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HUMAN CAPITAL ACCUMULATION AND INCOME DISTRIBUTION

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ABSTRACT

This paper analyzes the extent to which education will be subsidized when the subsidy rate is determined by majority voting. The analysis takes place in a framework where education is a discrete decision and all individuals would like to obtain an education because of its effect on future earnings. Individuals differ in their initial income levels. The non-existence of credit markets implies that initial income is a determinant of who actually obtains an education. We consider the outcome of a process in which income is taxed to provide subsidies for education, and taxes are chosen by majority voting. We characterize the outcome as a function of both the level and the distribution of income in the economy. In particular we derive conditions under which middle income individuals ally themselves with upper income individuals at the expense of lower income individuals, and vice versa. The analysis determines the relationship between human capital accumulation and distribution of income.

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1. Introduction

Societies intervene in the area of education in a variety of ways. That they should choose to do so is perhaps not surprising: plausible economic justifications for intervention are plentiful and range from the existence of market imperfections of various sorts (especially imperfect capital markets), to externalities from education both static and dynamic, to public goods arguments. The factors that determine the extent and the forms that these interventions take seem far less obvious, however. Heterogeneity among individuals, whether in terms of income, ability or locality, tends to generate conflicting preferences as to the kind of policies that are most desirable. Thus, there is reason to suspect that there may be substantial disagreement in the choice of, for example, the optimal degree of subsidization of education, the quality of education, the rules that should determine an individual's eligibility for particular subsidies (e.g. guaranteed student loans, scholarships, or financial aid), or the desirability of barriers to entry such as entrance examinations or enrollment restrictions. In the absence of a social planner that chooses policies to maximize a well-defined welfare function, an analysis is required to understand how heterogeneity and the political system interact to generate different features of the educational system.[1]

The focus of this paper is relatively narrow. We seek to examine how income heterogeneity in particular may affect the political feasibility of

[1]There is a growing literature that examines political forces as a determinant of economic outcomes. Early examples are Schumpeter (1947), Downs (1957), and Buchanan and Tullock (1962). More recent studies include Meltzer and Richard (1981), Alesina (1987), Persson and Tabellini (1990), and Fernandez and Rodrik (1991).

educational subsidies. To do so we deliberately simplify our framework, both economic and political, so as to highlight some of the fundamental tensions at work as a consequence of income heterogeneity and a system of majority rule.

At the economic level, individuals are assumed to be ex ante identical in every respect except for their initial income. Education is a discrete investment good and capital markets are imperfect. An individual's cost of acquiring an education is uniformly subsidized through a proportional income tax levied on the general public. The subsidy, however, is available only to those individuals who choose to acquire an education. Since we assume that all individuals would benefit from obtaining an education, an individual's income and the subsidized cost of education are the sole determinants of whether they will do so.

At the political level, we have chosen the equilibrium concept of majority rule, thus allocating each individual equal weight in determining the outcome. The tax rate, and hence the extent to which education is subsidized, is voted on directly and is therefore endogenously determined.

We derive the majority voting equilibrium of the economy. In equilibrium the degree of subsidization of education depends both on the wealth of the economy and on its distribution. Specifically, it depends on the economy's average income relative to the cost of education and on each income group's ability to extract revenue from another. The latter, we show, is determined by the relative income levels, by the allocation of the population among the different income groups, and by whether particular groups can afford to obtain an education at a zero subsidy.

Interpreting the results requires understanding how alliances are generated among income groups for different costs of education and different income distributions.[2] As will be shown, the middle class is often the pivotal group. There are cases, however, in which the poor and rich ally against the middle class, but these are relatively few for reasons that will be made clear in the paper. Loosely speaking, we find that the equilibrium tax rate tends to increase as an economy becomes wealthier as long as subsidization still plays a meaningful role (i.e. as long as some individuals still need a non-zero subsidy to afford an education). Interestingly, however, not all individuals necessarily prosper as society's wealth increases relative to the cost of education. The welfare of a poor individual conditional on a constant income, for example, is not a monotonically increasing function of society's average wealth. While in a poor society the equilibrium tax is zero, in a mediumly well-off one (in a sense that will be made precise in the paper) the tax rate is set at a level that ensures the maximum degree of exploitation of the poor, i.e. a tax rate which just prevents them from obtaining an education. In a richer economy with an equilibrium tax rate of one, they are best off. This contrasts in an interesting way with the results of Stiglitz (1974) who argued, for several variants of his model, that majority voting favors the prefences of the poor if the median income is less than the mean. This need not be the case in our framework.

Our paper is not, of course, the first to examine the determinants of an economy's degree of subsidization of education. Creedy and Francois (1990) examine an economy in which the growth rate is an increasing function of the

[2] Several authors have commented on and documented the tendency for the system of higher education financing to imply a transfer from the rich and the poor to the middle class (see e.g. Hansen and Weisbrod (1969), Hansen (1971), and Peltzman (1974)).

number of people educated. The authors assume that individuals are able to share in the benefits from growth regardless of whether they receive an education. Given income heterogeneity and complete capital markets they show that, for a particular income distribution, majority rule results in the subsidization of education despite the fact that the median voter is uneducated. G.E. Johnson (1984) provides a different motivation for why individuals who do not directly benefit from education may nonetheless wish to subsidize education. In this economy heterogeneity is in the level of skills possessed by a worker (high, medium, or low), rather than in initial income. Sufficient complementarity in the production function can provide low-skilled workers with an incentive to subsidize education for medium-skilled workers (which transforms the latter into high-skilled workers) although low-skilled workers themselves will not obtain an education.[3] Perotti's (1990) framework is perhaps the most similar to ours: capital markets are imperfect, education is the sole discrete choice variable, and majority vote determines the value of the tax rate. Taxation works differently than in our model, however. Tax revenue is redistributed independently of an individual's schooling decision, and there is also redistribution of second-period earnings. Second-period redistribution may induce the poor to subsidize education, even if only the rich individuals get educated.

