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Consumable Travel-for Local Public Good
and Equilibrium Stratification

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Abstract: This paper reexamine Tiebout's hypothesis of endogenous sorting in a competitive spatial equilibrium setup with both income and preference heterogeneity. Agents decide endogenously the number of trips to consume a travel-for congestable local public good. We show that the equilibrium configuration may be completely segregated, incompletely segregated or completely integrated, depending crucially on the scale of local public good services, relative market rents and the underlying income/preference/local tax parameters. Segregated equilibrium may feature endogenous sorting purely by income or by both income and preferences. Multiple equilibria may arise when the equilibrium configuration is incompletely segregated.

JEL Classification: D50, H41, R53.

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1 Introduction

In the postwar period, residential polarization as a result of racial and/or economic segregation has received much attention in both economics and sociology profession. Based on a widely used dissimilarity index, most of the 30 largest Metropolitan Statistical Areas (MSAs) in the U.S. have been highly stratified, especially since 1980 (see Table 1 for the year of 2000).[1] Jargowsky (1996) uses the Census tract data over the period from 1970 to 1990 to show the nation-wide, sharp increase in economic segregation in the 1980s accompanied by a small decline in racial segregation.

Accordingly, the main focus of this paper is on how the underlying non-racial economic forces may lead to various equilibrium configurations. In particular, we allow for both income and preference heterogeneities and study the possibility that endogenous sorting may be based on preferences for the local public goods. Moreover, we take into account of spatial factors explicitly by permitting individuals to determine the number of trips to the site of public facility. With these considerations, we establish conditions under which a segregated or an integrated equilibrium may arise in competitive spatial equilibrium.

Stratification has been argued to create significant disparities in socioeconomic status (SES), particularly in earned income, education, housing and social norms (cf. Weiss 1989; Jencks and Meyer 1990). While Wilson (1987) hypothesizes that the on-going increase in economic segregation plays a crucial role in the formation of urban ghettos, Ihlanfeldt (1994) suggests that the rapid suburbanization of jobs and workers and the formation of suburban business district (SBD) have expedited both economic and racial segregation. As a result, the ratio of central city income per capita to suburban income per capita in the 85 largest MSAs has dropped from 105% in 1960 to 84% in 1989 (cf. Barnes and Ledebur 1998). The associated

[1] The dissimilarity index, first constructed by Duncan and Duncan (1955), measures the degree to which two particular population characteristics of interest (such as white versus black or rich versus poor) are distributed differently within a population (such as residents in an MSA). For example, using the 2000 Census, DC-Baltimore had a shocking high dissimilarity index of 0.78, Detroit, 0.70, 8 MSAs in the 0.6 range (including New York and Chicago), 6 in the 0.5 range (including Boston and Los Angeles), 11 in the 0.4 range (including Miami and San Francisco), and only 3 below 0.4 (Dallas, Seattle and Portland).

neighborhood effect (cf. Bond and Coulson 1989) and urban labor-market networking effect (cf. O'Regan and Quigley 1993), in addition to changes in the commuting technologies (cf. Sassen 1991), further lead to spatial mismatch with high unemployment rates in central cities accompanied by high job vacancies in suburbs (see a discussion in Coulson, Laing and Wang 2001 and papers cited therein). Urban unemployment and immobility across generations have generated the unfortunate pathologies of central cities. Among many others, serious crimes per capita in central cities have far exceeded the comparable figures in suburbs (cf. Glaeser, Sacerdote, and Scheinkman 1996).[2]

What are the possible underlying forces of economic segregation? In addition to racial concerns, one may argue there are at least three important economic forces: human/non-human wealth, accessibility to capital markets, and individual preferences. For instance, Benabou (1996a) studies the effect of wealth and human capital investment on spatial segregation and economic growth, whereas Benabou (1996b) considers the roles of capital market imperfections and local spillover played in economic stratification. Much of the recent theoretical literature follows this vein of research. In a recent empirical investigation, however, Bayer, McMillan and Rueben (2002) document that heterogeneous preferences for housing and neighborhood characteristics are essential for endogenous stratification in urban housing markets where social interactions are present. It remains unexplored whether the local public good (LPG) or fiscal competition argument can be regarded as one of the major sources of the observed outcome of economic segregation, and whether the LPG preference and the conventional income/wealth factors reinforce each other, expediting the process of stratification.

The primary purpose of our paper is to address these open issues. Specially, we examine endogenous sorting by heterogeneous incomes as well as heterogeneous preferences for a consumable travel-for local public good in competitive spatial equilibrium.[3] We highlight in particular the nature of the travel-for local public good: agents decide endogenously the

[2] For example, Atlantic City reported 0.34 serious crimes per capita, while Ridgewood village only reported a low figure of 0.008.

[3] Fujita (1986) defines a traveled-for public good as a special type of Starrett's (1991) congestable local public good that is available to consumers only in specific locations to which they must travel to enjoy the service. The same concept can also be found in Thisse and Wildasin (1992).

number of trips to consume its service (patronization), bearing the commuting costs, where the quality of service is reduced by the degree of congestion measured by the total number of users. That is, we develop a multi-class competitive spatial equilibrium model of "sorting by foot" which determines endogenously the pattern of economic stratification. The idea that provision of local public good may induce "voting by foot" can be traced back to Tiebout (1956) and some important follow-up studies by Arnott and Stiglitz (1979), Ellickson (1979), Rose-Ackerman (1979) and Bewley (1981).[4] This insight, however, has not been applied to the studdings of economic segregation. Our paper can thus be regarded as a first attempt at filling this gap, complementing the conventional analysis of economic segregation mentioned above.

For any set of income, preference and local tax parameters as well as a given site of local public facility, we establish the range of relative market land rents within which agents may choose to be segregated or mixed in competitive spatial equilibrium. Extending the techniques developed in the multi-class locational equilibrium framework by Hartwick et al. (1976), we use the Negishi (1960) approach to show that a multi-class competitive spatial equilibrium exists. The equilibrium configuration may be completely segregated, incompletely segregated or completely integrated, depending crucially on the equilibrium scale of local public good services, the relative market rents and the three underlying parameters (preference heterogeneity, income heterogeneity and local tax progressiveness). Moreover, we find that segregated equilibrium may feature endogenous sorting purely by income or by both income and preference towards LPG consumption. Furthermore, multiple equilibria may arise when the equilibrium configuration is incompletely segregated, as either the rich or the poor may reside closer to the public facility site.

