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Economic Growth With Optimal Public Spending Compositional

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Economic Growth
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Abstract

This paper uses a one-sector, endogenous growth model to study optimal composition between public investment and consumption in government expenditure and its relationships with economic growth. Assuming a benevolent government which maximizes a representative household's lifetime utilities, the paper determines the unique, interior public investment share in government's budgets, which is determined by policy and structural parameters. It finds that the conventional determinants of economic growth now generate stronger growth effects, via their indirect impacts upon optimal public spending composition. The effects emerge from raising the marginal utility of private consumption, relative to the marginal utility of public consumption, thereby inducing public investment and increasing economic growth. Our quantitative results suggest that the growth effect is sizable. The large growth effect via optimal public investment in our model has implications to East Asian economic growth miracles where public investment share and economic growth are both higher than other area's countries.

Key words: public consumption, public investment, economic growth
JEL classification: H41, H54, O41

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I. Introduction

Since the work by Barro (1990), growth effects of public spending have been one of the popular topics in economics research.[1] This fashion results partly from the fact that the effects of fiscal policy have been an important area of policy debates in macroeconomics during the 1970s and earlier. It is natural for the growth effects of public spending to redraw attentions. Existing endogenous growth models specify public expenditure either as productive or as consumptive. While productive public spending is formulated as an investment to externally enhance private production, consumptive public spending is devised to externally increase households' utility. Recent works such as Futagami, Morita and Shibata (1993), Glomm and Ravikumar (1994), Fisher and Turnovsky (1998), and Chen (2003) have adopted the former fashion in specifying government spending, and other studies like Bianconi and Turnovsky (1997) and Devereux and Wen (1998) have used the latter modeling strategy. In addition, some analyses have incorporated both productive and consumptive aspects of public spending in their models, e.g., Barro (1990) and Turnovsky (2000).[2]

The composition between public consumption and public investment differs substantially across countries. Indeed, the share of public capital account in government spending is usually high in East Asian countries, low in North American and European countries, and much lower among Latin American countries.[3] Moreover, the set of countries with a large share of public investment in government spending has accomplished higher growth, while the set with a small share has experienced lower growth. Although it is clear to see a large share of productive public expenditure

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1 Another research on fiscal policy is the growth effects of taxation, made popular by Lucas (1990), including Rebelo (1991), Turnovsky (1996), Bond et al (1996) and Mino (1996), among others.

2 Other studies have included both productive and consumptive public spending to analyze macroeconomic adjustments; e.g., Baxter and King (1993) and Chang (1999).

3 For example, in the 1980s and early 1990s the share of public investment in government spending was above 15% in Korea and above 20% in Taiwan, and it was a little more than 5% in the U.S. and near 5% in the UK and France. In Brazil, it is less than 5%. See IMF (various years) and CEPD (2001).

associated positively with economic growth, the question is why some governments would choose a large fraction of productive spending in government budgets while others choose a small share. How does a benevolent government determine the composition between consumptive and productive expenditure optimally? What are the relationships between the determinants of public spending composition and economic growth? These are important questions, as their answers may uncover underlying factors enhancing long-run economic growth through a high optimal investment share in government budgets, a mechanism that has never been disclosed by existing works. This paper envisages these questions in a simple, one-sector endogenous growth model.

In this model, the government decides optimal spending composition under given income tax rates, in order to maximize representative households' lifetime utilities.[4] Given the optimal composition, representative households/producers then optimize their consumption and savings choices. Under this setup, the government optimization is only second-best optimum.[5] Under a reasonable condition, we have shown the existence of unique, interior optimal composition in government spending. Different from existing literature, our optimal public investment share is determined by all policy and other structural parameters. These factors raise the marginal utility from private consumption, relative to the marginal utility of public consumption, initiating government to allocate fewer budgets into consumptive spending and more into productive spending at optimum, and thereby yielding stronger positive long-run growth effects in equilibrium, than would be otherwise obtained conventionally. We calibrate the model into the Taiwan economy, and carry out various shocks to the benchmark economy. The numerical results indicate sizable economic growth effects,

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4 We could think of the decision procedure as the government sets an optimal tax rate first, and given the tax revenues, the government then determines the spending composition. Since the optimal tax rate and taxation policy is not the focus of this paper, we will simplify the analysis by treating the tax rate as a given parameter, overpassing the government's first-stage decision problem. This first-stage problem has been studied by Barro (1990) and Futagami, et. al. (1993), among others.

5 Due to the difficulties for a government to obtain first-best optimization, we only study second-best optimization Moreover, since there is only one kind of tax rates and no debt issue in our model, the first-best optimum cannot be replicated by a market equilibrium. See Turnovsky (1996) for this point.

when public spending composition responds to these shocks.

A related paper is one studied by Devarajan, et al. (1996), with two kinds of public investment without public consumption, but its main purpose is to use data for developing countries to empirically test the effects of government spending composition upon long-term growth. More related works are those with optimal public consumption and investment studied by Barro (1990), Lee (1992), Lau (1995), Turnovsky (2000) and Piras (2001).[6] While Lee (1992) finds two kinds of optimal composition, Barro (1990), Turnovsky (2000), Piras (2001) and Lau (1995) obtain unique optimal composition, and the optimal investment shares in Barro, Turnovsky and Piras, determined only by the degree of public capital externality, differ from ours. In addition to the time-preference rate, Lau's (1995) optimal public investment share is determined only by the elasticity of private capital in production, and his optimal consumption share is determined only by the private consumption share in utility. It is difficult to bear with the result that a parameter would increase shares for one kind of public spending without reducing shares of remaining kinds. Moreover, when government in these existing studies optimizes by increasing its consumption share, it takes its investment share as given.

This paper is different from and contributes to these above papers in two important ways. First, the optimal composition in our work is derived from general optimization in that the government in our model trades off its spending shares, so as to satisfy the government budget constraint. Therefore, we obtain a general form for the determinants of optimal public spending shares, a consistent result for parameters which increase shares for one kind of public spending, must reduce shares for remaining kinds. Secondly, and more important, these above papers do not investigate how

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6 While Lee (1992) extends Barro (1990) in analyzing optimal income tax rates and public spending shares among investment, consumption and income transfer, Lau (1995) compares the differences of optimal public investment share and public consumption share under growth-maximization and welfare-maximization. Turnovsky (2000) extends Barro to consider elastic labor supply and finds many different growth effects due to labor supply responses. Piras (2001) studies the effect of public consumption congestion upon economic growth and public investment share. We should mention that Lee (1992) and Piras (2001) employ public investment share in total government spending and are similar to ours, while the other three papers use the public investment share in income, a concept about the size of government, and are different from ours.

changes in underlying policy and economic structure affect economic growth through government's response to its changing investment and consumption share, entailing a growth mechanism that has never been studied in existing literature. This paper uncovers this channel, exhibiting steady-state growth effects of changes in underlying policies and economic structure which are stronger than those otherwise obtained from existing endogenous growth models. In particular, we quantitatively evaluate the effects of shocks upon economic growth via optimal public spending composition. Our simulation results not only confirm sizable growth effects, but also suggest that the induced, indirect growth effects through public spending composition are larger than the otherwise conventional growth effects. Our findings shed light on how the same determinants of growth lead to growth differentials between East Asia and other parts of the world, through their impacts on optimal public spending composition.