The main feature that distinguishes our work from those discussed above is our assumption that individuals are unable to share in others' gains from

[3] In a rather different vein, K.E. Lommerud (1989) discusses the role of relative income concerns as an additional justification for the provision of educational subsidies and J.R. Lott (1990) suggests that the public provision of schooling is undertaken by the state since it lowers the opposition to wealth transfers by indoctrinating students with the "correct" set of beliefs.

education. It is not that we consider such factors as the overall benefits from growth (Creedy and Francois), complementarities in production (Johnson), and increased tax revenues for redistribution (Perotti), to be unimportant motivators for society's willingness to subsidize education. Rather, we wish to understand, in the absence of such considerations, which groups would form an alliance favoring subsidization of education and to what degree. Who benefits and at whose expense (if anyone's)? How does the degree of subsidization relate to the distribution of income among different groups? How is it affected by the size of these groups? How does it relate to the cost of education? In what way does the overall wealth of an economy play a role? These are some of the questions that we are able to examine in our model without the additional impetus favoring the subsidization of education that would be introduced by any of the other factors previously mentioned. The fundamental tensions that we identify in this basic model are likely to be present in models that incorporate additional factors in their analyses.

2. The Model

In order to study in as stark and simple a framework as possible some of the interactions between income distribution, the political system and education, we choose to abstract away from considerations that may be generated by other factors such as income smoothing concerns, intergenerational bequest motivations, heterogeneity in preferences and abilities among individuals, etc.. We place emphasis instead on the affordability of education in an economy in which each individual would benefit by acquiring an education.[4]

The economy consists of a continuum of two-period lived agents with total mass equal to one. There is a single consumption good and individuals have a linear utility function defined over first and second-period consumption. There is no discounting. The agents belong to one of three groups, differentiated by their initial income (equivalently, endowment of the consumption good) which is assumed to take on the values y_1, y_2, or y_3.[5] We assume that $y_1 > y_2 > y_3$, and will often refer to the three groups of agents as rich, middle class, and poor respectively. The fraction of agents in group i is written as λ_i.

In the first period of her life, each agent decides whether to obtain an education. The choice is zero-one and the cost of obtaining an education (with zero subsidy) is E.[6] The benefit from education for an individual from group i is that second-period income equals $f(y_i)$. By contrast, an individual who does not obtain an education in the first period receives a second-period income equal to y_i. We assume that $f(y_i)-E>y_i$ for all i. This ensures that all individuals would like to obtain an education.

The market structure that we consider, however, does not necessarily permit all individuals to obtain an education. Individuals are assumed not to

[4]Affordability is not generally an issue at the primary and secondary level of education. Thus, this model is perhaps best viewed as concerning the subsidization of higher education.

[5]We have chosen to assume only three types of agents since it allows for closed form solutions and highlights the nature of the tensions among income groups.

[6]We have deliberately chosen to model the acquisition of education as a discrete choice. In terms of our results, what matters is that some individuals should find themselves at a corner with respect to their choice to invest in education.

have access to credit markets and hence cannot borrow against future earnings to finance expenditures on education when young.[7] It follows that period-one income will be a determinant of whether an individual obtains an education.[8]

A second factor that determines whether a given individual will receive an education is the extent to which education is subsidized. In our model education is (endogenously) a partially publicly provided private good that is subsidized solely via a proportional tax θ on period-one income.[9] The proceeds from taxation are distributed equally among all individuals that receive an education.

We study equilibria for the above economy in which the choice of the tax rate θ is endogenously determined by majority rule. Thus, by voting upon a tax rate individuals are simultaneously deciding two things: 1) the height of the "entry barrier" to education, i.e. the identity of the individuals who can afford to get educated, and 2) the degree to which educated individuals can extract revenue from those individuals who are not educated (and hence who do not receive a benefit from taxation). Of course, these two are simultaneously determined and thus cannot be chosen independently of one another. The barrier to entry aspect is central to our analysis and, in particular,

[7] Recent empirical work by Behrman, Pollak and Taubman (1989) finds evidence for the existence of credit constraints in the context of education. We do not model here the particular microfoundations underlying this market failure. This would merely complicate the model and its exact features are not critical. The essential feature is not that individuals cannot borrow at all, but rather that access to credit markets is such that initial income remains a determinant in an individual's decision to acquire education.

[8] Our specification implies that individuals may spend all their income on education. This could be generalized to include expenditures on other goods. What matters is that for some individuals income is a binding constraint on their purchase of education.

[9] Thus we are implicitly constraining our system from resorting to lump-sum taxation and other schemes.

distinguishes it from the bulk of voting models on education in which tax revenue is redistributed independently of an individual's education decision (e.g. Stiglitz (1974) and Perotti (1990)).

Before proceeding to the equilibrium analysis it is useful to analyze the relationship between values of the tax rate, individual actions and utilities. Consider first the relationship between the tax rate, the government subsidy to education and the fraction of the population that receives an education. With a tax rate equal to θ, tax revenues $T(\theta)$ are given by:

$$T(\theta) = \theta\sum\lambda_i y_i = \theta\mu \tag{1}$$

where μ is average income (which also equals total income since the mass of agents is one). If $N(\theta)$ represents the mass of agents who receive an education, then the per person subsidy $s(\theta)$ is given by

$$s(\theta) = \theta\mu/N(\theta). \tag{2}$$

The difficulty with these expressions is that N and s are jointly determined by θ. A simple procedure, however, allows us to determine the values of s and N that are mutually consistent.

