A Theory of Rational Choice Under Complete Ignorance

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ABSTRACT

This paper contributes to a theory of rational choice under uncertainty for decision-makers whose preferences are exhaustively described by partial orders representing "limited information". Specifically, we consider the limiting case of "Complete Ignorance" decision problems characterized by maximally incomplete preferences and important primarily as reduced forms of general decision problems under uncertainty.

"Rationality" is conceptualized in terms of a "Principle of Preference-Basedness", according to which rational choice should be isomorphic to asserted preference. The main result characterizes axiomatically a new choice-rule called "Simultaneous Expected Utility Maximization" which in particular satisfies a choice-functional independence and a context-dependent choice-consistency condition; it can be interpreted as the fair agreement in a bargaining game (Kalai-Smorodinsky solution) whose players correspond to the different possible states (respectively extremal priors in the general case).

Keywords: Ignorance, Ambiguity, Multiple Priors. Rational Choice, Incomple Preference, Robustness. Independence, Sure-Thing Principle, Context-Dependence. Choice Consistency.

1. INTRODUCTION

Decisions often have to be made on the basis of limited information. Sometime, this does not present any special difficulties to the decision maker; he may still be willing to rank all alternatives in a complete order and simply choose the best alternative. In other cases, this informational limitation may be perceived as a lack of adequate grounds for constructing such a ranking unambiguously; rather than arbitrarily declaring one of two alternatives superior, or both to be indifferent, it will seem more natural to acknowledge this lack and suspend judgment by asserting the *non-comparability*[1] of the two alternatives; the decision-maker's preferences are then to be described by a partial rather than a complete ordering.

In this paper, we deal with situations in which incompleteness arises from limited information about the likelihood of uncertain event?. In formal terms, we will consider partial orders R that satisfy all of the standard consistency conditions characteristic of Subjective Expected Utility (SEU) preferences, with the exception of the completeness axiom. Such partial orders can be represented as unanimity-relations (intersections) of the SEU-orders associated with convex sets of probability measures ("belief sets" of "acceptable priors")[2].

[1] Noncomparability is distinguished from genuine indifference by its lack of transitivity. Indeed, non-comparability is typically robust with respect to small (unambiguous) changes in the value of the alternatives. This is a typical feature of "hard" choices. For example, if you find it difficult to decide whether to accept a job-offer at a salary of x dollars per year, you will find it just as difficult to decide at $x + 1$ dollars, probably also at $x + 100$, maybe even at $x + 10000$ dollars. (While you will probably be able to tell the difference between x and $x + 10000$ dollars, this may not settle the matter for you, as money may simply not be the real issue.)

[2] This follows from standard representation theorems, e.g. Smith (1961) and Bewley (1986). Partial orders with the assumed structure have received a mathematically comprehensive and conceptually profound treatment in Walley's recent monograph "Statistical Reasoning with Imprecise Probabilities" (1991). Belief-functions and upper-and lower probabilities, other Frequently endorsed generalizations of the probability calculus, ran be viewed as special (and restrictive) instances of assessing such partial orders (see Walley (1991), ch. 4, especially p. 182-4 and 197-9).

For i tance, the extreme case of "complete ignorance" regarding the likely occurrence of uncertain events is represented by a maximally incomplete partial order in which the decision-maker weakly prefers one act over another if and only *if* the act generates a weakly better consequence in every state; this corresponds to a belief set that includes all possible priors.

For another example, a decision-maker who is a classical statistician may well be prepared to assume qualitative knowledge about the stochastic process generating the observations, but will not want to make probabilistic assumptions concerning parameter values. Such qualitative knowledge can be described by a partial order R, for instance in terms uf condition? of " xchangt ability"[3]; the corresponding belief set may include all priors consistent with the assumed qualitative knowledge.

The paper develops a theory of rational (or "optimal") choice for "decision-problems under uncertainty" (d.p.u.s) defined by a set of acts X and a partial order R on some universe of acts primarily focussing on d.p.u.s characterized by complete ignorance. "Optimality" for partial orders has been traditionally identified with the absence uf feasible superior alternatives ("admissibility"). By contrast, we will argue that optimality is nut exhausted by admissibility in that some admissible acts may be superior to others (in a context-dependent way) as *compromise* choices. The choice rule analyzed in this paper, "Simultaneous Expected Ctility Maximization" (SIMEU). makes this intuition of optimal choice as a best compromise formally precise and provides an axiomatic justification for it. As explained in detail in section **2.** SIMEU can be interpreted as Kalai-Smorodinsky bargaining solution representing a fair compromise among "alter egos" corresponding to the different extremal priors; it can also be seen as formalizing a notion of robustness."

[3]The classical reference is de Finrtti (1937); for a discussion of exchangeability in the context of partial orders, *see* Walley (ch. 9.5).

[4]It should he emphasized that the normative motivation of the axioms does not appeal to an intrinsic preference for robustness or to the intuitive force of the bargaining metaphor. If such

Of the full axiomatic theory underlying SIMEU, the present paper presents half, namely the limiting case of "maximally non-comparable" preferences characterized by all-inclusive belief sets; these turn out to correspond to the classical notion of "Complete Ignorance" (CI); for **a** still valuable introduction to the literature, which culminated in the early 1950's prior to Savage's "Foundations of Statistics" (1954), see Luce/Raiffa (1957), ch. **13.** CI problems can be viewed as reduced forms of general d.p.u.s, i.e. that a large class of choice rules defined on CI problems can be canonically extended to the class of general d.p.u.s. This has been shown in Nehring (1991, ch.2) and ds Nehring (11192) : how it works is briefly sketched in section 7.

The main conceptual innovation of the paper in a new rationale fur axiomatic restrictions on choices in CI problems, the "Principle of Preference-Basedness" (PPB), according to which the structure of choice should reflect the structure of preference. This Principle yiel a more convincing justification of the hallmark axiom of the classical literature (here fornulated as "Symmetry") that leads beyond expected utility maximization; Symmetry says that since CI preferences are symmetric with respect, to arbitrary event-permutations, CI choices must be symmetric in the same way.

The Principle of Preference-Basedness also gives rise to the other key axiom of "Consequence-Isomorphism" (CISO) which has no precedent in the cl sical literature. This axiom implies invariance of the choice rule with respect to positive affine transformations of consequence utilities event-by-event, and thus permits interpreting CI problems as bargaining problems. It makes little sense unless one views CI preferences as deliberately adopted (over other logically possible preference judgments) by the decision maker who is fully aware of their extreme character. as Theorem 1. the main result of the paper, characacterizes SIMEU as equivalent to the conjunction of Admissibility, Symmetry, Consequence Isomorphism and a context-dependent,

appeals were made, the solution could hardly stake a plausible claim of being canonical. Robustness and compromise simply turn out to describe qualitative features of the solution; they confirm but don't ground its normative claim.

choice-consistency condition WAREP

The classical literature was keenly aware of a conflict between context-independent. choice-consistency conditions and independence[5], and found itself unable to choose. Probably representatively, Arrow (1960, p. 72) concluded that a rational solution to complete ignorance problems is impossible: "Perhaps the most nearly definite statement is that of Milnor (1954) who showed in effect that every proposed ordering principle contradicts at least one reawnable axiom." Based on the Principle of Preference-Basedness which implies a choice-functional independence condition by way of Consequence-Isomorphism, SIMEU theory resolves the conflict in favor of independence over context-independent choice-consistency.[6] We will argue in section 6.3 that what appear to be "inconsistencies" of choice from a traditional perspective can be seen as natural consequences of asserted non-comparabilities.

That it is not the independence axiom that should give way is confirmed by obsewing that the must convincing direct justifications of independence derive from the logical structure of states as logically disjoint *ex* post. Yet ignorance is part of the decision situation *ex ante*, and, as a result, has no power per se to undermine the normative appeal of the independence axiom.' Completeness, on the other hand, and in its wake context-independent choice-consistency, seem natural casualties of "ignorance".

We conclude the introduction with some meta-remarks about the general approach that are important to an adequate understanding of what will follow. In the proposed theory, partial orders determine choices as well-defined wholes[R]; this is meaningful

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[5] often also referred to as "the sure-thing principle."

[6] In fact, CISO can be shown to be equivalent to an appropriate generalization of the "sure-thing principle" to d.p.u.s (see 6.2.6).

[7] Of course, unresolved ignorance may cause anxiety etc., and thereby lead to Ellsbergian uncertainty-aversion; this prima facie violation of independence can be accommodated by an appropriate re-description of consequences.

[8] I.e. the optimal choice may depend on every facet of the partial order.

only under an *exhaustive* interpretation, on which absence of weak preference (of x over y and of y over x) is equivalent to a judgment of non-comparability ("I decline to prefer one alternative over another"). rather than to mere "non-comparedness" as under an interpretation in terms of *partial elicitation* ("I have not made up my mind"). In terms of beliefs, non-comparability corresponds to *self-aware* ignorance, as in "I know that I don't know" (cf. section 6.1.4).[9]

In the terms of logical status, the partial order R represents an exhaustive list of preference/belief *judgments;* its status is thus akin to that of a statistical model, but differs in category from that of a psychological state or behavioral disposition. The envisaged theory determines the content of optimal choice for a wide class of hypothetically asserted partial orders R in the presence of general rationality conditions which are formalized as axioms. It is a "normative" *logic of choice given preference*, rather than "prescriptive" advice with the goal of improved decision making. Just as in the axiomatization of a social-choice rule, the issue of the decision-maker's competence or computational resources never arises.