This paper is organized as follows. We build a one-sector, endogenous growth model in Section II, and solve individuals' problem in Section III. Section IV studies optimal public spending compositions, and Section V investigates the relationships between optimal compositions and economic growth. Section VI provides numerical results. Finally, Section VII concludes the paper.

II. A Basic Model

Our model differentiates productive public spending from consumptive public spending. We follow Barro (1990) by assigning consumptive public expenditure entering households' instantaneous utilities, and productive public spending entering private production in an external fashion.[7] The economy is populated by a continuum of infinitely-lived, representative households. The population does not grow, and is normalized to be a unity. There is a continuum of representative firms, each of which has production technology and households entitle its shares. In addition, there is a government.

7 Early works modeling public capital as a factor of production function include Shell (1967) and Arrow and Kurz (1970). Empirically, a positive effect of public capital on private production has recently been documented by Aschauer (1989), in a study using the U.S. data. Lynde and Richmond (1993) find that public capital has played an essential role in enhancing the productivity growth of U.K. manufacturing. See Gramlich (1994) for a survey of the empirical literature, most of which is for the U.S..

A representative household is assumed to possess the following discounted lifetime utility:

$$U = \int_0^{\infty} e^{-\rho t} \frac{[c(t)^{\alpha} \; g_c(t)^{1-\alpha}]^{1-\sigma} - 1}{1-\sigma} \, dt, \quad \rho > 0, \; 0<\alpha<1, \; -\infty<1-\sigma< 1, \qquad \textbf{(1)}$$

in which the function $c(t)$ is the instantaneous private consumption in t, and $g_c(t)$ is the instantaneous public consumption. Parameter ρ is the instantaneous time-preference rate, α is the share of private consumption in households' utility, relative to public consumption, and σ is the reciprocal of the intertemporal elasticity of substitution for consumption. We assume $1\text{-}\sigma < 1$ so that the instantaneous utility function is strictly concave in its arguments.

The production technology is externally affected by public capital services:

$$y(t) = A k(t)^{\beta} g_I(t)^{1-\beta}, \; A>0, \; 0<\beta<1, \qquad \textbf{(2)}$$

in which $y(t)$ is the instantaneous output per capita, $k(t)$ is the instantaneous capital stock per capita, and $g_I(t)$ is the productive public service per capita in t. Parameter $1\text{-}\beta$ captures the externality to which public capital services affect private production, and A summarizes the productivity level. A Cobb-Douglas functional form ensures that each firm's profit-maximization problem is concave and well defined. For simplification, we assume zero depreciation for capital stock. Each firm is competitive in the goods and input markets.

Since the government provides both public consumption and public capital services free of charge, it is necessary to have sources of tax revenues. We assume that public spending is financed by income taxes. An income tax setup generates tax revenues consistent with a perpetual growth framework, and has been employed by Barro (1990) and many of his followers. Denoting τ as the income tax rate, then disposable income which is not spent becomes savings, and augments capital formation in a way as follows:

$$\dot{k} = (1-\tau) y(t) - c(t), \qquad \textbf{(3a)}$$

where a dot notation over a variable denotes the time derivative of that variable.

Finally, the government budget constraint must satisfy:

$$T(t) \equiv \tau y(t) = g_c(t) + g_I(t) = s\tau y(t) + (1-s)\tau y(t), \qquad \textbf{(3b)}$$

which says that the government devotes a fraction *s* of its tax revenues *T*(*t*) to public consumption, and the remaining fraction 1-*s* to public capital services.

III. Household-Firm's Problem

The representative household-firm's problem is to choose consumption flows and capital accumulation over time, in order to maximize the discounted present value of lifetime utilities (1), subject to production technology (2) and budget constraint (3a), given as follows: tax rate τ, public consumption spending g_c, and public investment g_I. Solving the problem, we define a current-value Hamiltonian equation, to derive the following first-order conditions:

$$\alpha c(t)^{\alpha(1-\sigma)-1} g_c(t)^{(1-\alpha)(1-\sigma)} = \lambda(t), \qquad \textbf{(4a)}$$

$$(1-\tau)A\beta k^{\beta-1} g_I^{1-\beta} = \rho - \frac{\dot{\lambda}}{\lambda}, \qquad \textbf{(4b)}$$

$$Lim_{t\to\infty}\, e^{-\rho t}\lambda(t)k(t) = 0, \qquad \textbf{(4c)}$$

where $\lambda(t)$ is the shadow price of private capital stock in *t*.

Equation (4a) equates the marginal utility of current consumption to the marginal utility of next period's consumption, the latter of which results from this period's savings as just the shadow price of private capital. Euler's equation (4b) equates the marginal net return of private capital stock to the marginal return on consumption, which is the time-preference rate adjusted for a capital gain. Finally, (4c) is the transversality condition.

Assuming perfect-foresight expectations, a perfect-foresight market equilibrium, given tax rate and public spending, is determined by household optimization (4a)-(4c), household and government budget constraints (3a)-(3b), and production technology (2). These equations can be used to solve for the following five variables: $\frac{\dot{k}}{k}$, $\frac{\dot{c}}{c}$, $\frac{\dot{\lambda}}{\lambda}$, $\frac{c}{k}$, and $\frac{T}{k}$. A feature of this type of model is that the equilibrium is always on a balanced growth path (BGP). Therefore, growth rates $\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\dot{y}}{y} = \frac{\dot{g}_c}{g_c} = \frac{\dot{g}_I}{g_I}$ always hold, referred to as ϕ, and great ratios $\frac{y}{k}$, $\frac{c}{k}$, $\frac{g_c}{k}$ and $\frac{g_I}{k}$ are constant in equilibrium. In order to solve for equilibrium, we first substitute $g_I = (1- s)\tau y$ into (2) to obtain:

$$y(t) = A^{\frac{1}{\beta}}[\tau(1-s)]^{\frac{1}{\beta}-1}k(t). \tag{5a}$$

Next, we substitute (5a) into $g_c = s\tau y$ and $g_I = (1- s)\tau y$, respectively, to give:

$$g_c(t) = A^{\frac{1}{\beta}}s(1-s)^{\frac{1}{\beta}-1}\tau^{\frac{1}{\beta}}k(t), \tag{5b}$$

$$g_I(t) = A^{\frac{1}{\beta}}(1-s)^{\frac{1}{\beta}}\tau^{\frac{1}{\beta}}k(t). \tag{5c}$$

It is obvious to see from (5a)-(5c) that for given s, great ratios $\frac{y}{k}$, $\frac{g_c}{k}$ and $\frac{g_I}{k}$ are constant. The constancy of $\frac{c}{k}$ can be obtained from dividing (3a) by k, together with the use of (5a).

