spurring entry by new firms. Finally, note that Proposition 3.1 follows from the discussion above: either cohort size falls for each k or the entrant favoring case obtains, and so the incumbent favoring case is not possible when marginal cost rises.

When fixed cost rises and Assumption 3 holds (e.g., when demand is inelastic), then if the number of firms rises for a cohort j it must rise for all older cohorts as well (i.e., those with $k < j$). Thus entrant favoring is not possible (Proposition 3.2). The intuition for why entrant favoring is impossible is that when fixed cost rises, market quantity falls (Proposition 2.1) but the scale of entry increases (Lemma 5 in the appendix). Thus the number of entrants must decrease.

When fixed cost rises, competitor neutrality and incumbent favoring are both possible and are shown in Figures 8 and 9. The direct effect is always -1, because an increase in ϕ directly shifts the threshold fixed cost down one for one. It is shown in the proof of Proposition 3.2 that the price effect for entrants (cohort age = 0 on the graphs) is always smaller than the direct effect when Assumption 3 holds, but then rises for older cohorts. If the direct effect is always larger than the price effect, as in Figure 8, then entry decreases, exit increases for all cohorts, and the competitor neutral case results. However, it can happen that the price effect dominates the direct effect for older cohorts, as in Figure 9. If exit decreases enough from these oldest cohorts, then the incumbent favoring case can result. In this case the increase in cost disproportionately hurts the small firms (including entrants), because their smaller scale leaves them more vulnerable to increases in fixed costs.[19]

Proposition 3 provides two implications from the model useful for empirical work. These are stated as a corollary, which follows directly from Proposition 3:

Corollary 4 *The following hold:*

[19] If Assumption 3 does not hold (for which elastic demand is necessary but not sufficient), however, then the price effect would slope down in the figures. As a result, if the number of firms rises for a particular cohort it would also rise for all younger cohorts, and incumbent favoring is not possible.

1. *Under Assumptions 2 and 3, the entrant favoring case can arise only from increases in marginal cost.*

2. *The incumbent favoring case can arise only from increases in fixed cost.*

The competitor-neutral outcome implies no restrictions on the nature of the cost increase, and in that sense is the least informative outcome. In the empirical companion paper (Prieger, 2004), we use these implications of the model to infer which elements of the ADA raised which costs. For example, we find that labor complaints about disability accommodation filed with the Equal Employment Opportunity Commission lead to entrant favoring impacts, and therefore must raise the marginal costs of retail stores. On the other hand, the number of disabled individuals in the area and accessibility lawsuits filed under Title III of the ADA lead to incumbent favoring outcomes. This suggests that fixed costs rose from these sources, which implies that accessibility suit costs are not proportional to output. Non-proportionality is consistent with the observed phenomenon of activists filing numerous ADA accessibility suits, without regard to firm size.[20]

The extension of the results to cost decreases instead of increases is immediate. The results of Proposition 2.1 would be reversed and the rest of the proposition unchanged. The changes in cohort size from equations (6)–(7) would be reversed. Proposition 3 would therefore be altered to state that when c decreases, the entrant favoring case is not possible, and when ϕ decreases, the incumbent favoring case is not possible.

Given our interest in immediate and short-term responses to regulatory changes, we have not discussed what the impacts are in the long run. The model has little to say about entry in the long run, because entry eventually ceases in all cases (Proposition 1.8). Based on the numerical explorations of the model (some of which are depicted in Figures 2–5), the long run effects of the cost increases on exit also appear to be minimal, because much of the adjustment to the shock

[20] Examples include Jarek Molski, a paraplegic who has filed close to 500 lawsuits since 2001 in Southern California, and George Louie, a wheelchair activist who has filed about 1,000 lawsuits since 1998 in Northern California (San Diego Union-Tribune web site, Sept. 12, 2004 <http://www.signonsandiego.com/uniontrib/20040912/news_1n12litigant.html>).

occurs within a few periods.

5 Conclusion

In this paper we have shown that increases in marginal and fixed costs may have interesting and non-obvious effects on entry and exit. The intuition suggested from static models of entry, that the number of entrants (i.e., firms in the market) must fall when costs rise, turns out to be only one of three possible cases. In addition to the "obvious" case in which entry declines and exit increases, it may also happen that entrants or incumbents as a group are relatively favored by the cost increase. The restrictions placed by the model on the conditions under which the various outcomes occur have practical implications for empirical work. For example, a particular regulation may have placed substantial costs on firms even if the number of firms in the market does not change much, because the aggregate may mask large compositional changes across cohorts. These implications are exploited in the empirical companion paper to the present work, an investigation into the impact of the ADA on the retail industry.

A remarkable aspect of our results is that a relatively simple model, with price-taking firms, generates such a rich set of possibilities. There are no strategic interactions among firms and no investment in the model. While leaving out game theory and investment simplifies the analysis, it also may limit application of the model to many of the commonly studied industries in the industrial organization literature, such as telecommunications services or heavy manufacturing, in which there are a small number of dominant firms and investment is a key feature of the industry. A potential avenue of future research is thus to add investment or non-atomistic firms to the model. Given the important role of fixed cost in generating incumbent favoring outcomes, it may be that firms have less incentive to reduce such costs through investment than previous static models have suggested.

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A Appendix

A.1 Proof of Proposition 1

Existence of a market clearing price p_t^* proceeds by induction. Assume p_{t-1}^* exists. Assumption 1 implies that if p_t is low enough, no firm will be able to stay in the market. At such a price demand exceeds supply. At $p_t = p_{t-1}^*$, supply exceeds demand. To see this, note that the equilibrium condition can be written $f_t(p_t) = \int s(i)_{t-1}Q_t + \Delta q(i)_t d\mu$, where integration is over the set of fixed costs for firms in the market at t Rewrite as $f_t(p_t)\left[1 - \int s(i)_{t-1}d\mu\right] = \int \Delta q(i)_t d\mu$. If $p_t = p_{t-1}^*$, no firm would plan to exit at that price and every firm would expand output. But without exit, $1 - \int s(i)_{t-1}d\mu = 0$ and $\int \Delta q(i)_t d\mu > 0$, and supply exceeds demand. Because f_t and Δq_t^* are continuous in p_t, there must be a price $p_t^* < p_{t-1}^*$ that clears the market. Since it is assumed that p_1^* exists, an equilibrium price path $\{p_t^*\}$ exists by induction.

