NBER WORKING PAPER SERIES

PENSION PLAN PROVISIONS AND RETIREMENT: MEN & WOMEN, MEDICARE, AND MODELS

Robin L. Lumsdaine

James H. Stock

David A. Wise

Working Paper No. 4201

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 1992

Financial support from the National Institute on Aging, and the Hoover Institution (Wise), is gratefully acknowledged. We also thank Jim Smith and John Rust for helpful comments. This paper is part of NBER's research program in Aging. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #4201
October 1992

PENSION PLAN PROVISIONS AND RETIREMENT: MEN & WOMEN, MEDICARE, AND MODELS

ABSTRACT

The ongoing analysis of the effects of pension plan provisions on retirement is pursued in this paper. A primary objective of this paper is to test the validity of models previously developed and estimated with data from a Fortune 500 company, here using data from a second large company. The evidence confirms that changes in the retirement rates by age correspond closely to provisions of the firm pension plan. There is essentially no difference in the retirement behavior of men and women. As in previous work, it is found that simpler "option value model" of retirement yields very similar results to the considerably more complex stochastic dynamic programming specification. Both fit the data well and predict rather well the effect on retirement of a special retirement window plan. Some consideration is also given to the effects of firm health insurance and median coverage on retirement.

Robin L. Lumsdaine
Department of Economics
Princeton University
Princeton, NJ 08544
and NBER

David A. Wise
NBER
1050 Massachusetts Ave.
Cambridge, MA 02138
and NBER

James H. Stock
Kennedy School of Government
Harvard University
Cambridge, MA 02138
and NBER

Our ongoing analysis of the effects of pension plan provisions on retirement is pursued in this paper. The work to date has emphasized the dramatic effect of employer-provided pension plan provisions on age of retirement and the enormous effects that can be had by changing the provisions. The work has also highlighted the important limitations of predictions of the effect on Social Security provisions on retirement, without accounting for the effect of employer pension plan provisions, which, for employees who have such plans, are typically much more powerful than Social Security provisions.

Two aspects of our work have guided the analysis as the research progressed. The first is that a new method has been used to model retirement decisions. The second is that the empirical analysis has been based on data from individual firms. Thus we have been led to consider whether the model provided accurate predictions of the effect of plan provisions on retirement and whether the behavioral implications of analysis based on data from one firm could be generalized to other firms with different plan provisions.

In two initial papers Stock and Wise [1990a, 1990b] developed an "option value" model of retirement. The central feature of this model is that in deciding whether to retire employees are assumed to compare the "value" of retiring now to the value of the maximum of the expected values of retiring at all future retirement ages. If the maximum of the future values is greater than the value of retirement now, the employee continues to work. We tested the predictive validity of this model in two ways: first, we considered the "within-sample fit" of the model, by comparing the actual pattern of retirement by age to the pattern predicted by the model, based on the data used for estimation. Second, in papers by Lumsdaine, Stock, and Wise [1990, 1991] we emphasized an external "out-of-sample" check of predictive validity, by considering how well the model predicted the effect on retirement of an unanticipated and temporary change in the pension plan provisions, occasioned by an early retirement window plan. In a subsequent paper, Lumsdaine, Stock, and Wise [1992] compared the predictive validity of the option value model and two versions of stochastic dynamic programming models. The stochastic dynamic programming model is close in spirit to the option value model, but the prediction of retirement is based on the comparison of the value of retirement now to the expected value of the maximum of the values of future retirement ages. The evidence was that the option value model predicted just as well as the stochastic dynamic programming models, but had the advantage of being much less complex numerically. Ausink [1991] pursued a similar comparison based on retirement from the military and found that the option value version was noticeably better than the stochastic dynamic programming versions.

All of these papers, with the exception of the work by Ausink, are based on data from a single firm. The use of firm data was motivated by the absence of information on pension plan provisions in standard data sources, like the Retirement History Survey, and by the realization that the incentives inherent in such plans could be very substantial and varied widely among firms, as shown in papers by Bulow [1981], Lazear [1983], and Kotlikoff and Wise [1985, 1987, 1988]. In principle, the ideal data source would provide retirement information and pension plan information for a random sample of employees, from a wide range of firms. Such information has not been available. The alternative we followed was to obtain data from several different firms. The hope was that similar results from different firms would tend to confirm the validity of the model, even though the firms themselves could not be considered a random sample of all firms. Thus there is a need to determine whether the results for the one firm are confirmed based on data from other firms.

Thus the first goal of this paper is to confirm that the age pattern of retirement from the firm corresponds to the pension plan provisions. Descriptive analysis confirms that this is the case.

A second emphasis in this paper is the comparison of the retirement behavior of men and women. It is sometimes proposed that women may tend to retire earlier than men because they are typically younger than their husbands and they may tend to retire when their husbands do. From descriptive analysis, it is clear that the retirement patterns of men and women are not appreciably different.

A third goal is to add another observation to the list of comparisons of the predictive validity of the option value versus the stochastic dynamic programming model. Predictive validity in this paper is judged by the model predictions of retirement under a special 1983 early retirement incentive plan. The models are estimated based on retirement decisions in 1982. The goal of the comparisons is to accumulate data on the extent to which the different models predict actual retirement choices and thus which specification best approximates the considerations that determine actual retirement decisions. The emphasis is not on which model best approximates the economists view of the "right" calculation, but rather which best approximates the calculations that the typical person makes. Or, better still, which predicts best the retirement decisions of the typical employee.

A fourth goal is to make limited inferences about the potential effect of medicare availability on retirement, especially at age 65. We have found in our previous work that model predictions of the age 65 retirement rate are typically much lower than the actual rate. We have attributed the high actual rate to an "age-65-retirement-effect." But our work, and we believe the work of others, has ignored the potential effect of medicare insurance that becomes available at age 65. The approach used here is to consider how retirement rates -- especially at 65 -- would be affected were medicare valued according to the average payments to the covered population.

The final goal is to compare parameter estimates based on data from two firms with different pension plans.

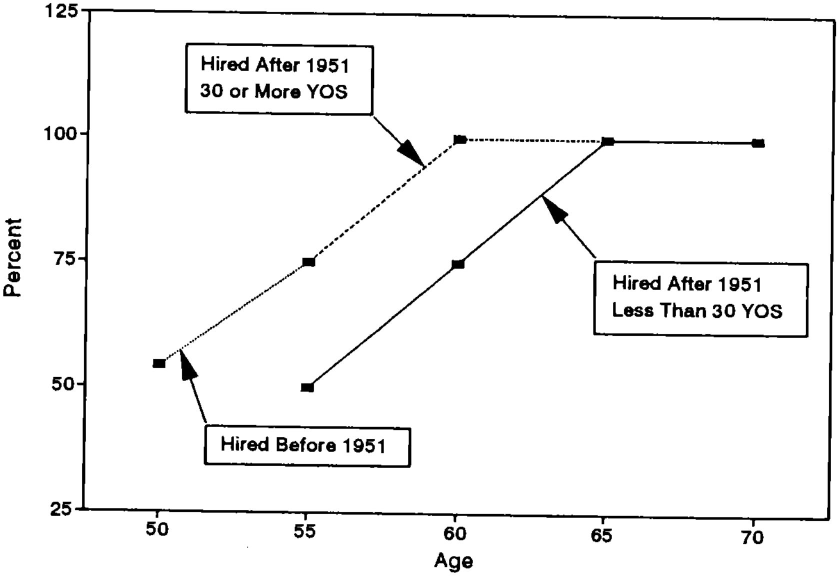
I. BACKGROUND.

A. THE FIRM II PLAN.

Employees are covered by a defined benefit pension plan with normal retirement at 65 and early retirement at 55. Cliff vesting occurs at 10 years of service. The normal retirement benefit at 65 depends on earnings, age, and years of service at retirement (that is, at the time of departure from the firm). A person can retire and elect to start receiving benefits before age 65 but the normal benefit will be reduced by 5 percent for each year that receipt of benefits precedes age 65, as shown in figure 1. A person who retired at age 55, for example, would receive 50 percent of the normal retirement benefit of a person who left the firm at age 65. (The normal benefit also depends on years of service at the time of retirement.)

However, if a person has thirty years of service at retirement, and if the person is age 60 or older, the person is eligible for 100 percent of the normal benefit. Benefits are reduced 5 percent for each year that retirement precedes age 60, if the person has 30 years of service. For example, a person

Figure 1. Early Retirement Benefit
As % of Normal Age 65 Benefit



who retired at age 55 with 30 years of service would receive 75 percent of the normal benefit.

Even a person who retires before age 55 and is vested can elect to receive benefits from the pension plan as early as 55, but like the post-55 retiree, benefits are reduced 5 percent for each year that receipt of the benefits precedes age 65. Of course, this person's benefits would be based on earnings, age, and years of service at the time of retirement, unadjusted for earnings inflation, and would thus be lower than the benefits of a person who retired later.

Employees who joined the firm before 1951, can retire as early as 50 and begin to receive benefits immediately, but at a reduced rate. An employee hired before 1951 had at least 31 years of service in 1982. The reduction for this group is indicated by the extended line that indicates benefits of 54.3 percent of the age 60 benefits for an employee who has 30 or more years of service at retirement.

To understand the effect of the pension plan provisions, figure 2a shows the expected future compensation of a person from our sample who is 51 years old and has been employed by the firm for 23 years. To compute the data graphed in figures 2a-2d, a 5 percent real discount rate and a 6 percent inflation rate are assumed. The discount rate is estimated in the empirical analysis and the inflation rate is assumed to be 6 percent. Total compensation from the firm can be viewed as the sum of wage

