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Main Flaws of The Collateralized Debt Obligation's Valuation Before And During The 2008/2009 Global Turmoil

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IES Working Paper: 1/2010

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January 2010

Abstract:

As a result of the 2008 financial crisis, the world credit markets stalled significantly and raised the doubts of market participants and policymakers about the proper and fair valuation of financial derivatives and structured products such as collateralized debt obligations (CDOs). The aim of the paper is to contribute to the understanding of CDOs and shed light on CDO valuation based on data before and during the current financial upheaval. We present the One Factor Model based on a Gaussian Copula and test five hypothesizes. Based on the results we discovered four main deficiencies of the CDO market. For our modelling we used data of the CDX NA IG 5Y V3 index from 20 September 2007 until 27 February 2009 and its quotes we appropriately transform into CDO quotes. Based on the results we discovered four main deficiencies of the CDO market: i) an insufficient analysis of underlying assets by both investors and rating agencies; ii) the valuation model was usually based only on expected cash-flows when neglecting other factors such mark-to-market losses or correlation risk; iii) mispriced correlation; and finally iv) the mark-to-market valuation obligation for financial institutions should be reviewed. Based on the mentioned recommendations we conclude that the CDO market has a chance to be regenerated. However, the future CDO market would then be more conscious, driven by smarter motives rather than by poor understanding of risks involved in CDOs.

1. Introduction

By 2007, mounting defaults in the US sub-prime mortgage market led to US market instability, unleashing a global fiscal contagion that spread around the world, roiling markets and causing world economic upheaval. This contagion led to, for example, the nationalization of big financial institutions, bank failures, the end of an era in investment banking, increased federal insurance on banking deposits, government bailouts and opportunistic investments by sovereign wealth funds (Teply,2009). Consequently, the world credit markets stalled significantly and raised the doubts of market participants and policymakers about the proper and fair valuation of financial derivatives and structured products such as collateralized debt obligations (CDOs).

A CDO is a credit-based derivative structured product which enables a transfer of a credit risk of a portfolio of assets from its issuer (protection seller) to an investor (protection buyer). CDOs employ securitization technology to pool assets and finance the purchase of those assets by the issuance of securities (Fabozzi and Kothari,2008). Some types of CDOs, and CDOs of underlying CDOs (CDO⁻) can be conceived as a form of insurance against a credit event of any of the underlying assets.

The aim of the paper is to contribute to the understanding of CDOs and shed light on CDO valuation based on data before and during the current financial upheaval. In our research we expand previous empirical and theoretical papers on CDO valuation (Bhansali et al. (2008), Longstaff and Rajan (2008), Scheicher (2008) or Tarashev and Zhu (2007)). Since CDOs rank among the more advanced structured products, the models used for their valuation are very complex and therefore investors often relied on assessment of rating agencies without a proper understanding of the model. After explaining the valuation model in context of the pending turmoil, we will be able to specify and demonstrate recent weaknesses of the CDO market and provide recommendations for the future existence and regulation of CDO markets. This paper is organized as follows: after a brief introduction, we describe basic principles of CDOs. In section three we present the one factor Gaussian copula model. Although it is a relatively simple model, which does not demand sophisticated mathematical and probability methods, it is understandable, straightforward and suitably illustrates the main sensitivities and key features of CDO valuation. The second part of this section develops the theoretical concept presented in the third section and explains the CDO valuation process when using the VBA program in MS Excel. In the fourth section, we test five hypothesizes related to CDO valuation models before the crisis. Based on the outcomes, we detect main flaws of the CDO valuation and make relevant recommendations that should help to restore confidence of the CDO market. Finally, in conclusion, we summarize the paper and state final remarks.

2. Basic Principles of CDOs

2.1 CDO basics

There is no unique definition of a CDO as it is a complex financial product which might take several forms (Figure 1). For instance, IMF (2008) offers the following definition of a typical CDO: "*A structured credit security backed by a pool of securities, loans, or credit default swaps, where securitized interests in the security are divided into tranches with differing repayment and interest earning streams.*" We can distinguish among various CDOs according to a variety of criteria such as mode of assets acquisition (cash, synthetic, hybrid),

underlying assets (high-yield, investment grade, emerging market, primary market, CDO squared), purpose (balance sheet, arbitrage), leverage structure (cash-flow, market value) or asset ramping (fully, partly, to be ramped up), asset management (static, managed) – for more details see Fabozzi et al. (2008), Fabozzi and Kothari (2008) or Mejstrik et al. (2008). Collectively, the unifying factor is that once structured, the resulting instrument is assigned a rating by the rating agencies that stands as a proxy for the instrument's credibility under normal circumstances.

Figure 1: CDOs as part of complex structured finance products
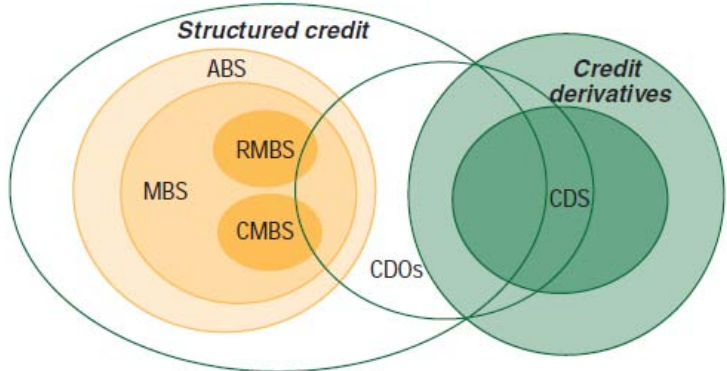
Source: IMF (2008)

Note: ABS = asset-backed security; MBS = mortgage-backed security; RMBS = residential mortgage-backed security; CMBS = commercial mortgage-backed security; CDS = credit default swap; and CDOs = collateralized debt obligations. Not proportionally representative.

Although CDOs were first presented in 1980s, their issuance registered an outstanding growth since 2001 when most CDOs received a rating as the rating agencies became more familiar with CDOs (Fabozzi et al., 2008). In 2004 the worldwide CDO issuance amounted to USD 97 billion and peaked in 2006 when it totaled USD 445 billion. As a result of the financial turmoil, the CDO issuance plummeted to mere USD 156 billion in 2008. Figure 2 demonstrates that structured CDOs (synthetic CDOs based on credit default swaps and/or CDOs squared) were the main source of CDO growth in the 2001-2008 period. The issuance of structured CDOs jumped from USD 12 billion in 2001 to USD 240 billion in 2006 and fell back to USD 26 billion in 2008. As of the end of 2008, total CDO outstanding amounted to USD 870 billion.

Figure 2: CDO issuance according to primary collateral type (USD millions)
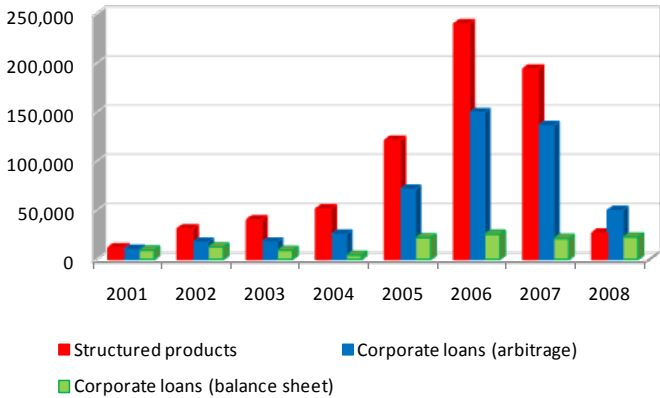
Source: www.abalert.com

The high growth in CDO activity in 2005 and 2006 was mainly due to arbitrage activity (i.e. profit motives) that replaced credit risk elimination (the initial motive of CDO issuance). The highest tranches often obtained the highest possible score from rating agencies and therefore were wrongly considered a safe investment (Teply and Cernohorsky, 2009). However, as a result of the US mortgage crisis in 2007, the issuance of CDOs fell dramatically and the premiums the issuers were willing to pay for credit protection skyrocketed. As of October 2008 the CDO market was frozen and 67% of the CDOs issued since late 2005 to middle 2007 were in formal state of default (Thomson Reuters, 2008). The problems with the CDO market started when mortgage backed securities' investors suffered losses. Since many CDOs were linked owned by these investors, their tranches were downgraded and a spiral of losses was then created, exacerbated by diminishing liquidity. Downgrades of CDO tranches triggered losses of CDO investors and since many CDOs were linked issued by to these investors, further CDO tranches had to be downgraded causing further losses and write-downs. As a result of mark-to-market cuts, total writedowns by banks and other financial institutions between mid-2007 and May 2009 are estimated at USD 685 billion (Asset-Backed Alert, 2009). Many institutional investors suffered massive write-downs (e.g. Citigroup, UBS or KBC), many of them were bailed-out (e.g. AIG, Royal Bank of Scotland or Northern Rock), while some were acquired by a stronger competitor (e.g. Bear Stearns, Merrill Lynch or Washington Mutual).