Proof of Prop. 1.1. Because $p_t^* < p_{t-1}^*$, it follows that $Q_t^* > Q_{t-1}^*$.

Proof of Prop. 1.2. By equation (3) and discussion following, $\Delta q(i)_t^k > 0$ if firm i stays in the market. The previous proof showed that i's market share and the market quantity to which it is applied both increase. Thus $q(i)_t^{k*} > q(i)_{t-1}^{k*}$.

Proof of Prop. 1.3. From (3) it is clear that Δq depends only on p_t, and so is the same each period for all firms remaining in the market. Thus Δq need not be indexed by i or k.

Proof of Prop. 1.4. From Prop. 1.3 all entrants at k produce the same amount in their first period and therefore have the same share. For $t = k + 1$, from (1) it is seen that because last period's share and this period's Δq_t^k are the same for all firms in cohort k, output must be identical as well. Continuing the argument as time goes on proves that $q(i)_t^{k*} = q_t^{k*}$ for all $t \geq k$. Since q and p are the same for all firms in the cohort (firms differ only by fixed cost), V is the same as well.

Proof of Prop. 1.5. Firms remain in the market if $\pi(i)_t^k = V_t^k - F \geq 0$. This defines the marginal firm as the one with fixed cost $F_t^k = V_t^k$. Firms with F below F_t^k remain in the market and others exit.

Proof of Prop. 1.6. From (3) and the assumption that $m'' > 0$ it is clear that as p_t^* falls over time (by Prop. 1.1) so does the scale of entry and output expansion Δq_t^*. If the scale of entry and p_t^* both fall over time, then it must be that V_t^{t*} falls over time, too. Since V_t^{t*} defines F_t^t (see proof of Prop. 1.5), F_t^t also falls over time.

Proof of Prop. 1.7. By Prop. 1.2 each firm remaining in the market increases its share each period. Without the exit of some firms this cannot happen.

Proof of Prop. 1.8. Because F_t^t falls over time, then because μ is monotonic the measure of entrants $\mu(F_t^t)$ falls over time. To show that entry eventually ceases, which is to say that there exists a $t < \infty$ such that no entry occurs after period t, first let $\mu(\underline{F}) = \hat{\mu}$, which is non-zero by assumption. Then, as long as $V_t^t > \underline{F}$, we have $\mu(F_t^t) \geq \hat{\mu} \ \forall \ t$ (if $V_t^t = \underline{F}$ and the lowest-cost firms are indifferent to entering, then because p falls in no future period will there be entry). Since the profit of an incumbent is higher than that of an entrant with the same F, it follows that if a non-zero measure of firms with $F = \underline{F}$ enter each period then no incumbent with $F = \underline{F}$ ever exits. Furthermore, the market shares of these incumbents grows in every period (Prop. 1.2). This cannot continue *ad infinitum*, however, because eventually the total market shares of the non-exiting incumbents and the entrants will account for the whole market, and further share growth will not be possible with some of them exiting. Without assuming a mass point at $\underline{F}$, one can only show that entry ceases in the limit (i.e., that $\lim_{t\to\infty} \mu(F_t^t) = 0$)

Proof of Prop. 1.9. Because there is exit each period (Prop. 1.7) and entry eventually ceases (Prop. 1.8), the proposition follows directly.

A.2 Proof of Proposition 2

Proof of Prop. 2.1. Consider a change in marginal cost first. In equilibrium demand equals supply, which is

$$f_t(p_t^*) = \sum_{k=1}^{t} \int_{\underline{F}}^{F_t^k} q(i)_t^{k*} d\mu \tag{9}$$

Total differentiation yields

$$f_t'(p_t)dp = \sum_{k=1}^{t} \left[\left(\frac{\partial F_t^k}{\partial c} q_t^k \mu'(F_t^k) + \int_{\underline{F}}^{F_t^k} \frac{\partial q_t^k}{\partial c} d\mu \right) dc + \left(\frac{\partial F_t^k}{\partial p} q_t^k \mu'(F_t^k) + \int_{\underline{F}}^{F_t^k} \frac{\partial q_t^k}{\partial p} d\mu \right) dp \right] \tag{10}$$

where asterisks are left out of the notation. The first term in each set of parentheses is the supply change coming from the marginal firms, and the second term is the supply change from the inframarginal firms. Solving for dp/dc yields:

$$\frac{dp}{dc} = \frac{\sum_{k=1}^{t} \left(\frac{\partial F_t^k}{\partial c} q_t^k \mu'(F_t^k) + \int_{\underline{F}}^{F_t^k} \frac{\partial q_t^k}{\partial c} d\mu \right)}{f_t'(p_t) - \sum_{k=1}^{t} \left(\frac{\partial F_t^k}{\partial p} q_t^k \mu'(F_t^k) + \int_{\underline{F}}^{F_t^k} \frac{\partial q_t^k}{\partial p} d\mu \right)} \tag{11}$$

The numerator is negative, because $\partial F_t^k/\partial c = \partial V_t^k/\partial c$ from (5), which is $\partial[q_t^k\,(p_t - c) - m(\Delta q_t)]/\partial c = -q_t^k < 0$ by the envelope theorem, q_t^k and μ' are positive, and $\partial q_t^k/\partial c = \partial\left[s_{t-1}^k Q_t + \Delta q_t\right]/\partial c = \partial \Delta q_t/\partial c = -m''(\Delta q_t)^{-1} < 0$ by assumption. The last equality follows from differentiating (3) with respect to c. The denominator is also negative. Letting $Q_t' \equiv f_t'(p_t)$, $N_t^k \equiv \int_{\underline{F}}^{F_t^k} d\mu$, and using the facts that $\partial F_t^k/\partial p = \partial V_t^k/\partial p = q_t^k + s_{t-1}^k Q_t'(p_t - c)$ and $\partial q_t^k/\partial p = s_{t-1}^k Q_t' + m''(\Delta q_t)^{-1}$, the denominator can be written

$$D = Q_t' - \sum_{k=1}^{t} \left[q_t^k + s_{t-1}^k Q_t'(p_t - c) \right] q_t^k \mu'(F_t^k) - \sum_{k=1}^{t} N_t^k \left[s_{t-1}^k Q_t' + m''(\Delta q_t)^{-1} \right] \tag{12}$$