Figure 2a. Future Compensation
For Person Age 51 & 23 YOS in 1982

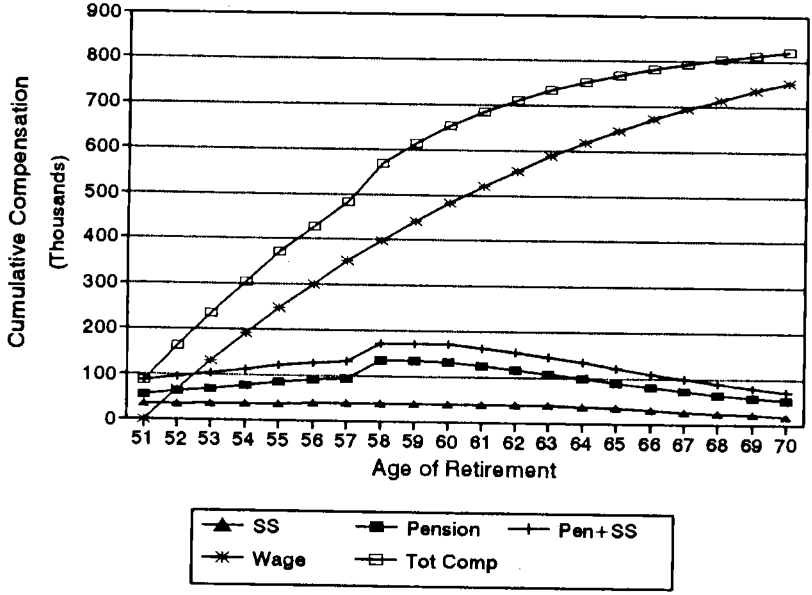


Figure 2b. Future Compensation
For Person Age 57 & 29 YOS in 1982

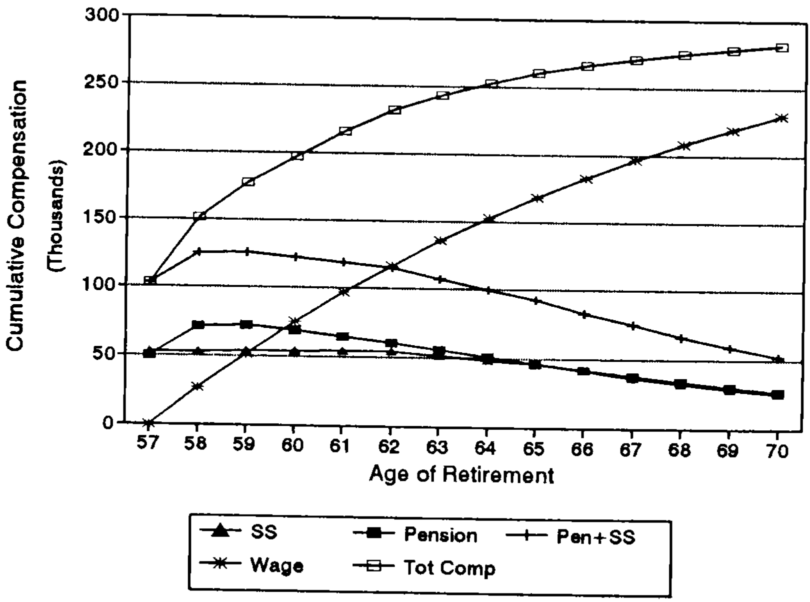


Figure 2c. Future Compensation
For Person Age 60 & 38 YOS in 1982

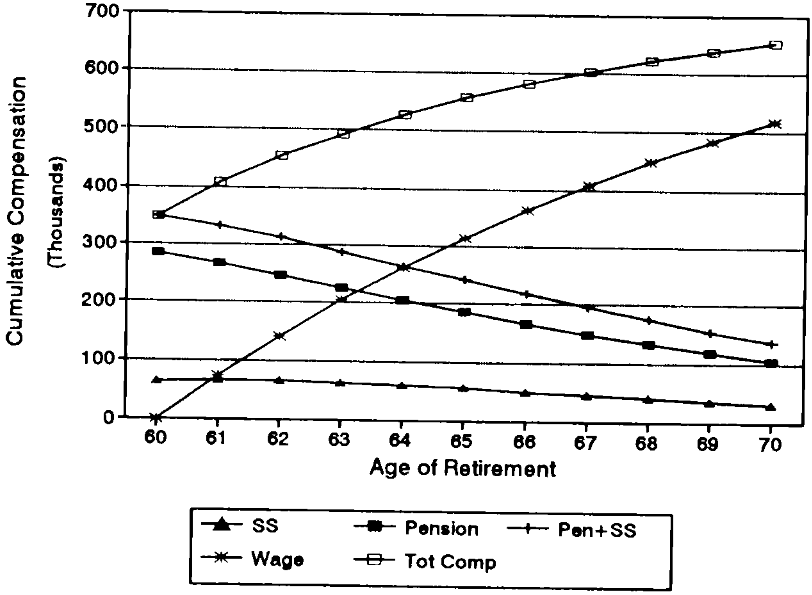
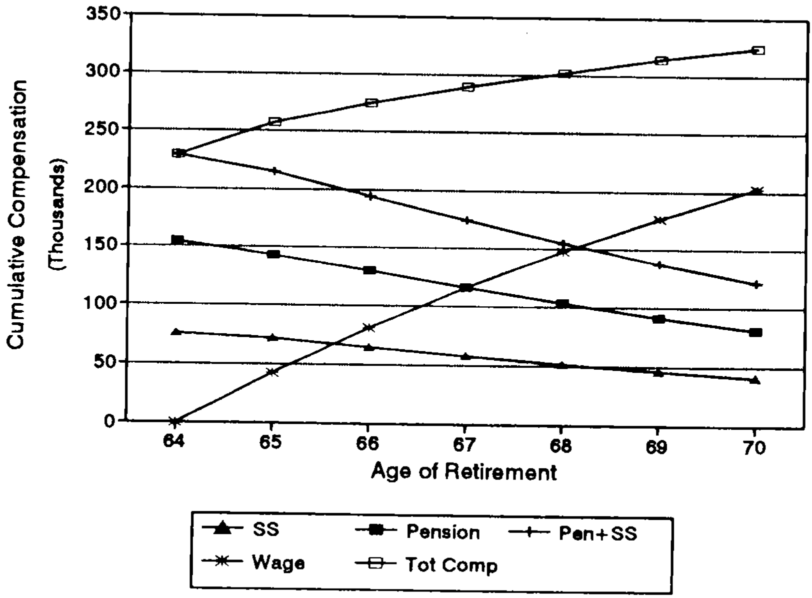


Figure 2d. Future Compensation
For Person Age 64 & 45 YOS in 1982



earnings, the accrual of pension benefits, and the accrual of Social Security benefits. (This omits medical and other unobserved benefits that should be included as compensation, but for which we have no data.) As compensation for working another year the employee receives salary earnings. Compensation is also received in the form of future pension benefits. The annual compensation in this form is the change in the present value of the future pension benefits entitlement, due to working an additional year. This accrual is comparable to wage earnings. The accrual of Social Security benefits may be calculated in a similar manner, and is also comparable to wage earnings. Figure 2a shows the present value at age 51 of expected future compensation in all three forms. Wage earnings represents cumulated earnings, by age of retirement from the firm (more precisely, by age of departure from the firm, since some workers might continue to work in another job). For example, the cumulated earnings of this employee between age 51 and age 60 were he to retire at age 60 would be about $482,000, discounted to age 51 dollars. The slope of the earnings line represents annual earnings discounted to age 51 dollars.

The pension line shows the accrual of firm pension benefits, again discounted to age 50 dollars. The shape of this profile is determined by the pension plan provisions. The present value of accrued pension benefit entitlements at age 51 is about $54,000. The present value of retirement benefits increases between 51 and 57 because years of service and nominal earnings increase. An employee could leave the firm at age 53, for example. If he were to do that, and if he were vested in the firm's pension plan he would be entitled to normal retirement pension benefits at age 65, based on his years of service and nominal dollar earnings at age 53. He could choose to start receiving benefits as early as age 55, the pension early retirement age, but the benefit amount would be reduced 5 percent for each year that the receipt of benefits preceded age 65. Because 5 percent is less than the actuarially fair discount rate, the present value of benefits of a person who leaves the firm before 55 are always greatest if receipt of benefits begins at 55.

Recall that a person who has accumulated 30 years of service and is 55 or older, is entitled to increased retirement benefits that would reach 100 percent of normal retirement benefits at age 60. No early retirement reduction is applied to benefits if they are taken then. So a person with 30 years of service who continues to work will no longer gain 5 percent a year from fewer years of early retirement reduction, as occurs before age 60. There is a jump in the benefits of a person who attains 30 years of service younger than 60. That accounts for the jump in the benefits of the person depicted in the figure 2a, when he attains 30 years of service at age 58.

The Social Security accrual profile is determined by the Social Security benefit provisions. The present value of accrued Social Security benefit entitlements at age 51 is about $33,000. Social Security benefits cannot begin until age 62. If real earnings do not change much between 51 and 62, then real Social Security benefits at 62 will not change much either. After age 62, the actuarial adjustment is such that the present value of benefits, evaluated at the age of retirement, does not depend on the retirement age. But the present value of the benefits discounted to the same age (51 in this case), declines. There is a further drop after age 65 because the actuarial adjustment is reduced from 7 percent to 3 percent.

The top line shows total compensation. For example, the wage earnings of an employee who left the firm at age 60 would increase $482,000 between ages 50 and 60, shown by the wage earnings line. Thereafter, the employee would receive firm pension plan and Social Security retirement benefits with a present value -- at age 50 -- of about $170,000. The sum of the two is about $652,000, shown by the top line. Compared to total compensation of $575,000 between 51 and 60, an average of $63,000 per year, total compensation between 60 and 65 would be only $100,000, or $23,000 per year. Thus the monetary reward for continued work declines dramatically with age.

Figures 2b through 2d show comparable compensation profiles for employees who are 57, 60, and 64 respectively in

1982; they have 29, 38, and 45 years of service respectively. The person depicted in figure 2b attains 30 years of service at age 58, and thus the jump in pension benefits at that age. The present value of pension plus Social Security compensation reaches a maximum at age 59 and declines thereafter. Were this employee to continue to work after 59, until 65, the present value of total retirement benefits would fall by $33,000, offsetting about 28 percent of the present value of wage earning over this period ($117,000). A similar prospect faces the employee depicted in figure 2c, but this employee is already entitled to 100 percent of normal retirement benefits and loses benefits for each year that he continues to work.

The employee who faces the figure 2d compensation profile is 64 and loses both pension and Social Security benefits for each year that retirement is postponed. At age 65, for example, about 54 percent of expected wage earnings would be offset buy a reduction in retirement benefits, if retirement were postponed.

B. THE 1983 WINDOW.

Under the window plan, that was in effect from January 1 to February 28, all employees were eligible for a separation bonus, but the most generous payments were available to persons 55 and older who had at least 21 years of service. Retirement benefits for this group were increased depending on age and years of service. For example, a person age 59 with 28 years of service, could receive 100 percent of normal retirement benefits, instead of 70 percent under the regular plan. That is, this person's retirement benefit would be increased by 43 percent. A person who was 55 with 21 years of service could receive 55 percent of the normal benefits, instead of 50 percent. Persons age 60 or older with 30 years of service were eligible for 100 percent of normal benefits under the regular plan.

In addition, all employees were eligible for a separation bonus equal to one week's pay for every year of service, with a minimum of 2 and a maximum of 26 weeks of pay. Thus even persons who were under 55 and those who were eligible for 100 percent of normal retirement benefits faced an added inducement to retire.

C. THE DATA.

The data used in the analysis are drawn from the personnel records of all persons employed by the firm at any time between 1979 and 1988. A year-end file is available for each year. Earnings records back to 1979 (or to the date of hire if after 1979) are available for each employee. In addition, the data contain some demographic information such as date of birth, sex, marital status, and occupational group. The retirement date of employees who retire is also known. (More generally, the date of any departure is known and the reason for the departure is recorded.) Thus we are able to determine whether a person who was employed at age a was also employed at age a+1, and if not, the exact age at which the employee left the firm.

The estimation of the retirement model in this paper is based on 1982 data, whether an employee left the firm in 1982. (To simplify the determination of age of retirement, only employees born in January and February and who had not retired before March 1, 1982 are used in this analysis.) The primary test of the predictive validity of the model is based on how well the model, estimated on 1982 data, predicts retirement under the 1983 window plan that substantially increased standard retirement benefits.

II. DEPARTURE RATES FOR MEN VERSUS WOMEN.

A. LIFECYCLE DEPARTURE RATES.

Firm departure rates for employees aged 20 through 70 are shown in figure 3. The graph reflects average departure rates over the years 1979 through 1982 After substantial turnover at younger ages, annual departure rates fall continuously to between 1 and 2 percent between ages 45 to 54. Employees start to leave the firm in larger numbers at age 55, the early retirement age.

Figures 4a through 4c compare the departure rates for men and women. Figure 4a pertains to employees with less than 10 years of service, who are not vested in the firm's pension

