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"Distributive class politics and the political geography of interwar Europe"

by

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1. Introduction

Since European socialist parties decided to participate in elections, around the turn of the century, many scholars have viewed European politics as the expression of 'the democratic class struggle.' That phrase was apparently first used in the title of a book by Dewey Anderson and Percy Davidson (1943), *Ballots and the Democratic Class Struggle*, with reference to the American experience, and the idea continues to resonate. Seymour Martin Lipset's (1959) classic *Political Man* is organized around the theme, with a chapter entitled "Elections: The expression of democratic class struggle," and the title of Adam Przeworski and John Sprague's (1986) *Paper Stones* is a phrase that early German socialists used to describe ballots: like real stones, casting them against the bourgeoisie could topple them from power[1]. The class analysis of elections is the hallmark of almost countless other studies: the best include Tingsten (1941), Abraham (1981), and Hamilton (1982).

These studies are written in the main by historians and political scientists; as is to be expected, the latter tend to abstract, somewhat more than the former, from thick historical particularity in favor of general explanation. Among the best of these is Gregory Luebbert's (1991) *Liberalism, Fascism, or Social Democracy*: *Social Classes and the Political Origins of Regimes in Interwar Europe*, in which the author attempts to explain what

[1] Other indicative titles are Walter Korpi, *The Democratic Class Struggle*, and Paul Nieuwbeerta, *The Democratic Class Struggle in Twenty Countries* .

determined the choice of regime in countries in interwar Europe, among the three types his title lists. Luebbert proposes that liberals failed to win, in this period, because they were not willing to use state power to intervene against what citizenries saw as the heartless effects of the unrestricted market, while both the socialists and the fascists were willing to do so. What, then, determined social democratic victories in some states and fascist victories in others? Luebbert proposes that the decisive factor was the status of class struggle in the countryside.

Think of the electorate as composed of four classes: the urban working class, the urban middle class, the rural peasantry (family farmers who own land), and the rural proletariat (landless laborers). The classes of large capitalists and large landowners may, of course, have been important in their influence on parties[2], but they were trivial as far as direct voting was concerned. The key to electoral victory, Luebbert claims, was the formation of an electoral alliance between the landed peasantry and one of the urban classes. *If* class struggle in the countryside was quiescent, then, he says, the social democrats were able to appeal to the landed peasantry and to construct an alliance between them and the urban workers, usually sufficient for electoral victory over the right. In the cases where rural class struggle was active, however, the socialists always took the side of the rural proletariat, alienating the landed peasantry (their employers), thus leaving the latter class open to appeals from the fascists, who were then able to construct an alliance between them and the urban middle class. The three countries in which this second scenario transpired (says Luebbert) were Germany, Italy, and Spain.

It is worth reproducing Luebbert 's prose on this point, as it is central to the present study.

> As we see, however, their [the German Social Democrats'] inability to make an effective alliance with the countryside had much more to do with their involvement in agrarian class conflict than it did with the burdens of membership in the initial Weimar coalition per se. The polarization of peasant-worker relations that ensued militated against an alliance that would have embraced measures to address the peasants' grievances as producers (Luebbert, p. 285).

[2] As Abraham (1981) strongly argues.

> When socialist parties did succeed in making regime-stabilizing coalitions with the peasantry, it was not because they had a superior grasp of the strategic requirements of the moment, but because they did not attempt to organize the rural proletariat (Luebbert, p. 287).
>
> Hence, the Spanish Socialists' campaign for land reform in the south of Spain antagonized peasants even in the north (Luebbert, p. 286).
>
> Socialists succeeded in making a coalition with family peasants wherever the agrarian proletariat had been mobilized by others before socialists had an opportunity to do so (Luebbert, p. 288).
>
> It was this entanglement [in rural class conflict] rather than conflicts between urban consumers and rural producers that distinguished socialist movements in Germany, Italy, and Spain from socialist movements in Norway, Sweden, Denmark, and Czechoslovakia (Luebbert, p. 300).
>
> Whatever the quibbles about precisely who voted for the Nazis in Germany, it is clear that the social core of that support came from the urban middle classes and the Protestant peasantry of the west (p. 301).
>
> Given the impuissance of liberal movements...the precondition of fascism was a working-class movement engaged in a defense of the rural proletariat. The coalitions of urban and rural middle classes that took shape in Spain, Italy, and Germany were premised on a common ambition to extirpate the socialist working-class menace. (p. 303).

My aim is to construct a model of party competition which can test Luebbert's theory. The model will consist of two parties, to be thought of as Socialists and Fascists, or Left and Right, each of which proposes a policy to voters. A *policy* will be a distribution of the national income among the four classes named above. If national income is fixed, as I shall assume, then the policy space is three-dimensional, since there are four classes. Giving each party the freedom to choose any income distribution (among the four classes) models the idea that both Left and Right parties were willing to intervene in the market: while the allocations of income that the market can deliver arguably constitute a small subspace of the relevant three-dimensional simplex, these parties, according to Luebbert, did not restrict themselves to that subspace. I will examine equilibria of the model under two 'treatments': first, that class conflict between the two rural classes is not an issue -- has been resolved-- and second, that it has not been resolved. The hypothesis I test is that the probability of a Left victory is greater in the first case than in the second case.

To carry out this plan, I require a model of competition between parties that possess preferences over policies (the Left favors the working classes, and the Right favors the propertied classes) on a multi-dimensional issue space, with a coherent notion of equilibrium. Recently, I have proposed such a model (Roemer [1998]). The innovation which delivers political equilibrium on a multi-dimensional domain of policies is the conceptualization of parties as consisting of factions -- in particular, of *opportunists* and *militants.* A political equilibrium consists of policies, one proposed by each party, which is Nash in the following sense: in neither party can the (internal) factions agree on a deviation from the proposed policy, given the other party's policy proposal.
This is called a *party-unanimity Nash equilibrium* (PUNE).

