NBER WORKING PAPER SERIES

FISCAL POLICIES, DEBT AND INTERNATIONAL ECONOMIC INTERDEPENDENCE

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Working Paper No. 1266

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 1984

Jacob A. Frenkel wishes to acknowledge support from the Sackler Institute of Advanced Studies at Tel-Aviv Univeristy, Israel, of which he was a Fellow while this paper was written. Assaf Razin wishes to acknowledge support from the Ross Endowment. Both authors thank Itzhak Gilboa for assistance in the computations. The research reported here is part of the NBER's research program in International Studies and Economic Fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

This paper deals with the relation between government spending and real rates of interest as well as with the international transmission of fiscal policies. The dependence of the patterns of consumption in one country on fiscal policies in the rest of the world are examined. For this purpose a general equilibrium model which is characterized by fully integrated world capital markets is constructed, economic agents behave rationally, and government policies are constrained to obey the intertemporal solvency requirements. It is shown that the effects of changes in countries' net debtor position as well as the effects of fiscal policies can be analyzed by reference to a multitude of "transfer problems criteria", which are familiar from the theory of international economic transfers. In the present case the impact of policies depends on the relations among the spending patterns of domestic and foreign private sectors; of domestic and foreign governments, as well as of domestic and foreign saving propensities.

The analysis draws a distinction between permanent and transitory policies as well as between current policies and expected future policies. A transitory current fiscal spending, must crowd out the foreign private sector and, thereby, result in a negative transmission. However, a transitory future rise in government spending induces an immediate increase in foreign private sector's consumption and thereby results in a positive current transmission. These responses are reflected in the current account of the balance-of-payments, in changes in the net debtor-creditor positions, and in complex changes in the term structure of interest rates. It is also shown that with full integration of capital markets, fiscal policies may exert different qualitative effects on real rates of interest in different countries since, depending on the structural parameters, the relative prices of non-traded goods, and thereby the price indices, might be negatively correlated between countries.

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I. INTRODUCTION

One of the major sources of recent friction between Europeans and Americans has been the interpretation of the economic implications of U.S. budget deficits. Theorists and policymakers on both sides of the Atlantic have differed in the analysis of the role of budget deficits in affecting key macroeconomic aggregates. Specifically, some have argued that large budget deficits are responsible for the recently observed high real rates of interest while others have claimed that budget deficits cannot be blamed for these real rates. The latter group claimed that theory does not predict a clear-cut relation between budget deficits and rates of interests and that the empirical record itself is very weak.

The increased integration of the world economy resulted in increased concern in each country over policy measures taken in the rest of the world. The complex pattern of the economic linkages within the interdependent world economy resulted in a variety of models of the transmission mechanisms. These models include those that highlight the implications of foreign trade multipliers, e.g. LINK models which build on a Keynesian structure, as well as those that highlight the role of the terms of trade along the lines of the

elasticity approach to the balance-of-payments.[1] In addition, some analyses have examined whether disturbances can be transmitted negatively to the rest of the world. For example, Laursen and Metzler have shown that in a model without capital flows, domestic autonomous government expenditures which raise domestic output, lower the level of output abroad, i.e. domestic spendings are transmitted negatively to the rest of the world.[2] Parallel to these developments there were examinations of the transmission mechanism along the lines of the Mundell-Fleming models.[3] These contributions were stimulated by the increased integration of world capital markets. They highlight the interdependence between the exchange-rate regime, the degree of capital mobility and the impact of macroeconomic policies. Here, the key factor in the transmission mechanism has been the interaction between the rate of interest and the rate of exchange. Thus, it was shown that in a model with capital mobility and flexible exchange rates, an expansionary monetary policy is transmitted negatively to the rest of the world whereas fiscal policy is transmitted positively. More recently the large volatility of nominal exchange rates and the associated changes in real exchange rates have stimulated modelling of exchange-rate dynamics which has focused on the role of expectations. This line of modelling also relied on the real rate of interest as a key factor in the transmission mechanism, but the focus on expectations introduced an important dimension of dynamic considerations.[4]

These theoretical studies were based on the premise that world capital markets were indeed highly integrated. A separate branch of research has examined this premise. In this context one line of inquiry tested the implications of capital market integration for international equality of rates

of return. These studies include tests of equality of real rates of interest as well as studies of covered interest arbitrage. By and large, the basic findings were favorable to the hypothesis that capital markets are integrated.[5] A second line of inquiry tested the implications of capital markets integration for the relation between national saving and national investment. These studies claimed that the positive correlation between national saving and investment suggests that the degree of capital market integration is somewhat limited.[6] While there may be some dispute over the exact degree of international capital mobility, there is no doubt that the mobility of capital serves an important role in linking world rates of interest and, thereby, in transmitting economic disturbances internationally.

The key issues underlying the international conflict over U.S. budget deficits and their implications for the state of the world economy have not yet been fully settled. Specifically, economic theory has not yet resolved the question of the precise relation between budget deficits and real rates of interest. Nor has economic theory fully settled the precise nature of the international transmission of fiscal policies.

This paper is an attempt to shed light on these issues. Specifically, we develop a model that is suitable for the analysis of (i) the linkage between fiscal spending and rates of interest, and (ii) the international transmission mechanism of fiscal policies. Our model is capable of generating a variety of patterns of links between fiscal spending and rates of interest, as well as a variety of patterns of international transmissions of fiscal policies.

The key characteristics of our model are: (i) A fully integrated world capital market; (ii) full rationality of all economic agents whose decisions

are based on self fulfilling expectations and are subjected to temporal and intertemporal budget constraints, and (iii) government behavior that is constrained by an intertemporal solvency requirement. The model is of a general equilibrium nature which implies that the various economies have access to, and are governed by, world markets. These markets determine both temporal prices (commodity terms of trade) and intertemporal prices (rates of interest).[7]

Some of the features of the model are (i) the prominent role that is being played by wealth (permanent income) and the minimal role that is being played by current income in determining the levels of private spending; (ii) the irrelevancy of the time pattern of taxes and government debt issue given the pattern of government spending and the requirement of intertemporal solvency[8]; (iii) the dependence of the current account of the balance of payments (and thereby the accumulation of national external debt) on the entire path of government spending rather than on the path of budget deficits. We show that the impact of fiscal policies and the dynamics of debt accumulation on the key macroeconomic aggregates at home and abroad depend on a comparison among various behavioral propensities. These comparisons generate a multitude of "transfer problem criteria" that are familiar from the theory of transfers in international trade.[9] In the present context the relevant transfers are among private sectors and government sectors through current and future taxes and government spendings, as well as between the home and the foreign countries through the process of accumulation of external assets and debts. As a result of these transfers the key factors determining the impacts of policies are comparisons among (i) domestic and foreign

patterns of private sectors' spending, (ii) domestic and foreign private sectors' saving propensities, and (iii) domestic and foreign patterns of governments' spending.

The focus on the entire path of the level and composition of government spending, also serves to highlight the importance of the distinction between permanent and transitory policies as well as between current and future policies. These distinctions along with the multitude of the "transfer problem criteria" are shown to be critical in accounting for the apparent ambiguities in the effects of government spending on rates of interest and the related question of the nature of the international transmission of fiscal shocks. Our analysis attempts to isolate the roles of these critical factors in determining the outcomes of fiscal policies.

In addition to providing a theoretical justification for the observed linkages, our model also has implications for the choice of relevant aggregate economic variables for empirical research. For example, our analysis has implications for the expected patterns of the dependence of the reduced form equations of each country's consumption function, on current and future fiscal spendings by the domestic as well as by the foreign governments. Similarly, our analysis has implications for the dependence of the reduced-form equations of world rates of interest and commodity prices on the paths of fiscal spendings.

Previous models of the international transmissions of fiscal policies share some of the features of our model. The novelty of the present analysis is the integration of these features into a consistent analytical framework. This integration, however, has not been obtained without cost. At the present stage our analysis is limited in several important dimensions. The concluding section of the paper outlines some of the limitations and suggests possible extentions.

II. ONE COMMODITY WORLD

In this part of the paper we analyze a model of the world economy which produces and consumes a single composite commodity. We first outline the analytical framework which is then applied to the analysis of the effects of fiscal policies on interest rates and the nature of the international transmission mechanism.

II.1 The Analytical Framework

The analytical framework of the model contains the specification of production, consumption, the government and the foreign economy which interact to determine the equilibrium of the world economy.

II.1.1 Production

Consider two countries each producing the same good. Let the path of the supply of output in each country be fixed at its full employment level. Denote the supply of output at period t by Y_t and Y_t^*, respectively, where an asterisk designates variables pertaining to the foreign country.

II.1.2 Consumption

Assume that the home country's utility function is logarithmic as in equation (1)

$$U = \sum_{t=0}^{\infty} \delta^t \log c_t \tag{1}$$

where δ denotes the subjective discount factor and where c denotes the rate of consumption. The infinitely lived representative individual is assumed to maximize the utility function subject to the following set of budget constraints:

$$c_0 = Y_0 - T_0 - (1 + r_{-1})B_{-1} + B_0$$

(2) $$c_1 = Y_1 - T_1 - (1 + r_0)B_0 + B_1$$

$$c_t = Y_t - T_t - (1 + r_{t-1})B_{t-1} + B_t$$

where T_t denotes taxes at period t, B_t denotes the one-period debt and where r_t denotes the one-period market interest rate. In addition to the budget constraints in equation (2) there are two additional constraints which govern the maximization problem. The first states the initial conditions according to which the initial commitment of interest and amortization (B) is historically given as in equation (3)

(3) $$B \equiv (1 + r_{-1})B_{-1}.$$

The second is the solvency requirement according to which the present value of debt must approach zero in the limit:

(4) $$\lim_{t \to \infty} \alpha_t B_t = 0$$

where α_t denotes the present-value factor which is composed of one-period rates of interest compounded up to period t. Thus:

$$\alpha_t \equiv (1 + r_0)^{-1}(1 + r_1)^{-1}\ldots(1 + r_{t-1})^{-1}$$

and obviously $\alpha_0 = 1$.