Let $\rho_i(\theta)$ be the fraction of individuals of type i that receive an education as a function of θ. For a given value of $s(\theta)$, an individual of type i can obtain an education if $(1-\theta)y_i - E + s(\theta) \geq 0$. Clearly, if an individual from group i can afford to be educated then so can all individuals from group j for all $j < i$. Consider the following expression for any fixed value of θ:

$$(1-\theta)y_j - E + \theta\mu/((\sum_{i<j}\lambda_i)+v\lambda_j) \tag{3}$$

First, set j=1 and v=1. If expression (3) is nonnegative, then $\rho_1(\theta)=1$. Otherwise, at the tax rate of θ it is not possible for all of group one to be educated and $\rho_1(\theta)$ is that value of v that sets (3) equal to zero. In this case $\rho_2(\theta)$ and $\rho_3(\theta)$ are equal to zero. If $\rho_1(\theta)=1$, this procedure is repeated for j=2. If $\rho_2(\theta)=1$, then it is repeated for j=3. This determines the values of $\rho_i(\theta)$ for any value of θ. Whenever $0<\rho_i(\theta)<1$, we assume that a fraction $\rho_i(\theta)$ of agents from group i is randomly selected to obtain an education and that the remaining fraction $1-\rho_i(\theta)$ does not.

Having determined the values of the ρ_i's, it is possible to express the expected utilities of each of the three groups as a function of the tax rate θ:

(4) $$EU_i(\theta)= (1-\theta)y_i + \rho_i(\theta)[s(\theta) - E + f(y_i)] + (1-\rho_i(\theta))y_i.$$

Since agents in this economy vote on the value of the tax rate to be instituted, it is essential to understand how their respective utilites are affected by different values of θ. A few preliminary results are helpful. First note that each of the functions $\rho_i(\theta)$ is non-decreasing in θ. Second, and following directly from the algorithm outlined above, if $0<\rho_i(\theta)<1$ the following relationship holds:

(5) $$E - s(\theta) = (1-\theta)y_i.$$

Some additional notation facilitates the characterization of the EU_i's. Let $\hat{\theta}_2$ be the maximum value of θ in [0,1] for which $\rho_2(\theta)$ is equal to zero or, this value failing to exist, equal to 0. $\hat{\theta}_2$ is, therefore, the value of θ at which for any strictly positive increase in the tax rate it becomes possible for some individuals of type two to obtain an education. Clearly, if $y_2\geq E$ then $\hat{\theta}_2$ equals zero. $\hat{\theta}_3$ is defined analogously. Lastly, define $\bar{\theta}_1$ to be the smallest value of θ for which $\rho_1(\theta) = 1$, whenever such a number exists in the unit interval.

The following proposition characterizes the expected utilities for each type of agent.

Proposition 1: $EU_i(\theta)$ is continuous and $EU_i(0) < EU_i(\bar{\theta}_i)$ for $\bar{\theta}_i \varepsilon (0,1]$ and for all i. Furthermore:

(i) $EU_1(\theta)$ is increasing and concave on $[0,\bar{\theta}_1]$, linearly increasing on $[\bar{\theta}_1,\hat{\theta}_2]$ with marginal utility of $(\mu/\lambda_1)-y_1$, linearly decreasing on $[\hat{\theta}_2,\bar{\theta}_2]$ with marginal utility y_2-y_1, linear on $[\bar{\theta}_2,\hat{\theta}_3]$ with marginal utility of $[\mu/(\lambda_1+\lambda_2)]-y_1$, linearly decreasing on $[\hat{\theta}_3,\bar{\theta}_3]$ with marginal utility y_3-y_1, and linearly decreasing on $[\bar{\theta}_3,1]$ with marginal utility $\mu-y_1$.

(ii) $EU_2(\theta)$ is linearly decreasing on $[0,\hat{\theta}_2]$ with marginal utility of $-y_2$, increasing and concave on $[\hat{\theta}_2,\bar{\theta}_2]$, linearly increasing on $[\bar{\theta}_2,\hat{\theta}_3]$ with marginal utility of $(\mu/(\lambda_1+\lambda_2])-y_2$, linearly decreasing on $[\hat{\theta}_3,\bar{\theta}_3]$ with marginal utility of y_3-y_2, and linear on $[\bar{\theta}_3,1]$ with a marginal utility of $\mu-y_2$.

(iii) $EU_3(\theta)$ is decreasing on $[0,\hat{\theta}_3]$ with marginal utility of $-y_3$, increasing and concave on $[\hat{\theta}_3,\bar{\theta}_3]$, and linearly increasing on $[\bar{\theta}_3,1]$ with marginal utility of $\mu-y_3$.

Proof: Continuity of the EU_i's follows directly from the fact that the $\rho_i(\theta)$'s are continuous. At $\bar{\theta}_i$, $EU_i=f(y_i)$. At $\theta=0$, $EU_i=2y_i$. Given $f(y_i)-E > y_i$ and $y_i<E$ (i.e. $\bar{\theta}_i>0$) it follows that $f(y_i)>2y_i$ and hence that $EU_i(0)<EU_i(\bar{\theta}_i)$ for $\bar{\theta}_i \varepsilon (0,1]$.