The remainder of the paper is organized as follows. In Section 2, we delineate a two-community economy with two types of agents who differ in both income and preferences

[4] See a comprehansive survey by Wildasin (1987) of the early research in this area. Recently, there has been a growing literature to formalize this Tiebout hypothesis in various dimensions. To name but a few, this includes Konish at al. (1997), using noncooperative equilibrium concepts, as well as Oates and Schwab (1991), Scotchmer (1994), Konish (1996), Nechyba (1997) and Conley and Wooders (2001), under the conventional competitive equilibrium or core setup.

for the LPG, where the LPG is financed by a local tax. Section 3 solves the individual optimization problem with respect to composite good consumption, the number of trips to patronize the local public facility, and the choice of residential location. We then establish in Section 4 the possibility of completely and incompletely segregated equilibrium under which there are four different equilibrium configurations depending on the relative distance to the site of the LPG facing the rich relative to the poor and their relative masses. The case of integrated equilibrium is examined in Section 5. Finally, Section 6 provides a summary of the main findings and potential avenues for future research.

2 The Model

We consider a simple model with two types of agents ($i = H, L$) of a total mass $N > 0$ and two communities ($j = 1, 2$). Type-H agents (of mass H) have higher income and higher preference towards public good consumption than type-L agents (of mass L), where $H + L = N$. Denoting a type-i agent's income as Y^i and letting her preference towards public good consumption be captured by γ^i, we have: $Y^H > Y^L > 0$ and $\gamma^H > \gamma^L > 0$. Throughout the paper, we will refer the type-H agents as "the rich" and the type-L agents as "the poor." For convenience, it suffices to normalize the total population in each community to be identical ($N/2$). Denote the (endogenously determined) population of type-i agents in community j as i_j ($i = H, L$; $j = 1, 2$). In the absence of vacant land, population balance conditions require ($j = 1, 2$):

$$H_j + L_j = N/2. \tag{1}$$

The population identities can be conveniently summarized by:

Population	1	2	Total
Type-H	H_1	H_2	H
Type-L	L_1	L_2	L
Total	$N/2$	$N/2$	N

A consumable local public good of size K is provided at community 1, where its service G depends on the degree of congestion from patronization by users measured M. Such a consumable local public good may include a community park, library, museum, theater hall, public swimming pool, exercise field or other recreational facility. Specifically, the service of the local public good inclusive of the congestion factor is given by:

$$G = \frac{K}{M^{\alpha}}, \tag{2}$$

where $\alpha \in [0, 1]$ reflects the degree of congestion – the local public facility is a nonexclusive pure public good if $\alpha = 0$ and a completely exclusive private good if $\alpha = 1$. Let $x_j \equiv \frac{H_j}{N_j}$ denote the fraction of the rich in community j and n_j^i measure the number of trips for a type-i agent residing at community j to patronize the local public facility. Thus, we have:

$$\sum_{j=1}^{2} \left[n_j^H x_j + n_j^L (1 - x_j)\right] N_j = M. \tag{3}$$

Denote the composite good consumption as C. Agents of each type are assumed to have an identical preference of the following form: $U = C + \gamma^i \cdot n^{\beta} \cdot \log G$, where $\beta \in (0, 1)$. This utility functional form extends the separable function proposed by Bergstrom and Cornes (1983) and Berliant, Peng and Wang (2002) by considering endogenous trips to patronize the local public facility. More specifically, an increase in the number of trips is assumed to lead to more enjoyment but at a diminishing rate. An increase in γ^i represents a shift in preferences away from the composite good towards the local public good. In the case where $\beta = 0$ and $\gamma^i = \gamma$, our utility specification reduces to that in Berliant, Peng and Wang (2002).

Denote R_j as the market rent (on land and housing property) in community j and $t(x_j)$ as the associated property tax rate that depends positively on the fraction of the rich (i.e., $t' > 0$). Let T measure the unit commuting cost to the site of the local public facility. Since the local public facility is at community 1, only do community 2 residents bear the commuting cost. We thus define an indicator function I_j taking values from {0,1} with $I_1 = 0$ and $I_2 = 1$ and use $T \cdot I_j$ to measure the commuting cost per trip facing each agent residing in community j. The local government charges a uniform fee (per user, per trip) at $\phi > 0$ for patronizing the local public facility and sets the local income tax rate at τ_j^i. Notably, $\tau_j^H > \tau_j^L$ reflects the fact that income taxes are progressive, whereas $\tau_1^i > \tau_2^i$ can be thought of as a tax surcharge imposed at the site of the local public facility.

Therefore, a representative agent of type-i will choose composite good consumption, the number of trips to patronize the local public facility and the residential location to maximize the utility subject to the budget constraint. Technically, this optimization problem can be divided into two steps. In step 1, for a given residential location j, a representative agent of type-i takes G, R_j, x_j and tax/user fee parameters as given to solve:

$$V_j^i = \max_{C,n} C + \gamma^i n^\beta \log G, \tag{4}$$

$$s.t. \quad C + R_j(1 + t(x_j)) + n(T \cdot I_j + \phi) = (1 - \tau_j^i)Y^i. \tag{5}$$

In step 2, a representative agent of type-i compares the values (indirect utilities) obtained in step 1 to choose residential location. That is, she will reside in community $h \in \{1, 2\}$ if and only if $V_h^i \geq V_j^i$ for $j \neq h$ and $j \in \{1, 2\}$.

