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**ABSTRACT**

Shimer (2005a) argues that the Mortensen-Pissarides equilibrium search model of unemployment explains only about 10% of the response in the job-finding rate to an aggregate productivity shock. Some of the recent papers inspired by his critique are reviewed and commented on here. Specifically, we suggest that the sole problem is neither the procyclicality of the wage nor the failure to account fully for the opportunity cost of employment. Although an amended version of the model, one that accounts for capital costs and counter cyclic involuntary separations, does much better, it still explains only 40% of the observed volatility of the job-finding rate. Finally, allowing for on-the-job search does not improve the amended models implications for the amplification of productivity shocks.

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# 1 Introduction

In this paper, we review Shimer's (2005a) critique of the equilibrium search model of unemployment. (See Mortensen and Pissarides (1994,1999a,1999b) and Pissarides (2000) for an extended development of the model and its implications.) Our purpose is to clarify and further the debate generated by Shimer's paper.<sup>1</sup>

Shimer documents the fact that volatility in unemployment is induced primarily by movements in the job-finding rate, the rate of transition from unemployment to employment, rather than the job-destruction rate. He then demonstrates that the magnitude of the response of unemployment, vacancies, and the job-finding rate to labor-productivity shocks predicted by the model explains less than 10% of the observed volatility in U.S. data when productivity shocks are assumed to be the sole driving force, given reasonable specification assumptions and parameter values.<sup>2</sup> A principal reason for this lack of explanatory power, he argues, is that the wage, set as the outcome of a bilateral wage bargain, responds procyclically to offset almost all the effects of productivity shocks on job creation.

We argue that a flexible wage *per se* is not the principal problem with the model. Rather, Shimer's results are due to 1) the large difference between labor productivity and the opportunity cost of a match implied by the assigned magnitudes of parameters and 2) the excessively strong feedback from the job-finding rate to the wage. Even if the wage were rigid, its level must be such that the future flow of quasi-rent attributable to the creation of a new job is very small, if the model is to account for the volatility of the job-finding rate observed in the data. As Hagedorn and Manovskii (2005) demonstrate, the model has no problem explaining vacancy and unemployment fluctuations if the parameters of the wage outcome function are set to match observed average profit rates and wage volatility. Unfortunately, the opportunity cost of employment required to explain the observed volatility in the job-finding rate is unrealistically high. Consequently, the calibrated model exhibits excessive sensitivity to small changes in labor-market policy, as Costain and Reiter (2005) point out. We show that the opportunity cost of a match is not only influenced by the opportunity cost of employment for the worker, but also depends on the cost of hiring and training workers. If

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<sup>1</sup>Hornstein, Krusell, and Violante (2005) also provide an analysis with a similar purpose.

<sup>2</sup>Costain and Reiter (2005) make a similar point in a less well-known, but independently developed paper.

these costs are significant, then the worker's opportunity cost of employment required to match observed volatility is much lower than that derived by Hagedorn and Manovskii.

The standard search model of unemployment is designed to account for the fact that it takes time to match jobs and workers. As a consequence of this friction, match-specific rents exist when a worker meets a prospective employer. The designers of the original model assume that these rents are shared according to Nash's (1950) axioms with the value of searching for an alternative job serving as the threat point. Hall (2005a) argues that any wage in the bargaining set, that is consistent with individual rationality for the employer-worker pair, should be regarded as a legitimate equilibrium candidate. He then proceeds to demonstrate by simulation that a rigid wage, one not conditioned on aggregate productivity, generally exists with the property that it is in the bargaining set. His argument, however, relies critically on the assumption that the only source of variability in the bargaining set is a small aggregate shock.

Hall and Milgrom (2005) argue that the outcome of a strategic bargaining game in which the disagreement payoff is delay rather than unemployed search, along the lines suggested by Binmore, Rubinstein and Wolinsky (1986), is a more realistic specification of a bargaining model. They also claim that the amended model substantially raises the implied degree of amplification because the alternative wage rule is less sensitive to productivity shocks. Although the solution to this wage-bargaining game is less volatile, the job-creation response to productivity shocks is not much larger if the value of delay is roughly equal to Shimer's value of the unemployment benefit. Alternatively, the Hall-Milgrom model explains the observed volatility of the job-finding rate only if the worker's benefit from delay plus the employer's cost, which is the opportunity cost of a match in their model, is equal to the required value derived by Hagedorn and Manovskii (2005) for the standard model.

The fact that a third of the variation in the unemployment rate is explained by job-destruction shocks and that job-to-job flows represent at least half of the flow of new hires are potentially important factors that are ignored in the simplest equilibrium unemployment model and in Shimer's analysis. We find that incorporating both fully accounts for the volatility in the job-finding rate when reasonable hiring costs are taken into account. As Nagypál (2005) has argued in detail, the reason for this result is that quits are procyclical and that employers profit more from employed rather than unemployed

workers when hiring costs are large enough because their average quit rate is smaller. Because the fraction of employed workers in the application flow increases on the upswing, employers' higher profit from hiring them both amplifies and propagates the effects of a positive productivity shock on job creation. Furthermore, we find that the same model with identical parameter values implies that vacancies and unemployment are almost perfectly negatively correlated, as Shimer (2005) finds in his data. One reason for this result is that job-destruction shocks induce negative co-movements between unemployment and vacancies when employed workers search for better jobs in sufficient numbers.

## 2 The Standard Model

In the version of the model Shimer (2005a) considers, all workers and jobs are respectively identical. Furthermore, all agents are risk-neutral wealth maximizers. For the sake of comparability, we use Shimer's notation when possible. Specifically, every job-worker match produces market output at flow rate  $p$ . Autocorrelated shocks to  $p$  occur from time to time. Hence, the current value of match productivity is an aggregate state variable. The possible dependency of any endogenous variable on the current value of productivity is represented by using  $p$  as a subscript. Following Shimer, we assume that the time sequence  $\{p_t\}$  is a jump process characterized by arrival rate  $\lambda$  and a conditional distribution of new values represented by the c.d.f.  $F : P \times P \rightarrow [0, 1]$  where  $P$  is the support of the process.

The opportunity cost of employment to the worker and the cost of posting a vacancy to the firm, measured in terms of output, are non-state-contingent parameters, denoted by  $z$  and  $c$ , respectively. Since all matches are identical, the flow of new matches is determined by a meeting function, denoted as  $m(u, v)$ , where  $u$  and  $v$  represent the number of unemployed workers currently looking for a job and the number of currently open job vacancies, respectively. By assumption, the meeting function is non-negative, increasing, concave, and homogeneous of degree one. As a consequence, the job-finding rate of workers,  $f(\theta) \equiv m(u, v)/u = m(1, \theta)$ , is positive, increasing, and concave in "market tightness," defined as the ratio of vacancies to unemployment,  $\theta \equiv v/u$ . Analogously, the rate at which vacancies are filled,  $m(u, v)/v = f(\theta)/\theta$ , is a positive, decreasing, and convex function of market tightness. Finally, matches are destroyed at the exogenous separation rate  $s$  and all

agents discount future income flows at the common rate  $r$ . The matching function  $m(\cdot)$ , the productivity process  $(\lambda, F)$ , and the set of parameters  $\{z, c, s, r\}$  fully characterize the environment of interest.

The wage in each aggregate state,  $w_p$ , as well as the levels of unemployment and vacancies are endogenous to the model. They are determined by the match surplus-sharing rule, free entry, and the law of motion for unemployment. To characterize these conditions, one needs to define the concept of match surplus.