Another (realistic) feature of the model is that there is a stochastic element in how citizens vote, and so the parties can only compute a probability that, given the two policies, one of them will win. As I said, I shall test whether the probability of Left victory, in the PUNE under the first treatment, is greater than the probability of Left victory in the PUNE under the second treatment.

This model, like any other, makes breath-taking simplifications of reality. In the conclusion, I will list a number of caveats due to them. An advocate of historical particularism will say that such a spare model can never provide an explanation for political history that arguably, in each country, depended on the infinite detail of personalities and events exogenous to the model (e.g., the consequences of the defeat in First World War, and an ignominious armistice, for German political behavior). In large part, Luebbert and the other authors I have referenced have already fought that methodological battle. While our method of explanation via abstraction cannot predict outcomes at any single point in the domain of time-space history, it can have power if its predictions are often true, at many points in that domain.

2. The concept of political equilibrium

The environment consists of two parties competing for votes of an electorate, where the parties have preferences over policies (they are *partisan),* and there is uncertainty concerning how voters will choose between a pair of policies proposed by the two parties. Denote the space of policies by T, its generic element by t, and the utility functions, defined on the domain T, representing (von Neumann-Morgenstern) preferences of the two parties, as v_1 and v_2. Given policies t_1 and t_2 proposed by the parties, there is a probability $\pi(t_1,t_2)$ that t_1 defeats t_2 in the election. (The function π is a datum of the problem; it is defined on T x T.) Thus, the expected utility of party i, for i=1,2, at the policy pair (t_1,t_2) is

$$\Pi_i (t_1,t_2) = \pi(t_1,t_2)v_i(t_1) + (1- \pi(t_1,t_2))v_i(t_2).$$

The standard concept of equilibrium for this environment (see Wittman (1981), Roemer (1997)) is:

Defintion[3] A policy pair (t_1,t_2) is a *reformist Nash equilibrium* if it is a Nash equilibrium in pure strategies of the game where party i has strategy space T and payoff function Π_i.

Reformist Nash equilibria typically exist in applications where T is a one-dimensional space; they generically fail to exist when T is multi-dimensional. Non-existence in the latter case is an analogue of, but not the same thing as, the non-existence of Condorcet winners with multi-dimensional policy spaces. The Condorcet winner is the natural concept of political equilibrium in two-party games where the parties are *Downsian,* meaning they have no preferences over policies, and desire only to maximize the probability, π, of victory.

Here, I briefly review the equilibrium concept introduced in Roemer(1999). From reading political history, one observes that parties have factions -- in particular, they typically contain *opportunist* , *militant* and *reformist* factions. Opportunists are those who wish to maximize the probability of victory, and militants as those who want to maximize $v_i(t)$ -- not $\Pi_i (t_1,t_2)$. Reformists maximize $\Pi_i (t_1,t_2)$. The opportunists are the sole faction

in Anthony Downs's (1957) conception of politics. Militants are those who are not primarily concerned with winning elections, but with advertising the preferences (or line) of the party. Reformists act like true agents of constituencies. Party histories are replete with descriptions of these three factions; for instance, Schorske (1955), in his classical history of German Social Democracy, discusses the factions of party bureaucrats, trade union leadership, and radicals. These correspond, respectively, to our opportunists, reformists, and militants.

Note that each of the three factions described has complete preferences over T x T. Given a policy pair (t_1,t_2) proposed by the two parties, say that party 1 *agrees to deviate to* t_1' *at* (t_1,t_2) if all three factions of the party weakly prefer (t_1',t_2) to (t_1,t_2), and at least one faction strictly prefers the former.

Defintion. A policy pair (t_1,t_2) is a *party unanimity Nash equilibrium (PUNE)* if it is case that there is no policy to which either party agrees to deviate at (t_1,t_2).

The motivation for calling such a policy pair a Nash equilibrium is this. Define the preferences of party i on T x T as the intersection[4] of the preferences of its three factions. Thus the party's preferences are typically incomplete. A PUNE is just a Nash equilibrium between the two parties with respect to the parties' preferences, so defined.

It is easy to see that the reformist factions can be dropped, with no change in the equilibria thus generated: that is, a party's three factions will agree to deviate at a point if and only if the militant and opportunist factions will agree to deviate at that point. We shall use this fact below.

One can also immediately see that reformist Nash equilibrium is a refinement of PUNE(or, equivalently, PUNE is an extension of reformist Nash equilibrium). Therefore, although reformist Nash equilibria typically fail to exist in multi-dimensional policy games,

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[3] This concept was introduced by Wittman (1981), but it was christened with this name in Roemer (1999).

we can hope that PUNE do exist. In fact, in the applications I have examined, there is typically a continuum of PUNEs.

Informally speaking, the concept of PUNE captures the fact of political struggle, within parties, between groups that are on the one hand primarily concerned with winning elections, and on the other, primarily concerned with ideological purity. One might argue that the PUNE conception is too stark: it rules out compromise between factions, in the sense that a party will only deviate from a proposed policy if all factions stand to (weakly) gain. But let it be noted that, whatever conception of factional compromise one might in addition elaborate, the equilibria so generated will in fact also be PUNEs: for it surely must be the case that, at an equilibrium with factional compromise, there exists no policy to which all factions in one party would agree to deviate. Thus, any theory of factional compromise will generate an equilibrium concept which is a *refinement* of PUNE. Retaining the PUNE concept is, therefore, equivalent to remaining agnostic about the precise nature of factional compromise.