We prove (i); the remaining statements can be demonstrated similarly. On $[0,\bar{\theta}_1]$, $EU_1(\theta)$ is given by $EU_1(\theta) = \rho_1(\theta)[f(y_1)] + (1-\rho_1(\theta))[(1-\theta)y_1+y_1]$ where $\rho_1(\theta) = \theta\mu/[\lambda_1(E-(1-\theta)y_1)]$. Note that $EU_1(\bar{\theta}_1)>EU_1(\theta)$ for all $\theta\varepsilon[0,\bar{\theta}_1)$. Calculation yields $dEU_1/d\theta|_{\theta=0} = [\mu(f(y_1)-2y_1)-\lambda_1y_1(E-y_1)]/((E-y_1)\lambda_1)$ which is positive since $f(y_1)> E+y_1 > 2y_1$ (if $\bar{\theta}_1 >0$) and $\mu>\lambda_1y_1$. Furthermore, $d^2EU_1(\theta)/d\theta^2 = -2\mu(E-y_1)y_1[f(y_1)-y_1-E]/[E-(1-\theta)y_1]^3\lambda_1 < 0$. Since EU_1 is increasing at zero, is concave throughout, and $EU_1(\bar{\theta}_1) > EU_1(\theta)$ for all θ ε

$[0,\bar{\theta}_1)$, it follows that EU_1 is increasing on the interval $(0,\bar{\theta}_1)$. On the interval $[\bar{\theta}_1,\hat{\theta}_2]$, EU_1 is given by $EU_1(\theta) = (1-\theta)y_1 - E + (\mu\theta/\lambda_1) + f(y_1)$. Differentiation gives: $dEU_1/d\theta = -y_1 + \mu/\lambda_1 > 0$. On the interval $[\hat{\theta}_2,\bar{\theta}_2]$, $EU_1(\theta)$ is given by $EU_1(\theta) = (1-\theta)y_1 - (1-\theta)y_2 + f(y_1)$, since for $0<\rho_2(\theta)<1$, $s(\theta) = E - (1-\theta)y_2$. Differentiation gives $dEU_1/d\theta = y_2-y_1 < 0$. On the interval $[\bar{\theta}_2,\hat{\theta}_3]$, EU_1 is given by $EU_1(\theta) = (1-\theta)y_1 - E + (\mu\theta/(\lambda_1+\lambda_2)) + f(y_1)$. Differentiation gives $dEU_1/d\theta = -y_1 + \mu/(\lambda_1+\lambda_2)$. Marginal utility in this region is positive if $y_1<\mu/(\lambda_1+\lambda_2)$ and negative if the reverse inequality holds. In the interval $[\hat{\theta}_3,\bar{\theta}_3]$ we have $EU_1(\theta) = (1-\theta)y_1-(1-\theta)y_3+f(y_1)$. Differentiation yields $dEU_1/d\theta = -y_1+y_3 < 0$. Finally, if θ lies in the interval $[\bar{\theta}_3,1]$, EU_1 is given by $EU_1(\theta) = (1-\theta)y_1-E+\mu\theta$. Differentiation gives $dEU_1/d\theta = -y_1+\mu$. This is negative since μ is simply the average of the y_i's. This completes the proof of (i). Parts (ii) and (iii) are demonstrated similarly.||

Figure 1 depicts EU_1 as a function of θ for a particular set of parameter values.

3. Majority Voting Equilibrium

The equilibrium is determined by majority voting on tax rates. We assume that individuals vote sincerely.

<u>Definition</u>: An equilibrium is a tax rate θ^*, $0 \le \theta^* \le 1$ such that for all $\theta'\varepsilon[0,1]$, the mass of agents with $EU_i(\theta^*) \ge EU_i(\theta')$ is strictly greater than .5.

Generically, each of the EU_i's has a unique maximizer on [0,1]. The discussion that follows assumes that the maximizers are unique, although the case where uniqueness does not obtain is easily handled. Denote the maximizer for group 1 by $\tilde{\theta}_1$. Note that uniqueness does not imply that individual preferences are single peaked; this is generically not true in this model (e.g. see Figure 1). Thus it is not possible to simply invoke the preferred tax rate of the median voter as the equilibrium tax rate.

As a first step in the characterization, note that $\tilde{\theta}_1$ necessarily corresponds to a local maximum of $EU_1(\theta)$, and hence Proposition 1 can be used to restrict the set of possible values of $\tilde{\theta}_1$. The possible values for $\tilde{\theta}_1$ are $\{\hat{\theta}_2, \hat{\theta}_3\}$, for $\tilde{\theta}_2$ are $\{0, \hat{\theta}_3, 1\}$, and for $\tilde{\theta}_3$ are $\{0, 1\}$. For all groups $EU_1(\bar{\theta}_1) >$ $EU_1(0)$ (for $\bar{\theta}_1 \varepsilon (0,1]$), so zero can be a global maximum for an individual of group 1 only if it is not feasible for all individuals in that group to obtain an education at any tax rate. Also, it is possible for a tax rate of one to be a global maximum for group one, but only if $\hat{\theta}_3=1$, i.e. only if there is no tax rate at which any individual of group three can obtain an education.

The following proposition helps establish which value is taken by $\tilde{\theta}_1$.

<u>Proposition 2</u>: Assume $y_2<E$. Then $EU_1(0) > EU_1(\bar{\theta}_2)$ if and only if $y_1 > E$ and $\mu/(\lambda_1+\lambda_2) < y_1$.