Figure 3. Departure Rates by Age
Men and Women Together

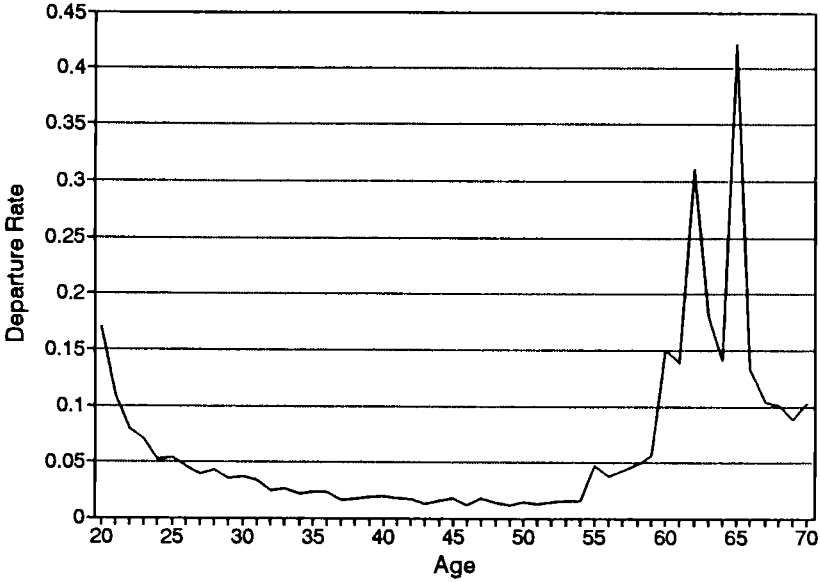


Figure 4a. Departure Rates by Age
Men & Women with Less Than 10 YOS

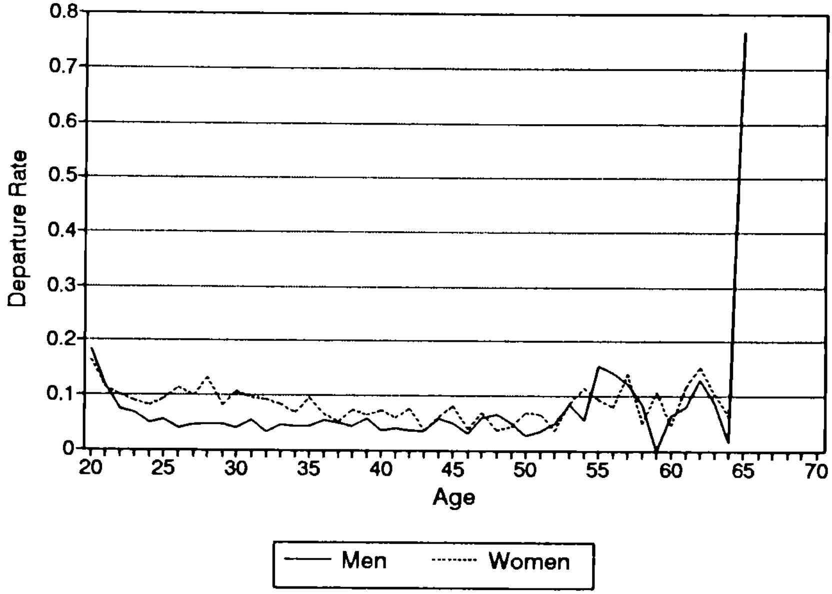


Figure 4b. Departure Rates by Age
Men & Women with 10 to 29 YOS

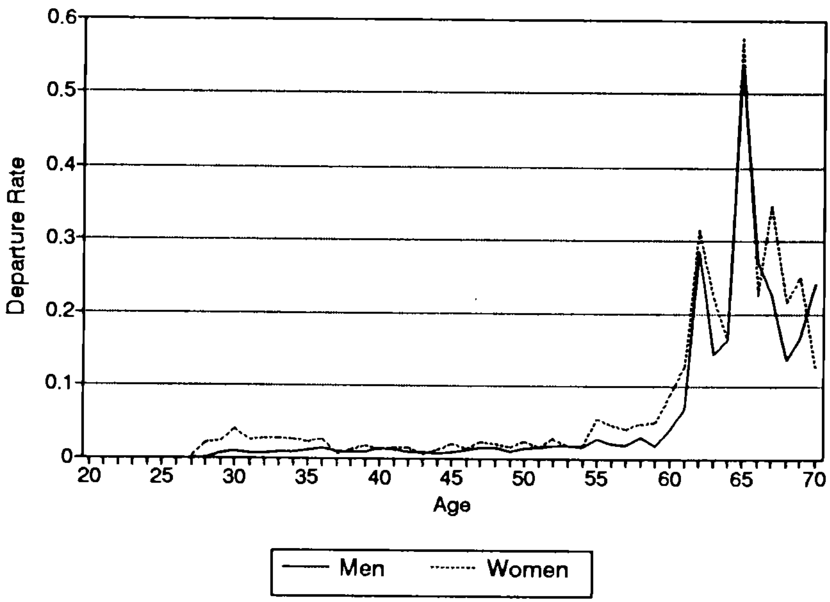
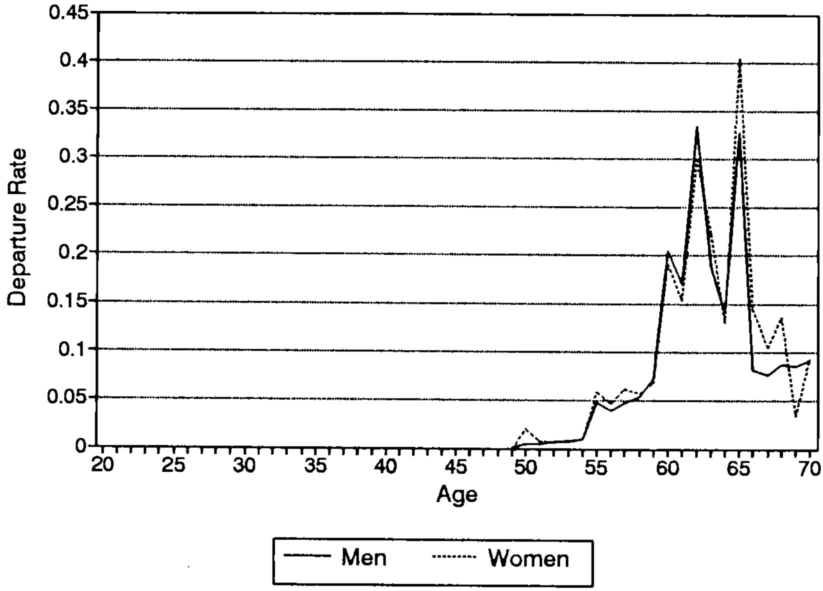


Figure 4c. Departure Rates by Age
Men & Women with 30 or More YOS



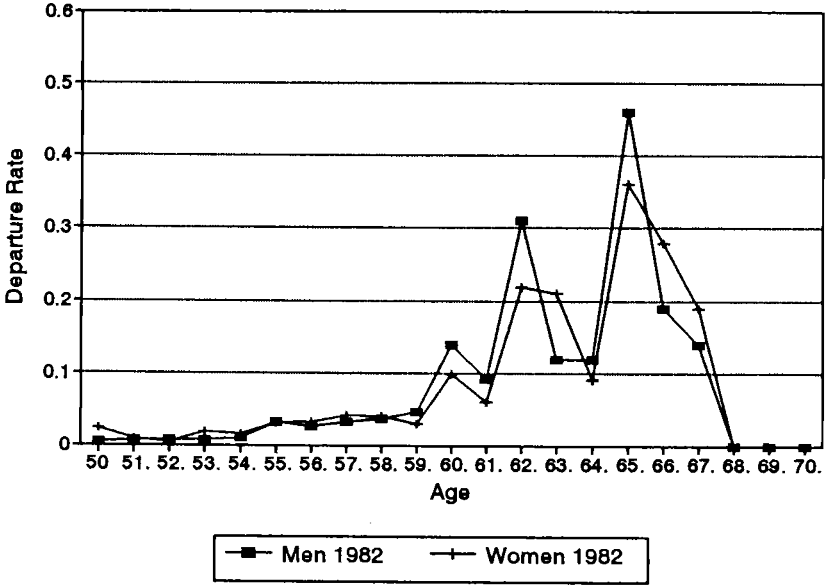
plan. Figure 4b pertains to employees with 10 to 29 years of service and figure 4c to those with 30 or more years of service. The striking aspect of the graphs is that there is virtually no difference between the departure rates of men and women, except at the principle child-bearing ages -- say 23 to 37. For example, between ages 37 and 54 the turnover rates of men and women with 10 to 29 years of service are almost identical. Among employees with less than 10 years of service. there is little difference in the departure rates of men and women between ages 37 and 65. Men and women with 30 or more years of service have almost identical departure rates at all ages.

B. RETIREMENT AGE DEPARTURE RATES.

The departure rates for persons 50 and above are shown in figure 5 for men and women. These rates are based on 1982 data only. There is a noticeable increase in departure rates at 55, from less than 1 percent for persons 50 to 54 to 3 or 4 percent for employees 55 to 59. Although the increase in the annual departure may seem small, the cumulative effect of the increase is substantial. For example, with a 4 percent annual departure rate, 19 percent of persons in the firm at 54 will leave before age 60. At a 1 percent annual rate, only 5 percent will leave.

There is also a sharp increase at age 60, the age at which persons with 30 years of service are entitled to 100 percent of normal (age 65) benefits. The sharp increases at ages 62 and 65

Figure 5. Departure Rates
Men and Women Compared, 1982



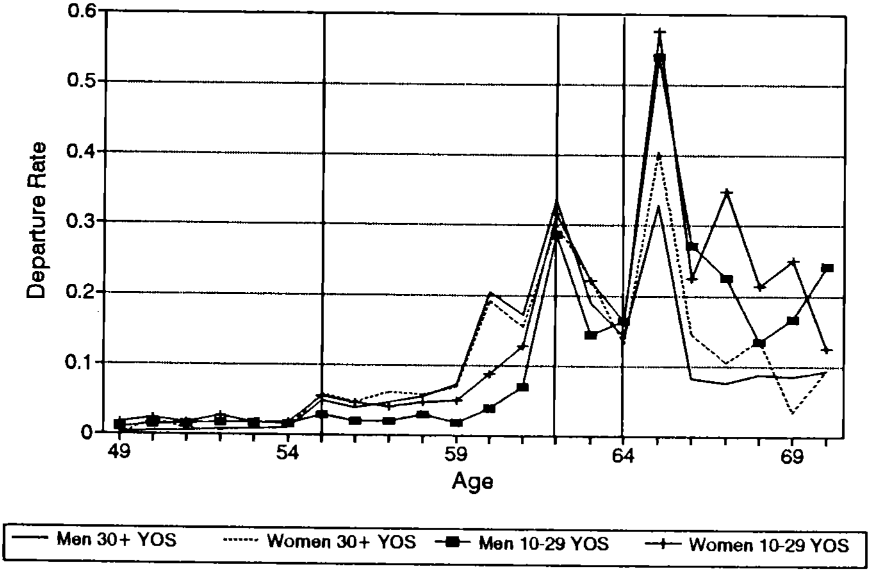
correspond to the Social Security early and normal retirement ages.

The plan provisions suggest that for employees aged 55 to 64, and especially those 55 to 60 or 61, the departure rate for persons with 30 or more years of service should be higher than the rate for persons with less than 30 years of service. The descriptive data are shown in figure 6. The departure rates for men with 30 or more years of service are higher in the 55 to 61 age range. They are also higher for women at age 60, but the differences at other ages are small. Women with less than 30 years of service appear more likely than men to take early retirement between the ages of 55 and 61.

These data also reveal what may be an individual-specific work effect. Employees with 30 or more years of service who have not retired before age 65 are thereafter less likely to retire than employees with less than 30 years of service.

In summary: even without formal analysis, the graphs make it clear that the pattern of departures reflects the provisions of the pension plan. The pattern is also consistent with Social Security provisions, but the magnitude of the age 65 departure rate seems much more abrupt than the reduction in Social Security benefits at 65 would suggest. These graphs also make it clear that there is little appreciable difference in the retirement patterns of men and women, with the possible exception of a greater likelihood of early retirement for women with less than

Figure 6. Departure Rates by Age & YOS
Men & Women Age 50 to 70



30 years of service. But in general, there is no evidence of a substantial difference in the retirement patterns of men and women.

C. WINDOW PLAN RETIREMENT RATES.

Departure rates under the 1983 window plan are shown for men in figure 7a and for women in figure 7b. These rates are contrasted with the average 1982 rates, shown on the same graph. Departure rates under the window plan were typically 3 to 5 times as large as the 1982 rates. Like 1982 departures, there was little difference in the departure rates of men and women under the window plan, as shown in figure 8. There is, however, some indication that women under 55 may have been more likely than men to accept the separation bonus.

III. FORMAL MODELS AND PREDICTION OF RETIREMENT.

A. MODELS.

Two models are compared during the course of the analysis: the "option value" model and a stochastic dynamic programming model. Both are described in Lumsdaine, Stock, and Wise [1992] and excerpts from that paper are included as an appendix to this paper. The models are explained only briefly here.