In the rest of the paper, I use PUNE as the concept of political equilibrium between parties.

3. The Luebbert model

According to Luebbert, interwar European politics were class politics: each party appealed to distinct classes by proposing policies which possessed clear consequences for the class distribution of income. Whether the Left or Right won was, he wrote, critically determined by whether class struggle in the countryside was settled or contested.

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[4] Formally, the intersection of the binary relations in $(TxT)^2$ which define the factional preferences.

A. The four class model

The four relevant classes are the workers (W), the middle class (M), the landed peasantry (L), and the agricultural proletariat (A) or landless laborers. I shall telescope political behavior by assuming that each party proposes a division of national income among the four classes, that is, a vector (w,m,ℓ,a), where $\omega w + \mu m + \lambda \ell + \alpha a = 1$, all components of which are non-negative, and ($\omega,\mu,\lambda,\alpha$), are the population proportions of the four classes. w is the income a worker will receive at this policy, and so on. The three-dimensional policy simplex is denoted by S_3.

This formulation of the policy space is, clearly, a vast simplification. In point of fact, parties proposed complex policies – involving nationalization, taxation, tariff policy, and land reform, to name several. I am assuming that voters interpret each set of actual policies as implying some distribution of income, (w,m,ℓ,a). That process of interpretation is here eclipsed. Furthermore, I am assuming that national income is fixed, and does not respond to different policies, and that no other constraints (except being in S_3) limit policies.

There are two parties, here called Left (L) and Right (R). The Left party 'represents' primarily the workers, and secondarily the agricultural proletariat; the Right 'represents' mainly the middle class and secondarily the landed peasantry. The (von Neumann-Morgenstern) utility functions of the reformists and militants of the parties are given by

$$v_L(w,m,\ell,a) = \text{Log } w + \beta_L \text{ Log } a \qquad \textbf{(3.1a)}$$

$$v_R(w,m,\ell,a) = \text{Log } m + \beta_R \text{ Log } \ell\,, \qquad \textbf{(3.1b)}$$

where β_L and β_R are in the interval [0,1]. Thus, each party cares primarily about 'its' urban class, and gives less weight to the welfare of 'its' rural class.

Indeed, if we dispense with the reformist factions, as I mentioned we can, in §2, then we need only view v_L and v_R as representing the ordinal preferences on S_3 of the two militant factions. (Militants do not evaluate lotteries.) Thus, equations (3.1) say that the militant faction in each party has Cobb-Douglas preferences over the distribution of income within a pair of classes – the Left's militants caring about the urban and rural workers, and the Right's caring about the urban and rural propertied classes.

We next propose a formulation of the probability function π, which is defined on $S_3 \times S_3$. Denote a policy proposed by Left as L = (w,m,ℓ,a) and by the Right as R = (w′,m′, ℓ′,a′).

Let $f(x) = \frac{1-x}{1+x}$. Note that f(∞) = -1, f(1) = 0, f(0) = 1, and f(1/x) = - f(x).

Define

$$\phi(L,R) = \omega\left(\frac{1}{2} + \frac{\varepsilon_1}{2} f\left(\frac{w'}{w}\right)\right) + \mu\left(\frac{1}{2} + \frac{\varepsilon_1}{2} f\left(\frac{m'}{m}\right)\right) + \lambda\left(\frac{1}{2} + \frac{\varepsilon_2}{2} f\left(\frac{\ell'}{\ell}\right)\right) + \alpha\left(\frac{1}{2} + \frac{\varepsilon_2}{2} f\left(\frac{a'}{a}\right)\right). \qquad (3.3)$$

ε_1 and ε_2 are i.i.d. random variables, uniformly distributed on [0,1]. I shall assume that ϕ(L,R) is the fraction of the population who vote for L against R, which depends upon the realization of ε_1 and ε_2. Note that if L = R, then $\phi = \frac{1}{2}$. The idea here is that, if $w > w'$, say, then more than one-half the worker vote for L, but there is a random element in exactly what fraction vote for L. The term $f\left(\frac{w'}{w}\right)$ will be close to one if w is much bigger than w' and close to -1 if w' is much bigger than w. Thus, the role of the function f is to cause the fraction of workers who vote Left to move from zero to one as w moves from being much smaller than to much larger than w'. The random element can be interpreted in various ways: perhaps there is some indeterminacy in how the workers will vote because they care about issues other than economic ones, such as religion; perhaps there is indeterminacy because different voters transform (or interpret) the actual policies into income distributions in different ways.

Similarly, the other terms in the expression (3.3) express the fractions of the other three classes that vote for Left. Ideally, one would include four random variables in this expression, one for each class; I have settled for two, because having two already generates a sufficiently complex probability function π, as we shall see, in which π has six different analytical definitions, in six regions of the domain $S_3 \times S_3$. (If we used only one random variable, there would in fact be no stochastic element in the outcome of the elections, so two is the minimum interesting number.)

It follows from assumption (3.3) that the probability that L defeats R is

$$\pi(L,R) = \text{Prob}\left[\phi(L,R) > \tfrac{1}{2}\right]. \qquad \textbf{(3.4)}$$

Letting $a_1 = f\left(\frac{w'}{w}\right)\omega + f\left(\frac{m'}{m}\right)\mu$, and

$$a_2 = f\left(\frac{\ell'}{\ell}\right)\lambda + f\left(\frac{a'}{a}\right)\alpha,$$

we must compute

$$\pi(L,R) = \text{Prob}[a_1\varepsilon_1 + a_2\varepsilon_2 > 0]. \qquad \textbf{(3.5)}$$

There are six cases.