Assumption 1 assures that the second term in D is negative, so it suffices to show that $Q_t' - \sum_{k=1}^{t} N_t^k \left[s_{t-1}^k Q_t' + m''(\Delta q_t)^{-1}\right] < 0$. The left side is $Q_t'\left[1 - \sum_{k=1}^{t} N_t^k s_{t-1}^k\right] - \sum_{k=1}^{t} N_t^k m''(\Delta q_t)^{-1}$ which is negative if $1 > \sum_{k=1}^{t} N_t^k s_{t-1}^k$, since $Q_t' < 0$ (demand slopes down). If no incumbents exit, then the right side, per-firm market shares last period weighted by number of firms in each cohort, would sum to 1. Since some incumbents exit by Prop. 1.7, the sum is indeed less than 1. Thus $D < 0$ and $dp/dc > 0$.

Now consider a change in fixed cost. Total differentiation of (9) yields an expression similar to (11) with ϕ replacing c wherever it appears. The numerator is negative, because $\partial F_t^k/\partial\phi = -1$ from (5) and $\partial q_t^k/\partial\phi = 0$ (changes in fixed cost affect marginal but not inframarginal firms' supply decisions). The denominator is again (12), which is negative, and thus $dp/d\phi > 0$. Since p_t^* rises in both cases, Q_t^* falls because demand slopes down.

Proofs of Props. 2.2–3. These are shown numerically; refer to the industry dynamics figures. Figure 3 shows that the number of entering firms can decrease and the number of incumbents can increase, while Figure 4 shows the opposite.

A.3 Proof of Proposition 3

Proof of Prop. 3.1. The proof proceeds by showing that when c increases, if N_t^k rises for any cohort, it rises for the youngest cohorts. Equations (6) and (8) imply that $N_t^k > 0$ when c rises iff

$$\frac{dp}{dc} > \left[\frac{s_{t-1}^k}{q_t^k}(p_t - c)\frac{dQ_t}{dp} + 1\right]^{-1} \tag{13}$$

Note first that assumption 1 ensures that the right side is positive. The only term varying with cohort entry period k is s_{t-1}^k/q_t^k, which can be written $\left(Q_t + \Delta q_t Q_{t-1}/q_{t-1}^k\right)^{-1}$ from (1) and $s_t^k = q_t^k/Q_t$. Propositions 1.2 and 1.6 imply that q_{t-1}^k decreases with k: younger cohorts produce less. Thus s_{t-1}^k/q_t^k decreases with k and therefore the right side of (13) also decreases with k. Therefore inequality (13) is easiest to satisfy for the youngest cohort. The incumbent favoring case, which requires that entry falls and the number of incumbents rises, is therefore not possible.

Proof of Prop. 3.2. The proposition is proved by showing that when assumptions 2 and 3 hold, entry must decrease. From the discussion of $dp/d\phi$ in the proof of Prop. 2.1 above it can be shown that (7) for entrants is

$$\frac{dN_t^t}{d\phi} = \left[\frac{-\Delta q_t \sum_{\ell=1}^t q_t^\ell \mu'(F_t^\ell)}{Q_t' - \sum_{k=1}^t \left(dV_t^k/dp\right) q_t^k \mu'(F_t^k) + N_t^k\left[s_{t-1}^k Q_t' + m''(\Delta q_t)^{-1}\right]} - 1\right]\mu'(F_t^t) \tag{14}$$

where $dV_t^k/dp = q_t^k + s_{t-1}^k Q_t'(p_t - c)$.Rearranging the right side shows that entry decreases if and only if

$$-\sum_{k=1}^{t}\left(s_{t-1}^k\left[Q_t+Q_t'(p_t-c)\right]\right)q_t^k\mu'(F_t^k)<-\sum_{k=1}^{t}Q_t'\left[1-\sum_{k=1}^{t}N_t^k s_{t-1}^k\right]+\sum_{k=1}^{t}\frac{N_t^k}{m''} \tag{15}$$

The left side is negative because assumption 3 ensures that $Q_t + Q_t'(p_t - c)$ is positive. Both terms on the right side are positive, the first because the total market share at $t-1$ of the firms remaining in the market at t must be less than one and the second because m is convex. Thus (14) is negative and entry must decrease.

To justify the slope of the price effect in the figures, we also show here that if N_t^k rises for any cohort it rises for the oldest cohorts. Under assumption 2, μ' is constant and the only term in (7) that varies with k is $A_t^k \equiv dV_t^k/dP = q_t^k + Q_t'(p_t - c)s_{t-1}^k$ in the price effect. To show that A_t^k increases with cohort age, for $k < t$ write $A_t^k - A_t^{k+1}$ as

$$-\Delta_k A_t^k = -\Delta_k q_t^k - Q_t'(p_t - c)\Delta_k - \Delta_k q_{t-1}^k/Q_{t-1} \tag{16}$$

where is the Δ_k forward difference operator in k and definition $s_t^k = q_t^k/Q_t$ is applied. From (1) it is seen that $q_{t-1}^k = (q_t^k - \Delta q_t)Q_{t-1}/Q_t$, and (16) is

$$-\Delta_k A_t^k = -\Delta_k q_t^k\left[1 - \varepsilon_t(p_t - c)/p_t\right] \tag{17}$$

Because $-\Delta_k q_t^k > 0$ (q_{t-1}^k decreases with k), (17) has the sign of the bracketed term. By assumption 3, $A_t^k - A_t^{k+1}$ is therefore positive and A_t^k increases with cohort age. Thus if N_t^k rises for any cohort it is for the oldest cohorts, and the price effect is as pictured in Figures (8)–(9).

A.4 Other results referred to in the text

Lemma 5 *When fixed costs rise, the scale of entry Δq_t increases.*

To see this, totally differentiate (3) and solve to yield

$$\frac{d\Delta q_t}{d\phi} = \frac{1}{m''(\Delta q_t)}\frac{dp_t}{d\phi} \tag{18}$$

Since m is convex and Prop. 2.1 shows that $dp/d\phi > 0$, the scale of entry must increase.