Consolidating the set of the budget constraints in (2) and using equations (3) and (4) yields

$$(5) \qquad \sum_{t=0}^{\infty} \alpha_t C_t = \sum_{t=0}^{\infty} \alpha_t Y_t - \sum_{t=0}^{\infty} \alpha_t T_t - B.$$

As can be seen, equation (5) is the present value constraint. It states that the sum of the present values of the rates of consumption in all periods must equal wealth (W_0), where the value of wealth consists of the sum of the present values of output streams (gross domestic products), minus the sum of the present values of taxes and the initial debt commitment. It is noteworthy that for the individual what matters is the present value of taxes rather than their precise distribution over time.

Maximizing the utility function (1) subject to the consolidated constraint (5), the competitive representative consumer is assumed to treat the market rates of interest (which determine α_t) as given. Further, since we assume full certainty and rational expectations, the future rates of interest as well as taxes and outputs are assumed to be known with perfect foresight.

The resultant consumption function is thus:

$$(6) \qquad C_t = (1 - \delta)W_t$$

where

$$(7) \qquad W_t = \frac{\delta^t}{\alpha_t} W_0 \ .$$

As is seen, the rate of consumption in period t is proportional to the value of wealth in that period with $(1-\delta)$ being the marginal (and the average) propensity to consume out of wealth. Thus, in this formulation, the subjective discount factor, δ, is also the marginal propensity to save out of wealth. In order to solve for the value of debt in each period and thereby gain some insight into the dynamics of debt accumulation, we use equations (6) and (7) in the budget constraints (2) and obtain

$$(8) \qquad B_t = \frac{1-\delta^{t+1}}{\alpha_t} W_0 - \sum_{\tau=0}^{t} \frac{(Y_\tau - T_\tau)}{\alpha_t} \alpha_\tau + \frac{B}{\alpha_t}$$

Equation (8) states that the value of debt in period t must make up for the difference between the present values (as of period t) of consumption rates from period zero to period t, $[(1-\delta^{t+1})/\alpha_t]W_0$, and that of disposable incomes, $\sum_{\tau=0}^{t}(Y_\tau - T_\tau)\alpha_\tau/\alpha_t$; in addition, the current debt must also cover payments of the (current value of the) initial debt commitment, B/α_t.

II.1.3 Government

The government is assumed to spend at the rate G_t. Spending can be financed by taxes and debt issue at the market rate of interest. Government behavior, however, is constrained by the solvency requirement according to which the present value of spending must equal the present value of taxes:

$$(9) \qquad \sum_{t=0}^{\infty} \alpha_t G_t = \sum_{t=0}^{\infty} \alpha_t T_t.$$

By substuting equation (9) into the private sector's consolidated budget constraint we can express the value of wealth, W_0, as:

$$(10) \qquad W_0 = \sum_{t=0}^{\infty} \alpha_t y_t - B$$

where

$$y_t \equiv Y_t - G_t.$$

In what follows we refer to y_t as output net of government spending.

II.1.4 The Foreign Economy

Individuals in the foreign country are also assumed to have a logarithmic utility function

$$(1') \qquad U^* = \sum_{t=0}^{\infty} \delta^{*t} \log c_t^*$$

where an asterisk (*) designates variables pertaining to the foreign country. Individuals in the foreign country are assumed to operate in the same world capital market as those of the home country and thus they face the same set of rates of interest. Analogously to equation (5), the consolidated foreign budget constraint is

$$(5') \qquad \sum_{t=0}^{\infty} \alpha_t c_t^* = W_0^*$$

where

$$(10') \qquad W_0^* = \sum_{t=0}^{\infty} \alpha_t y_t^* + B.$$

In equation (10') $y^* \equiv Y_t^* - G_t^*$; i.e., it denotes the value of foreign output net of government spending. Finally, it is relevant to note that, in our two-country world, the home country's debt,

B, appears as the foreign country's asset.

Maximization of the foreign utility function, (1'), subject to the constraint (10') yields the foreign consumption function

$$(6') \qquad c_t^* = (1-\delta^*)W_t^*$$

where

$$(7') \qquad W_t^* = (\delta^{*t}/\alpha_t)W_0^*$$

II.1.5 World Equilibrium and Debt

In our frictionless world, market clearing equilibrium requires that at each period of time world private sectors' demand for output equals the supply of world output net of government absorption. Using the consumption functions (6)-(6') along with (7) and (7') yields:

$$(11) \qquad (1-\delta)\frac{\delta^t}{\alpha_t} W_0 + (1-\delta^*)\frac{\delta^{*t}}{\alpha_t} W_0^* = \bar{y}_t$$

where

$$\bar{y}_t \equiv y_t + y_t^* .$$

From this equilibrium condition we can express the equilibrium present-value factors, α_t, as

$$(12) \qquad \alpha_t = \frac{(1-\delta)\delta^t W_0 + (1-\delta^*)\delta^{*t} W_0^*}{\bar{y}_t} \ .$$

Since $\alpha_0 = 1$ we can use (12) for $t = 0$ to obtain

$$(13) \qquad (1-\delta)W_0 + (1-\delta^*)W_0^* = \bar{y}_0 .$$

To gain insight into the dynamics of the rates of interest we substitute for W_0 from equation (13) into equation (12) and after some manipulations we obtain:

$$(14) \qquad \alpha_t = \delta^t + \frac{(1-\delta^*)}{y_t} W_0^* (\delta^{*t} - \delta^t) - g_t \delta^t$$

where

$$g_t \equiv \frac{\bar{y}_t - \bar{y}_0}{y_t} \ .$$

Likewise, we substitute for W_0^* from equation (13) into equation (12) to obtain:

$$(14') \qquad \alpha_t = \delta^{*t} - \frac{(1-\delta)W_0}{\bar{y}_t}(\delta^{*t} - \delta^t) - g_t \delta^{*t} \ .$$

Equations (14)-(14') show that the present-value factors, α_t, depend on three quantities: (i) the levels of the marginal propensities to save, δ and δ^*, (ii) the difference between these saving propensities, $\delta^* - \delta$, and (iii) the percentage growth of world output, g. As is seen, when the marginal propensities to save are equal to each other, $\alpha_t = \delta^t(1-g_t)$. In that case a rise in the saving propensity and a fall in the growth of output raise the present-value factors, i.e. lower the rates of

interest since both changes raise world savings at the initial rates of interest. If world output is stationary, i.e. if $g_t = 0$, the magnitude of the present-value factors are bounded between δ^{*t} and δ^t. In that case the rates of interest are bounded between the two countries' subjective rates of discount.

The solution for W_0, W_0^* and the sequence of the present value factors, α_t, can be obtained from equations (12)-(13) and either (10) or (10'); this latter degree of freedom is due, of course, to Walras Law.

Using (10'), (12) and (13) yields the solutions

$$(15) \qquad W_0^* = \frac{\bar{y}_0 \sum_{t=0}^{\infty} \delta^t \lambda_t^* + B}{\Delta}$$

$$(16) \qquad W_0 = \frac{(1-\delta^*)[\bar{y}_0 \sum_{t=0}^{\infty} \delta^{*t} \lambda_t - B]}{(1-\delta)\Delta}$$

$$(17) \qquad \alpha_t = \frac{(1-\delta^*)\{\bar{y}_0[\delta^{*t} \sum_{\tau=0}^{\infty} \delta^\tau \lambda_\tau^* + \delta^t \sum_{\tau=0}^{\infty} \delta^{*\tau} \lambda_\tau] + (\delta^{*t} - \delta^t)B\}}{\bar{y}_t \Delta}$$

where $\Delta = [1-(1-\delta^*)\sum_{t=0}^{\infty}(\delta^{*t}-\delta^t)\lambda_t^*]$,
and where λ_t^* denotes the share of foreign product net of government spending, i.e. $\lambda_t^* \equiv y_t^*/\bar{y}_t$; analogously λ_t denotes the corresponding share of the home country, i.e. $\lambda_t \equiv (y_t/\bar{y}_t) = 1 - \lambda_t^*$.

The requirement that the rates of consumption in both countries are positive imply that both W_0 and W_0^* are positive. Inspection of equations (15)-(16) reveals that these requirements imply restrictions on the

initial debt position, B, as well as on the values of λ_t and λ_t^* which reflect the patterns of government spending. These requirements imply that the value of Δ in the denominator of equations (15)-(17) is positive since, when the initial debt position is zero this restriction is necessary to ensure positive levels of consumption.[10] Given that the denominator in (15)-(17) is positive, the requirement that the initial values of wealth (and thereby the entire paths of wealth and consumption) are positive also imply a restriction on the maximal size of the initial value of debt. These restrictions may be inferred by inspection of the numerators of (15)-(17). It may be seen that the maximal size of the initial value of debt in the system depends positively on the value of world output, and on the marginal propensities to save, and it also depends on the patterns of fiscal policies.

The dynamics of the system is affected through changes in each country's debt position. The impact of these changes in debt positions can be analyzed and interpreted in terms of concepts familiar from the analysis of international transfers. To obtain insights into the economic factors governing the dynamics of the system we now turn to an examination of the effects of a redistribution of the world debt position.

Consider a transfer of assets from the home country to the foreign country. This transfer amounts to an increase in the value of B which measures the home country's debt. The impact of the transfer on each country's consumption and, thereby, on the process of further redistributions of debt through international transactions depends on its impact on both countries wealth as well as on the path of world interest rates. Differentiation of equations (15)-(17) with some manipulations yields

$$(15') \qquad \frac{dW_0^*}{dB} = \frac{1}{\Delta} > 0,$$

$$(16') \qquad \frac{dW_0}{dB} = -\frac{1-\delta^*}{(1-\delta)\Delta} < 0,$$

$$(17') \qquad \frac{d\alpha_t}{dB} = \frac{(1-\delta^*)(\delta^{*t} - \delta^t)}{\bar{y}_t \Delta} \quad .$$

We may conclude that, following a transfer from the home country, $dW_0^*/dB > 0$, $dW_0/dB < 0$, and $\text{sign}(d\alpha_t/dB) = \text{sign}(\delta^* - \delta)$.

