NORMATIVE PROPERTIES OF STOCK MARKET EQUILIBRIUM WITH MORAL HAZARD

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Normative Properties of Stock Market Equilibrium with Moral Hazard

Abstract

This paper presents a model of stock market equilibrium with a finite number of corporations and studies its normative properties. Each firm is run by a manager whose effort is unobservable and influences the probabilities of the firm's outcomes. The Board of Directors of each firm chooses an incentive contract for the manager which maximizes the firm's market value. With a finite number of firms, the equilibrium is constrained Pareto optimal only when investors are risk neutral and firms' outcomes are independent. The inefficiencies which arise when investors are risk averse, or when firms are influenced by a common shock, are studied and it is shown that under reasonable assumptions there is under investment in effort in equilibrium. The inefficiencies exist when the firms are not completely negligible, as is typical of the large corporations with dispersed ownership traded on public exchanges in the US. In the idealized case where firms of each type are replicated and replaced by a continuum of firms of each type with independent outcomes, the inefficiencies disappear.

1. Introduction

Economies with incomplete markets have been the focus of much study in general equilibrium theory during the last twenty-five years. Since asymmetry of information is one of the main sources of incompleteness of markets, there has recently been considerable interest in incorporating moral hazard and adverse selection into general equilibrium models, in particular into models with financial markets.[1] The focus of this paper is on the problem of moral hazard. In most general equilibrium models with moral hazard, beginning with Prescott-Townsend (1984 a,b) it is assumed that there is a continuum of agents of each type who are subject to independent shocks. Since in the real world, on any finite number of time periods, there is only a finite number of agents, a model with a continuum of agents must be interpreted as an approximation for a large but finite economy. The formal framework for showing that a continuum economy is the limit of finite economies has been presented by Hildenbrand (1974), and similar arguments justify studying economies with asymmetric information and a continuum of agents as the limit of finite economies.

In traditional general equilibrium theory, economies with a continuum of agents and finite economies have the same properties, except that the assumptions of convexity needed to obtain continuity of supplies and demands in finite economies—and hence existence of an equilibrium—are not needed with a continuum of agents: the convexifying effect of large numbers, expressed by the Lyapounov theorem, replaces the continuity of individual reaction functions. Otherwise continuum economies have the same normative properties as the finite economies which they approximate: as long agents (consumers and firms) are assumed to be price takers[2] a competitive equilibrium is Pareto optimal.

One role of the continuum in moral hazard economies is to solve the non-convexity problem created by the presence of incentive constraints in agents' budget sets so as to obtain existence of equilibrium. It might be thought that, as in traditional equilibrium theory, the assumption of the continuum plays no role in establishing the efficiency properties of an equilibrium. However this paper presents a class of general equilibrium models with moral hazard for which finite economies and continuum limit economies do not have the same normative properties: we study a model with a finite number of firms, each offering an incentive contract to its manager, and show that except under restrictive assumptions, the equilibrium is not constrained efficient. However when the firms are replicated and in the limit there is a continuum of firms of each type, the inefficiencies

[1] Kocherlakota (1998), Bisin-Gottardi (1999), Lisboa (2001), Magill-Quinzii (2002), Dubey-Geanakoplos (2002), Dubey-Geanakoplos-Shubik (2005), Acharya-Bisin (2005).

[2] The price taking assumption is precisely justified only in the continuum where agents are negligible: however the competitive analysis is useful only if the price taking behavior is a good approximation for large finite economies.

disappear. Thus the result of constrained efficiency typically obtained in equilibrium models with moral hazard (Prescott-Townsend (1984 a,b) and most of the papers in footnote 1) may depend crucially on the assumption that there is a continuum of agents of each type.

The firms that we have in mind are corporations: the characteristic feature of the corporate form of organization is that ownership is divided among a large number of shareholders, and control is vested in professional managers. The separation of ownership and control implied by the corporate form leads to the agency problem induced by the potential divergence of interests between managers and shareholders.[3] The principal-agent model is a useful way of formalizing the conflict of interest between managers and shareholders, and leads to the idea that CEOs of large corporations should be offered incentive contracts to align their interests with those of the shareholders. However, given that the CEO of a corporation is the "agent" of many principals, aligning the interests of CEOs with those of shareholders may create general equilibrium effects which are not apparent in the standard bilateral principal-agent model.

To study this problem we consider a two-period economy with two groups of agents, I investors (or shareholders) and K managers of K firms, in which managerial effort is not observable and influences the probabilities of the firms' outcomes. The assignment of managers to firms is taken as given. At date 0 there is trade on the financial markets and the the Board of Directors of each firm offers an incentive contract to the firm's manager. We make two simplifying assumptions: first the financial markets are complete relative to the possible outcomes of firms, and second, managers cannot undo the incentive contracts they are offered by trading on the financial markets. Moreover, to capture the constantly changing and widely dispersed ownership by the shareholders, we assume that the Board of Directors does not know the specific preferences of the shareholders, only that they are risk-averse and prefer more income to less from their ownership of the firm. As a result we assume that the Board of Directors chooses a contract for the firm's manager which maximizes the market value of the firm, which, as we shall see, is well defined in our setting.

This leads to a concept of equilibrium in which investors trade on the financial markets, choosing their holdings of equity shares in the firms, and managers are offered incentive contracts which maximize the market values of their firms. We study the normative properties of the equilibria of this model, and find that the conditions under which market-value maximization leads to constrained Pareto optimality are restrictive: investors must be risk neutral and firms' outcomes must be independent. Thus under the assumptions which best reflect the stylized facts about

[3] That the agency problem is inherent in the corporate form was the central thesis of the classic work of Berle and Means (1932).

equity markets—risk-averse investors and correlated outcomes of firms—the equilibrium levels of managerial effort are not socially optimal.

To clarify the sources of inefficiency we decompose the study of the model into two cases. In the first, investors are risk averse but firms' outcomes are independent; in the second, investors are risk neutral and firms' outcomes are affected by a common unobservable shock. The first source of inefficiency, linked to the risk aversion of the shareholders, comes from the fact that in the principal-agent model managerial effort affects the probabilities of firms' outcomes. When investors trade on the financial markets they evaluate the probabilities of outcomes—correctly under the assumption of rational expectations—and this evaluation influences security prices. But effort shifts probabilities across outcomes, and security prices do not accurately signal the value of such a shifting of probabilities: rather they provide a well-defined value for income in each outcome state, expressed by the stochastic discount factor which is used by the firms to maximize the present value of profit. We show that under these circumstances maximizing a weighted sum of expected utilities of the investors (what a planner does) and maximizing the present value of the firms' profits (what the equilibrium does) in general give different results, leading to under-provision of effort at the equilibrium.

The second source of inefficiency comes from the way an optimal contract makes use of available information. Although firms' outcomes are conditionally independent so that there is no direct externality, the optimal contract for a manager uses the information contained in the outcomes of other firms to infer how much of the manager's outcome can be attributed to the common shock and how much is attributable to the manager's effort, and it is this use of information in an optimal contract which induces an externality between the actions of the firms' managers.

In the last section of the paper we show that both sources of inefficiency disappear when firms are replicated and in the limit replaced by a continuum of identical firms of each type with independent, or conditionally independent, outcomes. Thus the inefficiency arises from the fact that firms are not completely negligible in the finite model.[4] The corporations whose shares are traded on public exchanges in the US are relatively small in number but contribute a significant share of aggregate output: our analysis highlights the potential inefficiencies created by such corporations. It would certainly be of interest to assess the magnitude of the inefficiency and how rapidly it increases as the economy departs from the continuum limit where all firms are negligible, but this is outside the scope of this paper.

[4]In Section 2 we argue that even if firms are not negligble the competitive assumptions embedded in the equilibrium concept are reasonable approximations provided the ownership of firms is sufficiently diffused and the compensation of a manager is only a small fraction of each firm's gross profit.

2. Stock Market Equilibrium and Constrained Pareto Optimality

The Model. Consider a one-good, two-period economy in which there are two groups of agents, I investors and K managers, and a collection of K firms, each run by one of the managers. The match between managers and firms is taken as given and, as in the standard principal agent model, we assume that manager k has an exogenously given outside option yielding a utility level ν_k. For each firm there is a finite number of possible outcomes and the probability of these outcomes is influenced by the entrepreneurial effort of its manager. Let $(y^k_{s_k})_{s_k \in S_k}$ denote the finite number S_k of possible outcomes[5] for firm k, where the outcomes are indexed in increasing order; that is $s_k > s'_k$ implies $y^k_{s_k} > y^k_{s'_k}$. An outcome for the economy at date 1 is a K-uple $s = (s_1, \ldots, s_K)$ describing the realized output (or profit) of each firm: we let $S = S_1 \times \ldots \times S_K$ denote the outcome space, $y_s = (y^1_{s_1}, \ldots, y^K_{s_K})$, $s \in S$, denoting the vector of outputs of the K firms in outcome s. The effort e_k of manager k influences the probabilities of the outcomes of firm k: $e_k \in \mathbb{R}_+$ is assumed to be unobservable. To permit common as well as idiosyncratic shocks to influence the outcomes of the firms let $p(s, e) = p(s_1, \ldots, s_K, e_1, \ldots, e_K)$ denote the joint probability of the outcomes, given the effort levels $e = (e_1, \ldots, e_K)$ chosen by the managers. The function p is assumed to be common knowledge for the agents in the economy. When we need to focus on a typical firm k, it will be convenient to use the notation $s = (s_k, s^{-k})$ and $e = (e_k, e^{-k})$, where $s^{-k} = (s_1, \ldots, s_{k-1}, s_{k+1}, \ldots, s_K)$ and e^{-k} is defined in the same way.

All agents in the economy, investors and managers, are assumed to have expected-utility preferences over date 1 consumption streams—at date 0 agents trade on financial markets and write contracts, but there is no date 0 consumption. Let $u_i : X_i \to \mathbb{R}$ denote the concave, increasing, VNM utility function of investor i, $i \in I$. When investors are risk averse $X_i = \mathbb{R}_+$, and when investors are risk neutral $X_i = \mathbb{R}$, since non-negativity constraints with linear utilities re-create risk aversion through the multiplier at zero consumption. Let $v_k : \mathbb{R}_+ \to \mathbb{R}$ be the concave, increasing VNM utility function of manager k, $k \in K$. The disutility of effort is assumed to enter additively[6] and is expressed by a convex, increasing cost function $c_k : \mathbb{R}_+ \to \mathbb{R}$.

Stock Market Equilibrium with Fixed Contracts. Investors trade on security markets, where the securities consist of contracts whose payoffs depend on the observable profits of the firms, securities such as equity, bonds, options on equity, or options on indices of equity contracts. Let J

[5] To economize on notation we use the same notation for the number of elements in a set and for the set itself.

[6] This assumption simplifies the analysis of the optimal contract in the principal-agent model but is not innocuous: see Bennardo-Chiappori (2003) and Panaccione (2005).

denote the set of securities, and let V_s^j, $s \in S$, $j \in J$, denote the payoff of security j in outcome s. If τ_s^k denotes the compensation paid to the manager of firm k in outcome s, then the vector of net profits for the K firms is

$$y_s - \tau_s = (y_s^1 - \tau_s^1 \ldots, y_s^K - \tau_s^K), \quad s \in S$$

The payoff of each security j is some function $\phi^j : \mathbb{R}^K \to \mathbb{R}$ of the observable vector of net profits of the K firms, $V_s^j = \phi^j(y_s - \tau_s), s \in S$. The first K contracts are the equity of the firms: $V_s^k = y_{s_k}^k - \tau_s^k$, $1 \leq k \leq K$. The firms' outcomes, indexed by $s \in S$, thus constitute the state space for the investors, and we assume that the security structure is sufficiently rich for the financial markets to be *complete* with respect to the outcome space S: thus if $V = [V_s^j, s \in S, j \in J\,]$ denotes the matrix of date 1 payoffs of the securities

$$\text{rank}\,(V) = S$$

Note that by making the payment to manager k depend on the entire vector of profits of all firms, summarized in s, we are assuming that the class of contracts in which the Board of Directors (BOD) of firm k chooses the contract of the manager includes all possible forms of relative performance compensation.

Firms are owned by the investors: $\delta_k^i \geq 0$ denotes agent i's initial ownership share of firm k, and the shares are normalized so that $\sum_{i \in I} \delta_k^i = 1, k \in K$. The investors have no initial holdings of the remaining securities which are in zero net supply: let $\delta^i = (\delta_1^i, \ldots, \delta_K^i, 0, \ldots, 0)$ denote investor i's vector of initial holdings of the securities. To facilitate the description of equilibrium, we first define an equilibrium assuming that the compensation of the managers and their effort levels $(\tau, e) = (\tau^k, e^k, k \in K)$ have been chosen. We then define the equilibrium concept that is studied in this paper, in which the portfolios of the investors and the contracts of the managers are chosen simultaneously (Definition 2).

Let $p_s, s \in S$, be the probabilities of the outcomes anticipated by the investors when they trade securities, let $q = (q_j)_{j \in J}$ denote the vector of security prices, and let $(x^i, z^i) = (x_s^i, s \in S, z_j^i, j \in J)$ denote the vector of consumption and the (new) portfolio of securities chosen by investor i. The date 0 and date 1 budget equations

$$q(z^i - \delta^i) \leq 0, \quad x^i = V z^i, \quad z^i \in \mathbb{R}^J \tag{1}$$

define the consumption streams that an investor can attain by trading on the financial markets: the portfolio z^i must finance the consumption stream x^i of the investor.