Finally, differentiating (4a) with respect to time, using (4b) and condition $\frac{\dot{c}}{c} = \frac{\dot{g}_c}{g_c} \equiv \varphi$, yields:

$$\varphi = \frac{(1-\tau)A^{\frac{1}{\beta}}\tau^{(\frac{1}{\beta}-1)^2}\beta(1-s)^{\frac{1}{\beta}-1} - \rho}{\sigma}. \tag{6a}$$

The above expression characterizes the equilibrium growth rate of consumption, which, using (3a), is equal to the equilibrium growth rate of capital and output. We consider:

Condition PB: $(1-\tau)A^{\frac{1}{\beta}}\tau^{(\frac{1}{\beta}-1)^2}\beta(1-s)^{\frac{1}{\beta}-1} > \rho > (1-\sigma)(1-\tau)A^{\frac{1}{\beta}}\tau^{(\frac{1}{\beta}-1)^2}\beta(1-s)^{\frac{1}{\beta}-1}$.

While the first inequality in Condition PB guarantees a positive ϕ, the second inequality assures a bounded lifetime utility.

To characterize (6a), we totally differentiate it to obtain:[8]

$$\varphi = \varphi(s; \tau, A, \beta, \sigma, \rho),\ \varphi_s<0,\ \varphi_\tau \gtrless 0,\ \varphi_A>0,\ \varphi_\beta>0,\ \varphi_\sigma<0,\ \varphi_\rho<0. \qquad \textbf{(6b)}$$

Relationship (6b) says that economic growth is decreasing in the share of public consumption in government spending. In a (s, φ) plane in Figure 1, Relationship (6b) is denoted as Locus BR (Best Response), which is downward slopping, and intersects φ axis at $\bar{\varphi} = \frac{1}{\sigma}[(1-\tau)\tau^{(1/\beta-1)^2}\beta A^{1/\beta} - \rho] > 0$ and s axis at $0 < \bar{s} = 1 - [\rho(1-\tau)^{-1}\tau^{-(1/\beta-1)^2}\beta^{-1}A^{-1/\beta}]^{\beta/(1-\beta)} < 1$. Moreover, for a given s, Locus BR shifts upward in A and β, and downward in σ and ρ, meaning that for given spending composition, steady-state growth rates are increasing in neutral productivity, private capital share and intertemporal elasticity of substitution, and decreasing in the time-preference rate. These properties are similar to those in existing literature. However, for a given s, a higher τ has an ambiguous effect on economic growth. Differentiating (6a) with respect to φ and τ gives

$$\frac{d\varphi}{d\tau} = \frac{\beta A^{1/\beta}(1-s)^{1/\beta-1}\tau^{(1/\beta-1)^2-1}}{\rho}\{(\frac{1}{\beta}-1)^2 - [1+(\frac{1}{\beta}-1)^2]\tau\} \gtrless 0 \ \ if\ \tau \lessgtr \hat{\tau} \equiv \frac{(1/\beta-1)^2}{1+(1/\beta-1)^2} < 1-\beta. \qquad \textbf{(6c)}$$

Condition in (6c) says that, for a given s, a higher income tax rate increases economic growth when it is below a threshold that is smaller than the externality of public capital. Our threshold is smaller that obtained in Barro (1990) and Futagami, et al. (1993), as these two works did not involve public consumption when their government chooses tax rates to maximize the economic growth rate.

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8 See mathematical Appendix A for derivation.

IV. Optimal Government Expenditure Composition

We now analyze the government's second-best optimization problem. We assume a benevolent government whose objective is to maximize the present discounted value of a representative household's lifetime utility by choosing public spending composition,[9] given production technology (2), resource constraints (3a)-(3b), and individuals' best responses (4a)-(4c). As individual's best responses, along with (2) and (3a)-(3b), are summarized in (6a), the government's problem becomes to maximize (1) subject to (6a). An easier way to solve this problem is to substitute (6a) into (1) to obtain households' indirect utility, and the problem becomes to maximize the indirect utility. The optimization proceeds in four steps. First, from (6a), we attain:

$$c(t) = c(0)\, e^{\varphi(s;\, \tau, A, \beta, \sigma, \rho)t}, \tag{7a}$$

and using (5b) and the result $\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \varphi$ implied by (6a) and (3a), we obtain:

$$g_c(t) = A^{\frac{1}{\beta}} s(1-s)^{\frac{1}{\beta}-1} \tau^{\frac{1}{\beta}} k(0) e^{\varphi(s;\, \tau, A, \beta, \sigma, \rho)t},\ k(0) > 0 \textit{ given}. \tag{7b}$$

Next, substituting (7a) and (7b) into (1) yields:

$$U = -\frac{1}{(1-\sigma)\rho} + \frac{[A^{1/\beta}\tau^{1/\beta-1}k(0)]^{1-\sigma}}{1-\sigma}\, \frac{s(1-s)^{(1/\beta-1)(1-\sigma)}c(0)^{\alpha(1-\sigma)}}{\rho-\varphi(1-\sigma)}, \tag{8a}$$

which can be rewritten as:

$$\hat{U} \equiv [U + \frac{1}{(1-\sigma)\rho}]\, \frac{1-\sigma}{[A^{1/\beta}\tau^{1/\beta-1}k(0)]^{1-\sigma}} = \frac{s(1-s)^{(1/\beta-1)(1-\sigma)}c(0)^{\alpha(1-\sigma)}}{\rho-\varphi(s;\, \tau, A, \beta, \sigma, \rho)\,(1-\sigma)}. \tag{8b}$$

Notice that maximization of U in (8a) with respect to s is equivalent to maximization of $\hat{U}$ in

9 The other is a growth-maximization government, which will lead to a trivial result with zero share of public consumption. We are not interested in this case.