In our paper we present and discuss a model for an arbitrage cash CDO. In a cash CDO, the assets are purchased by the CDO originator (in contrast to a synthetic CDO, where the originator does not buy the underlying assets but has an exposure to the assets through credit derivatives). Under arbitrage CDO scheme, the originator makes profit on spread between the return earned on the pool of assets that is the collateral for the CDO and funding costs of the CDO. An arbitrage cash CDO is a combination of the types mentioned above ("Model CDO"). The Model CDO can be understood as a type of insurance against default as it is a contract between an originator and an investor with specified maturity in which the originator commits to pay the investor regular premium payment until maturity. The investor in exchange promises to bear all the credit risk (all the principal risk in the event of a default). In the case of no default until maturity, the originator continues to regularly pay the investor the premium. In case of default until maturity, the investor compensates the originator the loss the originator suffered.

Moreover, the Model CDO represents a very diverse risk structure product. It offers the investor to choose the amount of credit risk he would absorb based on his risk profile and appetite. This feature lies in CDO tranching. The basic principle of the Model CDO is depicted on Figure 3. Imagine an underlying portfolio of 100 securities with the same nominal value. The Model CDO is divided into four tranches each absorbing a portion of the resulting cash flows or default impact; the first tranche receives the first flows and the remainder flows or "cascades" to the lower tranches. The investor of a first tranche receives the first impact, i.e. immediately when a first default occurs. He compensates the originator for first 5 percent of defaults. The second tranche investor does not have to make any loss payment to the originator in case of no more than 5 defaults (his tranche is not hit - he compensates the losses of more than five and less than 16 defaults). The same principle holds for higher tranches: they are impacted even after all more junior tranches lose 100%.

Figure 3: Arbitrage Cash CDO
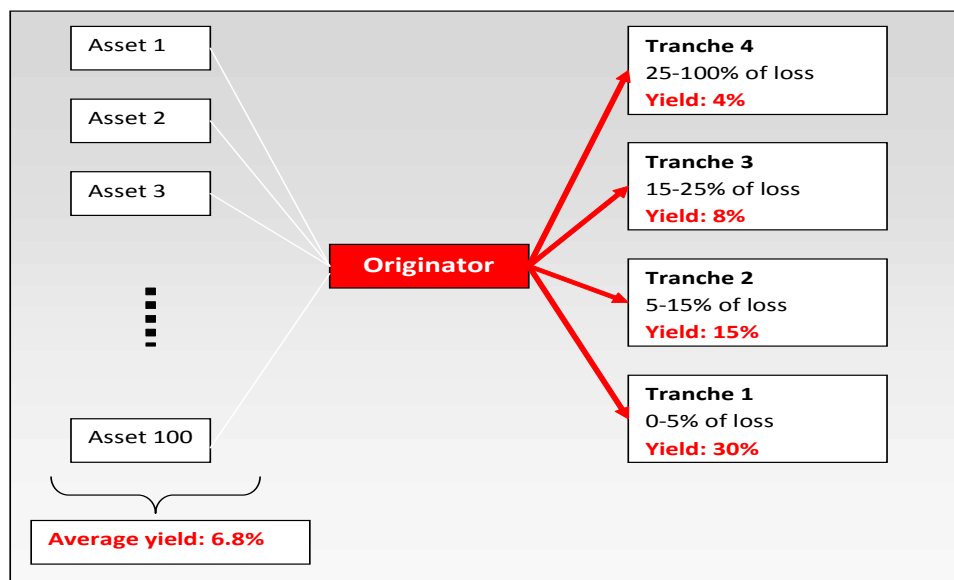
The higher the risk, the higher the premium received from the originator. Instead of investing in the whole portfolio yielding 6.8% to maturity, the structure allows the investor to choose a specified tranche with much more concrete risk profile and corresponding return.

2.2   CDO indices

CDOs rank equivalent to an over the counter (OTC) products and hence no official exchange exist. CDO indices were established during times of high CDO trade volume to achieve standardization in CDO trading. At present, there are two main groups of CDO indices: i) CDX and ii) iTraxx. CDX contains North American companies' names and was established by CDS Index Company and marketed by Markit Group Limited. On the other hand, iTraxx contains European and Asian companies' names and it was introduced by International Index Company. iTraxx is therefore divided into iTraxx Europe and iTraxx Asia. In 2007 Markit Group acquired CDS Index Company and International Index Company, thus it now manages both the CDX and iTraxx8 (see www.markit.com).

In our paper we use the CDX index data for our analysis. There are many advantages of using the CDX index compared to a single CDO for reasons such as diversification, transparency, standardization and liquidity that guarantees lower bid-offer spreads. Unlike a single CDO, the index provides a possibility to invest only in a selected geographical area or business sector. By investing in a tranche of an index, CDO investors can establish proxy hedges against the CDO tranches they hold. Index trades are also used to take positions and express outlook of credit markets. Indices are also widely used as a benchmark for credit risk assessment. Figure 4 shows an evolution of the CDX IG 5Y Series 3 index since 2004 and clearly reveals two main events on credit markets in the year 2008: Bear Stearns' bankruptcy in March 2008 and Lehman Brothers' collapse in September 2008. However, high values of the index implying loss of confidence of market players in credit markets persisted also in February 2009.

Figure 4: CDX IG 5Y index Series 3 (September 2004 – February 2009)
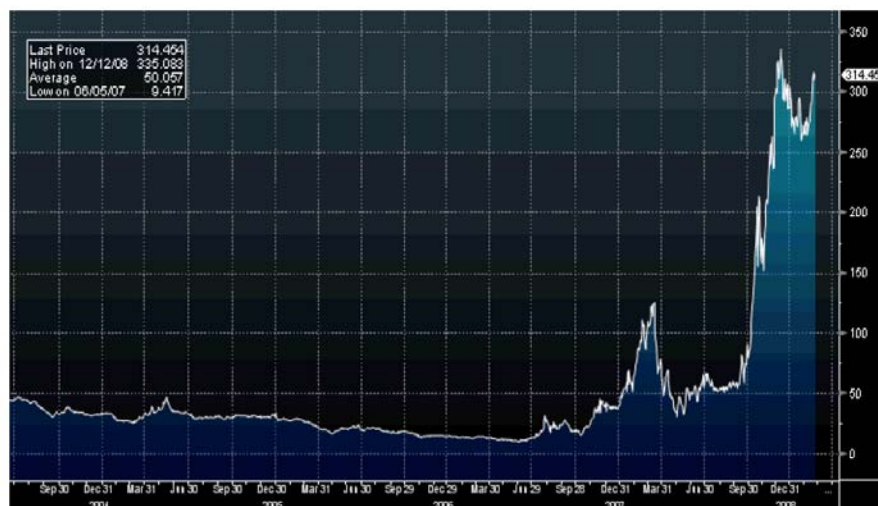
Source: Bloomberg

2.3 CDO risks

As each financial instrument, a CDO bears many risks for the investor such as interest rate risk, cross-currency risk, ramp-up risk and reinvestment risk (Fabozzi and Kothari, 2008); we will focus on the following two: i) correlation risk and ii) counterparty risk. By a correlation we mean the correlation between the defaults of underlying assets of a CDO. The higher the correlation, the more fragile the whole CDO structure. The role of correlation can be described using an example of a ferry (representing senior tranche) and a small wooden boat (representing an equity tranche) crossing a mine field. The ferry is destroyed after crossing ten mines whereas the boat is destroyed by one mine. Suppose there is a constant number of mines but they are differently distributed over the field. First, they are evenly distributed (i.e. no correlation), second, they all are in one small area whereas the rest of the field is mine free (i.e. high correlation). The probability that the ferry is destroyed is higher in case of high correlation of mines because in case of no correlation it wouldn't cross more than 9 mines. Therefore the senior tranche holder requires higher premium in case of high correlation of the default times. On the other hand, if there is no correlation the boat has nearly no chance of surviving compared to the high correlation and therefore the higher the correlation the lower the premium for an equity tranche. The correlation changes over time and depends on macroeconomic conditions. In times of a recession the correlation between assets tends to increase, whereas it is low in times of growth (Kakodkar, A. et al., 2003). This is a very important feature which is essential to understand when investing in a CDO. Correlation will be discussed in much more details in the following sections.