The paper is structured as follows: Section 2 presents and interprets the SIMEU choice rule for general d.p.u.s in the two-event case. In Section **3,** the formal framework is established. Sections 4 and 5 contain the key rationality postulates of the theory and axiomatize the SIMEU solution. **A** side result characterizes the lexicographic maximin-rule which is also shown to coincide with Barbera-Jackson's (1988) "protective criterion". The axioms are discussed extensively in section 6, with particular emphasis on the Principle of Preference-Basedness. The concluding section 7 briefly sketches the extensiun of SIMEU to general d.p.u.s . The appendix contains bits of extra material and the proofs.

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[9]In the language of epistemic logic, satisfaction of Negative Introspection is a key properly of complete ignorance, which therefore has nothing to do with "unawareness" in the sense of the recent literature on that topic, for which violation of Negative Introspection is deemed essential (cf. Modica-Rustichini (1994), Dekel-Lipman-Rustichini (1996)).

2. PRELIMINARY EXPOSITION OF SIMEU

This section is devoted to the explanation of the SIMEU choice rule for general partial orders in the two-event case. An act $x \in \mathbf{R}^2$ maps consequences to cardinal utilities[10]. A belief set Π is a closed convex subset of $\mathbf{A}^2$, the unit simplex of $\mathbf{R}^2$; its elements are called "acceptable", its extreme points π' and π'' "extremal" priors. A "consistent" partial order R on $\mathbf{R}^2$ is one that can be represented as the unanimity relation R_Π induced by a belief set Π:

$x \; R_\Pi \; y$ if and only if $\pi \cdot x \geq \pi \cdot y$ for all $\pi \in \Pi$.

Note that unanimity with respect to all extremal priors coincides with unanimity with respect to all acceptable ones.

A two-event decision-problem under uncertainty can then be specified as a pair (X, Π), where X denotes the choice-set, a convex and compact subset of $\mathbf{R}^2$; if $\Pi = \mathbf{A}^2$, the d.p.u. is one under complete ignorance. For simplicity, assume that X is strictly convex such that, for all $\pi \in \Delta^2$, there is a unique expected-utility maximizing act $x(\pi)$.

An undisputed necessary condition of the optimality of an act x is its "admissibility," i.e., the absence of any feasible alternative that is strictly preferred to it. In the two-dimensional case, the set of admissible acts $\mathcal{A}(X, \Pi) = \mathcal{A}(X, R_\Pi) = \{x \in X \mid$ for no y $\in$ X: $yR_\Pi x$ and not $xR_\Pi y\}$ traces out the boundary of X between x' and x'', the optimal acts under π' and π'' respectively[11]: see figure 1 below. $\mathcal{A}(X, \Pi)$ may be understood as the set of acts that compete for enactment. – While clearly necessary, is admissibility *sufficient* as a criterion of optimality for partial orders?

Note that a positive answer to this question would imply that it were always legitimate to arrive at a decision by selecting a complete order $R_{\{\pi\}}$ that extends the given partial order R_Π, with $\pi \in \Pi$. Since $\mathcal{A}(\cdot, R_{\{\pi\}}) \subseteq \mathcal{A}(\cdot, R_\Pi)$, any choice

[10] These can be derived from a standard representation theorem (rf. section 3).

[11] assumed to be unique for simplicity.

optimal under $R_{\{\pi\}}$ would then also he optimal under the original partial order R_Π. A decision-maker could thus never loose by adopting complete preferences: some decision must be made — some act will be chosen, after all — so what use is it to suspend judgment if you cannot suspend choice? At worst, some preference judgment might be arbitrary The concept of non-comparability would be useless for the purpose of decision-making, *pragmatically incoherent.*

To salvage the pragmatic coherence of assertions of non-comparability, it is thus necessary to show admissibility to be insufficient as exclusive criterion of optimality, by providing additional criteria. One such criterion is that of *robustness.* Intuitively speaking, an alternative lacks robustness as an optimal choice, if it is a very poor choice from the perspective of some extremal prior. In Figure 1, choices of x' or x'' exemplify failures of even "minimal robustness:" while each act performs best against some prior (π' respectively π''), it performs worst against its opposite (i.e., π'' respectively π') compared to any other admissible act. Robustness requires at a minimum choosing an act somewhere in between x' and x". An alternative is "optimal in terms of robustness" if it minimizes the risk of being a poor choice; the SIMEU choice rule axiomatized in this paper can be interpreted as making this notion precise. – It should be emphasized, however, that while the robustness interpretation helps to make sense of the proposed choice rule, the axioms themselves do not rely on the intuitively rather vague notion of "robustness;" instead, they rely on the much sharper concept of "structural isomorphism"

The "Simultaneous Expected-Utility Maximization" (SIMEU) rule σ is robust in the sense of "implementing" each extremal prior π' and π'' "to the same degree". It is based on a cardinal measure λ of the "degree of implementation" defined as follows.

$$\lambda(x,\pi;X,\Pi)=\frac{\pi\cdot x-\min\{\pi\cdot y\,|y\in\mathcal{A}(X,\Pi)\}}{\max\{\pi\cdot y\,|y\in\mathcal{A}(X,\Pi)\}-\min\{\pi\cdot y\,|y\in\mathcal{A}(X,\Pi)\}},$$

with $0/0=1$ by definition.

We will often suppress the arguments X and Π. In effect, $\lambda(\cdot,\pi)$ is the von Neumann-Morgenstern representation of the EU preferences induced by π such that $\max\{\lambda(y,\pi)\,|y \in \mathcal{A}(X,\Pi)\} = 1$ and $\min\{\lambda(y,\pi)\,|y \in \mathcal{A}(X,\Pi)\} = 0$. For example $\lambda(x'',\pi'') = 1$ and $\lambda(x'',\pi') = 0$.

The SIMEU choice rule σ is defined as the unique act that is admissible and implements both virtual probabilities to the same degree:

$$x \in \sigma(X,\Pi) \Longleftrightarrow x \in \mathcal{A}(X,\Pi) \text{ and } \lambda(x,\pi'') = \lambda(x,\pi').$$

It is easily verified that $\sigma(X,\Pi)$ can equivalently he defined as the unique maximin in degrees of implementation, i.e.,

$$\sigma(X,\Pi) = \arg\max_{x\in X}\ \min(\lambda(x,\pi'),\lambda(x,\pi'')).$$

Geometrically, σ can be constructed as follows:

Figure 1 about here

Define two reference points y^1 and y^0 where π'' and π' simultaneously achieve their maximal and minimal expected utilities. y^1 is thus defined by the conditions $\pi''\cdot y^1 = \pi''\,.\,x''$ and $\pi'\,.\,y^1 = \pi'\,.\,x'$, i.e., as intersection of the indifference-lines for π'' through x'' and for π' through x' . Similarly, y^0 is defined by $\pi''\,.\,y^0 = \pi''\,.\,x'$ and $\pi'\,.\,y^0 = \pi'\,.\,x''$.

By co truction, $\lambda(y^1,\pi'') = \lambda(y^1,\pi') = 1$ and $\lambda(y^0,\pi'') = \lambda(y^0,\pi') = 0$. By the affine definition of A, setting $y^\gamma = \gamma y^0 + (1-\gamma)y^1$, $\lambda(y^\gamma,\pi'') = \gamma = \lambda(y^\gamma,\pi')$; the straight line through y^1 and y^0 describes therefore the locus of acts that implement π'' and π' to the same degree. $\sigma(X,\Pi)$ is given as the intersection of this line and the admissible set $\mathcal{A}(X,\Pi)$.

It is easy to see from this construction that σ is formally identical to the Kalai-Smorodinsky (1975) solution to a bargaining problem with two players whose preferences are the EU preferences with respect to π' and to π''. Technically speaking.