Definition 1. A *stock market equilibrium with fixed contracts* (τ, e) is a pair $((\bar{x}, \bar{z}), \bar{q}) = ((\bar{x}^i, \bar{z}^i, i \in I), (\bar{q}^j, j \in J))$, consisting of actions by investors and security prices such that

(i) $\bar{x}^i \in \arg\max \left\{ \sum_{s \in S} p_s u_i(x_s^i) \;\middle|\; \bar{q}(z^i - \delta^i) \leq 0,\; x^i = V z^i,\; z^i \in \mathbb{R}^J \right\}$ with $\bar{x}^i = V\bar{z}^i$ for all $i \in I$.

(ii) $\sum_{i \in I} \bar{z}_k^i = 1,\; k \in K, \quad \sum_{i \in I} \bar{z}_j^i = 0, \quad j \in J,\; j > K.$

(iii) $V = \phi(y - \tau), \quad p_s = p(s, e),\; s \in S.$

Remark 1: A stock market equilibrium with fixed contracts assumes competitive financial markets—in (i) investors are price takers—and correct anticipations: in (iii) investors take as given the contracts τ proposed by the firms and correctly infer the vector of effort e that they induce from the managers. For the competitive assumption to be a good approximation, each investor should be "small": the initial ownership of the firms must be dispersed among many small shareholders, i.e. I must be sufficiently large and δ_k^i sufficiently small.

If $\bar{\lambda}^i = (\bar{\lambda}_0^i, \bar{\lambda}_1^i, \ldots, \bar{\lambda}_S^i)$ denotes the vector of multipliers for the $S+1$ constraints of investor i's maximum problem in (i), then the first-order conditions (FOC) in a stock market equilibrium are given by

$$p_s u_i'(\bar{x}_s^i) = \bar{\lambda}_s^i,\; s \in S, \quad \bar{\lambda}_0^i \bar{q}_j = \sum_{s \in S} \bar{\lambda}_s^i V_s^j,\; j \in J, \quad i \in I \tag{2}$$

If we let $\hat{\pi}^i = (\bar{\lambda}_s^i / \bar{\lambda}_0^i,\; s \in S)$ denote the vector of present values of income of investor i, then the second equation in (2) can be written as $\hat{\pi}^i V = \bar{q}$. Since the financial markets are complete with respect to the outcome states (rank $(V) = S$), given $\bar{q}$ and V there is a unique vector $\hat{\pi} \gg 0$ satisfying the equation $\hat{\pi} V = \bar{q}$, so that $\hat{\pi}^i = \hat{\pi}, i \in I$. The common vector

$$\bar{\pi} = (\bar{\pi}_s,\; s \in S) = \left(\frac{u_i'(\bar{x}_s^i)}{\bar{\lambda}_0^i},\; s \in S \right),\; i \in I \tag{3}$$

which factors out the probabilities of the outcome states from the vector of present values $\hat{\pi}$, i.e.

$$\hat{\pi}_s = \bar{\pi}_s p_s, \quad s \in S \tag{4}$$

is called the (common) *stochastic discount factor* of the investors at the stock market equilibrium. This decomposition of each present-value price into a product of a probability and a stochastic discount factor will play an important role in the analysis that follows.

The present-value vector $\hat{\pi}$ implied by the valuation $\bar{q}$ of the securities—which asserts that the price of security j is the present value of its future dividend stream—can be used to evaluate the present value of any income stream. In particular investor i's budget equations (1) can be written in present-value form, since $\hat{\pi}V = \bar{q}$ implies

$$\hat{\pi}x^i = \hat{\pi}Vz^i = \bar{q}z^i \leq \bar{q}\delta^i = \sum_{k \in K} \delta^i_k \hat{\pi}(y^k - \tau^k)$$

so that the present value of investor i's consumption must not exceed the present value of his initial holdings of firms. When this constraint is satisfied any income stream x^i can be created by appropriately trading on the security markets since rank $(V) = S$. The budget set, written in sequential form in (i) of Definition 1, can be written in the equivalent present-value form

$$\left\{ x^i \in X_i, \,|\, \hat{\pi}x^i \leq \sum_{k \in K} \delta^i_k \hat{\pi}(y^k - \tau^k) \right\} \tag{5}$$

Thus an equivalent and more condensed way of representing a stock market equilibrium when financial markets are complete is obtained by assuming that investors directly purchase income streams, the unit price of income in outcome s being $\hat{\pi}_s$. Since the matrix V is invertible and the equation $\hat{\pi}V = \bar{q}$ is satisfied at a stock market equilibrium, any asset price $\bar{q}$ defines a vector of present-value prices $\hat{\pi}$ and conversely. The market-clearing equations (ii) on the security markets are equivalent to the market-clearing equations

$$\sum_{i=1}^{I} \bar{x}^i = \sum_{k=1}^{K} (y^k - \tau^k) \tag{6}$$

on the markets for the good (income). It will sometimes be convenient to refer to the pair $(\bar{x}, \hat{\pi})$, where each investor chooses $\bar{x}^i$ to maximize expected utility over the budget set (5) and markets clear as expressed in (6), as a *reduced-form stock market equilibrium*.

Stock Market Equilibrium. In the above concept of equilibrium, whether expressed in terms of the asset prices $\bar{q}$ or of the present-value prices $\hat{\pi}$, we assumed that the contracts (τ, e) offered to the managers were fixed: we now explain how these contracts come to be determined. The assumption that the stock market is competitive requires that ownership be diffused among a large number of shareholders, each with small ownership shares. This, combined with the fact that shares are exchanged at date 0 among investors, suggests that it would be too costly for the Board of Directors to elicit precise information on the preferences of the shareholders. What shareholders receive from the firm is a share of the net profit $(y^k_s - \tau^k_s, s \in S)$: it seems natural, in the absence of specific information on the preferences of the shareholders, that the BOD would seek to maximize the present value of this profit $\hat{\pi}_s(y_s^k - \tau_s^k)$ or, what is equivalent, the market value $\bar{q}_k$ of the firm.

As indicated in (4) the present-value prices can be decomposed into a product $\hat{\pi}_s = \bar{\pi}_s p(s,e)$ of the stochastic discount factor of the investors and the probability of outcome s. The second competitive assumption that we make is that the BOD takes the stochastic discount factor $\bar{\pi} = (\bar{\pi}_s,\ s \in S)$ as given. Note that this is indirectly an assumption on the reservation utility levels $(\nu_k)_{k\in K}$ of the firms' managers. For firm k can influence the consumption $(x_s^i,\ i \in I)$ of the investors in outcome s only by changing the net aggregate output $\sum_{k\in K}(y_s^k - \tau_s^k)$. Since the total output of firm k in outcome s is fixed at y_{sk}^k, it can only affect net aggregate output through the choice of τ_s^k. If the level of compensation τ^k of the manager, which is determined by the manager's reservation utility ν_k, is only a small fraction of y^k, the possibility of influencing net aggregate output in outcome s will be negligible, even if the firm's output y^k is not a completely negligible fraction of total output.

The BOD of each firm now faces a principal-agent problem in a market setting. The BOD of firm k (the principal) chooses the incentive contract τ^k to offer its manager (the agent) which will induce the effort e_k which maximizes the market value of the firm (net of the payment to the manager). Since effort e_k is unobservable, the BOD must respect the incentive constraint that e_k is the optimal response of manager k to the contract τ^k. It must also ensure that the resulting expected utility of the manager net of the cost of effort meets the reservation utility level ν_k of the outside option.

The compensation package τ^k offered by the BOD to the manager works by exposing the manager to risk: if, after receiving the contract, manager k could turn to the financial markets to hedge against these risks then the incentives that τ^k was designed to provide may be eliminated. Some assumption regarding the trading opportunities available to the manager must therefore been made.[7] The assumption that we make, which is the simplest and most direct for the purpose of this paper is that the manager is not permitted to trade on financial markets: the manager's consumption stream thus coincides with his compensation and the full force of the incentives embodied in τ^k is retained.

[7] An alternative approach, introduced in our earlier paper Magill-Quinzii (2002) and further explored by Acharya-Bisin (2005), is to assume that a manager is given full access to financial markets, but that the trades are observable by the investors. In this setting investors deduce the optimal effort of a manager from his portfolio and the manager knows that his portfolio will influence the market value of the firm. Provided the manager has a sufficiently large initial stake in the firm (strictly speaking is full initial owner) the market, through the observability of trades, can act as an effective disciplining device i.e. leads to the same outcome as an optimal contract. Thus the fact that a manager can trade on the financial markets need not eliminate managerial incentives provided that these trades are observable.

In practice firms' managers do trade on financial markets but there are restrictions on how a manager can trade, in particular with respect of the securities related to his firm.[8] A more realistic approach to the design of an optimal contract by a principal (here the BOD) in the spirit of Bisin-Gottardi-Rampini (2006) would take in to account the cost of monitoring the trades of the manager, but this would lead to a more complex model in which it would be harder to exhibit clearly the inefficiencies that are the focus of this paper.

Assuming that the financial markets are complete with respect to the outcomes, and using the present-value representation of a stock market equilibrium, leads to the following concept.

Definition 2. A *stock market equilibrium* is a pair of actions and prices $(\bar{x}, \bar{\tau}, \bar{e}, (\hat{\pi}, \bar{\pi})) = ((\bar{x}^i)_{i\in I}, (\bar{\tau}^k, \bar{e}_k)_{k\in K}, (\hat{\pi}, \bar{\pi}))$ consisting of consumption streams for investors, contracts and effort levels for managers, present-value prices and stochastic discount factors, such that

(i) for $i \in I$, investor i chooses the optimal consumption stream

$$\bar{x}^i \in \text{argmax}\left\{\sum_{s\in S} u_i(x_s^i)p_s\right\}$$

subject to the present-value budget constraint

$$\left\{x^i \in X^i \;\middle|\; \sum_{s\in S} \hat{\pi}_s x_s^i \leq \sum_{k\in K} \delta_k^i \sum_{s\in S} \hat{\pi}_s (y_s^k - \bar{\tau}_s^k)\right\}$$

where

(ii) $p_s = p(s, \bar{e}), \quad s \in S$

(iii) $\hat{\pi}_s = \bar{\pi}_s p_s, \quad s \in S$

(iv) for $k \in K$, the BOD of firm k chooses $(\bar{\tau}^k, \bar{e}_k)$, the contract of the manager and the effort level to induce, which maximizes the market value of the firm:

$$\sum_{s\in S}(y_s^k - \tau_s^k)\bar{\pi}_s p(s, e_k, \bar{e}^{-k})$$

on the set of $(\tau^k, e_k) \in \mathbb{R}_+^S \times \mathbb{R}_+$ satisfying

$$\sum_{s\in S} v_k(\tau_s^k)p(s, e_k, \bar{e}^{-k}) - c_k(e_k) \geq \nu_k \tag{PC$_k$}$$

[8]Some authors have recently noted that new securities have been introduced which reduce the effectiveness of these restrictions. For interesting reviews of this literature, see Acharya-Bisin (2005) and Bisin-Gottardi-Rampini (2006).

$$e_k \in \operatorname{argmax}\left\{\sum_{s\in S} v_k(\tau_s^k)p(s,\tilde{e}_k,\bar{e}^{-k}) - c_k(\tilde{e}_k) \mid \tilde{e}_k \in \mathbb{R}_+\right\} \qquad (\mathrm{IC}_k)$$

(v) markets clear: $\sum_{i\in I}\bar{x}_s^i + \sum_{k\in K}\bar{\tau}_s^k = \sum_{k\in K} y_s^k, \quad s \in S.$

The same definition without the incentive constraints (IC_k) defines a *stock market equilibrium with observable effort*. If all the agents' consumption streams are in the interior of their consumption sets and all managers exert positive effort levels in the equilibrium, we will say that the equilibrium is *interior*.

REMARK 2: The concept of equilibrium in Definition 2 describes the interaction between financial markets and contracts in a setting where there is risk sharing among investors and moral hazard in the creation of incentives for managers of firms. (i), (ii) and (v) define a reduced-form stock market equilibrium for investors, in which they take the contracts and associated managerial effort levels $(\bar{\tau}, \bar{e})$ as given. As in models with rational expectations the investors are assumed to be well informed: they are aware of the contracts $\bar{\tau}$ offered by the BOD of each firm to its manager, and they know the risk aversion, cost of effort, and ability of each manager with sufficient precision to deduce the optimal effort levels $(\bar{e}_k)_{k\in K}$ and the associated probabilities $p(s,\bar{e})$ for the collective outcomes of the firms.[9]

The security markets in turn give the BOD of each firm information on the risk aversion of investors through the decomposition (iii) of the present-value prices $(\hat{\pi}_s)$ into the product of the probabilities $p(s,\bar{e})$ and the common stochastic discount factor $\bar{\pi}_s$ of the investors (equal by (3) to each agent's marginal value of income in outcome s). The stochastic discount factor $\bar{\pi}$ determines the risk premium on any security, and in particular on the equity of firm k. The objective for firm k in (iv) can be written[10] as

$$E_{e_k}(y^k - \tau^k) + \mathrm{cov}_{e_k}(\bar{\pi}, y^k - \tau^k)$$

where E_{e_k} and cov_{e_k} indicate that the probabilities $p(s, e_k, \bar{e}^{-k})$, used in calculating the expectation and covariance, take into account the effect of manager's k effort but take as given the effort levels of the managers of firms other than k. In determining the optimal effort of the manager, the BOD

[9] A discussion of the institutional features of security markets which justify making these strong assumptions on the information available to investors as a useful first approximation, can be found in Section 2.3 of Magill-Quinzii (2002).