A CDO investor is subject to a counterparty risk of both a CDO issuer and all underlying assets' issuers. Usually, each CDO tranche is classified by a rating. This rating, however, can theoretically be reviewed and changed by a rating agency at any time (although the rating agencies failed to exercise this option until too late during the crisis). In previous years the rating agencies' assessment of risk was taken as gospel by all its users and they acted as if the rating was once given and irreversible. In fact, a default of one underlying asset can cause a downgrade of all tranches of a CDO. Consequently, not only a junior tranche

investor is hit by the default but also a senior tranche investor suffers a loss – a mark-to-market loss - as the spread of the senior tranche soars. This happened in the crisis and that is why CDOs are often labeled as the main cause of the unthought-of losses resulting in numerous defaults or financial difficulties of often large and stable companies and financial institutions. The threat of downgrading of an asset and all its consequences based on numerical evidence will be further discussed in the fifth section.

Figure 5 illustrates the above explained risks and their consequences. On a right x-axis we it depicts institutions that issued more than USD 10 billion nominal of CDOs in the most successful year 2006. Merrill Lynch ranked the first with USD 54 billion, followed by Deutsche Bank with USD 35 billion and Citigroup with USD 33 billion CDO issuance.

Figure 5: Top CDO issuers and their writedowns as of February 2009 (in USD millions)
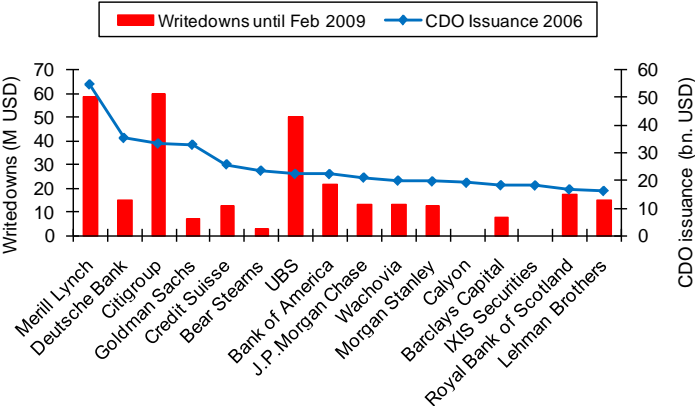
Source: Authors based on www.abalert.com

The columns in Figure 5 show the writedowns of the institutions since mid 2007 until February 2009. Citigroup with nearly USD 60 billion of writedowns was on the first place followed by Merrill Lynch and UBS. These writedowns have their roots in a high volume of subprime mortgages offered recklessly to households with a low credibility in the US. The mortgages were securitized into a mortgage backed securities (MBS) and then sold to institutional investors. In this way the credit risk of the mortgages was spread to the whole financial sector. After some mortgage defaults, many institutions involved in this process were hit and some of them even defaulted (such as Lehman Brothers or US mortgage agencies Fannie Mae and Freddie Mac). These defaults triggered first settlements of CDO contracts, downgrades of MBS holders, consecutive downgrades of CDO holders and massive writedowns of many counterparties included in MBS or CDO business. This also resulted in CDS spreads widening and further mark-to-market losses. Some CDOs had to be terminated before maturity creating even higher losses. Because of high interdependence within a financial sector and its strong link to all business sectors, a series of problems of underlying companies led to a serious financial crisis the world is now experiencing.

3. The One Factor Gaussian Copula Model and its Implementation

3.1 The One Factor Gaussian Copula Model

The One Factor Gaussian Copula Model is a basic model of a CDO valuation and based on a principle of correlation of default times firstly introduced by Li, 2000:3. The main idea behind all CDO valuation models lies in setting the spread of a tranche (i.e. such premium that ensures the present value of premium payments equals to the present value of the loss payments implying that the total present value of the contract remains zero). Both the loss

payment and the premium payment depend on a number of defaults in the future and their timing, which further determines a time distribution of loss. As none of this is known, losses are random variables whose expected value should be computed within the model. First, we should determine the probability of default of an obligor by time *t*. Then, having this distribution for each obligor and combining it with a correlation structure between the obligors, we identify the joint distribution function. The factor model enables to solve the problem with a correlation structure between obligors. The copula function approach introduces a e quantitative way how to cope with multidimensional distribution functions. After obtaining the joint distribution, it is straightforward to deduce the probability of number of defaults in each time period and the loss distribution. The model can be extended either by using multiple factors instead of one (Hull and White, 2004: 6), by assuming other than normal distribution of default times (Gregory and Laurent, 2004: 5) or by modifying assumptions of entry parameters such as recovery rate or correlation (Wang, Rachev and Fabozzi, 2006:19 or Krekel, 2008:4).

It is worth noting here that the abovementioned papers were written dealing with relatively many simplifying assumptions and improving the valuation from a mathematical point of view. But no one really predicted much more devastating consequences of several defaults after inclusion of a market insight (e.g. the liquidity risk or increased risk aversion of the markets).

Suppose a CDO with *n* underlying assets $i = 1, \ldots, N$. Using One factor Gaussian copula model our task is to set the premium payment V for each tranche so that the present value of premium payments equals to the present value of loss payments. Such premium V^* satisfies:

$$V^* = \frac{\sum_{k=1}^{T} B(t_0, t_k) \left[EL_{(K_A, K_D)}(t_k) - EL_{(K_A, K_D)}(t_{k-1}) \right]}{\sum_{k=1}^{T} B(t_0, t_k)(t_k - t_{k-1}) \left[1 - EL_{(K_A, K_D)}(t_k) \right]}, \quad (1)$$

where $B(t_0, t_k)$ is a discount factor discounting from time t_k, $k = 1, \ldots, T$ to t_0 ($0 = t_0 < t_1 < \cdots < t_T$), thus $B(t_0, t_k) = exp\left(-\int_0^k f(0, s) ds \right)$, where $f(0, s)$ is a spot forward interest rate.

To determine optimal premium V^* we need the expected loss function $EL_{(K_A, K_D)}(t_k)$ which is given by equation (2):

$$EL_{(K_A, K_D)}(t_k) = \frac{1}{K_D - K_A} \sum_{j=1}^{n} \left[min\left(\frac{L_j(t_k)}{A \cdot n}; K_D \right) - K_A \right]^+ \cdot P(N(t_k) = j) \quad (2)$$

K_A and K_D define tranches, i.e. an attachment resp. detachment point of a tranche. We suppose the same volume of each underlying asset in the CDO pool and denote it *A*. $L_j(t_k)$ is a cumulative loss on the whole portfolio by time t_k given *j* defaults. To put it simply, to reach $EL_{(K_A, K_D)}(t_k)$ we count loss on a tranche for all cases of $j = 0, 1, \ldots, n$ defaults $\left[min\left(\frac{L_j(t_k)}{A \cdot n}; K_D \right) - K_A \right]^+$ and sum them weighted by their probability $P(N(t_k) = j)$. As other parameters of equation (2) are known, only the probability of *j* defaults by time t_k $P(N(t_k) = j)$ is to be determined.

Therefore a random variable τ_i is introduced denoting a default time of *i*-th underlying asset, $i = 1, \ldots, n$. It is essential for our calculation to deduce its properties.

The derivation is not straightforward; we divide it in two steps. In the first step we condition the probability on one factor *M* which is supposed to be normally distributed:

$$X_i = \rho_i M + \sqrt{1 - \rho_i^2} \cdot \varepsilon_i \qquad (3)$$

where ε_i is a random variable with standard normal distribution, $i = 1, \ldots, n$. In One factor model ε_i and M are independent. Therefore also X_i is a random variable with normal distribution. ρ_i is a constant called loading factor, $|\rho_i| < 1$. Based on copula approach there is a link between X_i and τ_i. For a fixed i suppose $\Phi(x)$ is a distribution function of X_i and $F_i(t)$ is a distribution function of τ_i. If F_i is increasing, then there exists bilaterally unique correspondence between $t \in D_{F_i}$ and $x \in D_\Phi$[1] such that:

$$F_i(t) = P(\tau_i \leq t) = P(X_i \leq x) = \Phi(x) \quad or$$

$$t = F_i^{-1}\big(\Phi(x)\big) \quad resp. \quad x = \Phi^{-1}\big(F_i(t)\big) \qquad (4)$$

X_is are mapped to τ_i using a percentile-to-percentile transformation.

