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BANKING AND INSURANCE

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ABSTRACT

This paper studies the economic role of financial institutions in economies where agents' incomes are subject to privately observable, idiosyncratic random events. The information structure precludes conventional insurance arrangements. However, a financial institution--perhaps best viewed as a savings bank--can provide partial insurance by generating a time pattern of deposit returns that redistributes wealth from agents with high incomes to those with low incomes, resulting in a level of expected utility higher than that achievable in simple security markets. Insurance is incomplete because the bank faces a tradeoff between provision of insurance and maintenance of private incentives.

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I. Introduction

Economic analysis of institutions, including that of financial institutions such as banks, is entering an important third stage of development. Theoretical models of institutional organization are currently being developed on a wide variety of fronts[1] with a common emphasis on the importance of private information, incentives and contract enforcement. This third stage of economic analysis holds out the promise of providing theoretical explanations of the patterns of institutional structure and behavior catalogued in the initial stage, the detailed classificatory studies initiated early in the present century. Further, current theoretical explanations promise to be more intellectually satisfying and empirically relevant than earlier second-stage explanations that stressed amorphous transactions costs.

This paper investigates the nature of financial arrangements in an environment where individual agents' incomes are subject to idiosyncratic random shocks. Risk averse individuals desire insurance against such disturbances, but conventional insurance arrangements are not feasible because these income fluctuations are not publicly observed. A financial institution--perhaps best visualized as a savings bank--can provide partial insurance by generating a time pattern of deposit returns that redistributes income from agents with high incomes to those with low incomes, resulting in a level of expected utility for depositors that exceeds other market alternatives. Our development of this banking theory builds on the earlier analysis of Diamond and Dybvig [1983] and stresses the similarity of expected utility maximizing banking arrangements to optimal taxation of saving,

[1] See, for example, Radner [1981] and Fama-Jensen [1982].

analogous to discussions of income taxation by Mirrlees [1971]. As in the public finance literature, the bank faces a tradeoff between provision of insurance and maintenance of private incentives. Consequently, insurance is typically incomplete.

The organization of the remainder of the paper is as follows. In section II, we specify the economic environment and individuals' preferences. In section III, we compare three market institutions: full insurance, autarky and an ex post security market. The former is not feasible given the informational requirements of such a system, but serves as a useful "ideal." An ex post security market dominates autarky, but neither provides insurance against individual income shocks. In section IV, we discuss the nature of banking and its provision of partial insurance in our economy, with an explicit derivation and analysis of the nature of the optimal bank in a limited (linear) class of candidate institutions. In section V, we analyze two topics concerning the relationship between deposit banking and ex post security markets: (i) an equivalence between deposit banking and an ex post market in derivative claims and (ii) potential arbitrage opportunities occasioned by simultaneous operation of deposit banking and markets in underlying securities. In section VI, we further consider the insurance aspects of deposit banking in our model and the types of alterations that would yield more conventional insurance companies. We also provide a summary of our work and discussion of related ongoing research in section VII.

II. The Model Economy

The hypothetical economy that we study has three periods: a planning period (t=0) and two periods with production and consumption (t=1,2), as in Diamond and Dybvig (1983).

Preferences

There are an infinite number of agents, all of whom have the following identical valuation of consumption goods in $t = 1,2$.

$$(1) \qquad U = G(u)$$

where $u(c_1,c_2) = [c_1^{1-1/\sigma} + \beta c_2^{1-1/\sigma}]^{\sigma-1}$ and $G(u) = \frac{1}{1-\gamma} u^{1-\gamma}$. These preferences depend on three parameters: the discount factor $0 < \beta < 1$; the intertemporal elasticity of substitution in consumption, $\sigma > 0$; and relative risk aversion toward random variations in lifetime wealth at known prices, $\gamma \geq 0$.

Endowments

Each individual has an endowment of the single good in each period. At periods 0 and 2, all agents have identical endowments ϕ and y_2. At period 1, each agent receives a privately observable income level $y_1(\theta) = y_1 + \theta$, where y_1 is the level of per capita income at date 1. The idiosyncratic component of income, θ, has expected value of zero and is continuously distributed on $(\underline{\theta},\bar{\theta})$ with strictly positive density function $f(\theta)$. As our model has a continuum of traders, who may be indexed at date 1 by the realized value of θ, each value of the distribution is realized. That is, there are no 'aggregate shocks' in our model because per capita income is simply y_1.

Production Opportunities

In addition to these endowments, agents have intertemporal production opportunities--storage technologies--of two types. The first type (A) transforms a unit of goods stored at t into a unit of goods at t+1. The second type (B) transforms a unit of goods stored at t into R > 1 units in period t+2. The B storage technology is illiquid in that no output can be retrieved from a period t investment at period t+1. Table 1 indicates the time structure of production in our economy.

Table 1
Intertemporal Production Possibilities

	Process A investment at 0	Process B investment at 0	Process A investment at 1
t=0	-1	-1	
t=1	1	0	-1
t=2		R>1	1

Throughout our discussion, we denote the fraction of initial wealth (ϕ) invested in the process A as k.

Consumption Demand and Lifetime Utility

In the bulk of our discussion below, we will view our individual agent as facing sequential market opportunities with (i) the rate of return r_0 earned between t=0 and t=1; and (ii) the rate of return r_1 earned between t=1 and t=2. Thus, upon realization of θ at t=1, our agent maximizes (1) subject to the constraint

(2) $$c_1 + c_2/r_1 \le r_0\phi + y_1(\theta) + y_2/r_1 \equiv a(r_0,r_1,\theta).$$

Preferences represented by (1) imply that the consumption demands are proportional to wealth

(3) $$c_1^* = h(r_1)\ a(r_0,r_1,\theta)$$
$$c_2^* = r_1(1-h(r_1))\ a(r_0,r_1,\theta).$$

where $h(r_1) = [1+\beta^{\sigma} r_1^{\sigma-1}]^{-1}$ is the first-period propensity to consume out of wealth.

In these settings, because our agents face uncertainty about "lifetime wealth," we can separate the effects of attitudes toward risk aversion from those concerning the time pattern of consumption. That is, once individuals enter period 1, they face neither uncertain income nor risky assets. Lifetime utility, but not the consumption strategy, depends on the risk aversion parameter γ. To see this, recall that lifetime utility has the form $G(u) = u^{1-\gamma}/(1-\gamma)$, where $u(c_1,c_2)$ is the CES function specified above. The maximized value of u, which we denote v, is linear in wealth

(4) $$v = u(c_1^*,c_2^*) = \alpha(r_1)a(r_0,r_1,\theta)$$

where $\alpha(r_1) = h(r_1)^{1/(1-\sigma)}$ is the marginal effect of a change in wealth. Thus, lifetime realized utility--conditional on a value of θ--is

(5) $$G(v) = \frac{1}{1-\gamma}\ [\alpha(r_1)]^{1-\gamma}[a(r_0,r_1,\theta)]^{1-\gamma},$$

where (as discussed above) γ controls the individuals' aversion to bets on lifetime wealth. For $\gamma > 0$, individuals are risk averse.

III. Markets, Insurance and Liquidity

A useful starting point for our analysis is consideration of three alternative 'trading arrangements' that might arise in our economy.

Complete Insurance

The case of complete insurance is a useful benchmark case. In this benchmarking discussion, we assume that individual incomes/outputs are publicly observable at zero cost, although this strictly violates the character of our economic environment as detailed above. Since enforceable contracts must be contingent on observable variables, public knowledge of endowments allows standard insurance policies.

As idiosyncratic income (θ) has zero mean, each individual should be able to fully and costlessly insure against θ at date t=0. This insurance is desirable so long as individuals are risk averse.

With full insurance arranged at date t=0, individuals need not engage in any other market transactions. That is, they may directly invest in process A and process B to the points that are efficient given the returns implied by this technology ($r_0=1$ and $r_1=R$). These investments will yield the consumption levels shown in Figure 1, where $c_1 = h(R)a(1,R,0)$ and $c_2 = R[1-h(R)]a(1,R,0)$, where the post-insurance value of wealth is just $\phi + y_1 + y_2/R = a(1,R,0)$.[2]

Autarky

The polar extreme to the regime of perfect insurance is autarky, in which an individual agent cannot make any trades.[3] In autarky, our model implies that agents face two types of uncertainty as a result of θ, which are illustrated in Figure 2a. For a given value of k, the fraction of wealth

[2] In our analysis, here and below, we assume that ϕ is sufficiently large relative to y_1 and y_2 so that market equilibrium takes place "off the corner" at the aggregate level as shown in Figure 1. That is, individuals will want to save some portion of ϕ.