Match surplus is the difference between the expected present value of the future incomes that the two parties to a match earn and the expected present value of income that they forgo by participating in the employment relationship. Because the value of a vacancy is driven to zero by entry, match surplus is  $V_p \equiv J_p + W_p - U_p$  where the value of a match to the employer,  $J_p$ , the value of a match to the worker,  $W_p$ , and the value of unemployment,  $U_p$ , are recursively defined by the continuous-time Bellman equations

$$rU_p = z + f(\theta_p)(W_p - U_p) + \lambda(E_p U_{p'} - U_p) \quad (1)$$

$$rW_p = w_p - s(W_p - U_p) + \lambda(E_p W_{p'} - W_p) \quad (2)$$

$$rJ_p = p - w_p - sJ_p + \lambda(E_p J_{p'} - J_p), \quad (3)$$

where  $E_p$  represents the expectation operator conditional on the current state  $p$ . In all cases, these equations imply that the return on the value of an agent's state is equal to the income flow obtained plus the product of the change in value attributable to a state transition and the relevant transition rate summed over all possible transitions. In the case of an unemployed worker, the possible changes in state include a transition to employment as well as a transition to another aggregate productivity state. Similarly, changes in the value of employment and of a filled job occur when the match is destroyed and when the aggregate state changes. Notice that these equations are consistent with individual rationality only if  $W_p - U_p \geq 0$  and  $J_p \geq 0$  for all  $p$ . As Hall (2005a) emphasizes, any reasonable wage rule agreed to by an employer and a worker engaged in a match must satisfy these inequalities.

By summing Equations (2) and (3) and then subtracting the corresponding sides of (1), one obtains the following functional equation that the surplus value of a match must satisfy:

$$rV_p = p - z - f(\theta_p)(W_p - U_p) - sV_p + \lambda(E_p V_{p'} - V_p). \quad (4)$$

Given that each agent's threat point is assumed to be the value of not being matched, the generalized Nash solution to the bargaining problem that the

worker and the employer face upon meeting maximizes the so-called Nash product, the geometric average of their respective shares of the match surplus,  $(W_p - U_p)^\beta J_p^{1-\beta}$ , where the parameter  $\beta$  reflects the worker's "bargaining power". The resulting sharing rule is characterized by

$$\frac{W_p - U_p}{\beta} = V_p = \frac{J_p}{1 - \beta}. \quad (5)$$

It is usual to suppose that wages are renegotiated in each subsequent aggregate state so as to maintain Equation (5). Finally, the free-entry condition requires that the expected cost of posting a vacancy is equal to the expected return. That is, given that the average time to fill a vacancy is  $\frac{\theta}{f(\theta)}$ ,

$$\frac{c\theta_p}{f(\theta_p)} = J_p. \quad (6)$$

An equilibrium solution to the model is a vector of functions  $(\theta_p, w_p, U_p, W_p, J_p, V_p)$ , all defined on the set of possible values of productivity  $P$ , that satisfy Equations (1)-(6). To complete Shimer's analysis, we prove that a unique equilibrium exists and that all the functions increase with productivity, given reasonable technical restrictions on the matching function.

**Proposition 1** *If (i)  $p'$  is stochastically increasing in  $p$  and (ii)  $\theta/f(\theta)$  is a strictly increasing and concave function of  $\theta$  such that  $\lim_{\theta \rightarrow 0} \{\theta/f(\theta)\} = 0$ , then a unique equilibrium exists with the property that the equilibrium functions  $(\theta_p, w_p, U_p, W_p, J_p, V_p)$  are all strictly increasing in  $p$ .*

**Proof.** See the Appendix. ■

Note that the conditions of (ii) are all satisfied in the case of a Cobb-Douglas matching function.

The explicit equilibrium wage rule can be derived by noting that Equations (1), (2), (3), and (5) imply

$$\begin{aligned} & (1 - \beta)(r + s + \lambda)(W_p - U_p) \\ &= (1 - \beta)(w_p - z - f(\theta_p)(W_p - U_p) + \lambda E_p(W_{p'} - U_{p'})) \\ &= \beta(r + s + \lambda)J_p = \beta(p - w_p + \lambda E_p J_{p'}). \end{aligned}$$

Under the assumption that the wage is renegotiated after every aggregate shock, Equation (5) holds for all  $p'$ , so  $(1 - \beta)E_p(W_{p'} - U_{p'}) = \beta E_p J_{p'}$ .

Together with the free-entry condition (6), this implies that the wage function takes the form

$$w_p = \beta p + (1 - \beta)(z + \beta f(\theta_p)V_p) = \beta(p + c\theta_p) + (1 - \beta)z. \quad (7)$$

The wage depends on the current value of aggregate productivity and increases with its realized value through two channels: because current output is shared and because the value of search while unemployed is increasing in the job-finding rate, which in turn increases in market tightness.

Under the assumption that all workers desire employment and are either employed or unemployed, the unemployment rate adjusts according to the law of motion

$$\dot{u} = s(1 - u) - f(\theta_p)u,$$

where the size of the labor force is normalized at unity. Because productivity per worker is independent of employment and the matching function has constant return to scale, the unemployment rate is not an information-relevant state variable. Instead, unemployment simply converges toward the state-contingent target

$$u_p = \frac{s}{s + f(\theta_p)}. \quad (8)$$

Elsewhere, Shimer (2005b) argues that the speed of adjustment, equal to the sum of the separation and job-finding rate, is large enough in practice that the negative relationship between vacancies,  $v_p = \theta_p u_p$ , and unemployment that Equation (8) implies can be interpreted as the empirical Beveridge curve, the downward-sloping relationship between vacancies and unemployment commonly observed.

### 3 Volatility Implied by the Standard Model

Shimer's (2005a) principal claim is that the volatility of the job-finding rate and its determinant, the vacancy-unemployment ratio, is an order of magnitude larger in U.S. data than the value implied by the standard model for "reasonable" parameter values when fluctuations are induced by shocks to labor productivity. To show this point, we substitute appropriately from the free-entry condition in Equation (6) and use the Nash-bargaining outcome in Equation (5), to get that the Bellman equation in Equation (4) implies

$$(r + s + \lambda) \frac{c\theta_p}{f(\theta_p)} + c\beta\theta_p = (1 - \beta)(p - z + \lambda E_p V_{p'}). \quad (9)$$



The job-finding rate  $f(\theta_p)$  is determined by the solution to this equation. By taking logs and differentiating the result with respect to  $\ln p$ , one obtains

$$\frac{\partial \ln f(\theta_p)}{\partial \ln p} = \frac{\eta(\theta_p)(r + s + \lambda + \beta f(\theta_p))}{(1 - \eta(\theta_p))(r + s + \lambda) + \beta f(\theta_p)} \times \frac{p \left(1 + \lambda \frac{\partial E_p V_{p'}}{\partial p}\right)}{p - z + \lambda E_p V_{p'}},$$

where  $\eta(\theta) = \theta f'(\theta)/f(\theta)$  is the elasticity of the job-finding rate with respect to market tightness, which is equivalent to the elasticity of the matching function with respect to vacancies.

At this point, Shimer claims that the value obtained when there are no aggregate shocks ( $\lambda = 0$ ) serves as an adequate approximation for computational purposes. That is, suppressing the dependence on  $\theta$ ,

$$\frac{\partial \ln f}{\partial \ln p} = \frac{\eta(r + s + \beta f)}{(1 - \eta)(r + s) + \beta f} \times \frac{p}{p - z} \quad (10)$$

holds as an approximation. When evaluated at Shimer's choice of parameters, which are median labor productivity normalized to  $p = 1$ , quarterly rates  $r = 0.012$ ,  $s = 0.10$ , and  $f = 1.355$ , matching function elasticity  $\eta = 0.28$ , labor bargaining power  $\beta = 1 - \eta = 0.72$ , and opportunity cost of employment  $z = 0.4$ , the numerical value is

$$\frac{\partial \ln f}{\partial \ln p} = \frac{0.28 \times (0.112 + 0.72 \times 1.355)}{0.72 \times 0.112 + 0.72 \times 1.355} \times \frac{1}{1 - 0.4} = 0.481. \quad (11)$$

In contrast, Shimer finds that the volatility in the log of the job-finding rate relative to that of log productivity is over ten times as large in U.S. data. Namely,

$$\frac{\sigma_f}{\sigma_p} = \frac{0.118}{0.02} = 5.9 \quad (12)$$

given the data moments reported in Table 1 below (reproduced from Shimer (2005a)), where  $\sigma_x$  and  $\rho_{xy}$  represent the standard deviation of  $\ln x$  and the correlation between  $\ln x$  and  $\ln y$ , respectively, here and in the rest of the paper.