[10] To write the objective function of the firm in this form the present-value prices need to be normalized so that $\sum_s \hat{\pi}_s = 1$. This is equivalent to normalizing prices so that the interest rate is zero: since there is no consumption at date 0, there is no reason to have an interest rate different from zero in this model.

takes into account the covariance, or risk-premium, term in the valuation of firm k, under the competitive assumption discussed above that the effect of τ^k on $\bar{\pi}$ is ignored.

To study the normative properties of a stock market equilibrium we will compare it with the allocation that would be chosen by a planner seeking to maximize social welfare subject to the same incentive constraints as those faced by the firms' BODs.

Definition 3. An allocation $(x,\tau,e) = ((x^i)_{i\in I}, (\tau^k, e_k)_{k\in K}) \in \prod_{i\in I} X^i \times \mathbb{R}_+^{KS} \times \mathbb{R}_+^K$ is *constrained feasible* if

$$\sum_{i\in I} x_s^i + \sum_{k\in K} \tau_s^k = \sum_{k\in K} y_s^k, \quad s\in S \tag{RC$_s$}$$

and if for all $k\in K$ the effort level e_k is optimal for manager k given (τ^k, e^{-k}), i.e

$$e_k \in \operatorname{argmax}\left\{\sum_{s\in S} v_k(\tau_s^k)p(s,\tilde{e}_k, e^{-k}) - c_k(\tilde{e}_k) \mid \tilde{e}_k \in \mathbb{R}_+\right\} \tag{IC$_k$}$$

An allocation (x,τ,e) is *constrained Pareto optimal* (CPO) if it is constrained feasible and there does not exist another constrained feasible allocation which is weakly preferred by all agents, and strictly by at least one agent. The same definition without the incentive constraints (IC$_k$) defines a first-best optimum.

First-order conditions for Equilibrium and CPO. A natural approach to comparing equilibrium allocations $(\bar{x},\bar{\tau},\bar{e})$ with constrained Pareto optimal allocations is to compare the first-order conditions (FOCs) for equilibrium and constrained optimality. To derive the FOCs, consider a setting in which the incentive constraint (IC$_k$) can be replaced by the first-order condition for optimality of effort e_k

$$\sum_{s\in S} v_k(\tau_s^k)\frac{\partial p(s,e)}{\partial e_k} - c_k'(e_k) = 0 \tag{IC$'_k$}$$

Let $(\bar{x},\bar{\tau},\bar{e},\bar{\pi})$[11] be an interior equilibrium. To simplify notation[12] set $p(s,e) = p_s$. There exists a vector of multipliers $(\bar{\lambda},\bar{\beta},\bar{\mu}) = ((\bar{\lambda}_i)_{i\in I}, (\bar{\beta}_k,\bar{\mu}_k)_{k\in K}) \geq 0$ such that

[11] From now on we simplify the notation for a stock market equilibrium by omitting the present-value prices $\hat{\pi}$ which do not play a direct role in the analysis of the rest of the paper.

[12] Depending on the circumstances we will use the notation $p(s,e)$ or $p(s_k, s^{-k}, e_k, e^{-k})$, or, when the expressions become complex, the shorter notation $p_s(e)$ or just p_s.

$$
\begin{aligned}
&\text{(i) } u_i'(\bar{x}_s^i) = \bar{\lambda}_i \bar{\pi}_s, \quad s \in S,\ i \in I \\
&\text{(ii) } \left(\bar{\beta}_k + \bar{\mu}_k \frac{\frac{\partial p_s}{\partial e_k}}{p_s} \right) v_k'(\bar{\tau}_s^k) = \bar{\pi}_s, \quad s \in S,\ k \in K \\
&\text{(iii)} \sum_{s \in S} \bar{\pi}_s (y_s^k - \bar{\tau}_s^k) \frac{\partial p_s}{\partial e_k} + \bar{\beta}_k \left(\sum_{s \in S} v_k(\bar{\tau}_s^k) \frac{\partial p_s}{\partial e_k} - c_k'(\bar{e}_k) \right) + \\
&\qquad \bar{\mu}_k \left(\sum_{s \in S} v_k(\bar{\tau}_s^k) \frac{\partial^2 p_s}{(\partial e_k)^2} - c_k''(\bar{e}_k) \right) = 0,\ k \in K
\end{aligned}
\tag{FOC$_E$}
$$

where $\bar{\lambda}_i$ is the multiplier associated with the budget constraint in investor i's utility maximization problem, and $(\bar{\beta}_k, \bar{\mu}_k)$ are the multipliers associated with the participation constraint (PC$_k$) and the transformed incentive constraint (IC$'_k$) for manager k. If effort is observable, the incentive constraints do not exist (are not binding) and the FOCs are the same with $\bar{\mu} = 0$. If effort is unobservable and (IC$'_k$) is binding, the second term in (iii) is equal to zero.

If (x, τ, e) is an interior constrained Pareto optimal allocation then for some positive weights $(\alpha, \beta) \in \mathbb{R}^{I+K}_{++}$, it will maximize the social welfare function

$$
W_{\alpha,\beta}(x, \tau, e) = \sum_{i \in I} \alpha^i \sum_{s \in S} u_i(x_s^i) p(s, e) + \sum_{k \in K} \beta_k \left(\sum_{s \in S} v_k(\tau_s^k) p(s, e_k, e^{-k}) - c_k(e_k) \right)
$$

subject to the constraints

$$
\sum_{i \in I} x_s^i + \sum_{k \in K} (\tau_s^k - y_s^k) = 0, \quad s \in S \tag{RC$_s$}
$$

$$
\sum_{s \in S} v_k(\tau_s^k) \frac{\partial p(s, e)}{\partial e_k} - c_k'(e_k) = 0, \quad k \in K \tag{IC$'_k$}
$$

where the incentive constraints (IC$_k$) have been replaced by the first-order conditions (IC$'_k$). Thus there will exist non-negative multipliers $((\pi_s)_{s \in S}, (\mu_k)_{k \in K})$ such that

$$
\begin{aligned}
&\text{(i)}^* \quad \alpha_i u_i'(x_s^i) = \pi_s, \quad s \in S,\ i \in I \\
&\text{(ii)}^* \quad \left(\beta_k + \mu_k \frac{\frac{\partial p_s}{\partial e_k}}{p_s} \right) v_k'(\tau_s^k) = \pi_s, \quad s \in S,\ k \in K \\
&\text{(iii)}^* \quad \sum_{i \in I} \alpha_i \sum_{s \in S} u^i(x_s^i) \frac{\partial p_s}{\partial e_k} + \sum_{j \neq k} \sum_{s \in S} \left(\beta_j \frac{\partial p_s}{\partial e_k} + \mu_j \frac{\partial^2 p_s}{\partial e_j \partial e_k} \right) v_j(\tau_s^j) \\
&\qquad + \beta_k \left(\sum_{s \in S} v_k(\tau_s^k) \frac{\partial p_s}{\partial e_k} - c_k'(e_k) \right) + \mu_k \left(\sum_{s \in S} v_k(\tau_s^k) \frac{\partial^2 p_s}{(\partial e_k)^2} - c_k''(e_k) \right) = 0, k \in K
\end{aligned}
\tag{FOC$_{CP}$}
$$

where α_i (resp β_k) is the weight of investor i (manager k) in the social welfare function, π_s (or more accurately $\pi_s p_s$) is the multiplier associated with the resource constraint for outcome s, and

μ_k is the multiplier associated with the incentive constraint for manager k. As before, if effort is observable $\mu = 0$, while if effort is not observable the third term in (iii)* is equal to zero.

The FOCs (i), (ii) and (i)*, (ii)* which describe how risk is distributed between investors and managers so as to induce the appropriate effort on the part of the managers are the same, implying that the contracts which are optimal from the point of view of the shareholders to induce given effort levels of the managers are also the socially efficient way of inducing this effort. The FOCs (iii) and (iii)* however are different: while they evaluate the marginal cost of an additional unit of effort by manager k in the same way, they differ in the way they evaluate its marginal benefit. For the planner, the social benefit is measured by its effect on the expected utility of all other agents in the economy, namely all investors $i \in I$ and all managers $j \in K, j \neq k$, with incentive-corrected weights, while in equilibrium the marginal benefit of manager k's effort is measured by its effect on the profit of firm k. We will show however that these two distinct ways of measuring marginal benefit in fact coincide when investors are risk neutral ($u_i(x^i) = x^i$) and firms' outcomes are independent. Proving this property will then suggest that in all other cases the FOCs for optimal effort (iii) in equilibrium and (iii)* in a social optimum are different.

When Equilibrium is CPO. We say that the random outcomes of the firms are *independent* if for each $k \in K$ there exists a probability function $p_k(\cdot, e_k)$ on S_k, which depends on the effort of manager k, such that

$$p(s, e) = \prod_{k \in K} p_k(s_k, e_k)$$

Since the FOCs are necessary but, because of possible non-convexities, are not in general sufficient for constrained efficiency, we will show that under risk-neutrality and independence a stock market equilibrium is CPO without using the first-order conditions.

Proposition 1. *If all investors are risk neutral and firms' outcomes are independent, if the VNM utility indices of the managers are strictly concave and satisfy $v_k(c) \to -\infty$ as $c \to 0$, then a stock market equilibrium is constrained Pareto optimal.*

Proof. Let $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi}) \in \mathbb{R}^{IS} \times \mathbb{R}_+^{KS} \times \mathbb{R}_+^K \times \mathbb{R}_{++}^S$ be a stock market equilibrium. We first show that $\bar{\tau}^k(s_k, s^{-k})$ depends only on s_k and is independent of the realizations s^{-k} of the other firms. Suppose not, i.e. suppose that for two outcomes $s = (s_k, s^{-k})$ and $s' = (s'_k, s'^{-k})$, with $s_k = s'_k$, we have $\bar{\tau}^k(s) \neq \bar{\tau}^k(s')$. For a random variable $\xi : S \to \mathbb{R}$, let $E_e(\xi) = \sum_{s \in S} p(s, e)\xi(s)$ denote its expectation given the vector e of effort levels. By the independence assumption

$$E_{\bar{e}} v_k(\bar{\tau}^k) = \sum_{s_k \in S_k} p_k(s_k, \bar{e}_k) \sum_{s^{-k} \in S^{-k}} p(s^{-k}, \bar{e}^{-k}) v_k(\bar{\tau}^k(s_k, s^{-k})) \tag{7}$$

Define $\tilde{\tau}^k(s_k) = \sum_{s^{-k} \in S^{-k}} p(s^{-k}, \bar{e}^{-k}) \bar{\tau}^k(s_k, s^{-k})$. Since $\bar{\tau}^k(s) \neq \bar{\tau}^k(s')$, by strict concavity of v_k there exists $b(\cdot) \geq 0$ such that

$$v_k(\tilde{\tau}^k(s_k) - b(s_k)) = \sum_{s^{-k} \in S^{-k}} p(s^{-k}, \bar{e}^{-k}) v_k(\bar{\tau}^k(s_k, s^{-k})) \tag{8}$$

with $b(s_k) > 0$ for at least one s_k. If manager k is offered the contract $\tilde{\tau}^k(s_k) - b(s_k)$ for $s_k \in S_k$, independently of s^{-k}, by (8) the participation constraint is still satisfied and, since the coefficient of $p_k(s_k, \bar{e}_k)$ in (7) has not changed, $\bar{e}_k$ is still the optimal effort. However, since $E_{\bar{e}} b(s) > 0$, the expected cost of the contract is lower, contradicting profit maximization. Thus $\bar{\tau}^k(s_k, s^{-k})$ depends only on s_k.

