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One-Sector Growth Model

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FACTOR TAXATION AND LABOR SUPPLY IN A DYNAMIC ONE–SECTOR GROWTH MODEL

By

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Abstract

This paper studies a class of AK-type growth models with public capital stock and elastic labor supply. The government taxes both factor incomes and conduct expenditure. To rationalize the taxation, government expenditure affects the productivity of private sectors. It shows the existence of a unique balanced-growth path, near which there is only a transitional dynamic path leading the economy toward it. While a higher capital tax rate reduces economic growth in the short run, the long-term growth effect is ambiguous, and this long-term growth effect remains ambiguous even if the level of tax rate is larger than the degree of government externality. A higher labor income tax rate has equally ambiguous growth effects both in the short and long runs. However, if the intertemporal elasticity of substitution for labor supply is small enough, a higher labor tax rate always lowers economic growth in the long run, despite the existence of productive government taxation.

Key words: taxation, infrastructures, economic growth, transitional dynamics
JEL classification: E6, O4.

Comments from two anonymous referees are greatly appreciated.

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I. Introduction

Because of a renewed interest in economic growth research, much attention has been paid to the long-term effects of economic policies and among the many lines of research examining economic policies, is the growth effect from taxation.[1] Conventional wisdom modeled labor income taxation as better than capital income taxation from the growth point of view, and in extreme cases, set a zero tax rate for the capital income, raising all revenues from other sources, including the labor income.[2]

This paper examines the growth effect of labor income and capital income taxation in both the short term and the long term. In order to isolate the labor employment factor from other considerations, this paper will not introduce either the human capital accumulation, or the learning-by-doing effects via the labor employment. When labor is in the form of a reproducible human capital, it is less different from physical capital. Moreover, in order to rationalize the government taxation, we choose the framework used in Barro (1990) and extended by Futagami, Morita and Shibata (1993) under which productive government expenditure complements private capital in production. In these two existing studies, capital is the only variable private input, and thus, only the capital income is taxed.

In major departure of our work from Barro (1990) and Futagami, Morita and Shibata (1993), we include household elastic labor with labor and capital being complement to each other and government collection of labor income taxes, in addition to capital income taxes. In our model, the government taxes both capital and labor income, in order to finance flow infrastructures, which accumulate public infrastructure stock. Like Barro (1990) and Futagami, et al. (1993), the infrastructure stock is provided by the government free of charge. With the formation of government infrastructures, our one-sector model

[1] Contributions include Barro (1990), Lucas (1990), and Rebelo (1991), among others.

[2] See, for examples, Judd (1985), Chamley (1986), Lucas (1990) and Jones, Manuelli and Rossi (1997).

behaves like a two-sector Lucas (1988) model. Therefore, public infrastructure stock in our model is like human capital stock, and necessary for production. As public infrastructure public is produced using capital and labor income taxes and labor and capital income taxes directly discourage labor supply and capital formation, respectively, the two taxes discourage the input uses and thus growth. However, a higher capital tax rate also increases public infrastructures that enhance the marginal productivity of physical capital, resulting in more capital formation. Moreover, the increase in labor employment resulting from more public infrastructures, complements the marginal productivity of capital and enhances further capital formation. When these indirect effects are strong enough, they could negate the taxes' detrimental effect, thereby spurring healthy economic growth and physical capital accumulation. As a result, capital income taxation is not zero, and is like a capital user's cost.

For labor taxation, a higher labor income tax rate directly *reduce*s *the labor supply*, that lowers the marginal productivity of capital and thus, capital formation and economic growth. The higher labor income tax rate also generates an indirect positive effect on labor supply resulting from a higher shadow price of capital and thus, from lower consumption. Moreover, it has an indirect negative effect on labor demand via lower private capital, and also an indirect positive effect through higher public infrastructures. These indirect effects upon labor employment and economic growth are ambiguous. We have shown that if the intertemporal elasticity of substitution for labor supply is small enough, the net indirect effect is always negative and thus, a larger labor income tax rate reduces economic growth in the long run.

The major difference between capital and labor income taxes in our model lies in the fact that the former is a user's cost and the latter is not. As an outcome, there is an interior optimal capital income tax rate for economic growth and thus the growth effect of capital income tax is non-monotonic. Yet, there is no optimal labor income tax, so the growth effect of labor income tax is monotonic. Our calibration exercises based upon a set of parameter values representative of the U.S. economy, confirm monotonic negative growth effects of larger labor income tax rates and nonmonotonic growth effects of larger capital

income tax rates, with the latter effects dominating the former effects quantitatively, a result in line with Judd (1985) and others. A shift of income tax incidences from capital to labor by reducing capital tax rates by 5% while maintaining a constant fraction of tax revenue in income, has only a mild, positive growth effect, but has a large welfare effect, due mainly to a reduction in labor supply and thus, an increase in leisure. When the leisure is held at the benchmark level, such a shift in the tax incidence increases welfare only by 1%, about the size found by Lucas (1990).

Compared with existing literature, Turnovsky (2000) is the most compatible to our work in that it examines the effects of factor taxation and other changes within an AK-type model. Two major deviations differentiate our work from Turnovsky. First, it is the flow of government expenditure, not the stock, that affects the private sector. As a result, his model is intrinsically static and unable to analyze the short-run, dynamical and transitional growth effects from factor taxation. Second, Turnovsky examined the effects of factor taxation with rebated tax revenues in a lump-sum fashion, whereas we investigate the effects with the productive government expenditure. Since labor employment and public infrastructures both complement capital productivity, the growth effects of factor taxation are different.[3]

Elastic labor supply is considered among the existing endogenous growth models developed by Lucas (1990), Rebelo (1991), Jones, Manuelli and Rossi (1993), Stokey and Rebelo (1995), and Kim (1998).[4] Several factors differentiate our model from theirs. First, our model is a one-sector model, not a multiple-sector model. Second, while their government expenditure is just a lump-sum transfer, it is presented as a productive factor in our model. Finally, their main concern is on quantitative growth effects of tax reform, rather than focusing on the analytical effects from different factor taxation. Also in contrast to our model,

[3] For example, as an income tax rate increases together with the productive government expenditure, the RR locus in Figure 2 of Turnovsky (2000) shifts upwards. As a result, a larger labor income tax rate is probably growth enhancing.

[4] Although Rebelo (1991), Jones, Manuelli and Rossi (1993), and Stokey and Rebelo (1995) present several models, only a comparison with the model of elastic labor supply is made, as we compare them with our work below.

a labor income tax policy is considered beneficial to economic growth in Lucas (1990) and Manuelli and Rossi (1993), with only sector taxes, and not input taxes, modeled in Rebelo (1991) and Stokey and Rebelo (1995).