In interpreting these results consider the effect of the transfer on <u>world</u> savings at the initial rates of interest. The transfer raises foreign wealth by dB and lowers domestic wealth by the same amount. Consequently, at the prevailing rates of interest, world savings change by $(\delta^* - \delta)dB$. If $\delta^* > \delta$ the transfer induces positive world savings. Restoration of equilibrium with zero savings requires a fall in the rates of interest so as to discourage savings and stimulate current consumption through secondary changes in wealth. The secondary changes in wealth are brought about through the impact of the changes in the rates of interest on the valuation of wealth. When $\delta^* > \delta$ the rates of interest must fall, i.e., the values of α_t — the present-value factor — must rise as indicated by equation (17). The fall in the rates of interest raises each country's wealth. The secondary rise in wealth reinforces the initial effect of the transfer on foreign wealth whereas it mitigates the loss in wealth to the transferer. These mechanisms are illustrated by equations (15)-(17). As may be seen, when $\delta^* = \delta$,

$dW_0^*/dB = -dW_0/dB = 1$ whereas when $\delta^* \gtrless \delta$, $dW_0^*/dB \gtrless 1$ and $-dW_0/dB \lessgtr 1$, respectively.

The preceding analysis of the effect of the transfer on the initial wealth positions and on the path of the rates of interest provides the ingredients of the effects of wealth reallocation that occur during the dynamic processes of the interdependent economies. Specifically, in our world with no investment, one country's savings must equal the other country's dissaving. And the analysis of transfers reveals that if the marginal saving propensity of the country which is a net accumulator of assets exceeds that of the other country, then the subsequent path will be characterized by lower interest rates (i.e., higher present-value factors), a higher value of the accumulator's wealth, a lower value of the decumulator's wealth and a higher value of world's wealth.

Before turning to various comparative statics it is already evident that in this model fiscal policies affect the equilibrium only through their impact on net disposable incomes, $y_t \equiv Y_t - G_t$ and $y_t^* \equiv Y_t^* - G_t^*$. Therefore, their impact is equivalent to that of an exogenous change in gross domestic product.

Since individuals are only concerned with the present value of taxes which in turn are equal to the present value of government spendings, it follows that the details of government finance through either taxes or debt issue do not affect the equilibrium of the system as long as the government solvency requirements are met.

II.2 THE IMPACT OF FISCAL POLICIES

In this section we analyze the effects of fiscal spending in one country on world rates of interest and on the patterns of consumption in the rest of the world. The interdependence of private sector's spending on foreign fiscal spending results from the impact of fiscal policies on the entire path of rates of interest which, in turn, are common to both countries. Thus, in this interdependent world fiscal spending in one country is "financed" by crowding out of private spending in both countries.

Since private spending depends only on wealth position, it will be important to examine the impact of government spending on wealth. That impact in turn will depend on the effect of fiscal policies on disposable incomes as well as on the time pattern of the rates of interest. Since that pattern depends on the precise timing of government spending, we will analyze in detail situations in which changes in government spending are transitory as well as those in which they are permanent. To avoid repetitive examples we focus in the subsequent analysis on the impact of fiscal changes in the home country on world interest rates and on private consumption at home and abroad.

II.2.1 Transitory Changes

Consider the effect of a transitory reduction in domestic fiscal spending in period s on the rates of interest and on foreign and domestic current spending. Since consumption is proportional to wealth, we will seek to determine the effects of fixed spending on the levels of wealth. Suppose first that the change in fixed spending is expected to take place in the future (i.e. $s \neq 0$).

To solve for the effect of dy_s on the path of the present-value factors, α_t, we first differentiate equation (12) and use (13) to obtain:

$$(18) \qquad \frac{d\alpha_t}{dy_s} = (1-\delta^*)\,\frac{\delta^{*t} - \delta^t}{\bar{y}_t}\,\frac{dW_0^*}{dy_s} - \gamma_{t,s}\,\frac{\alpha_t}{\bar{y}_t} \quad ,$$

where

$$\gamma_{t,s} \equiv \begin{cases} 1 & \text{for} \quad t = s \\ 0 & \text{for} \quad t \neq s \end{cases}$$

We then differentiate the foreign wealth equation (10'), and obtain:

$$(19) \qquad \frac{dW_0^*}{dy_s} = \sum_{t=0}^{\infty} y_t^* \frac{d\alpha_t}{dy_s} \; .$$

Substituting (18) into (19) yields after some manipulations

$$(20) \qquad \frac{dW_0^*}{dy_s} = -\frac{\alpha_s \lambda_s^*}{\Delta} \quad .$$

Since $\bar{y}_0$ is given, oomestic and foreign wealth must change in opposite directions so as to ensure an unchanged value of world consumption in period 0. Thus from (13) and (20) it follows that:

$$(21) \qquad \frac{dW_0}{dy_s} = \frac{(1-\delta^*)\alpha_s \lambda_s^*}{(1-\delta)\Delta} > 0$$

Equations (20) and (21) imply therefore that a transitory future reduction in domestic fiscal spending raises current domestic private consumption and lowers current foreign consumption. As shown in these

equations, the magnitude of the change in current consumption depends on the timing of the given fiscal change, as well as on the relative size of the two economies. A given fiscal change that is expected to occur in the distant future will have a smaller impact on current consumption than a similar change that is expected to occur in the near future. This dependence is reflected in the value of α_s, the present value factor, which diminishes with the passage of time. Finally, it is noteworthy that the change in world wealth $[(dW_0+dW_0^*)/dy_s]$ depends only on the difference between the two saving propensities; if $\delta=\delta^*$ world wealth remains unchanged.

The solution of the effect of the future transitory fiscal change on the path of the present-value factors, is completed when we substitute equation (20) into (18) (for t = s) and obtain:

$$(18') \qquad \frac{d\alpha_s}{dy_s} = -\frac{\alpha_s}{\bar{y}_t^{\Delta}}[1-(1-\delta^*)\textstyle\sum_{t=s}^{\infty}(\delta^{*t}-\delta^t)\lambda_t^*] < 0;$$ [11/]

when $t \neq s$ the substitution of (20) into (18) yields

$$(18'') \qquad \frac{d\alpha_t}{dy_s} = -\frac{\alpha_s\lambda_s^*(1-\delta^*)}{\bar{y}_t^{\Delta}}(\delta^{*t}-\delta^t)$$

Equation (18') shows that a future transitory rise in net output occurring in period s, must lower the contemporaneous value of α_s which is the relative price of consumption in period s in terms of current consumption. This change in relative price is necessary to eliminate the incipient excess supply of goods in period s.

Equation (18'') shows that the effect of the future transitory rise in net output on the present-value factors in all other periods depend on the difference between the marginal propensities to save. The interpretation of this result can be given in terms of the previous analysis of debt transfer. Here, the rise in domestic wealth and the fall in foreign wealth (arising from the changes in y_s and α_s, imply that, at the prevailing present-value factors of all other periods ($t \neq s$) the difference between δ and δ^* determines whether world savings are positive or negative and, therefore, whether α_t must rise or fall.

To explore further the role of timing of the transitory shock suppose that the reduction in fiscal spending (or equivalently, the rise in net output) occurs at the present, i.e. $s = 0$. In that case, at the initial rates of interest, the rise in current domestic output raises domestic wealth by the same amount and, thereby, creates an excess supply of current goods. To restore equilibrium the relative prices of current consumption in terms of consumptions in all other periods must fall. Equivalently, the present-value factors, α_t, must rise. These changes imply that both domestic and foreign wealth must rise. Formally, using equations (12)-(13) and (19) we obtain after some manipulations:

$$(20') \qquad \frac{dW_0^*}{dy_0} = \frac{\sum_{t=1}^{\infty} \delta^t \lambda_t^*}{\Delta} > 0$$

which, together with equation (13) implies that

$$(21') \qquad \frac{dW_0}{dy_0} = 1 - \frac{(1-\delta^*)}{(1-\delta)} \frac{dW_0^*}{dy_0} = \frac{(1-\delta^*)\lambda_0}{\Delta} > 0.$$

Equations (20')-(21') show that a transitory reduction in domestic current fiscal spending (i.e. a rise in y_0) raises the values of current wealth and thereby the values of private consumption in both countries.

To summarize, it was shown that a future transitory change in fiscal spending results in opposite changes in current domestic and foreign wealth. In general, for all periods during which tne fiscal spending remains unchanged, market clearing requires that world private spending must also remain unchanged. Therefore, during these periods the fall in foreign wealth must be accompanied by a rise in domestic wealth. Whether the rates of interest linking the present period with other periods during which the fiscal spending remain unchanged, rise or fall, depends on whether the domestic propensity to save exceeds or falls short of the foreign savings propensity.

In contrast, during the period in which the fiscal change takes place, market clearing requires that the change in world private consumption must equal in (opposite sign) the change in government spending. The mechanism which brings about the necessary changes in private spending operates through changes in the rates of interest. For example a current transitory reduction in domestic fiscal spending creates, at prevailing interest rates, domestic (and therefore world) excess supply of present goods which is eliminated by a fall in interest rates linking the present period with the period during which the reduction in fiscal spending occurs. This fall in interest rates raises foreign and domestic wealth and serves to stimulate both domestic and foreign current private consumption. Thus, during the period in which the transitory fiscal changes occur, domestic and foreign wealth must move in the same

direction. The complex changes in the paths of the rates of interest are reflected in complicated alterations of the term structure of interest rates.

II.2.2 Permanent Changes

Consider now the effects of a permanent reduction in domestic fiscal spending on foreign and domestic private consumption. The effects of the permanent change on the current value of wealth can be computed from the previous expressions. For example, the effect of a permanent change in fiscal spending (dy) on foreign wealth is equivalent to the sum of the effects of all current (dy_0) and future transitory changes (dy_s) of equal magnitudes. Thus, adding (20') to the sum of the expressions in (20) for $t = 1,2,\ldots,$ yields the effect of an equivalent permanent change, as in (22).

$$(22) \qquad \frac{dW_0^*}{dy} \equiv \frac{dW_0^*}{dy_0} + \sum_{t=1}^{\infty} \frac{dW_0^*}{dy_t} = \frac{\sum_{t=1}^{\infty} (\delta^t - \alpha_t)\lambda_t^*}{\Delta}$$

The sign of dW_0^*/dy depends on the relations between δ^t and α_t which, as shown by equation (14) depends on the relation between the two countries' saving propensities and on the rate of growth of world output net of government absorption. Specifically, for the stationary case with $g_t = 0$, δ^t exceeds α_t if δ exceeds δ^*. In that case a permanent rise in y raises the current value of foreign wealth, and vice versa. On the other hand, when the saving propensities are equal to each other (i.e., when $\delta^* = \delta$), the permanent change in y alters foreign wealth only if $g_t \neq 0$. Thus when $\delta^* = \delta$, dW_0^*/dy is positive if world output (net of government absorption) exhibits positive growth, and vice versa.