Case 1. $a_1 > 0$, $a_2 < 0$, $\frac{-a_2}{a_1} < 1$.

Here we have

$$\pi = \text{Prob}\left[\varepsilon_1 > \tfrac{-a_2\varepsilon_2}{a_1}\right].$$

Clearly,
$$\pi = \int_0^1 \int_{\frac{-a_2\varepsilon_2}{a_1}}^1 d\varepsilon_1 d\varepsilon_2.$$

Integrating, we have $\pi = 1 + \frac{a_2}{2a_1}$.

In like manner, we can compute the probability of Left victory in the other five cases:

Case 2. $a_1 > 0$, $a_2 < 0$, $\frac{-a_2}{a_1} > 1$.

Here,
$$\pi = \frac{-1}{2}\,\frac{a_1}{a_2}.$$

Case 3. $a_1 < 0$, $a_2 > 0$, $\frac{-a_2}{a_1} < 1$.

Here, $\pi = -\frac{1}{2}\frac{a_2}{a_1}$.

Case 4. $a_1 < 0,\ a_2 > 0,\ \frac{-a_2}{a_1} > 1.$

Here, $\pi = 1 + \frac{a_1}{2a_2}$.

Finally we have obviously:

Case 5. $a_1 > 0, a_2 > 0$ $\pi = 1.$

Case 6. $a_1 < 0, a_2 < 0$ $\pi = 0.$

Thus we have defined the function π.

We now have all the information needed to define PUNE.

Our next task is to characterize the local conditions for a pair of policies' constituting a PUNE. These conditions derive from the separating hyperplane theorem -- more specifically, from Farkas' Lemma.

Define $g(w,m,\ell,a) = 1 - (\omega w + \mu m + \lambda \ell + \alpha a)$. The 'budget constraint' for each party is

$$g(L) \geq 0, \quad g(R) \geq 0.$$

Define the gradients

$$_L\pi = \left(\frac{\partial \pi}{\partial w}, \frac{\partial \pi}{\partial m}, \frac{\partial \pi}{\partial \ell}, \frac{\partial \pi}{\partial a}\right)$$

$$_R\pi = \left(\frac{\partial \pi}{\partial w'}, \frac{\partial \pi}{\partial m'}, \frac{\partial \pi}{\partial \ell'}, \frac{\partial \pi}{\partial a'}\right)$$

$$v_L = \left(\frac{1}{w}, 0, 0, \frac{\beta_L}{a}\right)$$

$$v_R = \left(0, \frac{1}{m}, \frac{\beta_R}{\ell}, 0\right).$$

Recall that a policy pair (L,R) is a PUNE if and only if the militants and opportunists in Left (Right) do not agree to deviate to any other policy. In particular, let $d \in \mathbb{R}^4$ be any direction such that

$$\nabla_L \pi(L,R) \cdot d > 0 \quad \text{and} \quad v_L(L) \cdot d > 0 ;$$

then it must be the case that $g(L) \cdot d < 0$. That is, any direction in which Left's opportunist and militant factions would agree to deviate must lead outside the feasible simplex of policies. This condition can be rewritten

$$d \in \mathbb{R}^4: \nabla_L \pi(L,R) \cdot d > 0 \quad \text{and} \quad v_L(L) \cdot d > 0 \quad \Rightarrow \quad -g(L) \cdot d > 0. \qquad \textbf{(3.6)}$$

But (3.6) implies, by Farkas' Lemma[5], that $-g(L)$ lies in the cone spanned by $\nabla_L \pi(L,R)$ and $v_L(L)$, which is equivalent to there being numbers $x_L, y_L \geq 0$ such that

$$-g(L) = x_L \cdot v_L(L) + y_L \cdot \nabla_L \pi(L,R). \qquad \textbf{(3.7a)}$$

[5] Farkas' Lemma states that if one vector (here $-g$) cannot be separated by some hyperplane from a set of other vectors (here $\nabla_L \pi$ and v_L), then the former must lie in the cone spanned by the latter.

In like manner, since the opportunists in Right want to maximize $-\pi$, if (L,R) is a PUNE, then there are non-negative numbers x_R, y_R such that

$$-\,g(R) = x_R\ \ v_R(R) - y_L\ \ {}_L\pi(L,R). \qquad \textbf{(3.7b)}$$

Equations (3.7a,b) are necessary and sufficient conditions for (L,R)'s being an interior <u>local</u> PUNE.[6] These equations would suffice to characterize (global) PUNE if the function π were quasi-concave – but, in fact, it is not. We can therefore not assert that the PUNEs we find are necessarily global.

Equations (3.7a), (3.7b) and equations

$$g(L) = 0 \qquad \textbf{(3.8a)}$$

$$g(R) = 0 \qquad \textbf{(3.8b)}$$

comprise ten equations in twelve unknowns $(w,m,\ell,a,w',m',\ell',a',x_L,y_L,x_R,y_R)$. Thus, they possess either no solution satisfying the non-negativity conditions or a continuum of such solutions.