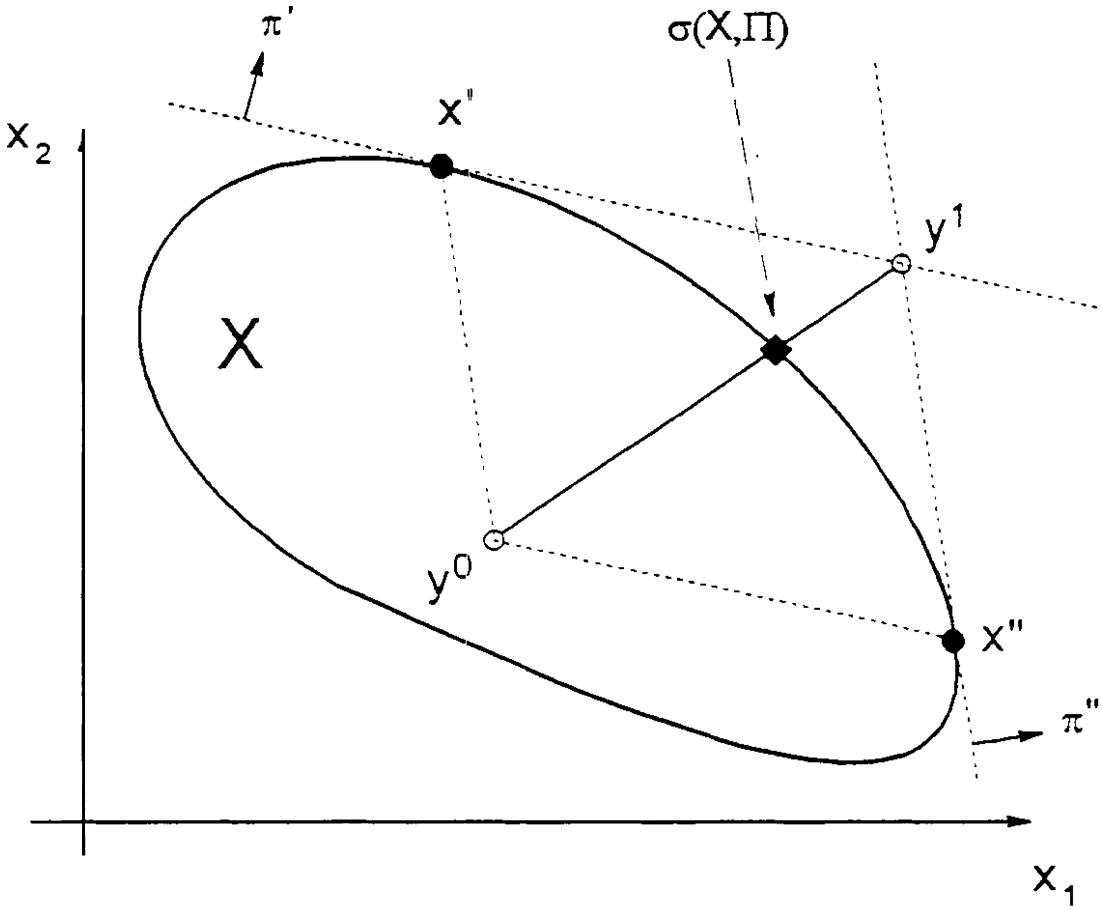

Figure 1

define a mapping $\Psi : \mathbf{R}^2 \rightarrow \mathbf{R}^2$, $\Psi(x) = (\pi' \cdot x, \pi'' \cdot x)$; Ψ maps into vectors of (expected) utilities and is one-to-one. If $\xi(Y, d)$ is defined as the Kalai-Smorodinsky solution for a feasible set of utilities Y and a "threat-point" d, σ can be characterized by

$$\xi(\Psi(X), \Psi(y^0)) = \Psi(\sigma(X, \Pi)).$$

Note that while y^0 is the threat-point (in act space), y^1 is the "ideal point" in the terminology of Kalai and Smorodinsky.

To establish comparability to the definition of σ, we shall also write ξ as $\bar{\xi}$ in terms of the primitives: the feasible set X and the set of player's preferences Π. I.e., we shall write $\bar{\xi}(X, \Pi)$ for $\Psi^{-1}(\xi(\Psi(X), \Psi(y^0)))$.

The equivalence can then be restated as

$$\sigma(X, \Pi) = \bar{\xi}(X, \Pi).$$

One can use this purely formal equivalence to interpret σ as the fair outcome of a bargaining between the different fictitious "alter egos" of the decision maker given by his extrenial priors. his different virtual Bayesian selves, as it were.

This interpretation of o as a fair bargaining solution extend? to the general (finite) case: one can define $\sigma(X, \Pi) = \bar{\xi}(X, \Pi)$, where $\bar{\xi}$ refers to the lexicographic variant of the KS-solution which has been defined and axiomatized by Imai (1983)[12].

3. FRAMEWORK AND NOTATION

Let Ω denote an infinite universe of states, and let $\mathfrak{F}$ be the set of finite partitions $F = \{S\}_{S \in F}$ of Ω into infinite subsets S. Note that.: by definition, any $F \in \mathfrak{F}$ is

[12] $\bar{\xi}$ provides an easy way to thematize the role of extremal priors. A plausible alternative to the definition of SIMEU as σ would be as $\sigma^{\infty}(X, \Pi)$ $\bar{\xi}(X, co\ \Pi)$; this is discussed in detail in Nehring (1991, ch.2.5), with arguments suggesting the superiority of lhr adopted specification of SIMEU as σ. For the moment, just note that while in higher dimensions the two specifications may easily differ, in two dimensions they are always identical; this has been shown in Nehring (1991, ch.2), proposition 6.

infinitely divisible in the sense that any event of any partition in $\mathcal{F}$ can be broken up into arbitrarily many subevents[13]; the role of this assumption is explained in remark 1 following theorem **2.**

An act x maps states to consequences $c \in K$: $x : \Omega \rightarrow K$. For expositional simplicity, we will assume $K = [0, 1]$, interpreting c as cardinal utility (normalized von-Neumann Morgenstern utility); such an interpretation can be justified by standard arguments along the lines of Anscombe-Aumann's (1963) two-stage "horse-lottery" approach[14]. In particular, in a world with only two final consequences ("winning" and "loosing", with winning preferred), x_ω can be identified with the objective probability of winning conditional on ω.

A well-defined *choice set* is assumed to be closed with respect to the inclusion of mixed acts, and is therefore formally represented as a convex set of acts $X \subseteq [0, 1]^\Omega$. To canonically include mixed acts is technically necessary and seems to be the more conservative way to proceed outside SEU-theory. Otherwise, standard choice rules recommend the decision-maker to give up utility in order to use a random device; this seems inappropriate since, presumably, he could just toss a coin in his head.

For $F \in \mathcal{F}$, let $[0,1]^F$ denote the class of F-measurable[15] acts, and denote $[0, 1]^{\mathcal{F}} = \bigcup_{F \in \mathcal{F}} [0,1]^F$, the class of simple acts . A choice-set X is *simple* if it is a compact convex subset of $[0, 1]^F$ for some $F \in \mathcal{F}$ ($[0, 1]^F$ being endowed with the Euclidean topology); let $\mathbb{X}$ denote the class of all simple choice-sets. Some additional notation: "$cl\ X$" is the closure of X, "$co\ X$" is the convex hull of X, and $[x, y] = co\,\{x, y\}$. "$x < y$" holds if $x \leq y$ and $x_\omega < y_\omega$ for some $\omega \in \Omega$, "$x \ll y$" if $x_\omega < y_\omega$ for all $\omega \in \Omega$; e^S denotes the indicator-function of S, i.e., $e^S_\omega = 1$ if $\omega \in S$, and $e^S_\omega = 0$ otherwise.

A *decision problem under Complete Ignorance* ("CI problem") is a pair $(X, R_\emptyset)$.

[13] I.e., for each $F \in \mathcal{F}$ and each $\#F$-tuple of natural numbers $(n_s)_{s \in F}$, there exists a refinement G of F in $\mathcal{F}$ such that $\#\{T \in G | T \subseteq S\} = n_s$.

[14] For an exposition of the theory that does not assume (but effectively reduces to) [0,1]-valued consequences, see Nehring (1995).

[15] x is F-measurable iff it is constant on each cell $S \in F$.

where X is a choice set and $R_\emptyset$ denotes the Complete Ignorance preference relation defined by

$$x \; R_\emptyset \; y \iff x_\omega \geq y_\omega \quad \text{for all } w \in \Omega.$$

Since $R_\emptyset$ is assumed fixed in almost all of the following, we will normally identify a CI problem $(X, R_\emptyset)$ with its choice set X, and define a choice function as a non-empty-valued mapping C on X such that $C(X) \subseteq X$ for all $X \in X$. We will write "$x \; P_\emptyset \; y$" for "$x \; R_\emptyset$ y and not y $R_\emptyset$ x", as well as "$x \; N_\emptyset \; y$" for "neither $x \; R_\emptyset$ y nor y $R_\emptyset \; x$".

In discussing various axioms, it will sometimes be helpful to refer to partial orders R other than $R_\emptyset$ contained in some universe R of hypothetical orders on $[0,1]^{\mathcal{F}}$. For this purpose, it suffices to think of R as a rich class of partial orders obtained as the intersections (unanimity relation?) of sets of expected-utility orders $R_{\{\pi\}}$, with π denoting a (finitely additive, say) probability measure on Ω and

$$x \; R_{\{\pi\}} \; y \iff \int x_\omega d\pi \geq \int y_\omega d\pi$$

The technical details are omitted; we just note that such classes can be axiomatized along the lines of standard representation theorems in the literature[1']. In such more general contexts, a d.p.u. is a pair $(X, R) \in$ X x R, and a choice function is defined on the domain X x R of such pairs.

4. SIMEU AND LEXIIMIN: DEFINITION AND BASIC PROPERTIES.

The following sections are devoted to an axiomatization of SIMEU for Complete-Ignorance problems, σ^{Cl}. Along the way, we also obtain a choice-functional characterization of the lexicographic maximin rule LM defined as follows.

$$LM(X) = \{x \in X \mid \text{For ally} \bullet X : \min_{\omega: x_\omega \neq y_\omega} x_\omega \geq$$

[16]See Smith (1961), Bewley (1986) and in great generality Walley (1991), as well as Nehring (1995) for a statement directly appropriate to SIMEU theory

$\min_{\omega: x_\omega \neq y_\omega} y_\omega\}$, with $\min\ \emptyset = -\infty$

As it reads, we have defined $LM(X)$ as Barbera-Jackson's (1988) "protective criterion". Since the following proposition shows it to coincide (on *convex* sets) with the lexicographic maximin, we will denote it by LM and refer to it by the latter, more informative name.