(8b) with respect to *s*. The optimal conditions, along with the use of (6a), yield:

$$\frac{d\log\hat{U}}{ds} = (1-\sigma)\left((1-\alpha)\frac{1-s/\beta}{(1-s)s} + \frac{\alpha}{c(0)}\frac{\partial c(0)}{\partial s} - (\frac{1}{\beta}-1)\frac{(\sigma\varphi+\rho)}{[\rho-(1-\sigma)\varphi]\sigma(1-s)} \right) = 0. \qquad \textbf{(9a)}$$

In order to derive $\frac{\partial c(0)}{\partial s}$ for (9a), we observe that (3a) must be satisfied in equilibrium, and therefore:

$$\frac{\dot{k}}{k(0)} = \varphi(s;\tau,A,\beta,\sigma,\rho) = (1-\tau)\frac{y(0)}{k(0)} - \frac{c(0)}{k(0)} = (1-\tau)A^{\frac{1}{\beta}}[\tau(1-s)]^{\frac{1}{\beta}-1} - \frac{c(0)}{k(0)}, \qquad \textbf{(3a}'\textbf{)}$$

and a differentiation of it, with the use of (6a), gives:

$$\begin{aligned}\frac{\partial c(0)}{\partial s} &= k(0)[-(\frac{1}{\beta}-1)(1-\tau)A^{\frac{1}{\beta}}\tau^{\frac{1}{\beta}-1}(1-s)^{\frac{1}{\beta}-2} - \frac{\partial\varphi}{\partial s}] \\ &= k(0)(\frac{1}{\beta}-1)\frac{\sigma\varphi+\rho}{(1-s)}(\frac{1}{\sigma}-\frac{1}{\beta}\tau^{(\frac{1}{\beta}-1)(2-\frac{1}{\beta})}) \gtrless 0 \;\; if \;\; \frac{\beta}{\sigma} \gtrless \tau^{(\frac{1}{\beta}-1)(2-\frac{1}{\beta})}.\end{aligned} \qquad \textbf{(3a}''\textbf{)}$$

Relationship (3a″) indicates that a larger share of public consumption in government spending reduces initial consumption via reducing initial output level (the first term in the first equality), and increases initial consumption via reducing economic growth $(\frac{\partial\varphi}{\partial s}<0)$. As a result, the initial consumption may jump downwards or upwards, in order for the equilibrium to move along the lower BGP. In general $\frac{\beta}{\sigma} < \tau^{(1/\beta-1)(2-1/\beta)}$ for a low intertemporal elasticity of substitution, and therefore, initial consumption level would jump downwards in response to a larger share of public consumption.

Thirdly, substituting (3a″) into (9a) yields:

$$\frac{1-\sigma}{\sigma(1-s)}\left(\sigma(1-\alpha)\frac{1-s/\beta}{s} - \alpha(\frac{1}{\beta}-1)(\sigma\varphi+\rho)(\frac{\sigma}{\beta}\tau^{(\frac{1}{\beta}-1)(2-\frac{1}{\beta})}-1)\frac{k(0)}{c(0)} - (\frac{1}{\beta}-1)\frac{\sigma\varphi+\rho}{\rho-(1-\sigma)\varphi} \right) = 0. \qquad \textbf{(9b)}$$

The above condition says that a larger share of public consumption in government spending creates a direct, positive effect on agents' lifetime utility (the first term in the parentheses), an indirect, negative effect through reducing initial private consumption (the second term), and another indirect, negative effect through reducing economic growth (the third term). At optimum, the marginal gains must equal the marginal costs. Using (3a′) and (6a) we derive:

$$\frac{k(0)}{c(0)} = \frac{1}{(1-\tau)A^{1/\beta}[\tau(1-s)]^{1/\beta-1}-\varphi} = \frac{\beta}{(\sigma\varphi+\rho)\,\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi}. \qquad \textbf{(9c)}$$

Finally, substituting (9c) into (9b) with rearrangement gives:

$$\sigma(1-\alpha)\left(\frac{1}{s}-\frac{1}{\beta}\right) = \left(\frac{1}{\beta}-1\right)(\sigma\varphi+\rho)\left(\frac{1}{\sigma\varphi+\rho-\varphi} + \frac{(\sigma\tau^{(\frac{1}{\beta}-1)(2-\frac{1}{\beta})}-\beta)\alpha}{(\sigma\varphi+\rho)\,\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi}\right). \qquad \textbf{(10a)}$$

Equation (10a) characterizes optimal public spending composition between consumption and investment. As the right-hand side in (10a) is positive, for the consistency it therefore requires $\beta > s$, so that the left-hand side is positive. Differentiating (10a) yields:[10]

$$s = s(\varphi;\,\tau,\,\beta,\,\alpha,\,\sigma,\,\rho),\;\; s_\varphi<0,\; \varphi_\tau<0,\; \varphi_\beta>0,\; \varphi_\alpha<0,\; \varphi_\sigma>0,\; \varphi_\rho>0, \qquad \textbf{(10b)}$$

in which $\frac{\partial s}{\partial \varphi} = -\frac{(1/\beta-1)s^2}{\sigma(1-\alpha)}\left(\frac{\rho}{(\sigma\varphi+\rho-\varphi)^2} + \alpha(\sigma\tau^{(\frac{1}{\beta}-1)(2-\frac{1}{\beta})}-\beta)\,\frac{\beta(1-\sigma\varphi)-\sigma\tau^{(\frac{1}{\beta}-1)(2-\frac{1}{\beta})}[(1-(\sigma\varphi+\rho)]}{[(\sigma\varphi+\rho)\,\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi]^2}\right) < 0$

if the second term in the large braces is positive, or if it is negative and is not too small relative to the first term, which is the case if α and σ are not too large. In a (φ, s) plane in Figure 1, Relationship (10b) is referred to as Locus OC (Optimal Composition), which is downward sloping, with a very large slope when s is very small and with $\hat{s} = \frac{\beta\sigma(1-\alpha)}{\sigma(1-\alpha)+(1-\beta)[1+\alpha-\alpha\beta\tau^{(1/\beta-1)(2-1/\beta)}]} < 1$ when $\varphi=0$.

Characterizing Locus OC, for a given φ, the locus shifts leftward in τ and α, and rightward in β, σ and ρ. Intuitively, other things being equal, a higher income tax rate reduces consumption and thus, increases the marginal utility of private consumption, and in optimum, the government reduces

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10 See mathematical Appendix B for derivation.

the share of public consumption in government spending, in order to raise the marginal utility of public consumption. Next, a larger share of private consumption directly increases the marginal utility of private consumption and decreases the marginal utility of public consumption, and in optimum, the share of public consumption is reduced. Thirdly, a larger share of private capital in production increases the marginal product of private capital, and thus income, that increases private consumption leading to a lower marginal utility of private consumption. In order to reduce the marginal utility of public consumption, the government optimally decreases the share of public investment and increases the share of public consumption. Finally, a smaller intertemporal elasticity of substitution and a larger time-preference rate both lower the marginal utility of future private consumption, and therefore, the government optimally increases the share of public consumption in government spending, in order to reduce the marginal utility of public consumption.