<u>Proof</u>: If $E>y_1$ then $EU_1(\theta)>EU_1(0)$ for $0<\theta\le 1$. If $E\le y_1$ then $EU_1(0)=f(y_1)+y_1-E$, and $EU_1(\bar{\theta}_2) = f(y_1) + (1-\bar{\theta}_2)y_1 + \bar{\theta}_2\mu/(\lambda_1+\lambda_2) - E$. Hence $EU_1(0)-EU_1(\bar{\theta}_2) = \bar{\theta}_2[y_1-\mu/(\lambda_1+\lambda_2))] \gtrless 0$ as $y_1 \gtrless \mu/(\lambda_1+\lambda_2)$. ||

Note that the condition above is identical to the one that determines whether $EU_1(\theta)$ is increasing on the interval $[\bar{\theta}_2, \hat{\theta}_3]$. The intuition underlying the proposition is easily understood. The net subsidy obtained by individuals in group one for a given θ is $\theta[(\mu/N(\theta))-y_1]$. At $\bar{\theta}_2$, $N(\bar{\theta}_2)=\lambda_1+\lambda_2$. Hence, for $\theta \in [\bar{\theta}_2, \hat{\theta}_3]$, the net subsidy will either monotonically increase or decrease depending upon whether $\mu/(\lambda_1+\lambda_2] \gtrless y_1$. The net subsidy at $\theta=0$ is 0. Thus, the sole determinant of whether $EU_1(0)$ is greater than $EU_1(\bar{\theta}_2]$ (given

$y_1 > E$) is likewise whether the net subsidy at $\bar{\theta}_2$ is positive or negative, i.e. the same condition as before. A necessary condition, therefore, for $\tilde{\theta}_1$ to equal $\hat{\theta}_3$ rather than $\hat{\theta}_2$ is $\mu/(\lambda_1+\lambda_2)>y_1$.

There is one case for which the equilibrium is immediate. If any λ_i is at least as great as .5, then this group's $\tilde{\theta}_i$ is clearly a majority voting equilibrium. The following analysis considers the case where λ_i < .5 for each i. Thus, the sum of any two of the λ_i's exceeds .5.

For a tax rate θ^* to be a majority voting equilibrium it must win against all alternatives, and, in particular, against all local alternatives. This observation leads to the following result:

<u>Theorem 1</u>: If θ^* is a majority voting equilibrium then at least one of the EU_i's has a local maximum at θ^*.

<u>Proof</u>: Assume that no group has a local maximum at θ^*. If θ^* equals zero, $EU_i(\theta)$ must be upward sloping at 0 for all i. But then there exists some $\theta >$ 0 which all three groups prefer to 0. If θ^* equals one, $EU_i(\theta)$ must be decreasing for all i as θ approaches 1. Again, there is some $\theta < 1$ which is preferred by all three groups. Now assume that $0 < \theta^* < 1$. Because θ^* is not a local maximum for any of the i's, either a small decrease or increase in θ must increase utility for at least two of the groups, which is sufficient to rule out θ^* as a majority voting equilibrium. ||

This theorem establishes that the potential majority voting equilibria all lie in the set $\{0, \hat{\theta}_2, \hat{\theta}_3, 1\}$. This set can be further reduced, however, by noting that an implication of Proposition 1 is that both groups two and three strictly prefer a tax rate of zero to a tax rate of $\hat{\theta}_2$ (for $\hat{\theta}_2$ not equal to zero). This follows directly from the fact that at $\hat{\theta}_2$ both groups two and three pay taxes but do not receive an education. This leaves $\{0, \hat{\theta}_3, 1\}$ as the only equilibrium candidates. In particular, there is only one possible interior equilibrium.[10]

The preceding propositions and theorem allow us to construct Table 1, which characterizes the equilibria that can result as a function of the values of the parameters of the economy.

As can be seen readily from Table 1, there are several cases where a majority voting equilibrium does not exist. In the present context this is perhaps not too surprising. Proposition 1 implies that preferences are typically not single peaked and, as is well known, in this case a majority voting equilibrium may fail to exist.[11]

The fact that preferences are not single peaked is due entirely to the discrete nature of the education decision. Although the resulting non-existence of equilibrium is unattractive, we believe that discreteness is inherent to the formulation of the question we study and not a feature to be assumed away. The resulting problem of non-existence has been dealt with by the majority voting literature in two related but distinct ways. One has been to impose a greater institutional structure on the collective choice mechanism than is implicit in the majority voting concept (see e.g. Shepsle and Weingast (1987)). The other is to propose criteria for what constitutes a "reasonable" set of outcomes without specifying particular rules to pick out one or more of the elements of this set (see, for example, the concept of uncovered set in McKelvey (1986) and Miller (1980)). Given the complexity and

[10] The existence of a sole interior equilibrium is an artifact of the three income group distribution; an entire range of interior equilibria can be obtained in the case of many discrete income groups.

[11] For an example in Table 1 see case 3Aa. Given the profile of preferred tax rates $(\hat{\theta}_2, \hat{\theta}_3, 0)$, $\hat{\theta}_2$ beats $\hat{\theta}_3$, 0 beats $\hat{\theta}_2$, and $\hat{\theta}_3$ beats 0.

diversity of the institutions involved in the educational system, and the potential arbitrariness of any particular set of rules to pick an equilibrium, our discussion will focus on the implications of majority voting for resolution of the educational subsidy problem. These outcomes provide a benchmark against which other procedural outcomes can be compared.

4. Equilibrium Outcomes

Table 1 lists the equilibrium outcomes for all configurations of the parameters describing the economy. This information can be used to address two issues. One is to examine how an economy's total income relative to the cost of education affects the choice of subsidization, holding the λ_i's constant. The second is how changes in the distribution of income affect this choice. Addressing each of these in turn serves as a useful way to organize the implications of the model.