We now turn to examine the effect of the permanent change in domestic fiscal spending on the current value of domestic wealth. Differentiating equation (13) and using equation (22) yields

$$(23) \qquad \frac{dW_0}{dy} = \frac{1 + (1-\delta^*)\sum_{t=0}^{\infty}(\alpha_t - \delta^{*t})\lambda_t^*}{(1-\delta)\Delta}$$

Analogously to the previous discussion, the sign of dW_0/dy seems to depend on the sign of $\alpha_t - \delta^{*t}$.

Further insight is obtained, however, in the stationary case when $g_t = 0$. Using equation (14') with $g_t = 0$ we can write (23) as:

$$(24) \qquad \frac{dW_0}{dy} = \frac{1 - [(1 - \psi_0\lambda^*)\delta + \psi_0\lambda^*\delta^*]}{1 - [1 - \lambda^*)\delta + \lambda^*\delta^*]} > 0.$$

where ψ_0 denotes the share of the home country's consumption (in period 0) in world output (net of government absorption), i.e., $\psi_0 \equiv (1-\delta)W_0/\bar{y} < 1$. Equation (24) demonstrates that a permanent rise in y must raise domestic wealth.

The effect of the permanent fall in domestic fiscal spending (i.e., a permanent rise in net output) on the rates of interest can be ascertained from the effect of this change on world savings at the prevailing rates of interest. The rise in net domestic output changes current domestic savings by $[dy_0 - (1-\delta)dW_0]$. In the stationary case, with the prevailing rates of interest, the percentage change in domestic output, $dy_0/y_0 \equiv \mu$, equals the percentage change in domestic wealth dW_0/W_0. In that case the incipient change in domestic savings is $[y_0-(1-\delta)W_0]$, where

the term in the brackets measures the initial value of domestic savings. Clearly at the prevailing rates of interest, foreign savings do not change. As is evident, the initial value of domestic savings is positive if the domestic marginal propensity to save exceeds the foreign propensity, i.e. if $\delta > \delta^*$ (in that case equilibrium requires that initially foreign savings were negative). Conversely, if $\delta < \delta^*$, domestic savings were negative. Thus, the permanent fall in domestic fiscal spending (the rise in y), raises world savings and induces a fall in the rates of interest if $\delta > \delta^*$, and vice versa.

In the preceding discussion we examined the effects of fiscal policies in the home country on the domestic and foreign levels of wealth and consumption as well as on the path of the world rates of interest. We assumed a single-commodity world or, equivalently, we assumed that the temporal relative price of goods was constant. This constancy permitted the construction of a single composite commodity, and resulted in an international transmission mechanism in which the intertemporal prices (the rates of interest) played the exclusive role. The subsequent section extends the analysis by incorporating the role of the temporal terms of trade.

III. TWO-COMMODITY WORLD

The analysis in Section II was confined to a world with a single composite commodity. In that world, therefore, the only relevant relative price was that of consumption in different periods, i.e. the rate of interest. In this section we extend the model so as to allow for two

different commodities. Therefore, in addition to the intertemporal terms of trade, the extended model also incorporates the role of the more conventional terms of trade, i.e. the relative price of importables in terms of exportables.

III.1 The Analytical Framework and World Equilibrium

Let the home country exportable good be denoted by x and its importable good by m. To simplify the analysis assume that each country is completely specialized in production. Thus good x is only produced in the home country at the level X and good m is only produced in the foreign country at the level M. Consumers on the other hand are assumed to consume both goods but since tastes may differ across countries, consumption patterns may differ. More formally, the expanded menu of goods is now incorporated into the utility function (1) by noting that c_t in (1) is a composite good which is defined as a Cobb-Douglas function of its components. Specifically

$$(1') \qquad U = \sum_{t=0}^{\infty} \delta^t \log c_t$$

where

$$\log c_t \equiv \beta \log c_{xt} + (1-\beta)\log c_{mt}.$$

In order to specify the budget constraint it is convenient to define current private expenditure by z_t where

$$z_t \equiv c_{xt} + p_t c_{mt} ,$$

thus, z_t measures private spending in units of good x in period t and p_t denotes the relative price of good m in terms of good x. Government spending also falls on both goods; G_{xt} denotes government spending on x and G_{mt} denotes government spending on m. Thus, the private sector's constraint, analogous to (5) becomes

$$(5') \qquad \sum_{t=0}^{\infty}\alpha_t z_t = \sum_{t=0}^{\infty}\alpha_t[X_t-(G_{xt}+p_tG_{mt})] - B.$$

In equation (5'), the right hand side defines W_0, the value of wealth in period 0. Equation (5') also incorporates the government budget constraint according to which the present value of government spending equals the present value of taxes.

Maximization of the utility function (1') subject to the budget constraint (5') yields the spending function z_t which is analogous to the consumption function which was obtained in the one-good world:

$$(6') \qquad z_t = (1-\delta)W_t$$

and, as before,

$$(7') \qquad W_t = (\delta^t/\alpha_t)W_0.$$

Equations (6')-(7') determines the pattern of <u>intertemporal</u> spending. In the present two-good world, individuals also need to determine the <u>temporal</u> allocation of spending between the two goods. The solution of the maximization problem yields:

$$(25) \qquad \begin{aligned} c_{xt} &\equiv \beta z_t \\ c_{mt} &= (1-\beta)\frac{z_t}{p_t} \end{aligned} \quad ;$$

As usual, the marginal propensities to spend on each good, β and $1-\beta$ respectively, are the exponents of the Cobb-Douglas composite good c_t. Before turning to the analysis of equilibrium it is relevant to emphasize that δ measures the marginal propensity to save out of wealth whereas β measures the marginal propensity to consume good x out of spending. In this framework as in the previous section, behavior is governed by permanent income (wealth) and the effect of current income in governing current behavior is only indirect.

The foreign country is modelled in an analogous fashion. Output in the foreign country is M_t, and the government is assumed to purchase G^*_{xt} of good x (which is imported from the home country) and G^*_{mt} of good m. The maximization problem is subjected to the private sector's initial wealth, W^*_0, and its solution yields the saving propensity, δ^*, and the marginal consumption propensities, β^* and $(1-\beta^*)$.

In order to facilitate the analysis of comparative statics we define x_t as domestic product net of the home country's government spending on domestic product, and m_t as the foreign product net of the foreign country's government spending on its product. Thus

$$x_t \equiv X_t - G_{xt}$$

$$m_t \equiv M_t - G^*_{mt}.$$

The equilibrium conditions are analogous to those in Section II except that, in the present expanded framework, equilibrium requires that the world market for <u>each</u> good clears. Using the demand functions the equilibrium conditions for both goods are:

$$(26) \qquad \beta(1-\delta)\frac{\delta^t}{\alpha_t}W_0 + \beta^*(1-\delta^*)\frac{\delta^{*t}}{\alpha_t}W_0^* = x_t - G_{xt}^*$$

$$(27) \qquad (1-\beta)(1-\delta)\frac{\delta^t}{\alpha_t p_t}W_0 + (1-\beta^*)(1-\delta^*)\frac{\delta^{*t}}{\alpha_t p_t}W_0^* = m_t - G_{mt}.$$

In period 0, $\alpha_0 = 1$, and therefore, the market clearing conditions for $t = 0$ become

$$(28) \qquad \beta(1-\delta)W_0 + \beta^*(1-\delta^*)W^*_0 = x_0 - G_{x0}^*$$

$$(29) \qquad (1-\beta)(1-\delta)\frac{W_0}{p_0} + (1-\beta^*)(1-\delta^*)\frac{W_0^*}{p_0} = m_0 - G_{m0}$$

where the values of wealth are

$$(30) \qquad W_0 = \sum_{t=0}^{\infty} \alpha_t x_t - \sum_{t=0}^{\infty} \alpha_t p_t G_{mt} - B$$

$$(31) \qquad W_0^* = \sum_{t=0}^{\infty} \alpha_t p_t m_t - \sum_{t=0}^{\infty} \alpha_t G_{xt}^* + B.$$

As before, by Walras Law, we may ignore one of the equations in the system.

Inspection of the equilibrium conditions (26)-(31) reveals that the relevant exogenous variables are x_t, m_t, G_{xt}^* and G_{mt}; given the

values of these variables the full solution can be obtained. This observation also suggests the particular channel through which government spending influences world equilibrium. Specifically, it is evident that government spending on goods produced in its own country exerts an identical effect on the equilibrium as an equivalent change in the level of domestic production of that good. Thus, a given rise in x_t yields the same effect whether it is caused by a rise in the level of production, X_t, or by a fall in government spending on that good G_{xt}. Further, in this two-good world the composition of government spending is crucial for the characteristics of world equilibrium. A given change in the total level of government spending affects world interest rates in a manner analogous to that of the one-good world. The key source for the interest-rate effect is that, in contrast with the government, which is essentially a zero saver, the private sector may either save or dissave parts of its income. Here, in addition to the interest rate effect, the composition of government spending also influences the relative price of the two goods since the pattern of government spending may differ from that of the private sector. Therefore, a given rise in G_{xt} may exert a different influence on world equilibrium than a rise in G_{mt} of equal value.

An explicit solution of the full system (26)-(31) is cumbersome. In order to highlight the key economic factors that affect the equilibrium, we now turn to a simplified version of the model. We assume that the paths of outputs and governments spending are stationary. Thus, let $x_t = x$, $m_t = m$, $G^*_{xt} = G^*_x$ and $G_{mt} = G_m$. By substituting these stationary values into equations (26)-(31) — as shown in Appendix I -- we obtain the following solutions:

$$(32) \qquad W_0^* = \frac{(x-G_x^*)[-\beta(xm-G_mG_x^*) + m(x-G_x^*) + \beta(1-\delta)(m-G_m)B]}{(x-G_x^*)\,[\beta^*(1-\delta^*)m-\beta(1-\delta)G_m] + \beta\beta^*(mx - G_x^*G_m)(\delta^*-\delta)}$$

$$(33) \qquad W_0 = \frac{(x-G_x^*)[\beta^*(xm-G_mG_x^*) - G_m(x-G_x^*) - \beta^*(1-\delta^*)(m-G_m)B]}{(x-G_x^*)[\beta^*(1-\delta^*)m - \beta(1-\delta)G_m] + \beta\beta^*(mx-G_x^*G_m)(\delta^*-\delta)}$$

To solve for p_0 we first substitute equation (28) into (29) and obtain

$$(34) \qquad p_0 = \frac{(1-\beta)(x-G_x^*) + (1-\delta^*)(\beta-\beta^*)W_0^*}{\beta(m - G_m)}$$

The explicit solution for p_0 can be obtained by substituting (33) for W_0^* into (34).