Table 1: Shimer’s Summary Statistics, Quarterly U.S. data, 1951-2003.

$x$		$u$	$v$	$v/u$	$f$	$s$	$p$
Standard Deviation		0.190	0.202	0.382	0.118	0.075	0.020
Autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
Correlation Matrix	$u$	1	-0.894	-0.971	-0.949	0.709	-0.408
	$v$	-	1	0.975	0.897	-0.684	0.364
	$v/u$	-	-	1	0.948	-0.715	0.396
	$f$	-	-	-	1	-0.574	0.396
	$s$	-	-	-	-	1	-0.524
	$p$	-	-	-	-	-	1

Source: Shimer (2005a), Table 1. All variables reported are log deviations from an HP trend with smoothing parameter  $10^5$ .

There are two points worth raising regarding the method of calculation used by Shimer. First, the empirical counterpart of the derivative computed in Equation (10) is

$$\rho_{fp}\sigma_f/\sigma_p = 0.396 \times 5.9 = 2.34. \quad (13)$$

When Shimer calculates the elasticity of the job-finding rate as the ratio of standard deviations,  $\sigma_f/\sigma_p$ , he is implicitly assuming that shocks to productivity are the only cause of fluctuations in vacancies and unemployment. Fluctuations in the interest rate and the rate of job destruction could also be sources of volatility, a point we return to in Section 5. Even with this qualification, though, the model does not measure up in the sense that it fails to explain the estimate of the empirical elasticity in Equation (13) as well. Nonetheless, the goal post for labor-productivity shocks is substantially lower when one allows for other sources of employment volatility.

Second, Shimer’s assertion that Equation (10) serves as an adequate approximation would seem to be inconsistent with the fact that the process that he actually fits to the U.S. productivity series and uses in his simulation has a very large arrival rate,  $\lambda = 4.0$  per quarter. Fortunately, the approximation also holds when the arrival rate is large, if the change in productivity is small. Formally, Shimer assumes that the change in “net productivity” defined as  $p - z$  is determined by

$$\ln(p' - z) = \ln(p - z) \pm \Delta \text{ with probability } \frac{1}{2} \left( 1 \mp \frac{\ln(p - z)}{n\Delta} \right). \quad (14)$$

At the estimated standard deviation parameter,  $\sigma = \sqrt{\lambda}\Delta = 0.0165$ ,  $\Delta = 0.0165/2 = 0.0083$  is small. Hence, the following result justifies the use of the approximation.

**Proposition 2** *Equation (10) holds in the limit as either  $\lambda \rightarrow 0$  or  $\Delta \rightarrow 0$ .*

**Proof.** See the Appendix. ■

This explains why the results of Shimer (2005a) from the full stochastic model conform well with the calculations based on the approximation in Equation (10).

Beyond these methodological points, Shimer’s result is also naturally affected by the values of the parameters he uses. We turn to the discussion of these values next.

### 3.1 The Elasticity of the Matching Function

The elasticity of the matching function with respect to vacancies determines the sensitivity of the number of new matches created to underlying changes in the number of vacancies, and thus is an important determinant of how strongly the model economy’s job-finding rate responds to changes in its driving forces.

Shimer’s value of the elasticity of the matching function with respect to vacancies,  $\eta = 0.28$ , is obtained by regressing the detrended log of his measure of the job-finding rate, derived from CPS data, on the detrended log of the ratio of vacancies, as reflected in the Conference Board Help Wanted index, to detrended CPS unemployment. The resulting estimate is somewhat outside the “plausible range” of 0.3 to 0.5 reported by Petrongolo and Pissarides (2001) in their review of the literature on the matching function.

There are alternative ways to estimate the elasticity of the matching function. In particular, Shimer’s data on vacancies and unemployment clearly imply that  $\ln v + \ln u$  is almost constant, and hence the Beveridge curve is close to a rectangular hyperbola. Specifically, the data moments in Table 1 imply that the OLS regression of (log) vacancies on (log) unemployment yield the coefficient estimate  $\rho_{vu}\sigma_v/\sigma_u = -0.894 \times 0.202/0.190 = -0.950$ . Given the rapid adjustment of unemployment to the state-contingent target value at rate  $s + f = 0.485$  per month, Equation (8) accurately represents the Beveridge curve relating vacancies and unemployment. Since this equation and

a Cobb-Douglas specification of the matching function,  $m(v, u) = \mu v^\eta u^{1-\eta}$ , can be written as

$$\ln \mu + \eta \ln v + (1 - \eta) \ln u = \ln s + \ln(1 - u),$$

the regression coefficient implied by Equation (8) is

$$\frac{\partial \ln v}{\partial \ln u} = -\frac{1}{\eta} \left( \frac{u}{1-u} + 1 - \eta \right).$$

Using the unemployment rate,  $u = 0.0687$ , implied by Equation (8) and the estimated regression coefficient  $\frac{\partial \ln v}{\partial \ln u} = -0.950$  gives an estimate of the elasticity of the matching function of  $\eta = 0.551$ , somewhat above the upper bound on the “plausible range” of Petrongolo and Pissarides (2001). Since the approximate elasticity of the job-finding rate in Equation (10) increases in  $\eta$ , this estimate suggests that Shimer’s measure of the volatility of the job-finding rate may be biased downward.

Notice, however, that even if one uses our alternative estimate of  $\eta$  and his parameter values, the elasticity of the job-finding rate with respect to productivity implied by Equation (10) is

$$\frac{\partial \ln f}{\partial \ln p} = \frac{0.551 \times (0.112 + 0.449 \times 1.355)}{0.449 \times 0.112 + 0.449 \times 1.355} \times \frac{1}{1 - 0.4} = 1.004.$$

This elasticity is twice as large as that implied by Shimer’s estimate of  $\eta$ , but it is still only a fraction of that observed in the data. Hence, disagreement regarding the magnitude of the elasticity of the matching function alone does not overturn Shimer’s conclusion given his choices of the other parameter values.

### 3.2 The Opportunity Cost of Employment

Shimer (2005a) sets  $z = 0.4$  as a “generous estimate” of the unemployment insurance replacement ratio. Hagedorn and Manovskii (2005) argue that Shimer’s choice of the opportunity cost of employment is too low because it does not allow for the “value of leisure” or “home production” forgone when employed above and beyond the unemployment insurance benefit. Moreover, they calibrate both the opportunity cost of employment and the bargaining share parameter to match the cyclical response of wages implied by the Solon,

Barsky, and Parker (1994) wage data series and the average profit rate computed by Basu and Fernald (1997). Their calibration results give  $z = 0.943$  and  $\beta = 0.061$ . Using these numbers and setting the other values equal to those of Shimer's calibration, the elasticity of the job-finding rate with respect to productivity is

$$\frac{\partial \ln f}{\partial \ln p} = \frac{0.28 \times (0.112 + 0.061 \times 1.355)}{0.72 \times 0.112 + 0.061 \times 1.355} \times \frac{1}{1 - 0.943} = 5.86.$$

In other words, with this calibration, the model exactly matches the relative magnitudes of the variability of the job-finding rate and labor productivity found in the data even when Shimer's estimate of  $\eta$  is used in the calculation and shocks to productivity are the only source of labor-market fluctuations.

Although the Hagedorn-Manovskii analysis does obey the letter of the law of the model's logic, one could argue that it violates its spirit. That is, they clearly demonstrate that the estimated elasticity of the wage with respect to labor productivity, which they report to be 0.47, and an estimated profit rate of 3%, which they interpret as the average value of  $(p - w_p)/p$ , require a small value of  $\beta$  and a large value of  $z$ . Hence, they conclude that the model is consistent with the data for the above values of these two parameters. While this is formally true, the economic implausibility of their solution is suggested by two implications.