B. The model when agricultural class struggle is resolved

We must now model the idea that in some countries, class struggle between the landed peasantry and the agricultural worker was not an issue. To do this simply, I propose to say that, when class struggle in the countryside is resolved, a division of the

agricultural product had been agreed upon; specifically, there exists a number $\gamma > 0$ such that all policies are constrained by

$$\ell = \gamma a. \qquad \textbf{(3.9)}$$

(One can interpret $\frac{\gamma}{1+\gamma}$ and $\frac{1}{1+\gamma}$ as the shares of the agricultural product going to landlord and tenant, respectively.) Thus, each party proposes a policy subject to two constraints: the 'budget' constraint (3.8), and (3.9). We may reformulate this by saying that each party proposes a policy (w,m,ℓ) subject to

$$\omega w + \mu m + \left(\lambda + \frac{\alpha}{\gamma}\right)\ell = 1, \qquad \textbf{(3.10)}$$

leaving 'a,' the agricultural workers' share, implicit. Thus policies are drawn for the simplex S_2 defined by (3.10).

Assumption (3.9) is equivalent to saying that each party will propose how much to give the rural sector, but it will leave the division of rural income to already existing contracts between family farmers and landless laborers.

The Right's utility function is defined on S_2 as

$$\hat{v}_R(w,m,\ell) = \text{Log } m + \beta_R \text{ Log } \ell ,$$

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[6] A local PUNE is a policy pair such that, in a sufficiently small neighborhood of each policy, the two factions in each party cannot find another policy to which they can agree to deviate.

as in (3.1b). The utility of Left on S_2 is, according to (3.1a), Log w + β_L Log $\frac{\ell}{\gamma}$: but these preferences can be equally well represented by

$$\hat{v}_L(w,m,\ell) = \text{Log } w + \beta_L \text{ Log } \ell ,$$

because γ is a constant.

By consulting (3.3), and using the constraint (3.9), we observe that the fraction of voters who vote Left is:

$$\hat{\phi}(w,m,\ell,w',m',\ell') = \omega\left(\tfrac{1}{2} + \tfrac{\varepsilon_1}{2} f\left(\tfrac{\omega'}{\omega}\right)\right) + \mu\left(\tfrac{1}{2} + \tfrac{\varepsilon_1}{2} f\left(\tfrac{m'}{m}\right)\right) + (\lambda + \alpha)\left(\tfrac{1}{2} + \tfrac{\varepsilon_2}{2} f\left(\tfrac{\ell'}{\ell}\right)\right);$$

hence the probability of victory is given by

$$\hat{\pi}(w,m,\ell,w',m',\ell') = \text{Prob}\left[\hat{\phi} > \tfrac{1}{2}\right].$$

The formulae defining $\hat{\pi}$ are hence given by the six cases discussed earlier, where now we define a_1 as previously, but newly define

$$a_2 = (\lambda + \alpha) f\left(\frac{\ell'}{\ell}\right).$$

Now define the budget equation as

$$\hat{g}(w,m,\ell) = 1 - \left(\omega w + \mu m + \left(\lambda + \frac{\alpha}{\gamma}\right)\ell\right).$$

Then a pair of policies L = (w,m,ℓ) and R = (w',m',ℓ') comprise a (local) interior PUNE iff there exist positive numbers x_L, y_L, x_R, y_R such that

$$-\hat{g}(L) = x_L\,\hat{v}_L(L) + y_L\,{}_L\hat{\pi}(L,R) \qquad \textbf{(3.11a)}$$

and

$$-\hat{g}(R) = x_R\,\hat{v}_R(R) - y_R\,{}_R\hat{\pi}(L,R)\,. \qquad \textbf{(3.11b)}$$

The analysis via Farkas' lemma again supplies these equations.

Equations (3.11a), (3.11b) and the budget constraints

$$\hat{g}(L) = 0 \qquad \textbf{(3.12a)}$$

$$\hat{g}(R) = 0 \qquad \textbf{(3.12b)}$$

now constitute eight equation in ten unknowns. Either there is no solution satisfying the non-negativity conditions, or there is a continuum of such solutions.

I call this model the <u>three-class model</u>.

Figure 1 illustrates Left's policy, L = (w,m, ℓ), in a local PUNE of the three-class model. The triangle ΔABC is the simplex S_2. The two curves illustrated in ΔABC are the indifference curves of Left's militants and Left's opportunists containing L. If L is in the simplex's interior, then these two curves must osculate, as illustrated, in the simplex, so that their upper contour sets are locally separated. In fact, the PUNE illustrated is global, from Left's viewpoint. Of course there is a similar figure for Right's policy, R = (w′,m′,ℓ′).

[Figure 1 here]

4. <u>Testing Luebbert's theory</u>

My method for testing Luebbert's theory is simple. I first describe the skeleton of the method, and then fill in the flesh. We study two countries, Germany and Sweden. In Germany, class struggle in the countryside was unresolved. Therefore, we model Germany with the four class model. We compute the (average) probability of Left victory in PUNEs of this model. Then we compute, counterfactually, the average probability of Left victory, had the three-class model described Germany. If the three-class-model probability of Left victory is greater than the four-class-model probability, Luebbert's theory is supported.

Sweden was a country in which, according to Luebbert, class struggle in the countryside was resolved. We compute the average probability of Left victory in PUNEs of the three-class model. Counterfactually, we also compute the average probability of victory in PUNEs of the four-class model, with Swedish parameters. Again, Luebbert predicts the first number is greater than the second.

Now for some details.

(i) Local versus global PUNEs

I have opted to use the local concept of equilibrium, for two reasons:

(a) We have an analytical representation of local PUNEs as the solution of a system of equations. No such representation exists for global PUNEs.

(b) In the four class model, there are many local PUNEs, but we did not succeed in finding any global PUNE. We did find global PUNEs in the three-class model, but not many.