The SIMEU rule σ^{Cl} modiies LM by normalizing ex-post utilities; the normalization yields "degrees of implementation" $\lambda_\omega(x)$ of x within X in state ω (respectively: "for each extremal prior e^ω ").

$$\lambda_\omega(x) = \frac{x_\omega - \inf_{y\in A(X)} y_\omega}{\sup_{y\in A(X)} y_\omega - \inf_{y\in A(X)} y_\omega}, \qquad \text{with } 0/0 = 1 \text{ by convention.}$$

Also, define

$$\sigma^{Cl}(X) = \{x \in X \mid \text{For all } y \in X : \min_{\omega:\lambda_\omega(x)\neq\lambda_\omega(y)} \lambda_\omega(x) \geq \min_{\omega:\lambda_\omega(x)\neq\lambda_\omega(y)} \lambda_\omega(y)\}.$$

Lastly, for the sake of comparison, the following lexicographic version ("LML") of the "minimax loss" rule first proposed by Savage (1951) is of some interest:

$$LML(X) = LM(X - m(X)), \qquad \text{with } m(X) = (\max_{x\in X} x_\omega)_{\omega\in\Omega}$$

Proposition 1 *i) If $X \in \mathcal{X}$, $LM(X)$ and $\sigma^{Cl}(X)$ are non-empty and single-valued*

ii) Moreover, if $x \in LM(X)$ and $y \in X\backslash\{x\}$,

$\min_{\omega:x_\omega\neq y_\omega} x_\omega > \min_{\omega:x_\omega\neq y_\omega} y_\omega$.

Similarly, if $x \in \sigma^{Cl}(X)$ and $y \in X\backslash\{x\}$,

$\min_{\omega:\lambda(x_\omega)\neq\lambda(y_\omega)} \lambda(x_\omega) > \min_{\omega:\lambda(x_\omega)\neq\lambda(y_\omega)} \lambda(y_\omega)$

Remark: Part ii) is crucial for the logical consistency of the subsequent axiomatization. The convexity-assumption on X is indispensable for its validity, as the counter-example of $X = \{(1,0),(0,1)\}$ shows."

[17]Taking any F such that $X \subseteq [0,1]^F$, and viewing $[0,1]^F$ as a fintie-dimensional unit-cube, thr proposition also implies that the unique $x \in LM(X)$ coincides with lexicographic maximin act as

5. AXIOMATIZATION OF SIMEU AND LEXIMIN

This section characterizes SIMEU and LM in complete ignorance problem?; while the relevant axioms are given a first-round motivation. a more extensive discussion of their meaning and plausibility is reserved for the next section.

The most basic rationality-requirement is compatibility with asserted preferences.

Axiom 1 (Admissibility, ADM) *For all* X $\in X$ *and* $x, y \in X$: $x\ P_\emptyset\ y$ implies $y \notin C(X)$.

If one rewrites the condition "$x\ P_\emptyset\ y$" in utility-terms as "for all $\omega \in \Omega$, $x_\omega \geq y_\omega$, and for some $\omega \in \Omega$, $x_\omega > y_\omega$", it is evident that ADM amounts to the standard concept of *strict* admissibility.

The two key axioms of the theory are axioms of structural equivalence. The first is based on the symmetry of $R_\emptyset$ in events. For any one-to-one map $\phi : F \to F'$ on event partitions E', $F' \in \mathcal{F}$, define an associated one-to-one map on acts $\Phi : [0.1]^F \to [0,1]^{F'}$ by $\Phi(x)_{\phi(s)} = x_s$, for $S \in$ F. $\Phi(x)$ is the act that results if the consequence x_S occurs in the event $\phi(S)$ instead of in the event S.

Axiom 2 (Symmetry, SY) *For all* $X \in \mathcal{X}$ *and* $\phi : F \to F$ *one-to-one* such *that* X is *F-measurable:* $\Phi(X) = X \Rightarrow C(X) = \Phi(C(X))$.

SY requires that symmetry of the choice set in events implies a corresponding symmetry of the chosen set. It is a weak version of the hallmark axiom of the CI literature *(see* remark 1 following theorem 1): it clearly rules out the representability of C by some (as-if) subjective probability, as shown by the following example.

Example 1: The following matrix describes the payoffs of four acts in terms of the event-partition $F^* = \{S_1, S_2, S_3\}$.

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defined **ordinarily for finite-dimensional Euclidean spaces.**

	S_1	S_2	S_3
w	1	0	1
x	1	1	0
y	0	1	1
z	1	0	0

Suppose C to be representable by the as-if subjective probability vector (π_1, π_2, π_3). SY applied to the choice set $[w,x]$, with $F = F^*$ and ϕ given by $\phi(S_1) = S_1$, $\phi(S_2) = S_3$, and $\phi(S_3) = S_2$, implies $x \in C([w,x]) \Leftrightarrow w \in C([w,x])$, and thus $\pi_2 = \pi_3$. An analogous application of SY to the choice set $[w,y]$ yields $\pi_1 = \pi_2$, and thus $\pi_1 = \pi_2 = \pi_3 = \frac{1}{3}$. However, applying SY to $[y,z]$ with $F = \{S_1, S_2 \cup S_3\}$ and ϕ given by $\phi(S_1) = S_2 \cup S_3$, and $\phi(S_2 \cup S_3) = S_1$ implies $y \in C([y,z]) \Leftrightarrow z \in C([y,z])$, and thus $\pi_1 = \pi_2 + \pi_3$, a contradiction. □

The conceptual basis of the symmetry axiom is clarified by viewing it as special instance of a more general condition of "Event-Isomorphism". A mapping $\Phi : [0,1]^F \to [0,1]^{F'}$ obtained from some bijection $\phi : F \to F'$ is an *event-isomorphism* with respect to R if, for all $x, y \in [0,1]^F : x \; R \; y \Leftrightarrow \Phi(x) \; R \; \Phi(y)$.

Condition 1 (EISO*, Event-Isomorphism) *For any event-isomorphism* $\Phi : [0,1]^F \to [0,1]^{F'}$ *with respect to* R *and any* $X \subseteq [0,1]^F : C(\Phi(X), R) = \Phi(C(X, R))$.

EISO' asserts, in words, that it does not matter per se which events yield certain consequences as the result of specific acts, as long as the partial preference ordering is unaffected by the substitution of events. Note that, due to its taking account of the underlying preference ordering R, EISO' is perfectly compatible with expected-utility maximization for complete orders. Indeed, it is obvious that in this case Φ is an event-isomorphism with respect to R if and only if ϕ preserves the subjective probability of events.

Since any Φ is an event-isomorphism with respect to $R_\emptyset$, one obtains as restriction of EISO' to $X \times \{R_\emptyset\}$ the following condition which implies SY.

Condition 2 (EISO) For all X $\in \mathcal{X}$ and $\phi : F \to F'$ one-to-one such *that* X *is* F-measurable: $C(\Phi(X)) = \Phi(C(X))$.

CI-proble are thus treated completely on par with other d.p.u.s by EISO*; they are special only in the extreme richness of symmetries of $R_\emptyset$ which makes the application of EISO' extraordinarily powerful.

SY and EISO' can be derived from a more general "Principle of *Preferences-Basedness*" (PPB), according to which the structure of choice should reflect the structure of asserted preference. **A** basic instance of this principle is the axiom of admissibility, which can be viewed as a requirement to map asymmetries of preference into asymmetries of choice. A complementary co equence of the principle is the requirement to map symmetries of preference into symmetries of choice. This can be formulated in term.; of conditions of "invariance with respect to order-preserving
ms
transformations," of which event-isomorphisms are a special case.

Based on a "dnal" class of order-preserving transformations, the PPB leads to a "dual" axiom called "Consequence-Isomorphsm" (CISO) which allows events to matter, but does not permit consequences to matter per se. The basis of the argument for
ns
CISO is that non-preference information properly defined, specifically: consequence-information, should be rationally irrelevant in the determination of choice.

To define CISO formally, let a *consequence-isomorphism* with respect to R be a mapping θ from $[0,1]^{\mathcal{F}}$ to $[0,1]^{\mathcal{F}}$ (not necessarily onto) that preserves order as well as mixture-information about acts and is separable in events, i.e., that satisfies

i) $\theta(x)$ R $\theta(y) \Leftrightarrow x$ R y $\quad \forall x, y \in [0,1]^{\mathcal{F}}$,
ii) $\theta(\lambda x + (1-\lambda)y) = \lambda\theta(x) + (1-\lambda)\theta(y) \quad \forall x, y \in [0,1]^{\mathcal{F}}$, $0 \leq \lambda \leq 1$, and
iii) There exist $(\theta_\omega)_{\omega\in\Omega}, \theta_\omega : [0,1] \to [0,1]$ such that $\theta(x) = (\theta_\omega(x_\omega))_{\omega\in\Omega}$.

Axiom 3 (CISO, Consequence-Isomorphism) *For* all $X \in \mathcal{X}$ and any *consequence-isomorphism θ with respect to $R_\emptyset$ such that $\theta(X) \in \mathcal{X}$, $C(\theta(X)) = \theta(C(X))$.*

Remark 1: It is easily verified that for *CI problems*, θ is a consequence-isomorphism if and only if each θ_ω is of the form $\theta_\omega(c) = \alpha_\omega c + \beta_\omega$, with $\alpha_\omega > 0$. Hence CISO requires invariance under positive affine transformations state-by-state,

Remark 2: The mixture-condition ii) reflects the need to preserve cardinal-utility information; as is well-known from bargaining theory, without it, no interesting theory could be developed. Note also that it is automatically satisfied by the event-isomorphisms considered in EISO.[18]

As discussed in more detail in the following section, the preceding three axioms are incompatible with traditional context-independent choice-consistency conditions such as WARP.

Axiom 4 (WARP) *For all* $x, y \in X \cap Y : x \in C(X) \Rightarrow [y \in C(Y) \Rightarrow x \in C(Y)]$

In words: if x is chosen in X, it is "revealed to be at least as choice-worthy as any alternative y in X, hence must be chosen in Y whenever y is.