First, the flow surplus enjoyed by an employed worker in the model for these parameter values is miniscule. Indeed, the wage at  $p = 1$  is

$$w = \beta(p + c\theta) + (1 - \beta)z = 0.061 \times (1 + 0.373) + (1 - 0.061) \times 0.943 = 0.969,$$

given the value of  $c\theta = 0.209$  implied by the free-entry condition (9), and the surplus flow when employed is

$$\frac{w - z}{z} = \frac{0.969 - 0.943}{0.943} = 0.028.$$

But, do workers work for a 2.8% surplus?

In their paper, Hagedorn and Manovskii respond to this point by arguing that a value of  $z$  near  $p$  is reasonable for the marginal worker. While this argument is correct, it is irrelevant in the context of this model, because job creation depends on the average value of  $z$ , not its value for the marginal worker.

To see this point more clearly, notice that, given heterogeneity in  $z$ , the value of unemployment for the marginal worker is equal to the value of non-participation. Thus the value of  $z$  for the marginal participant, denoted by  $\underline{z}$ , solves  $\underline{z} = r\underline{U}$  where  $\underline{U}$  represents the marginal worker's value of unemployed search. Because

$$\begin{aligned} rU(z) &= z + f(\theta)(W(z) - U(z)) \\ rW(z) &= w(z) - s(W(z) - U(z)) \end{aligned}$$

hold as steady-state approximations,

$$0 = r\underline{U} - \underline{z} = f(\theta)(\underline{W} - \underline{U}),$$

so  $\underline{U} = \underline{W}$  and  $\underline{z} = r\underline{U} = r\underline{W} = \underline{w}$ . This in turn implies that the surplus from the relationship is  $\underline{V} = \frac{\underline{W} - \underline{U}}{1 - \beta} = 0$ , which in turn necessitates that  $p = \underline{w} = \underline{z}$ . So not only is the marginal worker's value of leisure close to  $p$ , it is exactly equal to it.<sup>3</sup>

However, the incentive to create a job depends on the average value of  $z$ , not its marginal value. Indeed, as the value of a match with a worker of type  $z$  is

$$J(z) = \frac{p - w(z)}{r + s},$$

but the worker's type is not known when the employer posts the job, the free-entry condition is

$$\frac{cf(\theta)}{\theta} = E[J(z)|z \leq p] = \frac{p - E[w(z)|z \leq p]}{r + s} = \frac{(1 - \beta)(p - E[z|z \leq p]) - \beta c\theta}{r + s},$$

where the last equality can be derived from the wage equation as before. In sum, it is the average opportunity cost of unemployed participants that matters in the determination of market tightness.

A second problem with a high value of opportunity cost of employment is that the model predicts an implausibly large response to changes in economic policy variables, as noted by Costain and Reiter (2005). In particular, using the approximation of the free-entry condition in Equation (9) when  $\lambda = 0$  to derive the impact of changes in  $z$ , it follows that

$$\frac{\partial \ln f}{\partial \ln z} = -\frac{\partial \ln f}{\partial \ln(p - z)} \frac{z}{p - z} = -\frac{\partial \ln f}{\partial \ln p} \frac{z}{p}.$$

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<sup>3</sup>Adding search costs could drive a wedge between the value of leisure for the marginal worker and match output  $p$ .

Thus if  $z$  is close to  $p$ , the large response of the job-finding rate to changes in labor productivity also implies a large response of the job-finding rate (and consequently of unemployment) to changes in  $z$ . Assuming that half of  $z$  represents unemployment benefits, a 1% increase in unemployment benefits implies a 0.5% increase in  $z$ , which, using Hagedorn and Manovskii's numbers, results in a 2.76% decrease in the job-finding rate and a corresponding 2.57% increase in unemployment. This is a very large policy response that has no counterpart in any of the empirical studies of the response of unemployment to changes in the generosity of unemployment insurance reviewed by Costain and Reiter (2005).

### 3.3 Turnover Costs

One potentially important factor that Shimer's analysis abstracts from is turnover costs. As has been pointed out by several authors (see Braun (2005), Nagypál (2005), Silva and Toledo (2005), and Yashiv (2005)), the presence of fixed turnover costs makes firms' net payoff (after paying the fixed cost) more responsive to variation in the level of their gross payoff. In other words, allowing for hiring and firing costs increases the opportunity cost of the match without resorting to very high values of opportunity cost of employment for the worker.

For the sake of illustration, suppose that the cost of hiring and training a new worker is  $H$  and the termination cost is  $T$ . In this case, the steady-state value of filling a job solves

$$rJ = p - w - s(J + T)$$

and the free-entry condition is

$$\frac{c\theta}{f(\theta)} = J - H = \frac{p - w - sT}{r + s} - H = \frac{p - w - sT - (r + s)H}{r + s}.$$

Taking logs and taking derivatives with respect to  $\ln p$  gives

$$\frac{\partial \ln f(\theta)}{\partial \ln p} = \frac{\eta}{1 - \eta} \times \frac{p - w \frac{d \ln w}{d \ln p}}{p - w - sT - (r + s)H},$$

so, for a given level and elasticity of wages that determines  $w \frac{d \ln w}{d \ln p}$ , the model explains more of the relative volatility of the job-finding rate if the amortized

costs of hiring and firing are high enough. For example, introducing hiring costs equal to two quarters' of flow profits (which is less than a week of wages if the flow profit rate is 3%, as observed by Hagedorn and Manovskii) raises the elasticity by a factor of 1.29. Similarly, introducing firing costs equal to two quarters' of flow profits raises the elasticity by a factor of 1.25.

For this mechanism to work, however, it is important that these turnover costs are indeed fixed as the underlying driving force varies and depend neither on labor productivity nor on the level of wages in the economy. Whether this is indeed the case is an empirical question that deserves further attention.

It is worthwhile to mention that because of their dependence on the level of labor productivity, capital costs cannot be introduced as a fixed cost in the model. Doing so does increase the amount of amplification predicted by the model for the same reasons that fixed turnover costs do, but the assumption of fixed capital costs cannot be maintained given a standard neoclassical production function.<sup>4</sup>

## 4 Alternative Wage-Setting Mechanisms

The ingredient of the standard model that has been put to the most scrutiny in works following Shimer (2005a) has been the wage-setting mechanism. Several authors have argued that the model can match the empirically observed volatility of labor-market variables if the mechanism generates a less procyclical wage than implied by the model as usually formulated. However, a rigid wage is not enough, its level must also be high. Only when the wage is large and not too procyclical does the per-period profit that motivates job creation,  $p - w$ , respond strongly to changes in  $p$ . To understand this point, note that when  $\beta = 0$  in the standard model the wage is rigid and equal to  $z$  from Equation (7). However, given the values assigned by Shimer for the other parameters, the elasticity of the job-finding rate with respect to productivity would still only be

$$\frac{\partial \ln f(\theta)}{\partial \ln p} = \frac{\eta}{1 - \eta} \times \frac{p}{p - z} = \frac{0.28}{0.72} \times \frac{1}{0.6} = 0.648.$$

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<sup>4</sup>The presence of fixed capital costs goes some way towards explaining why Fujita and Ramey (2005) get a lot more amplification in their version of the standard model than Shimer (2005a).



## 4.1 Rational Wage Rigidity

Of course, if the wage level is high enough, then a rigid wage can easily account for the observed volatility. For example, suppose that the wage  $w$  is rigid as in Hall (2005a). In this case, the free-entry condition

$$\frac{c\theta}{f(\theta)} = J = \frac{p - w}{r + s}$$

holds as a steady-state approximation. Setting the rigid wage equal to the Nash solution at the median productivity,  $w = \beta(p + c\theta) + (1 - \beta)z = 0.983$ , as in Hall’s analysis, Shimer’s parameter values imply

$$\frac{\partial \ln f(\theta)}{\partial \ln p} = \frac{\eta}{1 - \eta} \times \frac{p}{p - w} = \frac{0.28}{1 - 0.28} \times \frac{1}{1 - 0.983} = 22.48.$$

This number is almost four times that needed to explain the observed response in the job-finding rate to productivity.