[3] This might be viewed as the outcome of an explicit restriction on trade or on information that rules out exchange of investments, securities, etc.

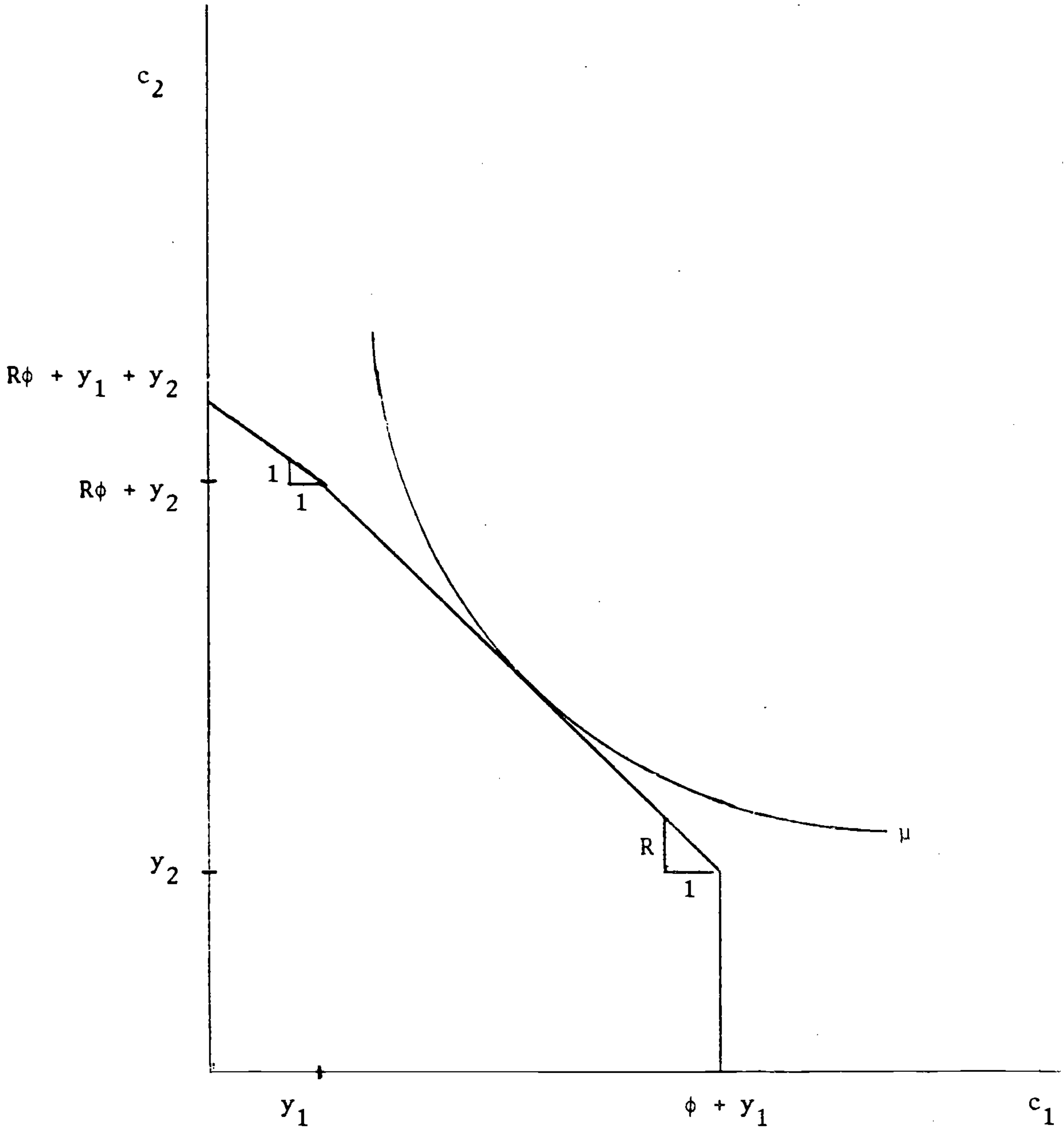

Figure 1

Equilibrium with θ Publicly Observable

Note: movements along the budget line reflect variations in k, the fraction of initial wealth invested in the long-term asset.

allocated to the 'liquid' asset, these are (i) uncertain income and (ii) uncertain liquidity. The first of these involves parallel shifts in the budget line (M_1 M_2) induced by variations in θ. The second of these involves variation in the position of the vertical segment (LL').

With a historically determined value of k, our individual may find that a particular realization of θ has confronted him with one of two possible situations, as illustrated in Figures 2b,c. In Figure 2b, the agent has a high draw of θ and 'regrets' that more was not invested in process B, as he faces a return of $1 < R$ in his current use of process A. In Figure 2c, the agent has a low draw of θ and 'regrets' that so much was invested in process B, as he would like to 'borrow' at the return of 1 reflected by the short-term process. Efficient selection of k involves trading off these costs and benefits, which we discuss in detail in Appendix A. We demonstrate two appealing results. First, in autarky, agents hold more of the liquid asset than under full information. Second, agents will always hold some of the illiquid asset ($k < 1$), trading off the consequent possibility of illiquidity for higher returns ($R > 1$), if initial wealth is sufficiently large.

Ex Post Security Market

In autarky, at date 1, there are some individuals who would like to sell part of their investments in technology B (those that have low values of θ and, hence, are liquidity constrained) and some who would like to buy units of B since these offer a superior return to the alternative of reinvesting in A (i.e., $R > 1$). We now introduce an *ex post* security market on which such trades can take place.

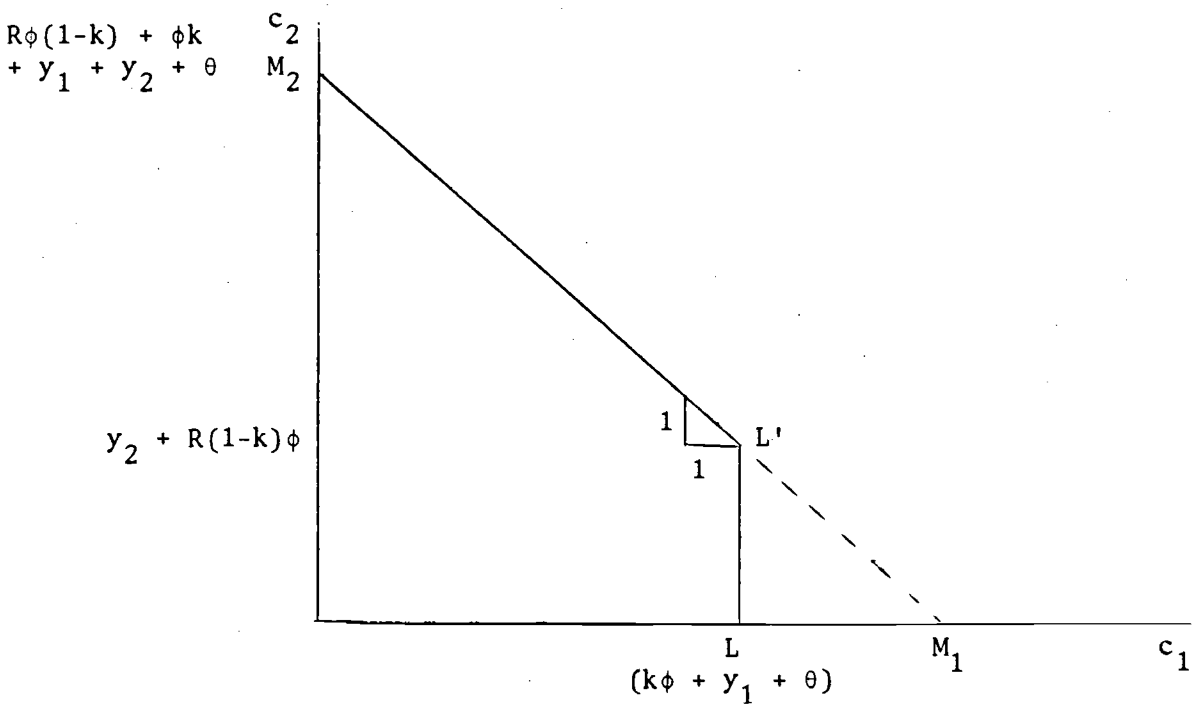

Figure 2a

Consumption Opportunities in Autarky
(conditional on specified k)