There are other related papers. Benhabib and Perli (1994) emphasize elastic labor, like in our study. Yet, they indicate it as very plausible to obtain local indeterminancy by a Lucas (1988)-type model, when considering elastic labor supply. In their paper, effects of taxation are not discussed nor examined. Caballé (1998) and Lin (1998) also investigate the effect of factor taxation on economic growth. In an overlapping-generations model with inelastic labor supply, Caballé (1998) constructs a formula for a threshold, above which zero-taxes on capital income (i.e., the income of the old) delivers faster economic growth, and below which taxing capital income leads to increased growth. Like our study, Caballe also believes that capital taxation may not have long term negative effects, but we render differences regarding the mechanisms which induce the end result. In another overlapping-generations model with human capital accumulation and inelastic labor supply, Lin (1998) finds that the positive or negative effect of labor taxation on economic growth, depends on whether savings are positively or negatively related to the labor tax rate. However, the growth effect of labor taxation in our model is entirely different from that of Lin. Finally, in an overlapping generations model, Uhlig and Yanagawa (1996) find a positive growth effect of capital income taxation on the old, as the taxation relieves the tax burden of the young and increases the savings. They do not examine the effect of labor income taxes.

Finally, King and Rebelo (1990), Bond, Wang and Yip (1994), Mino (1996), and Milesi-Ferretti and Roubini (1998) also analyze the growth effect of taxation. These works are two-sector models and, with the exception of Milesi-Ferretti and Roubini (1998), all focus on inelastic labor supply. All these papers extend that higher tax rates hurt economic growth, no matter whether they are in the form of sectoral taxes, as in King and Rebelo (1990) or in the form of factor taxation, as in Bond, Wang and Yip (1994), Mino (1996) and Milesi-Ferretti and Roubini (1998). The main reason is that while government expenditure is neutral,

taxation on labor, in the form of human capital, discourages the accumulation of human capital. As a result, labor taxation always diminishes economic growth. Barro and Sala-i-Martin (1992) also study the growth effect of taxation in several models with inelastic labor supply. They focus on which of the lump-sum taxation and the income taxation is the better way in financing government expenditure. They do not differentiate from different input income taxation.

As developed below, Section II sets up the model for examination. Section III analyzes the balanced-growth and transitional dynamic paths of the model in equilibrium. Section IV examines the short-run and long-run growth effect of factor taxation policies, and Section V offers conclusions as a result of the analysis.

II. A Basic Model

Our basic model draws on Barro (1990). Consider an economy populated by households and firms. Time (indexed by t) is continuous. There exists a continuum of infinite-lived representative households. There is no population growth, and the size of population is normalized to be unity. There exists a continuum of representative firms and each firm is endowed with a production technology. Additionally, there is a government.

1. Household's Problem

The representative household possesses a discounted lifetime utility of the following form:

$$\int_0^{\infty} e^{-\rho t} \left[\ln c(t) - \frac{l(t)^{1+\theta}}{1+\theta} \right] dt, \quad \rho > 0, \ \theta > 0,$$

in which $\rho>0$ is the instantaneous time-preference rate, and $\theta > 0$ is the reciprocal of the intertemporal elasticity of substitution for working/leisure. We assume $\theta > 0$ so that the marginal disutility of working increases in labor employment. The function $c(t)$ is the instantaneous private consumption expenditure in t, and $l(t)$ is the instantaneous labor supply. The intertemporal elasticity of substitution for consumption is

set to 1, in order to guarantee the existence of a balanced growth path in the steady state.[5]

Each representative household is endowed with one unit of labor in every period and supplies a fraction $l(t)$ of labor to work. The market wage rate is $w(t)$. A household also has wealth/capital $k(t)$ accumulated from the past. They lend capital to producers at the market interest rate of $r(t)$. Both earners of wage income and wealth/capital income must pay taxes, at rate τ_l and τ_k, respectively. Disposable income that is not consumed in each period will accumulate as wealth/capital in the next period. As a result, each representative household possesses the following budget constraint:

$$\dot{k} = (1-\tau_l)w(t)l(t) + (1-\tau_k)r(t)k(t) - c(t), \quad \textbf{(1)}$$

where a dot notation over a variable denotes the time derivative of that variable.

The representative household chooses the consumption flow $c(t)$, the labor supply $l(t)$, and the wealth/capital accumulation $k(t)$ over time, in order to maximize its total discounted present value of lifetime utility, subject to budget constraint in (1). To solve the dynamic optimization problem, we use present-value Hamiltonian to derive the following first-order conditions:

$$\frac{\dot{c}}{c} = (1-\tau_k)r(t) - \rho, \quad \textbf{(2a)}$$

$$l(t)^{\theta} = \frac{(1-\tau_l)w(t)}{c(t)}, \quad \textbf{(2b)}$$

$$\lim_{t\to\infty} e^{\rho t}\eta(t)k(t) = 0, \quad \textbf{(2c)}$$

where $\eta(t)$ is the shadow price of the wealth/capital in t.

Equation (2a) equates the period's marginal utility of consumption, and next period's marginal utility

[5] The same functional form is used by Benhabib and Peril (1994). An alternative form is the Lucas (1990) felicity, without separability in consumption and leisure. This alternative form generates the same results. In Appendix B, we derive the growth effects of labor income taxes under such a felicity.

of consumption, resulting from the savings from this period. This condition determines the optimal trade-off between the consumption flow and the accumulation of capital. While (2b) equates marginal disutility of labor supply, and net marginal revenue of labor supply, in order to determine the flow of labor supply,[6] (2c) is the transversality condition that guarantees the market value of the capital stock to be eventually bounded.[7]

2. Producer's Problem

As in Barro (1990), the production technology is affected by productive government infrastructure expenditure. The government expenditure has public-goods properties. The difference in our setting from that of Barro (1990) is that the stock of government infrastructures, and not the flow of government expenditures, affects private production in our model. This setup has been implemented in Futagami, Morita and Shibata (1993). Each representative firm owns the following technology:

$$y(t) = A k(t)^{1-\beta} l(t)^{\beta} g(t)^{\beta}, \qquad \textbf{(3)}$$

in which $y(t)$ is the instantaneous output per capita, $k(t)$ is the instantaneous capital stock per capita, and $g(t)$ is the per capita stock of government infrastructural services in t. The parameter β captures the degree of externality to which the government infrastructure affects private production, and $A>0$ is a productivity parameter summarizing the level of technology. The functional form of production technology ensures that the problem of profit maximization faced by each firm is concave and well-defined. Without loss of generality, we assume no depreciation of capital. Firms are competitive in the goods and input markets.

Facing given market rental rates and wage rates and the stock of government infrastructures, each representative producer, under endowed production technology in (3), determines demand for capital stock rental and demand for labor services in each period, in order to maximize its periodic profit flows. The

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[6] In deriving (2b), we have used the condition that $c(t)=e^{-\rho t}/\eta(t)$.

[7] To be precise, the transversality conditions also requires $\lim_{t\to\infty} e^{-\rho t}\eta(t)g(t) = 0$. As $g(t)$ is linear in $k(t)$ in steady state, (2c) is sufficient to guarantee transversality conditions.

necessary conditions of the optimization lead to the following two input demand schedules:

$$r(t) = A(1-\beta)\,(g(t)/k(t))^{\beta} l(t)^{\beta}, \tag{4a}$$

$$w(t) = A\beta k(t)\,(g(t)/k(t))^{\beta}\, l(t)^{\beta-1}, \tag{4b}$$

in which (4a) equates the rental rate of capital stock and the marginal productivity of capital stock, whereas (4b) equates the wage rate and the marginal productivity of labor employment.