Lemma 6 *When marginal costs rise, entry increases if and only if the scale of entry Δq_t also increases.*

When marginal cost increases, the number of entrants and the scale of entry both move with the margin $p_t - c$. Solving for $d\Delta q_t/dc$ as in (18) shows that

$$\frac{d\Delta q_t}{dc} = \frac{1}{m''(\Delta q_t)}\left(\frac{dp_t}{dc} - 1\right) \tag{19}$$

and thus Δq_t increases if and only if $dp_t/dc > 1$, which is to say that the margin rises. However, this is also the condition that is equivalent to the entry favoring case. From (6) for entrants we have

$$\frac{dN_t^{t*}}{dc} = \left[\Delta q_t \frac{dp_t^*}{dc} - \Delta q_t\right]\mu'(F_t^t) \tag{20}$$

which has the sign of $dp_t/dc - 1$, the same as for (19).

Lemma 7 *Assumption 1 is equivalent to assuming that*

$$\varepsilon_t(p_t - c)/p_t < 1 + \Delta q_t/(\Delta q_t + q_t^1), \tag{21}$$

and assumption 3 is stronger than assumption 1.

Since fixed cost is not affected by p, assumption 1 is equivalent to assuming that $dV_t^k/dp > 0$ for all t and $k \leq t$. Equation (8) implies that dV_t^k/dp has the sign of

$$\left[1 - \varepsilon_t \frac{(p_t - c)}{p_t}\right] + \frac{\Delta q_t}{s_{t-1}^k Q_t} \tag{22}$$

for $k < t$. The last term can be written $\Delta q_t/(\Delta q_t + q_t^k)$, which is smallest for $k = 1$, the oldest incumbents. If (22) is positive for $k = 1$, then it is also for all k. Thus assumption 1 is equivalent to (21). Since the last term in (22) is positive, and assumption 3 involves only the bracketed term in (22), assumption 3 is stronger than assumption 1.

Lemma 8 *When marginal cost rises, exit increases for the oldest cohort.*

Nomenclature	Entry of New Firms	Exit of Incumbents
competitor neutral	decreases	increases
incumbent favoring	decreases	decreases
entrant favoring	increases	increases

Table 1: Possible Changes in Entry and Exit in Response to cost increases

From (7) it suffices to show that (13) holds with $k = 1$ and the direction of the inequality reversed. Substituting (11) for dp/dc and simplifying as in that section leads to the condition to be shown:

$$\left[\sum_{k=1}^{t}\left(\left(\frac{s_{t-1}^{k}}{q_t^k}-\frac{s_{t-1}^{1}}{q_t^1}\right)(p_t-c)\left(q_t^k\right)^2\mu'\right)-\frac{s_{t-1}^{1}}{q_t^1}(p_t-c)\sum_{k=1}^{t}\frac{N_t^k}{m''}\right]<1-\sum_{k=1}^{t}N_t^k s_{t-1}^k \tag{23}$$

The right side is positive as discussed for (15). The second term on the left is negative because m'' is convex. Thus if $s_{t-1}^k/q_t^k - s_{t-1}^1/q_t^1 < 0$ the lemma is proved. Using $s_t = q_t/Q_t$, s_{t-1}^k/q_t^k can be written $\left[\left(\Delta q_t/q_{t-1}^k + Q_t/Q_{t-1}\right)Q_t\right]^{-1}$, which varies with k only through q_{t-1}^k. Since q_{t-1}^k decreases with k, the whole expression does, too, and therefore $s_{t-1}^k/q_t^k - s_{t-1}^1/q_t^1 < 0$.