From now on we suppose a homogenous portfolio – i.e. the default times of all obligors have the same distribution - $\tau_i = \tau$ for all $i = 1, \ldots, n$ and correlation among the default times is the same for each pair of obligors - $\rho_i = \rho$ for all $i = 1, \ldots, n$. The number of defaults at time t $N(t)$ follows a binomial distribution, therefore:

$$P(N(t) = j | M = m) = \binom{n}{j} \cdot P^j(\tau \leq t | M = m) \cdot \big(1 - P(\tau \leq t | M = m)\big)^{n-j}, \quad (5)$$

where $P(\tau \leq t | M = m)$ is derived from equation (6) using One Factor Gaussian Copula approach.

$$P(\tau \leq t | M = m) = \Phi\left(\frac{\Phi^{-1}(F(t) - \rho \cdot m)}{\sqrt{1 - \rho^2}}\right) \qquad (6)$$

The reason for conditioning in the first step is that based on the theory of copulas, the default times of obligors are mutually independent only conditionally on a factor value. Only having independent default times the binomial distribution in equation (5) can be used.

In the second step using integral over all M we derive the unconditional probability:

$$P(N(t) = j) = \int_{-\infty}^{\infty} \binom{n}{j} \cdot \left(\Phi\left(\frac{\Phi^{-1}(F(t) - \rho \cdot m)}{\sqrt{1 - \rho^2}}\right)\right)^j \cdot \left(1 - \Phi\left(\frac{\Phi^{-1}(F(t) - \rho \cdot m)}{\sqrt{1 - \rho^2}}\right)\right)^{n-j}$$
$$\cdot \phi(m) dm \qquad (7)$$

Given the mathematical background, in the following section we will present the valuation program we designed in the VBA program in MS Excel. Consequently parameters of the model will be chosen appropriately given recent improvements in CDO pricing. Finally, we will value the CDX index and its tranches, implement comparative statistics and assess sensitivity on each parameter in context with the current financial crisis.

3.2 Implementation of CDO valuation

In this section we will show how to implement the valuation of a CDO contract following the theoretical concept introduced in the previous part. All market data have been taken from Bloomberg. To implement the valuation it is necessary to adopt some assumptions about the entry parameters. The main is the distribution of τ and pairwise correlation ρ.

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[1] D is a standard notation of the definition scope

One useful measure of probability distribution of τ is hazard rate function $h(t)$ defined e.g. in Li (2000):

$$h(t) = \frac{f(t)}{1 - F(t)} \qquad (8)$$

where $F(t)$ is the distribution function of and is $f(t)$ the density of the default times. After some basic derivations we obtain:

$$f(t) = h(t)exp\left(-\int_0^t h(s)ds\right) \quad (9)$$

We suppose that the hazard rate function is constant at some level called hazard rate and denoted by λ. Based on this assumption the density of the default time simplifies to an exponential one:

$$f(t) = \lambda \cdot exp(-\lambda t) \qquad (10)$$

Hazard rate of an asset is then deduced from the market quotes of credit default swap (CDS).

Concerning correlation, the correlation parameter is defined as a loading factor in the One factor model in equation (3). We suppose that correlation is the same for each pair of assets. There are two approaches to correlation determination: implied correlation and base correlation. In both approaches the correlation is determined endogenously by the model. Implied correlation is defined as a correlation for which the net present value of a tranche equals zero. The base correlation approach is more complex. Suppose a CDX index with following tranching: (0-3)%, (3-7)%, (7-10)%, (10-15)% and (15-30)%. Now imagine a non-existing series of tranches (0-7)%, (0-10)%, (0-15)% and (0-30)%. Implied correlation is a value of correlation that sets the non-existing tranches' values to zero.

Theoretically, the implied and base correlation should be the same for all tranches and subsets of tranches. The discussion of values of correlation in imperfect markets will follow. Generally, it is observed that it differs through tranches and even through time - it tends to be lower in economic upturns rather than in downturns. For more discussion of correlation see Das (2005) or (Fabozzi and Kothari, 2008).

4. Results of The Model

The aim of this section is to illustrate CDO pricing when using the model presented in section 3.1 based on assumptions taken in section 3.2. We run the calculation introduced above to show market risk of a CDO and to model the mark-to-market loss of a holder of senior tranches (i.e. we demonstrate what were the consequences of massive and naïve investment in AAA rated tranches and why the CDO market nearly ceased to exist). At the beginning of this section we briefly discuss the data used for our research.

4.1 Data used

As we discussed above, the volume of CDO trades fell dramatically in 2008 after years of growth and the liquidity of the market disappeared. CDS index valuation will be implemented due to both difficulties to get market data and low level of liquidity of CDOs. We choose the most traded series of the main CDX index – 9 and maturity – 5 years (usually noted as CDX NA IG 5Y V3). This index originally counted 125 underlying bonds issued by North American companies. The effective date of the index was on 20 September 2007. Since then there have been 3 defaults of underlying companies (Federal Home Loan Mortgage

Corporation, Federal National Mortgage Association and Washington Mutual). After each event of default a new version of the index had to be launched so that new index investors had a starting position with no defaults and continuity of the index was sustained. First two defaults were settled at once, therefore a third version of this index was introduced in trading nowadays (named as the CDX.NA.IG S9 5Y Version 3).

Figure 6 shows the first CDX index (version 1) for the period from 20 September 2009 until November 2008 and describes reactions of an index to an announcement of defaults were enormous. For instance, the first version premium was decreasing since its issue date. While the index started trading at 30 bps, it reached 148 bps as of 5 September 2008. On 8 September 2008 the Federal Housing Finance Agency announced it put Fannie Mae and Freddie Mac under its conservatorship, which was considered as a credit event. Moreover, Washington Mutual nationalization was announced two weeks later. As a result, after the auction and settlement of the defaults the market quote skyrocketed to 3,000 bps in late November 2008.

Figure 6: The development of the CDX.NA.IG S9 5Y Version 1 from 20 September 2009 until November 2008
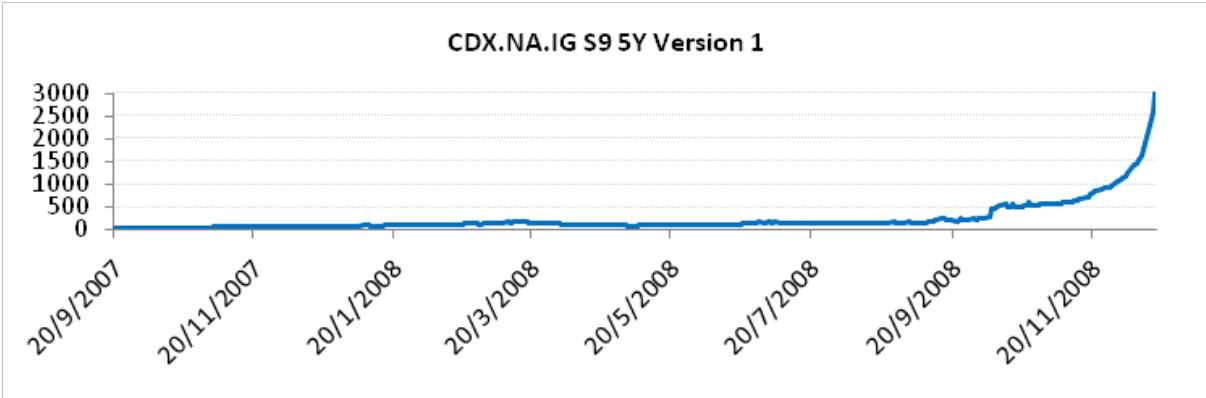

4.2 Hypotheses Testing

The section consists of three parts: in the first part, we determine a relation of each tranche's premium to correlation, hazard rate and event of default. In the second part we show these implications on recent data. As we have data of an index instead of a CDO, we estimate how a real CDO would have behaved in two recent years, which were affected by the financial crisis. The last part deals with an overall assessment of the CDO market, its weaknesses, role in the crisis and contribution to the crisis. The main flaws of the market are spotted and their correction is proposed. Moreover we test five hypothesizes about the entry parameters. The Hypothesis 1, 2 and 3 concerns correlation within the CDO model, the Hypothesis 4 deals with hazard rate and Hypothesis 5 raises the issue of mark-to-market valuation.