Suppose $(\bar{x}, \bar{\tau}, \bar{e})$ is not CPO. Then there exists an allocation $(\hat{x}, \hat{\tau}, \hat{e})$ such that

$$\sum_{i \in I} \hat{x}^i_s + \sum_{k \in K} \hat{\tau}^k_s = \sum_{k \in K} y^k_s, \quad s \in S \tag{9}$$

$\hat{e}_k$ is optimal for manager k given $\hat{\tau}^k$ and

$$E_{\hat{e}}(\hat{x}^i) \geq E_{\bar{e}}(\bar{x}^i),\ i \in I, \quad E_{\hat{e}}(v_k(\hat{\tau}^k)) - c_k(\hat{e}_k) \geq E_{\bar{e}}(v_k(\bar{\tau}^k)) - c_k(\bar{e}_k),\ k \in K \tag{10}$$

with strict inequality for some i or some k. By the same reasoning as above we know that there exists a contract $\tilde{\tau}^k$, which depends only on s_k such that $\hat{e}_k$ is optimal for this contract and

$$E_{\hat{e}} v(\tilde{\tau}^k) = E_{\hat{e}} v(\hat{\tau}^k), \quad \tilde{\tau}^k \leq \sum_{s^{-k} \in S^{-k}} p(s^{-k}, \hat{e}^{-k})(\hat{\tau}^k(s_k, s^{-k}))$$

Since $(\tilde{\tau}^k, \hat{e}_k)$ satisfy the (PC$_k$) and (IC$_k$) constraints, and since $\tilde{\tau}^k$ only depends on s_k, it could have been chosen in the maximization of expected profit. It follows that

$$E_{\bar{e}}(y_k - \bar{\tau}^k) = \sum_{s_k \in S_k} p_k(s_k, \bar{e}_k)(y_k - \bar{\tau}^k(s_k) \geq \sum_{s_k \in S_k} p_k(s_k, \hat{e}_k)(y_k - \tilde{\tau}^k(s_k) \geq E_{\hat{e}}(y_k - \hat{\tau}^k) \tag{11}$$

Suppose that in (10), it is investor i who is strictly better off, $E_{\hat{e}}(\hat{x}^i) > E_{\bar{e}}(\bar{x}^i)$. Then $\sum_{i \in I} E_{\hat{e}}(\hat{x}^i) > \sum_{i \in I} E_{\bar{e}}(\bar{x}^i) = \sum_{k \in K} E_{\hat{e}}(y_k - \bar{\tau}^k) \geq \sum_{k \in K} E_{\hat{e}}(y_k - \hat{\tau}^k)$, which contradicts the feasibility condition (9). Suppose that in (10), it is manager k who is strictly better off with $(\hat{\tau}^k, \hat{e}_k)$. Then the first inequality in (11) must be strict, once again contradicting the feasibility condition (9). For suppose that the first inequality in (11) holds with equality. Since manager k is strictly better off with $(\tilde{\tau}^k, \hat{e}_k)$, the (PC$_k$) constraint is not binding and $-\infty < v_k(\tilde{\tau}^k)$ implies $\tilde{\tau}^k \gg 0$. Thus for $\varepsilon > 0$ sufficiently small and for each outcome $s_k \in S_k$ the manager's reward can be decreased by $\Delta\tau^k(s_k)$ in such a way that

$$v_k(\tilde{\tau}^k(s_k) - \Delta\tau^k(s_k)) = v_k(\tilde{\tau}^k(s_k)) - \varepsilon,\ \ s_k \in S_k$$

The (PC$_k$) constraint is still satisfied, and since $E_e(v_k(\tilde{\tau}^k - \Delta\tau^k)) = E_e(v_k(\tilde{\tau}^k)) - \varepsilon$ for all e, the optimal effort is still $\hat{e}_k$. But the expected cost can be decreased by $E_{\hat{e}}(\Delta\tau^k)$, which contradicts profit maximization. □

Remark 3. Since an equilibrium with risk-neutral investors and independent firms is constrained Pareto optimal, the first-order conditions (i)-(iii) for an equilibrium must coincide with the first-order conditions (i)*-(iii)* for CPO, and it is instructive to understand why this is so. (i), (ii) and (i)*, (ii)* clearly coincide, so consider (iii) and (iii)*. Let $p'_k(s_k, \cdot)$ denote the derivative of the function $p_k(s_k, \cdot)$. By the independence assumption

$$\frac{\frac{\partial p_s(e)}{\partial e_k}}{p_s(e)} = \frac{p'_k(s_k, e_k)}{p_k(s_k, e_k)}$$

so that by (ii) the contract of manager k only depends on s_k and not on the realizations of other firms: this property was also derived directly in the proof of Proposition 1 without using the FOCs. The independence assumption also implies that $\dfrac{\partial^2 p_s}{\partial e_k \partial e_j} = \dfrac{\partial p_s}{\partial e_k}\dfrac{\partial p_s}{\partial e_j}/p_s$ so that the second term in (iii)* becomes

$$\sum_{s\in S}\sum_{j\neq k}\left(\beta_j \frac{\partial p_s}{\partial e_k} + \mu_j \frac{\partial p_s}{\partial e_k}\frac{\partial p_s}{\partial e_j}/p_s\right) v_j(\tau^j_s) = \sum_{s_k\in S_k} p'_k(s_k, e_k) \sum_{s^{-k}\in S^{-k}} \sum_{j\neq k}(\beta_j + \mu_j \frac{\partial p_s}{\partial e_j}/p_s) v_j(\tau^j_s) p(s^{-k}, e^{-k})$$

which is equal to zero since $\sum_{s_k\in S_k} p'_k(s_k, e_k) = 0$. Furthermore the third term in (iii)* is zero since the incentive constraint is binding. Since with linear preferences for the investors an interior allocation requires that all the weights of the investors be equal, (iii)* reduces to

$$\sum_{i\in I}\sum_{s\in S} x^i_s \frac{\partial p_s}{\partial e_k} + \mu_k\left(\sum_{s\in S}\frac{\partial^2 p_s}{(\partial e_k)^2} v_k(\tau^k_s) - c''(e_k)\right) = 0, \quad k \in K$$

The feasibility constraint can be written as

$$\sum_{i\in I} x^i_s = \sum_{j\neq k}(y^j_s - \tau^j_s) + (y^k_s - \tau^k_s), \quad s \in S$$

so that

$$\begin{aligned}\sum_{i\in I}\sum_{s\in S} x_s^i \frac{\partial p_s}{\partial e_k} &= \sum_{j\neq k}\sum_{s^{-k}\in S^{-k}} (y_s^j - \tau_s^j) p(s^{-k}, e^{-k}) \sum_{s_k\in S_k} p_k'(s_k, e_k) \\ &\qquad + \sum_{s_k\in S_k} (y_s^k - \tau_s^k) p_k'(s_k, e_k) \sum_{s^{-k}\in S^{-k}} p(s^{-k}, e^{-k}) \\ &= \sum_{s_k\in S_k} (y_s^k - \tau_s^k) p_k'(s_k, e_k)\end{aligned} \tag{12}$$

since $\sum_{s_k\in S_k} p_k'(s_k, e_k) = 0$ and $\sum_{s^{-k}\in S^{-k}} p(s^{-k}, e^{-k}) = 1$, and, since risk neutrality implies $\pi_s = 1,\ s \in S$, (12) coincides with the first term of (iii), so that (iii)* coincides with (iii).

Since risk neutrality and independence play an essential role in showing the equivalence of (iii) and (iii)*, it seems likely that this equivalence will fail if either risk aversion or independence is not satisfied: let us show that this is indeed the case and examine the consequences.

3. Local Analysis

Whenever a competitive equilibrium is not constrained Pareto optimal, it is a sign that some form of externality—whether pecuniary or direct—is present which has not been internalized at equilibrium. In the analysis that follows we examine the nature of the externalities whose effects are not fully internalized when the assumptions of Proposition 1 are not satisfied. Whenever possible, we sign the bias in the provision of managerial effort at equilibrium.

The procedure that we adopt to determine whether there is under or over provision of effort at equilibrium is based on a comparison of the first-order conditions $(\text{FOC})_E$ and $(\text{FOC})_{CP}$ at an equilibrium and a constrained Pareto optimum respectively. More precisely the general procedure is as follows. Suppose $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an interior stock market equilibrium. Under assumptions which will be spelled out below, the first-order approach (replacing the incentive constraints by the first-order condition (IC_k')) is valid and there exist multipliers $(\bar{\lambda}, \bar{\beta}, \bar{\mu}) = ((\bar{\lambda})_{i\in I}, (\bar{\beta}_k, \bar{\mu}_k)_{k\in K}) \geq 0$ such that (i)-(iii) in $(\text{FOC})_E$ are satisfied. To evaluate the optimality of the equilibrium, consider the social welfare function $W_{\bar{\alpha},\bar{\beta}}(x, \tau, e)$ defined in the previous section where the investors' weights $\bar{\alpha}_i = 1/\bar{\lambda}_i,\ i \in I$, are the inverse of the marginal utilities of income and the managers' weights $\bar{\beta}_k,\ k \in K$, are the multipliers of the participation constraints (PC_k). Let $RC_s(x, \tau)$ and $IC_k'(\tau, e)$, denote the functions which permit the resource and incentive constraints (RC_s) and (IC_k') in the previous section to be written as $RC_s(x, \tau) = 0, s \in S$ and $IC_k'(\tau, e) = 0,\ k \in K$. Consider the Lagrangian function $\bar{\mathcal{L}}(x, \tau, e)$ defined by

$$\bar{\mathcal{L}}(x, \tau, e) = W_{\bar{\alpha},\bar{\beta}}(x, \tau, e) - \hat{\pi} RC(x, \tau) + \bar{\mu} IC'(\tau, e)$$

where the multipliers $(\hat{\pi}, \bar{\mu})$, with $\hat{\pi}_s = \bar{\pi}_s p_s(\bar{e})$, are evaluated at the equilibrium. With this choice of weights $(\bar{\alpha}, \bar{\beta})$ and multipliers $(\hat{\pi}, \bar{\mu})$, it is clear that the first-order conditions $(FOC)_E$ (i)-(ii) and $(\text{FOC})_{CP}$ (i*)-(ii*) coincide so that

$$D_x \bar{\mathcal{L}}(\bar{x}, \bar{\tau}, \bar{e}) = 0, \quad D_\tau \bar{\mathcal{L}}(\bar{x}, \bar{\tau}, \bar{e}) = 0$$

If we can sign the gradient of $\bar{\mathcal{L}}$ with respect to e, then we can deduce, at least locally, if there is under or over-provision of managerial effort at equilibrium.

Proposition 2. *If $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an interior stock market equilibrium and if $D_e \bar{\mathcal{L}}(\bar{x}, \bar{\tau}, \bar{e}) \gg 0$, then there exists a constrained feasible marginal reallocation*

$$(\bar{x}, \bar{\tau}, \bar{e}) \longrightarrow (\bar{x} + \Delta x, \bar{\tau} + \Delta \tau, \bar{e} + \Delta e)$$

with $\Delta e > 0$ which is Pareto improving.

Proof: It is convenient to introduce the following more condensed vector notation: let $p(e) = (p_s(e))_{s \in S}$, $u_i(x^i) = (u_i(x^i_s))_{s \in S}$, $v_k(\tau^k) = (v_k(\tau^k_s))_{s \in S}$ and for a pair of vectors $x, y \in \mathbb{R}^S$, let $x \circ y = (x_s y_s)_{s \in S}$ denote the vector in $\mathbb{R}^S$ obtained by component-wise multiplication. Consider any semi-positive[13] marginal change in the vector of effort levels of the managers $\bar{e} \to \bar{e} + \Delta e$ with $\Delta e = (\Delta e_1, \ldots, \Delta e_K) > 0$. Choose a change $\Delta \tau^k$ in the reward of each manager $k \in K$ such that the utility level of the manager is unchanged and the incentive constraint (IC'_k) stays satisfied to terms of first order. Thus for each k we must find $\Delta \tau^k \in \mathbb{R}^S$ such that

$$\begin{aligned} p(\bar{e}) \circ v'_k(\bar{\tau}^k) \Delta \tau^k + D_e p(\bar{e}) \Delta e \cdot v_k(\bar{\tau}^k) - c'(\bar{e}_k) \Delta e_k &= 0 \\ D_{e_k} p(\bar{e}) \circ v'_k(\bar{\tau}^k) \Delta \tau^k + D^2_{e, e_k} p(\bar{e}) \Delta e \cdot v_k(\bar{\tau}^k) - c''(\bar{e}_k) \Delta e_k &= 0 \end{aligned}$$

The vector $p(\bar{e}) \circ v'_k(\bar{\tau}^k)$ is positive and, since $\sum_{s \in S} \dfrac{\partial p_s}{\partial e_k} = 0$, the vector $D_{e_k} p(\bar{e}) \circ v'_k(\bar{\tau}^k)$ has positive and negative elements. Thus the two vectors are linearly independent, so that a solution $\Delta \tau^k \in \mathbb{R}^S$ to this pair of equations always exists for each $k \in K$.