To accommodate the PPB, it seem.. natural to contain the impact of context-dependence by restricting WARP to pairs of decision problems for which it is unproblematic. A move of this kind is quite standard in bargaining theory (see, for instance, Roth (1977)).

X and X' are *range-equivalent* if proj, $cl\ \mathcal{A}(X) =$ proj, $cl\ \mathcal{A}(X')$ for all $\omega \in \Omega$, that is, if they agree on the set of "admissible consequences" in each state.

Axiom 5 (WAREP) *For any mnge-equiualent* $X, X' \in X$ *and* $x, x' \in X \cap X' : x \in C(X) \Rightarrow (x' \in C(X') \Rightarrow x \in C(X'))$.

[18] It would seem to be desirable to unify EISO and CISO in a general axiom of invariance with respect to order-preserving information. We leave this to future work, as it is mathematically not entirely trivial, raises further subtle issues and since a unified axiom would not seem to simplify the demonstration of the results

While WAREP does not seem to rest on quite as compelling a foundation as the other axioms, it has the definite merit of leading to a tractable and nicely interpretable solution. Moreover, it is weak in the sense of being satisfied by all major CI-solutions proposed in the literature, and also in that it does not determine the qualitative character of the choice rule, for which SY and CISO are responsible.

Theorem 1 σ^{CI} is *uniquely characterized* by Admissibility, Symmetry, *Consequence-*Isomorphism and *WAREP*.

If one insists on preserving context-independence, at least one of the other axioms has to go. If one drops CISO, a charac eriz ιtion of leximin is obtained by a much simplified proof.

Theorem 2 *LM is uniquely characterized by Symmetry, Admissibility* and WARP.

Remark 1: Theorems 1 and **2** appear to be unique in the literature in using only symmetry besides the shared assumptions of admissibility and choice-consistency (as well as CISO in the case of theorem 1). From Milnor **(1954)** on ***(see*** also Luce/Raiffa (1957)), most use in addition an axiom based on some idea of description-invariance. This conceptually not unproblematic requirement can be dispensed with due to the infinite-divisibility assumption on the partitions $F \in \mathcal{F}$. It has been the main reason for making that assumption in the first place.

Remark 2: The two theorems are the first in the literature that make Symmetry and *strict* Admissibility compatible without an ad-hoc qualification of the axioms. The problem of their apparent incompatibility has in fact been (at least implicitly) a major issue of the CI-literature in the 80's. Maskin (1979), the first contribution to that issue, had to impose an ad-hoc restriction on the applicability of "Column Duplication", Barbera/Jackson (1988) in effect restrict the requirement of preference completeness. Lastly. Cohen and Jaffray (1980, 1983) felt forced to demand only "approximate satisfaction" of certain conditions.

Remark 3: Theorems 1 and 2 are also unique among axiomatizations of "maximin-type" solutions in that they do not make any assumption of "uncertainty-aversion," be it in the form of a quasi-concavity condition on preferences, as Milnor (1954) and Barbera/Jackson (1988) do, or of a convex-valuedness assumption, which would be the choice-functional equivalent. We are enabled to drop such a condition by lemma 2 for which strict Admissibility is crucial.

In the literature, Complete Ignorance is defined in terms of finite universes of events; part 1 of the appendix shows how the characterization theorems of this section apply to finite universes via an embedding argument.

6. DISCUSSION

6.1 Symmetry and Event-Isomorphism

1. The PPB can be understood as a "*second-order consequentialism*" tying choices to asserted (consequentialist) preferences over acts and, as a result, to the valuation of consequences and expectations of events contained in those preferences. It rules out additional "security considerations" as in Isaac Levi's (1980, ch.7) theory. Nor is it compatible with the use of (non-preference) information about events, such as the existence of "ultimate and indivisible events" as in Keynes (1921, p.64) or the structure of language as in Carnap (1952).

Note that even if one grants the philosophical meaningfulness of such pieces of information, it is unclear why they should and how they could be relevant for decision-making. In particular, if a decision-maker is prepared to accept their relevance, should this not be reflected in his preference-judgments directly?

2. EISO* is appealing because it excludes the use only of information that typically is irrelevant anyway, namely information about the nature of events; its interpretation

requires care, however, since seemingly counterintuitive implications for CI-problems arise almost at once. Consider, for instance, example **1** of section **5.** SY implies both $x \in C([x,y]) \Leftrightarrow y \in C([x,y])$ and $z \in C([z,y]) \Leftrightarrow y \in C([z,y])$. This simultaneous equivalence might be viewed as conflicting with the evident superiority of x over z (due to the dominance $x > z$); that is, the argument would run, one should assert $\{x\} = C([x,y])$.

Yet, since the partial order $R_\emptyset$ already captures this dominance relation, an argument from dominance is simply an argument based on the decision-maker's own preferences. As such, it lacks force because first, the implied superiority of x to y conflicts with the asserted non-comparability of x and y, and secondly, because the argument is based on a transitivity principle: if z is indifferent to/weakly preferable to **y,** x is strictly preferable to z, then x must be strictly preferable to y. However, invoking this principle shows only via modus tollens that the equivalence of chuice $x \in C(X) \Leftrightarrow y \in C(Y)$ cannot he interpreted as indifference; but again, this simply reflects the fact that $R_\emptyset$ asserts non-comparability of x and y, not indifference, and that the non-comparability $N_\emptyset$ is inherently Intransitive

3. Note also the essential *context-dependence* of the asserted equivalence: for instance, SY does not imply $z \in C(co\ \{x,y,z\}) \Leftrightarrow y \in C(co\ \{x,y,z\})$; this context-dependence, combined with that of the companion axiom CISO, naturally engenders a context-dependent choice rule. A context-independent version of SY (i.e. one that asserts equivalences of the form "$z \in C(X) \iff y \in C(X)$, for all $X \ni y, z$") would conflict with *(strict)* Admissibility[19]. Such context-independent versions come with the very set-up in contributions which define alternatives as sets of possible consequences[20], as well as in Jaffray's **(1989)** mixture-space approach.

[19] as well as with the conjunction of weaker conditions of Admissibility and Independence (see 6.2.5. below)

[20] The must recent contribution to that literature, Nehring-Puppe (1995), contains further references

4. SY may also be viewed as expressing a "principle of insufficient reason". It is desirably weaker than the classical Laplacian version by merely asserting context-dependent equivalences of choice, not indifferences or equiprobabilities. This makes it possible to apply this principle to arbitrary event partition? simultaneously and thereby to genuinely reflect ***complete*** ignorance.[21]

It may seem that even it its revised version some knowledge on part of the decision-maker must implicitly be assumed to obtain any determinate restriction on choice.[22] Indeed, it *is* "assumed" that when asserting $R_\emptyset$ the decision-maker acknowledges and, in this sense, "knows of" his complete ignorance about events. In other words, the symmetry axiom and indeed the axiomatic approach as a whole are meaningful only on an exhaustive interpretation of $R_\emptyset$ as an incomplete preference relation.

6.2 Consequence-Isomorphism

1. Consider a typical instance of CISO

Example 2:

	S_1	S_2
x	1	0
y	0	1
y^ϵ	0	ϵ

Let $X = [x, y], X^\epsilon = [x, y^\epsilon]$, and assume $0 < \epsilon < 1$. Since $x\ N_\emptyset$ y as well as $x\ N_\emptyset$ y', and as X^ϵ can be obtained from X by replacement of consequences, CISO implies $y \in C(X)$ ***a*** $y^\epsilon \in C(X^\epsilon)$.

[21] Dating hark to the nineteenth century, there has been a long tradition of criticism of the principle **in** *its Laplacian form* which has been revived in recent years under the name of "non-informative Bayesian priors" (see Berger (1985, ch.3) lor a review and Walley (1991, ch.5) for an extended critique of non-informative priors).

[22] Otherwise, "something" would appear to come from "nothing". However, complete ignorance is not "nothing"; rather, it corresponds to an extreme and committed agnositicism

On first blush, this implication seems wild, since it holds for arbitrarily small positive ε. While it seems perfectly reasonable to choose y in X, who would not choose x over y^ϵ in X^ϵ? After all, x might be much better than y^ϵ (in S_1) which at best might only be slightly better (in S_2). Such a reaction forgets, however, that the decision-maker could have asserted this preference himself, but explicitly declined to do so by asserting x $N_\emptyset$ y^ϵ. There is no reason to patronize him and override his asserted preference.

2. In effect, CISO asserts the inappropriateness of inter-event comparisons of ex-post utility, whether in terms of utility levels as in maximin and its variants, or in terms of utility differences as in the minimax loss rule. CISO is thus responsible for the bargaining interpretation of the proposed choice rule.[23]

3. The case for CISO rests entirely on the Principle of Preference-Basedness; this principle is what justifies CISO's exclusion of evidently meaningful and prima facie important information such as the utility-differences between acts in different states. The PPB implies that while such differences are undoubtedly crucial in rational preference-formation, they are irrelevant once preferences have been formed – once preferences are controlled for, as one might put it in another jargon.

EISO, by contrast, does not involve the exclusion of any "apparently relevant" information, as argued above. In the light of this difference it becomes clear why no version of CISO has ever appeared in the traditional CI-literature while EISO-type axioms occupy such a central place.