3. The Government's Problem

The government behaves passively in this model. It collects both the labor income taxes and the capital income taxes in each period, and then spends the total amount of taxation in accumulating public infrastructure stock. Like the capital stock, we assume no depreciation for the stock of government infrastructures. To simplify the analysis, we assume flat tax rates, and do not analyze optimal tax rates. As a consequence, the formation of government infrastructure stock is:

$$\dot{g} = \tau_l w(t)\, l(t) + \tau_k r(t)\, k(t). \tag{5}$$

III. Equilibrium

In equilibrium, the markets for commodities and the two inputs must be clear in each period. The model economy exhibits perpetual growth and hence we cannot simply analyze the economic aggregates without transforming the perpetual growing variables into stationary ratios. Denote $x(t) \equiv \frac{c(t)}{k(t)}$ and $z(t) \equiv \frac{g(t)}{k(t)}$. The market equilibrium condition for capital can be obtained by substituting the demand for capital stock in (4a) into the supply of capital stock in (2a):

$$\frac{\dot{c}}{c(t)} = (1-\tau_k)A(1-\beta)\, z(t)^{\beta} l(t)^{\beta} - \rho. \tag{6}$$

Similarly, substituting the labor demand in (4b) into the labor supply in (2b) yields the labor market clearing condition:

$$l(t)^{1+\theta-\beta} = (1-\tau_l)A\beta z(t)^{\beta} x(t)^{-1}. \quad \textbf{(7)}$$

The commodity market is automatically satisfied if we combine the household budget constraint in (1) and the government budget constraint in (5), together with (4a), (4b) and (3).

We are now ready to define the equilibrium.

Definition: *A perfect foresight equilibrium (PFE) is a tuple* $\{r(t), l(t), w(t)/k(t), y(t)/k(t), c(t)/k(t)\ g(t)/k(t)$ $\dot{c}/c(t), \dot{k}/k(t), \dot{g}/g(t)\}$ *such that:*

(i) *the representative household budget (1) satisfies;*

(ii) *the representative household optimizations (2a)-(2b) and transversality condition (2c) satisfy;*

(iii) *the technology (3) and the optimization of producers (4a)-(4b) satisfy;*

(iv) *the government budget (5) balances;*

(v) *the capital market (6) and the labor market (7) clear.*

Define $\tau \equiv \beta\tau_l + (1-\beta)\tau_k$ and $D(\tau_l) \equiv [A\beta(1-\tau_l)]^{\frac{1}{1+\theta-\beta}}$. Then, (7) can be rewritten as:

$$l(t) = D(\tau_l)\left(\frac{z(t)^{\beta}}{x(t)}\right)^{\frac{1}{1+\theta-\beta}}. \quad \textbf{(8)}$$

Next, we divide both sides of (1) by $k(t)$, and of (5) by $g(t)$, and then take a difference of these two equations, together with (4a), (4b) and (8), to yield:

$$\frac{\dot{z}}{z(t)} \equiv \frac{\dot{g}}{g(t)} - \frac{\dot{k}}{k(t)} = A\tau D(\tau_l)^{\beta}\frac{1}{z(t)}\left(\frac{z(t)^{(1+\theta)}}{x(t)}\right)^{\frac{\beta}{1+\theta-\beta}} - A(1-\tau)D(\tau_l)^{\beta}\left(\frac{z(t)^{(1+\theta)}}{x(t)}\right)^{\frac{\beta}{1+\theta-\beta}} + x(t). \quad \textbf{(9)}$$

Finally, dividing both sides of (1) by $k(t)$, and then take a difference between (6) and the resulting (1), together with (4a), (4b) and (8), leads to:

$$\frac{\dot{x}}{x(t)} \equiv \frac{\dot{c}}{c(t)} - \frac{\dot{k}}{k(t)} = -A\beta(1-\tau_l)D(\tau_l)^{\beta}\left(\frac{z(t)^{1+\theta}}{x(t)}\right)^{\frac{\beta}{1+\theta-\beta}} + x(t) - \rho. \qquad \textbf{(10)}$$

Equation (9) states the evolution in government infrastructure difference and in capital accumulation, (10) describes the evolution in household consumption difference and in capital accumulation choice.

With these transformations, the economic system is recursive and easy to solve. Equations (9) and (10) can be used to solve equilibrium $z(t)$ and $x(t)$, and after substituting the resulting equilibrium $z(t)$ and $x(t)$ into (8), gets equilibrium $l(t)$. After solving for these three variables, we can substitute $z(t)$ and $l(t)$ into (4a), (4b) and (3) to obtain $r(t)$, $w(t)/k(t)$ and $y(t)/k(t)$. We can also substitute $r(t)$ into (2a), substitute $w(t)/k(t)$, $l(t)$ and $x(t)$ into (1), and substitute $r(t)$, $w(t)/k(t)$, $l(t)$ and $z(t)$ into (5) to obtain $\dot{c}/c(t)$, $\dot{k}/k(t)$ and $\dot{g}/g(t)$ respectively. Therefore, all the endogenous variables are solved in equilibrium.

1. Steady State

We now start with the solution of the economic system in the steady state. A steady state is a balanced growth path (BGP) of a PFE under which r, l, w/k, y/k, c/k and g/k are constant, and $\dot{c}/c$, $\dot{k}/k$ and $\dot{g}/g$ are all constant and equal over time. In solving for BGP, we begin with the economic system in (9) and (10). Since $\frac{\dot{c}}{c}$, $\frac{\dot{k}}{k}$ and $\frac{\dot{g}}{g}$ are constant and equal along the BGP, it must be that $\frac{\dot{x}}{x} = 0$ and $\frac{\dot{z}}{z} = 0$ along the BGP. While the relationship in (10) under $\frac{\dot{x}}{x} = 0$ can be written as:

$$x^{\frac{1+\theta}{1+\theta-\beta}} - \rho x^{\frac{\beta}{1+\theta-\beta}} = A[A\beta(1-\tau_l)]^{\frac{\beta}{1+\theta-\beta}}\beta(1-\tau_l)\, z^{\frac{(1+\theta)\beta}{1+\theta-\beta}}, \qquad \textbf{(11)}$$

the relationship in (9) under $\frac{\dot{z}}{z} = 0$ can be expressed as:

$$x^{\frac{1+\theta}{1+\theta-\beta}} = A[A\beta(1-\tau_l)]^{\frac{\beta}{1+\theta-\beta}}\left(1-(\frac{1}{z}+1)[\beta\tau_l+(1-\beta)\tau_k]\right) z^{\frac{(1+\theta)\beta}{1+\theta-\beta}}. \quad \textbf{(12)}$$

For convenience, we will call the relationship in (11) as Locus CK (consumption capital evolution) in the (z, x) plane, and that in (12), Locus GK (government-capital evolution). To guarantee nonnegative, steady-state values of x and z, the left-hand side of (11) and the right-hand side of (12) must be positive, which implies $x>\rho$ and $z-(1+z)\tau>0$, respectively. The two loci are nonlinear in the (z, x) plane. Locus CK intersects the vertical axis at $x=\rho$, and is upward-slopping and concave. Similarly, Locus GK intersects the horizontal axis at $z=\tau/(1-\tau)$, and is upward-slopping and concave. See Figure 1.