For each investor $i = 2, \ldots, I$ choose a change in consumption $\bar{x}^i \to \bar{x}^i + \Delta x^i$ such that the utility of investor i is unchanged

$$p(\bar{e}) \circ u'_i(\bar{x}^i) \Delta x^i + D_e p(\bar{e}) \Delta e \cdot u_i(\bar{x}^i) = 0$$

Finally, for agent 1 choose Δx^1 such that the resource constraints are satisfied, $\sum_{i \in I} \Delta x^i + \sum_{k \in K} \Delta \tau^k = 0$. Let $\bar{\mathcal{L}} = \mathcal{L}(\bar{x}, \bar{\tau}, \bar{e}; \hat{\pi}, \bar{\mu})$; the change in $\bar{\mathcal{L}}$ induced by the change $(\Delta x, \Delta \tau, \Delta e)$

[13] For $z \in R^K$, z is *semi-positive* (we write $z > 0$) if $z \geq 0$ and $z \neq 0$).

in the allocation satisfies

$$\Delta\bar{\mathcal{L}} = D_x\bar{\mathcal{L}}\Delta x + D_\tau\bar{\mathcal{L}}\Delta\tau + D_e\bar{\mathcal{L}}\Delta e > 0$$

since $D_x\bar{\mathcal{L}} = D_\tau\bar{\mathcal{L}} = 0$ and $D_e\bar{\mathcal{L}} \gg 0$. Since $(\Delta x, \Delta\tau, \Delta e)$ has been chosen so that $\Delta RC = 0$, $\Delta IC' = 0$, and the utility of all managers and investors except for investor 1 is unchanged, it follows that $\Delta\bar{\mathcal{L}} = \Delta W_{\bar{\alpha},\bar{\beta}} = \alpha_1\Delta(p(\bar{e})u_1(\bar{x}_1)) > 0$, so that the reallocation $(\bar{x}, \bar{\tau}, \bar{e}) \to (\bar{x} + \Delta x, \bar{\tau} + \Delta\tau, \bar{e} + \Delta e)$ is Pareto improving. □

We analyze the effect of removing the assumptions of investor risk neutrality and of independence of firms' outcomes separately. We begin by studying the effect of risk aversion of investors.

4. Effect of Risk Aversion

The approach to modeling uncertainty for the principal-agent problem, originally proposed by Mirrlees (1976)—by which the effort of the agent influences the probability of the outcome—inevitably brings with it a built-in external effect, since the agent's action affects the expected utility of the principal. In our setting the effort e_k of manager k affects the expected utility $\sum_{s\in S} p(s,e)u^i(x_s^i)$ of each investor. It is akin to an externality of firm k on all the consumers in the economy. Given the Mirrlees' approach to modeling uncertainty, the externality is always present; however given additional assumptions on the characteristics of the economy, it may or may not create an inefficiency. In Section 1 we saw that if investors are risk neutral there is no (constrained) inefficiency: this is because the expected utilities of the investors coincide with their expected income, which is precisely what the BOD maximizes. In this case the criterion of present-value maximization ensures that the externality is internalized.

When investors are risk averse, their expected utilities $\sum_{s\in S} p(s,e)u^i(x_s^i)$, $i \in I$, no longer coincide with the market values of their consumption streams. In this case, as we show below, the externality creates an inefficiency;[14] furthermore we show that the sign of the bias in managerial effort at equilibrium can be determined. Under reasonable assumptions market-value maximization systematically under-values risk, so that increasing managerial effort would lead to a Pareto improvement.

To establish this result we retain the assumption that firms' outcomes are independent, so that $p(s,e) = \prod_{k=1}^{K} p_k(s_k, e_k)$. Since the analysis is based on an examination of first-order conditions at equilibrium, we introduce sufficient conditions which ensure that the incentive constraint of each manager can be characterized by a single equation, and that the associated multiplier can be signed.

[14] In the bilateral principal-agent model, maximization of the principal's expected utility subject to the participation and incentive constraints of the agent internalizes the externality.

A1. The utility functions $(v_k)_{k\in K}$ of managers are differentiable, increasing, strictly concave on $\mathbb{R}_+$ and $v_k(c) \to -\infty$ as $c \to 0$, for all $k \in K$.

A2. The utility functions $(u_i)_{i\in I}$ of investors are differentiable, increasing, strictly concave on $\mathbb{R}_+$ and $u_i'(c) \to \infty$ as $c \to 0$, for all $i \in I$.

A3. Firms' outcomes are independent.

A4. For all $k \in K$ and $e_k > 0$, $\dfrac{p_k'(s_k, e_k)}{p_k(s_k, e_k)}$ is an increasing function of s_k.

A5. For all $k \in K$, and $\min_{s_k}(y^k_{s_k}) \leq \alpha < \max_{s_k}(y^k_{s_k})$, $1 - F_k(\alpha, e_k) \equiv \sum_{\{s_k | y^k_{s_k} > \alpha\}} p_k(s_k, e_k)$ is a concave, increasing function of e_k.

A4 is the Monotone Likelihood Ratio Condition (MLRC) which requires that for $e_k > e_k'$ the likelihood ratio

$$\frac{p(s_k, e_k)}{p(s_k, e_k')} = \exp\left(\int_{e_k'}^{e_k} \frac{p'(s_k, t)}{p(s_k, t)} dt\right)$$

is increasing in s_k: higher effort makes higher outcomes more likely. A5 implies that the probability that $y^k_{s_k}$ is greater than any fixed value α increases with effort, but at a decreasing rate: it is either called Stochastic Decreasing Returns to Effort or Convexity of the Distribution Function (since under A5 $F(\alpha, e_k)$ is convex in e_k). Rogerson (1985) showed that under A1, A4, A5 the first-order approach, which consists in replacing the incentive constraint (IC_k) by the first-order condition (IC_k') is valid. Also, since the paper of Grossmann-Hart (1983), A4 and A5 have been used to derive properties of the optimal incentive contract. The paper of Jewitt (1988) emphasized that A5 is restrictive, but recently Li-Calzi and Spaeter (2003) exhibited large classes of distribution functions $F(\alpha, e_k)$ satisfying A4 and A5.

Proposition 3. *Let A1-A5 be satisfied. If $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an interior stock market equilibrium (with or without observable effort) such that for all $k \in K$ and all $s^{-k} \in S^{-k}$, $y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k})$ is positive and increasing in s_k, then $D_e\bar{\mathcal{L}}(\bar{x}, \bar{\tau}, \bar{e}) \gg 0$.*

Proof. Let $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ be a stock market equilibrium. Assumptions A1, A4, A5 imply that in the case where effort is not observable and the incentive constraints have to be taken into account, the first-order approach is valid (Rogerson (1985)) so that the first-order conditions $(\text{FOC})_E$ and $(\text{FOC})_{CP}$ are satisfied at equilibrium and at a CPO respectively.

Since at the equilibrium (iii) of $(\text{FOC})_E$ holds, $D_e\bar{\mathcal{L}} \gg 0$ is equivalent to $A_k(\bar{x}, \bar{\tau}, \bar{e}) > 0$ for all

k, where

$$A_k(\bar{x},\bar{\tau},\bar{e}) = \frac{\partial \bar{\mathcal{L}}}{\partial e_k} - \bar{\pi}\circ\frac{\partial p(\bar{e})}{\partial e_k}\cdot(y^k-\bar{\tau}^k) - \bar{\beta}_k\left(v_k(\bar{\tau}^k)\frac{\partial p}{\partial e_k} - c_k'(\bar{e}_k)\right) - \bar{\mu}_k\left(\frac{\partial^2 p(\bar{e})}{\partial e_k^2}v_k(\bar{\tau}^k) - c''(\bar{e}_k)\right)$$

i.e. A_k is obtained by subtracting (iii) from (iii)*, and the notation is that introduced in the proof of Proposition 2. Evaluating $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_k}$ and canceling terms gives

$$A_k(\bar{x},\bar{\tau},\bar{e}) = \frac{\partial p(\bar{e})}{\partial e_k}\cdot\left[\sum_{i\in I}\bar{\alpha}_i u_i(\bar{x}^i) + \sum_{j\neq k}\left(\bar{\beta}_j + \bar{\mu}_j\frac{\frac{\partial p(\bar{e})}{\partial e_j}}{p(\bar{e})}\right)\circ v_j(\bar{\tau}^j) - \bar{\pi}\circ(y^k-\bar{\tau}^k)\right]$$

where we have used the fact that under Assumption A3 of independence $\dfrac{\partial^2 p_s}{\partial e_k\partial e_j} = \dfrac{\partial p_s}{\partial e_k}\dfrac{\frac{\partial p_s}{\partial e_j}}{p_s}$, and where $\dfrac{\frac{\partial p(\bar{e})}{\partial e_j}}{p(\bar{e})}$ denotes the vector of likelihood ratios $\dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}$, $s\in S$. Also note that $\dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})} = \dfrac{p_j'(s_j,\bar{e}_j)}{p_j(s_j,\bar{e}_j)}$, so that it only varies with s_j.

For $s^{-k}\in S^{-k}$, consider the function $V_{s^{-k}}:\mathbb{R}_{++}\to\mathbb{R}$ defined by

$$V_{s^{-k}}(\xi) = \max\left\{\sum_{i\in I}\bar{\alpha}_i u_i(\xi_i) + \sum_{j\neq k}\bar{\alpha}_j v_j(\xi_j)\;\middle|\;\sum_{i\in I}\xi_i + \sum_{j\neq k}\xi_j = \xi\right\} \tag{13}$$

with $\bar{\alpha}_j = \bar{\beta}_j + \bar{\mu}_j\frac{p_j'(s_j,\bar{e}_j)}{p_j(s_j,\bar{e}_j)}$. Thus $V_{s^{-k}}$ is the maximized social welfare function for all agents except manager k, with managers weighted by their "incentive weights" $\bar{\alpha}_j$.[15] In view of A1 this function is differentiable, increasing and strictly concave. If a vector $(\xi_i^*, i\in I, \xi_j^*, j\neq k)$ is such that $\sum_{i\in I}\xi_i^* + \sum_{j\neq k}\xi_j^* = \xi$ and there exists a vector ρ such that $\bar{\alpha}_i u_i'(\xi_i^*) = \bar{\alpha}_j v_j'(\xi_j^*) = \rho$, then $(\xi_i^*, i\in I, \xi_j^*, j\neq k)$ is a solution to the maximum problem (13), so that $V_{s^{-k}}(\xi) = \sum_{i\in I}\bar{\alpha}_i u_i(\xi_i^*) + \sum_{j\neq k}\bar{\alpha}_j v_j(\xi_j^*)$. In addition, $V_{s^{-k}}'(\xi) = \rho$ (see e.g. Magill-Quinzii (1996, p. 192)).

For any $s^{-k} = (s_j)_{j\neq k}\in S^{-k}$, let $Y_{s^{-k}} = \sum_{j\neq k} y_{s_j}^j$ denote the production of all firms excluding k. In outcome $s = (s_k, s^{-k})$, the investors and the managers other than k share the output $Y_{s^{-k}} + y_{s_k}^k - \bar{\tau}_s^k$, and the first-order conditions (i) and (ii) in $(\mathrm{FOC})_E$ imply that

$$V_{s^{-k}}\left(Y_{s^{-k}} + (y_{s_k}^k - \bar{\tau}_s^k)\right) = \sum_{i\in I}\bar{\alpha}_i u_i(\bar{x}_s^i) + \sum_{j\neq k}\bar{\alpha}_j v_j(\bar{\tau}_s^j)$$

and $V_{s^{-k}}'\left(Y_{s^{-k}} + (y_{s_k}^k - \bar{\tau}_s^k)\right) = \bar{\pi}_s = \bar{\pi}(s_k, s^{-k})$. Thus $A_k(\bar{x},\bar{\tau},\bar{e})$ can be written as

$$\begin{aligned} A_k(\bar{x},\bar{\tau},\bar{e}) = & \sum_{s^{-k}\in S^{-k}} p(s^{-k},\bar{e}^{-k})\sum_{s_k\in S_k} p_k'(s_k,\bar{e}_k)\Big[V_{s^{-k}}\left(Y_{s^{-k}} + y_{s_k}^k - \bar{\tau}^k(s_k,s^{-k})\right) \\ & -V_{s^{-k}}'\left(Y_{s^{-k}} + y_{s_k}^k - \bar{\tau}^k(s_k,s^{-k})\right)(y_{s_k}^k - \bar{\tau}^k(s_k,s^{-k}))\Big], \quad k\in K \end{aligned} \tag{14}$$

[15] If effort is observable, $\mu_j = 0$ for all $j\in K$, and the weights of the managers are just their weights in the social welfare function associated with the equilibrium.

Define $\phi(\chi) = V_{s^{-k}}(Y_{s^{-k}} + \chi) - V'_{s^{-k}}(Y_{s^{-k}} + \chi)\chi$. Then $\phi'(\chi) = -V''_{s^{-k}}(Y_{s^{-k}}) + \chi)\chi > 0, \forall \chi > 0$ since $V_{s^{-k}}$ is strictly concave, so that ϕ is an increasing function. The monotone likelihood ratio condition A4 implies that if $\bar{e}_k > \tilde{e}_k$, the distribution function $F(\sigma, \bar{e}_k) = \sum_{s_k \leq \sigma} p_k(s_k, \bar{e}_k)$ first-order stochastically dominates $F(\sigma, \tilde{e}_k)$ (see Rogerson (1985)). It follows that if $y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k})$ is an increasing function of s_k then

$$\sum_{s_k \in S_k} p_k(s_k, \bar{e}_k)\phi(y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k})) > \sum_{s_k \in S_k} p_k(s_k, \tilde{e}_k)\phi(y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k}))$$

and in the limit when $\tilde{e}_k \to \bar{e}_k$, $\sum_{s_k \in S_k} p'_k(s_k, \bar{e}_k)\phi(y^k_{s_k} - \bar{\tau}^k(s_k, s^{-k})) > 0$. Thus $A_k(\bar{x}, \bar{\tau}, \bar{e}) > 0$ and the proof is complete. □

Remark 4. Proposition 3 requires that the payoff to the shareholders be an increasing function of the firm's output (profit). If the model is viewed as a discrete version of the model with continuous outcomes then the condition requires that the slope $d\tau^k/dy^k$ of the reward schedule $\tau^k(y^k)$ of the manager of firm k be less than 1. This is a condition which is intuitively reasonable and is certainly satisfied in practice for the observed compensation of CEOs. Murphy (1999) studies the compensation of CEOs for a large sample of leading US corporations during the 1990's and in particular examines how CEO compensation increases (on average) when shareholder wealth increase by 1000\$: the maximum reported number is 35\$ or a slope of 0.035. But of course we cannot be sure that the observed compensation schemes are optimal or close to being optimal. For the model studied in this paper it is easy to specify outputs $(y^k_{s_k})$, probability functions $p_k(s_k, e_k)$, preferences (u_i) and (v_k, c_k), and reservation utility (ν_k) for the managers, so that the resulting equilibrium compensation $(\bar{\tau}^k)$ schedules satisfy this condition: but we have not found simple clear-cut restrictions on the parameters of the model ensuring that it is always true in equilibrium.