4.2.1. Correlation Evaluation (Hypotheses 1, 2 and 3)

Hypothesis 1: The higher the asset correlation, the lower the risk premium for a junior tranche and the higher the risk premium for a senior tranche.

A correlation parameter shows the correlation between each pair of underlying assets. Figure 7 shows what the role of correlation in each tranche. We fixed the hazard rate at 0.07 and calculated the premium of a tranche for a varying level of correlation on vertical axis.

Figure 7: Tranche premium for a given level of correlation (as of 28 February 2009)
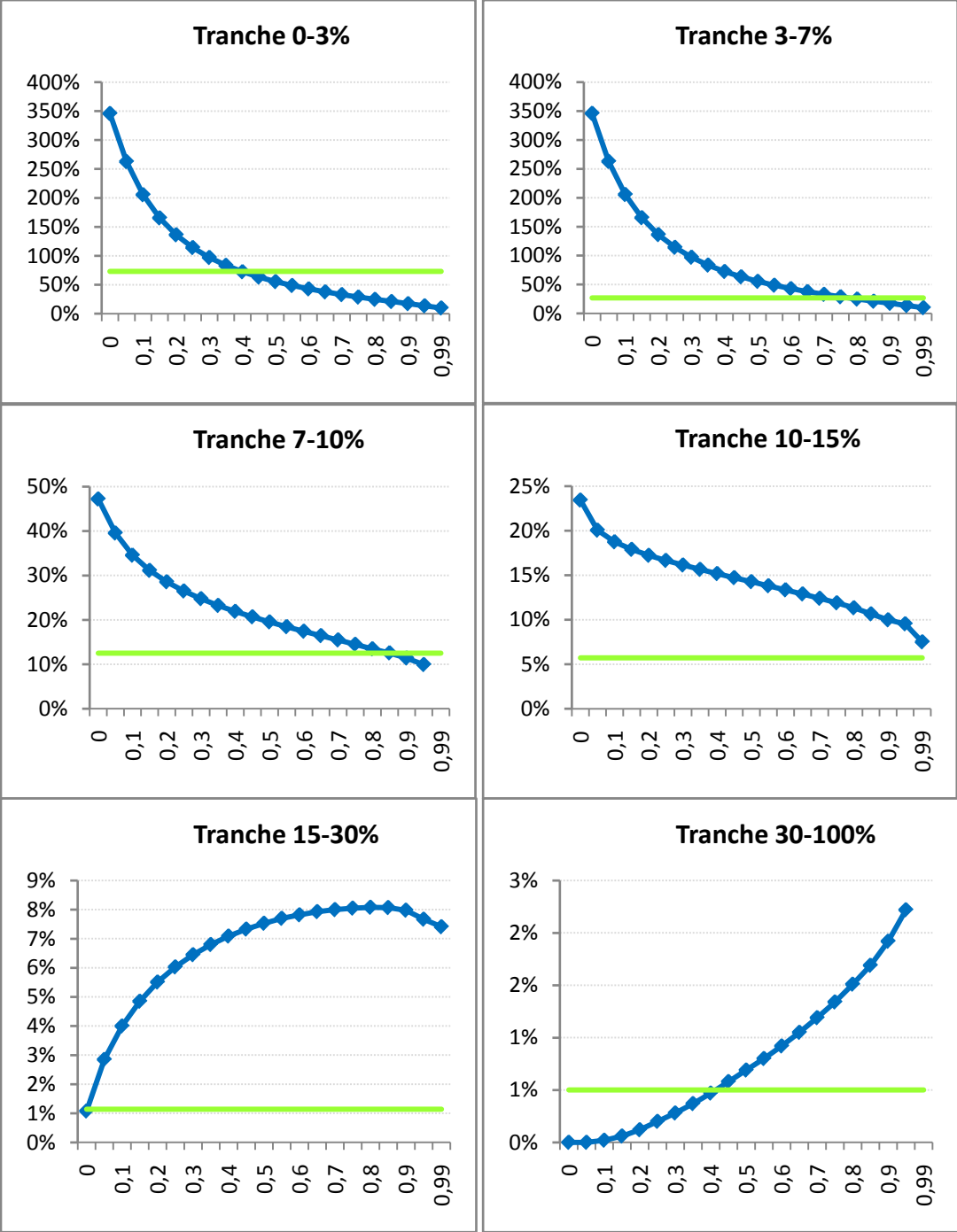
For the most junior tranche (0-3%) the premium is a decreasing function of correlation whereas for the most senior tranche the premium increases with correlation. The mezzanine tranches are less sensitive to correlation. Moreover, the relation between correlation and premium does not always have to be monotonic (see results for the 15-30% tranche). Higher correlation has a lower value for someone who buys protection on the equity tranche as he is

willing to pay less to the protection buyer. The opposite holds for the senior tranches investors for whom higher correlation has a higher value. To conclude, Hypothesis 1 cannot be rejected.

For proper CDO valuation we need to determine the level of correlation. First, we will use the implied correlation. See Table 1 for the results of the valuation based on different values of correlation. In the first row there is a market quote of CDX tranches on 28 February 2009. For the three lowest tranches the values displayed are already recalculated to the upfront payment quotes. Market quote of the tranche is also incorporated in Figure 7 by a light green line and its intersection point with the blue line determines the implied correlation.

Table 1: Results of valuation on 28 February 2009 with hazard rate of 0.07

	Tranche 0-3%	Tranche 3-7%	Tranche 7-10%	Tranche 10-15%	Tranche 15-30%	Tranche 30-100%
Market 28/2/2009	**81.72%**	**53.85%**	**22.97%**	**7.72%**	**1.14%**	**0.50%**
correlation						
0.00	98.02%	92.86%	86.68%	23.45%	**1.08%**	0.00%
0.05	97.46%	90.37%	75.91%	20.07%	2.85%	0.00%
0.10	96.59%	85.87%	67.40%	18.76%	4.00%	2.00%
0.15	95.21%	80.89%	61.18%	17.91%	4.85%	0.06%
0.20	93.28%	76.01%	56.34%	17.24%	5.51%	0.12%
0.25	90.88%	71.39%	52.34%	16.67%	6.03%	0.20%
0.30	88.08%	67.03%	44.91%	16.15%	6.45%	0.28%
0.35	84.95%	62.98%	45.88%	15.66%	6.80%	0.37%
0.40	**81.54%**	59.12%	43.12%	15.19%	7.09%	**0.47%**
0.45	77.87%	55.44%	40.56%	14.73%	7.33%	0.58%
0.50	73.98%	**51.89%**	38.15%	14.28%	7.53%	0.69%
0.55	69.87%	48.45%	35.84%	13.82%	7.70%	0.80%
0.60	65.53%	45.07%	33.59%	13.36%	7.82%	0.92%
0.65	60.97%	41.72%	31.36%	12.90%	7.93%	1.05%
0.70	56.15%	38.36%	29.13%	12.41%	8.00%	1.19%
0.75	51.02%	34.94%	26.79%	11.90%	8.05%	1.34%
0.80	45.52%	31.40%	24.34%	11.34%	8.08%	1.51%
0.85	39.46%	27.51%	**22.10%**	10.66%	8.07%	1.69%
0.90	32.68%	22.44%	18.82%	9.99%	7.98%	1.92%
0.95	24.52%	17.62%	15.12%	9.54%	7.67%	2.22%
0.99	15.30%	12.67%	8.71%	**7.51%**	7.41%	2.64%
Implied correlation	**0.4**	**0.5**	**0.85**	**0.99**	**0.0**	**0.4**

Table 1 displays that the correlation differs substantially among tranches. The difference between implied correlations is a usual outcome of CDO valuation models (Hull and White (2004) or Amato and Gyntelberg (2005)). This is sometimes called "correlation smile" and points to both imperfection of the model and the fact that market quotes comprise other factors that are not included in the model. In our case the variation is significant, what was due to the financial crisis and false market valuation.