Figure 7a. Departure Rates for Men
1982 Versus 1983 Window

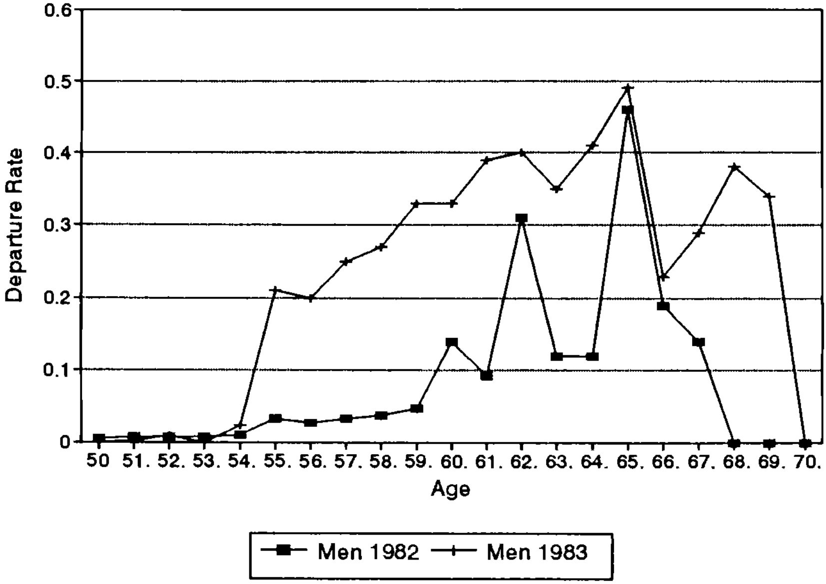


Figure 7b. Departure Rates for Women
1982 Versus 1983 Window

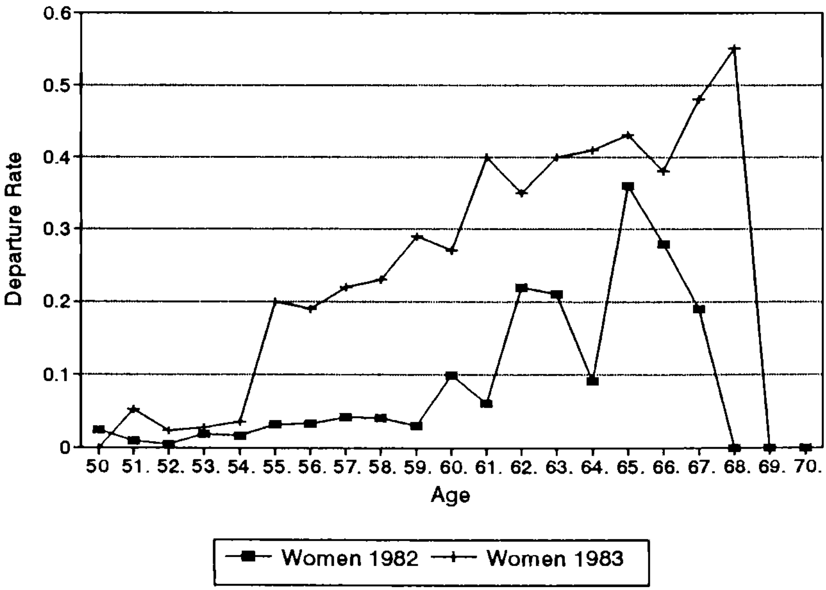
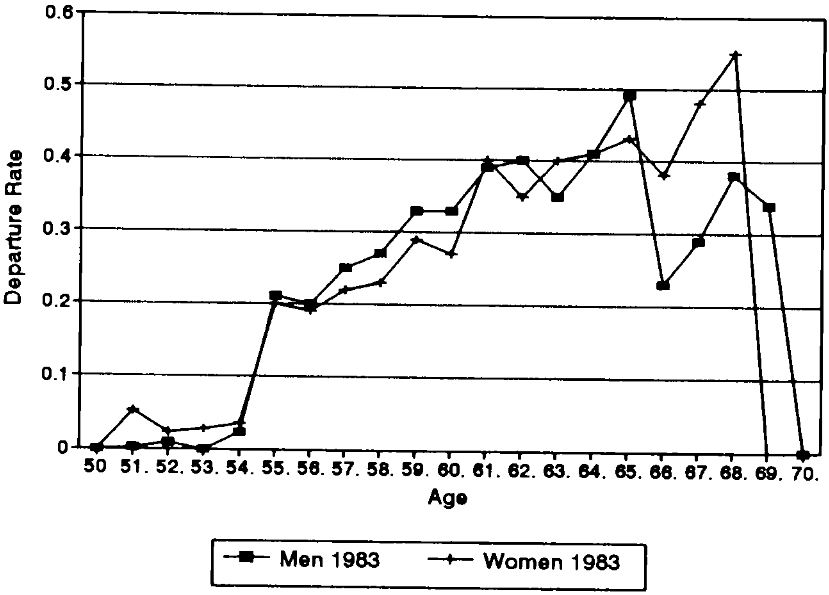


Figure 8. Departure Rates

Men and Women Compared, 1983 Window



1. The Option Value Model.

At any given age, based on information available at that age, it is assumed that an employee compares the expected present value of retiring at that age with the value of retiring at each age in the future through age 70. The maximum of the expected present values of retiring at each future age, minus the expected present value of immediate retirement is called the option value of postponing retirement. A person who does not retire this year maintains the option of retiring at a more advantageous age later on. If the option value is positive, the person continues to work; otherwise she retires. With reference to figure 1, for example, at age 51 the employee would compare the value of the retirement benefits that she would receive were she to retire then -- approximately $87,000 -- with the value of wage earnings and retirement benefits in each future year. The expected present value of retiring at 60 (discounted to age 50), for example, is about $652,000. Future earnings forecasts are based on the individual's past earnings, as well as the earnings of other persons in the firm. The precise model specification follows.

A person at age t who continues to work will earn Y_s in subsequent years s. If the person retires at age r, subsequent retirement benefits will be $B_s(r)$. These benefits will depend on the person's age and years of service at retirement and on his earnings history; thus they are a function of the retirement age.

We suppose that in deciding whether to retire the person weighs the indirect utility that will be received from future income. Discounted to age t at the rate ß, the value of this future stream of income if retirement is at age r is given by

(1) $$V_t(r) = \Sigma_{s=t}^{r-1} \beta^{s-t} U_w(Y_s) + \Sigma_{s=r}^{S} \beta^{s-t} U_r(B_s(r)),$$

where $U_w(Y_s)$ is the indirect utility of future wage income and $U_r(B_s(r))$ is the indirect utility of future retirement benefits. It is assumed that the employee will not live past age S.

The expected gain, evaluated at age t, from postponing retirement until age r is given by

(2) $$G_t(r) = E_t V_t(r) - E_t V_t(t).$$

Letting r* be the age that gives the maximum expected gain, the person will postpone retirement if the option value, $G_t(r^*)$, is positive,

(3) $$G_t(r^*) = E_t V_t(r^*) - E_t V_t(t) > 0 .$$

The utilities of future wage and retirement income are parameterized as

(4a) $$U_w(Y_s) = Y_s^{\gamma} + \omega_s$$

(4b) $$U_r(B_s) = (kB_s(r))^\gamma + \xi_s$$

where ω_s and ξ_s are individual-specific random effects, assumed to follow a Markovian (first order autoregressive) process

(5a) $$\omega_s = \rho\omega_{s-1} + \epsilon_{\omega s} \, , \, E_{s-1}(\epsilon_{\omega s}) = 0 \, ,$$

(5b) $$\xi_s = \rho\xi_{s-1} + \epsilon_{\xi s} \, , \, E_{s-1}(\epsilon_{\xi s}) = 0 \, .$$

The parameter k is to recognize that in considering whether to retire the utility associated with a dollar of income while retired may be different from the utility associated with a dollar of income accompanied by work. Abstracting from the random terms, at any given age s, the ratio of the utility of retirement to the utility of employment is $[k(B_s/Y_s)]^\gamma$.

2. The Stochastic Dynamic Programming Model.

The key simplifying assumption in the Stock-Wise option value model is that the retirement decision is based on the maximum of the expected present values of future utilities if retirement occurs now versus each of the potential future ages. The stochastic dynamic programming rule considers instead the expected value of the maximum of current versus future options. The expected value of the maximum of a series of random variables will be greater than the maximum of the expected values. Thus to the extent that this difference is large, the

Stock-Wise option value rule underestimates the value of postponing retirement. And to the extent that the dynamic programming rule is more consistent with individual decisions than the option value rule, the Stock-Wise rule may undervalue individual assessment of future retirement options. Thus we consider a model that rests on the dynamic programming rule.

It is important to understand that there is no single dynamic programming model. Because the dynamic programming decision rule evaluates the maximum of future disturbance terms, its implementation depends importantly on the error structure that is assumed. Like other users of this type of model, we assume an error structure -- and thus a behavioral rule -- that simplifies the dynamic programming calculation.[1] In particular, although the option value model allows correlated disturbances, the random disturbances in the dynamic programming model are assumed to be uncorrelated. Thus the two models are not exactly comparable. Whether one rule is a better approximation to reality than the other may depend not only on the basic idea, but on its precise implementation. In the version of the dynamic programming model that we implement here the disturbances are assumed to follow an extreme value distribution.

---

[1]See the appendix for a more complete description of the error structure.

In most respects our dynamic programming model is analogous to the option value model. As in that model, at age t an individual is assumed to derive utility $U_w(Y_t) + \epsilon_{1t}$ from earned income or $U_r(B_t(s)) + \epsilon_{2t}$ from retirement benefits, where s is the retirement age. The disturbances ϵ_{1t} and ϵ_{2t} are random perturbations to these age-specific utilities. Unlike the additive disturbances in the option value model, these additive disturbances in the dynamic programming model are assumed to be independent. Future income and retirement benefits are assumed to be nonrandom; there are no errors in forecasting future wage earnings or retirement benefits.

B. RESULTS.

Parameter estimates are shown in table 1. The effect of the special plan provisions for the pre-1951 hires is considered first (columns (1) and (2)). Estimates for men versus women and stochastic dynamic programming versus option value estimates are then considered (columns (3) through (8)). Finally, estimates are also presented with the "value" of medicare and firm retiree health insurance benefits counted as equivalent to Social Security benefits, and with firm current employee health insurance benefits counted as equivalent to wage earnings (column (9)).

Table 1. Parameter Estimates, by Method and Sample

	Pre-1951 Provisions		Option Value Versus SDP Model & Men Versus Women						Medicare	Firm I Comparison
Parameter	SDP Model Pre 51 Ignore	SDP Model Pre 51 Correct	OV Model Men	OV Model Women	OV Model Men & Women	SDP Model Men	SDP Model Women	SDP Model Men & Women	SDP Model Men & Women	SDP Model Men
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Gamma	1.045	1.018	0.599	0.850	0.656	0.898	0.866	0.839	0.789	1.018
	(0.245)		(0.155)	(0.178)	(0.080)	(0.134)	(0.245)	(0.098)	(0.127)	
K	1.605	1.932	2.516	1.656	2.408	1.858	1.446	1.877	1.892	1.881
	(0.147)		(0.722)	(0.358)	(0.272)	(0.250)	(0.362)	(0.035)	(0.073)	
Beta	0.185	0.600	0.973	0.872	0.963	0.549	0.301	0.564	0.630	0.620
	(0.337)		(0.053)	(0.114)	(0.020)	(0.135)	(0.398)	(0.032)	(0.032)	
Sigma	0.224	0.170	0.114	0.095	0.120	0.252	0.224	0.229	0.203	0.303
	(0.054)		(0.031)	(0.026)	(0.027)	(0.031)	(0.045)	(0.024)	(0.034)	(0.024)
Log Likelihood:										
At Maximum	122.839	121.233	380.59	100.41	485.64	385.86	101.84	489.69	489.34	387.00
Age	127.051	127.051	362.55	91.26	461.30	362.55	91.26	461.30	461.30	
Chi Square:										
Fitted Data	13.059	11.453	28.24	14.30	30.92	64.30	17.12	59.17	61.78	78.89
Window	78.997	46.491	74.04	39.84	86.52	49.59	15.60	68.69	86.79	89.87
Figure	9a-9c	10a-10c	11a-11c	—	—	12a-12c	—	—	13a-13c	—

1. Pre-1951 Pension Plan Provisions.

The estimates in column (1) are based on the assumption that the pre-1951 hires face the same pension plan provisions as later hires. These estimates, as well as those in column (2), are based on a sample of 400 employees. Taken literally, the estimated value of gamma (1.045) suggests that with respect to retirement income employees are essentially risk neutral. The estimated value of K is 1.605, implying that a dollar of retirement benefit income -- unaccompanied by work -- is valued at 60 percent more than a dollar of income that is accompanied by work. These estimates are very similar to those obtained in our previous work. The estimated value of beta, however, is extremely small. If taken literally, it would suggest that in making retirement decisions, future income is given very little weight, compared to income in the current year. Indeed, a value of zero would imply that the decision to retire is based only on the comparison of wage income versus retirement benefits -- the replacement ratio, without concern for future possibilities. When the immediate ratio is large enough, the person retires. [Based on our experience elsewhere, we are not inclined to believe this estimate.]