The intuition for a positive-sloped CK locus is that, when the government infrastructure-capital ratio increases, output will increase. Under given tax rates, both the growth rates of consumption and capital increase, and since the effect on the capital growth rate dominates that of consumption, the consumption-capital ratio decreases. In order to return to steady state, the consumption-capital ratio must increase, thereby reducing the growth rate of consumption and raising the growth rate of capital. The reason for a positive-sloped GK locus is that, a larger public to private capital ratio increases both the growth rate of government infrastructures and that of capital stock for given tax rates, thereby lowering the ratio of public to private capital over time. In the steady state, the consumption-capital ratio needs to increase, in order to (i) raise the growth rate of government infrastructures, and (ii) decrease the growth rate of capital stock, moving the public to private capital ratio back to a constant level.

Given the positive slope and the non-linearity and concavity of the CK and the GK loci, the two loci may not intersect, or may intersect more than once. To see their intersection, rewrite (11) as:

$$AD(\tau_l)^{\beta}\left(\frac{z^{1+\theta}}{x}\right)^{\frac{\beta}{1+\theta-\beta}} = \frac{x-\rho}{\beta(1-\tau_l)}, \quad \textbf{(11}'\textbf{)}$$

next, substitute the above relationship into (12) to obtain:

$$z = \frac{\beta\tau_l + (1-\beta)\tau_k}{1-\beta-(1-\beta)\tau_k}\frac{x-\rho}{x-\Phi\rho}, \qquad \textbf{(12}'\textbf{)}$$

where $\Phi \equiv \frac{1-\beta\tau_l-(1-\beta)\tau_k}{1-\beta-(1-\beta)\tau_k} > 1$, and finally, substitute the above z expression back into (11) to yield:

$$AD(\tau_l)^{\beta}\left[\frac{\beta\tau_l+(1-\beta)\tau_k}{1-\beta-(1-\beta)\tau_k}\right]^{\frac{\beta(1+\theta)}{1+\theta-\beta}}\left[\frac{x^*-\rho}{x^*-\Phi\rho}\right]^{\frac{\beta(1+\theta)}{1+\theta-\beta}} x^{\frac{-\beta}{1+\theta-\beta}} = \frac{(x^*-\rho)}{\beta(1-\tau_l)}. \qquad \textbf{(13)}$$

Define $x_{inf} \equiv \frac{(1-\tau)\rho}{(1-\beta)(1-\tau_k)} > 0$. Then, it implies $x_{inf} = \frac{(1-\beta\tau_l)-(1-\beta)\tau_k}{(1-\beta)-(1-\beta)\tau_k}\rho > \frac{(1-\beta)-(1-\beta)\tau_k}{(1-\beta)-(1-\beta)\tau_k}\rho = \rho$. It is obvious that the left-hand side (LHS) of the above expression is negative for $x \in (\rho, x_{inf})$, and is positive and monotonically decreasing in x from infinity for $x > x_{inf}$, as $1+\theta > 1 > \beta$. The right-hand side (RHS) is positive and linearly increasing in x for $x > \rho$. We denote them as the LHS locus and the RHS locus, respectively. Because the LHS has $x = x_{inf}$ and the horizontal axis as the asymptotes, and the RHS is linear in x, Loci LHS and RHS must intersect uniquely, as illustrated in Figure 2. In light of this, the GK locus must intersect the CK locus only once, as illustrated by Point E in Figure 1.

[Insert Figures 1 and 2 here]

After determining the unique steady-state x^* and z^* using the GK and CK loci, all other equilibrium values in steady state can be easily obtained. We can substitute x^* and z^* into (8) and (4a) to solve for l^* and r^*, respectively. In particular, from (2a) the balanced rate of economic growth is $(\dot{c}/c)^* = (1-\tau_k)r^* - \rho$.

To see whether the balanced-growth rate is positive, substitute l^* in (8) into (4a) to yield:

$$\frac{r^*}{1-\beta} = AD^{\beta}\left(\frac{z^{*1+\theta}}{x^*}\right)^{\frac{\beta}{1+\theta-\beta}}. \qquad \textbf{(14)}$$

As the right-hand side of the above equation equals the left-hand side of (11′), we obtain:

$$r^* = \frac{(1-\beta)}{\beta(1-\tau_l)}(x^* - \rho). \qquad \textbf{(15)}$$

Denote $r_{inf} \equiv \frac{(1-\beta)}{\beta(1-\tau_l)}(x_{inf} - \rho)$. Then, $r_{inf} \equiv \frac{\rho}{1-\tau_k}$, and $\left(\frac{\dot{c}}{c}\right)_{inf} = (1-\tau_k)r_{inf} - \rho = 0.$ Since the balanced growth solution of x must exceed x_{inf}, nondegenerate growth is therefore ensured

Summarizing the above results, we obtain:

Theorem: (Existence and Uniqueness of Steady State) *There exists a unique balanced-growth path with positive economic growth.*

2. Transitional Dynamics

We now study the transitional dynamics of economic system in the neighborhood of a balanced growth path. We linearize the system in (9) and (10) around the steady-state point (z^*, x^*) to obtain:

$$\begin{pmatrix} \dot{z} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} z - z^* \\ x - x^* \end{pmatrix}, \qquad \textbf{(16)}$$

where $a_{11} = -\frac{\tau x^*}{z^* - (1+z^*)\tau} - \frac{(1+\theta)\beta}{1+\theta-\beta}x^* < 0,$

$a_{12} = \frac{(1+\theta)}{1+\theta-\beta}z^* > 0,$

$a_{21} = -\frac{(1+\theta)\beta}{1+\theta-\beta}\frac{(x^* - \rho)x^*}{z^*} < 0,$

$a_{22} = \frac{(1+\theta)x^* - \beta\rho}{1+\theta-\beta} > 0.$[8]

The local dynamic properties are characterized by two eigenvalues of the Jacobian matrix in (16).

[8] The reason for $a_{11} < 0$ is $z^*-(1+z^*)\tau = z^*(1-\tau)-\tau > 0$, which is guaranteed by the positive sign in the right-hand side of (12). The reason for $a_{22} > 0$ is $1+\theta > 1 > \beta$ and $x^* > \rho$.