The rigid wage assumption is difficult to swallow, however. Since aggregate shocks are common knowledge, why wouldn’t negotiated wages reflect the fact that the worker’s outside search option is procyclical as the Nash bargaining solution implies? Hall (2005a) argues, as many others have done in the past, that the solution to a bilateral monopoly problem is simply indeterminate. According to Hall, any solution in the bargaining set should be regarded as a legitimate equilibrium. Furthermore, under these circumstances, it is reasonable to suppose that the wage set in previous bargains with other workers will serve as either a “norm” or a “focal point” for the outcome of any current bargain in every state for which this solution is jointly rational, that is when both  $W_p - U_p \geq 0$  and  $J_p \geq 0$  hold.<sup>5</sup> He then proceeds to show that the shocks to aggregate productivity required to explain the volatility of unemployment are so small that this condition is always satisfied in simulations.

There are two problems with Hall’s argument. First, to maintain that the rigid wage is jointly rational, only small aggregate shocks can affect the employment relationship of workers and firms in the economy. This assumption is greatly at odds with the extent of gross flows in the labor

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<sup>5</sup>Wage norms have not been the only way to rationalize the existence of a rigid wage. Kennan (2005), Menzio (2005), and Moen and Rosen (2005) all develop contracting models with asymmetric information that deliver a rigid wage as an equilibrium outcome.

market that reflect the importance of idiosyncratic variability. Second, the limited empirical evidence available does not support the claim that wages of workers in new employment relationships are rigid over the business cycle, in fact, these wages have been found to be more cyclically sensitive than wages of workers in continuing relationships.<sup>6</sup>

## 4.2 Infrequent Wage Bargaining

Since the work of Taylor (1980) and Calvo (1983), the idea that nominal prices and wages are revised infrequently has played an important role in macroeconomics. Gertler and Trigari (2005) have shown that this mechanism also has the power to generate substantial labor-market volatility in a matching model. Although we abstract from their assumption that wage negotiations are at the firm level and are staggered, the basic point can be made within our simple framework. That is, all workers in all matches receive the same wage which is revised from time to time to reflect new information about aggregate productivity.

By assumption, a new wage bargain is made with all workers, old and new, infrequently for whatever reason.<sup>7</sup> In the Calvo version of the model, wage renegotiation takes place with Poisson frequency  $\alpha$ . Hence, the value of a new match depends on the currently prevailing wage as well as future anticipated wages. Indeed, at the moment a new productivity shock  $p$  is realized, the Bellman equation can be approximated as

$$rJ_p(w) = p - w - sJ_p(w) + \alpha(J_p - J_p(w)).$$

where  $w$  is the wage inherited from the last renegotiation. Hence,  $J_p(w)$  is the value of the match when worker and employer meet and  $J_p$  is the value after the wage is renegotiated in the future. That is

$$rJ_p = p - w_p - sJ_p = \frac{p - w_p}{r + s},$$

where  $w_p$  represents the bargained wage given that common match productivity is  $p$ . Equivalently,  $J_p = J_p(w_p)$  is the value of the match after the wage has been adjusted to reflect the value of  $p$ .

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<sup>6</sup>See Vroman (1977), Bils (1985), and Barlevy (2001).

<sup>7</sup>For the Gertler and Trigari story to work, it is imperative that the wages of new employees are negotiated only at the time that all wages of worker in the firm are revised and not when the new worker is hired.

When a wage is renegotiated, it reflects prevailing demand conditions as in the original model. Specifically, Equation (7)

$$w_p = z + \beta(p - z) + \beta c\theta_p$$

holds by virtue of the Nash sharing rule and the free-entry condition at the renegotiation date as in the original model. Therefore,

$$J_p = \frac{p - w_p}{r + s} = \frac{(1 - \beta)(p - z) - \beta c\theta_p}{r + s}$$

as before, but

$$J_p(w) = \frac{p - w + \alpha J_p}{r + s + \alpha} = \frac{p - w}{r + s + \alpha} + \frac{\alpha}{r + s + \alpha} \frac{(1 - \beta)(p - z) - \beta c\theta_p}{r + s}.$$

Note that this formulation nests the standard model in which wages are continually revised ( $\alpha = \infty$ ) and the rigid wage model ( $\alpha = 0$ ).

Finally, the free-entry condition given the current wage  $w$  is

$$\frac{c\theta}{f(\theta)} = J_p(w) = \frac{r + s}{r + s + \alpha} \frac{p - w}{r + s} + \frac{\alpha}{r + s + \alpha} \frac{(1 - \beta)(p - z) - \beta c\theta_p}{r + s},$$

where  $c\theta_p/f(\theta_p) = J_p(w_p) = J_p$  determines market tightness once the wage is renegotiated. Suppose, as Gertler and Trigari do, that wages are renegotiated once every three quarters on average,  $\alpha = 0.33$ . When evaluated at a wage equal to that prevailing in the median productivity state  $w = \beta(p + c\theta) + (1 - \beta)z = 0.983$  and  $\frac{\partial \ln f(\theta_p)}{\partial \ln p} = 0.481$  from Equation (11),

$$\begin{aligned} \frac{\partial \ln f(\theta)}{\partial \ln p} &= \frac{\eta p}{1 - \eta} \left( \frac{r + s + \alpha \left( 1 - \beta - \frac{\beta c\theta_p}{\eta p} \frac{\partial \ln f(\theta_p)}{\partial \ln p} \right)}{(r + s)(p - w) + \alpha [(1 - \beta)(p - z) - \beta c\theta_p]} \right) \\ &= \frac{0.28}{0.72} \left( \frac{0.112 + 0.33 \times \left( 0.28 - \frac{0.72 \times 0.209 \times 0.481}{0.28} \right)}{0.112 \times (1 - 0.983) + 0.33 \times (0.28 \times (1 - 0.4) - 0.72 \times 0.209)} \right) \\ &= 6.05. \end{aligned}$$

In this model, the initial impact of the productivity shock on the job-finding rate is somewhat larger than the relative volatility of the job-finding rate observed in the data. However, note that in the future when the wage is actually renegotiated, the wage will jump up from  $w$  to its equilibrium value  $w_p$  in response to a positive productivity shock. The effect is a reduction in market tightness at that point in time to a level below its initial response. That is, the implied response in the job-finding rate to productivity shocks is large on impact but then dissipate as wages adjust to the shock with a lag.

### 4.3 Strategic Wage Bargaining

The Nash bargaining wage-setting mechanism of the standard model is often justified by noting that the wage outcome it implies is equivalent to the outcome of an alternating-offers strategic bargaining game (characterized by Binmore, Rubinstein and Wolinsky (1986)). Hall and Milgrom (2005) note that for this equivalence to hold, one needs to assume not only that the outside option of the worker (the payoff she gets in case negotiations break down) but also the disagreement payoff of the worker (the payoff she gets when agreement is delayed) is equal to the value of search. They take issue with the second assumption and argue that unemployed search is not the relevant payoff from delay during negotiations. When the disagreement payoff is fixed rather than tied to the value search, the wage agreed to is more rigid than that implied by the standard sharing rule.

For the sake of illustration, suppose that the worker receives payoff  $z$  and the employer incurs no cost while bargaining continues and that they renegotiate the division of the match product  $p$  whenever it changes. In this case, the outcome of an alternating-offers game is

$$w_p = z + \beta(p - z). \quad (15)$$

As

$$\frac{c\theta}{f(\theta)} = \frac{p - w_p}{r + s} = (1 - \beta) \frac{p - z}{r + s} \quad (16)$$

holds in this case, the value of  $z$  required to match the volatility of the job-finding rate, the solution to

$$\frac{d \ln f(\theta)}{d \ln p} = \frac{\eta}{1 - \eta} \times \frac{p}{p - z} = \frac{0.28}{1 - 0.28} \times \frac{1}{1 - z} = 5.9,$$

is  $z = 0.934$ . In other words, the worker's delay benefit must be over 93% of median match output. Just like a very high value of leisure, this possibility also seems implausible. This solution does have the attractive feature, though, that it allows one to bring in driving forces other than labor productivity as a plausible source of fluctuations, as we show in Section 5.