The model fits the data rather well, however, as shown in figures 9a and 9b. The principle discrepancy between actual and predicted rates occurs at age 55, where the jump in the predicted rates is noticeably less than the jump in the sample

Figure 9a. Predicted Versus Actual
Annual Departure Rates

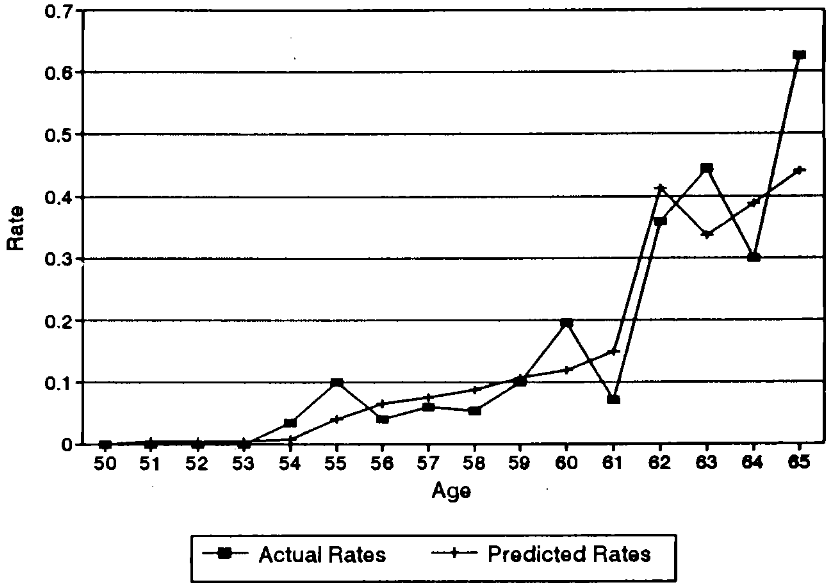


Figure 9b. Predicted Versus Actual
Cumulative Departure Rates

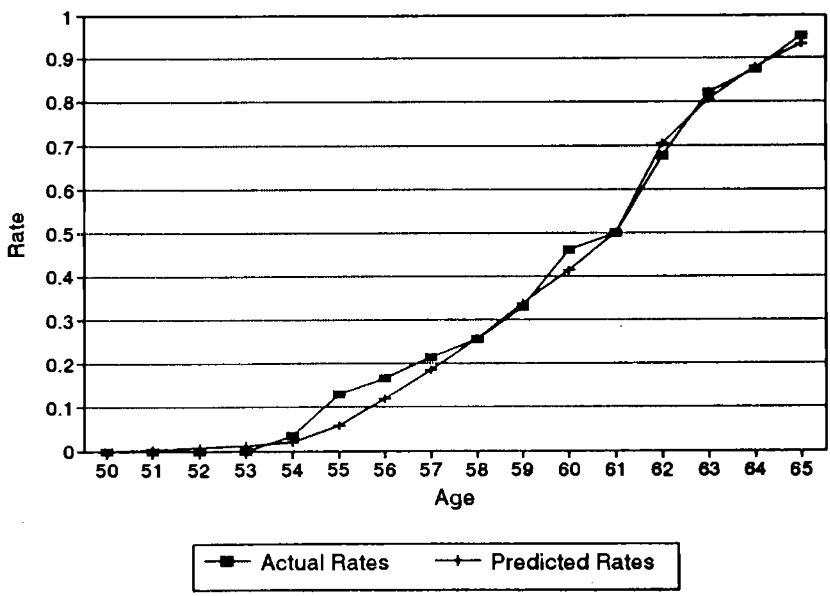
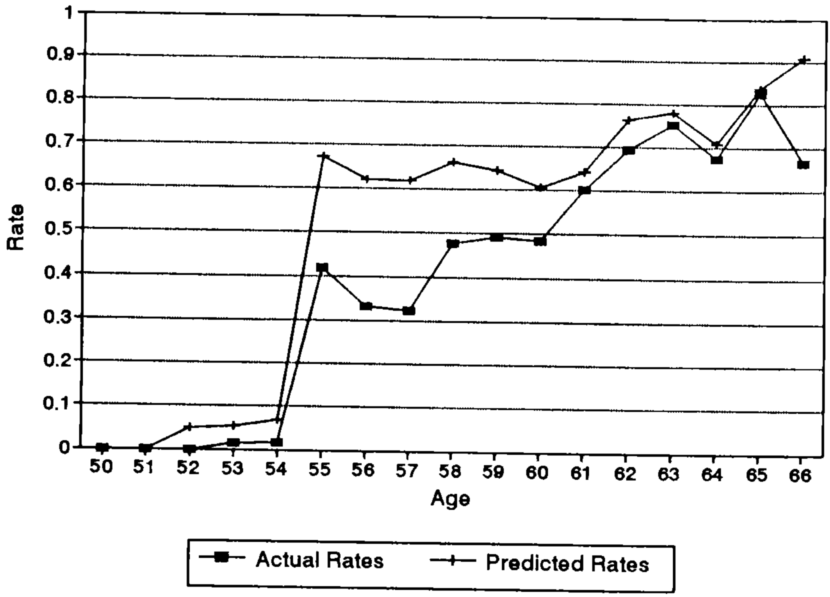


Figure 9c. Predicted Versus Actual
Window Departure Rates



rates at 55. The sample data show a 10 percent departure rate at 55, which is twice as large as the rates shown in the graphs above, based on larger sample sizes. The predicted and actual cumulative departure rates are very close. Based on the likelihood values, the model fits the data better than a model with dummy variables for each age -- that is, better than predictions based on average retirement rates by age. The model does not allow directly for an effect of age on retirement.

The primary test of the predictive validity of the model is how well it predicts retirement rates under the 1983 window plan. The model predictions capture the general pattern of retirement under the window, but substantially overpredict retirement rates between 55 and 60, as shown in figure 9c. The model also predicts some retirements among employees 52 to 54, whereas the actual data show essentially no retirements in this age group.

The estimates in column 2 are obtained if the special pension plan provisions that pertain to employees hired before 1951 are used to determine their options. (Because there are so few retirements among employees younger than 55, we have questioned whether these provisions translate into visible alternatives that are actively considered by older employees, or whether in practice these older employees consider their options to be the same as employees of the same age who are covered by the current plan provisions. Thus we have obtained the two sets of estimates.) The estimated parameter values are very similar to those reported in column 1, although to hasten convergence the discount factor is set in this case. Predicted versus actual rates are shown in figures 10a through 10c. In general, the fit to actual values is close. The major exception is at age 55 and in this case the actual sample rate is abnormally high; the predicted rate is more in line with typical retirement rates at this age. The difference in the age 55 retirement rate is reflected in the difference between the actual and predicted cumulative rates through age 60. The model predictions of the effects of the 1983 window are very accurate, with the exception of predictions for employees 53-54 and 56-57. The actual sample rates for the 56-57 ages are abnormally low; the typical rates are more like the model predictions. Thus for these ages at least, the model predictions give a more accurate indication of actual behavior than the actual sample values.

2. Stochastic Dynamic Programming Versus "Option Value" Estimates.

The two sets of parameter values are shown for men (columns (3) and (6)), for women (columns (4) and (7)), and for men and women combined (columns (5) and (8)). In general, the estimated parameters are similar. The most noticeable difference is that the SDP estimated values of beta are lower than the option value estimates. For men and women combined, for example, the SDP estimate is .565 and the option value estimate .953.

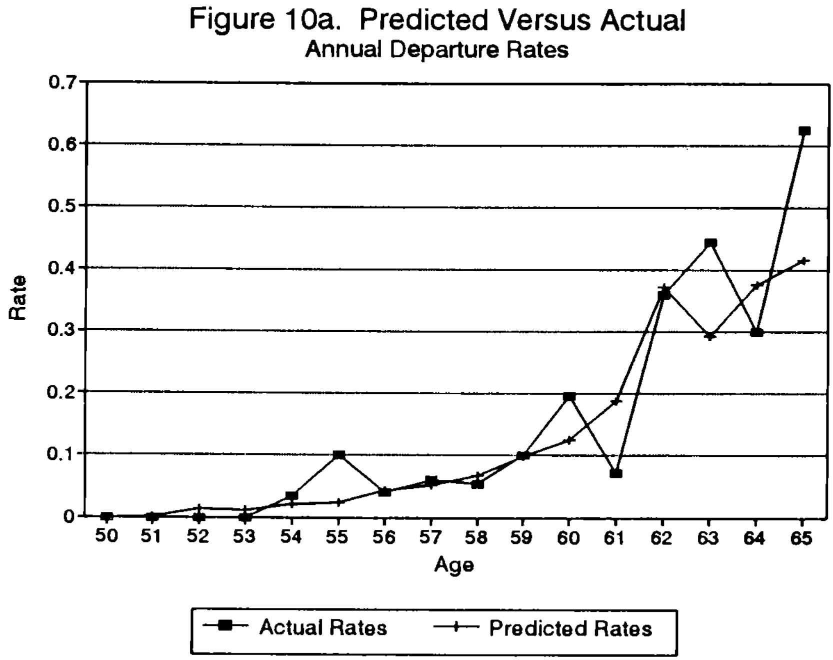


Figure 10a. Predicted Versus Actual
Annual Departure Rates
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Rate
50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65
Age
Actual Rates
Predicted Rates

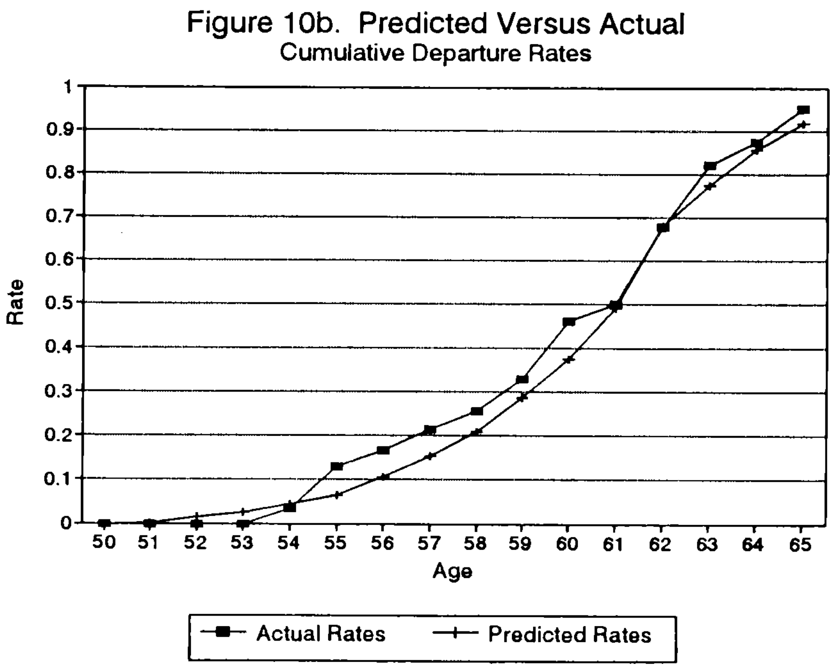
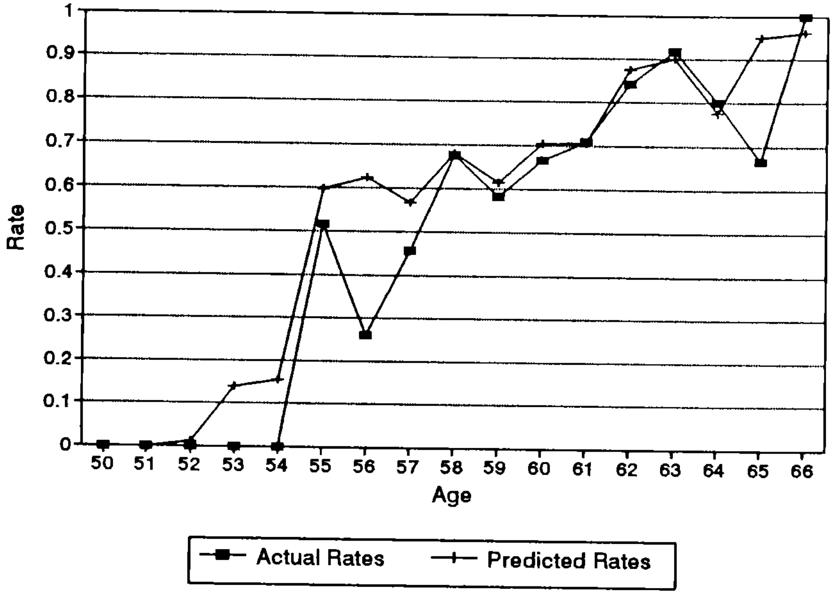


Figure 10b. Predicted Versus Actual
Cumulative Departure Rates
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Rate
50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65
Age
Actual Rates
Predicted Rates

Figure 10c. Predicted Versus Actual
Window Departure Rates



The estimated value of K based on the OV model is somewhat larger than the SDP estimate (2.231 versus 1.446) and the estimated value of gamma is somewhat smaller (0.656 versus 0.839).