Thus, we shall resign ourselves to saying that parties are only capable of evaluating small deviations of their policies from 'equilibrium.' [7]

(ii) Data

The data of the models are the sets $\{\omega,\mu,\lambda,\alpha,\beta_L,\beta_R,\gamma\}$. The vectors $(\omega,\mu,\lambda,\alpha)$ are computed from Przeworski et al. (1978), who assembled them from census data[8]. They are:

For Germany in 1933: $(\omega,\mu,\lambda,\alpha) = (0.4242, 0.3501, 0.1584, 0.0673)$,

For Sweden in 1930: $(\omega,\mu,\lambda,\alpha) = (0.4225, 0.2385, 0.2149, 0.1244)$.

Thus the propertied classes constitute 51% of the adult population in Germany and 45% in Sweden. Germany is more urbanized than Sweden: 77% of the population live in cities, versus 66% in Sweden. In sum, Sweden is less urbanized but more proletarianized than Germany.

We chose various values for the remaining data (β_L,β_R,γ) in our calculations.

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[7] A superior alternative would be to construct a function π that is quasi-concave. I have found no reasonable formulation of the stochastic element which yields quasi-concavity in this model.

[8] Przeworski et al (1978) report the detailed occupational distribution of the population of various European countries for various years. We partitioned the occupations into our four classes.

(iii) The calculations of PUNE

First, I describe the four-class model. Our problem is to find non-negative solutions of equations (3.7a), (3.7b), (3.8a), and (3.8b) in the twelve unknowns. These equations cannot be solved analytically. We first reduce the ten equations in twelve unknowns to four equations in six unknowns, as follows. Solve the first two equations of (3.7a) for (x_L,y_L) – this is easy, as the equations are linear in (x_L,y_L). Likewise, solve the first two equations of (3.7b) for (x_R,y_R). Solve (3.8a) for one Left policy variable and solve (3.8b) for one Right policy variable. This leaves four equations in six unknowns.

We now generate values for two of these six unknowns randomly, and solve the four equation system in the remaining four unknowns, via Newton's method. We then check the values of all twelve variables: if they are non-negative, we have found a local PUNE. This and the previous paragraph describe one iteration.

Because we have two free variables to choose, it will generically be the case that, if solutions exist, they will project onto a set of positive measure in the co-ordinate plane associated with two of the policy components. Thus, our method should find solutions, if they exist.

A slight complication is introduced because there are six possible formulae for the function π, and hence for the gradients $_L\pi$ and $_R\pi$. We therefore in fact investigated four cases, corresponding to the four cases listed above in which $0 < \pi < 1$. For instance, we carried out the above procedure letting π be defined by case 1

$\pi(L,R) = 1 + \frac{a_2(L,R)}{2a_1(L,R)}$. Our procedure was to compute 500 iterations for each case: thus, 2,000 iterations for each experiment.

The method for solving the three-class model is analogous. First, solve two equations of the system (3.11a) for (x_L,y_L), then solve two equations of (3.11b) for (x_R,y_R), then solve (3.12a) for one Left policy variable and (3.12b) for a Right policy variable. This leaves two equations in four unknowns. We generate two of these unknowns randomly, and solve for the remaining two. We check the 'case' constraint on π, and non-negativity of the solution. We performed 2,000 iterations for each experiment.

Tables 1 and 2 (Tables 3 and 4) report the average probability of Left victory (π) in the PUNEs we found for the Swedish (German) parametrs, and the standard deviation of these probabilities (σ_1). They also report the *average expected vote* for Left, which is defined as

$$\nu = E(\phi(L,R)) = \tfrac{1}{2} + \tfrac{1}{4}\left(\omega f\left(\tfrac{w'}{w}\right) + \mu f\left(\tfrac{m'}{m}\right) + \lambda f\left(\tfrac{\ell'}{\ell}\right) + \alpha f\left(\tfrac{a'}{a}\right)\right).$$

We report as well the standard derivation of ν for the PUNEs found (σ_2).

Figures 2a and 2b graph two projections of the set of PUNEs found, for Run 8, Sweden, the data being {country, year, β_L , β_R, γ} = {Sweden, 1930, 0.5, 0.5, 1.5}. Figure 2a graphs the projection of the simplex S_2 onto the w-ℓ plane of Left policies, and figure 2b shows the projection of S_2 onto the m' - ℓ' plane of Right policies.

(iv) Observations

(1) From Figures 2a-2b, we see that, although we are dealing with a continuum of equilibria, they appear to be quite localized in the domain simplices. So the average values of the policies given in Tables 2 and 4 are quite indicative of what the typical equilibrium looks like.

(2) In all local PUNEs, the expected vote is very close to 0.5; note how small the standard deviations (σ_2) are. In fact, in all local PUNEs we found, $0.48 < \nu < 0.52$. Contrast this with the probability of Left victory, which varies a great deal across experiments. Thus it is not inconsistent to say that Left has a high probability of victory and that we expect approximately one-half the population to vote for Left. Paraphrased, the expectation that the vote will be close is different from the expectation that each party has an even chance of winning.

(3) To test Luebbert, we compare the average probability of Left victory in the three-class and four-class models. For Sweden (see Table 1), all experiments are consistent with Luebbert's theory except for the case $(\beta_L,\beta_R) = (0.1, 0.5)$ [see runs 8 – 12]. In that exceptional case, the Left has a higher probability of victory if there is class struggle in the countryside.

Turning to Germany (see Table 3), we see that all experiments are consistent with Luebbert's theory, except $(\beta_L,\beta_R) = (0.5, 1)$ [see runs 5 and 6], although the case $(\beta_L,\beta_R) = (0.5, 0.1)$ [see runs 11 and 12] is inconclusive as we found only one local PUNE in the four-class model for that parameter vector.