To examine the relationship between total income and the equilibrium subsidy, consider the patterns that emerge as we move from panel 1 to panel 4. This movement corresponds to increases in total income relative to the cost of education. In panel 1 the economy is sufficiently poor that even if it devoted all of its resources to subsidizing education, at most only all the rich would be able to attend. Consequently, both group two and three's preferred tax rate is zero which is then the majority voting equilibrium.

In panel 2 the economy is wealthier, although total income does not yet permit any group 3 individual to attend school even were society to devote all of its resources to education. This situation is potentially more interesting than that of panel 1; it is now economically feasible for the middle class to share in the benefits of taxation. One may, therefore, be inclined to believe that a positive rate of subsidization will be chosen. As Table 1 shows, however, no such majority voting equilibrium will emerge; the only possible majority voting equilibrium is zero. Should y_2 individuals desire a positive tax rate, this rate must be strictly greater than $\hat{\theta}_2$ since at this rate the middle class is worst off. (At $\hat{\theta}_2$ the exploitation of the middle class by the rich is at its maximum since this is the largest tax rate compatible with no middle class individual obtaining an education.) But, there is always a lower tax rate (e.g. $\hat{\theta}_2$ itself) which is preferred by both y_1 and y_3 since y_1's preferred tax rate is $\hat{\theta}_2$ and EU_3 is monotonically decreasing on [0,1].

In panel 3 the economy has enough income to send the rich, the middle class, and at least some of the poor to school. In the extreme case it is economically feasible to provide education for all individuals, though this option would exhaust all income. The main feature of the economy is that in equilibrium no group three individual obtains an education. There are many subcases in panel 3. These correspond to two divisions: (1) whether group two individuals can afford an education independently of the value of the subsidy and (2) whether y_1 is greater or smaller than $\mu/(\lambda_1+\lambda_2)$ (for the significance of this inequality, see Proposition 2). Group two's preferred tax rate is always $\hat{\theta}_3$. In 3A, with one possible exception discussed later, whenever an equilibrium exists it is $\hat{\theta}_3$ and supported by an alliance of the rich and middle class. The rich switch between $\hat{\theta}_3$ and $\hat{\theta}_2$, generating an equilibrium of $\hat{\theta}_3$ or nonexistence respectively. In B, the rich prefer either $\hat{\theta}_3$ or zero and, unlike in any of the cases discussed before, an alliance between the rich and poor is feasible. The reason for this alliance in case Bb is that there is not enough income to ensure that a sufficiently large number of the poor get educated and the rich are made worse off by any strictly positive tax rate (since all revenue must be shared with the middle class and $\mu/(\lambda_1+\lambda_2)<y_1$). This yields zero as the equilibrium.

Panel 4 covers the case of economies with $\mu>E$, i.e., economies with sufficient income to educate everyone and still have resources left over. There are also many subcases in panel 4. These correspond to the following divisions: (1) whether group two or group three individuals can afford an education independently of the value of the subsidy, (2) whether y_1 is greater or smaller than $\mu/(\lambda_1+\lambda_2)$ and (3) whether the median income (y_2) is greater or smaller than the mean. In case C the poor can afford an education independently of any subsidy. Taxation, therefore, is simply a means of redistributing income among all members of society; the barrier to entry aspect no longer plays a role in generating preferences over taxes. The main feature of our model effectively disappears: preferences are single peaked and the median voter result applies. Consequently the equilibrium tax rate is either zero or one according to whether y_2 is greater or smaller than μ. Henceforth our discussion will focus on cases A and B.

Whenever $\tilde{\theta}_2=1$, it is the equilibrium since one is always the preferred tax for group 3. Group 2, however, may find it profitable to restrict education to y_1 and y_2 individuals (i.e. $\tilde{\theta}_2=\hat{\theta}_3$). If it does, the equilibrium (when it exists) is either $\hat{\theta}_3$ or 0. It is $\hat{\theta}_3$ when the rich and the middle class both find it in their best interests to tax so as to extract as much revenue as possible from y_3 without allowing the latter to obtain an education.

In panel 4, whenever an equilibrium exists it is the preferred tax rate of the middle class (with one exception, discussed later). If $y_2<E$ or $\mu>y_2\geq E>y_3$ and $\mu/(\lambda_1+\lambda_2)>y_1$ the preferred tax rate of the middle class may be either $\hat{\theta}_3$ or 1. Because these two outcomes have very different implications for the resulting pattern of educational attainment, it is of interest to ask what factors influence the value of $\tilde{\theta}_2$ in these cases. We can express the difference between the two levels of utility as:

(6) $EU_2(1)-EU_2(\hat{\theta}_3) = (\mu-y_2) - (E-y_3)(\mu-(\lambda_1+\lambda_2)y_2)/(\mu-(\lambda_1+\lambda_2)y_3)$.

Note first that if $\mu-y_2 > E-y_3$, then $\tilde{\theta}_2=1$ since $[\mu-(\lambda_1+\lambda_2)y_2]/[\mu-(\lambda_1+\lambda_2)y_3]<1$. Similarly, if $\mu-y_2 < E-y_3$ and $y_3\mu > y_2E$ then $\tilde{\theta}_2 = \hat{\theta}_3$.

Differentiation and some algebraic manipulation can be used to show the following:

<u>Proposition 3:</u> The difference in expected utilities in (6) is:

(a) decreasing in E

(b) increasing in λ_1 holding λ_3 constant

(c) increasing in λ_1 holding λ_2 constant if $y_1>2y_3$

(d) increasing in λ_2 holding λ_1 constant if $y_2>2y_3$

(e) increasing in y_1

(f) increasing in y_3

(g) ambiguous with respect to y_2.