To gain insight into the determinants of p_0 we note that the equilibrium price must equate one country's trade balance surplus with the other's trade deficit or equivalently, the equilibrium price must ensure that the value of <u>world</u> spending on goods equals the value of <u>world</u> output. Thus, the equilibrium price p_0 must satisfy equation (35):

$$(35) \qquad [(1-\delta)W_0 + G_x + p_0G_m] - X = p_0M - [(1-\delta^*)W_0^* + G_x^* + p_0G_m^*]$$

where the left-hand-side of (35) measures the home country's trade balance deficit and the right-hand-side measures the foreign country's trade balance surplus. To ensure that the market for <u>each</u> good clears we use equation (28) and substitute for W_0 into (35). The solution for p_0 yields equation (34).

III.2. The Impact of Debt Redistribution

The equilibrium of the system described in section III.1 was conditional on the prevailing allocation of debt, and, as before, the dynamics of the system is effected through changes in each country's debt position. Like in the one-good world, the dynamic process may be associated with changes in interest rates, depending on the effect of debt redistribution on world savings. In the two-goods world the dynamic process may also be associated with changes in relative prices, depending on the effect of debt redistribution on excess demands for goods. When such changes in relative prices occur the impact of changes in real rates of interest on wealth and thereby on consumption also depends on the path of relative prices. In this section we deal with these issues.

Consider first the impact of a transfer of assets from the home to the foreign country. From equations (32)-(33) we note that the rise in the value of B raises foreign wealth by a factor proportional to $\beta(1-\delta)(m-G_m)$ and lowers domestic wealth by a factor proportional to $\beta^*(1-\delta^*)(m-G_m)$. These changes in wealth alter the demand for goods, and, as seen from equations (32) and (34), induce a change in the relative price. The direction of the change in the relative price depends on the sign of $(\beta-\beta^*)$. Thus, conforming with the well-known transfer problem criterion, a redistribution of wealth towards the foreign country raises the relative price of foreign goods if $(1-\beta^*)$, the foreign marginal propensity to spend on these goods, exceeds $(1-\beta)$, the home country's marginal propensity to spend on foreign goods or, equivalently, if $\beta > \beta^*$.

To determine the effect of the transfer on real consumption in the presence of changes in relative prices we need to define a price index which will then be used for evaluating the real values of consumption and wealth, as well as in real rates of interest. We define the price index associated with one unit of the consumption bundle c_t as $p_t^{1-\beta}$ and, correspondingly, the foreign price index is defined as $p_t^{1-\beta^*}$. Formally, the domestic price index is obtained by minimizing the cost $c_{xt} + p_t c_{mt}$ associated with obtaining one unit of the consumption bundle $c_{xt}^{\beta} c_{mt}^{1-\beta}$; the resulting index is the (utility based) true consumer price index. Analogously, the foreign price index is obtained by minimizing the cost of obtaining one unit of the foreign consumption bundle.

Using the price index, the values of <u>real</u> consumption, c_t and c_t^* are related to private spendings, z_t and z_t^* according to $c_t = z_t/p_t^{1-\beta}$ and $c_t^* = z_t^*/p_t^{1-\beta^*}$ which are also equal to $(1-\delta)W_t/p_t^{*-\beta}$, and to $(1-\delta^*)W_t^*/p_t^{1-\beta^*}$, respectively. Therefore, in order to evaluate the impact of debt transfer on real consumptions we need to evaluate its impact on the values of real wealth.[12] Since the value of real wealth depends on the paths of the present-value factors and prices, we first examine the impact of the transfer on the present value factor which expresses the rates of interest in terms of good x.

Using equations (26), (28) and (32) and assuming that initially government spendings are zero, the present-value factor measured in terms of good x can be written as:

$$\alpha_t = \delta^t + \frac{(1-\delta^*)[(1-\beta)x + (1-\delta)\beta B](\delta^{*t} - \delta^t)}{x[1 - (1-\beta)\delta^* - \beta\delta]} \tag{36}$$

As may be seen, the effect of a transfer on α_t depends on the relation between δ and δ^*. A rise in B lowers α_t (i.e., raises the rate of interest in terms of good x) if $\delta > \delta^*$. In that case the interest rate effect of the transfer operates to lower the recipient country's wealth.

To compute the present value-factor in terms of good m we use equations (27), (32) and (33) and obtain:

(36')

$$\alpha_t p_t = \frac{(1-\beta)x[\beta^*(1-\delta)\delta^t + (1-\beta^*)(1-\delta^*)\delta^{*t}]+(1-\delta^*)(1-\delta)[\beta(1-\beta^*)(\delta^{*t}-\beta^*(1-\beta)\delta^t]B}{\beta^* m[1 - (1-\beta)\delta^* - \beta\delta]}$$

As may be seen, the effect of a transfer on $\alpha_t p_t$ depends on the relation between the propensities to save, δ and δ^*, and the propensities governing the patterns of spending, β and β^*. As is evident, when $\beta = \beta^*$, the transfer does not alter the commodity terms of trade and the effects of the transfer depend only on the relation between the marginal propensities to save. In that case the present value factors in terms of x and m move in the same direction.

Having examined all the ingredients of the real values of wealth, we now turn to examine the effects of the transfer on the real values of consumption starting with the receiving country's consumption. Using equations (32) and (34) it is shown in Appendix II that around an initial equilibrium with zero government spendings and zero initial debt:

$$(37) \qquad \frac{d\log c_0^*}{dB} = \frac{(1-\delta)}{(1-\beta)}x\left[\beta-(1-\beta^*)(1-\delta^*)\frac{(\beta-\beta^*)}{(1-(1-\beta^*)\delta^*-\beta^*\delta)}\right]$$

An examination of (37) reveals that when $\beta = \beta^*$, i.e., when a transfer does not alter the commodity terms of trade, then a receipt of a transfer must raise real consumption. In this case, all goods may be aggregated into a single composite commodity and the analysis reduces to that of the one-commodity world of Section II. Likewise, when $\delta = \delta^*$, i.e., when a transfer at the initial terms of trade does not alter world savings, then a receipt of a transfer will also necessitate a rise in real consumption. In this case expenditures must equal income in each country so as to eliminate intertemporal trade. Thus, this case corresponds to the traditional static transfer problem analysis where it is known that a receipt of a transfer must raise real consumption. In the general case both $\beta \neq \beta^*$ and $\delta \neq \delta^*$, and the transfer alters the commodity terms of trade -- the temporal relative price of goods -- and the rates of interest -- the intertemporal terms of trade. It is noteworthy, however, that even in cases for which the transfer results in a higher consumer price index and in higher rates of interest (measured in terms of good x) the value of the receiving country's real wealth must rise along with the value of its real consumption.[13]

We now turn to examine the effect of the transfer on the value of the paying country's real consumption. Using equations (32)-(34) it is also shown in Appendix II that around the initial equilibrium with zero government spendings and zero initial debt:

$$(38) \qquad \frac{d\log c_0}{dB} = -\frac{(1-\delta^*)}{x}\left[1 + \frac{(1-\delta)(\beta-\beta^*)}{(1-(1-\beta^*)\delta^*-\beta^*\delta)}\right]$$

Analogously to the analysis of the effect of the transfer on the receiving country's consumption, it can be shown that independent of the configurations of β, β^*, δ and δ^*, $d \log c_0/dB < 0$. Thus, even if the payment of the transfer results in a lower price index and lower rates of interest, which tend to raise the real value of wealth, the net result must be a reduced level of real consumption.

The preceding analysis determined the effect of a change in debt on the initial levels of real consumption in both countries (each in terms of its own price index). In that discussion the cause for the change in debt was an exogenous transfer. In general, however, the international reallocation of debt results from the dynamic processes which characterize each country's income-spending decisions and which are reconciled through the equilibrium condition that world savings must be zero. The expressions in equations (37) and (38) can also be used to determine at each point the impact of current account surpluses and deficits (i.e. of changes in debt holdings) on the current levels of real consumption. Since from (7') $w_t = (\delta^t/\alpha_t)W_0$ and $W_t^* = (\delta^{*t}/\alpha_t)W_0^*$, it follows that whether the home country accumulates or decumulates wealth, i.e. whether its debt falls or rises, depends on whether δ^t/α_t rises or falls with the passage of time. As may be seen from equation (36), over time the home country's debt increases and the foreign country's debt decreases if $\delta^* > \delta$ and vice versa. It follows, therefore, that over time the home country's real consumption falls and the foreign country's real consumption rises.

III.3. The Effects of Fiscal Policies

In this section we analyze the effects of fiscal spending on the patterns of consumption in the various countries. The two-good world contains additional channels of interdependence that were not present in the one-good world of Section II. Since the relative prices of goods reflect the pattern of spending in both countries, the analysis of fiscal policies needs to specify the spending patterns of the government. In general it will be seen that the impact of policies depends on relations between various behavioral propensities. Specifically, the key factors determining the outcomes of policies are differences among the spending patterns and the saving propensities of four groups: foreign and domestic private sectors as well as foreign and domestic governments. These differences govern the evolution of relative prices and rates of interest following fiscal changes.