Remark 5. The key to the proof of Proposition 3 is that the planner in determining the optimal effort e_k of manager k takes into account the change in the expected social welfare[16] $V(Y + y^k_{s_k} - \bar{\tau}^k_{s_k})_{s_k \in S_k}$ arising from the shift in probability across the stream of net outputs $(y^k_{s_k} - \bar{\tau}^k_{s_k})_{s_k \in S_k}$, while the market evaluates the increment to the expected value of $V'(Y + y^k_{s_k} - \bar{\tau}^k_{s_k})(y^k_{s_k} - \bar{\tau}^k_{s_k})_{s_k \in S_k}$. Since V is a concave function, $V(Y + \chi) - V'(Y + \chi)\chi$ is increasing for $\chi > 0$, and the function $V(Y + \chi)$ varies more than its "marginal function" $V'(Y + \chi)\chi$, in the sense that

$$V(Y + \chi_2) - V(Y + \chi_1) > V'(Y + \chi_2)\chi_2 - V'(Y + \chi_1)\chi_1, \text{ whenever } \quad \chi_2 > \chi_1 \qquad (15)$$

[16]To simplify we write V rather than $V_{s^{-k}}$ and Y instead of $Y_{s^{-k}}$.

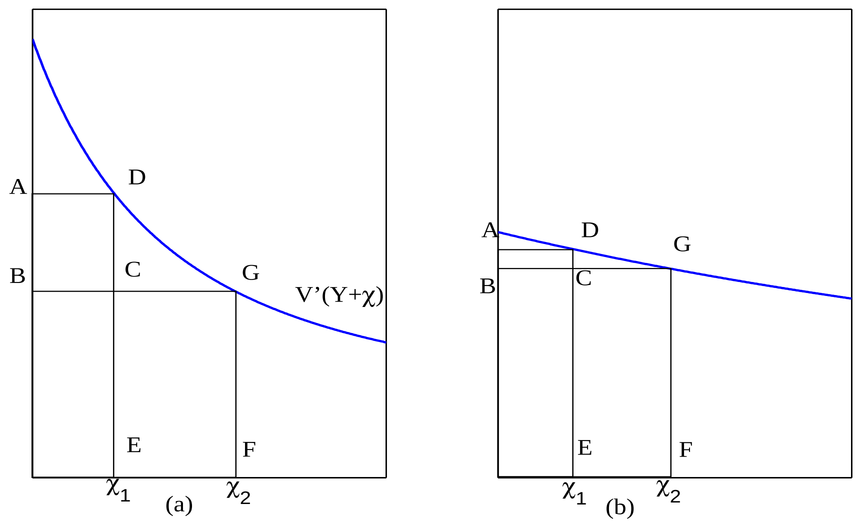

Figure 1: Difference between planner and market evaluation (area ABCGD)

Thus the shift in the probabilities arising from an increment to the effort e_k of manager k creates greater gains in the welfare function of the planner than in the equilibrium profit function, so that the effort chosen by the planner is greater than that in the equilibrium. The difference between the planner's and the market's evaluation in (15) is shown in Figure 1. $V(Y+\chi_2)-V(Y+\chi_1)$ is the area DCEFG, while $V'(\bar{Y}+\chi_2)\chi_2 - V'(\bar{Y}+\chi_1)\chi_1$ is the area CEFG minus the area ABCD, and

$$\text{area CEFG}-\text{area ABCD}<\text{area CEFG}<\text{area DCEFG}$$

The error in the market evaluation is ABCGD. As Figure 1b illustrates, the flatter the marginal function $V'(Y+\chi)$, either because Y is large or because agents are less risk averse, the smaller the difference between the planner's and the market's evaluation, and hence the smaller the underinvestment in effort at equilibrium.

Remark 6. Proposition 3 holds as soon as effort influences probability and does not depend on the non-observability of effort: it comes from the fact that market value is linear, while maximizing weighted expected utilities requires that it be nonlinear. If the model is presented in the framework of the general equilibrium model with incomplete markets (GEI) in which primitive states of nature are explicitly modelled, there must be more primitive states than outcomes so that, even with the assumption of complete markets with respect to the firms' outcomes, markets are incomplete with respect to the primitive states of nature. The problem here is not however of the same nature as the problem of the objective of the firm with incomplete markets studied by Ekern and Wilson (1974), Radner (1974), Drèze (1974) and Grossman and Hart (1979), where the problem is the indeterminacy of the stochastic discount factor used in the present-value calculation.[17]

5. Effect of Common Shock

In this section we analyze the setting where there is mutual dependence between the outcomes of the firms induced by the presence of a common shock. To isolate the effect of such a mutual dependence on the efficiency of the equilibrium we revert to the case where investors are risk neutral, so that the source of inefficiency studied in the previous section disappears.

The common shock is modeled as a random variable η with distribution function $G(\eta)$. We assume that, conditional on the value of η, firms' outcomes are independent so that there is no direct externality among firms: the effort of manager k only affects the probabilities of firm k's outcomes. For each firm k, let $\rho_k(s_k, e_k, \eta)$ denote the probability of the outcome $y^k_{s_k}$, given the effort level e_k and given a shock η. Then the probability of the joint outcome $s = (s_1, \ldots, s_K)$ given the vector of effort levels $e = (e_1, \ldots, e_K)$ and the shock η is given by

$$\rho(s, e, \eta) = \prod_{k \in K} \rho_k(s_k, e_k, \eta)$$

If the shock η were observable then all the variables could be indexed by η and the argument of Proposition 1 would continue to hold, so that a stock market equilibrium would be constrained efficient. But to make the assumption that the common shock is observable would not be in keeping with the basic tenet of the principal-agent model that primitive states are not observable. We thus assume that the shock η is not observable and cannot be deduced with certainty from the observed outcomes of the firms, so that contracts cannot be directly written conditional of the value of η. Furthermore we assume that investors and managers are symmetrically uninformed about the value of the common shock so that their information is restricted to the knowledge of its distribution

[17] A simple example will clarify the relation between the model studied here and its GEI counterpart. Suppose there is a single firm with two outcomes (y_g, y_b) and the effort of the manager (e_H or e_L) influences the probability of these outcomes: $p(y_g|e_H) = 2/3$, $p(y_g|e_L) = 1/3$. There must be at least three primitive states in a GEI model to generate this statistical description: for example let the primitive states be (η_1, η_2, η_3), each with probability $1/3$, and suppose the production function influenced by effort is $f(\eta_1, e_H) = f(\eta_2, e_H) = y_g,\ f(\eta_3, e_H) = y_b$ while $f(\eta_1, e_L) = y_g,\ f(\eta_2, e_L) = f(\eta_3, e_L) = y_b$. Since there are only two outcomes, there can be at most two independent securities based on the firm's outcomes: thus the GEI model has incomplete markets. However with two independent securities which associate discount factors to the outcomes y_g and y_b, the objective function of the firm is well defined despite the incompleteness of the markets. Suppose the firm has chosen e_H. Suppose security 1 with price q_1 has payoff $(y_g, y_g, 0)$, giving the stochastic discount factor π_g defined by $q_1 = 2/3\pi_g$, and security 2 with price q_2 has payoff $(0, 0, y_b)$, giving π_b defined by $q_2 = 1/3\pi_b$. Then the present value of output (y_g, y_b, y_b) with effort e_L would be $1/3\pi_g y_g + 2/3\pi_b y_b$, so that the present value of output can be compared in the two cases.

function G: thus for any agent in the economy the probability of an outcome $y_s = (y^1_{s_1}, \ldots, y^K_{s_K})$ given the effort levels $e = (e_1, \ldots, e_K)$ is given by

$$p(s,e) = \int_{\mathbb{R}} \rho(s,e,\eta) dG(\eta)$$

As usual we will use either the notation $p(s,e)$, or $p_s(e)$, or just p_s, depending on the complexity of the expression.

Since η is not observable, the contract of manager j will depend on the realized outputs of the other firms since these realizations give information on the value of the common shock and, by inference, on the likelihood that the outcome of firm j comes from a high or a low effort of manager j. The dependence of the contract of manager j on the outcome of firm k introduces a dependence of this contract on the effort of manager k, and hence an externality. A (constrained) planner will take this externality into account, while the markets will not. Thus a stock market equilibrium is typically not Pareto optimal. However, as we shall see, the sign of the bias is less clear than in the previous section.

In this section we make use of the following assumptions on the characteristics of the economy:

B1. The utility functions $(v_k)_{k \in K}$ of managers are differentiable, increasing, strictly concave, and $v_k(c) \to -\infty$ as $c \to 0$, for all $k \in K$.

B2. Investors are risk neutral: $u_i(c) = c$, for all $i \in I$.

B3. $p(s,e) = \int_{\mathbb{R}} \prod_{k \in K} \rho_k(s_k, e_k, \eta) dG(\eta)$, for some distribution function G.

B4. For all $k \in K$, $e_k > 0$, and $\eta \in \mathbb{R}$, $\dfrac{\frac{\partial}{\partial e_k} \rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e_k, \eta)}$ is an increasing function of s_k.

B5. For all $k \in K$, $\eta \in \mathbb{R}$, and $\min_{s_k}(y^k_{s_k}) \leq \alpha < \max_{s_k}(y^k_{s_k})$, $\sum_{\{s_k | y^k_{s_k} > \alpha\}} \rho_k(s_k, e_k, \eta)$ is a concave, increasing function of e_k.

B6. For all $k \in K$, $e_k > 0$, and $\eta \in \mathbb{R}$, $\dfrac{\frac{\partial}{\partial \eta} \rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e_k, \eta)}$ is an increasing function of s_k.

B7. For all $k \in K$, $e_k > 0$, and $s_k \in S_k$, $\dfrac{\frac{\partial}{\partial e_k} \rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e_k, \eta)}$ is a decreasing function of η.

B3 defines the probability structure: firms' outcomes are affected by the common shock η but, conditional on the value of η, their outcomes are independent random variables. B4 and B5 are the standard properties assumed in the principal-agent model, namely the monotone likelihood ratio property and stochastic decreasing returns to effort, which are assumed to hold for every value of the common shock. B6 is the condition which ensures that a higher value of η is favorable to high outcomes: it is equivalent to the property that, if $\eta > \eta'$, the ratio of the likelihood of $y^k_{s_k}$ with η to the likelihood of $y^k_{s_k}$ with η', namely

$$\frac{\rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e_k, \eta')} = \exp\left(\int_{\eta'}^{\eta} \frac{\frac{\partial}{\partial \eta}\rho_k(s_k, e_k, \theta)}{\rho_k(s_k, e_k, \theta)} d\theta\right)$$

increases with s_k. B7 is an assumption on the interaction between the effect of managerial effort and the common shock: it is equivalent to the property that, for $e_k > e'_k$, the likelihood ratio

$$\frac{\rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e'_k, \eta)} = \exp\left(\int_{e'_k}^{e_k} \frac{\frac{\partial}{\partial e_k}\rho_k(s_k, t, \eta)}{\rho_k(s_k, t, \eta)} dt\right)$$

decreases with η. The shock and effort are in essence substitutes since increasing η decreases the likelihood that $y^k_{s_k}$ can be attributed to a high rather than a low effort. If η were observable, the compensation of manager k would decrease as η increases. When η is not observable but B6 holds, the outcomes of firms $j \neq k$ give information on the likelihood that η has been high or low, and this leads to a monotone dependence of manager k's compensation on the outcomes of other firms $j \neq k$. We say that manager k's compensation $\tau^k(s_k, s^{-k})$ is decreasing in s^{-k} if for all pairs of outcomes $s^{-k} = (s_j)_{j\neq k}$ and $\tilde{s}^{-k} = (\tilde{s}_j)_{j \neq k}$, with $s_j \geq \tilde{s}_j$ for all $j \neq k$ and at least one strict inequality, $\tau^k(s_k, s^{-k}) < \tau^k(s_k, \tilde{s}^{-k})$.

Lemma 1. *Under the assumptions B1-B7, if $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an interior stock market equilibrium , then for any $k \in K$ and $s_k \in S_k$, the contract $\bar{\tau}^k(s_k, s^{-k})$ is decreasing in s^{-k}.*

The proof is given in Magill-Quinzii (2006), as well as examples which do and do not satisfy B7. Assumption B7 is satisfied when the probability $\rho_k(s_k, e_k, \eta)$ depends additively on e_k and η.