Denote J the Jacobian matrix, and λ_1 and λ_2 the two eigenvalues. Then, λ_1 and λ_2 satisfy:

$$\lambda_1\lambda_2 = Det(J) = a_{11}a_{22} - a_{21}a_{12} = -\frac{\tau}{z^* - (1+z^*)\tau}\frac{(1+\theta)x^* - \beta\rho}{1+\theta-\beta} - \frac{(1+\theta)\beta\rho}{1+\theta-\beta} < 0.$$

Because the system in (16) involves only one predetermined state variable, a negative eigenvalue indicates that the economic system is saddle-path stable. Thus, there is a uniquely, locally transitional dynamic path in equilibrium, leading the economy toward the steady state (See Figure 1).[9]

Summarizing the above results, we have:

Proposition 1. *There exists a unique, transitional dynamic path leading the economy toward the unique balanced-growth path.*

IV. Effects of Tax Policies

We now characterize the PFE by conducting comparative-static effects of the changes in tax policies upon the consumption-capital ratio, the government infrastructure-capital ratio, and in particular the labor employment and the economic growth rate. We start with the capital taxation.

1. Capital Taxation

When the capital income tax rate is raised, the CK locus is not affected, as its detrimental effect on consumption growth and capital accumulation cancels each other exactly. The GK locus, on the other hand, shifts rightwards, as a larger capital tax rate increases government tax revenues and expenditure, and reduces disposable income and capital accumulation. (See the G′K′ locus in Figure 3).

[Insert Figure 3 here]

[9] The negative determinant also implies Locus GK being steeper than Locus CK, as $\partial x/\partial z\big|_{\dot{z}=0} \equiv -a_{11}/a_{12} > -a_{21}/a_{22} \equiv \partial x/\partial z\big|_{\dot{x}=0}$.

As the result of a larger capital tax rate, the consumption-capital ratio will increase instantaneously. Intuitively, a larger capital tax discourages savings and encourages consumption, thus inducing a larger consumption-capital ratio. Moreover, larger consumption reduces marginal utility of consumption, which also reduces the shadow price of wealth and thus, lowers marginal revenue of labor supply. As a result, the labor supply is decreased, reducing labor employment in equilibrium. This employment effect can be illustrated by differentiating (7) with respect to τ_k, evaluated at steady state, to obtain

$$\frac{1}{l^*}\frac{dl}{d\tau_k} = \frac{1}{1+\theta-\beta}\left(\underset{(+)}{\frac{\beta}{z^*}\frac{dz}{d\tau_k}} - \underset{(+)}{\frac{1}{x^*}\frac{dx}{d\tau_k}}\right), \tag{17}$$

in which the second term in the parentheses is the instantaneous, negative employment effect through a higher consumption capital ratio. Since labor input complements marginal productivity of physical capital, a lower labor employment therefore, reduces the interest rate. In the short run, both capital stock and the government infrastructure stock are not affected, leaving the government infrastructure-capital ratio unchanged.[10] The growth effect may be analyzed by differentiating (6), evaluated at steady state, to obtain

$$\frac{d(\dot{c}/c)}{d\tau_k} = -[(\frac{\dot{c}}{c})^*+\rho]\frac{1}{1-\tau_k} + [(\frac{\dot{c}}{c})^*+\rho]\left(\underset{(+)}{\frac{1}{z^*}\frac{dz}{d\tau_k}} + \underset{(+)}{\frac{1}{l^*}\frac{dl}{d\tau_k}}\right). \tag{18}$$

While a higher capital income tax rate generates a direct negative growth effect (first term), the resulting instantaneous higher consumption capital ratio that reduces employment creates an indirect, negative growth effect. As a result, economic growth is lower in the short run.

Over time, the smaller quantity of savings under a higher capital tax rate reduces physical capital accumulation, which will further increase the consumption-capital ratio. A higher capital tax rate, on the

[10] More specifically, the short run here means the instantaneous run, where only flow variables change while stock variables remain the same.

other hand, increases government expenditure, which in turn increases the government infrastructure-capital ratio over time. As a result of a higher public to private capital ratio, an indirect, positive growth effect emerges as illustrated by the first term in the parentheses in (18). Moreover, while a larger consumption to private capital ratio reduces labor employment, a higher public to private capital ratio increases labor employment(cf. (17)), as it increases marginal productivity of labor. We have shown in Appendix 1-A that the positive effect always dominates the negative effect in the steady state and therefore labor employment always increases (cf. second term in the large parentheses in (18)). As the direct growth effect of capital taxation is negative and the indirect growth effect via a higher public to private capital ratio and larger labor employment is positive, the net growth effect is ambiguous.

To summarize the effects on labor employment and economic growth, we obtain

Proposition 2. (Capital Taxation) *While a larger capital income tax rate reduces labor employment and economic growth in the short run, it raises labor employment, with ambiguous effects on economic growth in the long run.*

While most existing works find a negative, long-run growth effect of capital taxation, we obtain an ambiguous long-run growth effect. This difference is because we include productive government expenditure in the model. In Barro (1990) and Futagami, Morita and Shibata (1993) where productive government expenditure is considered, the growth effect of capital taxation is negative, when the tax rate is evaluated at the degree of government expenditure externality. Yet, it is not the case in our model. In order to see this, we derive the effect of a higher capital tax rate on the output growth rate in steady state, evaluating both capital income and labor income tax rates at the government externality, i.e., $\tau_l=\tau_k=\tau=\beta$. We find that the growth effect remains ambiguous. Therefore, the consideration of elastic labor supply alters the otherwise held property.

2. Labor Income Tax

When labor income tax rate is raised, both the GK and the CK loci shift downwards, with the intersection of the CK locus and the vertical axis remaining unchanged, and a higher intersection of the GK locus and the horizontal axis. The reason for the downward shift of the GK locus is the same as that for a higher capital income tax rate as discussed above. The CK locus shifts downward because a higher labor income tax rate reduces disposable income, and thus also the savings and growth rate of capital stock. In steady state, the consumption-capital ratio needs to decrease to render an increase in the growth rate of consumption, in order to bring the consumption-capital ratio to a constant steady-state level. (See Figure 4.)

[Insert Figure 4 here]

As a result of a higher labor tax rate, the consumption-capital ratio is reduced instantaneously. The effect on labor employment involves, in addition to a direct negative effect, an indirect positive effect through a lower consumption-capital ratio and an indirect effect via an ambiguous effect on public to private capital ratio. This can be seen if we differentiate (7) to obtain

$$\frac{1}{l^*}\frac{dl}{d\tau_l} = \frac{1}{1+\theta-\beta}\left(\underset{(-)}{\frac{-1}{1-\tau_l}} + \underset{(?)}{\frac{\beta}{z^*}\frac{dz}{d\tau_l}} - \underset{(-)}{\frac{1}{x^*}\frac{dx}{d\tau_l}}\right), \tag{19}$$

and as a result, the effect on labor employment is ambiguous.