The option value model fits the sample data considerably better than the SDP model, based on the likelihood and chi-square values pertaining to the fitted data. This is revealed graphically for men in figures 11a and b versus 12a and b. On the other hand, the SDP model predictions of retirement under the 1983 window fit actual retirement rates better than the option value model. This can be seen by comparing figures 11c and 12c and in the chi-square values pertaining to the window. Thus, in general, there is no reason to prefer one model over the other.

3. Separate Estimates for Men and Women.

Based on the option value model, the estimates for men and women are statistically different but based on the SDP model they are not, judged by likelihood ratio tests. The option value model chi-squared statistic is 9.28 (and with 4 degrees of freedom the .05 significance level is 9.49). The SDP model chi-squared statistic is only 3.98, however, which is not statistically significant. The t-statistics for the individual parameters also suggest that the estimates for men and women are not statistically different. Thus the formal estimates appear to be consistent with the graphical evidence in figures 4, 5, 6,

Figure 11a. Predicted Versus Actual
Annual Departure Rates--OV-Men

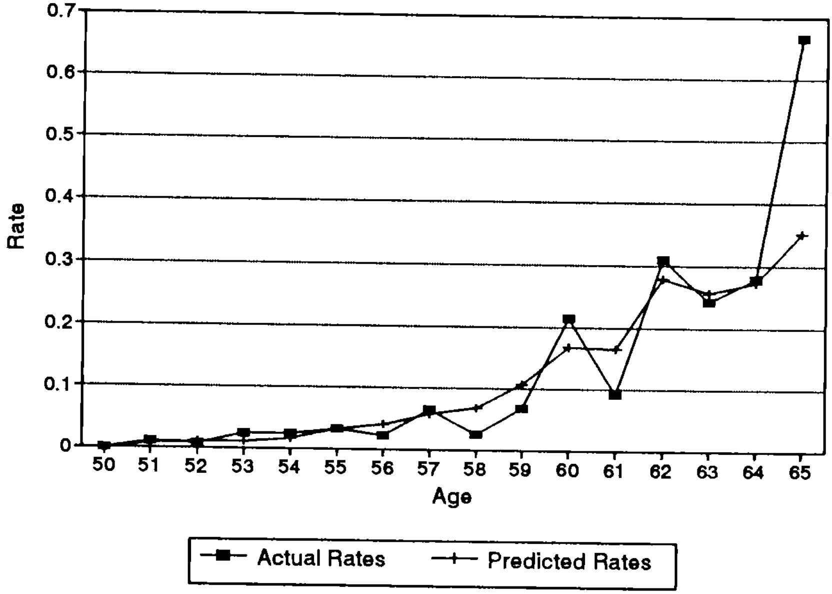


Figure 11b. Predicted Versus Actual
Cumulative Departure Rates--OV-Men

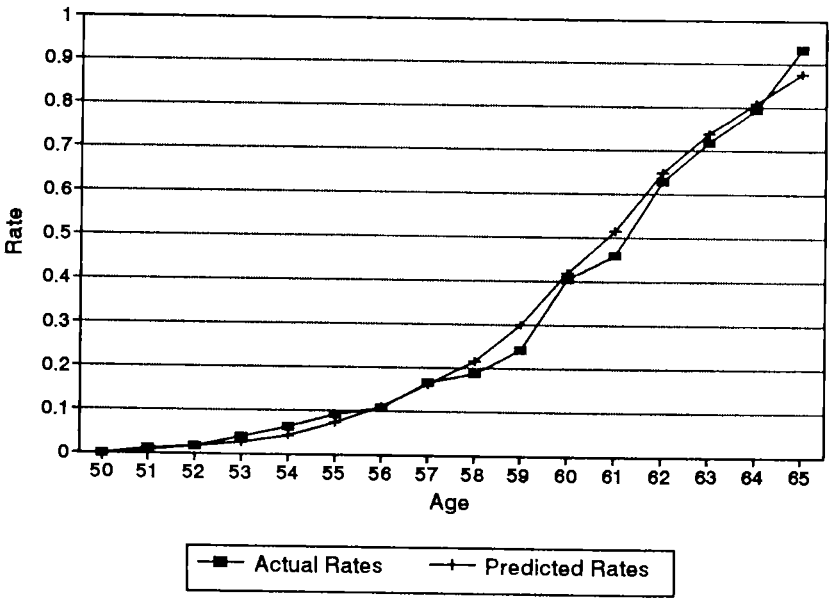


Figure 11c. Predicted Versus Actual
Window Departure Rates--OV-Men

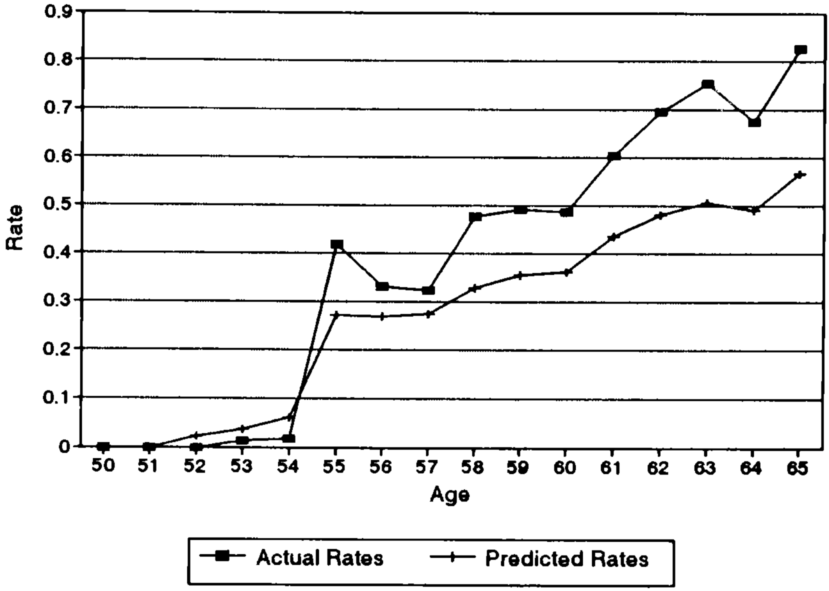


Figure 12a. Predicted Versus Actual
Annual Departure Rates--SDP-Men

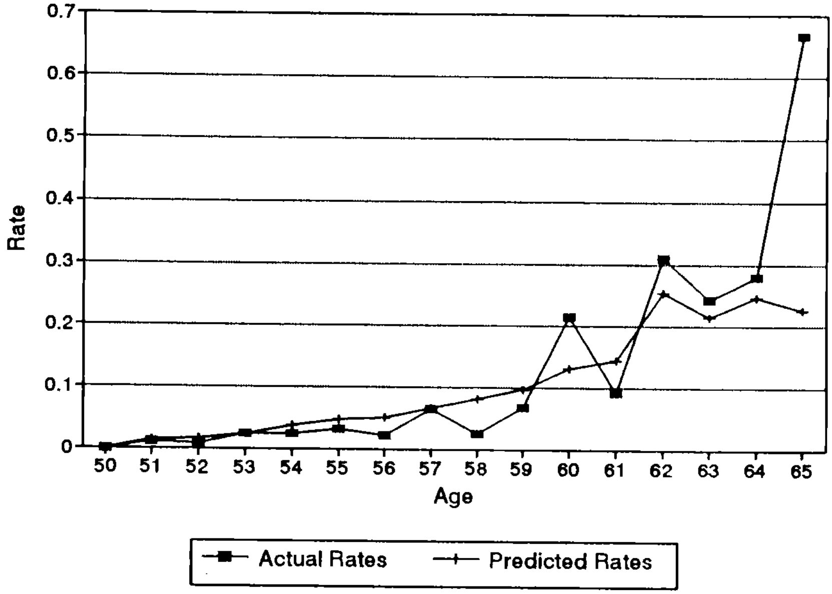


Figure 12b. Predicted Versus Actual
Cumulative Departure Rates--SDP-Men

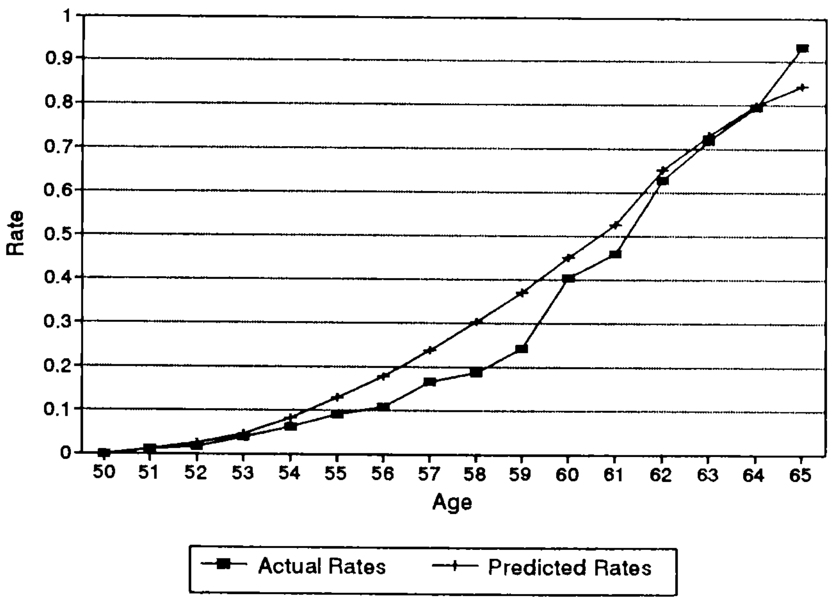
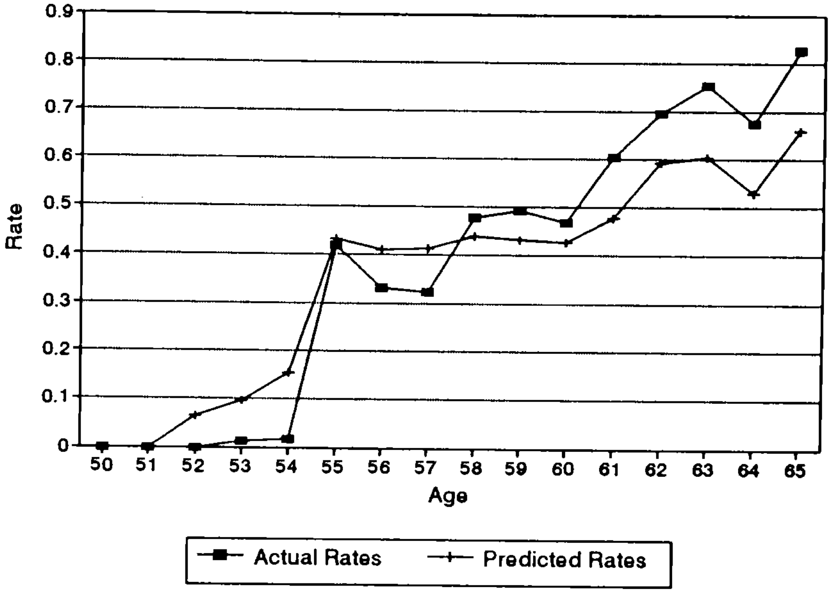


Figure 12c. Predicted Versus Actual
Window Departure Rates--SDP-Men



and 8 showing that departure rates for men and women are virtually indistinguishable after age 40.