## 5 Other Sources of Fluctuations

### 5.1 Discount Rate

Clearly, one important source of fluctuations missing from the above analysis is variation in the discount rate. Qualitatively, the question is whether variations in the discount rate amplify or dampen the response of labor-market variables, which in turn depends on whether the discount rate is pro- or countercyclical. When Hall (2005b) embeds his version of the matching model in a DSGE framework, he finds that increases in the discount rate induced by an increase in productivity dampen labor-market amplification. If the source of variation in the match output,  $p$ , are monetary policy shocks, however, as in Braun (2005), the discount rate moves countercyclically, and labor-market responses are amplified by variation in the discount rate. Quantitatively, Yashiv (2005), using observed values of the discount rate in his version of the matching model, finds that variations in the discount rate have a small role at best in explaining the volatility of the job-finding rate. This is because the pure discount rate  $r$  makes up a small fraction of the total rate of “discounting” that firms apply to match profits,  $r + s$ , both in terms of levels and in terms of volatility.

### 5.2 Job-Destruction Rate

Given that the job-destruction rate makes up the bulk of firms’ total discount rate,  $r + s$ , it is natural to consider volatility in the job-destruction rate as a potential driving force. Shimer (2005a) argues that volatility in the job-destruction rate cannot be an important source of fluctuations in a matching model, because it induces a counterfactual positive correlation between vacancies and unemployment and thus does not have a noticeable effect on market tightness or the job-finding rate. He therefore concludes that only labor productivity shocks can be plausible sources of fluctuations in the standard model. His results critically hinge on the fact that any decrease in the job-finding rate following an upward shock to the destruction rate puts a very strong downward pressure on wages. This downward pressure on wages then stimulates vacancy creation through its effect on flow profits, which in turn reverses the decrease in the job-finding rate.

To see this, notice that the wage derived in Equation (7) implies that, at

Shimer's parameter values,

$$\begin{aligned} \left. \frac{\partial \ln(p-w)}{\partial \ln f} \right|_{p \text{ fixed}} &= -\frac{1}{\eta} \frac{\beta c \theta}{(1-\beta)(p-z) - \beta c \theta} = \\ &= -\frac{1}{0.28} \frac{0.72 \times 0.209}{0.28 \times 0.6 - 0.72 \times 0.209} = -31.1. \end{aligned}$$

Hence, any significant change in the job-finding rate that does not come from changes in  $p$  has a huge impact on the flow profit of firms which is not sustained in equilibrium.

As is clear from Table 1, however, changes in the job-destruction rate,  $s$ , are present in the data and are negatively correlated with labor productivity. These changes, which are implied by the Mortensen and Pissarides (1994) model with endogenous job destruction, serve as an important source of volatility in unemployment. Moreover, because countercyclical movements in the job-destruction rate imply that the rate at which firms discount match profits falls in a boom and rises in a bust, these shocks tend to also amplify the effects of productivity shocks on vacancy creation and the job-finding rate if the feedback from the job-finding rate to the wage is weak.

To see these points more clearly, consider the simplified version of the Hall-Milgrom strategic wage bargain discussed above, where wages do not directly depend on the job-finding rate, and assume that the model is subject to two shocks: productivity and job-destruction rate shocks. Given the free-entry condition as stated in Equation (16) and the fact that  $\eta$  is the elasticity of the job-finding rate function  $f(\theta)$ ,

$$\frac{1-\eta}{\eta} \Delta \ln f = a \Delta \ln p - b \Delta \ln s \quad (17)$$

holds as a linear approximation, where  $\Delta \ln x$  is the difference between  $\ln x$  and its mean in the data and

$$\begin{aligned} a &= \frac{\partial \ln(p-z)}{\partial \ln p} = \frac{p}{p-z} \\ b &= \frac{\partial \ln(r+s)}{\partial \ln s} = \frac{s}{r+s} \end{aligned}$$

are the indicated partial derivatives. Since Equation (17) implies

$$E(\Delta \ln f)^2 = \sigma_f^2 = \left( \frac{\eta}{1-\eta} \right)^2 \left( a^2 \sigma_p^2 - 2ab\rho_{sp}\sigma_s\sigma_p + b^2\sigma_s^2 \right),$$

it follows that

$$\frac{\sigma_f}{\sigma_p} = \frac{a\eta}{1-\eta} \left( 1 - \frac{2b}{a} \frac{\rho_{ps}\sigma_s}{\sigma_p} + \left( \frac{b\sigma_s}{a\sigma_p} \right)^2 \right)^{1/2}.$$

Given Shimer's values of the parameters

$$\frac{\sigma_f}{\sigma_p} = \frac{1.667 \times 0.28}{1 - 0.28} \left( 1 + \frac{2 \times 0.893 \times 0.524 \times .075}{1.667 \times .02} + \left( \frac{0.893 \times .075}{1.667 \times .02} \right)^2 \right)^{1/2} = 1.732. \quad (18)$$

Although the model still fails to account adequately for the observed magnitude of fluctuations in the labor market, taking account of job-destruction shocks roughly triples the model's implied volatility of the job-finding rate relative to that of labor productivity. This result is driven by the fact that countercyclical changes in the job-destruction rate imply that the expected length of a newly formed relationship is significantly shorter in a recession, which discourages firms from creating vacancies. As we'll see in Section 6, the conclusion that a higher job-destruction rate in a recession implies a shorter expected length of employment relationships critically depends on abstracting from another form of termination of employment relationships: job-to-job transitions.

## 6 Search on the Job

Shimer (2005a) and subsequent authors abstract from job-to-job flows. Considering these flows is important for two reasons. First, employed workers represent well over half of those hired in any period. Furthermore, because their fraction among new hires is strongly procyclical (Nagypál (2006)), the payoff from meeting an employed worker has an important influence on the incentives to create vacancies. Second, in the presence of job-to-job flows, the rate that firms use to discount match profits is determined by the total separation rate (the sum of the quit and the job-destruction rate) and not the job-destruction rate alone. Since the total separation rate is less volatile than the job-destruction rate, the calculations of Section 5 are altered in the presence of job-to-job flows.<sup>8</sup> In this section, we extend our analysis to

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<sup>8</sup>The relative acyclicity of the total separation rate does not justify dismissing changes in the job-destruction rate as irrelevant (as in Hall (2005a)), since, as we mentioned in

include flows of workers from job to job.

## 6.1 The Model

The standard explanation for job-to-job flows is search on the job motivated by match heterogeneity. These models generally imply that the quit rate is procyclical, an effect that offsets that of the countercyclical job-destruction rate on the firm's discount rate. However, the matching technology is also different when workers search on the job. Indeed, if all workers contact jobs at the same rate, which is the benchmark case considered here, then the job-finding rate is a function of vacancies alone rather than the vacancy-unemployment ratio. Because vacancies are less volatile than the vacancy-unemployment ratio, the implied estimate of the elasticity of the job-finding rate is larger when search on the job is taken into account.

Suppose for simplicity that all jobs pay the same wage but that each job match has an idiosyncratic value to the worker which varies across matches.<sup>9</sup> Specifically, let the flow value of a job to a specific worker equal  $w+x$  where  $w$  is the common wage paid in all jobs and  $x$  is a random variable representing an idiosyncratic taste component characterized by the c.d.f.  $F : [\underline{x}, \bar{x}] \rightarrow [0, 1]$ . By assumption,  $x$  is i.i.d. across matches. This form of heterogeneity will induce worker movements from matches with lower to higher values of  $x$ .

To illustrate simply the differences between the standard model and this simple perturbation, we assume that workers generate job offers at rates that are independent of employment status. (For a more general analysis, see Nagypál (2005).) Because the measure of searching workers is equal to the labor force in this case, the rate at which workers, employed or not, meet jobs is simply a function of the number of vacancies. Formally, the aggregate meeting rate is

$$f(v) = m(1, v), \tag{19}$$

and the vacancy filling rate is  $f(v)/v$ . Furthermore, because the opportunities to search are the same whether employed or not, the reservation value of the idiosyncratic component  $x$  is the one that compensates the worker for any

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Section 5, changes in the job-destruction rate account for over a third of the variation in the unemployment rate.