The effect upon economic growth is

$$\frac{d(\dot{c}/c)}{d\tau_l} = \beta[(\frac{\dot{c}}{c})^*+\rho]\left(\underset{(?)}{\frac{1}{z^*}\frac{dz}{d\tau_l}} + \underset{(?)}{\frac{1}{l^*}\frac{dl}{d\tau_l}}\right) = \frac{\beta[(\dot{c}/c)^*+\rho]}{1+\theta-\beta}\left(\underset{(-)}{\frac{-1}{1-\tau_l}} + (1+\theta)\underset{(?)}{\frac{1}{z^*}\frac{dz}{d\tau_l}} - \underset{(-)}{\frac{1}{x^*}\frac{dx}{d\tau_l}}\right)$$

$$= \frac{\beta[(\dot{c}/c)^*+\rho]}{1+\theta-\beta}\left(\frac{-1}{1-\tau_l} - \frac{[(1+\theta)(1-\tau_l)(1-\tau)-\beta(1-\tau)-\beta(1+\theta-\beta)(1-\tau_l)]z^* - [\tau-\beta(1+\theta-\beta)(1-\tau_l)-\beta\tau]}{\tau(1+\theta)(x^*-\rho)+\tau\rho(1+\theta-\beta)+[z^*-(1+z^*)\tau](1+\theta)\beta\rho}\right). \tag{20}$$

Because of the first and the third terms in (19), the instantaneous growth effect via labor employment

is ambiguous. Over time, public and private capital stocks change. While a higher labor income tax rate reduces disposable income and thus private capital formation, it may increase or decrease public infrastructure depending upon whether labor employment is decreased or not. As a consequence, the public to private capital ratio is ambiguous, which results in an ambiguous growth effect. Substituting (19) into (20) leads to the second equality, which includes a direct negative growth effect of labor income taxes, and an indirect positive effect via a lower consumption to private capital ratio and an ambiguous effect through public to private capital ratio. Combining the two indirect effects, we obtain the third equality in (20). It suffices to consider:

Condition LT. (Labor Taxation) $\theta \geq \dfrac{\beta(1-\beta)\tau(1-\tau_l)-\tau_l(1-\tau)}{1+\tau_l-2\tau}$.

Under Condition LT, the numerator of the second term in the third equality is positive and thus, the net indirect growth effect is negative. Therefore, labor income taxes lower economic growth. Condition LT requires the intertemporal elasticity of substitution for labor supply, and thus for leisure (i.e., $1/\theta$), to be small enough. The condition is easy to meet, in particular when τ_l and τ_k are close to each other, then $\beta(1-\beta)\tau(1-\tau_l)-\tau_l(1-\tau)<0$. Summarizing the employment and growth effects, we obtain:

Proposition 3. (Labor Taxation) *A higher labor income tax rate has an ambiguous effect on employment and economic growth in the short run. Under Condition LT, economic growth is reduced in the long run.*

It is interesting to compare the above growth effect with existing studies. Taxation on human capital has been found detrimental to economic growth (e.g., Bond, Wang and Yip, 1996 and Mino,1996). In these models, an economic growth reduction occurs mainly because the taxation on human capital discourages human capital accumulation, which is the engine of economic growth. In our model, capital accumulation is the engine of growth, but raising the tax rate on labor input that is not the engine of growth, always deters economic growth. Moreover, even though government expenditure is productive in the model, labor taxation

creates a strong possibility of reduced economic growth. These are surprising results.

The reason for a negative growth effect of labor income taxes is as follows. While a higher labor income tax rate may increase government expenditure and thus fasten private capital formation, it directly reduces labor employment, which indirectly lowers the marginal productivity of capital and then diminishes private capital accumulation. When the intertemporal elasticity of substitution for labor supply is small enough, labor income taxes lead to an increase in leisure, and a reduction in employment, so much so that the indirect effects discourage economic growth.

3. Calibration Analysis

Further insights into the growth effects of capital and labor income taxes can be obtained by carrying out calibration analysis. We begin by characterizing a benchmark economy, by calibrating the model based on the following parameter values representative of the U.S. economy:

$$\rho=4\%,\ \theta=1,\ \beta=10\%,\ \tau_k=\tau_l=28\%,\ \dot{c}/c=2\%,\ A=99.4409\%.^{11}$$

With the first five parameter values, coefficient A is calibrated in order to be consistent with the 2% long-term, real economic growth rate in the U.S.. These parameter values result in the following benchmark long-term equilibrium: consumption private capital ratio x= 4.6667%, public-private capital stock ratio z=129.617%, the fraction of labor allocated to employment l= 37.7963%, economic growth rate ċ/c=2%, and the welfare of representative household U=10.7143.

[Insert Table 1]

Rows 1-6 in Table 1 report the effects of changing the capital income tax rates from the benchmark. When the capital income tax rate is raised from 28% to 35%,and then 40%, economic growth rate is reduced by 0.334 and then 0.59 percentage points, respectively, and welfare is lowered by 20.132 percent and 35.566

[11] While the values of ρ, τ_k and τ_l are taken from Turnovsky (2000), β=10% is 2% is higher than that in Turnovsky (2000). The chosen θ=1 lies in the range (0.14, 10) used in Benhabib and Perli (1994).

percent, respectively, from the benchmark. Consumption-capital ratio, public to private capital ratio and labor employment all increase. On the other hand, when the capital income tax rate is reduced to 20%, and then 15%, both economic growth and welfare are enhanced. However, when the capital income tax rate is reduced further to 10%, and then 0%, both economic growth and welfare decline. Thus, the quantitative growth and welfare effects for lowering capital tax rates from the benchmark rate is not monotonic and is concave in τ_k, indicating an optimal capital tax rate between 10% and 15%.

Rows 7-12 in Table 1 are the effects of changing the labor income tax rate from the benchmark tax rate. When the labor income tax rate is increased to 35% and 40%, growth rates are increased, like that of a higher capital income tax rate. When the labor income tax rate is decreased, the growth rates are decreased monotonically, different from the nonmonotonic growth effects of decreasing the capital income tax rates. The quantitative growth effects for the labor income tax rate are milder than those under the capital income tax rate. For welfare, a lower labor income tax rate reduces welfare, because it increases labor supply and thus, reduces leisure. Thus, taxation of labor income is bad for economic growth, but not bad for welfare.

We next conduct some robustness checks. We examine three cases, by reducing values of θ, ρ and β by 50%, respectively. Thus, θ is reduced from 1 to 0.5, ρ from 4% to 2%, and β from 0.1 to 0.05. Value of *A* in each case is calibrated so that the model economy generates 2% long-term real per capita economic growth rate. Where the resulting *A* is reported on the top of each of the three cases in Table 2, the resulting equilibrium under each case is reported in the note to Table 2. Case 1 is for lowering θ by 50%. The growth effects for changing the capital income tax rates and those for changing the labor income tax rates from the benchmark tax rates, are reported in Rows 1-5 and 6-9 under Case 1. The quantitative results are clearly similar to those in Table 1. Results for lowering ρ and β in Cases 2 and 3, are also similar.