The optimal public expenditure composition s^* in (10a) is affected by an expected economic growth rate φ, which is characterized by private sectors' best response (6a). Thus, (6a) and (10a) together determine an optimal public consumption share and an economic growth rate in a steady-state equilibrium. As Loci BR and OC are downward sloping, in order to assure the existence of an interior optimal public consumption share, we impose $\hat{s} < \bar{s}$ (Figure 1). This requires:

Condition S: $$\frac{\beta\sigma(1-\alpha)}{\sigma(1-\alpha)+(1-\beta)[1+\alpha-\alpha\beta\tau^{(1/\beta-1)(2-1/\beta)}]} < 1-[\frac{\rho}{(1-\tau)\tau^{(1/\beta-1)^2}\beta A^{1/\beta}}]^{\frac{\beta}{1-\beta}}$$

which for a small ρ is easily met. This condition demands the optimal share of public consumption when economic growth is zero, to lead to a positive equilibrium economic growth rate, so that the government responds by reducing the optimal share of public consumption, as indicated by arrows.

It is clear that the optimal share of public consumption in government expenditure, and thus the optimal share of public investment, is determined not only by the degree of public capital externality, but also by all the underlying policy and other structural parameters in (6a) and (10a). Therefor, the optimal composition is different from, and is more general than, those in existing works.

To summarize, we obtain:

Proposition 1. *Under Conditions PB and S, there exists an interior, optimal share of public consumption in government spending and a positive economic growth rate. The optimal share of public consumption is determined by all policy and structural factors*.

V. Optimal Composition and Economic Growth

As the composition of public spending is optimized, parameters underlying policies and economic structure tend to affect economic growth through via public spending compositions. This section investigates how these factors influence the optimal public spending composition and the resulting growth effects. The factors can be classified into three aspects, and we start with the size of government, followed by supply-side factors, and finally, by demand-side factors.

1. Size of Government

The size of government in this model is represented by the average income tax rate τ. Most cross-country empirical studies find that the average growth rate of GDP per capita tends to be negatively associated with the size of government consumption expenditure (e.g., Barro and Sala-i-Martin, 1995). It is interesting to analyze how the size of government affects the optimal composition between public consumption and public capital, and the consequent effects upon economic growth.

A larger size of government may shift Locus BR upwards or downwards, depending upon the initial government size relative to threshold $\hat{\tau} \equiv \frac{(1/\beta - 1)^2}{1 + (1/\beta - 1)^2}$. When the initial government size is equal to or above the threshold, a larger government size shifts Locus BR downwards to B′R′ (in Figure 2), which for a given *s*, reduces economic growth (E_1). Moreover, the government optimally increases the share for public consumption in response to a larger government size, and therefore, Locus OC shifts rightward (see O′C′). Economic growth is thereby reduced further (E_2). As a corollary, under the condition that the initial government size is larger than the threshold, lowering government size shifts Locus BR upwards to B″R″, which for a given *s* increases economic growth (E_3), and as a smaller share of public consumption is accompanied, Locus OC shifts leftward to O″C″ and therefore, economic growth is increased further (E_4).

[Insert Figure 2 here]

Proposition 2. *When the government optimizes its spending composition and its size is above the threshold, lowering the government size reduces the share of public consumption, and increases economic growth more than the case of exogenous or unresponsive public spending composition.*

2. Supply-side Factors

The supply-side factors here include the productivity coefficient and the private capital share. When production productivity is higher (a higher *A*), Locus BR shifts upwards to B′R′ in Figure 3. Therefore, equilibrium changes from E to E_1, and as a result, economic growth increases, as in existing literature (e.g., Barro, 1990). Moreover, as economic growth increases, the government is induced to optimally reduce the public consumption share indirectly (along Locus OC), thereby moving equilibrium to E_2. As a consequence, economic growth is enhanced further. Intuitively, higher production productivity increases marginal productivity of private capital, leading to a direct growth effect, and resulting in higher discounted marginal utility from future consumption. The government therefore, optimally reduces public consumption share, in order to increase the marginal utility of public consumption. Public investment share is thus increased, which complements the productivity of private capital and leads to faster capital accumulation in a steady-state equilibrium. This allows for the economy to grow even more rapidly.

[Insert Figure 3 here]

Similarly, a higher private capital share in production (a higher β) shifts Locus BR upward to B′R′ in Figure 3, just as the case of higher production productivity. Under given government spending composition, equilibrium changes from E to E_1, resulting in higher economic growth. As the degree of government productive externality is smaller (i.e., a lower 1-β), for a given economic growth rate the government reduces the share of public investment in its spending, thus shifting Locus OC rightward to O′C′. Nevertheless, higher economic growth indirectly induces the government to reduce the share of public consumption, so the net effect depends upon whether the direct or the indirect effect dominates. Therefore, economic growth may be enhanced or mitigated. As our calibration in the next section suggests, the indirect effect dominates the direct effect, and therefore, the share of public consumption is reduced, and the economic growth rate is increased further (see E_3).

Proposition 3. *Higher productivity results in a lower consumption share in public spending, thereby leading to a stronger growth effect than the case of exogenous or unresponsive public spending compositions. Higher private capital share has similar effects when the indirect effect dominates.*

3 Demand-Side Factors

There are three demand-side parameters. First, when a private consumption share is higher (a higher α), relative to public consumption, Locus BR is not affected. Nevertheless, Locus OC shifts leftward to O″C″(see Figure 3), because a higher private consumption share reduces the marginal utility of public consumption and increases the marginal utility of private consumption, and thus the government optimally raises public investment and lowers public consumption, driving a leftward shift of Locus OC and thereby relocating equilibrium from E to E_4. As a result, optimal public consumption share decreases and economic growth increases. This indirect positive growth effect differs from those in conventional wisdom. For example, a higher private consumption share does not have any direct or indirect growth effect in Barro (1990), whereas it causes a direct negative growth effect in Turnovsky (1996, 2000), and a direct, but not an indirect, positive growth effect in Piras (2001).[11] These differences lie in the situation that the government in our model optimally

11 While the direct negative growth effect in Turnovsky (1996a) and the direct positive growth effect in Piras (2001) both come from the congestion in public consumption service, the direct negative growth effect in Turnovsky (2000) roots in disutility from the reduction in leisure.

increases the share of public investment in reaction to a higher share of private consumption, which complements private capital formation and thereby enhances economic growth.