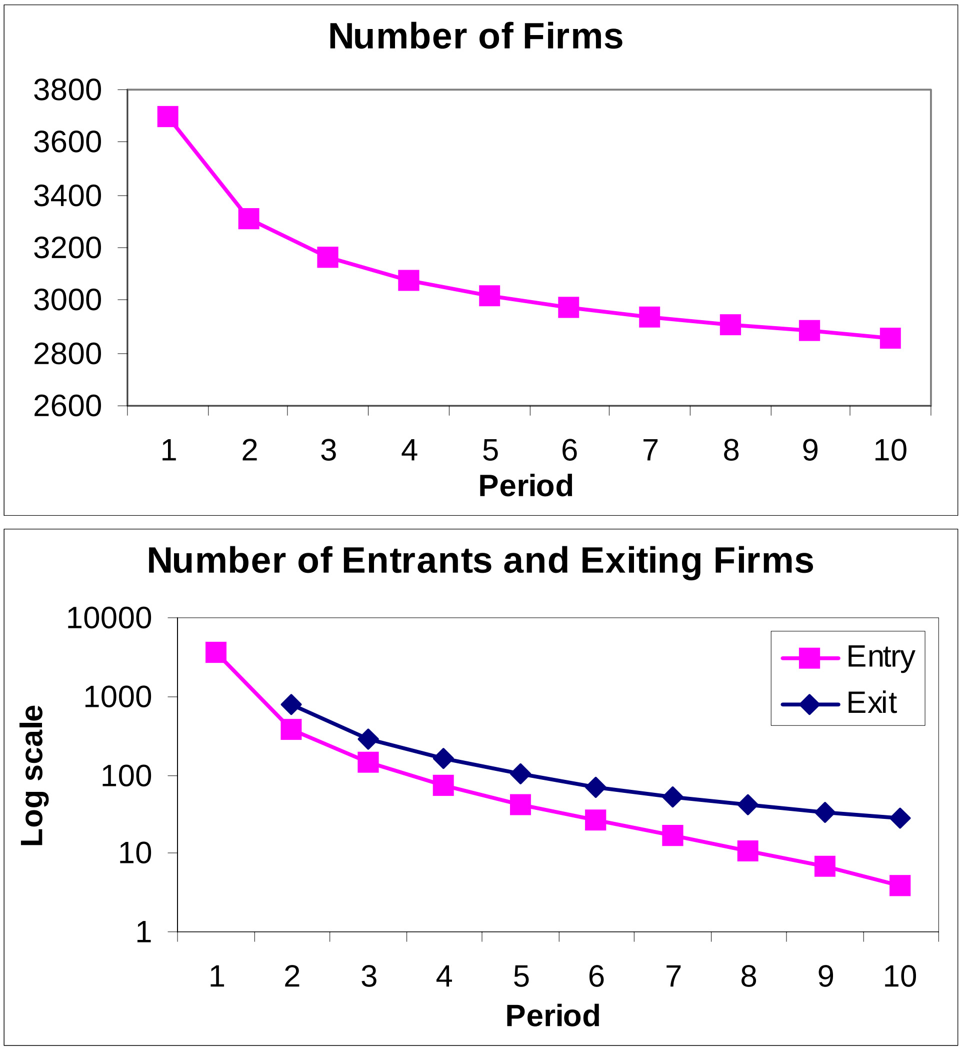

Figure 1: Industry Dynamics for a Baseline Case

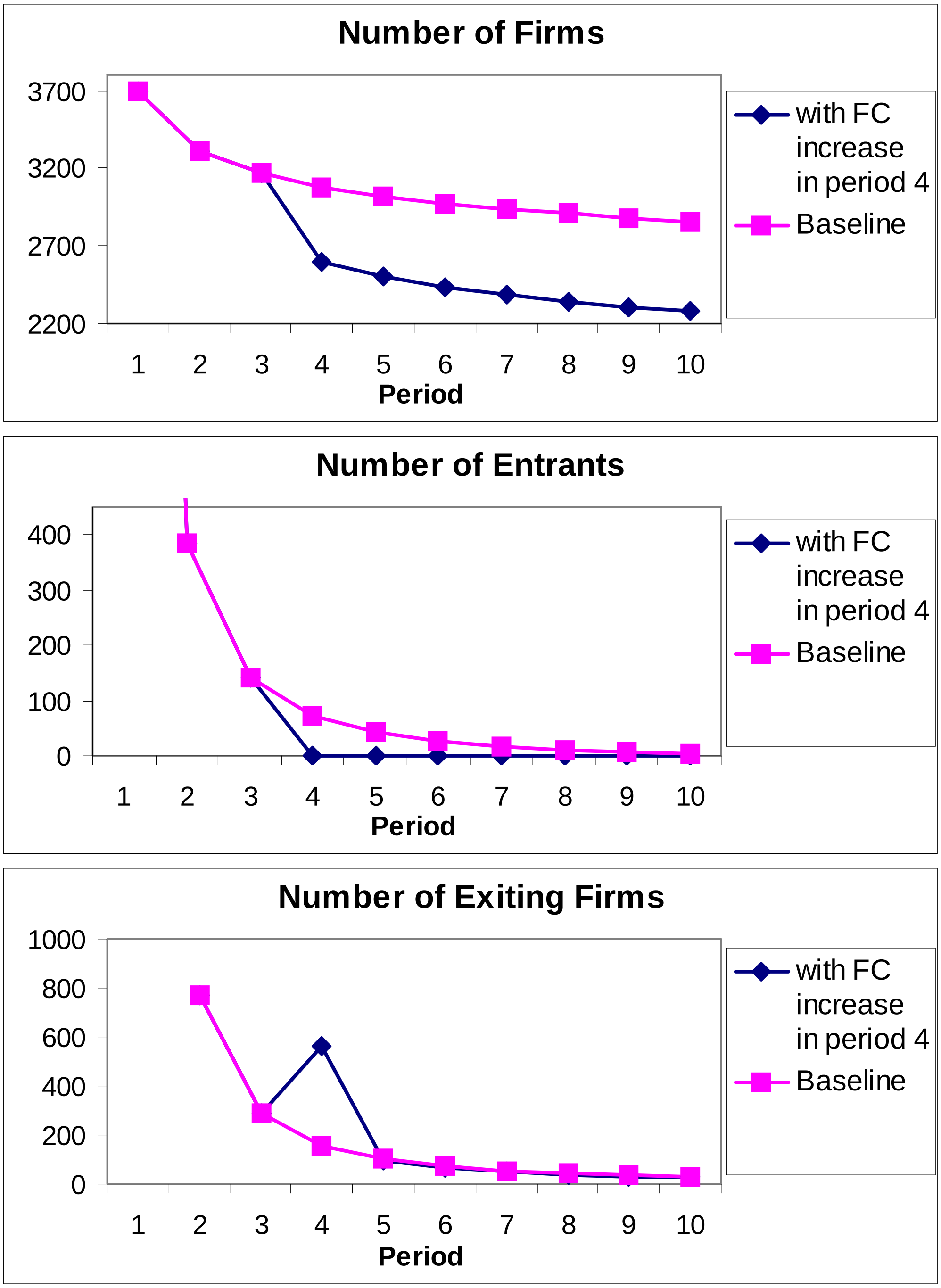

Figure 2: The Competitor Neutral Case Arising from an Increase in Fixed Cost

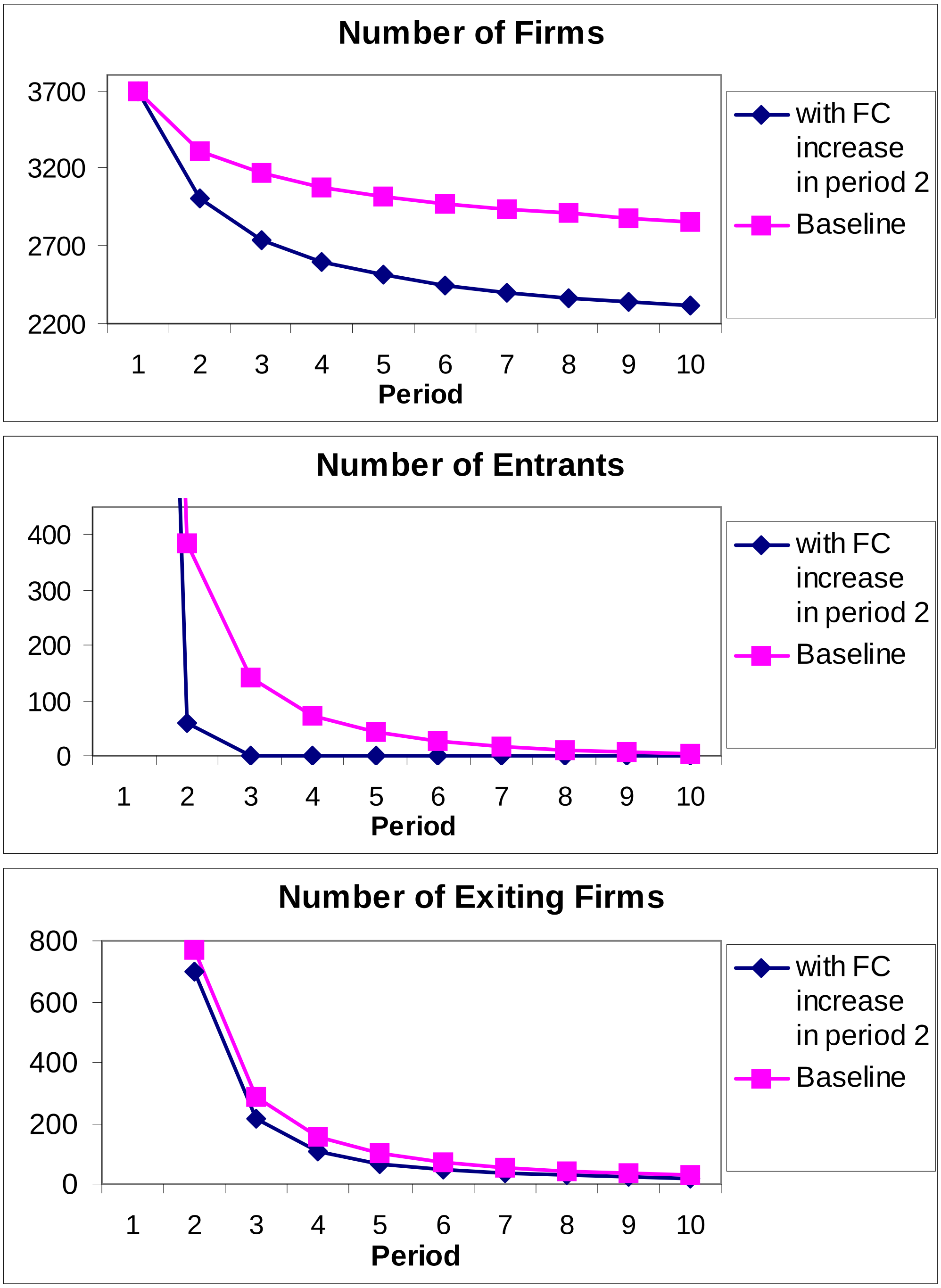