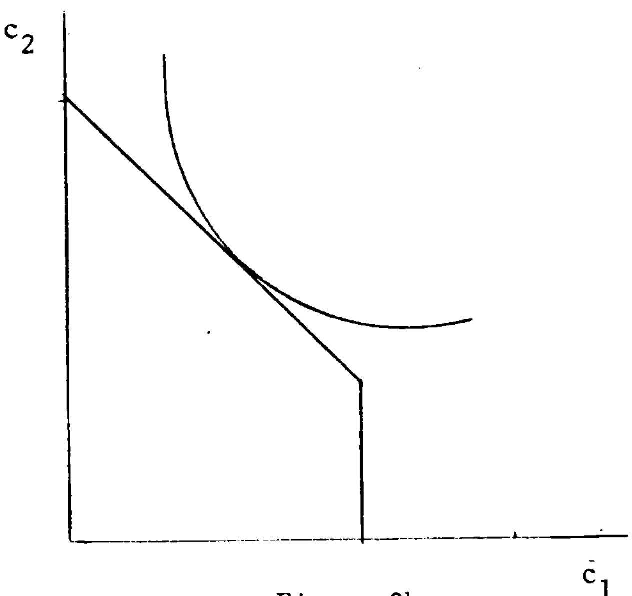

Figure 2b

Liquidity Constraint
Not Binding

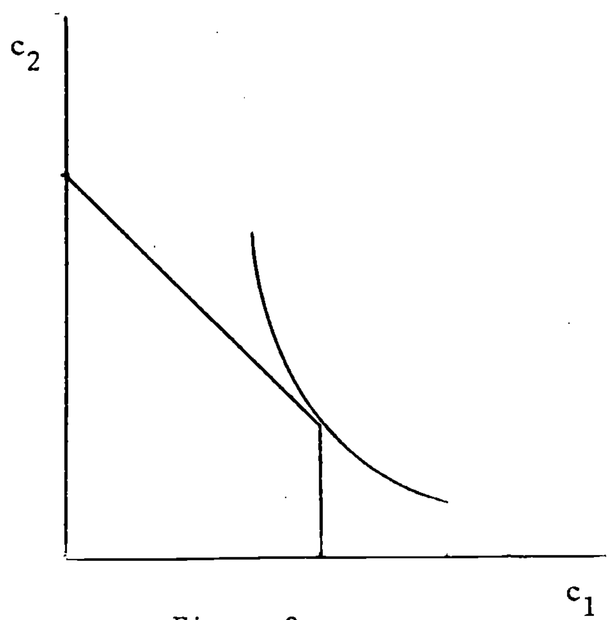

Figure 2c

Liquidity Constraint Binding

On this market, claims on type B storage may be bought and sold, so that the strategy of storing at date 0 and selling off at date 1 generates another 'liquid asset' for the agents in this economy. As a convention, let one share be a claim to a physical unit stored at date 0 and let P denote its price at date 1.

At date t=1, agents have the following budget constraint,

$$(6) \qquad c_1 + Pq = \phi k + P(1-k) + y_1 + \theta$$

in which k is the predetermined fraction invested in the short term asset and in which q is holdings of the long term asset chosen at t=1. Thus, consumption at t=2 is given by $c_2 = y_2 + Rq$. Equivalently, the two period consumption opportunities are given by

$$(7) \qquad c_1 + \frac{P}{R} c_2 = \phi k + P\phi(1-k) + y_1 + \frac{Py_2}{R} + \theta.$$

Solving for a rational expectations equilibrium involves four stages: (i) finding an agent's optimal consumption profile, subject to the date 1 budget constraint, at given P; (ii) finding the P that clears the asset market, given the supply of long term assets, $(1-k)\phi$; (iii) finding an optimal date 0 choice of k given that agents know the value of P that will prevail; and (iv) requiring that date 0 market equilibrium occur. A modest amount of intuition, however, suggests that our economy will have the following solution: P=1 and $k=k_I$, where k_I is full information portfolio share discussed above.[4] That is, with our CES preference specification, the individual income distribution is unimportant for the determination of date 1 prices and date 0 investments.

[4] Notice that in equilibrium, individual portfolio choice is indeterminate, because the strategy of buying a long-term asset at 0 and selling it at date 1 has the same return as the short term technology. However, market equilibrium requires that a 'representative agent' hold k_I.

The *ex post* securities market banishes liquidity risk, as individuals can trade claims to B storage at P=1, as illustrated in Figure 3. However, since agents are subject to income risk, expected utility $E\{G[\alpha(R)a(1,R,\theta)]\}$ is lower than under the full insurance scheme.

IV. Alternative Banking Structures

In this section, we explore some alternative banking structures that could arise in the economic environment outlined in section 1 above. One important object is to develop the welfare implications of alternative arrangements, as our presumption is that competition produces banks that maximize the expected utility of the representative consumer.

Throughout our discussion, we imagine banks operating in the following way. Individuals 'deposit' the initial endowment ϕ at date 0 with the bank, which will pay an interest rate r_0 from period 0 to 1 and r_1 from period 1 to 2 on deposits held during these periods.

Conditional on realization of θ, the individual decides on an amount of funds to be withdrawn from the bank. From our discussion in section II above, we know that we can write the optimal withdrawal pattern as

$$(8) \qquad w^*(r_0,r_1,\theta) = c_1^*(r_0,r_1,\theta) - y_1(\theta)$$
$$= h(r_1)[r_0\phi + y_1(\theta) + y_2/r_1] - y_1(\theta).$$

where the second equality follows from (3) above.

For the bank, assets must be structured so as to meet the deterministic pattern of withdrawals, i.e., investment in the short-term technology so as to meet period one withdrawals