Finally, higher intertemporal elasticity of substitution in consumption (a lower σ) and lower time-preference rates (a lower ρ) both increase the growth rate of consumption, shifting Locus BR upward (see Loci B′R′ and B″R″, respectively in Figure 4). For given government spending composition, equilibrium changes to E_1 and E_2, respectively, and economic growth is higher. Moreover, both these changes raise the marginal utility of future consumption, and in optimum, the government reduces public consumption shares shifting Locus OC leftward toward O′C′. It follows that equilibrium moves to E_3 and E_4, respectively, thereby increasing economic growth. Therefore, indirectly through increasing optimal public investment shares by the government, resulting in larger private capital formation, a higher intertemporal elasticity of substitution in our model raises economic growth higher than that obtained in existing studies with only a direct effect (e.g., Barro, 1990), and a lower time-preference rate strengthens economic growth than the proposed growth from existing literature (e.g., Palivos and Yip, 1995).

Proposition 4. *Higher private consumption share and intertemporal elasticity of substitution, and a lower time-preference rate all increase optimal public investment shares in government expenditure, resulting in a stronger positive growth effect, as compared with public spending compositions which are exogenous or unresponsive to these factors*.

VI. Some Numerical Results

Further insights into the growth effects of policy and structural parameters through adjustments in public expenditure compositions can be obtained by carrying out numerical analysis of the model. We begin by characterizing a benchmark economy, by calibrating the model using the following parameter values representative of the Taiwan economy:[12]

$$\rho=0.04,\ \sigma=2.5,\ \tau=0.15,\ \beta=0.92,\ A=0.305983,\ \alpha=0.80227.$$

We should note that calibration into other economies, e.g., the U.S., will generate similar results. While parameter value for the time preference rate, ρ, is taken from Turnovsky (2000), the coefficient for risk aversion, σ, is chosen as 2.5 so that the intertemporal elasticity of substitution ($1/\sigma$) is smaller than one (e.g., Jones, et al., 1993). We choose a time-preference rate higher than 2% used for the U.S., because the calibrated Taiwan economy has a higher real economic growth rate per capita, and a higher time-preference rate assures the bounded lifetime utility. As we calibrate the model to the Taiwan economy whose s is 0.8, the consistency in equation (10a) requires $\beta>s$, and therefore the parameter for the share for public capital, $1-\beta$, must be less than 0.2. We choose $1-\beta=$ 8%, which lies between the range for the documentation of high externality (e.g., Aschauer, 1989) and low externality (e.g., Barro and Sala-i-Martin, 1995) of public capital. With the chosen $\beta=0.92$, the threshold of tax rates for the maximal economic growth rate is $\hat{\tau} \equiv \frac{(1/\beta-1)^2}{1+(1/\beta-1)^2}=0.751\%$. We use average tax burden 15% in Taiwan to measure the flat tax rate and thus, the size of government. Finally, based on these parameter values, we calibrate both the values for productivity coefficient, A, and for the share of public expenditure contributed to household's utility, $1-\alpha$, so that the model economy is consistent with both the real per capita GDP growth rate $\phi=5.8\%$ and the share of public capital in government expenditure $1-s=20\%$ in Taiwan in 1952-1999 (CEPD, 2001). Using economic system (6a) and (10a), we thus obtain $A=0.30598$ and $\alpha=0.80229$. These parameters lead to the following steady-state equilibrium values: $s=80\%$ and $\phi=5.8\%$. See Row 1 in Table 1.

[Insert Table 1 here]

Rows 2-7 in Table 1 describe various shocks from the benchmark. Row 2 reports the effects of reducing the size of government. Notice that the calibrated $\tau=15\%$ is larger than the threshold for the maximal economic growth rate, $\hat{\tau}=0.751\%$. Therefore, as the size of government is reduced by

12 For economic growth in Taiwan between the 1950s and the early 1990s, see Tallman and Wang (1994).

1 percentage point to 14%, under exogenous public spending composition economic growth is increased mildly by dϕ= 0.083 percentage points to ϕ=5.883%. This yields a weak, negative association between the size of government and the economic growth rate. As the government optimizes its spending composition, public consumption spending share is lowered to s_1=60.48% and public investment share is increased to 39.52%. This leads to faster capital accumulation, and hence, economic growth is raised further by ϕ_1-ϕ=0.451 percentage points to ϕ_1=6.334%. Consequently, the negative relationship between the size of government and economic growth is more apparent.

Rows 3-4 are shocks to the supply side. Under exogenous or unresponsive public spending composition, a neutral productivity increase (a higher *A*) from 0.305983 to 0.32 raises the economic growth rate to 6.169%, whereas an increase in private capital share (a higher β) from 0.92 to 0.93 accompanies a larger economic growth rate (6.486%). Under optimal government spending composition, a higher productivity and a larger capital share in production, reduce optimal public consumption share from 80% to s_1=60.41% and s_1=63.82%, respectively, and as a result, the economic growth rate increases further by 0.485 percentage points to 6.664% for the larger *A*, and by 0.354 percentage points to 6.514% for the larger β.

Finally, Rows 5-7 are changes in the demand side. While a higher private consumption share in household's utility (α) is increased from 0.80227 to 0.82, it does not change the equilibrium economic growth in the case of exogenous public spending composition (Row 5). Under optimal public spending composition, however, it reduces optimal public consumption shares contributing to higher public capital services. As a consequence, the economic growth rate is increased by ϕ_1-ϕ=0.488 percentage points to 6.288%. Finally, for an increase in the intertemporal elasticity of substitution from 1/2.5 to 1/2.4 (i.e., a higher 1/σ) and a reduction in the time-preference rate (ρ) from 0.04 to 0.039, the economic growth rate is increased mildly by dϕ=0.242 and dϕ=0.04 percentage points, respectively, when public spending composition is exogenous (Rows 6 and 7). When public spending composition is optimized, both changes lead to higher optimal public investment shares.

The economic growth rate is raised by 0.477 percentage points to 6.519% for the smaller σ, and by 0.451 percentage points to 6.291% for the smaller ρ.

In summary, when the public spending composition between consumption and investment is optimized, our numerical results indicate sizable growth effects for shocks to policies and structural parameters. In particular, except for a change in β, the induced, indirect growth effect through public spending composition (Column ϕ_1-ϕ in Table 1) is larger than the conventional direct growth effect (Column dϕ). These quantitative results lend supports to the importance of taking public spending composition into consideration when investigating the engines of economic growth.

VII. Concluding Remarks

This paper extends existing endogenous growth models into incorporating both public consumption and public investment, to optimize public spending compositions and investigates their relationships with economic growth. It builds a simple, one-sector growth model to study these issues. It derives a unique, interior optimal public investment share of total government budget, and thus a unique interior, optimal public consumption share, which is determined by policy and structural parameters.