Proposition 4 summarizes the results that can be obtained on the sign of $\frac{\partial \bar{\mathcal{L}}}{\partial e_k}$ at an equilibrium.

Proposition 4. (i) *Let B1-B5 be satisfied. If $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an interior stock market equilibrium, then, for all $k \in K$, $D_{e_k}\mathcal{L}(\bar{x}, \bar{\tau}, \bar{e}) = D_k + I_k$, where*

$$D_k = \sum_{j \neq k}\sum_{s\in S} \bar{\alpha}^j_s \left(v_j(\bar{\tau}^j_s) - v'_j(\bar{\tau}^j_s)\bar{\tau}^j_s\right)\frac{\partial p_s(\bar{e})}{\partial e_k} \qquad \textit{with} \;\; \bar{\alpha}^j_s = \bar{\beta}_j + \bar{\mu}_j \frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}$$

$$I_k = \sum_{j \neq k}\sum_{s\in S} \bar{\mu}_j p_s(\bar{e}) \frac{\partial}{\partial e_k}\left(\frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right) v_j(\bar{\tau}^j_s)$$

(ii) If in addition B6 and B7 are satisfied and the utility functions $(v_k)_{k\in K}$ are such that

$$v_k(\bar{\tau}_s^k) - v_k'(\bar{\tau}_s^k)\bar{\tau}_s^k > 0, \quad \forall\, s \in S, \quad \forall\, k \in K \tag{16}$$

then $D_k < 0$ and $I_k > 0$.

Proof: (i) Let $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ be an interior stock market equilibrium. Under Assumptions B1-B5 the first order approach is valid and let $(\bar{\lambda}, \bar{\beta}, \bar{\mu})$ be the multipliers associated with the equilibrium for which $(\mathrm{FOC})_E$ hold. Since investors are risk neutral we can assume that $\bar{\pi}_s = 1$ for all $s \in S$ and $\bar{\alpha}_i = \frac{1}{\lambda_i} = 1$ for all $i \in I$. Since at the equilibrium the FOC for optimal effort, (iii) of $(\mathrm{FOC})_E$, is satisfied for each firm it follows that

$$D_{e_k}\mathcal{L}(\bar{x}, \bar{\tau}, \bar{e}) = \sum_{i\in I}\sum_{s\in S} \bar{x}_s^i \frac{\partial p_s(\bar{e})}{\partial e_k} + \sum_{j\neq k}\sum_{s\in S}\left(\bar{\beta}_j \frac{\partial p_s(\bar{e})}{\partial e_k} + \bar{\mu}_j \frac{\partial^2 p_s(\bar{e})}{\partial e_j \partial e_k}\right) v_j(\bar{\tau}_s^j) - \sum_{s\in S}(y_s^k - \bar{\tau}_s^k)\frac{\partial p_s(\bar{e})}{\partial e_k}$$

From the market clearing conditions, it follows that $\sum_{i\in I} \bar{x}_s^i - (y_s^k - \bar{\tau}_s^k) = \sum_{j\neq k}(y_s^j - \bar{\tau}_s^j)$, for all $s \in S$. Let us show that $\sum_{s\in S} y_s^j \dfrac{\partial p_s(\bar{e})}{\partial e_k} = 0$, for each $j \neq k$. Using the notation $\rho^{-k}(s^{-k}, \bar{e}^{-k}, \eta) = \prod_{j\neq k} \rho_j(s_j, \bar{e}_j, \eta)$

$$\sum_{s\in S} y_s^j \frac{\partial p_s(\bar{e})}{\partial e_k} = \int_{\mathbb{R}} \sum_{s^{-k}\in S^{-k}} \rho^{-k}(s^{-k}, \bar{e}^{-k}, \eta) y_{s_j}^j \left(\sum_{s_k\in S_k} \frac{\partial \rho_k(s_k, \bar{e}_k, \eta)}{\partial e_k}\right) dG(\eta) = 0$$

since $\displaystyle\sum_{s_k\in S_k} \frac{\partial \rho_k(s_k, \bar{e}_k, \eta)}{\partial e_k} = 0$. It follows that

$$D_{e_k}\mathcal{L}(\bar{x}, \bar{\tau}, \bar{e}) = \sum_{j\neq k}\sum_{s\in S}\left[\left(\bar{\beta}_j \frac{\partial p_s(\bar{e})}{\partial e_k} + \bar{\mu}_j \frac{\partial^2 p_s(\bar{e})}{\partial e_j \partial e_k}\right) v_j(\bar{\tau}_s^j) - \bar{\tau}_s^j \frac{\partial p_s(\bar{e})}{\partial e_k}\right] \tag{17}$$

Adding and subtracting the terms $\bar{\mu}_j \dfrac{\frac{\partial p_s(\bar{e})}{\partial e_j}\frac{\partial p_s(\bar{e})}{\partial e_k}}{p_s(\bar{e})} v_j(\bar{\tau}_s^j)$ and using equation (ii) in $(\mathrm{FOC})_E$ with $\bar{\pi}_s = 1$, gives the decomposition

$$D_{e_k}\mathcal{L}(\bar{x}, \bar{\tau}, \bar{e}) = D_k + I_k, \quad D_k = \sum_{j\neq k} D_{j,k}, \quad I_k = \sum_{j\neq k} I_{j,k}$$

with

$$D_{j,k} = \sum_{s\in S}\left(\bar{\beta}_j + \bar{\mu}_j \frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right)\left(v_j(\bar{\tau}_s^j) - v_j'(\bar{\tau}_s^j)\bar{\tau}_s^j\right)\frac{\partial p_s(\bar{e})}{\partial e_k}$$

$$I_{j,k} = \sum_{s\in S} \bar{\mu}_j \left(\frac{\partial^2 p_s(\bar{e})}{\partial e_j \partial e_k} - \frac{\frac{\partial p_s(\bar{e})}{\partial e_j}\frac{\partial p_s(\bar{e})}{\partial e_k}}{p_s(\bar{e})}\right) v_j(\bar{\tau}_s^j)$$

Note that $\frac{\partial}{\partial e_k}\left(\frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right) = \frac{\frac{\partial^2 p_s(\bar{e})}{\partial e_j \partial e_k} p_s(\bar{e}) - \frac{\partial p_s(\bar{e})}{\partial e_j}\frac{\partial p_s(\bar{e})}{\partial e_k}}{p_s(\bar{e})^2}$, so that $I_{j,k}$ can also be written as

$$I_{j,k} = \sum_{s\in S} \bar{\mu}_j p_s(\bar{e}) \frac{\partial}{\partial e_k}\left(\frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right) v_j(\bar{\tau}_s^j)$$

(ii) Let us assume B1-B7 and show that we can sign $D_{j,k}$ and $I_{j,k}$.

Sign of $D_{j,k}$: Since $a_j v_j + b_j$ for $a_j > 0$ represents the same preferences for manager j as v_j and since the consumption vector $\bar{\tau}_s^j$ is bounded, we can assume without loss of generality that b_j is chosen such that (16) holds. As we saw in the proof of Proposition 3, $x \to v_j(x) - v'(x)x$ is an increasing function of x. Since by Lemma 1 $\tau^j(s_k, s^{-k})$ is decreasing in s_k, the function $v_j(\bar{\tau}_s^j) - v_j'(\bar{\tau}_s^j)\bar{\tau}_s^j$ is decreasing in s_k. Since $1 = \left(\bar{\beta}_j + \bar{\mu}_j \frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right) v_j'(\bar{\tau}_s^j)$, and v_j' is decreasing, $\bar{\tau}_s^j$ decreasing in s_k is equivalent to $\frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}$ decreasing in s_k. Thus the product

$$H_j(s_k, s^{-k}) = \left(\bar{\beta}_j + \bar{\mu}_j \frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}\right)\left(v_j(\bar{\tau}_s^j) - v_j'(\bar{\tau}_s^j)\bar{\tau}_s^j\right)$$

is a decreasing function of s_k as a product of positive decreasing functions of s_k, and $D_{j,k}$ can be written as

$$D_{j,k} = \int_{\mathbb{R}} \sum_{s^{-k}\in S^{-k}} \rho^{-k}(s^{-k}, \bar{e}^{-k}, \eta) \sum_{s_k \in S_k} H_j(s_k, s^{-k}) \frac{\partial \rho(s_k, \bar{e}_k, \eta)}{\partial e_k} dG(\eta)$$

The monotone likelihood ratio condition B4 implies that if $e_k > e_k'$ the distribution function generated by $\rho(s_k, e_k, \eta)$ first-order stochastically dominates the distribution function generated by $\rho(s_k, e_k', \eta)$, which implies that $\sum_{s_k\in S_k} H_j(s_k, s^{-k}) \frac{\partial \rho(s_k, \bar{e}_k, \eta)}{\partial e_k} < 0$ since $H_j(s_k, s^{-k})$ is decreasing in s_k. Thus $D_{j,k} < 0$.

Sign of $I_{j,k}$: Let us show that $\frac{\partial^2 p_s(e)}{\partial e_j \partial e_k} > \frac{\frac{\partial p_s(e)}{\partial e_j}\frac{\partial p_s(e)}{\partial e_k}}{p_s(e)}$, for all $s \in S$ and all $e \gg 0$. Since $v_j(\bar{\tau}_s^j) > v_j'(\bar{\tau}_s^j)\bar{\tau}_s^j > 0$, this will imply that $I_{j,k} > 0$. Note that

$$\frac{1}{p_s(e)} \frac{\partial^2 p_s(e)}{\partial e_j \partial e_k} = \int_{\mathbb{R}} L_j(s_j, e_j, \eta) L_k(s_k, e_k, \eta) a(s, e, \eta) dG(\eta) \tag{18}$$

where $L_k(s_k, e_k, \eta) = \frac{\frac{\partial}{\partial e_k}\rho_k(s_k, e_k, \eta)}{\rho_k(s_k, e_k, \eta)}$ is the local likelihood function of manager k and where $a(s, e, \eta) = \frac{\rho(s, e, \eta)}{\int_{\mathbb{R}} \rho(s, e, \eta) dG(\eta)}$ is a density function for the measure $dG(\eta)$. Let G_a denote the distribution function induced by the density a with respect to dG. The integral (18) is the expectation of the product of the random variables L_j and L_k with respect to dG_a so that

$$\frac{1}{p_s(e)}\frac{\partial^2 p_s(e)}{\partial e_j \partial e_k} = E_a(L_j L_k) = E_a(L_j)E_a(L_k) + \text{cov}_a(L_j, L_k) = \frac{\partial p_s(e)}{\partial e_j}\frac{\partial p_s(e)}{\partial e_k} + \text{cov}_a(L_j, L_k)$$

Thus the sign of the difference $\dfrac{\partial^2 p_s(e)}{\partial e_j \partial e_k} - \dfrac{\frac{\partial p_s(e)}{\partial e_j}\frac{\partial p_s(e)}{\partial e_k}}{p_s(e)}$ is the sign of the covariance term. By B7 the random variables L_j and L_k are decreasing functions of η, and are thus positively dependent random variables with respect to dG_a. This in turn implies that $\text{cov}_a(L_j, L_k)$ is positive (see e.g. Magill-Quinzii (1996, p.170)). □

The general principle underlying an incentive contract is that the agent undertaking the effort should be paid more when the realized outcome is more likely to have occurred with high effort, and should be paid less when the outcome is more likely with low effort. When outcomes are the combined result of effort and a common shock—and when the shock is not observable but also affects other firms—then the realized outcomes of these other firms provide information on the shock, and this in turn provides information on the likelihood that a given outcome for the firm is due to high or low effort on the part of its manager: this point was emphasized by Holmstrom (1982) and Mookherjee (1984). Since the outcomes of other firms are also influenced by the effort of their managers, the fact that observed outcomes are used to infer information about the unobservable common shock introduces a dependence between the effort of manager k and the compensation of manager $j \neq k$. The contract of manager k in equilibrium only takes into account the effect of his effort on the expected profit of the firm and his expected utility, but ignores its effect on the compensation, and hence the expected utility, of the managers of the other firms. Proposition 4 can be interpreted as a description of the two additional effects that a planner would take into account when deciding on the effort to induce from manager k.

The first, $D_k = \sum_{j \neq k} D_{j,k}$, which we call the direct effect, is similar to the difference (14) studied in the proof of Proposition 3, the welfare difference terms being restricted to the managers other than k since the investors are risk neutral. D_k expresses the difference between the effects of a marginal change Δe_k in manager k's effort on the weighted expected utility of the other managers—which would enter the objective of the planner—and on the weighted market value of their consumption—which enters in the objective of profit maximization. As in Remark 5, since v_j is stricly concave, the function $v_j(\tau_s^j) - v_j'(\tau_s^j)\tau_s^j$ is increasing in τ_s^j when $\tau_s^j > 0$. When B6 and B7 hold, Lemma 1 implies that $\tau^j(s_k, s^{-k})$ is decreasing in s_k and, as we have seen in the proof, $\alpha^j(s_k, s^{-k})$ is also decreasing in s_k. Thus by an argument similar to that in Remark 5, but this time with a decreasing function, decreasing the effort e_k of manager k shifts probability towards lower values of s_k and hence increases the weighted expected utility of consumption of manager j more than it increases the market value of his consumption.