Market conditions have changed substantially since 20 September 2007, when the CDOs trading was relatively low. Figure 8 compares implied correlation of tranches on 28 February 2009 and 20 September 2007 and demonstrates that implied correlation was more stable through tranches in   September 2007. Therefore we conclude that distressed markets and inappropriate valuation of tranches caused current huge variations of implied correlation among tranches in February 2009.

Figure 8: Implied correlation of tranches on 28 February 2009 and 20 September 2007
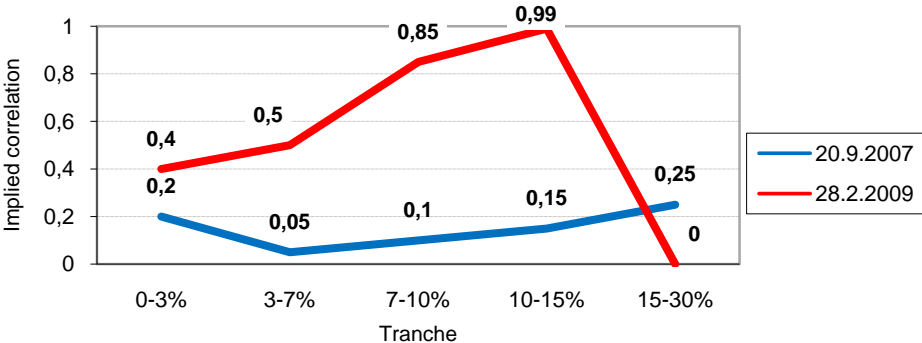

Hypothesis 2: Base correlation is more stable measure of correlation than implied correlation.

As Figure 9 depicts, base correlation recorded less variance than the implied correlation as of 28 February 2009. To illustrate this, suppose an investment in all tranches of a CDO totaling USD 1 million. The distribution of the notional among tranches is given by their attachment and detachment points (e.g. we invest USD 30,000 in equity tranche, USD 40,000 in 3-7% tranche etc.).

First we start with the equity tranche where the base correlation equals the implied correlation, then we need to evaluate the base correlation for the two lowest tranches. Taking correlation of 0.4 the present value of the 0-3% tranche equals 0 and present value of the 3-7% tranche is negative. Both these tranches' premiums are decreasing in correlation (see Figure 7). Therefore, we run the calculation with a higher value of correlation given market quotes increases the present value of both tranches. With the correlation of 0.44 and we obtain the present value of the 0-3% tranche worth USD +3,200 and the present value of the 3-7% tranche worth USD -3,200. Therefore 0.44 is the base correlation for the two lowest tranches.

Figure 9: Base and implied correlation for tranches on 28 February 2009
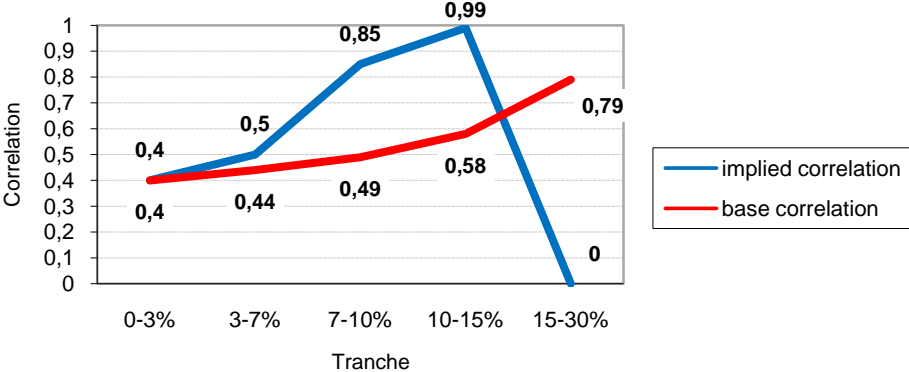

However, the concept of base correlation is not that intuitive as it seems to be. To illustrate it, we show how the correlation is appointed for the 15-30% tranche. Figure 7 shows that all first four tranches are decreasing in correlation; consequently the base correlation has

to increase while adding more senior tranches up to the detachment point 15%. Then, after arriving at correlation of 0.56, we add the 15-30% tranche which has a premium increasing in correlation. Now it depends on its implied correlation which determines its present value. Figure 9 depicts that as of 28 February 2009 the implied correlation is approaching 0 thus for the correlation of 0.56 the present value of the tranche is deeply negative. Decreasing correlation leads to an increase of the present value of the 15-30% tranche from negative values but also to a decrease of all more junior tranches' present values from positive values. On the other hand, increasing correlation would decrease present value of the 15-30% tranche to more negative values but it also increases the present value of other tranches. Now, it depends on the sensitivity of the tranches to correlation and on the width of the tranches which effect would be stronger and lead to a 0 sum of present values. Finally, we conclude that it would be the case of an increase in correlation. The base correlation is therefore monotonically increasing in correlation and more stable than the implied correlation. In conclusion, Hypothesis 2 cannot be rejected.

Hypothesis 3: Correlation and hazard rate changed substantially between 20 September 2007 and 28 February 2009

In case of a standard CDO, the most senior tranche (30-100%) is not sold to protection sellers and is usually retained by the issuer and therefore not included into our calculations. Figure 10 shows the evolution of base correlation during the financial crisis and clearly demonstrates that the base correlation changed in the observed period. As a result, the first part of Hypothesis 3 concerning the correlation cannot be rejected. Neither the second part of the hypothesis regarding the hazard rate can be rejected as detailed discussed in section 4.2.2.

Figure 10: Base correlation of tranches on 28 February 2009 and 20 September 2007
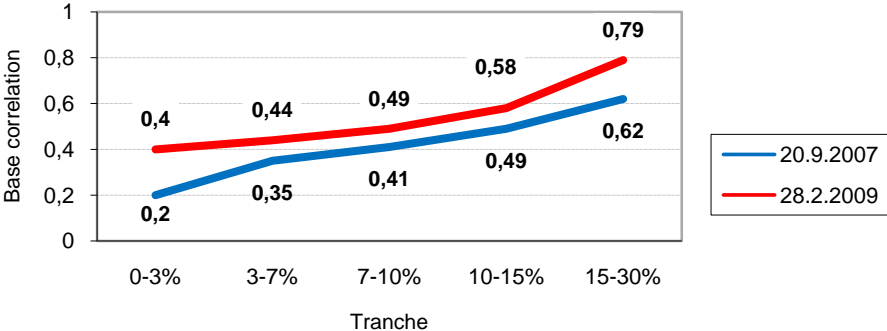
4.2.2. Hazard Rate and Tranche Premium Evaluation (Hypothesis 4)

Hypothesis 4: Higher hazard rate increases the premium of all tranches more than proportionally

Hazard rate for an asset is calculated from the CDO quote and recovery rate. Higher premium of a CDO implies to a higher credit risk of an asset. The premium of any CDO tranche based on a pool of assets also increases. Higher recovery rate adverts to lower loss given default and therefore the premium of a CDO tranche would be lower (Figure 11). We assume constant correlation of 0.4.

Figure 11: Tranche's premium with respect to a hazard rate on 28 February 2009
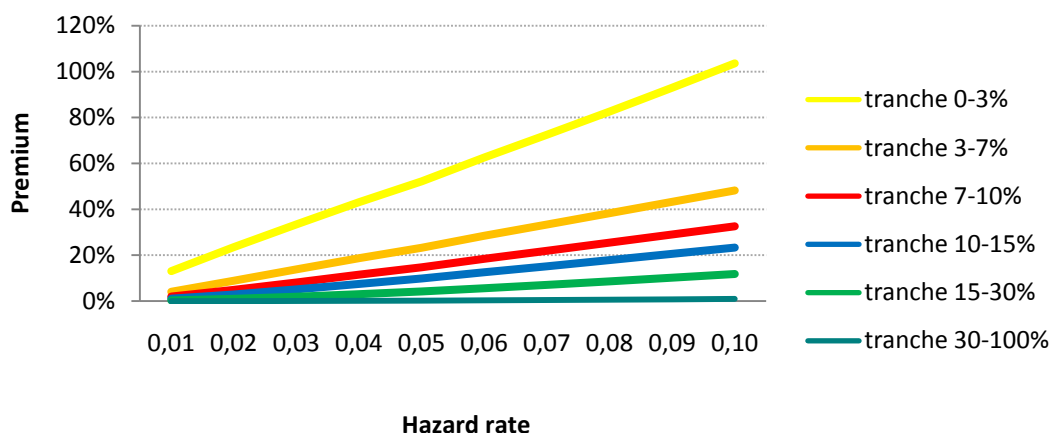

Figure 11 displays that the higher the seniority of the tranche, the more convex the relation between the premium and hazard rates. The higher the hazard rate, the higher the compensation in form of tranche's premium has to be to offset increased credit risk. Accordingly, the mark-to-market loss on the senior tranche in case of an increase of hazard rate has to be expected higher for higher starting level of the hazard rate. As a result, Hypothesis 4 cannot be rejected.