In order to avoid tedious derivations we will focus in this section on the effects of permanent fiscal policies by the home country. Since the effects of fiscal policies depend on the patterns of government spending, we will consider the cases in which spending falls on domestic goods and those in which it falls on foreign goods. In these cases we will examine the effects of the policies on the values of real consumption at home and abroad which, to recall, are defined as:

$$c_t = (1-\delta)W_t/p_t^{1-\beta} \quad \text{and}$$

$$c_t^* = (1-\delta^*)W_t^*/p_t^{1-\beta^*}.$$

Using equations (32)-(34) it is shown in Appendix III that the logarithmic derivatives of real consumptions with respect to changes in government spending on domestic goods, G_x, and on foreign goods, G_m, evaluated around an initial equilibrium with zero government spendings and zero initial debt are:

$$(39) \qquad \frac{d \log c_0^*}{dG_x} = -\frac{\beta^*}{x}$$

$$(40) \qquad \frac{d \log c_0^*}{dG_m} = \frac{1}{m}\left[\frac{\beta(1-\delta)}{\beta^* Q} - (1-\beta^*)\frac{1+RS}{1+R}\right]$$

$$(41) \qquad \frac{d \log c_0}{dG_x} = -\frac{\beta}{x}$$

$$(42) \qquad \frac{d \log c_0}{dG_m} = \frac{1}{m}\left[\frac{\beta(1-\delta) - Q}{\beta^* Q} - (1-\beta)\frac{1+RS}{1+R}\right]$$

where

$$Q \equiv 1 - (1-\beta)\delta^* - \beta\delta > 0$$

$$R \equiv \frac{(\beta-\beta^*)(1-\delta^*)}{\beta^* Q}$$

$$S \equiv \frac{\beta(1-\delta)-\beta^*(1-\delta^*) - \beta\beta^*(\delta^*-\delta)}{\beta^* Q}$$

As may be seen, a rise in fiscal spending on domestic goods lowers the values of real consumption at home and abroad. The reduction in foreign consumption thus "finances" part of the increased government spending. The proportional reduction in foreign real consumptions is β^*, i.e., the relative share of foreign spending on good x. Analogously, the proportional

reduction in the home country's private sector's real consumption is β. Thus, when the home country's government increases spending on the domestic good the extent of the reduction in the values of real consumptions of the private sectors in both countries depends on the importance of that good in private sectors' budgets. The precise effects of a rise in government spending on foreign goods depends on the magnitudes of the various propensities as may be seen from equations (40) and (42).

In order to highlight the role of the government's spending propensities we define the government spending function G as:

$$(43) \qquad G \equiv G_x + pG_m$$

and we assume that the government spending propensities are β_G on good x, and $(1-\beta_G)$ on good m. Thus:

$$G_x = \beta_G G \quad \text{and} \quad pG_m = (1-\beta_G)G,$$

which implies that, formally, G_m becomes a function of G_x, $G_m = G_m(G_x)$. It follows that around an initial equilibrium with zero government spending:

$$(44) \qquad dG_m = \frac{\gamma}{p} dG_x$$

where $\gamma = (1-\beta_G)/\beta_G$.

Using this specification of government spending we note that

$$(45) \qquad \frac{d \log c_0^*}{\partial G} = \frac{\partial \log c_0^*}{\partial G_x} + \gamma \frac{\partial \log c_0^*}{p \partial G_m} .$$

To obtain insights into the economic factors which are at play, we now turn to examine some special cases. These cases correspond to specific assumptions about some of the marginal propensities. We start with the case in which the domestic and foreign marginal propensities to save are the same, i.e., $\delta = \delta^*$. In that case using equations (39)-(40) and the specification of government spending which is embodied in (45) along with the solution for p_0 (from equation (A-8) in Appendix II), we obtain:

$$(46) \qquad \frac{d\log c_0^*}{dG} = \frac{\beta^*}{\beta_G \beta(1-\beta)x}\left[\beta(\beta-\beta_G) + 2(1-\beta^*)(1-\beta_G)(\beta-\beta^*)\right].$$

Equation (46) reveals that when $\delta = \delta^*$ the effect of domestic fiscal spending on foreign real consumption depends only on differences among the patterns of spending of domestic private and public sectors, $\beta-\beta_G$, as well as between domestic and foreign private sectors, $\beta-\beta^*$. The economic interpretation of this result is as follows. When $\delta-\delta^*$ we know from equation (36) that in both countries the interest rate equals the subjective discount rate. Therefore, in each country savings are zero. In that case, neither the rates of interest, nor the total level of world spending are altered in response to government spending since the government, like the private sectors, is a zero saver. It follows that the potential effect of fiscal policies can only operate through changes in relative prices. In conformity with the standard analysis of economic transfers, such changes can occur only if the spending patterns differ among private and public sectors.

To demonstrate the role of the spending patterns consider the following special cases:

(i) When all spending patterns are identical, i.e., when $\beta=\beta^* = \beta_G$, we note from equation (46) that $d\log c_0^*/dG = 0$. Thus, in this special case, the effects of fiscal policy are not transmitted internationally and only the domestic private sector is crowded out. In that case aggregate behavior in the various markets are not affected by the fiscal policy and, therefore, there are no changes in relative prices.

(ii) When the spending patterns of the domestic and foreign private sectors are identical, i.e. when $\beta=\beta^*$, we note from equation (46) that the direction of the change in foreign real consumption following a rise in domestic fiscal spending depends only on the difference between β, the private sectors' spending pattern, and β_G, the corresponding government spending pattern. If $\beta > \beta_G$, a rise in government spending creates an excess supply of good x and an excess demand for good m at the initial relative price. Equilibrium is restored through a rise in p, the relative price of good m. This rise in p raises the real value of foreign wealth and, thereby, raises real consumption. The preceding discussion also implies that in the extreme case in which $\beta_G = 1$, i.e., when government spending falls entirely on good x, foreign real consumption must fall (as was already shown in equation (39)). At the other extreme, when $\beta_G = 0$, i.e., when government spending falls entirely on good m, (the case which corresponds to equation (40)), the value of foreign real consumption must rise.

The effect of the rise in government spending on the home country's real consumption must always be negative independent of the patterns of government

spending. Thus, in conformity with the traditional results of economic transfers, the secondary gain that might occur through an improvement of the terms of trade cannot offset the primary loss which in the present case is the tax levied to finance government spending.

The preceding analysis was confined to the case in which each country's income equaled its spending since δ was assumed to equal δ^*. As a result, the international transmission of fiscal policies operated entirely through the effects of these policies on the relative price of goods without any impact on the rates of interest. We turn now to examine the case in which the saving propensities differ, i.e. $\delta \neq \delta^*$. In order to isolate the effects of differences between private and public spending patterns we will assume that $\beta=\beta^*$. Analogously to the previous derivation we obtain in this case:

$$(46') \qquad \operatorname{sign} \frac{d \log c_0^*}{dG} = \operatorname{sign} \left\{ \frac{(1-\beta_G)}{\beta_G} - \frac{(1-\beta)[1-(1-\beta)\delta^* - \beta\delta]}{(1-\delta)-(1-\beta)[1-(1-\beta)\delta^*-\beta]} \right\}$$

if the denominator of the second ratio on the right-hand-side of (46') is positive and vice versa. As may be seen, the sign in (46') depends on the relation between the spending patterns of the private sectors and the government, β and β_G, as well as on the relation between the saving propensities δ and δ^*. In the special case for which $\beta = \beta_G$, equation (46') becomes

$$(46'') \qquad \operatorname{sign} \frac{d \log c_0^*}{dG} = \operatorname{sign}(\delta^* - \delta).$$

In that case the patterns of world spending on goods are independent of fiscal policies since $\beta = \beta^* = \beta_G$. Therefore the two goods can be aggregated into a composite commodity and the analysis reduces to that of the one-good world. And, as seen in equation (46") the key factor determining the effect of fiscal spending on foreign real consumption is the relation between δ^* and δ. Analogously to the analysis of equation (22) (where we analyzed the effect of a rise in output net of government absorption), the interpretation of this result is in terms of the effect of government spending on the rate of interest. If $\delta^* > \delta$, the foreign country saves part of its income whereas the domestic economy dissaves; thus the domestic country's marginal propensity to save out of <u>income</u> is negative. In that case, a rise in the home country's government spending amounts to transferring income from a dissaver (the home country's private sector) to a zero saver (the government), and, thereby creating (at the prevailing rates of interest) excess world savings. To restore equilibrium the rates of interest must fall. The fall in the rates of interest raises foreign wealth and real consumption. The opposite occurs when $\delta^* < \delta$. In that case government spending lowers world savings and raises interest rates and, thereby, lowers foreign wealth and real consumption. In that case, the effect of government spending on the home country's real consumption is always negative and the analysis is analogous to that of the one-commodity world equation (24). As usual, the secondary gain arising from a fall in the rate of interest (that occurs when $\delta^* > \delta$) cannot outweigh the primary loss from the tax that is levied to finance government spending.

The preceding analysis focused on the nature of the international transmission of domestic fiscal policies. One of the central mechanisms through which the transmission is effected operates through the effects of fiscal spending on real rates of interest. In what follows we elaborate on the effects of fiscal policies on the real rates of interest.

The example underlying equation (46'') assumes that the patterns of spending are identical among domestic and foreign governments and private sectors. The assumption that $\beta=\beta^*$ implies that the real rates of interest are equal across countries. The additional assumption that $\beta=\beta_G$ implies that the effects of changes in fiscal spending on the real rates of interest depend only on the differences between the domestic and the foreign saving propensities.

The example underlying equation (46'), assumes that the spending patterns of the domestic and foreign private sectors differ from those of the government, i.e. that $\beta_G \neq \beta = \beta^*$. In that case real rates of interest are equal across the world but the impact on the real rates of government spending depend not only on the relation between the two countries' saving propensities but also on the relation between β and β_G. In the more general case for which all spending patterns differ, real rates of interest differ across countries and the effect of government spending on these rates is more complex.

To illustrate, consider the special case in which government spending falls entirely on the domestically produced good, i.e. $\beta_G = 1$. The effects of government spending on the real rates of interest can be computed from their effects on the present-value factors measured in terms of the consumption baskets. The domestic and foreign real present-value factors are $\alpha_t p_t^{1-\beta}$ and $\alpha_t p_t^{1-\beta^*}$, respectively. It is shown in Appendix IV that:

$$(47) \qquad \frac{d\log(\alpha_t p_t^{1-\beta})}{d\log x} = -(1-\beta)$$

and

$$(47') \qquad \frac{d\log(\alpha_t p_t^{1-\beta^*})}{d\log x} = -(1-\beta^*)$$

Thus, a reduction in government spending G_x (i.e. a rise in x) lowers the real present-value factors in both countries and raises the corresponding real rates of interest. In contrast with the case of equality among private and public spending patterns in which the direction of changes in real rates of interest depended on the relation between saving propensities, here these propensities play no role. In the present case with $\beta_G = 1$ domestic and foreign real rates of interest move in the same direction. The extent of their response to fiscal changes depends on the relative share of the good which is not consumed by the government (good m) in private sector's spending.