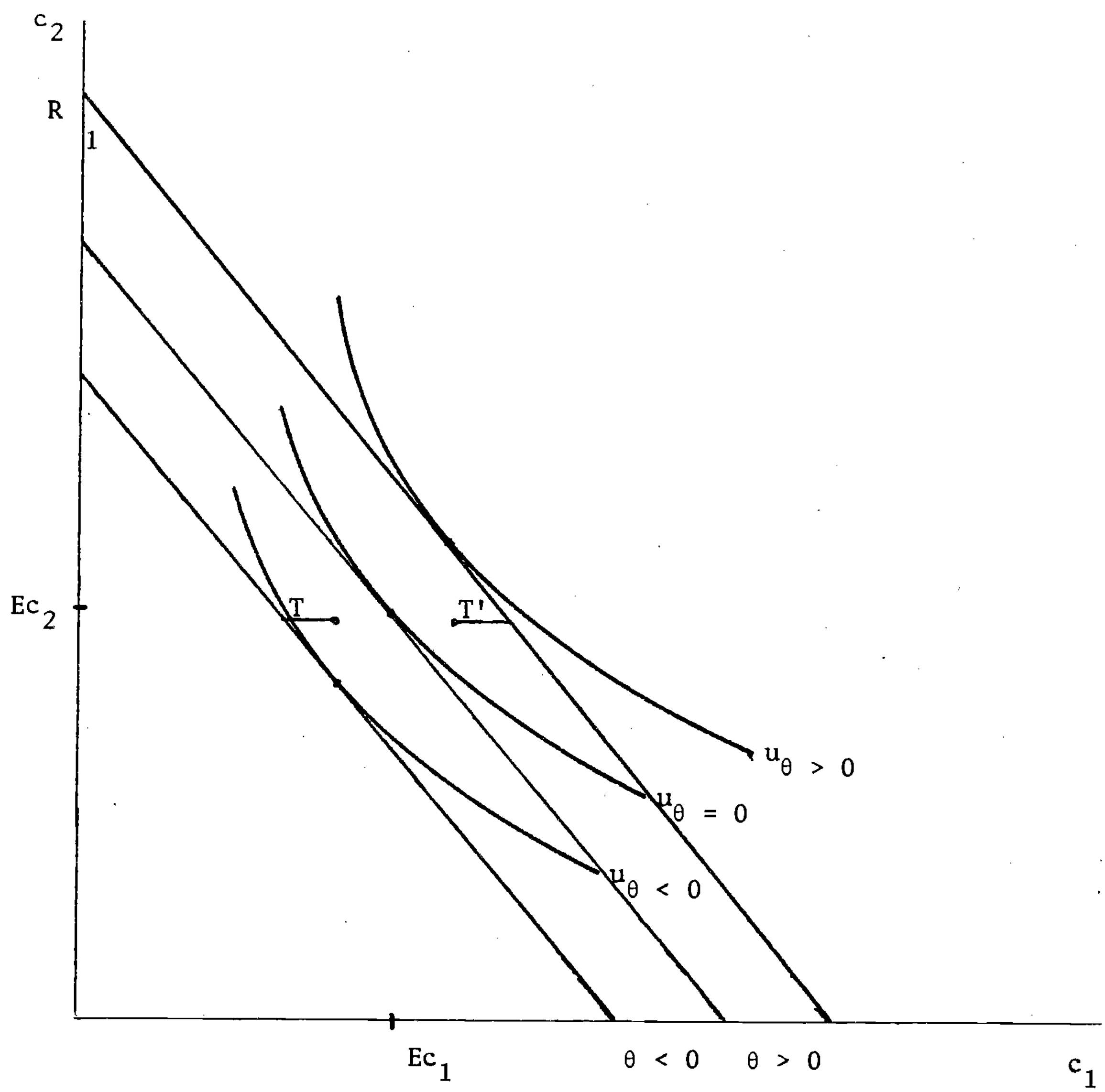

Figure 3

Trade in the
<u>Ex Post</u>
Securities Market

Agents with low incomes ($\theta<0$) sell securities worth T to agents with high incomes ($\theta>0$).

$$(9a)\qquad k_B\phi = \int_{\underline{\theta}}^{\bar{\theta}} w^*(r_0,r_1,\theta)f(\theta)d\theta$$

and investment in the long-term technology so as to meet period two withdrawals

$$(9b)\qquad (1-k_B)\phi R = \int_{\underline{\theta}}^{\bar{\theta}} r_1[r_0\phi - w^*(r_0,r_1,\theta)]f(\theta)d\theta.$$

That is, the bank creates a demand deposit that may be fully withdrawn at date 1. Yet, conditional on the withdrawal behavior $w^*(r_0,r_1,\theta)$, the bank structures assets so as to satisfy the demands that will actually materialize. Clearly, (9a,b) restrict the range of feasible deposit returns (r_0,r_1).

Banking Without Income Insurance

It is possible to interpret the *ex post* securities market as a deposit banking institution, which provides a convenient starting point for our discussion. That is, selecting the deposit rates $r_0 = 1$ and $r_1 = R$, it follows that agents simply accomplish their consumption patterns just as above, withdrawing more or less depending on θ. These individual withdrawal fluctuations were represented as security trades in the *ex post* market.

Thus, this basic banking arrangement has some features that are typically employed in analysis of banking markets: (i) interest rates on deposits are equal to the returns on underlying securities over the relevant horizon ($r_0 = 1$ and $r_1 = R$) and (ii) individual circumstances cause changes in deposit balances that are stochastic, but cancel out across depositors. In the current setup, however, these individual "risks" involve imply a lower level of expected utility at date 0 than that obtained in the full insurance case, so long as individuals are risk averse ($G'' < 0$). Thus, other banking arrangements can potentially dominate this basic structure.

In order for any banking structure to improve welfare in the current environment, it must redistribute wealth from the lucky ($\theta > 0$) to the unlucky ($\theta < 0$). The limiting case of such redistribution is, of course, the full information redistribution scheme outlined above. A market banking institution, however, must accomplish such a redistribution subject to informational constraints, i.e., the private character of idiosyncratic income, θ. Consequently, a tension arises between redistribution and individual incentives that is analagous to the tradeoffs in the analysis of principal-agent problems (e.g., Ross [1973]) and optimal income taxation (as initially investigated by Mirrlees [1971] and with a good overview provided by Atkinson and Stiglitz [1980, sections 13-3,4]).

<u>Deposit Rates and Income Insurance</u>

By raising the interest rate r_0 and lowering r_1, the bank can induce a redistribution from agents with higher incomes to those with lower incomes. We begin by considering a small increase in r_0 and a small decrease in r_1. The bank, of course, must respect its budget constraint, which is obtained from (9a,b) as $\phi = E[w^*(r_0,r_1,\theta)] + \{r_1(r_0\phi - E[w^*(r_0,r_1,\theta)])\}/R$ or, equivalently, as $\phi = r_0\phi + (r_1^{-1} - R^{-1})[y_2 - Ec_2^*(\theta,r_0,r_1)]$. This constraint requires that

$$(10) \qquad 0 = dr_0[\phi - (r_1^{-1} - R^{-1})E\frac{\partial c_2^*}{\partial r_0}]$$

$$- dr_1[(y_2 - Ec_2^*) + (r_1^{-1} - R^{-1})E\frac{\partial c_2^*}{\partial(1/r_1)}]/r_1^2.$$

When we evaluate at $r_1 = R$, feasibility simply requires $dr_0\phi = dr_1(y_2 - Ec_2^*)/r_1^2$. Since $Ec_2^* > y_2$, it follows that a small increase in r_0 requires a decrease in r_1.

The effects on expected utility can similarly be calculated by differentiation.

$$(11) \qquad dU = E\{G'\frac{\partial v}{\partial r_0}\}dr_0 + E\{G'\frac{\partial v}{\partial r_1}\}dr_1$$
$$= E\{G'\alpha(r_1)\}\phi dr_0 - E\{G'\alpha(r_1)(y_2 - c_2^*(\theta,r_0,r_1))\}dr_1/r_1^2.$$

The form of this expression reflects that increases in r_0 have an identical wealth effect on all consumers, where α is the marginal benefit (in u units) of a unit of period 1 wealth. As discussed above, α is invariant to θ in our CES example. By contrast, the wealth effect of an increase in r_1 is largest for individuals who are the greatest lenders in period 1, i.e., for those whom $y_2 << c_2^*(\theta)$. Imposing the condition that dr_0 and dr_1 be feasible and rearranging the resulting expression, we get

$$(12) \qquad dU = \alpha(R)E\{G'(c_2^* - Ec_2^*)\}dr_1/r_1^2.$$

With risk aversion, $G'' < 0$, the covariance term is unambiguosly negative, so that a small decline in r_1 raises welfare. Intuitively, by raising r_0 and lowering r_1, the bank has shifted wealth from those with high θ's to the average individual. In effect, at date 0, the bank offers an individual security that has a certain date 1 expected return (ϕdr_0) and pays off negatively when high θ's occur, so reducing individual risks.

Valuing Fractional Insurance

The extent of the demand for insurance in our economy is best measured by the price that individuals would pay for artificial assets with individual-specific returns. That is, for any individual at date 0, wealth is a random variable $a(r_0,r_1,\theta)$ because of the idiosyncratic component of individual income. Expected utility is $E\{G[\alpha(r_1)a(r_0,r_1,\theta)]\}$, where $\alpha(r_1)$ was defined above.

Now, imagine constructing the artificial asset that has date 1 "returns" that are a function of θ, $x(\theta)$ with $Ex(\theta) = 1$. Our individual would have expected utility unchanged if an infinitesimal amount of this claim was traded for p_x units of output in period 1 if

(13) $$E\{G'\}p_x - E\{G'x(\theta)\} = 0.$$

Equivalently, the price must be less than unity if the covariance of $x(\theta)$ with marginal (lifetime) utility of wealth is negative.

(14) $$p_x = \frac{E\{x(\theta)\}E\{G'\}+E\{G'(x(\theta)-Ex(\theta))\}}{E\{G'\}} = \frac{\text{cov}\{G',x(\theta)\}}{E\{G'\}} + 1.$$

That is, our agent would 'price' such an asset by principles that accord with the theory of finance (Fama and Miller, [1972]) and, in particular, with recent theories that stress covariance with marginal utility of lifetime wealth, (Breeden [1979] and Grossman-Shiller [1982]). In our context, however, the private character of information and absence of aggregate shocks implies that (i) risk arises from the idiosyncratic disturbance, θ, and (ii) that no securities with these direct characteristics could be traded on markets.