4.2.3 Loss Evaluation (Hypothesis 5)

Hypothesis 5: There has been a substantial loss even on the most senior tranche without a necessity to be hit directly by a default of this tranche.

We already mentioned that an expressive increase of credit risk during the financial upheaval. The premiums of the tranches often multiplied in the crisis even despite the fact that after each default the index was recalculated. In this part, we will transform the CDX to a CDO, i.e. we will abolish the feature of the new version of the index following each default. The loss after a default can be separated to three parts according to a consequence of: i) increased perception of credit risk; ii) new definition of tranche attachment and detachment point and iii) settlement of the defaulted asset. The first part points to a pure increase of fear on the market. Numerically it is expressed by an increase of hazard rate and correlation between assets. The second part of the loss is numerically expressed by a decrease of underlying assets with a fixed volume and decrease of attachment and detachment points' absolute values by the notional of the defaulted assets, both with fixed hazard rate and correlation. The third part of the loss bears only one tranche depending on sequences of the default.

In our calculation we suppose a CDO tranche buyer who entered the CDO contract on 20 September 2007 and hold the CDO until 28 February 2009. This implies that his CDO suffered three defaults during its life. Our task is to evaluate his loss on 28 February 2009 based on the difference between the premium he agreed and the current fair premium based

on expected cash-flows. First, we evaluate the CDO as of the issue date. Consequently, we evaluate it on the valuation date with new parameters (Table 2).

Table 2: Changes in parameters of the model between 28 February 2009 and 20 September 2007

	20.9.2007	28.2.2009
Correlation	0.32	0.44
Hazard rate	0.01	0.07
Number of assets	125	122
Notional invested	USD 100 mil.	USD 97.6 mil.
AP and DP		-2.4 percentage points

Note: AP = attachment point, DP = attachment point

The hazard rate was deduced from the CDO spreads of the underlying assets based on 0.39 recovery rate and it has increased seven times since autumn 2008. Therefore Hypothesis 3 cannot be rejected. The correlation is set as an average base correlation for three lowest tranches (Table 3).

Table 3: Mark-to-Market loss on a CDO tranche on 28/2/09 with 10 M USD initial investment
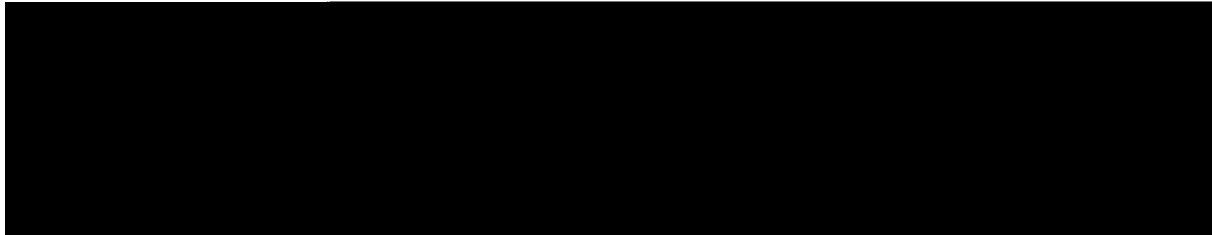
We should note that what we call the mark-to-market loss is in fact the loss based on changed values of expected cash-flows (i.e. the loss based on mark-to-market change of entry parameters, then the tranches are still valued by the model). The real mark-to-market loss would have to be derived from the market value of an instrument based on the fair value accounting principle. There are no available market data to a particular CDO but we can deduce from Figure 10 that this loss would be much higher. In fact, we might assume that such CDO contract would have to be terminated before our valuation date. The reason is that we did not fully included others risks such as liquidity risk and market sentiment (despite that these factors are partly included in the hazard rate).

The outcomes of our model using the expected cash-flows are alarming. The premium on the most senior tranche increased 44 times in the observed period, while the loss on this tranche amounted 26.50% of the notional amount. We should mention that these senior tranches usually got the highest possible rating scores indicating poor risk assessment of CDOs from rating agencies. As a consequence, Hypothesis 5 cannot be rejected.

Even though only the equity tranche investors were factually hit by the defaults, all tranches were hit indirectly - in form of mark-to-market losses - regardless of their rating. Although it is unlikable that more than 18 defaults occurred resulting in hit of the 15-30% tranche, its mark-to-market loss remained high between as of 26 February 2009 (Table 3).

Therefore, even if a financial institution (as the CDO investor) intended to hold the tranche until maturity, it should report a significant loss in its accounting.

4.3 Main flaws of the CDO market

As a consequence of the financial turmoil, financial institutions have suffered from massive writedowns (Figure 5). It is worthwhile to note that the absolute majority of these writedowns resulted from only seven credit events as of February 2009 (Washington Mutual Inc., Lehman Brothers Holdings Inc., Fannie Mae, Freddie Mac, Glitnir, Kaupthing and Landsbanki). After these credit events numerous downgrades by rating agencies followed. The agencies downgraded the tranches of CDOs whose underlying assets came from any of these seven companies (for example, Standard & Poor's downgraded 791 tranches of CDOs during one week in December 2008). As a result, the companies that held positions in these CDOs had to be downgraded too (for instance, AIG, MBIA or Ambac were downgraded due to CDS hedging their CDO positions losses). Frequently these companies were also included in CDOs and therefore caused further downgrades and mark-to-market losses. In consequence, the mortgage crisis was no doubt the trigger of the following complex credit crunch. Standard & Poors estimated that 3,000 CDO contracts were issued as of October 2008 (Thomson Reuters, 20081), while 75% of synthetic CDOs sold swaps on Lehman Brothers. Moreover, 376 contracts included Kaupthing, Glitnir or Landsbanki, 1,500 contracts incorporated Washington Mutual and 1,200 contracts encompassed both Fannie Mae and Freddie Mac. In Europe, 75% of all CDO deals contained at least one of the 7 defaulted companies, what implies that the CDO market lacked real diversification at that time. Non-diversified CDOs' portfolios, low cohesion of international financial markets together with the spiraling out of control of mark-to-market losses and downgrades resulted in disastrous consequences. In Figure 12 we suggest main weaknesses of the CDO market, their effects and lessons that should be learnt.

First, CDO investors did not undertake a deeper analysis of CDO underlying assets. However, in many cases the volume of outstanding bonds (included as underlying assets of a CDO) was much lower than the total volume of this bond included in this CDO. Rating agencies should have also reflected low diversification and the threat of CDO market breakdown after even a few defaults (due to advanced complexity of the CDO market).

Second, the valuation model comprehension was often incomplete. Neither the basic model introduced in Section 3 was comprehended by the investors as they relied on ratings and did not care why a bond rated AAA by S&P yielded less than a CDO tranche with the same rating. The model introduced by David X. Li is based on many simplifying assumptions. Since then papers were written to present more precise valuation models as explained in section 3.1., but no one really predicted or warned about the devastating results of an extreme case of a series of defaults that followed in 2008 after inclusion of not only a mathematical but also a market insight (e.g. the liquidity risk or increased risk aversion of the markets).

The valuation models are based on future expected cashflows (vs. default probabilities) and show the value for investors who hold a CDO to maturity and hence should not be valued on the mark-to-market principle. If an investor bought a senior tranche, even after three defaults the chance of being hit was still very low and his cashflow would remain unchanged and therefore the basic idea of the model was correct. However, the model did not take into account mark-to-market losses. This should have been understood by the majority of investors that have to disclose the mark-to-market value of their assets. Stress tests on changing input parameters – hazard rate and correlation – in combination with credit events should have been run (see above). Last but not least, the resulting losses based on model quotes can be only considered as the lower limit of losses, because the market quotes tend to overreact in bad

markets. This complex analysis would lead to a better risk assessment of a CDO and higher premiums required from a CDO seller.