[23] Indeed, from the formal point of view, the axiomatization of SIMEU under Complete Ignorance can be viewed as simply another characterization of the Kalai-Smorodinsky bargaining solution (with endogenous threat point) in the context of a variable (or infinite) number of agents. However, within the horizon of bargaining theory, our result seems to be of limited interest, since the axiom that is mainly responsible for the qualitative character of the choice rule ("Symmetry" in the infinite, and "Embedding" in the finite version) lacks appeal, for it amounts to assuming that the solution does not depend on the number of agents holding a particular preference ordering over social states.

4. It also follows that justified acceptance of CISO must be accompanied by acceptance of EISO. Thus, the class of hargaining solutions that make sense in the present context is severely restricted; in particular, EISO implies that the solution cannot depend on the number of players with identical preferences, as for instance adaptations of the Nash solution would imply. When WAREP is assumed in addition, the lexicographic Kalai-Smorodinsky solution is already uniquely singled out. Thus, its privileged status does not hinge implicitly on a special egalitarian concept of fairness between alter egos, as would be the case for instance in an axiomatization that relies on a Kalai-Smorodinsky type "monotonicity" axiom.

5. As EISO, CISO can be viewed as an instance of a general axiom of consequence-isomorphism CISO* for arbitrary R.

Condition 3 (CISO') *For any* $(X, R) \in \mathcal{X} \times \mathcal{R}$ *and any consequence-isomorphism* θ *with respect to* R, $C(\theta(X), R) = \theta(C(X, R))$

Observation 1 *i) For any* $R \in \mathcal{R}$, *any* θ *of the form* $\theta_\omega(c) = \alpha c + \beta_\omega$, *with* $\alpha > 0$, *is order-preserving with respect to* R.

ii) If R *is complete, any consequence-isomorphism w.r.t.* R *is of this form.*

Part ii) of the observation shows the compatibility of CISO* with expected-utility maximization for complete R. Part i) implies that choice-functions satisfying CISO' must satisfy the following choice-functional independence condition.

Condition 4 (IND, Independence) *For all* $X \in \mathcal{X}$, $x \in [0,1]^{\mathcal{F}}$ *and* $0 < \lambda \leq 1$: $C(\lambda X + (1-\lambda)x, R) = \lambda C(X, R) + (1-\lambda)x$.

6. In combination with the reduction condition CIR described in section 7, CISO implies, beyond independence, a "sure-thing principle" (STP) which determines for a particularly simple class of decision problems how choices respond to the "conditioning" of preferences that results from a partial resolution of the uncertainty. It is

shown in Nehring *(1991,* ch.1), that in the presence of CIR, STP and CIS0 are in fact equivalent.

8.3 On the Rationale for Context-Independence

It follows easily from examples 1 and 2 that for single-valued choice-functions the conjunction of EISO and CIS0 implies

$$x \; N_{\emptyset} \; y \Rightarrow C\left([x, y]\right) = \{\frac{1}{2}x + \frac{1}{2}y\} \; . \tag{1}$$

This "coin-flip property" (1) endows judgments of non-comparability with well-defined *operational* menning. It also entails that one cannot hope to reconcile these axioms with traditional context-independent choice–consistency conditions such a WARP. Indeed, the coin-flip property (1) violates even "contraction consistency" α.

Condition 6 ("α") *For all* $X, Y \in$ X **such** *that* $X \supseteq Y$ and $x \in Y : x \in C(X) \Rightarrow x \in C(Y)$.

To see how essential context-dependence is to SIMEU, consider in figure *1* of section *2* the subset X' of all acts in X above the straight line through y^0 and y^1. While $\sigma(X,\Pi)$ is still feasible in X', it is now worst against π' within the set of admissible acts $\mathcal{A}(X',\Pi) = \mathcal{A}(X,\Pi) \cap X'$; as a result, to preserve even minimal robustness. $\sigma(X',\Pi)$ must be "to the left" of $\sigma(X,\Pi)$, with lower payoff in state one and higher payoff in state two, thus violating condition α.

Interestingly, similar phenomena of context-dependence have been observed in a multi-attribute context[24] quite systematically in consumer-choice experiments (see Simonson-Tversky *(1992)* and Tversky-Simonson *(1993))*;there is even a significant overlap with SIMEU theory in the way these authors describe and explain their

[24]For an extension of the theory to a multi-attribute context, *see* Nehring (1995, s.9)

findings psychologically (independently from us), in particular in their use of notions such as "compromise effect" and "extremeness aversion".

In the present non-comparability-based approach, the necessity of violating WARP should come as no surprise. Indeed, since CISO and EISO express the requirement that the choice-function take proper account of the (non-transitive) non-comparability inherent in the structure of the underlying partial order $R_\emptyset$, WARP's radical incompatibility with these axioms is simply tantamount to its inappropriateness. By contrast, the case against WARP had been less clear in the traditional symmetry-based approach to Complete Ignorance in which Symmetry and Independence were motivated by entirely different considerations, rather than being unified by the PPB.

The inherent context-dependence of SIMEU allows to resolve an apparent tension between the assumed exhaustive interpretation of the underlying partial order and the single-valnedness of the derived choice-rule: how can an act x be legitimately chosen over another (y) when the decision maker has deliberately suspended judgment between them? The answer is that suspension of judgment involves abstention only from expressing a definite preference of x over y, and thus, given ADM, abstention from context-independent choice of x over y. However, it is not difficult to show that for any x, y such that $xN_\emptyset y$, any choice of x over y is context-dependent, i.e. that there exist $X', X'' \supseteq \{x, y\}$ such that $\{x\} = C(X')$ and $\{y\} = C(X'')$. Intuitively, non-comparability rules out the choice of one act over another as intrinsically better, but is compatible with the choice of one act over another as a superior compromise in the context of a particular choice-set.

A particularly clear-cut instance of this distinction occurs in the choice among just two alternatives, where SIMEU recommends the flipping of a fair coin. Clearly, the only conceivable advantage of such randomization is the symmetric treatment of both alternatives; this may not seem much. On the other hand, it seems obvious that given the assumed suspension of judgment one cannot really hope to do better. Psychologically, some dissatisfaction may still remain. But perhaps such dissatisfaction reveals just how hard it is to honestly face genuine ignorance and to suspend judgment accordingly. In this vein, Elster (1989, p. 5459) argues that as a rule there is a psychological bias against its acknowledgment. He makes a strong case for the existence of a human tendency to exaggerate the support of many decisions by "reasons," summarizing (on p. 58): "The toleration of ignorance, like the toleration of ambiguity more generally, does not come easily."[25]

6.4. WAREP

The only axiomatization in the literature of a choice rule that reconciles Symmetry with Independence is Milnor's (1954) axiomatization of the minimax loss rule. Milnor assumes that a complete ordering[26] of acts can be established that may vary across choice-sets; while his approach implies WAREP, it assumes much more.

Technically, WAREP falls far short of implying the existence of a complete transitive ordering of acts in range-equivalent problems, due both to the convexity assumption on choice-sets and the range-equivalence restriction inherent in WAREP. Conceptnally, the statns of context-dependent orderings is unclear since nothing operational (no hypothetical choices) corresponds to them. The comparative weakness of WAREP[27] implies also that significantly more careful constructions are required

[25] Elster also supports the "Solomonic" use of randomization in situations of ignorance.

[26] "Ordering" refers here to an ordering generating the choice-function, rather than to the underlying preference relation $R_\emptyset$.

[27] Two remarks on the technical definition of WAREP:

1. One might consider defining range-equivalence alternatively by: "$\forall\omega \in \Omega : \text{proj}_\omega X = \text{proj}_\omega X'$" However, this would make the choice rule highly dependent on the addition or deletion of strictly dominated arts. The present formulation avoids this, implying the condition "$\mathcal{A}(X) = \mathcal{A}(X') \Rightarrow C(X) = C(X') \quad \forall X, X'$".

2. It would he preferable to specify range-equivalence without using the topological concept of closure, i.e., as "$\forall\omega \in \Omega$: $\text{proj}_\omega\ \mathcal{A}(X) = \text{proj}_\omega\ \mathcal{A}(X')$". This is not possible in general, since compactness of X fails to imply that of $\mathcal{A}(X)$ in more than two dimensions (See Arrow et al. (1953).

to obtain a characterization result.

7. CONCLUDWG REMARKS

1. Due to their rich structure, the analysis of Complete Ignorance problems is quite easy and fruitful. Their conceptual simplicity **makes** them also appealing to intuition. Yet, this is the simplicity of a logical extreme case. As such, it naturally tends to generate extreme implications. Their frequent apparent contrariness to common sense reflects the fact that in most situations it is simply unreasonable to assert CI preferences $R_\emptyset$. Contemplating what rationally would have to be chosen if one *were* completely ignorant brings to Light that one generally has beliefs over many events, that is: that one is prepared to bet if betting one must. Complete Ignorance problems are thus relevant primarily because they can be viewed as "reduced forms" of general d.p.u.s

2. The axiomatically grounded reduction of general d.p.u.s is brought about by a condition of "Complete Ignorance Reduction" (CIR)[28]. In the two-event case, it reads **as** follows (using the notatiun of section **2).**

Condition 6 **(CIR)** $C(X,\Pi) = \Psi^{-1}(C(\Psi(X),\Delta^2))$

CIR associates to each d.p.u. an equivalent CI problem "in expected utility profiles"; these are obtained from taking the expected utility of an act with respect to each extremal prior.