<sup>9</sup>For a specification of an alternating-offers bargaining game with asymmetric information that supports this wage outcome, see Nagypál (2005).



forgone income or its equivalent when unemployed. That is, the reservation value, denoted as  $\hat{x}$ , solves

$$\hat{x} = \begin{cases} z - w & \text{if } z - w > \underline{x} \\ \underline{x} & \text{if } z - w \leq \underline{x} \end{cases} . \quad (20)$$

As is well known, in any model with on-the-job search, the distribution of employed workers over any job characteristic generally differs from the distribution over vacant jobs as a consequence of selection. Specifically, because employed workers only move to jobs with higher values of  $x$ , and workers only accept jobs above the reservation value  $\hat{x}$ , the measure of workers employed in jobs with idiosyncratic component less than or equal to  $x$ , denoted as  $G(x)$ , and the measure of unemployment, represented as  $u$ , satisfy the following steady-state conditions that arise from equating flows into and out of the relevant pool of workers:

$$\begin{aligned} (s + f(v)(1 - F(x))) G(x) &= f(v)(F(x) - F(\hat{x})) u \\ s(1 - u) &= f(v)(1 - F(\hat{x})) u. \end{aligned}$$

Thus

$$G(x) = \frac{uf(v)(F(x) - F(\hat{x}))}{(s + f(v)(1 - F(x)))}, \quad (21)$$

where

$$u = \frac{s}{s + f(v)(1 - F(\hat{x}))}. \quad (22)$$

Bargaining over a match's value is problematic when workers search on the job, particularly if the worker's idiosyncratic component of its value is not observable.<sup>10</sup> One simple alternative is to suppose that commitment is not possible so that bargaining takes place continuously over the division of the net match product. Specifically, we continue to assume that Equation (15) characterizes the wage given the current value of match product  $p$ .

As the worker quits a match with idiosyncratic component  $x$  at rate  $f(v)(1 - F(x))$ , the value of a filled job with idiosyncratic component equal to  $x$  solves

$$rJ(x) = p - w - (s + f(v)(1 - F(x))) J(x).$$

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<sup>10</sup>For a discussion and analysis of the problem, see Shimer (2005c).

Given the wage rule of Equation (15),

$$J(x) = \frac{(1 - \beta)(p - z)}{r + s + f(v)(1 - F(x))}.$$

Notice that  $s$  in the employer's discount rate is now replaced with the total separation rate,  $s + f(v)(1 - F(x))$ , which includes the quit rate.

As unemployed workers accept any job with a value of  $x$  above the reservation value and employed workers accept an alternative job when it yields a higher value of  $x$ , the probability that a job characterized by  $x$  will be accepted is  $A(x) = 0$  if  $x < \hat{x}$  and

$$A(x) = u + G(x) = \frac{s}{s + f(v)(1 - F(x))} \text{ if } x \geq \hat{x},$$

where the second equality is obtained after using Equations (21) and (22) to eliminate  $u$  and  $G(x)$ . Notice that the acceptance probability is counter-cyclical, implying that, conditional on meeting a worker, it is harder for a firm to hire a worker in a boom than in a recession.

Free entry equalizes the expected cost and return to job creation. That is, given that firms pay hiring costs of  $H$  when forming an employment relationship,

$$\begin{aligned} \frac{cv}{f(v)} &= \int_{\hat{x}}^{\bar{x}} A(x) [J(x) - H] dF(x) & (23) \\ &= \int_{\hat{x}}^{\bar{x}} \frac{(1 - \beta)(p - z)s - [r + s + f(v)(1 - F(x))]sH}{[r + s + f(v)(1 - F(x))] [s + f(v)(1 - F(x))]} dF(x) \\ &= \int_{F(\hat{x})}^1 \frac{(1 - \beta)(p - z)s - [r + s + f(v)(1 - y)]sH}{[r + s + f(v)(1 - y)] [s + f(v)(1 - y)]} dy \\ &= \frac{(1 - \beta)(p - z)s}{rf(v)} \left( \ln \left( \frac{r + s}{s} \right) - \ln \left( \frac{r + s + f(v)(1 - F(\hat{x}))}{s + f(v)(1 - F(\hat{x}))} \right) \right) \\ &\quad - \frac{s}{f(v)} \ln \left( \frac{s + f(v)(1 - F(\hat{x}))}{s} \right) H, \end{aligned}$$

where the third equality follows by changing the variable of integration to  $y = F(x)$ . Notice that without hiring costs ( $H = 0$ ) the expected return to job creation, the right-hand side, is decreasing in the job-finding rate for two reasons: the probability of acceptance declines with  $f$  and the employer's total discount rate increases with  $f$ . As pointed out by Nagypál (2005), both of these effects temper the impact of shocks on the equilibrium number of vacancies.

## 6.2 Job-Finding Rate Volatility: No Hiring Cost

Market tightness is defined as the ratio of vacancies to searching workers. As employment is procyclical by definition, this ratio is less volatile when employed workers search on the job. In the case in which employed workers contact vacancies at the same rate as unemployed workers, market tightness is simply proportional to vacancies. Given Shimer’s data as reported in Table 1, the elasticity of the job-finding rate with respect to “market tightness” as measured by vacancies is

$$\eta = \frac{\rho_{fv}\sigma_f}{\sigma_v} = \frac{0.897 \times 0.118}{0.202} = 0.524,$$

rather than Shimer’s value of 0.28.<sup>11</sup>

Because the interest rate  $r$  is small relative to both the estimates of the destruction rate  $s$  and the job-finding rate  $f$ , the right-hand side of the free-entry condition (23) can be approximated by its limit as  $r$  tends to zero. By L’Hôpital’s rule

$$\begin{aligned} \frac{s}{f} \lim_{r \rightarrow 0} \left( \frac{\ln \left( \frac{r+s}{s} \right) - \ln \left( \frac{r+s+f(1-F(\hat{x}))}{s+f(1-F(\hat{x}))} \right)}{r} \right) &= \frac{s}{f} \lim_{r \rightarrow 0} \left( \frac{\frac{1}{r+s} - \frac{1}{r+s+f(1-F(\hat{x}))}}{1} \right) \\ &= \frac{1 - F(\hat{x})}{s + f(1 - F(\hat{x}))}. \end{aligned}$$

Hence, the approximate free-entry condition when  $\hat{x} = \underline{x}$ <sup>12</sup> is

$$\frac{cv}{f(v)} = \frac{(1 - \beta)(p - z)}{s + f(v)} \quad (24)$$

when there are no hiring costs ( $H = 0$ ).

<sup>11</sup>This alternative specification of market tightness implies that the job-finding rate is a function of vacancies rather than the vacancy-to-unemployment ratio. Since the correlations of the job-finding rate with each of the two variables are roughly equal (see Table 1), the evidence does not distinguish between the two hypotheses.

<sup>12</sup>This means that unemployed workers accept all matches they encounter. According to Devine and Keifer (1991), this is a reasonable empirical approximation. Moreover, if  $\underline{x}$  is not too low, then this is implied by Equation (20). Notice that if  $\hat{x} = z - w > \underline{x}$ , then  $\hat{x}$  becomes countercyclical, which raises the response of the job-finding rate,  $f(v)(1 - F(\hat{x}))$ , to shocks.