To close the model, we assume that the local public facility is financed entirely by revenues from local property taxes, user fees and local income taxes:

$$K = \sum_{j=1}^{2} t(x_j) R_j + M\phi + \sum_{i=H,L} \sum_{j=1}^{2} \tau_j^i Y^i. \tag{6}$$

3 Optimization and Equilibrium

We are now prepared to solve the optimization problem in step 1. Define the Lagrangian function $L(C, n, \lambda^i)$ as:

$$L \equiv C + \gamma^i n^\beta \log G + \lambda^i \left[(1 - \tau_j^i)Y^i - C - R_j(1 + t(x_j) - n(T \cdot I_j + \phi) \right].$$

Straightforward differentiation yields:

$$\frac{\partial L}{\partial C} = 1 - \lambda^i = 0, \tag{7}$$

$$\frac{\partial L}{\partial n} = \beta \gamma^i n^{\beta - 1} \log G - \lambda^i (T \cdot I_j + \phi) = 0. \tag{8}$$

Using (2), (3) and (6), we can express the logged level of the local public good service $\log G$ as a function of $(n^i_j; R_j, x_j; Y^i)$, where

$$\frac{d\log G}{dn^i_j} = \frac{d\log K}{dn^i_j} - \alpha\frac{d\log M}{dn^i_j} = x_j N_j(\frac{\phi}{K} - \frac{\alpha}{M}), \tag{9}$$

$$\frac{d\log G}{dx_j} = \frac{1}{K}t'(x_j)R_j + (\frac{\phi}{K} - \frac{\alpha}{M})\frac{dM}{dx_j} = \frac{1}{K}t'(x_j)R_j, \tag{10}$$

$\frac{d\log G}{dR_j} = \frac{d\log K}{dR_j} = \frac{t(x_j)}{K}$, and $\frac{d\log G}{dY^i} = \frac{d\log K}{dY^i} = \frac{\tau^i_j}{K}$.

Utilizing the above results, we can solve from (7) and (8) the number of patronizing trips:

$$n^i_j = \left[\frac{\beta\gamma^i \log G}{TI_j + \phi}\right]^{\frac{1}{1-\beta}} \equiv q(n^i_j; R_j, x_j; Y^i, \gamma^i), \tag{11}$$

which constitutes a fixed point mapping in n^i_j. Since $\gamma^H > \gamma^L$, $I_1 = 0$ and $I_2 = 1$, we can rank: $n^H_1 > \max\{n^H_2, n^L_1\} > \min\{n^H_2, n^L_1\} > n^L_2$. Moreover, straightforward differentiation yields:

$$\frac{\partial q}{\partial n^i_j} = -\frac{1}{1-\beta}\Gamma^i_j, \tag{12}$$

where $\Gamma^i_j \equiv \frac{n^i_j}{\log G} x_j \cdot N_j \cdot (\frac{\phi}{K} - \frac{\alpha}{M})$ captures the positive user fee revenue effect and the negative congestion effect from an additional trip of a user on the demand for patronizing the local public facility. Without loss of generality, we impose:

Assumption 1: $\Gamma^i_j = 0$.

This assumption can be justified by imagining that the underlying local government is revenue-maximizing with respect to the patronization of its local public facility.

We can then establish:[5]

Lemma 1: *Under Assumption 1, the optimal number of trips to patronizing the local public facility (n^i_j) is increasing in the market rents (R_j), the fraction of rich (x_j), the level of income (Y^i) and the preference for the local public good (γ^i).*

Proof: All proofs are relegated to the Appendix. ■

[5] The assumption is innocuous as our results are robust as long as the direct effect dominates these two opposing indirect effects, i.e., $1 - \frac{\partial q}{\partial n^i_j} > 0$.

Next, substituting (11) into (5), we have:

$$C_j^i = (1-\tau_j^i)Y^i - R_j(1+t(x_j)) - (\beta \cdot \gamma^i \cdot \log G)^{-\frac{1}{\beta-1}}(T \cdot I_j + \phi)^{-\frac{\beta}{1-\beta}}, \tag{13}$$

which can be substituted into (4) to yield the indirect utility function:

$$V_j^i(R_j, x_j;\ I_j) = (1-\tau_j^i)Y^i - R_j(1+t(x_j)) + (1-\beta)\gamma^i(n_j^i)^\beta \log G. \tag{14}$$

We can now define the bid-rent Ψ as the slope of the indifference curves in (R_j, x_j) space:

$$\Psi(R_j,\ x_j,\ \gamma^i, Y^i) \equiv \frac{dR_j}{dx_j}\mid_{V_j^i=\overline{V}} = -\frac{dV_j^i/dx_j}{dV_j^i/dR_j}.$$

Denote $S_j^i \equiv [\gamma^i(n_j^i)^\beta \frac{1}{K} - 1]$, which captures all the effects of γ^i and Y^i on the bid rent. To ensure positive bid rents, we need: $S_j^i < 1/t(x_j)$ for all i and j. Recall that n_j^i is strictly increasing in γ^i and $n_1^i > n_2^i$. We thus have $S_1^H = \max_{i,j}\{S_j^i\}$. Moreover, $t(x_j)$ is strictly increasing in x_j, implying $1/t(1) = \min_{x_j}\{1/t(x_j)\}$. Utilizing (11), a sufficient condition can therefore be established,

Assumption 2: $(\gamma^H)^{\frac{1}{1-\beta}}\left[\frac{\beta \log G}{\phi}\right]^{\frac{\beta}{1-\beta}} < \left[1 + \frac{1}{t(1)}\right]K.$

We can then obtain:

Lemma 2: *Under Assumptions 1 and 2, the bid rents are positive, satisfying:*

$$\Psi(R_j,\ x_j,\ \gamma^i,\ Y^i) = \frac{t'(x_j)R_j}{(S_j^i)^{-1} - t(x_j)} > 0. \tag{15}$$

Utilizing Lemmas 1 and 2, we then establish:

Proposition 1: (Bid Rent Functions) *Under Assumptions 1 and 2, the bid rents possess the following properties:*

$$\frac{d\Psi}{d\gamma^i} = \frac{1}{K(1-\beta)}(n_j^i)^\beta \frac{t'(x_j)R_j}{\{1-t(x_j)S_j^i\}^2} > 0, \tag{16}$$

$$\frac{d\Psi}{dY^i} = \frac{\gamma^i(n_j^i)^\beta \tau_j^i t'(x_j)R_j}{K^2 \log G[1-t(x_j)S_j^i]^2}\left(\frac{\beta}{1-\beta} - \log G\right) \gtreqless 0 \ \text{ if } \ \log G \gtreqless \frac{\beta}{1-\beta}. \tag{17}$$

Thus, the bid rent is always increasing in individuals' preferences towards the local public good. It is increasing in individuals' incomes only when the scale of local public good services is small. Intuitively, income generates two opposing effects. On the one hand, there is a *patronization effect*: higher income encourages the use of the local public facility (higher n_j^i), thereby raising the bid rent. On the other, it creates a *diminishing-return effect*: higher income leads to a larger tax revenue and a greater provision of the local public good (K), which, by diminishing marginal utility, results in a lower bid rent. When the scale of local public good services is small, the diminishing return effect is dominated by the patronization effect. As a consequence, the bid rent depends positively on income.