Figure 3: The Incumbent Favoring Case Arising from an Increase in Fixed Cost

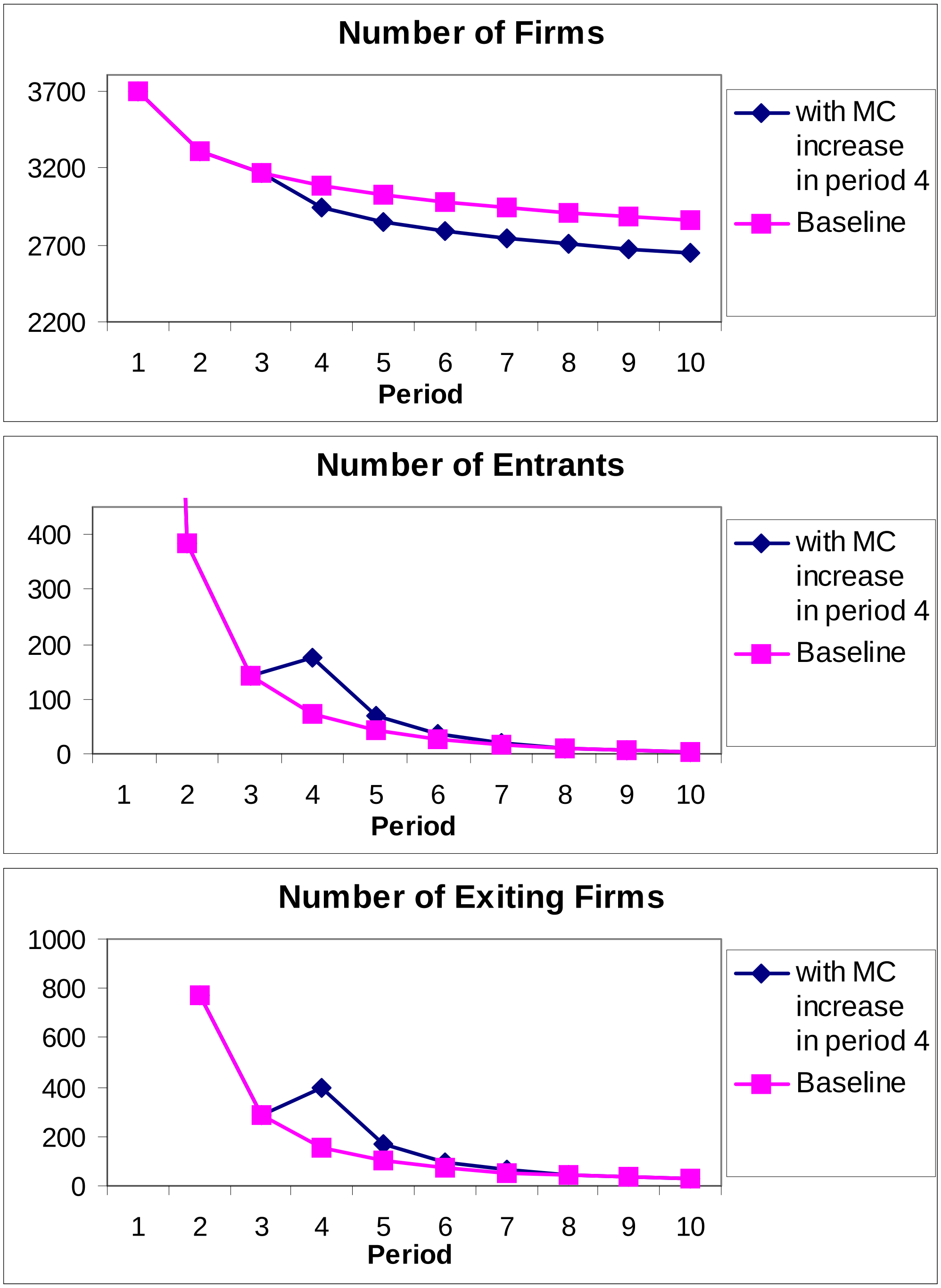

Figure 4: The Entrant Favoring Case Arising from an Increase in Marginal Cost