The second effect which the planner would take into account is that the effort of manager k influences the local likelihood ratios $\frac{\frac{\partial p_s(\bar{e})}{\partial e_j}}{p_s(\bar{e})}$, and hence the informativeness of the outcomes of other firms. When Assumption B7 holds, an increase Δe_k in manager k's effort increases the likelihood of high outcomes for firm k. As a result a high value of y^k becomes a less informative signal of the value of η and the the outcome y^j becomes more informative on the value of e_j so that the welfare of manager j in the social welfare function increases. Since this effect occurs through the likelihood ratio, or the information that can be inferred from a given realization of firm j, we call it the information effect.

Example. The following example, which satisfies Assumptions B1-B7, is instructive for studying which of the two effects dominates, i.e. whether there is under or over-provision of effort at equilibrium. Let $K = 2$, $S_1 = \{g_1, b_1\}$, $S_2 = \{g_2, b_2\}$, $S = S_1 \times S_2$, $v_k(c) = \frac{1}{1-\alpha} c^{1-\alpha}$, $0 < \alpha \neq 1$, and let the probabilities be affine in effort and the shock

$$\rho_k(g_k, e_k, \eta) = a_k + b_k e_k + d\eta, \quad 0 < a_k + b_k + d < 1, \quad \rho_k(b_k, e_k, \eta) = 1 - \rho_k(g_k, e_k, \eta), \quad k = 1, 2$$

where η is uniformly distributed on $[0, 1]$ and the cost functions $c_1(e_1)$, and $c_2(e_2)$ are such that e_1 and e_2 always lie in $(0, 1)$, i.e. $c_k(0) = 0$, $c_k(e_k) \to \infty$ as $e_k \to 1$.

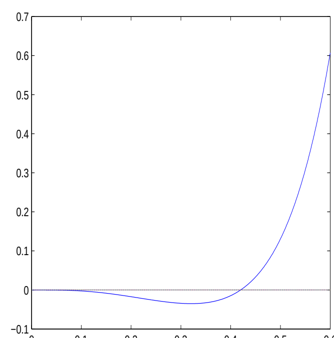

Figure 2: $\frac{\partial \bar{\mathcal{L}}}{\partial e_2}$ as a function of d, which parameterizes the impact of the common shock η on the probabilities.

To compute an equilibrium we need in addition to specify the outputs $y^k = (y^k_{g_k}, y^k_{b_k})$ of the two firms ($k = 1, 2$), the outside options (ν_1, ν_2) and the cost functions (c_1, c_2) of the two managers. However since the expression (17) that we want to study only depends indirectly on these characteristics through the resulting equilibrium values $(\bar{e}_k, \bar{\beta}_k, \bar{\mu}_k)$, $k = 1, 2$, it is more convenient to study (17) by treating the equilibrium values as parameters. For once $(a_k, b_k, d, \bar{e}_k, \bar{\beta}_k, \bar{\mu}_k)$, $k = 1, 2$ have been chosen, there exist characteristics (y^k, ν_k, c_k) , $k = 1, 2$, consumption streams, contracts and prices $(\bar{x}, \bar{\tau}, \bar{\pi})$ such that $(\bar{x}, \bar{\tau}, \bar{e}, \bar{\pi})$ is an equilibrium. Clearly $\bar{\pi}_s = 1, \ \forall s \in S$, and $\bar{\tau}^k$ is such that

$$\bar{\tau}_s^k = \left(\bar{\beta}_k + \bar{\mu}_k \frac{\frac{\partial p_s(\bar{e})}{\partial e_k}}{p_s(\bar{e})} \right)^{\frac{1}{\alpha}}$$

where $p_s(\bar{e}) = \int_0^1 \rho_1(s_1, \bar{e}_1, \eta)\rho_2(s_2, \bar{e}_2, \eta)d\eta$. Calculating $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_j}$, $j = 1, 2$, and varying the parameters $(\alpha, a, b, d,)(\bar{e}, \bar{\beta}, \bar{\mu})$, we find that the typical graph of $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_j}$ as a function of d—the coefficent which measures the magnitude of the impact of the shock η on the probability of the outcomes of each firm—has the form shown in Figure[18] 2.

When there is no common shock ($d = 0$) the equilibrium is efficient. For small magnitudes of d, the direct externality effect dominates and $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_j}$ is negative: managers over invest in effort. When d is sufficiently large, the information effect—which, as we saw in the proof of Proposition 4, is a positive covariance term between two random variables jointly influenced by η—becomes strong enough to dominate. To the extent that in practice the outcomes (profits) of firms are often strongly correlated, it seems natural within the framework of this model to adopt a relatively large value of d, so that the latter scenario seems more likely. Since, as we saw in Proposition 3, investors' risk aversion also makes $\dfrac{\partial \bar{\mathcal{L}}}{\partial e_j}$ positive, the effect of risk aversion when combined with that of a common unobservable shock seems likely to lead to under-provision of effort in equilibrium, in the sense of Proposition 2.

6. Continuum of Firms

Most models of general equilibrium with moral hazard in which, as in this paper, effort influences the probability of the outcomes, make the assumption that there is a continuum of agents of each type with independent shocks (Prescott-Townsend (1984a), (1984b), Kocherlakota (1998), Lisboa (2001)). The papers just cited reach the conclusion that an equilibrium is CPO, while we show that typically a stock market equilibrium is not CPO. Thus it is instructive to see what happens

[18]Figure 2 uses the following values of the parameters: $a = (0.25, 0.25)$, $b = (0.2, 0.2)$, $\alpha = 0.5$, $\bar{e} = (0.2, 0.2), \bar{\beta}_1 = 100$, $\bar{\mu}_1 = 50$.

in our model if we replicate the firms and, in the limit, have a continuum of firms of each type. We will not write out the details of the model for the continuum case, but rather indicate, using the structure of our model, why the inefficiencies studied in Sections 4 and 5 disappear when there is a continuum of firms of each type.

Consider first the model of Section 4 and let us change the model by assuming that $k \in K$ represents a type of firm and that there is a continuum of mass 1 of identical firms of each type. We assume that the probabilities of the outcomes of any two firms (whether of the same or of different types) are independent, and that firms of the same type k have identical managers (same (v_k, ν_k, p_k)). Assuming that all the managers of the same type are offered the same contract and choose the same effort, in equilibrium as well as in the planner's problem, the probabilities $p_k(s_k, e_k)$, $s_k \in S_k$ of the outcomes of firms of type k become the proportion of firms of this type with output s_k, so that the total output $\sum_{s_k} p_k(s_k, e_k) y^k_{s_k}$ of the firms of type k is non-random, and increases with e_k. The continuum of firms eliminates risk and thus the effect of risk aversion studied in Section 4. Another way of explaining the result is to note that the trade-off between the cost of providing incentives and the probability of good outcomes faced by an individual firm becomes, at the aggregate level, a trade-off between cost of incentives and quantity of output, and the marginal value of output is correctly evaluated by the market.

For the model of Section 5 with a common shock, satisfying the assumptions B1-B7, consider adding a continuum of firms of each type $k \in K$, assuming that the probabilities of the outcomes of any two firms are independent conditional on the value of η. The continuum removes the idiosyncratic shocks of firms from the aggregate: since the optimal effort e_k of a representative manager can be deduced from the incentive contract of firms of type k, and since the proportion of the firms with output s_k can be observed, the probabilities $\rho(s_k, e_k, \eta)$ can be inferred, and from this the value of η can be deduced. Thus the continuum in essence transforms the unobservable η into an observable or inferrable η, and this solves the information problem without introducing an externality. Given Assumption B6 which implies that if $\eta > \eta'$ the distribution function induced by $\rho_k(s_k, e_k, \eta)_{s_k \in S_k}$ first-order stochastically dominates the distribution function for $\rho_k(s_k, e_k, \eta')_{s_k \in S_k}$, the total output $\sum_{s_k} \rho_k(s_k, e_k, \eta) y^k_{s_k}$ of the firms of type k is an increasing function of η. Thus the optimal contract for the representative manager of a type k firm when η is known can equivalently be expressed as a contract which depends on the total output of the firms of type k or the economy-wide aggregate output. Thus even if there is a common shock, if there is a continuum of firms of each type and investors are risk neutral, a stock market equilibrium is constrained Pareto optimal.

In Sections 4 and 5 we have separated the effect of risk aversion and the informational problem induced by the unobservability of the common shock. In the case where there is a common shock and investors are risk averse, constrained Pareto optimality will be obtained with a continuum of firms if there are appropriate markets which permit the aggregate risk induced by η to be optimally shared.

References

Acharya, V.V. and A. Bisin (2005), "Managerial Hedging, Equity Ownership and Firm Value", NYU Discussion Paper.

Bennardo, A. and P.A. Chiappori (2003), "Bertrand and Walras Equilibria under Moral Hazard", *Journal of Political Economy*, 111, 785-817.

Berle, A.A. and G.C. Means (1932), *The Modern Corporation and Private Property*, New York: Harcourt, Brace and World.

Bisin, A. and P. Gottardi (1999), "General Competitive Analysis with Asymmetric Information", *Journal of Economic Theory*, 87, pp. 1-48.

Bisin, A., Gottardi, P. and A.A. Rampini (2006), "Managerial Hedging and Portfolio Monitoring", Discussion Paper.

Drèze, J. (1974), "Investment under Private Ownership: Optimality, Equilibrium and Stability", in J. Drèze, *Allocation Under Uncertainty: Equilibrium and Optimality*, New York: Wiley, 129-65.

Dubey, P. and J. Geanakoplos (2002), "Competitive Pooling: Rothschild-Stiglitz Reconsidered", *Quarterly Journal of Economics*, 117, 1529-1570.

Dubey, P., J. Geanakoplos and M. Shubik (2005), "Default and Punishment in General Equilibrium", *Econometrica*, 73, 1-37.

Ekern, S. and R. Wilson (1974), "On The Theory of the Firm in an Economy with Incomplete Markets", *Bell Journal of Economics and Management Science*, 5, 171-180.

Grossman, S. and O. Hart (1979), "A Theory of Competitive Equilibrium in Stock Market Economies", *Econometrica*, 47, 293-330.

Grossman, S. and O. Hart (1983), “An Analysis of the Principal-Agent Problem”, *Econometrica*, 51, pp. 7-45.

Hildenbrand, W (1974), *Core and Equilibria in Large Economies*, Princeton University Press.

Holmström, B. (1982), “Moral Hazard in Teams”, *Bell Journal of Economics*, 13, 324-340.

Jewitt, I., “Justifying the First-Order Approach to Principal-Agent Problems”, *Econometrica*, 1988, 56, 1177-1190.

Kocherlakota, N.R., “The Effect of Moral Hazard on Asset Prices When Financial Markets are Complete”, *Journal of Monetary Economics*, 1998, 41, pp. 39-56.

Li-Calzi, M. and S. Spaeter (2003), “Distribution for the First-Order Approach to the Principal Agent Problems”, *Economic Theory*, 21, 167-173.

Lisboa, M.B., “Moral Hazard and General Equilibrium in Large Economies”, *Economic Theory*, 2001, 18, 555-575.

Magill, M and M. Quinzii (1996), *The Theory of Incomplete Markets*, Cambridge, Massachusets: MIT Press.

Magill, M and M. Quinzii (2002), ”Capital Market Equilibrium with Moral Hazard”, *Journal of Mathematical Economics,* 38, 149-190.

Magill, M and M. Quinzii (2006), “Common Shocks and Relative Compensation”, *Annals of Finance*, 2, 407-420.

Milgrom, P. (1981), “Good News and Bad News: Representation Theorems and Applications”, *Rand Journal of Economics*, 12, 380-391.

Mirrlees, J.A. (1975), “The Theory of Moral Hazard and Unobservable Behaviour: Part I”, published in 1999 in *The Review of Economic Studies*, 66, 3-21.

Mirrlees, J.A. (1976), “The Optimal Structure of Incentives and Authority within an Organization”, *Bell Journal of Economics*, 7, 105-131.

Mookherjee, D. (1984), “Optimal Incentive Schemes with Many Agents”, *Review of Economic Studies*, 51, 433-446.

Murphy, K.J.(1999), "Executive Compensation", *Handbook of Labor Economics, Volume 3B*, O. Ashenfelter and D. Card eds, North-Holland.

Panaccione, L. (2005), "Constrained Inefficiency of Competitive Equilibria with Hidden Action", mimeo, CORE, Université Catholique de Louvain.

Prescott, E.C. and R.M. Townsend (1984a), "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard", *Econometrica*, 52, 21-46.

Prescott, E.C. and R.M. Townsend (1984b), "General Competitive Analysis in an Economy with Private Information", *International Economic Review*, 25, 1-20.

Radner, R. (1974), "A Note on the Unanimity of Stockholders' Preferences Among Alternative Production Plans: A Reformulation of the Ekern-Wilson Model", *Bell Journal of Economics and Management Science*, 5, 181-184.

Rogerson, W.P. (1985), "The First-Order Approach to Principal-Agent Problems", *Econometrica*, 53, 1357-1367.