This paper also finds that economic factors which affect economic growth in conventional wisdom, now yield stronger growth effects, from government's optimal response through its spending share adjustments between investment and consumption. These effects emerge because these economic factors change the marginal utility of private consumption, relative to the marginal utility from public consumption, and therefore, induce the government to redistribute its budget between investment and consumption spending. Our numerical results indicate large growth effects through this mechanism, and in most cases the indirect growth effect through public spending composition is larger than the direct growth effect.

Finally, many existing cross-sectional empirical growth studies have investigated the growth effects of productive and public consumption spending. These studies normally use the ratios of public consumption spending to gross domestic products (e.g., Barro and Sala-i-Martin, 1995, Ch. 12), or to total government expenditure (e.g., Devarajan, et al, 1996), as a regressor against the long-run growth rate of real per capita GDP, and estimate and test the effects of public spending upon long-term economic growth. Moreover, observations indicate that East Asian countries have had higher fractions of public investment in government spending and higher economic growth rates than other area's countries, among other differences. To the extent that a government optimizes its spending shares, the shares are determined by underlying economic structure. Our results suggest that high public investment shares in East Asian countries come from their governments' adjustment toward fundamental economic structures. Therefore, a high public investment share, and thus a low public consumption share, itself may not be the main underlying reason for explaining cross-country growth differentials. They are the result of the government's optimal choices.

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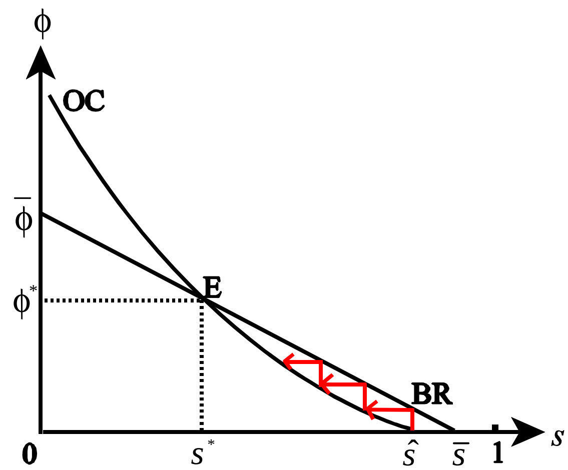

Figure 1. Optimal Government Expenditure Composition

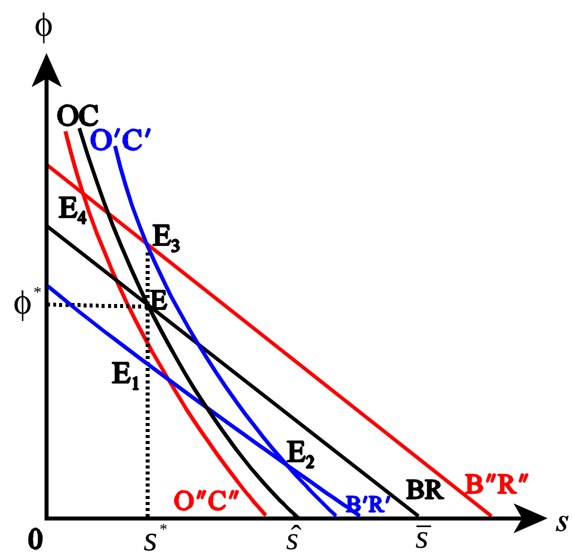

Figure 2. Effects of a Larger Size of Government

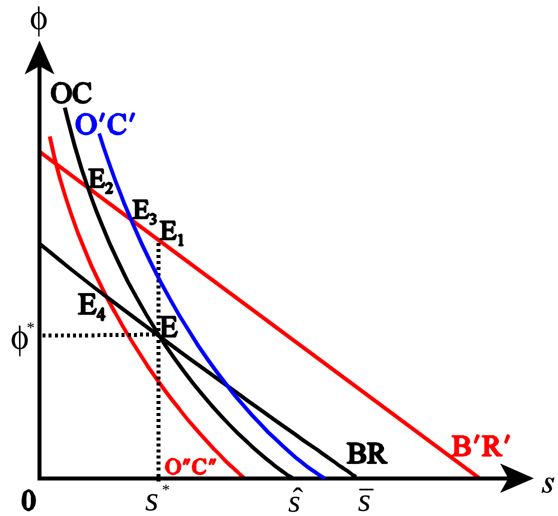

Figure 3. Effects of Higher A, β and α

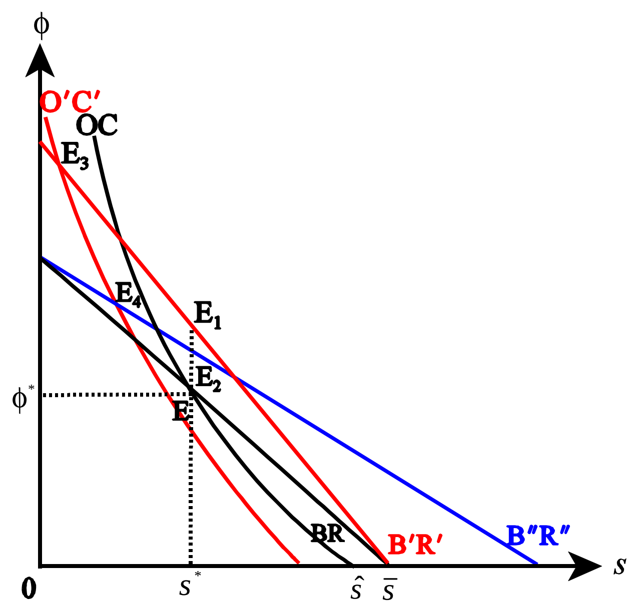

Figure 4. Effects of Lower σ and ρ

Table 1. Some Simulation Results

%

	ϕ	$d\phi$	s_1	ϕ_1	$\phi_1 - \phi$
benchmark	5.8	0	80	5.8	0
τ =0.14	5.88316	0.08316	60.48	6.33397	0.45081
A =0.32	6.169	0.369	60.41	6.66445	0.48545
β =0.93	6.15657	0.35657	63.82	6.51409	0.35392
α =0.82	5.8	0	58.32	6.28792	0.48792
σ =2.4	6.04167	0.24167	60.08	6.51915	0.47748
ρ = 0.039	5.84	0.04	60.49	6.29133	0.45133

Note: Parameter values for the benchmark case are: τ =0.15, A=0.305983, β =0.92, α =0.80227, σ =2.5 and ρ= 0.04. Notation ϕ indicates the real economic growth rate per capita and $d\phi$ its changes when s is fixed at 80%, whereas ϕ_1 denotes the real economic growth rate per capita when s is optimized as s_1.

Appendix: (**Not To Be Published**)

This appendix derives some comparative-static results.