A type i person will choose location of region $h \in \{1, 2\}$ if

$$V_h^i(I_h, R_h, t(x_h)) \geqslant V_j^i(I_j, R_j, t(x_j)) \qquad \forall \quad j \neq h, \tag{18}$$

where the relevant indirect utilities are given by (14). This may be referred to as the *optimal locational choice* condition. Following Negishi (1960), we must find equilibrium price support (market rents) for any equilibrium configuration to be established. We can now define the concept of equilibrium with endogenous sorting,

Definition 1: *A* ***multi-class competitive spatial equilibrium*** *(MCSE) is a tuple* $\{n_j^i, C_j^i, I_j, \Psi_j, G, M, K, H_j, L_j\}$ *and the market land rents* $\{R_j\}$ *such that the following conditions are satisfied:*

(i) *given the residential location, each individual maximizes her utility subject to her budget constraint, i.e., (5), (11) and (13) are met;*

(ii) *the market rents are in appropriate range such that the optimal locational choice conditions (18) are met for all individuals and that each individual only resides in one location* ($I_1 + I_2 = 1$)*;*

(iii) *the bid rents satisfy (15);*

(iv) *the service of the local public good is captured by (2), the number of users patronizing the local public good is captured by (3), and the government budget is balanced as given by (6);*

(v) *the population balance conditions (1) are met for both communities.*

In the next two sections, we will characterize the MCSE. There are two types of MCSE: segregated and integrated. In a segregated equilibrium, at least one of the two communities must have residents of a homogeneous type. Depending on the relative size of the population of the rich to the poor, however, a segregated equilibrium may have both types residing in one of the two communities. When each community only contains one type of residents, the equilibrium is called completely segregated. In an integrated equilibrium, both communities must be populated by residents of both types. Formally, these equilibrium configurations can be defined as:

Definition 2: *A multi-class competitive spatial equilibrium* $\{n_j^i, C_j^i, I_j, \Psi_j, G, M, K, H_j, L_j; R_j\}$ *is called,*

(i) ***completely segregated*** *if* $H_j \cdot L_j = 0$ *for both* $j = 1$ *and* $j = 2$*;*

(ii) ***incompletely segregated*** *if* $H_j \cdot L_j = 0$ *for either* $j = 1$ *or* $j = 2$ *(but not both);*

(iii) ***integrated*** *if* $H_j + L_j = N/2$ *and* $H_j, L_j > 0$.

We will establish conditions on the underlying parameters and ranges of market rents under which each type of equilibrium configuration is supported. Obviously, from the population balance condition, a completely segregated equilibrium is possible only when $H = L = N/2$. To simplify the analysis, we assume throughout the remainder of the paper that the property tax rate is proportional to the fraction of rich, that is,

Assumption 3: $t(x_j) = t_0 \cdot x_j$, with $t_0 > 0$.

4 Segregated Equilibrium

We begin by examining the case of segregated equilibrium. From the bid rent function (15), this type of equilibrium requires that the bid rent be increasing in Y^i. That is, the willingness to pay for a higher market land (property) rent must be matched by the ability to pay. By utilizing (17), this condition is equivalent to:

Condition S: $\log G < \frac{\beta}{1-\beta}$.

There are two cases to consider: (i) equal population of the rich and the poor and (ii) unequal population of the rich and the poor. While the former case may results in complete segregation, the stratification between the rich and the poor in the latter case must be incomplete. Depending on the parameters, it is possible that the rich reside in community 1 where the local public facility is provided or in community 2 where they need to travel in order to consume the local public good service. If we regard community 1 as city center with publicly provided museums and activity facilities and community 2 as suburbs, the former case captures many Asian and European cities (e.g., London, Paris, Rome, Taipei and Tokyo), whereas the latter is consistent with the configurations of typical American cities (such as Atlanta, Chicago, Detroit, Los Angeles and Washington DC).

4.1 Complete Segregation

Consider the case where the rich and the poor are of equal size: $H = L = N/2$. We will show when the market rents are within an appropriate range, the equilibrium feature a complete segregation in the sense that the rich all reside in one community and the poor in another. Since the rich may all reside in community 1 or in community 2 depending on the underlying set of parameters, each case must be analyzed accordingly, to which we now turn.

4.1.1 The Rich are Closer to the Site of Public Facility

The first case is that all the rich reside in community 1 and the poor travel for public good services. Specifically, we have: $x_1 = 1$ and $x_2 = 0$. To ensure this is an optimal locational choice to every individual, it requires: $V_1^H > V_2^H$ and $V_1^L < V_2^L$.

K

Population	1	2	Total
Type-H	H		H
Type-L		L	L
Total	$N/2$	$N/2$	N

4.1.2 The Poor are Closer to the Site of Public Facility

The second case is that all the poor reside in community 1 and the rich travel for public good services. That is, we have: $x_1 = 0$ and $x_2 = 1$. Optimal locational choice requires: $V_1^H < V_2^H$ and $V_1^L > V_2^L$.

K

Population	1	2	Total
Type-H		H	H
Type-L	L		L
Total	$N/2$	$N/2$	N

4.1.3 Completely Segregated Equilibrium

Define,

$$R_0(G) \equiv (1-\beta)(\log G)^{\frac{1}{1-\beta}}(\frac{\beta}{\phi})^{\frac{\beta}{1-\beta}}[1-(\frac{\phi}{T+\phi})^{\frac{\beta}{1-\beta}}]. \tag{19}$$

To accept different types of completely segregated equilibrium, we consider:

Assumption 4a: $\left[(\gamma^H)^{\frac{1}{1-\beta}} - (\gamma^L)^{\frac{1}{(1-\beta)}}\right] R_0(G) > \Delta\tau^H Y^H - \Delta\tau^L Y^L$.