To provide intuition for some of the results, consider what happens when the cost of education increases. At a tax rate of 1, the amount transferred from the rich to the middle class is unchanged (though E has gone up, this is also true at $\hat{\theta}_3$). $\hat{\theta}_3$, on the other hand, has increased since y_3 individuals now need a larger subsidy in order to afford an education. This allows y_1 and y_2 individuals to "exploit" y_3 individuals more than before since it takes a greater tax rate than previously for y_3 individuals to be able to afford an education. Hence, an increase in E makes $\hat{\theta}_3$ relatively more attractive than one.

When the proportion of the population that is rich increases at the expense of the middle class, the economics underlying the relative attractiveness of 1 versus $\hat{\theta}_3$ is more subtle. The increase in μ brought on by this change serves to make a tax rate of 1 relatively more attractive since the net subsidy received by a y_2 individual is greater than before. The increase in μ also decreases $\hat{\theta}_3$ since, for the same tax rate as before, more revenue is generated. Recalling expression (5) it is easy to show that the net subsidy at $\hat{\theta}_3$ decreases. These two effects unambiguously make one more attractive relative to $\hat{\theta}_3$. Similarly, when either y_1 or y_3 increase, μ increases and $\hat{\theta}_3$ falls. Again it can be shown unambiguously that this change relatively favors a tax rate of 1 over a tax rate of $\hat{\theta}_3$.

In the preceding discussion the analysis was restricted to those cases in which the equilibrium was the preferred tax rate for two groups. There are four exceptions to this. When $\mu/\lambda_1 > E \geq \mu/(\lambda_1+\lambda_2)$ and $y_1>E$ (panel 2), then $\tilde{\theta}_1=\hat{\theta}_2$, $\tilde{\theta}_2=1$, and $\tilde{\theta}_3=0$. In this case an equilibrium may exist at $\theta=0$, the preferred tax rate for a member of group three, since there may not exist any tax rate that both group one and two both prefer to zero. There is no coalition that is able to block $\theta=0$ in favor of some other tax rate. This is also possible in the case 3Ab where $y_1\geq\mu/(\lambda_1+\lambda_2)>E\geq\mu$, and $y_2<E$. Lastly, in cases 3Bb and 4Bbii where $y_1\geq\mu/(\lambda_1+\lambda_2)>y_2>\mu>E>y_3$, we have $\tilde{\theta}_1=0$, $\tilde{\theta}_2=\hat{\theta}_3$, and $\tilde{\theta}_3=1$. Zero, group one's preferred tax rate, cannot be ruled out as an equilibrium.

5. Discussion

Having discussed the outcomes presented in Table 1 in some detail, it is instructive to note some general points that arise in the analysis.

1. There is no simple ranking of preferred tax rates among groups, e.g. it is not the case that the rich are typically for low taxes and the poor for high taxes, as would be the case if individuals voted over pure redistribution schemes rather than the subsidization of education.

2. A majority voting equilibrium in this framework is essentially a coalition between (at least) two of the three groups. The alliance that emerges is a function of the total income in the economy, its distribution, and the cost of education. In the poorest economies (panels 1 and 2), the poor and middle classes form an alliance. In panel 3, however, the alliance first switches to the rich and middle class but, as the economy becomes even wealthier (3Bb), the rich may form an alliance with the poor. In panel 4 the alliances are either between the middle class and the rich or the middle class and the poor.

3. Although decreases in E never reduce aggregate second-period income, utility for a particular group may strictly decrease. For example, a decrease in E that results in a change from an equilibrium of zero (in panel 1 or 2) to an equilibrium of $\hat{\theta}_3$ in panel 3 makes group three individuals strictly worse-off. Similarly, if a decrease in E changes the equilibrium from $\hat{\theta}_3$ in panel 3Aa to one in panel 4A, group 1 can be made strictly worse-off. Note, furthermore, that equilibrium levels of indivdual utilities are discontinuous functions of E.

4. Clearly, Table 1 illustrates that income distribution matters. An important message that emerges, however, is that the dependence of the equilibrium rate of subsidization is not likely to be captured by looking at a few simple statistics describing income distribution, such as the ratio of median to mean income, or the Gini coefficient. The model predicts that the

relationship between income distribution and subsidization is quite subtle, a conclusion that should be kept in mind in empirical work.

6. Conclusion

Governmental support for education varies greatly both internationally and across states and localities within the United States. One possible cause of this variation is the interaction of income distribution and the political system. This paper has explored this hypothesis within a simple setting in which individuals are able to affect the private cost of education via subsidization. Rather than repeat the findings here, we outline what we believe to be several important avenues for future research aimed at producing models with sufficient richness to analyze actual data on education funding. First, the cost and quality of education have been taken as exogenous. In reality, the quality of educational systems vary as do their costs. In the case of higher education both affordability and quality are important attributes of the system. In primary and secondary education, tuition is typically zero and the variation is primarily in quality. These features need to be incorporated into the analysis. The (endogenous) existence of private alternatives is also an important feature to introduce, since this alternative will necessarily affect people's preferences over the allocation of resources to public education.[12] In the case of primary and secondary education it is also important to develop models in which location is stressed since much of this education is both locally financed and consumed.[13] Finally, the analysis here has taken place in a static setting. It is also of interest to study

[12] Glomm and Ravikumar (1990) study a model in which purely public and purely private systems are compared.