4. Valuing Medicare.

Both the option value and the SDP models underpredict retirement at age 65 for both men and women. A possible reason for the underprediction is that medicare insurance becomes available at 65 and that provides an inducement to retire similar to the Social Security inducement. But employees at this firm have health insurance while working. And after retirement the same coverage is provided, at no cost to the retiree. For example, a person who retired at age 60 would be covered by retiree health insurance until age 65. After age 65, medical costs up to the medicare limit would be paid by medicare and any additional costs -- that are covered by the firm plan -- would be paid by the firm retiree insurance. A simple assumption, albeit one that is unlikely to be precisely true, is that medical insurance is valued at its cost, which is treated by employees as comparable to wage or pension benefit compensation. Following this rule, there are three parts to medical coverage: first, while employed at the firm, health benefits are valued at the cost of insurance to the firm.[2] Second, if the person retires before 65,

---

[2]This cost was estimated by the average cost at large firms for group insurance with coverage like the plan offered by our firm -- $105 and $247 for individual and family coverage respectively in 1989 dollars. These costs were deflated to 1982 dollars based on a constructed index of Blue Cross Blue Shield premiums per insured

firm pension benefits are increased by the cost of insurance with coverage comparable to the retiree health insurance.[3] After age 65, Social Security benefits are increased according to the average payment to persons covered by medicare.[4] Estimates incorporating these assumptions and based on the SDP model for men and women are reported in column (9).

The parameter estimates are affected very little, relative to comparable estimates without these adjustments, shown in column (8) of table 1. The likelihood value and the fitted data chi-square statistic are almost the same as the comparable column (8) estimates, that do not account for the value of medical insurance. In particular, the addition of these measures of the value of medical insurance does nothing to explain the departure rate at 65, as can be seen in figure 13a. The actual rate is .636

---

person, obtained from the U.S. Health Insurance Association (1991).

[3]The value of this insurance was estimated by increasing the basic group insurance premium according to age, by 5.4 percent per year for each year after age 50. This rate is based on the annual premium costs by age reported by the Congressional Research Service (1988).

[4]The costs were estimated based on the average 1986 Medicare payments by age to married and single persons reported in Shoven, Topper, and Wise (1992) in this volume. The 1986 values were deflated to 1982 dollars based on the Blue-Cross-Blue-Shield index described in the footnote above. Linear interpolation was used to convert the payments by age interval reported by Shoven, Topper, and Wise to payments for each age.

Figure 13a. Predicted Versus Actual
Annual Dep Rates--SDP-M&W-Medicare

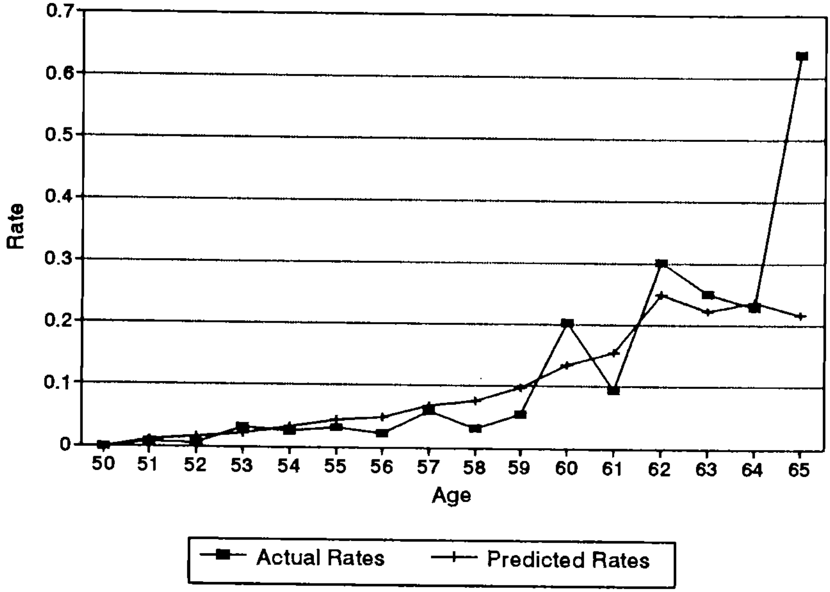


Figure 13b. Predicted Versus Actual
Cumulative Dep Rates--SDP-M&W-Medicare

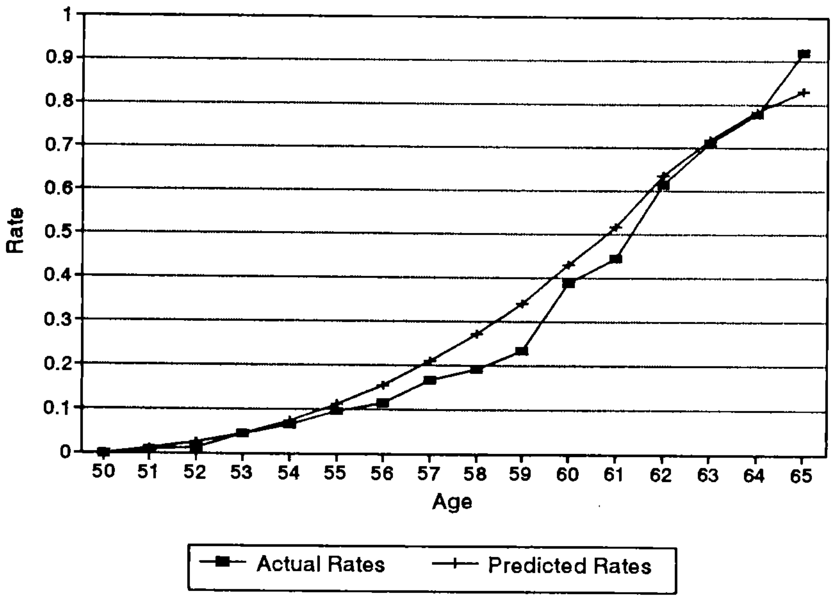
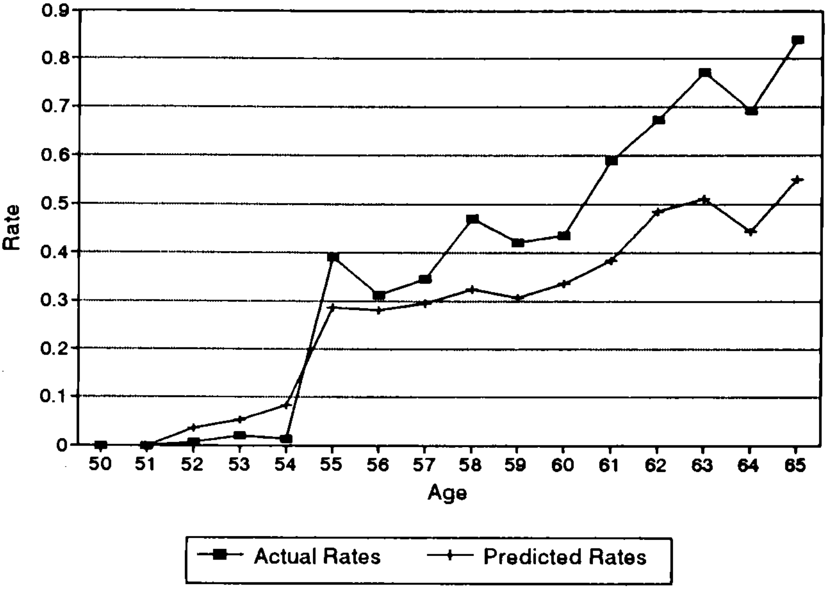


Figure 13c. Predicted Versus Actual
Window Dep Rates--SDP-M&W-Medicare



but the predicted rate is only .215, somewhat lower than the predicted rate without accounting for medical insurance (.227, based on column (8) estimates). In addition, the model yields worse predictions of retirement under the window plan, judging by the window chi-square statistic. Thus these results lend no support to the conjecture that retirement at 65 is strongly affected by the availability of medicare at that age. However, these exploratory results should not be interpreted to mean that medicare doesn't matter. It may well be that the rough specification that we experimented with does not capture the effect of medicare, but that a more careful treatment of the value of medical coverage would show an effect. For example, the assumption that medical insurance is valued at its cost may be incorrect.

5. Estimates From Firms I and II Compared.

The parameter estimates on Firm II data are surprisingly close to those based on Firm I data. Results, for Firm II, with parameters estimates set to those that we obtained for Firm I [1992] are shown in column (10) of table 1. By comparing the estimates in columns (6) and (10), it can be seen that the Firm I estimates for men are very close to the estimates for Firm II, based on the stochastic dynamic programming specification. The hypothesis that the parameters are the same cannot be rejected, based on a likelihood ratio test. From the chi-square statistics, however, it is clear that the Firm I parameter estimates do not fit the data or predict departure rates under the window plan quite as well as the Firm II estimates. Option value model estimates for the two firms (not shown) are also similar but not as close as the stochastic dynamic programming estimates and the hypothesis that the estimates are the same is rejected at the 5 percent level. Again, based on chi-square statistics, the Firm I estimates do not fit the data or predict window departure rates as well as the Firm II parameter estimates. On balance, however, the results provide strong confirmation that employees in these two firms react similarly to the incentives inherent in pension plan provisions.

IV. SUMMARY AND CONCLUSIONS.

The data for Firm II confirm a principle conclusion based on Firm I data. It is clear that the changes in retirement rates by age correspond closely to provisions of the pension plan. And, like the results based on Firm I, we find that the option value and the stochastic dynamic programming models yield similar results. There is no apparent reason to choose one over the other, except based on numerical simplicity. In this case, the option value model fits the sample data better than the stochastic dynamic programming model, but the SDP model predicts the window plan retirement rates better than the option value model. We also find that there is essentially no difference in the retirement behavior of men and women. There is some indication that women may be slightly more likely than men to take early retirement between 55 and 60. But at most ages, the annual retirement rates of men and women are very close. In addition, we explored the possibility that retirement at 65 is induced by Medicare benefits that become available at that age. Our method of incorporating medical insurance, however, did little to explain the large retirement rates at 65. Thus we are still left with an "age-65-retirement-effect" that is not explained by monetary gain .

REFERENCES

Ausink, John A. 1991. "The Effect of Changes in Compensation On a Pilot's Decision to Leave the Air Force." Ph.D. Dissertation, Harvard University.

Bulow, J. 1981. "Early Retirement Pension Benefits." National Bureau of Economic Research Working Paper no. 654.

Congressional Research Service. 1988. "Costs and Effects of Extending Health Insurance Coverage."

Kotlikoff, Laurence J. and David A. Wise. 1985. "Labor Compensation and the Structure of Private Pension Plans: Evidence for Contractual Versus Spot Labor Markets," in D. Wise (ed.), Pensions, Labor and Individual Choice. Chicago: University of Chicago Press.

________________________. 1987. "The Incentive Effects of Private Pension Plans," in Z. Bodie, J. Shoven, and D. Wise (eds.), Issues in Pension Economics. Chicago: University of Chicago Press, pp. 283-339.

________________________. 1988. "Pension Backloading, Wage Taxes, and Work Disincentives," in L. Summers (ed.), Tax Policy and the Economy, Vol. 2. Cambridge, MA: MIT Press.

Lazear, Edward P. 1983. "Pensions as Severance Pay," in Z. Bodie and J. Shoven (eds.), Financial Aspects of the

United States Pension System. Chicago: University of Chicago Press.

Lumsdaine, Robin L., James H. Stock, and David A. Wise. 1990. "Efficient Windows and Labor Force Reduction," *Journal of Public Economics* 43, pp. 131-159.

______________________. 1991. "Windows and Retirement," *Annales d'Economie et de Statistique* 20/21, pp. 219-242.

______________________. 1992. "Three Models of Retirement: Computational Complexity versus Predictive Validity," in D. Wise (ed.), Topics in the Economics of Aging. Chicago: University of Chicago Press, pp. 19-57.

Rust, John P. 1989. "A Dynamic Programming Model of Retirement Behavior," in D. Wise (ed.), The Economics of Aging. Chicago: University of Chicago Press, pp. 359-398.

Shoven, John B., Michael D. Topper, and David A. Wise. 1992. "The Impact of the Demographic Transition on Government Spending."

Stock, James H. and David A. Wise. 1990a. "Pensions, The Option Value of Work, and Retirement," *Econometrica*, Vol. 58, no. 5, September, pp. 1151-1180.

____________________. 1990b. "The Pension Inducement to Retire: An Option Value Analysis," in D. Wise (ed.), Issues in the Economics of Aging. Chicago: University of Chicago Press, pp. 205-224.

U.S. Health Insurance Association. 1991. "Sourcebook of Health Insurance Data, 1990."

Appendix

The "option value" and stochastic dynamic programming models used in the analysis are described.