(4) When $\beta_L = \beta_R$, we should expect $\pi > \frac{1}{2}$ in the three-class model [see runs 1,2, and 15 of Table 1and 1,2, and 13 of Table 3]. This is because the utility functions of the two militant factions are in this case the same, except that the Left cares about workers primarily, and the Right cares about the middle class primarily, and $\omega > \mu$. This expectation is borne out in the observations.

(5) When there is class struggle in the countryside (four-class model, Sweden and Germany, in the cases $(\beta_L,\beta_R) = (0.5, 0.5)$ [see runs 3 and 4 of Tables 2 and 4] and $(\beta_L,\beta_R) = (0.5, 1)$ [see runs 6 and 7 of Table 2 and run 6 of Table 4]), the two parties polarize in opposite ways vis-à-vis the rural population: the Left dramatically favors the agricultural workers over the small farmers, and the Right dramatically favors the landed peasantry over the agricultural workers. In these two cases, $\pi < \frac{1}{2}$. These are the clearest illustrations of Luebbert's mechanism. The Left champions the cause of the agricultural proletariat, while the Right favors the small farmers, which increases the Right's probability of victory, because $\lambda > \alpha$, over what it would be in the three-class model.

(6) The case $(\beta_L,\beta_R) = (0.1, 0.5)$ is ambiguous. For Germany [see runs 9 and 10 of Table 2], the same intuition holds as in observation (5): in the four-class model the Left and Right polarize in opposite ways vis-à-vis the rural classes, and the probability of Left victory falls, in comparison to the three-class model. But in Sweden [see runs 11 and

12 of Table 4], although there is some polarization in what the parties offer the rural classes, the probability of Left victory remains high -- in fact, higher than in the three-class model. This is inconsistent with Luebbert's theory.

(7) With the exception of the case $(\beta_L,\beta_R) = (1, 1)$ in both Sweden and Germany, both parties offer the small farmers income greater than their per capita share (one) in the three-class model. The irony is that, in the one case where parties do count the welfare of the peasantry as heavily as the welfare of their primary class ($\beta = 1$), the peasantry get less. I have no intuition for this.

(8) The middle class is 50% larger in Germany than in Sweden. One might expect that this would cause the Left to offer them more in Germany than in Sweden: there is, however, no such pattern observed. The agricultural proletariat is twice as large in Sweden as in Germany. One might expect that, in the four-class model, this would cause the Right to offer them more in Sweden than in Germany, an expectation that is borne out by our observations.

(9) I next summarize the observations in a different way. Define

$$a_{LU} = \frac{\omega w + \mu m}{\omega + \mu} \quad , \quad a_{RU} = \frac{\omega w' + \mu m'}{\omega + \mu}$$

$$a_{LW} = \frac{\omega w + \alpha a}{\omega + \alpha} \quad , \quad a_{RW} = \frac{\omega w' + \alpha a'}{\omega + \alpha} \ ,$$

for two policies (w,m,ℓ,a) and (w′,m′,ℓ′,a′). a_{LU} is the fraction of per capita national income that Left proposes to give to the urban classes, a_{LW} is the fraction of per capita national income that Left proposes to give to the working classes, etc. Table 5 presents mini-tables of the form

	Urban	*Workers*
Left	a_{LU}	a_{LW}
Right	a_{RU}	a_{RW}

for every 'Run' reported in Table 2 (Sweden), and Table 6 presents the mini-tables for every run in Table 4 (Germany).

The second columns of all the mini-tables in Tables 5 and 6 offer no surprises: the Left (Right) always proposes to give the working (propertied) classes more than their share of national income. But the first columns of the mini-tables are not so intuitive. After all, both parties in all cases (at least weakly) favor 'their' urban class over 'their' rural class, and so one might conjecture that all the column-one entries in the mini-tables would be greater than one: but this is far from the case. The main reason seems to be that, when class struggle has been resolved in the countryside, the parties are effectively competing directly for the votes of landed peasantry -- cf. their utility functions -- which raises what they offer the landed peasantry. Consider Runs 1-4 for Sweden, in which $(\beta_L,\beta_R) = (0.5, 0.5)$. In Runs 1 and 2, corresponding to the three-class model, the rural sector receives more than its share of national income, but in Runs 3 and 4, the four-class model, they

receive less -- as it 'should be.' The same pattern holds in Runs 5-7. However, the pattern breaks down in Runs 8-12 -- here, the rural sector does not receive more in the three class model than in the four-class model, although it does receive more than its share of national income in all five runs (except for the marginal Run 9, Left). In Runs 15 and 16, where each party weights the welfare of 'its' urban and rural class equally, the urbanites receive more than their share of national income. Here, the aforementioned pattern reverses: in Run 16 (four class model), the rural sector receives a larger share of national income than in Run 15 (three-class model).

Next consider the German story (Table 6). How well does the conjecture, that the rural sector does better when class struggle is resolved than when it isn't, hold? It holds in Runs 1-4, and in Runs 7-10. In Runs 5-6 it also holds, but perhaps not with statistical significance; Runs 13 and 14 appear to show no significant difference in what the peasants are offered, and the small sample of Run 12 eliminates Runs 11-12 from consideration.[9]

The lesson we might cautiously draw from Tables 5 and 6 is that the rural population should receive, from both parties, a larger share of national income when class struggle among them is resolved. The political logic here seems clear.

A final remark is in order with regard to the 'test' of Luebbert's model we have carried out. What we have done is to check whether Left victory is more probable when agricultural class struggle is quiescent, than when it is active, and we have calibrated the model with the frequency distribution of classes from Germany and Sweden in the 1930s.