Assumption 4b: $\left[(\gamma^H)^{\frac{1}{1-\beta}} - (\gamma^L)^{\frac{1}{1-\beta}}\right] R_0(G) < \Delta\tau^H Y^H - \Delta\tau^L Y^L$.

We can now establish:

Proposition 2: (Completely Segregated MCSE) *Under Assumptions 1-3 and Condition S, a multi-class competitive spatial equilibrium with complete segregation exists.*

(a) *If we impose additionally Assumption 4a, then in this equilibrium the rich are closer to the site of public facility given,*

$$\frac{(\gamma^L)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^L Y^L + R_2}{1+t_0} < R_1 < \frac{(\gamma^H)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^H Y^H + R_2}{1+t_0}. \tag{20}$$

(b) *If we impose instead Assumption 4b, then in this equilibrium the poor are closer to the site of public facility given,*

$$(\gamma^H)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^H Y^H + R_2(1+t_0) < R_1 < (\gamma^L)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^L Y^L + R_2(1+t_0). \tag{21}$$

Notably, while Assumption 4a ensures the inequalities in (20) to be valid, Assumption 4b guarantees the validity of (21).Assumptions 4a and 4b are mutually exclusive, thereby ruling out the possibility of multiple equilibrium configurations.

Focusing first on part (a) of Proposition 2, it requires that the preference differential must be sufficiently large to overcome the differences in income and income tax progressiveness. In this case, endogenous sorting is by *both income and preferences towards local public good consumption.* Interestingly, the inequality in Assumption 4a is more likely to hold if the level of provision of the local public good is high (as R_0 is larger), provided that Condition S is still satisfied. Notice that in equilibrium one can only pin down the relative market rent between the two communities. Should we fix R_2 at the agricultural land rent or as an exogenous multiple of the agricultural land rent, the range of R_1 is fully determined by (20).[6]

Turning next to part (b), the inequality in Assumption 4b is met if the preference differential is small and the level of local public good provision is low. When this inequality holds, endogenous sorting to a completely segregated equilibrium is *entirely driven by income heterogeneity*, as those with stronger preferences towards the local public good (the type-H) are now residing *away from* the site of public facility.

[6] In a close city model with a finite number of communities, the land rent at the boundary need not equal the agricultural land rent.

4.2 Incomplete Segregation

When the rich and the poor have different population masses, one of the two communities must have mixed residents of both types. There are two cases to be studied: (i) more rich than poor ($H > L$, or, $\frac{H}{N} > \frac{1}{2}$) and (ii) more poor than rich ($\frac{H}{N} < \frac{1}{2}$). In the first case, community 2 is mixed with both types, whereas in the second, community 1 is mixed.

4.2.1 More Rich Than Poor

Since the population of the rich exceeds that of the poor, some of the rich must reside with the poor by construction. Depending on the underlying parameters, the poor may reside in community 1 or community 2.

A. The Rich are Closer to the Site of Public Facility In this case, we have $x_1 = 1$ and $x_2 = \frac{H-N/2}{N/2} = \frac{2H}{N} - 1$. With some of the rich residing in community 1 and others in community 2, locational equilibrium requires: $V_1^H = V_2^H$ and $V_1^L < V_2^L$.

K

Population	1	2	Total
Type-H	H_1	H_2	H
Type-L		L	L
Total	$N/2$	$N/2$	N

B. The Poor are Closer to the Site of Public Facility In this case, we have $x_1 = \frac{2H}{N} - 1$ and $x_2 = 1$. Again, locational equilibrium requires: $V_1^H = V_2^H$ and $V_1^L > V_2^L$.

K

Population	1	2	Total
Type-H	H_1	H_2	H
Type-L	L		L
Total	$N/2$	$N/2$	N

C. Incompletely Segregated Equilibrium with More Rich than Poor Define,

$$\Lambda \equiv \frac{\Delta\tau^H Y^H - (\frac{\gamma^H}{\gamma^L})^{\frac{1}{1-\beta}} \Delta\tau^L Y^L}{(\frac{\gamma^H}{\gamma^L})^{\frac{1}{1-\beta}} - 1}.$$

As shown in the appendix, we can establish:

Proposition 3: (Incompletely Segregated MCSE) *Under Assumptions 1-3 and Condition S, a multi-class competitive spatial equilibrium with incomplete segregation exists.*

(a) *In this equilibrium, the rich are closer to the site of public facility iff*

$$R_1 > \frac{1}{1+t_0} \left\{ \Lambda + \left[1 + t_0 \left(\frac{2H}{N} - 1 \right) \right] R_2 \right\}. \tag{22}$$

(b) *In this equilibrium, the poor are closer to the site of public facility iff*

$$R_1 < \frac{1}{1 + t_0(\frac{2H}{N} - 1)} \left[\Lambda + (1+t_0) R_2 \right]. \tag{23}$$

(c) *The equilibrium is indeterminate in which either the rich or the poor may be closer to the site of public facility iff*

$$\frac{1}{1+t_0} \left\{ \Lambda + \left[1 + t_0 \left(2H/N - 1 \right) \right] R_2 \right\} < R_1 < \frac{1}{1 + t_0(2H/N - 1)} \left[\Lambda + (1+t_0) R_2 \right]. \tag{24}$$

The incompletely segregated equilibrium configuration with the rich closer to the public facility site is supported by relatively high market rent in the community with the public facility (community 1), whereas that with the poor closer to the public facility site is supported by relatively low market rent in community 1. When market rents fall in the range (24), multiple equilibria arise as the poor may reside either in community 1 or in community 2, depending on the self-fulfilling prophecies.

4.2.2 More Poor Than Rich

Now, the population of the poor exceeds that of the rich, thereby forcing some of the poor to reside with the rich. Again, depending on the underlying parameters and relative market rents, the rich may reside in community 1 or community 2.