Since capital income tax rates have quantitatively larger growth effects, how beneficial is a shift from capital income taxation to labor income taxation? To see this, one could decrease the capital income tax rate and increase labor income tax rate, in order to obtain a constant fraction of tax revenue, and thus a constant

fraction of flow government expenditures, in income.[12] We experiment with a reduction of τ_k by 5%, namely τ_k= 23%, and in order to maintain $\tau = \beta\tau_l + (1-\beta)\tau_l$ =28% and with β=0.1, τ_l needs to increase to τ_l= 73%. The effects of such a change are reported in the bottom row of Table 1. As we can see, economic growth is increased by a mild 0.0678 percentage point from the benchmark and welfare is increased by 14.109%. Yet, the welfare increase mainly results from a reduction in labor supply and thus, an increase in leisure. Holding disutility from labor supply at the benchmark level, welfare is increased by 1.167% from the benchmark level. This magnitude is about the same size in Lcuas (1990, Table 3), where he experiments lowering capital tax rates by 5%, and increasing labor/human capital income tax rates by 1%, in order to maintain the same fraction of government consumption.

To summarize the calibration results, a labor income tax rate has negative growth effects, while a capital income tax rate has positive growth effects at small tax rates initially and negative growth effects at large tax rates. The numerical, detrimental growth effect of higher labor income tax rates supports our theoretical results and differentiates our model from existing studies. The quantitative growth effect of capital income taxation dominates that of labor income taxation, and a shift of income tax incidences from capital to labor while maintaining a constant fraction of tax revenue in income is beneficial, a result in line with the findings in Judd (1985), Chamley (1986) and Lucas (1990).

V. Concluding Remarks

This main objective of this paper is to examine the growth effect of factor taxation. In order to rationalize the taxation, we allow for the productive government expenditure. In order to consider both capital taxation and labor taxation, we allow households to decide their savings behavior and labor supply. To isolate the labor supply decision from other factors, we do not consider the human capital accumulation or the learning-by-doing effects of labor employment. We have shown that, while a larger capital taxation

[12] Using (5), we obtain $\dot{g} = [\beta\tau_l + (1-\beta)\tau_k]y$.

reduces economic growth in the short run, its long-run growth effect is ambiguous. This long-run growth effect remains ambiguous even if tax rates are larger than degree of government externality. We also find that regardless of the level of labor income tax rate, when the intertemporal elasticity of substitution for labor supply is small enough, a larger labor taxation always lowers economic growth in the long run. The above two results arise mainly from elastic labor supply, and the complementarity of physical capital with labor employment and government expenditure.

Existing wisdom considers labor taxation as better than capital taxation, from the economic growth point of view. Although we do not analyze the optimal factor taxation, our results indicate that labor taxation is always detrimental under a mild condition, while capital taxation may be better, which suggest that for economic growth, labor taxation is not always better than capital taxation especially when the response of labor supply is taken into consideration.

There are possible extensions of the model. A natural extension is the welfare analysis of different factor taxation. This extension will be of course, very difficult, given that the expectations on households' consumption, labor supply choices, producers' capital demand, and labor demand behavior, affect the choices of tax rates. Therefore, some kinds of simplification need to be made. The analysis of this extension is even more complicated if the issues of time inconsistency are taken into account. Would new governments in the future choose to continue the taxation policies enacted by current government? Another extension is to consider a public production sector. Instead of buying final goods from the market, the government uses a production technology to produce the public goods. In order to produce the public goods, the public sector uses both capital and labor inputs. It would be interesting to contemplate the growth effects of factor taxation under this framework in the future.

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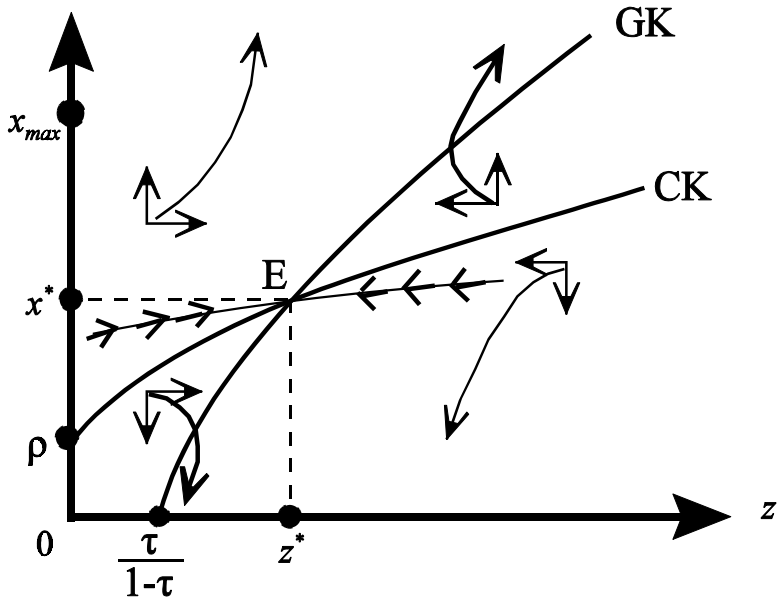