3. As far as we know, the only other approach of extending choices in CI-problem to a reawnably general class of d.p.u.s is Jaffray's (1989) mixture-space approach

Compactness of $\mathcal{A}(X)$ is guaranteed, **on the other hand, if X is a polyhedron.**

[28]See **Nehring** (1992), **for a** brief published statement, **and Nehring** (1991), ch.2 **for a more extensive** discussion; **it is also** effectively **shown there (in a slightly different** setting) **that a** choice rule **defined on** the class **of** CI problems h a a CIR **extension** if **and only if it satisfies EISO.**

(MSA) taken up by Hendon et al. (1994). We note the following differences between the mixture-space approach and ours. The MSA applies only to "belief-functions" which correspond to a rather restrictive class of belief sets. Since, moreover, the MSA describes acts in terms of *marginal* belief-functions on consequences, axioms that rely on an a Savagelike event-based definition of acts such as CISO or ADM cannot even be *stated* within the MSA; for the same reason, the PPB itself cannot be meaningfully invoked to guide rational choice, nor can choice-rules such as LML or SIMEU even be *defined* within the MSA. There is also an important conceptual difference in terms of interpretation. While the MSA takes the underlying belief-function (respectively lower probability) as representing *given* evidence, an agent's incomplete preference relation is viewed here as the outcome of the agent's judgment, and, in this sense, as something chosen. The appeal to the agent's active suspension of judgment (inherent in the very definition of CI preferences on an exhaustive interpretation) has been central to our justification of the key axioms Symmetry and CISO in section 6.

4. Finally, the approach adopted in this paper of determining the choice implications of hypothetically asserted partial orders poses subtle questions regarding the logical status of such partial orders. In particular, it is not obvious to what extent (if any) this involves sacrificing the traditional *identification* of preference and binary choice. While a more detailed analysis is left to future work[29], it should be pointed out that in a straightforward but important sense, no such sacrifice is involved. For the theory itself establishes a tight connection between preference and choice. In particular, as mentioned in section **6.3**, *for SIMEU maximizers* asserting two acts to be non-comparable is co-extensive to randomized choice between them (with equal probabilities)

[29] For a first attempt in this direction, see Nehring (1995) which distinguishes three levels of this issue.

APPENDIX

A1. Extension of **Theorems** 1 and 2 to Finite Universes

To derive versions of theorems 1 and **2** for finite universes, one has to interpret $\mathcal{F}$ as a class of conceivable "universes" F described by finite sets of "states" (atomic events); each F may be thought of as a "framework of description" related by the common "language" Ω.

A CI-problem is now defined as a pair $(X, R_\emptyset^F)$ such that $F \in \mathcal{F}$ and X is a compact convex subset of $[0,1]^F$. Let $D^F = \{(X, R_\emptyset^F) \mid X \subseteq [0,1]^F\}$; a solution is defined on the class of such problems $D = \bigcup_{F \in \mathcal{F}} D^F$. The axioms are now applied to each subdomain separately.

The subdomains can be linked by an embedding condition

Axiom **6** (EMB) If $X \subseteq [0,1]^F$ *and* G *is a refinement of* F, $C(X, R_\emptyset^F) = C(X, R_\emptyset^G)$.

EMB can be read as saying that if a given frame F with complete ignorance $R_\emptyset^F$ is refined to G, that refinement should not affect the chosen set per se, i.e., as long as no preference is asserted beyond those affirmed by $R_\emptyset^F$ and implied by the consistency axioms on preferences. Following the terminology of Walley (1991, ch. 3.1). this may be described as "Natural Extension" property . Noting that for any $F, G \in \mathcal{F}$ there exists $H \in \mathcal{F}$ that is a refinement of both F and G, EMB implies that

$$C(X, R_\emptyset^F) = C(X, R_\emptyset^G)\ ,\ whenever\ X \subseteq [0,1]^F \cap [0,1]^G.$$

C may thus be viewed as defined on X only, and, with EMB in place, the axioms defined on $\bigcup_{F \in \mathcal{F}} D^F$ turn out to be equivalent to those defined on X x $\{R_\emptyset\}$. It follows that theorems 1 and **2** carry over.

Remark: Although one now needs to refer to CI-problems in alternative hypothetical universes of events, as the traditional CI-literature does, the present approach still has the significant conceptual advantage that it does not make the assumption that the frame of reference is irrelevant. Such an assumption is implicit in the traditional treatment of events as "generic events without names" which can be formalized in the current setting by:

" For all $F, G \in \mathcal{F}$ and any one-to-one map $\Phi : F \to G$: $\Phi(C(X, R_\emptyset^F)) = C(\Phi(X), R_\emptyset^G)$ ".

A2. Proofs

For future reference, a set $X \subseteq [0,1]^\Omega$ is called *normalized* if, for all $\omega \subseteq \Omega$, $\mathrm{proj}_\omega\, cl\, \mathcal{A}(X) = [0,1]$ or $\mathrm{proj}_\omega\, cl\, \mathcal{A}(X) = \{1\}$.

Proof of Proposition 1:

Since LM and σ^{Cl} agree on normalized choice-sets, it evidently suffices to prove the proposition for LM. Let $F \in \mathcal{F}$ be any partition such that X is F−measurable.

For $G \subseteq F$, define $\mu(X,G) = \max_{x \in X} \min_{S \in G} x_S$ and $MM(X,G) = \arg\max_{x \in X} \min_{S \in G} x_S$. The key to the proof is the following lemma.

Lemma 1 *If X is convex, then then exists $T \in G$ such that.* for *all* $x \in X$: $x \in MM(X,G) \Rightarrow x_T = \mu(X,G)$.

Proof of lemma.

The following simple fact will be used repeatedly:

$$\text{For any } x \in MM(X,G) \text{ and } S \in G : x_S \geq \mu(X,G). \tag{2}$$

Suppose the claim of the lemma to be false , i.e. that for every $T \in G$ there exists $z^T \in MM(X,G)$ such that $z_T^T > \mu(X,G)$. Then, setting $z' = \sum_{T \in G} \frac{1}{\#G} z^T$ ($\in X$ by convexity), in view of (2), $\min_{S \in G} z'_S > \mu(X,G)$, a contradiction. □

Let $F(0) = F$, $X^{(0)} = X$, and $n = \#F$.

For $k = 0, \dots, n-1$, define inductively $X^{(k+1)} = MM(X^{(k)}, F^{(k)})$, and $F^{(k+1)} = F^{(k)} \setminus \{S^{(k)}\}$, where $S^{(k)}$ is any $T \in F^{(k)}$ satisfying the property asserted in the lemma for $(X^{(k)}, F^{(k)})$.

It is easily verified by induction that for all $k \leq n-1$: $X^{(k)}$ is non-empty, compact and convex. Fix some $\xi \in X^{(n-1)}$, and consider any $y \in X \setminus \{\xi\}$.

We will show that

$$\min_{\omega:\xi_\omega \neq y_\omega} \xi_\omega > \min_{\omega:\xi_\omega \neq y_\omega} y_\omega. \tag{3}$$

This implies $y \notin LM(X)$, and , since y is arbitrary and $LM(X)$ is non-empty, indeed $LM(X) = \{\xi\}$, from which the asserted properties of LM follow in view of (3).

To show (3), assume that $y_S > \xi_S$ for some $S \in F$; otherwise (3) is satisfied trivially. Let $\nu = \min_{S \in F}\{\xi_S \mid \xi_S < y_S\}$, and let k^* be the largest integer k such that $\xi_{S^{(k)}} \leq \nu$.

We will show that for some $k \leq k^*$, $y_{S^{(k)}} < \xi_{S^{(k)}}$. From this (3) follows, since $k \leq k'$ implies, for any k, k', $\mu(X^{(k)}, F^{(k)}) \leq \mu(X^{(k')}, F^{(k')})$ (by definition) which in turn implies $\xi_{S^{(k)}} \leq \xi_{S^{(k')}}$ by lemma 1.

Suppose that the last claim is false. i.e. that

$$\text{for all } k \leq k^*, y_{S^{(k)}} \geq \xi_{S^{(k)}}. \tag{4}$$

Let $z^\varepsilon = \varepsilon \cdot y + (1 - \varepsilon) \cdot \xi$. For sufficiently small but strictly positive ε, the following three properties are satisfied:

i) $z^\varepsilon_{S^{(k)}} \geq \xi_{S^{(k)}}$, for all $k \leq k^*$.

ii) $z^\varepsilon_{S^{(k)}} > \xi_{S^{(k)}}$, for some $k \leq k^*$.

iii) $z^\varepsilon_{S^{(k)}} > \nu$, for all $k > k^*$.

i) is straightforward from (4); ii) follows from the definition of k^* and **(4)**;iii) finally follows from the fact that $\xi_{S^{(k)}} > \nu$, for all $k > k^*$ if ε is chosen sufficiently small.

i) and iii) imply $z^\varepsilon \in X^{(k)}$, for all $k \leq k'$. But then ii) contradicts lemma 1, the desired contradiction. ■

<u>Proof of Theorem 2:</u>

Necessity of the first three properties is straightforward, and that of WAREP is implied by part ii) of proposition 1.

To show sufficiency, note first that WAREP implies the following property IDA ("Independence of Dominated Alternatives"):

$$\textbf{(IDA)} \qquad \mathcal{A}(X) = \mathcal{A}(X') \Rightarrow C(X) = C(X') \quad \forall X, X' \in \mathcal{X}.$$

It thus involves no loss of generality to restrict attention to normalized choice-sets. A choice set $Y \subseteq [0,1]^F$ will be called *F-comprehensive* if $x' \leq x$, $x \in Y$, and $x' \in [0,1]^F$ imply $x' \in Y$.