A. The Rich are Closer to the Site of Public Facility In this case, we have $x_1 = \frac{2H}{N}$ and $x_2 = 0$. As some of the poor now reside in community 1 and others in community 2, locational equilibrium requires: $V_1^H > V_2^H$ and $V_1^L = V_2^L$.

K

Population	1	2	Total
Type-H	H		H
Type-L	L_1	L_2	L
Total	$N/2$	$N/2$	N

B. The Poor are Closer to the Site of Public Facility In this case, we have $x_1 = 0$ and $x_2 = \frac{2H}{N}$, which require: $V_1^H < V_2^H$ and $V_1^L = V_2^L$.

K

Population	1	2	Total
Type-H		H	H
Type-L	L_1	L_2	L
Total	$N/2$	$N/2$	N

C. Incompletely Segregated Equilibrium with More Poor than Rich Parallel to the case with more rich than poor, we can obtain the following:

Proposition 4: (Incompletely Segregated MCSE) *Under Assumptions 1-3 and Condition S, a multi-class competitive spatial equilibrium with incomplete segregation exists.*

(a) *In this equilibrium, the rich are closer to the site of public facility iff*

$$R_1 > \frac{1}{1 + t_0 \frac{2H}{N}} (\Lambda + R_2). \tag{25}$$

(b) *In this equilibrium, the poor are closer to the site of public facility iff*

$$R_1 < \Lambda + R_2 (1 + t_0 \frac{2H}{N}). \tag{26}$$

(c) *The equilibrium is indeterminate in which either the rich or the poor may be closer to the site of public facility iff*

$$\frac{1}{1+t_0(2H/N)}(\Lambda + R_2) < R_1 < \Lambda + R_2[1 + t_0(2H/N)]. \tag{27}$$

Since the intuition behind Proposition 4 resembles that underlying Proposition 3, it is omitted for the sake of simplicity. Again, when the market rents fall in the range of (27), the economy features multiple equilibria in which the rich may reside in community 1 or 2.

5 Integrated Equilibrium

We now study the case of integrated equilibrium under which both communities must be populated by residents of both types.

K

Population	1	2	Total
Type-H	H_1	H_2	H
Type-L	L_1	L_2	L
Total	$N/2$	$N/2$	N

From the bid rent functions (15), this type of equilibrium requires that the bid rents be decreasing in income, or, using (17), the following condition must be met: $\log G > \frac{\beta}{1-\beta}$. That is, the service of the local public good is sufficiently large so that the diminishing-return effect dominates the patronization effect. In an integrated equilibrium, it requires: $V_1^i = V_2^i$ for $i = H, L$.

Define,

$$\Omega \equiv \frac{\Delta\tau^H Y^H - \Delta\tau^L Y^L}{(\gamma^H)^{\frac{1}{1-\beta}} - (\gamma^L)^{\frac{1}{1-\beta}}},$$

which is positive because $\gamma^H > \gamma^L$, $Y^H > Y^L$ and $\Delta\tau^H > \Delta\tau^L$. In order to satisfy $\log G > \frac{\beta}{1-\beta}$, it requires (see the Appendix):

Condition I: $\frac{\phi^\beta}{\beta^\beta(1-\beta)^{1-\beta}}\left[\frac{\Omega}{1-(\frac{\phi}{T+\phi})^{\frac{\beta}{1-\beta}}}\right]^{1-\beta} > \frac{\beta}{1-\beta}$.

We will show below that for each pair of market rents within a specific range, there is a corresponding value of x_1 (and $x_2 = \frac{2H}{N} - x_1$) which falls within the unit interval, $(0, 1)$.

Proposition 5: (Integrated MCSE) *Under Assumptions 1-3 and Condition I, a multi-class competitive spatial equilibrium with an integrated configuration exists iff*

$$\frac{1}{1+t_0}\left\{\left[1+t_0\left(\frac{2H}{N}-1\right)\right]R_2+\Lambda\right\} < R_1 < R_2\left(1+t_0\frac{2H}{N}\right)+\Lambda, \tag{28}$$

under which the level of local public good services is determined by,

$$\log G = \frac{\phi^\beta}{\beta^\beta(1-\beta)^{1-\beta}}\left[\frac{\Omega}{1-(\frac{\phi}{T+\phi})^{\frac{\beta}{1-\beta}}}\right]^{1-\beta}. \tag{29}$$

Condition I indicates that in order for an integrated equilibrium to exist, we need the preference heterogeneity not too large (other things being equal). Notice that when $R_1 = R_2 + \frac{\Lambda}{1+t_0 H/N}$, we have: $x_1 = x_2 = \frac{H}{N}$. This features a *symmetric* integrated equilibrium where both communities have an identical fraction of rich. In general, the proportion of rich need not be equalized across the two communities, depending crucially on the relative market rent.

6 Conclusions

We have characterized competitive spatial equilibrium with endogenous sorting where the equilibrium configuration may be completely segregated, incompletely segregated or completely integrated, depending on the relative market rents and the underlying parameters (especially, preference heterogeneity, income heterogeneity and local tax progressiveness). We have established equilibrium featuring endogenous sorting by both income and preference towards local public good consumption or by income only. Multiple equilibria may arise when the equilibrium configuration is incompletely segregated.

The main results established in our paper can be best summarized as follows (see also Table 2 for a brief outline of the findings concerning segregated equilibrium outcomes).

(i) Under Condition S ($\log G < \frac{\beta}{1-\beta}$):

1. $H = L$

   (a) under $R_0(G) > \frac{\Delta\tau^H Y^H - \Delta\tau^L Y^L}{(\gamma^H)^{\frac{1}{1-\beta}} - (\gamma^L)^{\frac{1}{1-\beta}}}$ and $[(\gamma^L)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^L Y^L + R_2]/(1+t_0) < R_1 < [(\gamma^H)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^H Y^H + R_2]/(1+t_0)$: complete segregation with the rich closer to the public facility site (endogenous sorting by both income and preferences towards local public good consumption),

   (b) under $R_0(G) < \frac{\Delta\tau^H Y^H - \Delta\tau^L Y^L}{(\gamma^H)^{\frac{1}{1-\beta}} - (\gamma^L)^{\frac{1}{1-\beta}}}$, and $(\gamma^H)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^H Y^H + R_2(1+t_0) < R_1 < (\gamma^L)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^L Y^L + R_2(1+t_0)$: complete segregation with the poor closer to the public facility site (endogenous sorting purely by income).