In the intermediate case for which government spending falls on both goods but the relative share β_G exceeds the corresponding shares of the private sectors, β and β^*, the effect of government spending on the real rates of interest reflects the influence of both spending patterns and saving propensities. For example, if $\delta^* > \delta$ a reduction in government spending lowers world savings (as shown in Section II) and thereby reinforces the effects embodied in equations (47)-(47'). On the other hand, if $\delta < \delta^*$, the impact of government spending on world savings and, thereby, on the real rates of interest tends to mitigate and may reverse the effects operating via equations (47)-(47').

We have shown in this section that the effects of government spending on world rates of interest and the nature of the international transmission mechanism depend on the multitude of "transfer problem criteria". These

include the relations between saving propensities and spending patterns of domestic and foreign governments and private sectors.

IV. CONCLUDING REMARKS

In this paper we attempted to analyze the relation between government spending and real rates of interest as well as the international transmission of fiscal policies. Specifically, we examined the dependence of the patterns of consumption in one country on fiscal policies in the rest of the world. For this purpose we developed a general equilibrium model which was characterized by fully integrated world capital markets. Economic agents were assumed to behave rationally and government policies were constrained to obey the intertemporal solvency requirement.

We demonstrated that the effects of changes in countries' net debtor position as well as the effect of fiscal policies can be analyzed by reference to a multitude of "transfer problem criteria", which are familiar from the theory of international economic transfers. In the present case the impact of policies depended on comparisons among the patterns of domestic and foreign private sectors' and government spending as well as between domestic and foreign saving propensities.

For example, we showed that when spending patterns are similar across countries, the effect of a permanent rise in government spending on the rate of interest depends on whether the country in question is a saver or a dissaver in the world economy. If the country is a dissaver a rise in government spending lowers domestic consumption and rates of interest and induces a rise

in foreign wealth and consumption. Thus, transmission in that case is positive. The opposite holds when the country is a net saver. In the more general case the exact nature of the transmission and of the effect of government spending on interest rates depends also on comparisons among the various spending patterns which, in turn, determine the impact of fiscal policies on the terms of trade. The impact of permanent fiscal policies on the rates of interest and, thereby, on the nature of the transmission mechanism also depends on the percentage growth of world net output. Specifically, the rates of interest which result from expansionary fiscal policies tend to be higher, and the international transmission tends to be more negative, the faster is the percentage growth of world output.

Our analysis also draws a distinction between permanent and transitory policies as well as between current policies and expected future policies. It was shown that a transitory current fiscal spending, in addition to crowding out the domestic private sector, must also crowd out the foreign private sector and, thereby, result in a negative transmission. However, a transitory future rise in government spending must induce an immediate increase in foreign private sector's consumption and thereby result in a positive current transmission. In both cases, of course, domestic private sector's consumption must fall. These patterns of response reflect themselves in the current account of the balance-of-payments and in changes in the various countries' net debtor-creditor positions.

The distinction between permanent and transitory policies also reflects itself in complex changes in the term structure of interest rates. For example, it was shown that a future transitory fiscal expansion lowers the

rate of interest linking the present period with the period during which the transitory fiscal change occurs. The impact on the rates of interest linking the present period with all other periods depends on the difference between domestic and foreign marginal propensities to save. In contrast, it was shown that a transitory current rise in fiscal spending must raise the rates of interest pertaining to all maturities. The rise in short-term rates of interest exceeds the rise in long-term rates and, thereby, the slope of the yield curve is lowered.

The analysis of the impact of fiscal policies on real rates of interest revealed that even when capital markets are highly integrated, real rates of interest may differ if spending patterns differ across countries. With such differences in spending patterns, fiscal policies exert different quantitative effects on real rates of interest in the various countries. In the presence of non-traded goods, fiscal policies may also exert different qualitative effects on real rates of interest in different countries since, depending on the nature of the transmission mechanism and on the patterns of government and private sectors' spending, the relative prices of non-traded goods, and thereby the price indices, might be negatively correlated between countries. Under these circumstances it might be preferable to test for the degree of integration of international financial capital markets by reference to tests of covered interest arbitrage rather than by direct comparisons of aggregate real rates of interest.

The examination of the response to future changes is applicable to the analysis of the impact of the discovery of North-Sea oil. That discovery changes current wealth, but its impact on the actual flow of oil is expected

to occur only in the future. The rise in current wealth stimulates consumption and is observed to induce a process of de-industrialization, as resources move from manufactures to services -- a process referred to as the "Dutch Disease". General equilibrium considerations, such as those incorporated in our analysis suggest that the "Dutch Disease" in the oil discovering country is accompanied by an opposite process, namely, an industrialization process, in the rest of the world. The channel of transmission in that case is the rise in world interest rates induced by the oil discovery.

Our analysis is subject to several limitations which stem from some of the simplifying assumptions. We assumed that the output levels were given exogeneously. An extension would allow for a process of investment which responds to rates of interest and which changes the paths of outputs. Such an extention would modify the pattern of the current account and debt accumulation.[14] The endogeneity of output could also be introduced through the incorporation of some Keynesian features such as price rigidities. Under such circumstances government spending would alter the level of economic activity and would be transmitted internationally through the mechanisms similar to those of the foreign-trade multipliers.

Further extension would modify the assumption of full certainty. The incorporation of uncertainty might raise additional issues such as the role of bankruptcy and default. In that case, a relevant question would be the impact of fiscal policies on the likelihood of bankruptcies and defaults in the international capital markets.

One of the key features of our model has been the central role played by the path of government spending and the irrelevancy of the path of government budget deficits. One of the reasons responsible for this feature was the assumption that all taxes were of the non-distorting variety. As a result the time pattern of taxes and government debt issue was irrelevant once the pattern of government spending was given. An extension would allow for distorting effects of taxes.[14] Such distortions would introduce new considerations associated with the determination of the optimal paths of the various means of government finance.

Finally, our two-country world with exogenous government spendings enables exploitation of monopoly-monopsony powers in goods and capital markets. An extension would determine the optimal pattern of government spending along with the optimal trade-cum-capital-flows tax structure along the lines of the optimal tariff literature. Such a strategic behavior could then be incorporated into a more elaborate game-theoretic world equilibrium. In such a set-up, government spendings and its means of finance would become endogenous variables that are determined in the context of world equilibrium.

APPENDIX

THE TWO-COMMODITY STATIONARY SYSTEM

I. The Solution of the System

The solutions of the system for the stationary case can be obtained as follows. We first express the definitions of wealth in equations (30)-(31) as

$$W_0 = x \sum_{t=0}^{\infty} \alpha_t - G_m \sum_{t=0}^{\infty} \alpha_t p_t - B \tag{A-1}$$

$$W_0^* = m \sum_{t=0}^{\infty} \alpha_t p_t - G_x^* \sum_{t=0}^{\infty} \alpha_t + B. \tag{A-2}$$

Substituting (A-1)-(A-2) into (28)-(29) yields:

$$\beta(1-\delta)\left[x\sum_{t=0}^{\infty} \alpha_t - G_m \sum_{t=0}^{\infty} \alpha_t p_t - B\right] \tag{A-3}$$

$$+ \beta^*(1-\delta^*)\left[m \sum_{t=0}^{\infty} \alpha_t p_t - G_x^* \sum_{t=0}^{\infty} \alpha_t + B\right] = x - G_x^*$$

$$(1-\beta)(1-\delta)\left[x \sum_{t=0}^{\infty} \alpha_t - G_m \sum_{t=0}^{\infty} \alpha_t p_t - B\right] \tag{A-4}$$

$$+ (1-\beta^*)(1-\delta^*)\left[m \sum_{t=0}^{\infty} \alpha_t p_t - G_x^* \sum_{t=0}^{\infty} \alpha_t + B\right]$$

$$= p_0(m - G_m)$$

We then multiply both sides of (26) by α_t, sum both sides over all t (from 0 to ∞) and substitute (A-1) and (A-2) for W_0 and W_0^*. The resulting equation is then:

$$(A\text{-}5) \qquad \beta[\textstyle\sum_{t=0}^{\infty}\alpha_t - G_m\sum_{t=0}^{\infty}\alpha_t p_t - B] + \beta^*[m\ \textstyle\sum_{t=0}^{\infty}\alpha_t p_t - G_x^*\ \sum_{t=0}^{\infty}\alpha_t + B] = (x-G_x^*)\textstyle\sum_{t=0}^{\infty}\alpha_t.$$

Equation (A-3), (A-4) and (A-5) constitute the relevant system for the case of stationary outputs and fiscal policies. The economic interpretation of the equation is as follows: Equations (A-3) and (A-4) describe the equilibrium in the markets for the two goods in period 0. It is evident that this is a rational expectations equilibrium since the demand functions are based on the fully expected realization of future values of rates of interest and prices. Equation (A-5) requires that the sum of the present values of world demand for good x equal the corresponding sum of world supply. As may be noted we have used Walras Law to ignore the analogous requirement for good m.

Using this system we may solve for the three unknowns: The sum of the present value factors in terms of good x -- $\sum_{t=0}^{\infty}\alpha_t$; the sum of the present value factors in terms of good m -- $\sum_{t=0}^{\infty}\alpha_t p_t$; and the relative price of good m in terms of x in the first period -- p_0. These solutions can then be used in (A-1) and (A-2) to obtain the values of W_0 and W_0^*. These are the solutions that are reported in the text.