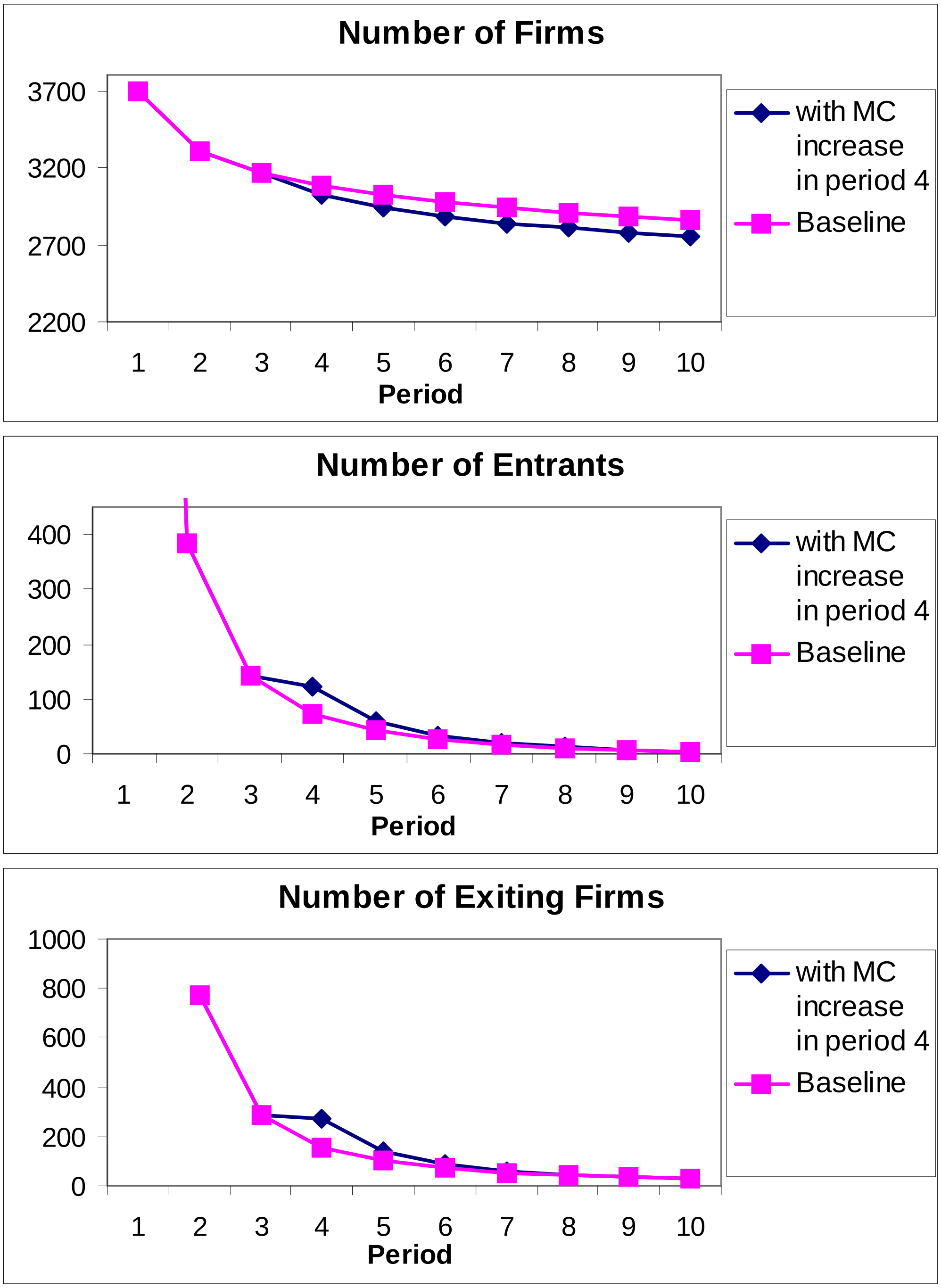

Figure 5: Another Entrant Favoring Case Arising from an Increase in Marginal Cost

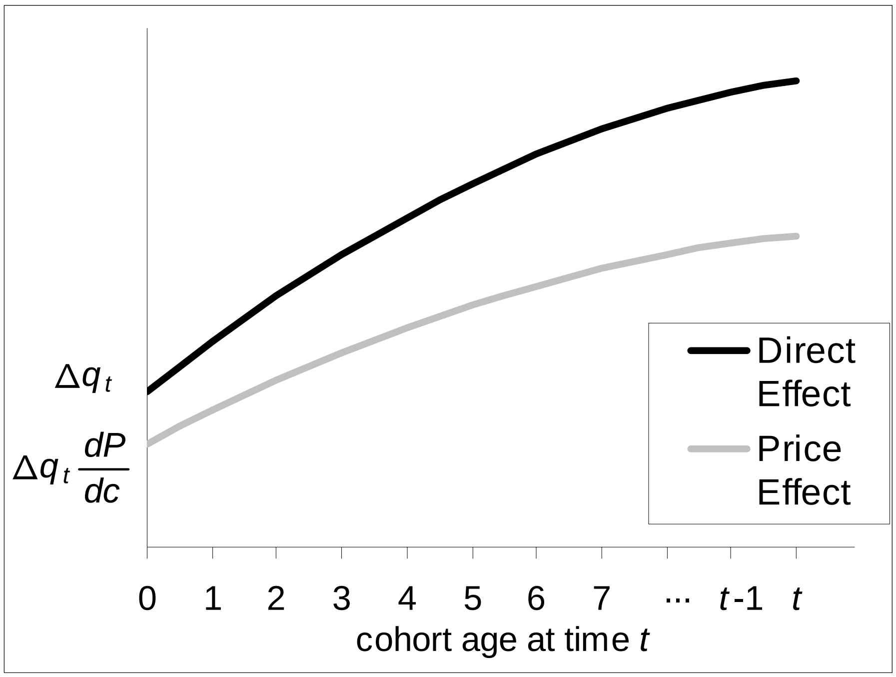

Figure 6: The Competitor Neutral Case Resulting from an Increase in Marginal Cost