We have simply stipulated various values for the parameters β_L, β_R, and γ. A desirable alternative would be to use data on the class pattern of voting in Germany and Sweden in the 1930s to calibrate the model (i.e., to calculate the values β_L and β_R that would yield a best fit of the model to the data). Then, using the model so calibrated, we would calculate the probability of Left victory under the two possible scenarios. Unfortunately, reliable data of this kind do not exist (there are data for class voting in Germany by city [see Hamilton (1982)], but their accuracy is questionable).

5. Concluding remarks

Formal politico-economic analysis has, almost universally, been heretofore limited to problems which postulate a unidimensional policy space. Moreover, somewhat less universally, the equilibrium concept used has been that of a Condorcet winner in policy space, known as a Downsian equilibrium. The present paper departs from these conventional practices on both counts: it uses a concept of political equilibrium between *parties with policy preferences* whose leaders are not concerned solely with winning office, and it assumes *multi-dimensional policy spaces.* Because, in the real world, political competition is almost always carried out between partisan parties which announce policies on a variety of issues, the present analysis marks a theoretical advance.

I suggest that readers focus upon these two improvements over the conventional analysis, rather than on the many remaining shortcomings of the present one. Perhaps the main shortcoming is that our equilibrium concept produces a continuum of equilibria in

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[9] Recall, our runs are sampling from the full set of PUNEs, so statistical analysis is, in principle, possible.

the party-competition game. Rather than try to refine that concept to narrow the set of equilibria, I have been content to observe that these equilibria are quite spatially concentrated, and so we do not lose much by looking at average values of policies over our sampled equilibria. Refining the equilibrium concept is not a trivial matter, for if one posits something like Nash bargaining between the factions, with the threat point being dissolution of the party, one ends up inducing a complete preference order for each party on the space TxT, and then, as with reformist Nash equilibrium, equilibria of multi-dimensional games will generically fail to exist[10].

A second limitation of the present analysis is its reliance on simulation. Again, I suggest that the reader look at the other side of the coin. We have gained in our modeling of reality by injecting factions into political parties and many dimensions into policy competition. It is, perhaps, too much to ask that we do these things and *in addition* be able to do comparative statics in a fully satisfactory (i.e., analytical) manner.

Thirdly, we have assumed that voters are interested only in their economic fortunes: we have relegated non-economic interests to their influence on an unstudied random variable that affects the fractions of classes that vote for policies.

Fourthly, we have assumed that there are only two parties, which is an historical inaccuracy. The Communists, in particular, were extremely important in Germany. A model of three party politics with proportional representation, however, is an order of magnitude more difficult than the model in this paper, for we must propose a theory of

[10] This claim is not supposed to be obvious, but it is beyond this paper's scope to pursue it.

coalition formation to form a government, and model citizens as voting strategically, not sincerely.

Modulo these caveats, the main result of our analysis is to provide strong but not conclusive support for Luebbert's theory of regime choice in European countries during the interwar period. The support is strong because Luebbert's claim is true for most parameter vectors of our model, and moreover, Luebbert's mechanism appears also to hold -- that is, the Left sustained lower probabilities of victory when class struggle in the countryside was active *because* it proposed to give a lot to the agricultural proletariat and only a little to the landed peasantry. The support is not conclusive because there are parameter values of our model for which Luebbert's claim is false.

References

Abraham, David. 1981. *The collapse of the Weimar Republic,* Princeton: Princeton University Press

Anderson, Dewey and Percy Davidson .1943. *Ballots and the Democratic Class Struggle*, Stanford: Stanford University Press

Claudin, Fernando. 1975. *The Communist Movement from Cominform to Comintern, Part One*. New York: Monthly Review Press

Downs, Anthony. 1957. *An Economic Theory of Democracy,* New York: HarperCollins

Hamilton, Richard F. 1982. *Who Voted for Hitler?* Princeton: Princeton University Press

Korpi, Walter. 1983. *The Democratic Class Struggle*, London: Routledge & Kegan Paul

Lipset, Seymour Martin. [1959] 1994. *Political Man* , Baltimore: The Johns Hopkins University Press

Luebbert, Gregory. 1991. *Liberalism, Fascism, or Social Democracy,* New York: Oxford University Press

Nieuwbeerta, Paul. 1995. *The Democratic Class Struggle in Twenty Countries, 1945-1990,* Amsterdam: Thesis Publishers

Przeworski, Adam and John Sprague. 1986. *Paper Stones,* Chicago: University of Chicago Press

Przeworski, Adam, Ernest Underhill, and Michael Wallerstein. 1978 *The Evolution of class structure in Denmark, 1901-1960; France, 1901-1968; Germany 1882-1933 and 1950-1961; and Sweden, 1900-1960: Basic Data Tables,* University of Chicago (xerox)

Roemer, John E. 1997. "Political-economic equilibrium when parties represent constituents: The uni-dimensional case," *Social Choice and Welfare* 14, 479-502

Roemer, John E. 1998. "Why the poor do not expropriate the rich: An old argument in new garb," *Journal of Public Economics* 70, 399-424

Roemer, John E.1999. "The democratic political economy of progressive taxation," *Econometrica* 67, 1-19

Schorske, Carl. 1993[1955]. *German Social Democracy, 1905-1917*, Cambridge, MA: Harvard University Press

Tingsten, Herbert. [1941] 1973. *The Swedish Social Democrats,* Totawa, NJ: Bedminster Press

Wittman, Donald. 1983. "Candidate motivation: A synthesis of alternative theories," *American Political Science Review* 77, 142-57