Third, the correlation was obviously mispriced in the model. As explained in Section 4, both the implied and base correlations are derived from CDO market quotes. In the previous paragraph we argued that the tranches were mispriced and therefore neither the correlation value was correct. Only after the market proper valuation of a CDO the actual value of correlation can be derived. Technically, market participants underestimated the high level of correlation among global markets and the speed of the contagion to even unaffected markets, leading to a "whiplash" effect among the participants.

Finally, the mark-to-market valuation principle according to the US law should be reconsidered. After the mentioned defaults the CDO market froze and the quotes of tranches plummeted. However, all financial institutions still had to value their assets according to these market quotes, in spite of their intention to hold CDOs until maturity. As a consequence, this obligation has induced multiple losses. In October 2008 the Emergency Economic Stabilization Act (often referred to as a bailout of the US financial system) was pronounced in the US. Primarily, the Act set apart USD 700 billion for purchase of distressed assets and capital injection of the US banks. Additionally, it also allowed in some cases suspending the mark-to-market accounting. Instead, the value of a distressed asset can be derived from the expected value of cash-flows, i.e. it can be valued according to the model. As discussed in section 5.2.2, such valuation would cause huge losses after the default and change of input parameters but in a lesser extent compared to the use of distressed market quotes.

Figure 12: Main flaws of the CDO market

What went wrong?

	1.	2.	3.	4.
Mistakes	**Insuficient analysis of underlying assets**	**Misunderstanding of the valuation model**	**Mispriced correlation**	**Use of mark-to-market valuation principle**
Explanation	- low diversification of underlying assets - volume of CDOs issued on one bond exceeded the issued volume of the bond - few bonds included in a majority of CDOs	- model based on expected cashflows - mark-to-market loss not considered - reliance on the rating score without understanding the model	base and implied correlation derived from the market quotes which were set on lower than fair levels	obligation to mark assets to market quotes even if held to maturity
Effects	chain reaction of massive downgrades and losses after just one default	- huge mark-to-market losses triggered dy the downgrades - lower premiums required from investors: mispricing	biased CDO pricing	market quotes' overreaction and freeze of the CDO market resulted in multiplied writedowns
Lessons	- deeper analysis done by investors - better diversification - use of more complex methods by rating agencies	stress-testing: changing a hazard rate, correlation and number of defaults for a better appraisal of risks	correlation can be only priced fairly if market values fairly CDO tranches	introduction of a possibility to value assets by the expected cash-flows in some cases

5. Conclusion

The world CDO market has undoubtedly experienced a serious shock since late 2007. In this paper we research the main flaws of the CDO market that caused extensive writedowns from CDOs for many financial institutions worldwide. We present the One Factor Model based on a Gaussian Copula and develop a simple valuation program in VBA/MS Excel, in which we run simulations to test five hypothesizes. Based on the results we discovered four main deficiencies of the CDO market and make our recommendation for their elimination. Specifically, for our modelling we used the CDX index data from 20 September 2007 until 27 February 2009 and its quotes we appropriately transform into CDO quotes. Next, we run own valuation model with varying entry parameters to show the sensitivities of all tranches. Finally, we compare the model value of tranches before and after the crisis based on changed expected cash-flows made by CDO investors. We conclude that this loss constitutes a lower bound of real mark-to-market losses incurred by the investors.

The first identified deficiency was an insufficient analysis of underlying assets by both investors and rating agencies. The fact that seven financial institutions that defaulted since September to December 2008 were included in 75% of all European synthetic CDOs should be alarming. Such a poor diversification has resulted in chain reaction of losses and downgrades of institutions and CDO tranches after these few defaults. Therefore, a deeper analysis of diversification effects and quality of underlying assets should be implemented in the future. Low understanding of the valuation model caused the second deficiency. Since the structure and the valuation of a CDO remains quite sophisticated, investors relied on a high rating of senior CDO tranches without understanding the main underlying risks. The model was usually based only on expected cash-flows when neglecting other factors such mark-to-market losses or correlation risk. The possibility of mark-to-market losses of the tranches should have been included in the pricing. Results of stress-testing of tranches should increase the expected premium payments, what should better reflect higher credit risk involved. The third deficiency we found was the fact that also correlation was mispriced. Both implied and base correlation derives from the market quotes were artificially lowered by improper market optimism. Only after a deep understanding of the CDO valuation model the correlation should have been priced correctly. Finally, as we numerically demonstrated, the mark-to-market valuation obligation for financial institutions should be reviewed and it should be possible to back out of it in cases of a frozen market when risk premiums explode. Accordingly, the expected cash-flows valuation should be considered, especially if the instrument is held to maturity. Otherwise a next set of writedowns and downgrades may be triggered.

Based on the mentioned recommendations we conclude that the CDO market has a chance to be regenerated. However, that was not the case of past couple of years. The future CDO market would then be more conscious. The future CDO market would then be more conscious, driven by smarter motives rather than by poor understanding of risks involved in CDOs.

6. References

[1]   Amato, J. and Gyntelberg, J. (2005). "CDX Index Tranches and the Pricing of Credit Risk Correlations", *BIS Quarterly Review*, March 2005

[2]   Asset-Backed Alert (2009). "*The Weekly Update on Wordlwide Securitization*", 14 May 2009

[3]   Bhansali, V., Gingrich, R. and Longstaff, F. (2008). "Systemic credit risk: What is the market telling us?,"Working Paper, UCLA.

[4]   Fabozzi, F. J., Kothari, V. (2008). *Introduction to Securitization*, John Willey & Sons, Inc.: Singapore, ISBN 978-0-470-37190-9

[5]   Fabozzi, F. J. et al. (2008). *Subprime Mortgage Credit Derivatives*, John Willey & Sons, Inc., ISBN 978-0-470-24366-4

[6]   Gregory, J. and Laurent, J. (2004). "In the Core of Correlation", *Risk*, Vol. 17, No. 10, pp. 87-91, available on 13/5/2009 at http://www.defaultrisk.com/pp_corr_53.htm

[7]   Hull, J. and White, A. (2004). "Valuation of a CDO and an n-th to Default CDS Without Monte Carlo Simulation," *Journal of Derivatives*, Vol. 12, No. 2, pp. 8-23, available on 13/5/2009 at http://www.defaultrisk.com/pp_crdrv_14.htm

[8]   Kakodkar, A. et al. (2003). "Correlation Trading, A New Asset Class Emerges," Merrill Lynch, *Credit Derivatives Handbook*, 2003

[9]   Li, David X. (2000). "On Default Correlation: A Copula Function Approach", *Journal of Fixed Income*,   Vol.   9,   No.   4,   p.   43-54,   available   on   13.5.2009   at http://www.defaultrisk.com/pp_corr_05.htm

[10]   Longstaff, F. and Rajan, A. (2008). "An empirical analysis of the pricing of Collateralized Debt Obligations," *Journal of Finance*, 64, 529 - 563.

[11]   Markit (2008): "*Markit iTraxx Europe Indices, Series 10*", Markit Power Point presentation, available   on   27.2.2009   at   http://markit.com/en/products/data/indices/credit-and-loan-indices/itraxx/itraxx.page

[12]   Mejstrik, M., Pecena, M. and Teply, P. (2008). *Basic Principles of Banking*, Karolinum Press, Prague, Czech Republic.

[13]   Scheicher M. (2008). How has CDO market pricing changed during the turmoil? Evidence from CDS index tranches, BIS Working Papers No. 910, June 2008

[14]   Teply, P. and Cernohorsky, J. (2009). "Risk Management Lessons from The Current Financial Crisis", ISSN 978-80-7248-504-8, conference proceedings, *Silesian University in Karvina*, Czech Republic

[15]   Teply, P., Cernohorsky, J. and Cernohorska, L. (2009). "Strategic Implications of The 2008 Financial Crisis", ISSN 1947-2195, *Global Strategic Management, Inc.*, Michigan, USA

[16]   Teply, P. (2009). *Three essays on risk management and financial stability.* Dissertation thesis, Charles University in Prague, Czech Republic

[17]   Thomson Reuters (2008). "Credit News on 22 October 2008"

[18]   Tang, D. and Yan, H. (2007). "Liquidity and credit default swap spreads," *Working paper, Kennesaw State University and University of South Carolina*.

[19]   Wang, D., Rachev, S. T., Fabozzi, F. J. (2006): Pricing Tranches of a CDO and a CDS Index: Recent Advances and Future Research, *Working paper* available on 13.5.2009 at http://www.defaultrisk.com/pp_cdo_44.htm

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