A. The Option Value Model.

Given the specification as described through equation (5) in the text, the function $G_t(r)$ can be decomposed into two components

$$(6) \qquad G_t(r) = g_t(r) + \phi_t(r)$$

where $g_t(r)$ and $\phi_t(r)$ distinguish the terms in $G_t(r)$ containing the random effects, ω and ξ, from the other terms. If whether the person is alive in future years is statistically independent of his earnings stream and the individual effects ω_s and ξ_s, $g_t(r)$ and $\phi_t(r)$ are given by

$$(7a) \qquad g_t(r) = \Sigma_{s=t}^{r-1} \beta^{s-t} \pi(s|t) E_t(Y_s^\gamma)$$
$$+ \Sigma_{s=r}^{S} \beta^{s-t} \pi(s|t)[E_t(kB_s(r))^\gamma]$$
$$- \Sigma_{s=t}^{S} \beta^{s-t} \pi(s|t)[E_t(kB_s(t))^\gamma]$$

$$(7b) \qquad \phi_t(r) = \Sigma_{s=t}^{r-1} \beta^{s-t} \pi(s|t) E_t(\omega_s - \xi_s),$$

where $\pi(s|t)$ denotes the probability that the person will be alive in year s, given that he is alive in year t. Given the random Markov assumption, $\phi_t(r)$ can be written as

$$(8) \qquad \phi_t(r) = \Sigma_{s=t}^{r-1} \beta^{s-t} \pi(s|t) \rho^{s-t} (\omega_t - \xi_t)$$

$$= K_t(r) \upsilon_t ,$$

where $K_t(r) = \Sigma_{s=t}^{r-1} (\beta\rho)^{s-t} \pi(s|t)$ and $\upsilon_t = \omega_t - \xi_t$. The simplification results from the fact that at time t the expected value of $\upsilon_s = \omega_s - \xi_s$ is $\rho^{s-t}\upsilon_t$, for all future years s. (The term $K_t(r)$ cumulates the deflators that yield the present value in year t of the future expected values of the random components of utility. The further r is in the future, the larger is $K_t(r)$. That is, the more distant the potential retirement age, the greater the uncertainty about it, yielding a heteroskedastic disturbance term.)

$G_t(r)$ may thus be written simply as

$$(9) \qquad G_t(r) = g_t(r) + K_t(r) \upsilon_t .$$

If the employee is to retire in year t, $G_t(r)$ must be less than zero for every potential retirement age r in the future. If $r_t^\dagger$ is the r that yields the maximum value of $g_t(r)/K_t(r)$, the probability of retirement becomes

$$(10) \qquad \Pr[\text{Retire in year t}] = \Pr[g_t(r_t^\dagger)/K_t(r_t^\dagger) < -\upsilon_t] .$$

If retirement in only one year is considered, this expression is all that is needed.

More generally, retirement decisions may be considered over two or more consecutive years. In this case the retirement probabilities are simply an extension of equation (10). The probability that a person who is employed at age t will retire at age $\tau > t$ is given by

$$(11) \quad \Pr[R=\tau] = \Pr[g_t(r_t^\dagger)/K_t(r_t^\dagger) > -\upsilon_t, \ldots, \\ g_{\tau-1}(r_{\tau-1}^\dagger)/K_{\tau-1}(r_{\tau-1}^\dagger) > -\upsilon_{\tau-1}, \\ g_\tau(r_\tau^\dagger)/K_\tau(r_\tau^\dagger) < -\upsilon_\tau] \; .$$

The probability that the person does not retire during the period of the data is given by

$$(12) \quad \Pr[R>T] = \Pr[g_t(r_t^\dagger)/K_t(r_t^\dagger) > -\upsilon_t, \ldots, \\ g_{T-1}(r_{T-1}^\ddagger)/K_{T-1}(r_{T-1}^\ddagger) > -\upsilon_{T-1}, \\ g_T(r_T^\ddagger)/K_T(r_T^\ddagger) > -\upsilon_T] \; .$$

This is a multinomial discrete choice probability with dependent error terms υ_s.

Finally, we assume that υ_s follows a Gaussian Markov process, with

(13) $\upsilon_s = \rho\upsilon_{s-1} + \epsilon_s, \quad \epsilon_s$ i.i.d. $N(0,\sigma_\epsilon^2)$,

where the initial value, υ_t, is i.i.d. $N(0,\sigma^2)$ and is independent of ϵ_s. The covariance between υ_τ and $\upsilon_{\tau+1}$ is $\rho var(\upsilon_\tau)$, and the variance of υ_τ for $\tau > t$ is $(\rho^{2(\tau-t)})\sigma^2 + (\Sigma_{j=0}^{\tau-t-1} \rho^{2j})\sigma_\epsilon^2$.

The estimates in this paper are based on retirement decisions in only one year and the random terms in equation (5) are assumed to follow a random walk, with $\rho = 1$. In this case, the covariance between υ_τ and $\upsilon_{\tau+1}$ is $var(\upsilon_\tau)$, and the variance of υ_τ for $\tau \geq t$ is $\sigma^2 + (\tau-t)\sigma_\epsilon^2$. Prior estimates show that one- and multiple-year estimates are very similar.[1]

B. The Stochastic Dynamic Programming Model.

The dynamic programming model is based on the recursive representation of the value function. At the beginning of year t, the individual has two choices: retire now and derive utility from future retirement benefits, or work for the year and derive utility from income while working during the year and retaining the option to choose the best of retirement or work in the next year. Thus the value function W_t at time t is defined as

1. Estimates based on several consecutive years and with ρ estimated are shown in Stock and Wise [1990a]. These generalizations have little effect on the estimates.

$$(15)\quad W_t = \max\{E_t[U_w(Y_t) + \epsilon_{1t} + \beta W_{t+1}],$$
$$E_t[\Sigma_{\tau=t}^{S} \beta^{\tau-t}(U_r(B_\tau(t)) + \epsilon_{2\tau})]\},$$
$$\text{with } W_{t+1} = \max\{E_{t+1}[U_w(Y_{t+1}) + \epsilon_{1t+1} + \beta W_{t+2}],$$
$$E_{t+1}[\Sigma_{\tau=t+1}^{S} \beta^{\tau-t-1}(U_r(B_\tau(t+1)) + \epsilon_{2\tau})]\},$$

etc. ...

where β is the discount factor and, as in the option value model, S is the year beyond which the person will not live.

Because the errors ϵ_{it} are assumed to be i.i.d., $E_t\epsilon_{it+\tau} = 0$ for $\tau > 0$. In addition, in computing expected values, each future utility must be discounted by the probability of realizing it, i.e., by the probability of surviving to year τ given that the worker is alive in year t, $\pi(\tau|t)$. With these considerations, the expression (15) can be written as

$$W_t = \max\{\bar{W}_{1t} + \epsilon_{1t}, \bar{W}_{2t} + \epsilon_{2t}\}\text{ , where}$$

$$(16)\quad \bar{W}_{1t} = U_w(Y_t) + \beta\pi(t+1|t)E_tW_{t+1}$$
$$\bar{W}_{2t} = \Sigma_{\tau=t}^{S} \beta^{\tau-t}\pi(\tau|t)U_r(B_\tau(t)).$$

The worker chooses to retire in year t if $\bar{W}_{1t}+\epsilon_{1t} < \bar{W}_{2t}+\epsilon_{2t}$; otherwise he continues working. The probability that the individual retires is $\Pr[\bar{W}_{1t}+\epsilon_{1t} < \bar{W}_{2t}+\epsilon_{2t}]$.

If a person works until the mandatory retirement age (70), he retires and receives expected utility $\bar{W}_{2t_{70}}$.

2. Recursions and computation.

With a suitable assumption on the distribution of the errors ϵ_{it}, the expression (16) provides the basis for a computable recursion for the nonstochastic terms $\bar{W}_{it}$ in the value function. The extreme value and normal distribution versions of the model are considered in turn.

a. Extreme Value Errors. Following Berkovec and Stern [1988], the ϵ_{it} are assumed to be i.i.d. draws from an extreme value distribution with scale parameter σ. Then, for the years preceding mandatory retirement, these assumptions together with equation (16) imply that

$$
\begin{aligned}
E_t W_{t+1}/\sigma &\equiv \mu_{t+1} \\
&= \gamma_e + \ln[\exp(\bar{W}_{1t+1}/\sigma) + \exp(\bar{W}_{2t+1}/\sigma)] \qquad (17)\\
&= \gamma_e + \ln[\exp(U_w(Y_{t+1})/\sigma)\exp(\beta\pi(t+2|t+1)\mu_{t+2}) \\
&\qquad + \exp(\bar{W}_{2t+1}/\sigma)]
\end{aligned}
$$

where γ_e is Euler's constant. Thus (17) can be solved by backwards recursion, with the terminal value coming from the terminal condition that $\mu_{t_{70}} = \bar{W}_{2t_{70}}$.

The extreme value distributional assumption provides a closed form expression for the probability of retirement in year t:

$$(18) \quad \Pr[\text{Retire in year t}] = \Pr[\bar{W}_{1t} + \epsilon_{1t} < \bar{W}_{2t} + \epsilon_{2t}]$$
$$= \exp(\bar{W}_{2t}/\sigma)/[\exp(\bar{W}_{1t}/\sigma) + \exp(\bar{W}_{2t}/\sigma)].$$

b. Gaussian Errors. Following Daula and Moffitt [1989], the ϵ_{it} are assumed to be independent draws from an $N(0,\sigma^2)$ distribution. The Gaussian assumption provides a simple expression for the probability of retiring:

$$(19) \quad \Pr[\text{Retire in year t}] = \Pr[(\epsilon_{1t}-\epsilon_{2t})/\sqrt{2}\sigma$$
$$< (\bar{W}_{2t}-\bar{W}_{1t})/\sqrt{2}\sigma] = \Phi(a_t),$$

where $a_t = (\bar{W}_{2t}-\bar{W}_{1t})/\sqrt{2}\sigma$. Then the recursion (16) becomes:

$$(20) \quad E_t W_{t+1}/\sigma \equiv \mu_{t+1} = (\bar{W}_{1t+1}/\sigma)(1-\Phi(a_{t+1}))$$
$$+ (\bar{W}_{2t+1}/\sigma)\Phi(a_{t+1}) + \sqrt{2}\phi(a_{t+1})$$

where $\phi(\bullet)$ denotes the standard normal density, and $\Phi(\bullet)$ denotes the cumulative normal distribution function. As in (19), $\Phi(a_t)$ is the probability that the person retires in year t and receives utility $\bar{W}_{2t}$, plus utility from $E(\epsilon_{2t} \mid \epsilon_{1t}-\epsilon_{2t} < \bar{W}_{2t}-\bar{W}_{1t})$. The latter term, plus a comparable term when the person continues to work, yields the last term in equation (20).

3. Individual-specific effects.

Individual-specific terms are modeled as random effects but are assumed to be fixed over time for a given individual. They enter the two versions of the dynamic programming models in different ways. Each is discussed in turn.

a. Extreme Value Errors. Single year utilities are

(21a) $U_w(Y_t) = Y_t^\gamma$

(21b) $U_r(B_t(s)) = (\eta k B_t(s))^\gamma$

where ηk is constant over time for the same person but random across individuals. Specifically, it is assumed that η is a lognormal random variable with mean one and scale parameter λ: $\eta = \exp(\lambda z + \frac{1}{2}\lambda^2)$, where z is i.i.d. N(0,1). A larger λ implies greater variability among employee tastes for retirement versus work; when $\lambda = 0$ there is no variation and all employees have the same taste.

b. Normal Errors. In this case, the unobserved individual components are assumed to enter additively, with

(22a) $U_w(Y_t) = Y_t^\gamma + \zeta$

(22b) $U_r(B_t(s)) = (kB_t(s))^\gamma$

where γ and k are nonrandom parameters, as above, but ζ is a random additive taste for work, assumed to distributed $N(0,\lambda^2)$. When $\lambda = 0$, there is no taste variation.

In summary: the dynamic programming models are given by the general recursion equation (15). It is implemented as shown in equation (17) under the assumption that the ϵ_{it} are i.i.d. extreme value, and as shown in equation (20) under the assumption that ϵ_{it} are i.i.d. normal. The retirement probabilities are computed according to equations (18) and (19) respectively. The fixed effects specifications are given by equations (21) and (22). The unknown parameters to be estimated are $(\gamma,k,\beta,\sigma,\lambda)$. Because of the different distributional assumptions, <u>the scale parameter σ is not comparable across option value or dynamic programming models, and λ is not comparable across the two dynamic programming models</u>.