Essential to the proof are the following two lemmas:

Lemma 2 *If Y is F-measurable and Y is symmetric with respect to all $\Phi: [0,1]^F \rightarrow [0,1]^F$ that leave events outside $G \subseteq F$ invariant (i.e. such that $\Phi(x)_T = x_T \; \forall T \in F \backslash G$), then any $x \in C(X)$ is constant on $\cup G$.*

Proof. By CISO and IDA, Y can assumed to be normalized and F-comprehensive. The proof is by contradiction: suppose that $C(Y)$ contains an act ξ that is not constant on $\cup G$. Let $\nu = \min_{S \in G} \xi_S$, and let S_0 be any $S \in G$ such that $\xi_S = \nu$. Also, let $F' \in \mathcal{F}$ be any partition obtained from F by splitting S_0 into $\{S_1, S_2\}$: $F' = \{S \in F \mid S \neq S_o\} \cup \{S_1, S_2\}$.

Define $\eta : [0,1]^F \rightarrow [0,1]^{F'}$ by

$$\eta(x)_S = \begin{cases} \min_{T \in G} x_T & \text{if } S = S_1 \\ & \text{if } S = S_2 \\ & \text{otherwise.} \end{cases}$$

$Z \subseteq [0,1]^{F'}$ as

$$Z = co\ \left(\{\eta(x) \mid x \in Y\} \cup e^{S_1}\right),$$

and let Y' $= \{x \in [0,1]^{F'} | x \leq y$ for some $y \in Y\}$, the "F'-comprehensive hull" of Y.

Z has the following propel-ties:

i) $\xi \in Z \subseteq Y'$.

ii) $\forall S \in \mathbf{F}'$: $\text{proj}_S\ cl\ \mathcal{A}(Z) = \text{proj}_S\ cl\ \mathcal{A}(Y) = \text{proj}_S\ cl\ \mathcal{A}(Y') = [0,1]$.

iii) Z is symmetric w.r.t. all event-isomorphisms $\Phi : [0,1]^{F'} \to [0,1]^{F'}$ that leave **all** events in $(F \backslash G) \cup \{S_1\}$ invariant.

Note that i) follows from the definition of S_0 , ii) hinges on the inclusion of e^{S_1} in Z, and iii) follows from the symmetry assumption on Y.

Since $\mathcal{A}(Y') = \mathcal{A}(Y)$, from IDA,

$$\xi \in C(Y'). \tag{5}$$

Hence, using properties i) and ii) of Z, by WAREP also

$$\xi \in C(Z). \tag{6}$$

Since ξ is non-constant. fur some $S_3 \in G : \xi_{S_0} < \xi_{S_3}$. Let $\phi : F \to F$ permute S_0 and S_3, leaving other events invariant, and let $\phi' : \mathbf{F} \to F'$ permute S_2 and S_3, leaving other events invariant, with associated Φ respectively Φ'. By property iii) of Z, $\Phi'(Z) = Z$; using SY, it thus follows from (6) that

$$\Phi'(\xi) \in C(Z). \tag{7}$$

By WAREP, from (5), (7) and properties i) and ii) of Z also

$$\Phi'(\text{f}) \text{ E } C(Y'). \tag{8}$$

However, by the symmetry assumption on Y, Y and hence Y' contain also $\Phi(\xi)$. Noting $\Phi(\xi)_{S_1} = \xi_{S_3} > \xi_{S_1} = \Phi'(\xi)_{S_1}$ and $\Phi(\xi)_{-S_1} = \Phi'(\xi)_{-S_1}$, one has $\Phi(\xi) > \Phi'(\text{f})$. By admissibility. $\Phi'(\xi) \notin C(Y')$, in contradiction to (8). □

Lemma 3 *Consider any normalized X, $y \in X$, and F such that X is F-measurable. If then: exists $z \in X$ such that:*

i) $z_S > 0 \quad \forall S \in F$,

ii) z *is constant on* $\{S \in F \mid z_S \neq y_S\}$, *and*

iii) for some $S \in F : z_S > y_S$,

then $\quad y \notin C(X)$.

Proof: Take any X, F and $y, z \in X$ with the properties assumed in the statement. of the lemma. Partition F into the following three collections of events , fixing some S' such that $z_{S'} > y_{S'}$.

$$F' = \{S'\},$$
$$F'' = \{S \in F \setminus \{S'\} \mid z_S \neq y_S\} \text{ , and}$$
$$F''' = \{S \in F \mid z_S = y_S\}$$

It is clear that events S such that $\#\text{proj}_S X = 1$ make no difference; hence, assume w.l.o.g. that there are no such events. Take any sufficiently large integers l and m such that

$$m > \frac{2}{\min_S z_S} \text{ and } l \geq \frac{\#F \cdot m}{(z_{S'} - y_{S'})} \tag{9}$$

Let $G \in \mathcal{F}$ be a refinement of F such that S' is "replicated" l times (i.e. such that $\#\{T \in G \mid T \subseteq S'\} = l$) and any $S \neq S'$ is replicated m times. Also, let G' (resp. G'', G''') denote the corresponding refinement of F' (resp. F'', F''').

Let ϕ^* be the class of permutations ϕ of G that leave events outside $G' \cup G''$ invariant (i.e. events such that $T \notin G' \cup G'' \Rightarrow \phi(T) = T$). Likewise, let ϕ^{**} be the class of those permutations ϕ of G such that, for all $T \in G$, $\phi(T)$ is a "replica" of the same event in F as T (ie. such that $\forall T \in G, \forall S \in F : S \supseteq T \Rightarrow S \supseteq \phi(T)$), and let Φ^*, Φ^{**} denote the associated classes of event-isomorphisms $\Phi : [0,1]^G \to [0,1]^G$.

Define a choice-set Z as follows:

$$Z = co\left(\{z\} \cup \{\Phi(y)\}_{\Phi \in \Phi^*} \cup E\right),$$
$$\text{with } E = \left\{e^H \mid H = T_1 \cup T_2 \text{ for some } T_1, T_2 \in G, T_1 \neq T_2\right\}$$

If $F'' = 0$, the claim follows directly from admissibility; assume thus $F'' \neq \emptyset$ which implies $z_{s'} < 1$ in view of assumption ii). Hence any $e^H \in E$ such that $H \cap S' \neq 0$ is admissible, which implies $\mathrm{proj}_T\ \mathcal{A}(Z) = [0,1]\ \forall T \in G$. Z is thus range-equivalent to X.

Take w $\in C(Z)$ and express w as convex combination:

$$w = \lambda_z z + \sum_{\Phi\in\Phi^*} \lambda_\Phi \Phi(y) + \sum_{e^H\in E} \lambda_H e^H.$$

For any $S \in F$, Z is symmetric under all permutations ϕ : G $\rightarrow$ G leaving events outside S invariant. By lemma 2, w must thus be constant on each $S \in F$, i.e. F-measurable.

It is also not difficult to verify that, for any F-measurable act x, $x = \sum_{\Phi\in\Phi^{**}} \frac{1}{\#\Phi^{**}} \Phi(x)$, and, in view of (Q), that $z > \frac{2}{m} e^\Omega \geq \sum_{\Phi\in\Phi^{**}} \frac{1}{\#\Phi^{**}} \Phi(e^H)$ for all $e^H \in E$.

Thus, by the admissibility of w, $\lambda_H = 0$ for all H such that $e^H \in E$ (for otherwise $w < (\lambda_z + \sum_{e^H\in E} \lambda_H) z + \sum_{\Phi\in\Phi^*} \lambda_\Phi \Phi(y)$, contradicting the admissibility of w).

This shows $w \in Z' = co\left(\{z\} \cup \{\Phi(Y)\}_{\Phi\in\Phi^*}\right)$.

By the admissibility of w in Z', the fact that for any $x \in Z'$: $x_{-\cup(G'\cup G'')} = z_{-\cup(G'\cup G'')}$, and the convexity of Z', it follows from a standardsupporting-hyperplane argument that w must maximize $\sum_{T\in G'\cup G''} \pi_T x_T$ in Z' for appropriate non-negative coefficients π_T. Since Z' is symmetric under all permutations $\phi \in \phi^*$ by construction, w must be constant on $\cup(G' \cup G'')$ by lemma 2; moreover, the π_T can assumed to be constant (= 1) as well; it follows that w must in fact maximize $\sum_{T\in G'\cup G''} x_T$ in Z'. Since this is uniquely done by z in view of the assnmption on 1 in (9), it follows $C(Z) = \{z\}$, and in particular y $\notin C(Z)$. Since $z \in X$,the claim then follows from WAREP. □

Proof of theorem 1, ctd.: Fix any F such that X is F-measurable. By IDA, X can be assumed to be F-comprehensive. Let $\sigma^{Cl}(X) = \{\xi\}$.

Take any $y \neq \xi$, and define z by $z_\omega = \begin{cases} y_\omega & \text{if } y_\omega = \xi_\omega \\ \min_{\omega' :\, y_{\omega'} \neq \xi_{\omega'}} \xi_{\omega'} & \text{if } y_\omega \neq \xi_\omega \end{cases}$

By proposition 1, for some $S \in F$, $z_S > y_S$. Since $z \leq \xi$ and by the F$-$comprehensiveness of X, it follows that $z \in$ X. Thus X, y, z, F satisfy the properties assumed by lemma 3 which yields $y \notin C(X)$. It follows that $C(X) = \sigma^{C1}(X)$ by the non-emptiness of C. ■

Proof of Theorem 2:

Theorem **2** can be demonstrated using a significantly simplified version of the proof of the Theorem 1. ■

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