Figure 1. Steady State and Transitional Dynamics

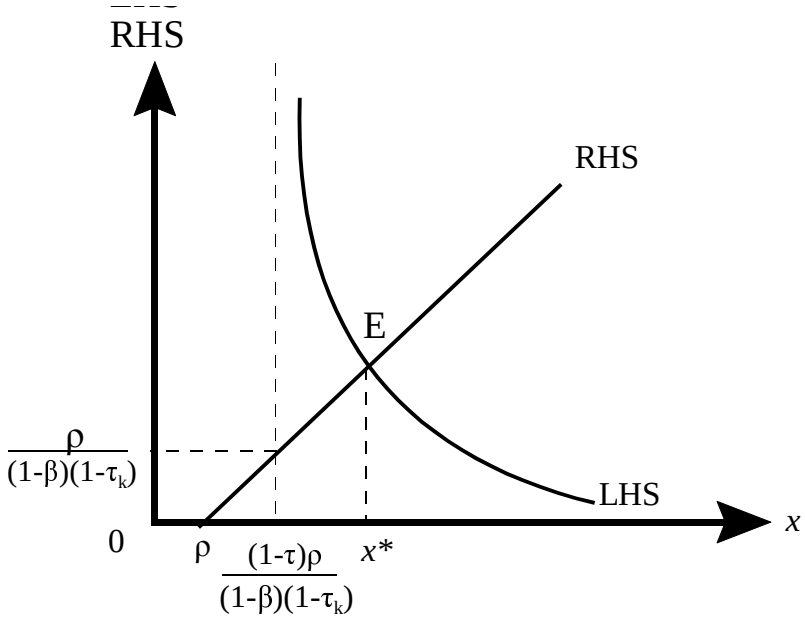

Figure 2: Determination of Steady State for x

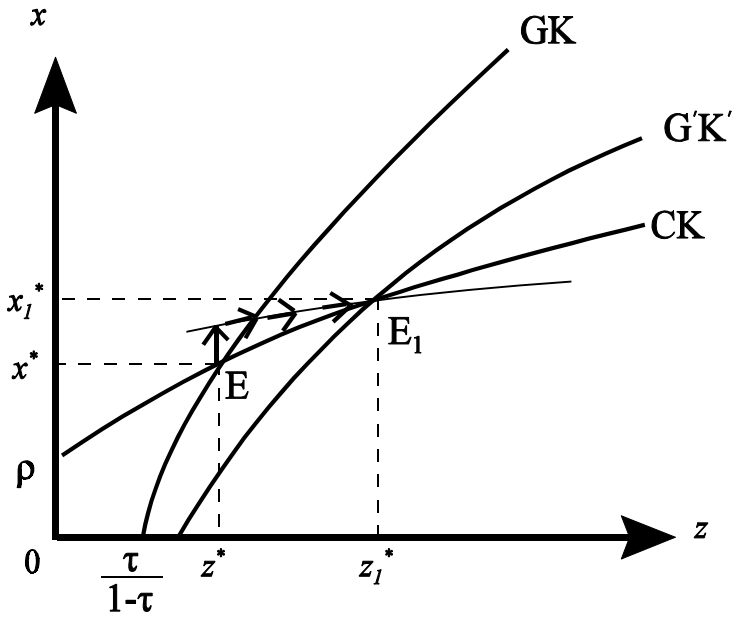

Figure 3. Effect of A Higher Capital Tax Rate

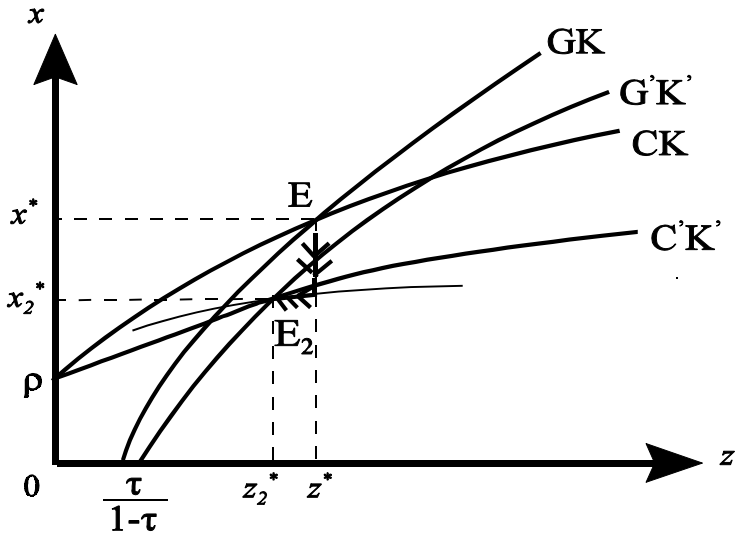

Figure 4. Effect of A Higher Labor Tax Rate

Table 1 Simulation Results

%

		θ=100, ρ=4, β=10,			
	$\dot{c}/c$	U	x	z	l
τ_k=0.4	- 0.59	- 35.566	1.170	112.721	3.418
τ_k=0.35	- 0.334	- 20.132	0.656	53.872	1.943
τ_k=0.2	+0.326	19.802	- 0.731	- 39.384	- 2.224
τ_k=0.15	+0.733	43.077	- 3.229	- 39.208	- 0.888
τ_k=0.1	+0.582	36.172	- 1.751	- 71.344	- 5.492
τ_k=0	+0.27	19.638	- 3.531	- 93.377	- 11.658
τ_l=0.4	- 0.02	1.370	- 2.421	4.999	- 7.734
τ_l=0.35	- 0.01	0.851	- 1.412	2.870	- 4.386
τ_l=0.2	+0.008	- 1.097	1.607	- 3.098	4.645
τ_l=0.15	+0.011	- 1.848	2.610	- 4.986	7.370
τ_l=0.1	+0.014	- 2.656	3.615	- 6.880	9.966
τ_l=0	+0.02	- 4.340	5.608	- 10.469	14.839
τ_k=0.23 τ_l=0.73	+0.0678	14.109 (1.167)	- 9.221	- 8.523	- 37.500 (0.0)

Note. The numbers are percentage changes from benchmark values, except for numbers for $\dot{c}/c$, which are changes from $\dot{c}/c$=2.0%. For benchmark, θ=100%, ρ=4%, β=10%, τ_k=τ_l=28%, and A is calibrated to be consistent with $\dot{c}/c$=2% real per capital economic growth rate. Equilibrium in the benchmark is $\dot{c}/c$=2%, U=10.7143, x=0.046667, z=1.29617 and l=0.377963.

Table 2 Robustness Simulation Results

%

	$\dot{c}/c$	U	x	z	l
		Case 1: θ=50, A=10.2716			
τ_k=0.4	- 0.585	- 37.792	1.185	112.090	4.632
τ_k=0.2	+0.322	20.964	- 0.737	- 39.336	- 2.995
τ_k=0.15	+0.476	31.173	- 1.224	- 57.017	- 5.021
τ_k=0.1	+0.57	37.758	- 1.770	- 71.276	- 7.350
τ_k=0	+0.25	20.712	- 3.570	- 93.335	- 15.420
τ_l=0.4	- 0.034	1.492	- 2.447	5.470	- 10.302
τ_l=0.2	+0.016	- 1.268	1.631	- 3.387	6.316
τ_l=0.1	+0.031	- 3.077	3.664	- 7.391	13.678
τ_l=0	+0.041	- 5.083	5.692	- 11.214	20.516
		Case 2: ρ=2, A=6.82083			
τ_k=0.4	- 0.437	- 24.587	1.252	89.546	2.748
τ_k=0.2	+0.237	13.449	- 0.847	- 36.691	- 1.939
τ_k=0	+0.201	13.140	- 4.435	- 93.129	- 11.047
τ_l=0.4	- 0.013	0.731	- 3.081	4.635	- 7.421
τ_l=0.2	+0.005	- 0.617	2.045	- 2.979	4.415
τ_l=0	+0.009	- 2.480	7.126	- 10.200	13.997
		Case 3: β=5, A=9.26858			
τ_k=0.4	- 0.771	- 41.951	0.336	139.465	2.088
τ_k=0.2	+0.478	26.011	- 0.206	- 42.878	- 1.321
τ_k=0	+0.943	52.116	- 1.219	- 97.170	- 8.160
τ_l=0.4	- 0.018	0.297	- 1.237	2.757	- 8.279
τ_l=0.2	+0.009	- 0.310	0.827	- 1.777	5.058
τ_l=0	+0.028	- 1.321	2.892	- 5.868	16.450

Note. The numbers are percentage changes from benchmark values, except for numbers for $\dot{c}/c$, which are changes from $\dot{c}/c$=2.0%. Parameter value θ is lowered by 50% in Case 1, ρ is lowered by 50% in Case 2, and β is lowered by 50% in Case 3, all from the benchmark case in Table 1, and the resulting A is calibrated so as to be consistent with $\dot{c}/c$ = 2%. Benchmark equilibrium is $\dot{c}/c$=2%, U=10.119, x=0.046666, z=1.29656 and l=0.273278 in Case 1, $\dot{c}/c$=2%, U=45.4545, x=0.024444, z=0.864284 and l=0.426405 in Case 2, and $\dot{c}/c$=2%, U=11.5853, x=0.043157, z=1.22877 and l=0.270504 in Case 3.

Number	Author(s)	Title	Date
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