The Optimal Linear Bank

The tradeoff between insurance and incentives emerges when one considers changes in r_0 and r_1 that are not small. But the principal economic institution behind the previous local results extends to the optimal linear banking structure, which we derive in Appendix B. In particular, the optimal level of r_1 satisfies

$$(15) \qquad r_1 = R[\varepsilon_2 + \delta_2 \frac{\partial c_2^*}{\partial a}]/[\varepsilon_2 + \delta_2 \frac{\partial c_2^*}{\partial a} + R\delta_2]$$

$$\equiv R\bullet Z(\varepsilon_2, \delta_2, \frac{\partial c_2^*}{\partial a}).$$

The determinants of this optimal level of r_1 are as follows. First, there is a measure of intertemporal substitution, specifically ε_2 is the compensated semi-elasticity of second period consumption demand with respect to its price, $p_2 \equiv 1/r_1$. That is, $\varepsilon_2 = -\frac{1}{c_2}\frac{\partial c_2^*}{\partial p_2}|_u > 0$. Second, $\frac{\partial c_2^*}{\partial a}$ is the effect of a wealth increment on second period consumption. Third, δ_2 is the extent of the 'risk premium' imposed by a representative private agent on a bet of the form $c_2^*(\theta)/Ec_2^*(\theta)$, which has expected return of one but covaries negatively with lifetime marginal utility, $\delta_2 = -\{cov(G', c_2^*(\theta))/EG'Ec_2^*(\theta)\} > 0$.

Initially, note that $r_1 < \bar{R}$ so long as agents are risk averse ($\delta_2 > 0$), which preserves the flavor of the local results above. Further, the higher the risk premium δ_2, the lower the level of r_1, i.e., $\partial z/\partial\delta_2 < 0$. This accords with the idea, developed above, that it is a demand for insurance--reflected in δ_2--that motivates decreases in r_1 and consequent increases in r_0. Two further elements enter the formula that were not present above: higher values of second consumption response to the interest rate or to wealth raise the efficient value of r_1, i.e., $\partial z/\partial\varepsilon_2 > 0$ and $\partial z/\partial(\frac{\partial c^*}{\partial a}) > 0$. These additional effects can readily be explained by returning to the bank's feasibility condition and exploring the effects of changes in r_0 and r_1 laid out in (15) above. The bank receives a return R on funds held over from t=1 to t=2, but must pay out only r_1, yielding a 'surplus' that can be distributed as initial returns r_0. The extent of this surplus can be illustrated by writing the budget constraint as

$$(r_0-1)\phi = (\frac{R-r_1}{R})[r_0\phi - Ew^*(r_0,r_1,\theta)]$$

where the right-hand side is the surplus. Now, as there is a higher substitution response (ε_2) in second period consumption, changes in r_1 erode the deposit 'base' $[r_0\phi - Ew^*(r_0,r_1,\theta)]$ more rapidly. Thus, the bank picks a higher value of r_1. Furthermore, wealth-induced changes in consumption enter via r_0 because an increase in r_0 raises wealth and, hence, second period consumption by the amount $\frac{\partial c_2^*}{\partial r_0} = \frac{\partial c_2^*}{\partial a}\phi$. With a large $\frac{\partial c_2^*}{\partial a}$, consumers spend most of the wealth increase in period 2, initially saving it. Because the bank receives R on these deposits, the "surplus" has increased. Conversely, a low $\frac{\partial c_2^*}{\partial a}$ implies consumers withdraw much of the wealth, leaving the bank with a small deposit base and little surplus, able to pay only a low r_1.

In fact, this idea of deposit base erosion brings us naturally to discussion of a constraint not explicitly imposed in derivation of formula (15) above. That is, in order for the income redistribution to work, it is necessary that $r_1 \geq 1$, so that depositors do not have an incentive to withdraw at date t=1 so as to invest in technology A from period t=1 to t=2.

Figure 4 illustrates the effects of banking on the welfare and consumption levels of an 'average' agent, i.e., one with $\theta = 0$. The budget constraint facing this agent reflects the fact that $r_1 < R$ under deposit banking and induces a substitution from consumption in period 2 to period 1. The 'tax on saving' that the banking system imposes lowers the welfare of this average agent, since his consumption must fundamentally be constrained by the (per capita) nature of social opportunities. Although the welfare of the 'average' agent depicted in Figure 4 declines, the expected utility of agents (prior to the realization of θ) is increased by deposit banking, because of a reduction in the range of variability of individual circumstances.

Nonlinear Schedules and Ex Post Arbitrage

A priori, nothing restricts the bank to a linear pay-off schedule. Indeed, throughout our discussion, we have employed the analogy between deposit banking and distributive taxation: Mirrlees [1971] and others have explored non-linear tax structures in some detail.

Although we have so far not made it explicit in our discussion and in particular have not formally incorporated this constraint into Appendix B, we assume that ex post arbitrage by groups of depositors would rule out any bank contracts that made the interest rates r_0 and r_1 functions of withdrawal amounts. That is, even if depositors were limited to one withdrawal per period, then nonlinearities in the bank schedules would make coalition formation feasible and desirable.

V. Deposit Banking and Ex Post Security Markets

The purpose of this section is to clarify several aspects of the relationship between the deposit banking arrangements outlined previously and ex post security markets.

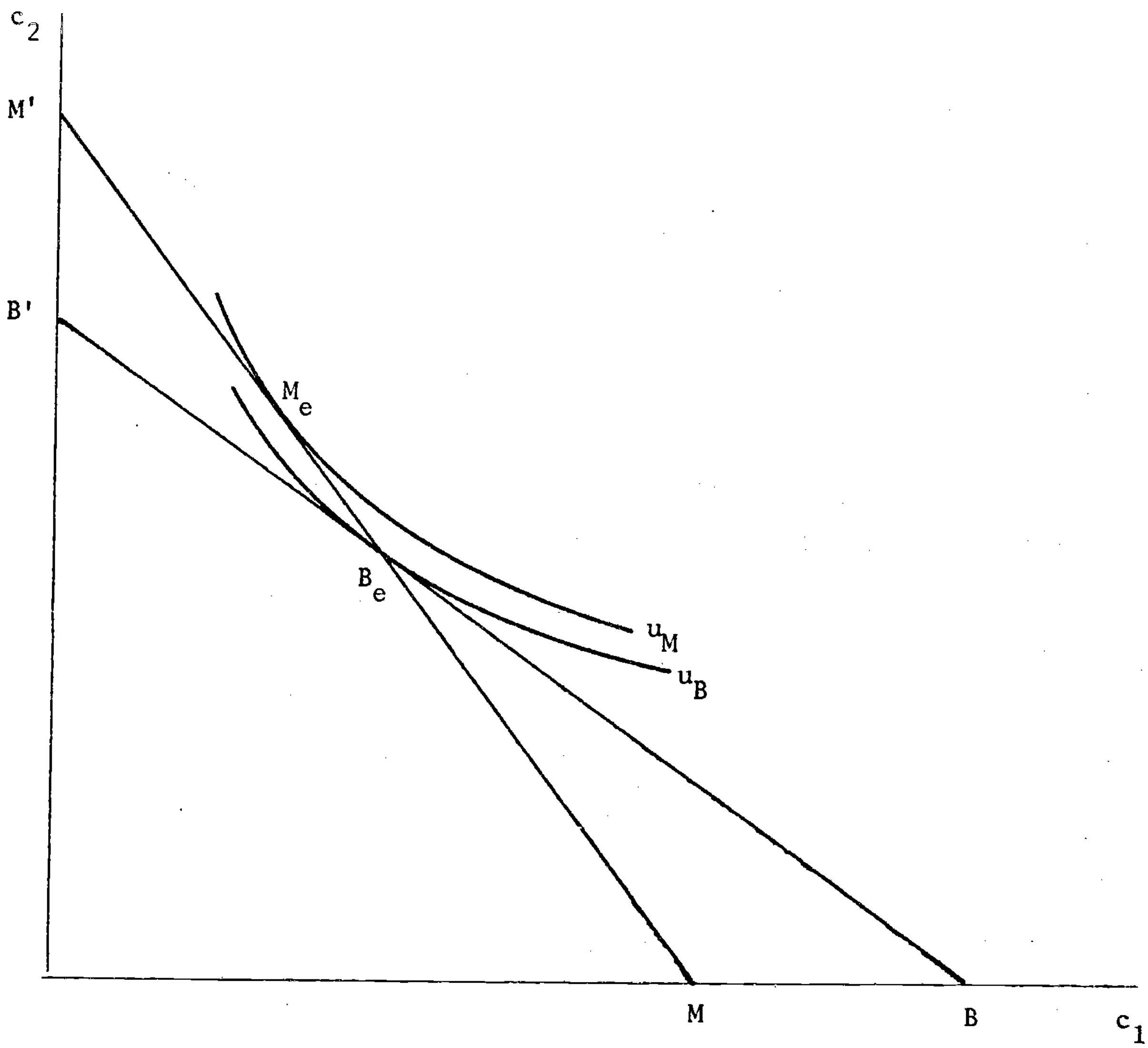

Figure 4

An Average Agent Under Simple and Linear Banking Systems

MM' is $c_1 + R^{-1}c_2 = y_1 + R^{-1}y_2 + \phi$

BB' is $c_1 + r_1^{-1}c_2 = y_1 + r_1^{-1}y_2 + r_0\phi$

Markets in Derivative Claims

The deposit banking institution discussed above utilized no resources but was a pure intermediary, purchasing one set of claims and issuing another. This observation suggests that similar insurance could be delivered by a particular type of financial instrument without the demand deposit characteristic.