II. The Effects of Debt on Real Consumptions

In this part of the Appendix we derive the effects of debt transfers on the real values of domestic and foreign consumptions. These results are computed around an initial equilibrium with zero debt and zero government spendings. Around such an equilibrium, the values of W_0^*, W_0 and p_0 are (from (32)-(34)):

$$(A\text{-}6) \qquad W_0^* = \frac{(1-\beta)x}{\beta^* D} ,$$

$$(A\text{-}7) \qquad W_0 = \frac{x}{D}$$

$$(A\text{-}8) \qquad p_0 = \frac{(1-\beta)x}{\beta^* m} \frac{[1-(1-\beta^*)\delta^* - \beta^* \delta]}{D}$$

where

$$D = 1-(1-\beta)\delta^* - \beta\delta > 0.$$

From (32) we obtain

$$(A\text{-}9) \qquad \frac{dW_0^*}{dB} = \frac{\beta(1-\delta)}{\beta^* D}$$

which can be divided by (A-6) to yield:

$$(A\text{-}10) \qquad \frac{d\log W_0^*}{dB} = \frac{\beta(1-\delta)}{(1-\beta)x} .$$

Analogously, by differentiating (34) with respect to B and using (A-9) we obtain:

$$(A\text{-}11) \qquad \frac{dp_0}{dB} = \frac{(1-\delta^*)(1-\delta)(\beta-\beta^*)}{m\beta^* D}$$

Dividing (A-11) by p_0 from (A-8) yields

$$(A\text{-}12) \qquad \frac{d \log p_0}{dB} = \frac{(1-\delta^*)(1-\delta)(\beta-\beta^*)}{(1-\beta)x[1-(1-\beta^*)\delta^*-\beta^*\delta]} \quad .$$

Recalling that:

$d \log c_0^*/dB = d \log W_0^*/dB - (1-\beta^*)\, d \log p_0/dB$ we may multiply (A-12) by $(1-\beta^*)$ and use (A-10) to obtain equation (37) in the text.

The effect of the transfer on domestic real consumption is obtained analogously. We first note that for equation (28):

$$(A\text{-}13) \qquad dW_0 = - \frac{\beta^*(1-\delta^*)}{\beta(1-\delta)} dW_0^*$$

and thus, by using (A-9) and (A-7) we get

$$(A\text{-}14) \qquad \frac{d \log W_0}{dB} = - \frac{(1-\delta^*)}{x}.$$

Multiplying (A-12) by $(1-\beta)$ and subtracting from (A-14) yields equation (38) in the text.

III. The Effects of Fiscal Spending on Real Consumptions

In this part of the Appendix we derive the effects of fiscal policies on the real values of domestic and foreign consumptions. As before these results are computed around an initial equilibrium with zero debt and zero government spendings. From (32) we obtain

$$(A\text{-}15) \qquad \frac{dW_0^*}{dx} = \frac{1-\beta}{\beta^* D}$$

$$(A\text{-}16) \qquad \frac{dW_0^*}{dG_m} = \frac{x}{m} \frac{\beta(1-\beta)(1-\delta)}{\beta^{*2} D^2}$$

and from (A-13) and (A-15)-(A-16) we obtain

$$(A\text{-}17) \qquad \frac{dW_0}{dx} = \frac{1}{D}$$

$$(A\text{-}18) \qquad \frac{dW_0}{dG_m} = \frac{x[\beta(1-\delta)-D]}{\beta^* m D^2} .$$

Differentiating (34) with respect to x and using (A-15) yields

$$(A\text{-}19) \qquad \frac{dp_0}{dx} = \frac{(1-\beta)-(1-\delta^*)(\beta-\beta^*)\frac{(1-\beta)}{\beta^* D}}{\beta m} .$$

To convert (A-19) into a logarithmic derivative we first substitute (A-6) for W_0^* in equation (34) to obtain:

$$(A\text{-}20) \qquad p_0 = \frac{(1-\beta)x}{\beta m}\left[1 + \frac{(1-\delta^*)(\beta-\beta^*)}{\beta^* D}\right] ,$$

and dividing (A-19) by (A-20) yields

$$(A\text{-}21) \qquad \frac{d\log p_0}{dx} = \frac{1}{x} .$$

Multiplying (A-21) by $(1-\beta^*)$ and subtracting from the logarithmic derivative of W_0^* with respect to x (obtained from (A-15) and (A-6)) we get $d\log c_0^*/dx$ which is equation (39) in the text, with a minus sign (since $dx = -dG_x$).

Analogously, dividing (A-17) by (A-7) yields the logarithmic derivative of W_0 with respect to x, from which we subtract the product of $(1-\beta)$ and (A-20), to obtain $d \log c_0/dx$ which is equation (41) in the text (with a minus sign).

To compute the effects of changes in G_m on real consumptions we first note that from (34) and (A-16):

$$\text{(A-22)} \qquad \frac{dp_0}{dG_m} = \frac{(1-\beta)}{\beta} \frac{x}{m^2}\{1 + \frac{(\beta-\beta^*)(1-\delta^*)}{\beta^{*2}D^2}[\beta(1-\delta)-\beta^*(1-\delta^*)-\beta\beta^*(\delta^*-\delta)]\} \quad ,$$

and dividing by (A-21) yields

$$\text{(A-23)} \qquad \frac{d \log p_0}{dG_m} = \frac{1}{m} \frac{1 + RS}{1 + R}$$

where R and S are defined in equation (42) in the text. Multiplying (A-23) by $(1-\beta^*)$ and subtracting from the ratio of (A-16) and (A-6) yields equation (40) in the text.

Analogously, dividing (A-18) by (A-17) and subtracting the product of $(1-\beta)$ and (A-23) yields equation (42) in the text.

IV. The Effects of Fiscal Spending on Real Rates of Interest

In this part of the Appendix we compute the effects of domestic permanent fiscal spending on domestic and foreign real rates of interest. For simplicity we focus on the case in which government spending falls entirely on good x, i.e. the case for which $\beta_G = 1$.

Equation (26) in text around an initial equiibrium with zero government spending and initial debt implies that

$$(A\text{-}24) \qquad \alpha_t = 1/x[\beta(1-\delta)\delta^t W_0 + \beta^*(1-\delta^*)\delta^{*t} W_0^*],$$

and its logarithmic derivative is:

$$(A\text{-}25) \qquad \frac{d\log \alpha_t}{dx} = \theta\frac{d\log W_0}{dx} + (1-\theta)\frac{d\log W_0^*}{dx} - \frac{1}{x}$$

where, using (A-6)-(A-7)

$$\theta = \frac{\beta(1-\delta)\delta^t}{\beta(1-\delta)\delta^t + (1-\beta)(1-\delta^*)\delta^{*t}} \quad .$$

Using (A-17) and (A-7) for $d\log W_0/dx$, and using (A-15) and (A-6) for $d\log W_0^*/dx$, we obtain

$$(A\text{-}25') \qquad \frac{d\log \alpha_t}{dx} = 0.$$

Analogously, from equation (27) we obtain:

$$(A\text{-}26) \qquad \frac{d\log p_t}{dx} = \tilde{\theta}\frac{d\log W_0}{dx} + (1-\tilde{\theta})\frac{d\log W_0^*}{dx} - \frac{d\log \alpha_t}{dx} \quad .$$

where

$$\tilde{\theta} = \frac{\beta^*(1-\delta)\delta^t}{\beta^*(1-\delta)\delta^t + (1-\beta^*)(1-\delta^*)\delta^{*t}}$$

Using (A-17) and (A-7) for $d\log W_0/dx$, and using (A-15) and (A-6) for $d\log W_0^*/dx$, we obtain after substituting (A-25'):

$$(A\text{-}26') \qquad \frac{d\log p_t}{dx} = \frac{1}{x}$$

As may be observed by comparing (A-26') with (A-21), the effect of x on the initial price p_0 is the same as its effect on the entire path of prices, p_t. This result reflects the finding in (A-25') where it was shown that the change of x does not alter the entire path of interest rates.

Finally, logarithmic differentiation of the real present-value factors, $(\alpha_t p_t^{1-\beta})$ and $(\alpha_t p_t^{1-\beta^*})$ with x, and using (A-25') and (A-26'), yields equations (47)-(47') in the text.

FOOTNOTES

1. For surveys of some of this literature see Fair (1979) and Mussa (1979) and the references therein.
2. See Laursen and Metzler (1950) and Harberger (1950).
3. See contributions by Mundell (1968) and Fleming (1962).
4. See Dornbusch (1976) and Frenkel and Rodriguez (1981).
5. See, for example, Mishkin (1983), Huizinga and Mishkin (1981), Mark (1983) and Cumby and Obstfeld (1984) for studies of real interest rate equality. On covered interest arbitrage see Frenkel and Levich (1977).
6. For this hypothesis see Feldstein and Horioka (1980) and Feldstein (1983), and for some discussions see Harberger (1980) and Tobin (1983).
7. The basic model of these characteristics is developed in Helpman and Razin (1982) who study the implications of its monetary counterpart for the analysis of exchange rate dynamics.
8. On this see Barro (1974).
9. On the role of "transfer problem criteria" see Mussa (1969).
10. As is evident from (15)-(17), the denominator Δ is always positive if $0 < \lambda_t^* < 1$. This statement is obvious when $\delta^* < \delta$. When $\delta^* > \delta$ it is also true since

$$[1-(1-\delta^*)\textstyle\sum_{t=0}^{\infty}(\delta^{*t}-\delta^t)\lambda_t^* > 1 - (1-\delta^*)\bar{\lambda}^*\sum_{t=0}^{\infty}(\delta^{*t} - \delta^t) =$$

$$= [1-(1-\bar{\lambda}^*)\delta - \bar{\lambda}^*\delta^*]/(1-\delta),$$

where $\bar{\lambda}^*$ is the maximal value of λ_t^*. The weighted average $[(1-\bar{\lambda}^*)\delta + \bar{\lambda}^*\delta^*] < 1$ since δ and δ^* are smaller than unity. Thus

also in this case the value of Δ is positive. The only problem may arise if, for some periods, government spending exceeds gross domestic product so as to result in values of λ_t^* that either exceed one or are negative. In that case we impose the requirement that the values of wealth are positive.

11. Equation (18') must be negative for considerations similar to those in footnote 10.

12. In general, to determine the effect of a transfer on the real value of consumption in terms of the consumer price index, we need to determine the change in the real value of wealth as well as in the real interest rate (in terms of the consumer consumption bundle). In the present case, however, since we have used logarithmic utility function, the marginal saving propensity is independent of the rate of interest and, therefore, the entire effect of transfers and other policies on real consumption operate through their effects on real wealth.

13. This result follows by noting that the denominator in equation (37) is positive, and the minimal value of the numerator cannot fall below a positive number.

14. For an analysis of the role of investment in determining the path of the current account see Sachs (1981).

15. For an analysis of these effects see Barro (1979), Kydland and Prescott (1976) and Razin and Svensson (1983).

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