2. $H > L$

   (a) under $R_1 > \frac{\Lambda + (1+t_0)R_2}{1+t_0(2H/N-1)}$: incomplete segregation with the rich closer to the public facility site (endogenous sorting by both income and preferences towards local public good consumption),

   (b) under $\frac{\Lambda + [1+t_0(2H/N-1)]R_2}{1+t_0} < R_1 < \frac{\Lambda + (1+t_0)R_2}{1+t_0(2H/N-1)}$: incomplete segregation with either the rich or the poor closer to the public facility site (multiple equilibria),

   (c) under $R_1 < \frac{\Lambda + [1+t_0(2H/N-1)]R_2}{1+t_0}$: incomplete segregation with the poor closer to the public facility site (endogenous sorting purely by income).

3. $H < L$

   (a) under $R_1 > \Lambda + R_2[1 + t_0(2H/N)]$: incomplete segregation with the rich closer to the public facility site (endogenous sorting by both income and preferences towards local public good consumption),

(b) under $\frac{\Lambda+R_2}{1+t_0(2H/N)} < R_1 < \Lambda + R_2[1 + t_0(2H/N)]$: incomplete segregation with either the rich or the poor closer to the public facility site (multiple equilibria),

(c) under $R_1 < \frac{\Lambda+R_2}{1+t_0(2H/N)}$: incomplete segregation with the poor closer to the public facility site (endogenous sorting purely by income).

(ii) Under Condition I ($\log G > \frac{\beta}{1-\beta}$): integration (no income or preference sorting).

There are at least three possible avenues for future research. First, we may abandon the assumption of positive correlation between income and preferences for the local public good, i.e., agents may be divided into four categories: high income and high preferences for the local public good, low income and high preferences for the local public good, high income and low preferences for the local public good, and low income and low preferences for the local public good. One may ask whether pure stratification by preferences towards local public good consumption can now occur. Second, it may be interesting to consider an alternative equilibrium concept based on the club theory (especially the "clubs and the market" framework developed by Ellickson, Grodal, Scotchmer, and Zame 1999 and 2002). In particular, we can view different stratified classes as clubs and establish the formation of these clubs. Third, one may introduce the political economy issues, establishing voting or other mechanisms to determine endogenously the level of local public good provision or the setting of the fiscal instruments (user fee as well as local income and property tax rates). Of course, one must bear in mind the such a political equilibrium need not exist in general. Finally, our paper conducts exclusively a positive analysis concerning equilibrium sorting by income and preferences for the travel-for consumable local public good. It may be interesting to undertake a normative analysis, examining, on the basis of efficiency and equity, which policy may achieve higher welfare for the local economy as a whole.

Appendix

Proof of Lemma 1: Totally differentiating (11) leads to,

$$(1-\frac{\partial q}{\partial n_j^i})dn_j^i = \frac{\partial q}{\partial R_j}dR_j + \frac{\partial q}{\partial x_j}dx_j + \frac{\partial q}{\partial Y^i}dY^i + \frac{\partial q}{\partial \gamma^i}d\gamma^i,$$

where under Assumption 1, $\Gamma_j^i = 0$, and hence $\frac{\partial q}{\partial n_j^i} = 0$. Using (6), it is trivial that $\frac{\partial K}{\partial M} = \phi$, $\frac{\partial K}{\partial R_j} = t(x_j)$ and $\frac{\partial K}{\partial x_j} = t'(x_j)R_j$. Thus, we have:

$$\frac{\partial q}{\partial R_j} = \frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{1}{K}t(x_j), \tag{A1}$$

$$\frac{\partial q}{\partial x_j} = \frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{1}{K}t'(x_j)R_j, \tag{A2}$$

$$\frac{\partial q}{\partial Y^i} = \frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{1}{K}\tau_j^i, \tag{A3}$$

$$\frac{\partial q}{\partial \gamma^i} = \frac{1}{1-\beta}\frac{n_j^i}{\gamma^i}. \tag{A4}$$

From (A1)-(A4), the following results can be established:

$$\frac{dn_j^i}{dR_j} = \frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{1}{K}t(x_j) > 0, \tag{A5}$$

$$\frac{dn_j^i}{dx_j} = \frac{1}{K}t'(x_{j)}R_j] > 0, \tag{A6}$$

$$\frac{dn_j^i}{dY^i} = \frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{\tau_j^i}{K} > 0, \tag{A7}$$

$$\frac{dn_j^i}{d\gamma^i} = \frac{1}{1-\beta}\frac{n_j^i}{\gamma^i} > 0, \tag{A8}$$

which complete the proof. ■

Proof of Lemma 2: From (14), (A5), we obtain:

$$\begin{aligned}\frac{dV_j^i}{dx_j} &= -R_j t'(x_j) + (1-\beta)\gamma^i(n_j^i)^\beta \frac{1}{K}t'(x_j)R_j \\ &\quad +\beta(1-\beta)\gamma^i(n_j^i)^{\beta-1}(\log G)\frac{1}{1-\beta}\frac{n_j^i}{\log G}\frac{1}{K}t'(x_j)R_j\end{aligned}$$

which can be simplified to,

$$\frac{dV_j^i}{dx_j} = \frac{1}{K}t'(x_j)R_j[\gamma^i(n_j^i)^\beta\frac{1}{K} - 1]. \tag{A9}$$

Similarly, from (10), (14), and (A6), we have:

$$\frac{dV_j^i}{dR_j} = -(1 + t(x_j)) + \gamma^i(1-\beta)(n_j^i)^\beta. \tag{A10}$$