That is, suppose that at date 0 individuals can only purchase shares on a "fund" that combines k_B units of the short-term asset and $(1-k_B)$ units of the long-term asset, where k_B is the amounted invested by the optimal linear bank discussed in the preceeding section. To make this conform as closely to our previous discussion, let the "fund" announce that its policy will be to pay share-holders interest equal to (r_0-1) per share at date 1 and use the remainder of its income to repurchase its shares at the going market price p_F. Let the amount of this repurchase in shares be η so that $(r_0-1)\phi + p_F\eta = k_B\phi$. Further, let the fund announce that it will pay shareholders r_1 (principal plus interest) at date 2 with its available funds $r_1\{\phi-\eta\} = (1-\kappa_B)\phi R$. Now, suppose that the market price (p_F) is one and that $\eta = [Ew^*(r_0,r_1,\theta)-(r_0-1)\phi]$ is the announced repurchase amount. Then, it is direct that the levels of bank rates r_0,r_1 are feasible from the perspective of the "fund". Further, $p_F = 1$ is a rational expectations equilibrium price in the market for "fund" shares since supply and demand for "fund" shares will be equated at that price.

Notice that the "fund" does not have a policy of "pegging" the price of its shares at one, which is one interpretation of the bank's policy in section IV above. Similarly, it is commonly observed that banks have date 1 liabilities $[(1+r_0)\phi]$ that exceed their date 1 cash flows $[k_B\phi]$, which is sometimes asserted to lead to banking instability. The "fund" described above should not have such a problem.

The fact that a market in this type of derivative security can replicate the allocations of the optimal linear bank has important consequences for the analysis of Diamond and Dybvig (1983). These authors suggest that the demand characteristic is a necessary outcome of environments where individuals circumstances are subject to privately observable random shocks.[5] They go on to argue that banking runs--viewed as self-fulfilling expectations that all agents will demand funds in period 1--are a consequent potential outcome of this sort of environment. The _ex post_ market in the derivative security outlined above would not be susceptible to runs and thus would dominate the sort of bank deposit scheme considered by Diamond and Dybvig (1983).

Derivative Claims and Ex Ante Incentives

Throughout the preceeding discussion, we confronted individual agents with extreme choices of date 0: remain in autarky or purchase claims at date 0 from the bank (by depositing one's funds) or from the "fund". Now, we want to imagine a single agent pursuing the following strategy: (i) at date 0, invest a unit (part of ϕ) in the long-term asset and (ii) at date 1, sell this to a depositor from the bank at a price that the depositor will find yields a higher return than the bank deposit, i.e., $p_\varepsilon = R/r_1 - \varepsilon$, for some $\varepsilon > 0$.

[5] Diamond and Dybvig (1983) consider a two state model with preference shocks and a slightly different technology (see fn 6 below) but these differences are inessential for the present discussion.

Is this strategy profitable from the stand-point of our agent? The answer turns on whether the rate of return from 0 to 1 exceeds the bank deposit rate r_0. That is, we want to know whether $p_\varepsilon > r_0$ for some $\varepsilon > 0$. To discuss this, we need to return to the bank's budget constraint in present value form, as that links feasible r_0 and r_1, which is $\phi = Ew^*(r_0,r_1,\theta) + r_1[r_0\phi - Ew^*(r_0,r_1,\theta)]/R$. This may be rewritten as

$$R/r_1 - r_0 = [\phi(r_0-1)/(\phi-Ew^*)],$$

where the right-hand term is positive for our bank as $r_0 > 1$ and $Ew^*(r_0,r_1,\theta) < \phi$. Thus, it is possible to construct a price p_ε that satisfies

$$r_0 < p_\varepsilon < R/r_1$$

for any $0 < \varepsilon < \phi(r_0-1)/(\phi-Ew^*)$, with the choice of ε reflecting the division of the gains from trade between the two agents.

Thus, there are incentives for individual agents to (i) not join the bank if others do and to (ii) induce bank members to withdraw balances to finance asset accumulation rather than consumption. Basically, this result reflects the fact that the bank is engaged in insurance (redistribution) so that its returns do not correspond to those given by the technology (in particular, the return $r_1 \neq R$). Similar <u>ex</u> <u>ante</u> arbitrage opportunities (i.e., based on decisions at date 0) would also occur in the "fund" case discussed above. The incentives for individuals to avoid joining the bank in period 0 mean that without binding contracts, the bank would not arise. This points out the importance of refining the equilibrium concept employed in our analysis, a subject which is on our agenda for further research.

V. Comparisons of Alternative Structures

In autarky, each individual agent is subject to two types of private risks stemming from individual income fluctuations (θ): (i) pure income risk and (ii) illiquidity risk, which arise from the interaction of the information technology (the private character of θ) and the production technology (process B is irreversible).

Ex post security markets remove the illiquidity risk from fluctuating individual circumstances (θ) by enabling agents to sell claims against the returns from long term investment projects. Consequently, in our setup, ex post security markets dominate autarky, according to the expected utility (pareto) criterion at date 0.[6] Equivalently, a simple banking arrangement, could provide the necessary 'pooling' of liquidity risks, as it replicates the ex post security market. Such possibilities for pooling of liquidity risks induce agents to invest more in long-term projects than they would under autarky.

The optimal linear banking structure provides agents with a higher level of expected utility than such a simple banking structure or an ex post security market, however, as it partially insures agents against income risks as well as fully against liquidity risks. The provision of insurance is typically incomplete because the bank faces a tradeoff between insurance and

[6] This result contrasts to Diamond-Dybvig [1983], who show an equivalence between autarky and ex post security markets, in a model where agents are subject to individual (privately observable) preference shocks. If our technology B is modified to be partly reversible, with payoffs (1-b) at t=1 and R at t=2, then ex post security markets will dominate autarky so long as $b > 0$, i.e., there is a positive opportunity cost to investing in B rather than A. If b=0, the only technology B will be employed and agents will face no "illiquidity risks" in autarky. Thus, there will be an equivalence of autarky and ex post security markets as only pure income risks will be relevant in each case. Further, no trade will occur on these ex post markets.

incentives for saving. Relative to ex post security markets, banks offer higher short term yields ($r_0>1$) and lower long term yields ($r_1<R$). The banking-insurance mechanism induces substitutions that imply that more investment will take place in projects of shorter duration. Without income uncertainty (if $\delta_2=0$), an optimal bank would set $r_0 = 1$ and $r_1 = R$, thus serving no economic purpose.

In viewing an economic role of banking as providing insurance against unobservable private risks, it is useful to consider two related insurance concepts. First, in environments where there are substantial idiosyncratic private risks, borrowing and lending can serve as 'buffer' that permits long-lived individuals to smooth consumption and, hence, reduce the risks of individual shocks. However, this insurance role of saving is bounded as long as agents have finite lives or are sufficiently impatient.[7] Thus, we expect that our results would be robust to extensions to more time periods. Second, it is natural to ask why one sees both cooperative institutions such as savings banks and companies providing traditional insurance. We believe that traditional insurance typically arises when individual circumstances can be verified but not costlessly. (Townsend, [1979].) Consequently, equilibrium insurance contracts involve 'deductability,' i.e., the decision by the insured agent that particular events are not sufficiently costly to warrant payment of verification costs. By contrast, the insurance provided by our 'savings bank' scheme does not require state verification. Thus, these institutions coexist because there are many types of idiosyncratic risks, with varying costs of state verification.