[13] Recent work along these lines includes Benabou (1991) and Durlauf (1991).

dynamic versions of these models that permit one to understand how the educational system and income distribution will evolve over time and the nature of their interactions.[14]

[14]Building on the earlier work of Becker and Tomes (1979) and Loury (1981), recent papers by Galor and Zeira (1990) and Ljungqvist (1991) study the evolution of income distribution in dynamic models in which credit constraints affect educational attainment. In these models, however, there is no endogenous choice of policy; all features of the educational system are taken to be exogenous. Durlauf (1991) is an exception.

Table 1 Voting Equilibria

Parameter Restrictions	$\tilde{\theta}_1$	$\tilde{\theta}_2$	$\tilde{\theta}_3$	Equilibrium
1. $\mu/\lambda_1 \leq E$	1	0	0	0
2. $\mu/\lambda_1 > E \geq \mu/(\lambda_1+\lambda_2)$	$\hat{\theta}_2$	0	0	0
	$\hat{\theta}_2$	1	0	0 or NE
3. $\mu/(\lambda_1+\lambda_2) > E \geq \mu$				
A. $y_2<E$				
a. $y_1<\mu/(\lambda_1+\lambda_2)$	$\hat{\theta}_2$	$\hat{\theta}_3$	0	NE
	$\hat{\theta}_3$	$\hat{\theta}_3$	0	$\hat{\theta}_3$
	$\hat{\theta}_2$	$\hat{\theta}_3$	1	NE
	$\hat{\theta}_3$	$\hat{\theta}_3$	1	$\hat{\theta}_3$
b. $y_1>\mu/(\lambda_1+\lambda_2)$	$\hat{\theta}_2$	$\hat{\theta}_3$	0	0 or NE
	$\hat{\theta}_2$	$\hat{\theta}_3$	1	NE
B. $y_2\geq E$				
a. $y_1<\mu/(\lambda_1+\lambda_2)$	$\hat{\theta}_3$	$\hat{\theta}_3$	0	$\hat{\theta}_3$
	$\hat{\theta}_3$	$\hat{\theta}_3$	1	$\hat{\theta}_3$
b. $y_1>\mu/(\lambda_1+\lambda_2)$	0	$\hat{\theta}_3$	0	0
	0	$\hat{\theta}_3$	1	0 or NE

4. $\mu>E$				
A. $y_2<E$	$\hat{\theta}_2$	1	1	1
	$\hat{\theta}_3$	1	1	1
	$\hat{\theta}_2$	$\hat{\theta}_3$	1	NE
	$\hat{\theta}_3$	$\hat{\theta}_3$	1	$\hat{\theta}_3$
B. $y_2\geq E$ $y_3<E$				
a. $y_1<\mu/(\lambda_1+\lambda_2)$				
i. $y_2<\mu$	$\hat{\theta}_3$	$\hat{\theta}_3$	1	$\hat{\theta}_3$
	$\hat{\theta}_3$	1	1	1
ii. $y_2\geq\mu$	$\hat{\theta}_3$	$\hat{\theta}_3$	1	$\hat{\theta}_3$
b. $y_1>\mu/(\lambda_1+\lambda_2)$				
i. $y_2<\mu$	0	$\hat{\theta}_3$	1	NE
	0	1	1	1
ii. $y_2\geq\mu$	0	$\hat{\theta}_3$	1	0 or NE
C. $y_3\geq E$				
a. $y_2>\mu$	0	0	1	0
b. $y_2<\mu$	0	1	1	1

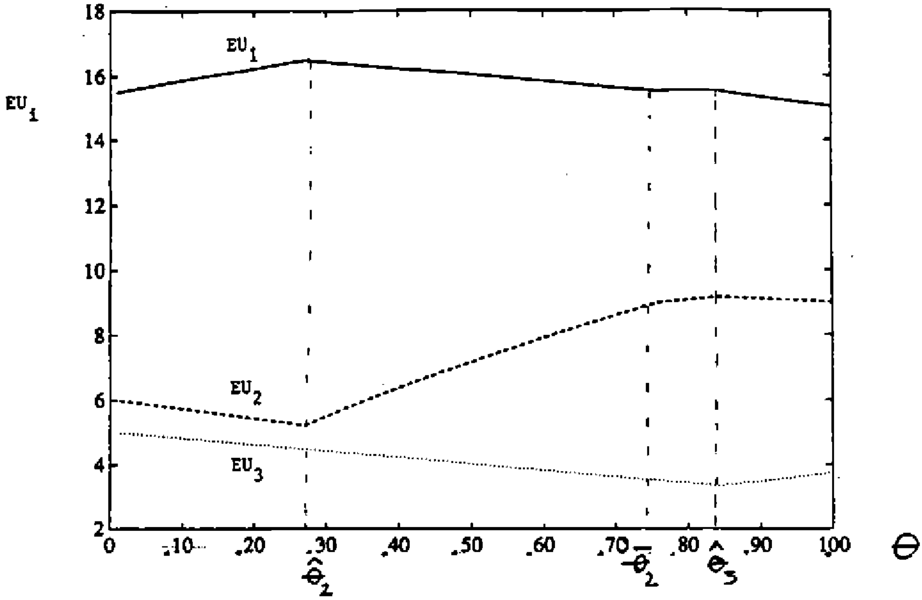
EU$_i$
18
16
14
12
10
8
6
4
2
EU$_1$
EU$_2$
EU$_3$
0
.10
.20
.30
.40
.50
.60
.70
.80
.90
100
$\hat{\theta}_2$
$\bar{\theta}_2$
$\hat{\theta}_3$
θ

FIGURE ONE

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