Combining (A9) and (A10) gives,

$$\Psi(R_j,\ x_j,\ \gamma^i,\ Y^i) = -\frac{\gamma^i(n_j^i)^\beta t'(x_j)R_j - Kt'(x_j)R_j}{\gamma^i(n_j^i)^\beta t(x_j) - Kt(x_j)R_j},$$

and hence the result in (15). ■

Proof of Proposition 1: Straightforward differentiation of the bid rent function (15) yields,

$$\begin{aligned}
\frac{d\Psi}{dS_j^i} &= \frac{t'(x_j)R_j}{\{1 - t(x_j)S_j^i\}^2} > 0, \\
\frac{dS_j^i}{d\gamma^i} &= \frac{1}{K}(n_j^i)^\beta \frac{1}{(1-\beta)} > 0, \\
\frac{dS_j^i}{dY^i} &= \frac{\gamma^i}{K^2}(n_j^i)^\beta \frac{\tau_j^i}{\log G}[\frac{\beta}{1-\beta} - \log G] \gtreqless 0,
\end{aligned}$$

thus giving the properties in the proposition. ■

Proof of Proposition 2: We first consider part (a). From (14), the indirect utility differential for a type-i agent between the two communities is:

$$V_1^i - V_2^i = (1-\beta)(\gamma^i \log G)^{\frac{1}{1-\beta}}(\frac{\beta}{\phi})^{\frac{\beta}{1-\beta}}[1 - (\frac{\phi}{T+\phi})^{\frac{\beta}{1-\beta}}] + [R_2 - R_1(1+t_0)] - \Delta\tau^i Y^i,$$

where $\Delta\tau^i \equiv \tau_1^i - \tau_2^i$. Thus, $V_1^H - V_2^H > 0$ requires:

$$(\gamma^H)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^H Y^H > R_1(1+t_0) - R_2, \tag{A11}$$

where $R_0(G)$ is defined in (19). Similarly, $V_1^L - V_2^L < 0$ requires:

$$(\gamma^L)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^L Y^L < R_1(1+t_0) - R_2. \tag{A12}$$

Combining (A11) and (A12) yields the required condition of Proposition 2(a) under Assumption 4a.

Similarly, for part (b), we utilize (14) and repeat the same steps as before, to establish that $V_1^H - V_2^H < 0$ and $V_1^L - V_2^L < 0$, respectively, require:

$$(\gamma^H)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^H Y^H < R_1 - R_2(1+t_0), \tag{A13}$$

$$(\gamma^L)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^L Y^L > R_1 - R_2(1+t_0). \tag{A14}$$

Combining (A13) and (A14), we obtain the required condition Proposition 2(b) under Assumption 4b. ■

Proof of Proposition 3: Consider part (a). Following similar steps as in the proof of Proposition 2, we can express $V_1^H - V_2^H = 0$ as:

$$(\gamma^H)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^H Y^H = R_1(1+t_0) - R_2(1+t_0 x_2), \tag{A15}$$

and $V_1^L - V_2^L < 0$ as:

$$(\gamma^L)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^L Y^L < R_1(1+t_0) - R_2(1+t_0 x_2). \tag{A16}$$

Substituting $1 + t_0 x_2 = 1 + t_0(\frac{2H}{N} - 1)$ into the above expressions, we can then use (A15) to eliminate $R_0(G)$ in (A16) to obtain the result.

Consider next part (b). By straightforward manipulation, $V_1^H - V_2^H = 0$ implies:

$$(\gamma^H)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^H Y^H = R_1(1+t_0 x_1) - R_2(1+t_0),$$

whereas $V_1^L - V_2^L > 0$ requires:

$$(\gamma^L)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^L Y^L > R_1(1+t_0 x_1) - R_2(1+t_0).$$

These together yield the required inequalities.

Part (c) is a direct consequence of parts (a) and (b) by comparing the equilibrium ranges of market rents. ■

Proof of Proposition 4: To prove part (a), repeating the same exercises, $V_1^H > V_2^H$ and $V_1^L = V_2^L$ require, respectively, as follows:

$$\begin{aligned} (\gamma^H)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^H Y^H &> R_1(1+t_0 x_1) - R_2, \\ (\gamma^L)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^L Y^L &= R_1(1+t_0 x_1) - R_2. \end{aligned}$$

By eliminating $R_0(G)$, we then derive (25).

For part (b), the requirements that $V_1^H < V_2^H$ and $V_1^L = V_2^L$ imply, respectively,

$$\begin{aligned} (\gamma^H)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^H Y^H &< R_1 - R_2(1+t_0 x_2), \\ (\gamma^L)^{\frac{1}{1-\beta}} R_0(G) - \Delta\tau^L Y^L &= R_1 - R_2(1+t_0 x_2). \end{aligned}$$

Repeating the same exercise to eliminate $R_0(G)$ gives (26).

Finally, part (c) is shown by comparing the equilibrium ranges of market rents in (25) and (26). ■

Proof of Proposition 5: Equalizing the indirect utility specified in (14) for each type across two communities yields:

$$(\gamma^H)^{\frac{1}{(1-\beta)}} R_0(G) - \Delta\tau^H Y^H = R_1(1 + t_0 x_1) - R_2(1 + t_0 x_2), \tag{A17}$$

$$(\gamma^L)^{\frac{1}{(1-\beta)}} R_0(G) - \Delta\tau^L Y^L = R_1(1 + t_0 x_1) - R_2(1 + t_0 x_2). \tag{A18}$$

Combining (A17) and (A18) to eliminate market rents, we obtain:

$$R_0(G) = \Omega. \tag{A19}$$

It is easily seen that for any set of income and local tax parameters, Ω is decreasing in the degree of heterogeneity in preferences towards the local public good. Substituting $x_2 = \frac{2H}{N} - x_1$ and (A19) into (A18) leads to:

$$x_1 = \frac{R_2}{R_1 + R_2}\frac{2H}{N} + \frac{1}{t_0(R_1 + R_2)}\left[\Lambda - (R_1 - R_2)\right]. \tag{A20}$$

Since $x_1 \in (0, 1)$, we can manipulate (A20) to obtain the following two inequalities:

$$\begin{aligned} R_1 - R_2 &< t_0 R_2 \frac{2H}{N} + \Lambda, \\ t_0 R_2(\frac{2H}{N} - 1) + \Lambda &< (1 + t_0)R_1 - R_2. \end{aligned}$$

These can be combined to obtain (28). Finally, from the definitions of $R_0(G)$ and Ω and (A19), we get (29), which completes the proof. ■

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