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[7] Grossman and Weiss [1982] suggest that the bond market can provide perfect insurance, but a careful reading of their discussion makes clear that incomes must be observable to obtain such a result.

VI. Summary and Conclusions

In this paper, we have discussed how a private financial institution--such as a savings bank--can provide partial insurance against fluctuations in individual incomes that are private information. In our model, fluctuations in individual incomes cause income and illiquidity risks in autarky. *Ex post* security markets or simple banking structures that are pure intermediaries can remove illiquidity risks, but insurance against income fluctuations requires that bank deposit returns differ from those on individual securities. Insurance is partial because the cooperative banking institution faces a tradeoff between insurance and (intertemporal) allocative efficiency.

As macroeconomists, our principal interest in this framework is to have a secure foundation for aggregate analysis and, consequently, several comments are in order. First, our formal model is the standard sort that most macroeconomists have in mind when concerned with 'micro structure': all agents have identical preferences that can be aggregated, but individual agents differ due to fluctuations in individual conditions that average out across individuals (see, for example, Friedman [1969]).[8] Yet, risk aversion and the private character of individual income fluctuations, however, imply that banking institutions of the sort discussed in section IV dominate conventional market structures. Second, one might conjecture that responses to aggregate disturbances--such as shifts in per capita incomes (y_1,y_2) or rates of return (R) in our framework--would be unaffected by the institutional structure, since no risk pooling can occur against these shocks. But, in extensions to our analysis, such an invariance to micro structure requires that aggregate disturbances not be associated with changes in the dispersion of individual

[8] The trivial aggregation makes it easy for us to analyze *ex post* security markets, etc.

income fluctuations. In fact, Haubrich [1983] demonstrates that the existence of banks alters the temporal structure of equilibrium interest rates and consumption-investment quantities if there are effects of aggregate disturbances on the income distribution. Consequently, taking these two observations together, explicit analysis of factors that lead to development of specific institutional structures--such as private information--may also alter our conclusions about aggregate responses.

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Appendix A

Portfolio Composition and Consumption in Autarky

We discuss optimal decisions of the autarkic individual using the typical dynamic programming method of recursive optimization.

Decisions at Date 1

At date 1, our individual's portfolio composition (k) is predetermined. The agent maximizes utility G(u) subject to the following two constraints:

(A1) $$c_1 \leq y_1 + \theta + k\phi \equiv L(\theta,k)$$

(A2) $$c_1 + c_2 \leq y_2 + (1-k)R\phi + (k\phi+y_1+\theta) \equiv A(\theta,k).$$

The former of these constraints is the liquidity constraint. It does not bind an agent if $c_1 \equiv h(1)A(\theta,k) \leq L(\theta,k)$. With the liquidity constraint not binding, an agent has indirect utility $v_{NB}(\theta,k) = \alpha(1)A(\theta,k)$. Correspondingly, with the liquidity constraint binding, an agent has indirect utility $v_B(\theta,k) = [L^{1-1/\sigma} + \beta(A-L)^{1-1/\sigma}]^{\frac{\sigma}{\sigma-1}}$. In Figure A-1, we graph the indirect utility function against θ. Notice that v is not linear in current income, θ, when the constraint is binding, i.e., when $\theta < \hat{\theta} \equiv c_1^* - y_1 - k\phi$, and that agents have a higher marginal utility of θ over this range.

Decisions at Date 0

At date 0, the autarchic individual selects k so as to maximize expected utility, which is (A3) in autarky, where $\hat{\theta}$ is the value of k at which the constraint just binds.

(A3) $$EU = \int_{\underline{\theta}}^{\hat{\theta}} G(v_B(\theta_1,k))f(\theta)d\theta + \int_{\hat{\theta}}^{\bar{\theta}} G(v_{NB}(\theta,k))f(\theta)d\theta$$

The marginal expected utility of investment in the liquid asset is (A4). For an interior optimum ($0 < k < 1$), this must be set to zero.

$$\text{(A4)} \quad \frac{\partial EU}{\partial k} = G(v_B(\theta,k))f(\theta)\frac{d\hat{\theta}}{dk} + \int_{\underline{\theta}}^{\hat{\theta}} \frac{\partial}{\partial k}G(v_B(\theta,k))f(\theta)d\theta$$
$$- G(v_{NB}(\theta,k))f(\theta)\frac{d\hat{\theta}}{dk} + \int_{\hat{\theta}}^{\bar{\theta}} \frac{\partial}{\partial k}G(v_{NB}(\theta,k))f(\theta)d\theta = 0.$$

From Figure A-1, we know that $v_B(\theta) = v_{NB}(\theta)$, so that the first and third terms just cancel.

Efficient Investment in the Liquid Asset

First, we demonstrate that an individual will subject himself to some illiquidity risk in autarky, providing--as assumed in the main text--that wealth is sufficiently large. Specifically, we require that if income is at its lowest level, $y_1 + \underline{\theta}$, that

$$\text{(A5)} \quad \phi + y_1 + \underline{\theta} > c_1^* = h(1)[(\phi + y_1 + \underline{\theta} + y_2].$$

Now, consider evaluating $\partial EU/\partial k$ at $k = 1$. By virtue of assumption (A5) above, $\hat{\theta}(k = 1) < \underline{\theta}$. Consequently,

$$\text{(A6)} \quad \frac{\partial EU}{\partial k}\Big|_{k=1} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial}{\partial k}G(v_{NB}(\theta))\Big|_{k=1} f(\theta)d\theta$$
$$= -(R-1)E\{\alpha(1)G'\}\phi < 0$$

since $R > 1$. This is a variant of Arrow's famous proposition [1964] that a risk averse agent will always accept a small amount of a bet with positive expected return.

Second, we want to demonstrate that an individual will hold more of the liquid asset in autarky than with an ex post security market. Consider the value k_E that is efficient in an ex post security market. This satisfies

$$\text{(A7)} \quad \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial}{\partial k} G(v_{NB}(\theta,K))|_{k_E} f(\theta)d\theta = 0.$$

Now, consider the crossover value θ_E implied by k_E. Above θ_E, agents are equally well off in autarky and in an ex post security market. Evaluate (A4) at k_E. Then, it follows that

$$\text{(A8)} \quad \frac{\partial EU}{\partial k}|_{k_E} = \int_{\theta_E}^{\bar{\theta}} [\frac{\partial}{\partial k} G(v_B(\theta,k) - \frac{\partial}{\partial k} G(v_{NB}(\theta,k))]|_{k_E} f(\theta)d\theta$$

The utility derived from a marginal increment of investment in the liquid asset is always higher with the constraint binding for two reasons: (i) as may be seen from Figure A-1, the level of unconstrained "u unit" utility (v_B) lies above the constrained level (v_{NB}) and the strict concavity of G insures that G' is diminishing in v; and (ii) the marginal "u unit" utility obtained from a unit of period are resources is higher when the constraint is binding, i.e., $\partial v_B/\partial k > \partial v_{NB}/\partial k$. Thus, it follows that $\partial EU/\partial k$ is negative at k_E. It is tedious but straightforward to show that (A3) is strictly concave in k, so that the efficient proportion of investment in the liquid asset under autarky (k_A) must be greater than k_E.