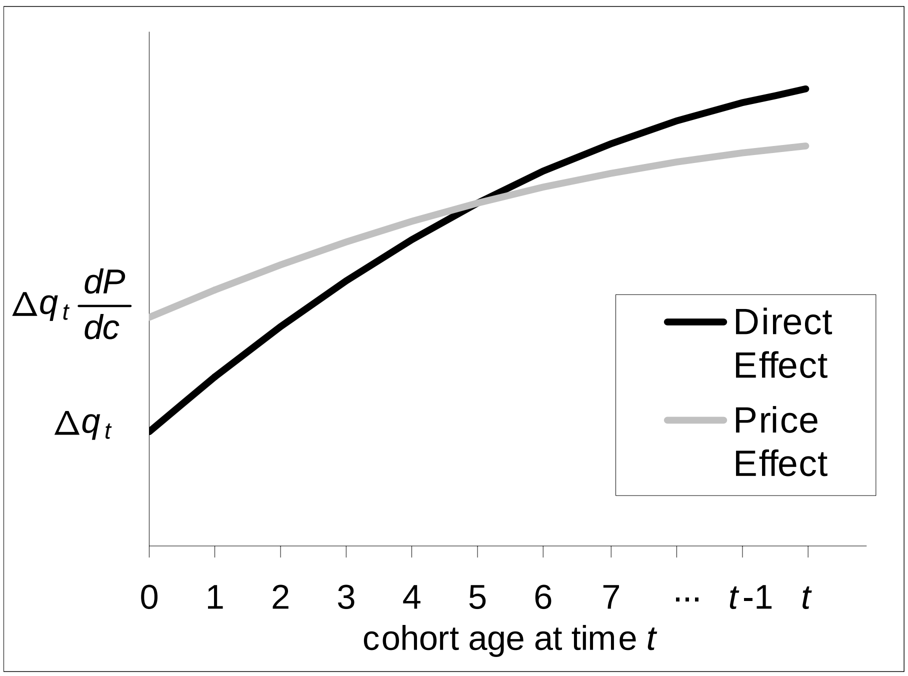

Figure 7: Entrant Favoring Resulting from an Increase in Marginal Cost

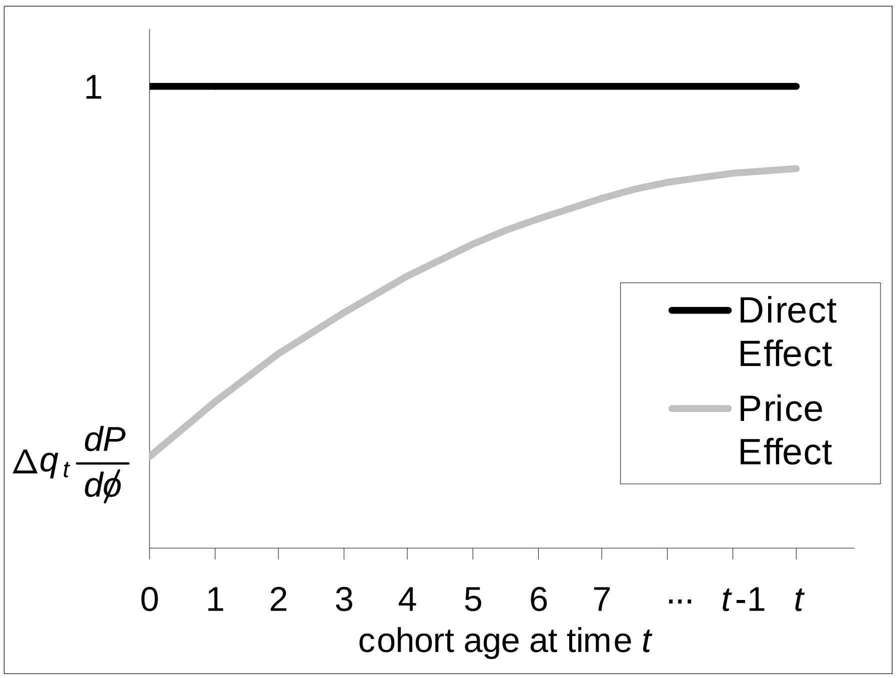

Figure 8: The Competitor Neutral Case Resulting from an Increase in Fixed Cost

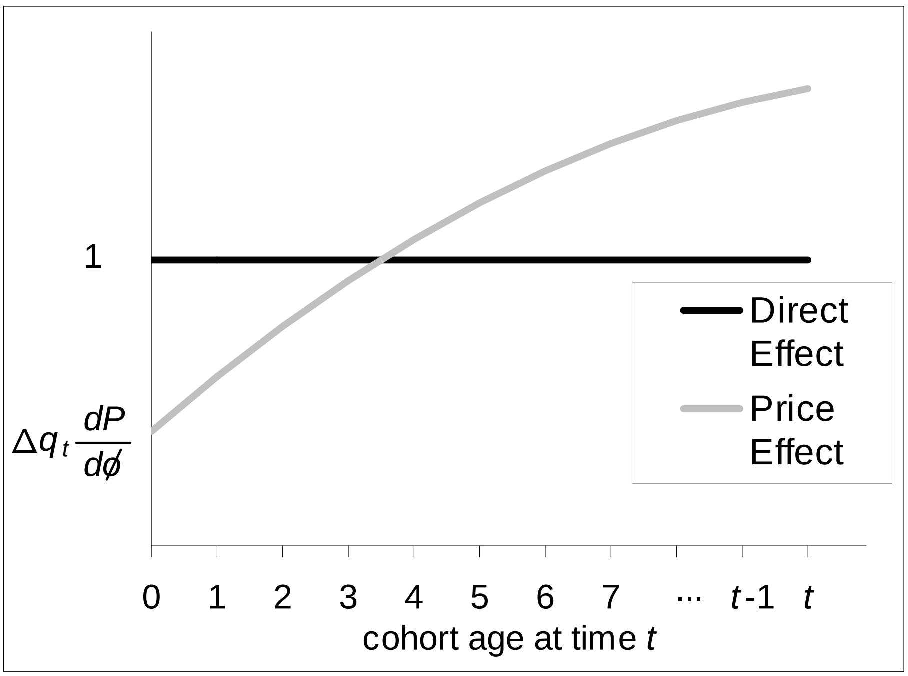

Figure 9: Incumbent Favoring Resulting from an Increase in Fixed Cost