A Derivation of the comparative-static results in equation (6b).

Differentiation of (6a) with respect to ϕ, s,τ, A, β, σ and ρ leads to

$$\frac{\partial\varphi}{\partial s} = -(\frac{1}{\beta}-1)\frac{(1-\tau)\tau^{(\frac{1}{\beta}-1)^2}\beta A^{\frac{1}{\beta}}(1-s)^{\frac{1}{\beta}-2}}{\sigma} = -(\frac{1}{\beta}-1)\frac{\sigma\varphi+\rho}{\sigma(1-s)} < 0, \tag{A1}$$

$$\begin{aligned}\frac{\partial\varphi}{\partial\tau} &= \frac{\beta A^{1/\beta}(1-s)^{1/\beta-1}\tau^{(1/\beta-1)^2-1}}{\sigma}\{(\frac{1}{\beta}-1)^2-[1+(\frac{1}{\beta}-1)^2]\tau\} \\ &= \frac{\sigma\varphi+\rho}{\sigma(1-\tau)}\{(\frac{1}{\beta}-1)^2-[1+(\frac{1}{\beta}-1)^2]\tau\} \gtrless 0 \;\; if \;\; \tau \lessgtr \hat{\tau} \equiv \frac{(1/\beta-1)^2}{1+(1/\beta-1)^2} < 1-\beta,\end{aligned} \tag{A2}$$

$$\frac{\partial\varphi}{\partial A} = \frac{(1-\tau)\tau^{(\frac{1}{\beta}-1)^2}\beta A^{\frac{1}{\beta}-1}(1-s)^{\frac{1}{\beta}-1}}{\beta\sigma} = \frac{\sigma\varphi+\rho}{\beta\sigma A} > 0, \tag{A3}$$

$$\frac{\partial\varphi}{\partial\beta} = \frac{1}{\beta}\{1+\frac{1}{\beta}[2(-\log\tau)(\frac{1}{\beta}-1)-\log(1-s)]\}\frac{\varphi\rho+\sigma}{\sigma} > 0, \;\; as \;\; -\log\tau > 0 \;\; and \;\; -\log(1-s) > 0, \tag{A4}$$

$$\frac{\partial\varphi}{\partial\sigma} = -\frac{(1-\tau)\tau^{(\frac{1}{\beta}-1)^2}\beta A^{\frac{1}{\beta}}(1-s)^{\frac{1}{\beta}-1}-\rho}{\sigma^2} = -\frac{\varphi}{\sigma} < 0, \tag{A5}$$

$$\frac{\partial\varphi}{\partial\rho} = \frac{-1}{\sigma} < 0. \tag{A6}$$

B. Derivation of the comparative-static results in equation (10b).

Differentiation of (10a) with respect to s,ϕ, τ, A, β,α, σ and ρ leads to:

$$\frac{\partial s}{\partial\varphi} = -\frac{(1/\beta-1)s^2}{\sigma(1-\alpha)}\left(\frac{\rho}{(\sigma\varphi+\rho-\varphi)^2}+\alpha(\sigma\tau^{(\frac{1}{\beta}-1)(2-\frac{1}{\beta})}-\beta)\frac{\beta(1-\sigma\varphi)-\sigma\tau^{(\frac{1}{\beta}-1)(2-\frac{1}{\beta})}[(1-(\sigma\varphi+\rho)]}{[(\sigma\varphi+\rho)\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi]^2}\right) < 0, \tag{B1}$$

$$\frac{\partial s}{\partial \tau} = -\frac{s^2}{\sigma(1-\alpha)} \frac{(1/\beta-1)^2(2-1/\beta)(\sigma\varphi+\rho)\alpha\tau^{(1/\beta-1)(2-1/\beta)-1}\beta(\sigma\varphi+\rho-\varphi)}{[(\sigma\varphi+\rho)\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi]^2} < 0, \quad \text{(B2)}$$

$$\frac{\partial s}{\partial A} = 0, \quad \text{(B3)}$$

$$\frac{\partial s}{\partial \beta} = \frac{s^2}{\beta^2} + \frac{s\alpha(\beta-s)}{\beta^2\sigma(1-\beta)} + \frac{s^2(\sigma\varphi+\rho)(1-\beta)}{\sigma(1-\alpha)\beta} \frac{(1-\sigma\varphi)\tau^{(1/\beta-1)(2-1/\beta)}(3-2/\beta)/\beta\log\tau + (2\sigma\varphi+\rho)\sigma\tau^{(1/\beta-1)(2-1/\beta)} - 2\beta\varphi}{[(\sigma\varphi+\rho)\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi]^2} > 0, \quad \text{(B4)}$$

$$\frac{\partial s}{\partial \alpha} = -\frac{s^2}{\sigma(1-\alpha)}\left(\frac{\sigma}{s} + (\frac{1}{\beta}-1)(\sigma\varphi+\rho)\frac{\alpha\tau^{(1/\beta-1)(2-1/\beta)-1} - \beta)}{(\sigma\varphi+\rho)\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi}\right) < 0, \quad \text{(B5)}$$

$$\frac{\partial s}{\partial \sigma} = \frac{s}{\sigma} + \frac{(1-\beta)s^2}{\beta\sigma(1-\alpha)}\left(\frac{\varphi^2}{(\sigma\varphi+\rho-\varphi)^2} + \frac{\alpha\varphi\tau^{(1/\beta-1)(2-1/\beta)}(\sigma\tau^{(1/\beta-1)(2-1/\beta)}-\beta)}{[(\sigma\varphi+\rho)\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi]^2}\right.$$
$$\left. - \frac{\alpha[(2\sigma\varphi+\rho)\tau^{(1/\beta-1)(2-1/\beta)-1}-\beta\varphi][(\sigma\varphi+\rho)\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi]}{[(\sigma\varphi+\rho)\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi]^2}\right) > 0, \quad \text{(B6)}$$

$$\frac{\partial s}{\partial \rho} = \frac{(1/\beta)s^2}{\sigma(1-\alpha)}\left(\frac{\varphi}{(\sigma\varphi+\rho-\varphi)^2} + \frac{\alpha\beta\varphi(\sigma\tau^{(1/\beta-1)(2-1/\beta)}-\beta)}{[(\sigma\varphi+\rho)\tau^{(1/\beta-1)(2-1/\beta)}-\beta\varphi]^2}\right) > 0. \quad \text{(B7)}$$

Number	Author(s)	Title	Date
03-A001	Chung-Ming Kuan Wei-Ming Lee	A New Test of the martingale Difference Hypothesis	11/03
03-A002	Chung-Ming Kuan, Yu-Lieh Huang Ruey S. Tsay	A Component-Driven Model for Regime Switching and Its Empirical	11/03
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