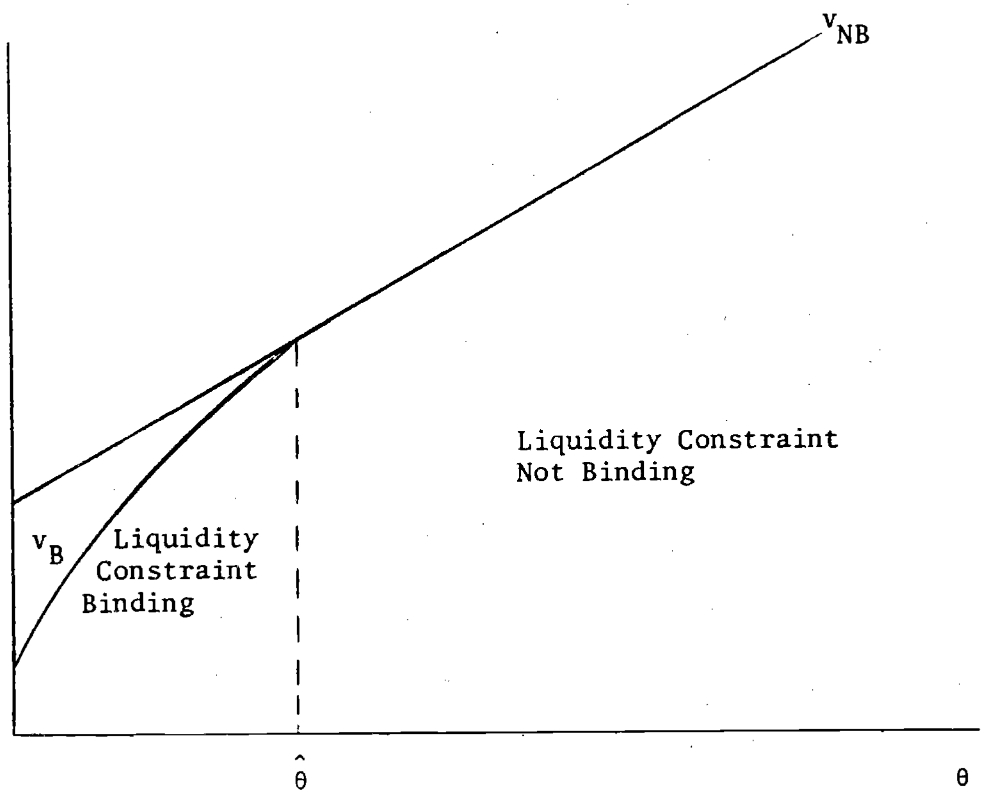

Figure A-1

Indirect Utility (in u units) and θ

Appendix B

In this appendix, we present a detailed treatment of an optimal linear banking structure or, equivalently, an optimal linear tax on saving.

An individual private agent, with given value of period 1 income, chooses an optimal consumption plan so as to maximize utility.

(B1) $$\max_{c_1, c_2} u(c_1, c_2) \text{ subject to } c_1 + p_2 c_2 \le T + y_1(\theta) + p_2 y_2 ,$$

where $p_2 = (r_1)^{-1}$ and $T = r_0\phi$. The outcome of this maximization is an indirect utility function, $v(\theta, p_2, T)$, and a set of optimal decisions $c_1^*(\theta, p_2, T)$ and $c_2^*(\theta, p_2, T)$. These are not the same functions as those discussed in the text, but it is easy to move from one to the other.

In the CES case, $u(c_1, c_2) = [c_1^{1-1/\sigma} + \beta\, c_2^{1-1/\sigma}]^{\sigma/\sigma-1}$, and the relevant value function and decision rules are given by

(B2a) $$c_1^* = h(p_2^{-1})[y_1(\theta) + T + p_2\, y_2]$$

(B2b) $$c_2^* = p_2^{-1}[1-h(p_2^{-1})][y_1(\theta) + T + p_2\, y_2]$$

(B2c) $$v = \alpha(p_2^{-1})[y_1(\theta) + T + p_2\, y_2].$$

where $h(p_2^{-1}) = [1 + \beta^\sigma p_2^{-(\sigma-1)}]^{-1}$ and $\alpha(p_2^{-1}) = h(p_2^{-1})^{1/1-\sigma}$ is independent of θ. Further, we record for future use that the compensated semi-elasticity

(B3) $$\varepsilon_2 = \frac{1}{c_2^*} \frac{\partial c_2^*}{\partial p_2}\Big|_u > 0$$

is independent of the scale of wealth, i.e., $[y_1(\theta) + T + p_2\, y_2]$.

The implied withdrawal behavior of an individual is $w^*(\theta) =$

$c_1^*(\theta) - y_1(\theta)$. Thus, it follows that $w^*(\theta,p_2,T) = h(p_2^{-1})(p_2\, y_2 + T) - (1-h(p_2^{-1}))y_1(\theta)$.

Prior to the realization of θ, in period 0, an expected utility maximizing bank or government will maximize

$$\text{(B4)}\qquad E\{G(v(\theta,p_2,T))\}$$

with respect to $p_2 = r_1^{-1}$ and $T = r_0\phi$ subject to (i) a resource constraint and (ii) the depositor's decision rules, $c_1^*(\theta)$, $c_2^*(\theta)$, and $w^*(\theta)$. The basic resource constraints for a bank are

$$\text{(B5a)}\qquad \phi k = Ew^*(\theta,p_2,T)$$

$$\text{(B5b)}\qquad R\phi(1-k) = E[r_1(r_0\phi - Ew^*(\theta,p_2,T))]$$

For current purposes, it is convenient to combine and rewrite these as

$$\text{(B6)}\qquad p_2[\phi - Ew^*(\theta,p_2,T)] - R^{-1}[T - Ew^*(\theta,p_2,T)] = 0$$

Using the fact that $w^*(\theta,p_2,T) = T + p_2(y_2 - c_2^*(\theta,p_2,T))$, the resource constraint is

$$\text{(B7)}\qquad [(\phi-T)] - (p_2 - R^{-1})(y_2 - Ec_2^*(\theta,p_2,T)) = 0.$$

Forming the 'Lagrangian' for this problem,

$$\text{(B8)}\qquad H = E\{G(v(\theta,p_2,T))\} + \lambda[\phi-T - (p_2 - R^{-1})(y_2 - Ec_2^*(\theta,p_2,T))]$$

and differentiating with respect to T and p_2, we get the following necessary conditions for an optimal linear bank.

$$\text{(B9a)}\qquad \frac{\partial H}{\partial T} = E\{G'\frac{\partial v}{\partial T}\} - \lambda + \lambda(p_2-R^{-1})(E\,\frac{\partial c_2^*}{\partial T}) = 0$$

(B9b) $\frac{\partial H}{\partial p_2} = E\{G'\frac{\partial v}{\partial p_2}\} - \lambda(y_2 - Ec_2^*) + \lambda(p_2 - R^{-1})\frac{\partial Ec_2^*}{\partial p_2} = 0.$

The first of these conditions may be reorganized as

(B10) $E\{b(\theta)\} = 1$

where $b(\theta) \equiv \{G'\alpha(\theta)/\lambda + (p_2 - R^{-1})\frac{\partial c_2^*}{\partial T}\}$. Analagously, using $\partial v/\partial p_2 = \alpha(p_2^{-1})(y_2 - c_2^*(\theta,p_2^{-1},T))$ and the Slutzky decomposition

$$\frac{\partial c_2^*}{\partial p_2} = \frac{\partial c_2^*}{\partial T}(y_2 - c_2^*) + \frac{\partial c_2^*}{\partial p_2}\Big|_u ,$$

it follows that the second condition may be written as

(B11) $E\{(b(\theta)-1)(y_2 - c_2^*)\} + (p_2 - R^{-1})E\{\frac{\partial c_2^*}{\partial p_2}\big|_u\} = 0$

These conditions correspond to those of Atkinson and Stiglitz [1980, pp. 407-8] in the optimal linear income tax case, who provide the relevant interpretations in that setting.

Solving the second necessary condition, and simplifying slightly, we find

(B12) $p_2 - R^{-1} = \dfrac{\mathrm{cov}(b(\theta),c_2^*(\theta))}{E\{\frac{\partial c_2^*}{\partial p_2}\big|_u\}}$

In our framework, some further reorganization of this condition is useful. As α, $\partial c_2^*/\partial T$ and ε_2 are independent of θ, we may write this as

(B13) $p_2 - R^{-1} = [\frac{-\mathrm{cov}(G',c_2^*)}{EG'Ec_2^*}](\frac{\alpha EG'}{\lambda})\frac{1}{\varepsilon_2}$

$$= (\frac{\alpha EG'}{\lambda})\,\delta_2/\varepsilon_2$$

where δ_2 is the individual's implicit risk premium on the ficticious security, $c_2^*(\theta,p_2,T)/Ec_2^*(\theta,p_2,T)$, which has a unit mean, and $\varepsilon_2 = -\frac{1}{c_2^*}\frac{\partial c_2^*}{\partial p_2}\big|_u$ which is the compensated semi-elasticity of second period consumption with respect to its price $p_2 = r_1^{-1}$. Further simplification may be obtained by noting that $E\{b(\theta)\} = 1$ implies that $E\{G'\frac{\alpha}{\lambda}\} + (p_2 - R^{-1}) \frac{\partial c_2^*}{\partial T} = 1$. Consequently, we may write

$$\text{(B14)} \qquad p_2 - R^{-1} = \delta_2/\{\varepsilon_2 + \delta_2 \frac{\partial c_2^*}{\partial T}\}.$$

The formula discussed in the text is a straightforward transformation of this expression.