Next, we compute the volatility of the job-finding rate under the assumption that vacancies are determined by two random driving forces,  $p$  and  $s$ . As  $\Delta \ln f = \eta \Delta \ln v$  given Equation (19), Equation (24) implies that

$$\frac{1-\eta}{\eta} \Delta \ln f = \tilde{a} \Delta \ln p - \tilde{b} \Delta \ln s - \tilde{c} \Delta \ln f \quad (25)$$

holds as a linear approximation, where the coefficients

$$\begin{aligned} \tilde{a} &= \frac{\partial \ln(p-z)}{\partial \ln p} = \frac{p}{p-z} = \frac{1}{0.6} = 1.667 \\ \tilde{b} &= \frac{\partial \ln(s+f)}{\partial \ln s} = \frac{s}{s+f} = \frac{0.1}{1.455} = .0687 \\ \tilde{c} &= \frac{\partial \ln(s+f)}{\partial \ln f} = \frac{f}{s+f} = \frac{1.355}{1.455} = 0.931 \end{aligned} \quad (26)$$

represent the indicated partial derivatives. Equation (25) then implies

$$E(\Delta \ln f)^2 = \sigma_f^2 = \left( \frac{\eta}{1-\eta+\tilde{c}\eta} \right)^2 \left( \tilde{a}^2 \sigma_p^2 - 2\tilde{a}\tilde{b}\rho_{sp}\sigma_s\sigma_p + \tilde{b}^2 \sigma_s^2 \right),$$

so the volatility of the job-finding rate relative to that of productivity implied by the model is

$$\frac{\sigma_f}{\sigma_p} = \frac{\tilde{a} \times \eta}{1-\eta+\tilde{c}\eta} \left( 1 - \frac{2\tilde{b}\rho_{ps}\sigma_s}{\tilde{a}\sigma_p} + \left( \frac{\tilde{b}\sigma_s}{\tilde{a}\sigma_p} \right)^2 \right)^{1/2}.$$

Given our estimated value of  $\eta = 0.524$  and Shimer's values for the other parameters, the implied relative volatility of the job-finding rate is

$$\frac{\sigma_f}{\sigma_p} = \frac{1.667 \times 0.524}{1 - 0.524 + 0.931 \times 0.524} \left( 1 + \frac{2 \times .0687 \times 0.524 \times .075}{1.667 \times .02} + \left( \frac{.0687 \times .075}{1.667 \times .02} \right)^2 \right)^{1/2} = 0.986. \quad (27)$$

Although the elasticity of the job-finding rate,  $\eta$ , is larger when workers search on the job, movements in the job-finding rate dampen the incentive to create vacancies because the discount rate includes the quit rate. The latter effect induces both a smaller value for the parameter  $\tilde{b}$  and a positive value for the parameter  $\tilde{c}$ . Overall, the impact of the quit rate dominates, in the sense that the relative volatility of the job-finding rate is smaller than that obtained in the case of no search on the job (see Equation (18)).

### 6.3 Job-Finding Rate Volatility: Positive Hiring Cost

As Nagypál (2005) emphasizes, the reduction in amplification in the presence of on-the-job search is due to the fact that employers reap lower benefits from contacting employed workers than from contacting unemployed workers, because they have lower acceptance rates. A lower benefit from contacting employed workers is always present when a standard model of on-the-job search is embedded into a matching model. Nagypál shows that in this context, there are two channels through which firms can have a higher benefit from contacting employed workers. First, the presence of hiring costs can ensure that the payoff from meeting employed workers is higher because the expected duration of the job created is longer. Second, ex-ante heterogeneity in worker productivity that is correlated with employment status in equilibrium can also give rise to a higher payoff from contacting employed workers. While the exposition of the second channel is beyond the scope of this paper, the first channel can be easily demonstrated in our simple framework.

As demonstrated above, in the presence of hiring costs, the free-entry condition when  $\hat{x} = \underline{x}$  is approximated by

$$\begin{aligned} \frac{cv}{f(v)} &= \frac{(1-\beta)(p-z)}{s+f(v)} - \frac{s}{f(v)} \ln\left(\frac{s+f(v)}{s}\right) H \\ &= \frac{(1-\beta)(p-z) - (s+f(v))\frac{s}{f(v)} \ln\left(\frac{s+f(v)}{s}\right) H}{s+f(v)} = \pi(p, s, f). \end{aligned} \quad (28)$$

Note that the first term on the right of the first equality is decreasing in the job-finding rate, primarily because the quit rate increases with the job-finding rate, while the absolute value of the second term, the expected cost of hiring per worker contacted, falls with the job-finding rate, because the fraction of the applicants who are employed rises with  $f$ . Of course, the second effect dominates if the cost of hiring is large enough.

The second equality of Equation (28) illustrates another view of the same facts. As pointed out above, the cost of turnover, the product of the separation rate and the expected cost of hiring, augments the unemployment benefit to determine the overall opportunity cost of employment. Because employed workers quit, this cost is much higher for a given value of  $H$  than in the standard model. As already noted, large values of the opportunity cost of employment imply large responses in vacancy creation to a shock for a given value of the job-finding rate. For this reason, relatively small positive values

for the hiring cost parameter can imply large response in the job-finding rate even if the net effect of an increase in  $f$  reduces the expected present value.

Equation (24) can be linearly approximated by

$$\frac{1-\eta}{\eta} \Delta \ln f = \hat{a} \Delta \ln p - \hat{b} \Delta \ln s - \hat{c} \Delta \ln f \quad (29)$$

where

$$\begin{aligned} \hat{a} &= \frac{\partial \ln \pi}{\partial \ln p} = \frac{(1-\beta)p}{\pi(s+f(v))} \\ \hat{b} &= -\frac{\partial \ln \pi}{\partial \ln s} = \frac{s}{\pi} \left[ \frac{(1-\beta)(p-z)}{[s+f(v)]^2} + \left( \frac{1}{f(v)} \ln \left( \frac{s+f(v)}{s} \right) - \frac{1}{s+f(v)} \right) H \right] \\ \hat{c} &= -\frac{\partial \ln \pi}{\partial \ln f} = \frac{f}{\pi} \left[ \frac{(1-\beta)(p-z)}{[s+f(v)]^2} - \frac{s}{f(v)} \left( \frac{1}{f(v)} \ln \left( \frac{s+f(v)}{s} \right) - \frac{1}{s+f(v)} \right) H \right]. \end{aligned} \quad (30)$$

Equation (29), then, implies

$$E(\Delta \ln f)^2 = \sigma_f^2 = \left( \frac{\eta}{1-\eta+\hat{c}\eta} \right)^2 \left( \hat{a}^2 \sigma_p^2 - 2\hat{a}\hat{b}\rho_{sp}\sigma_s\sigma_p + \hat{b}^2 \sigma_s^2 \right),$$

so the volatility of the job-finding rate relative to that of productivity implied by the model is

$$\frac{\sigma_f}{\sigma_p} = \frac{\hat{a} \times \eta}{1-\eta+\hat{c}\eta} \left( 1 - \frac{2\hat{b}}{\hat{a}} \frac{\rho_{ps}\sigma_s}{\sigma_p} + \left( \frac{\hat{b}\sigma_s}{\hat{a}\sigma_p} \right)^2 \right)^{1/2}.$$

Note that the coefficient  $\hat{a}$  is larger than its corresponding value  $\tilde{a}$  defined by the equations of (26) when  $H$  is positive because  $\pi$  is smaller. Furthermore, the ratio  $\hat{b}/\hat{a}$ , which captures the impact of the job-destruction shock, is larger than  $\tilde{b}/\tilde{a}$ . Finally, the coefficient reflecting the dampening effect of the procyclical quit rate reflected in the ratio  $\hat{c}/\hat{a}$  is smaller than its counterpart  $\tilde{c}/\tilde{a}$  and can be negative if  $H$  is large enough. For all of these reasons, positive hiring costs imply more volatility of the job-finding rate.

Notice that what matters in the determination of the above coefficients is the relative magnitude of the hiring costs compared to flow profits,  $\frac{H}{p-w}$ . Holding this ratio constant, changing the level of wages (by varying  $\beta$ ) has no impact on the coefficients, thus on amplification. The reason for this is

simple: what the firm cares about is turnover and the length of time it has to recoup the hiring costs in the form of flow profits.

Although there is a consensus that hiring costs are important, there is no authoritative estimate of their magnitude. Still, it is reasonable to assume that in order to recoup hiring costs, the firm needs to employ a worker for at least two to three quarters. When wages are equal to their median level in the standard model ( $w = 0.983$ ), hiring costs of this magnitude correspond to less than a week of wages. Given the same values of  $r$ ,  $s$ ,  $\eta$ , and  $z$  that we used to calculate Equation (27) in Section 6.2, a hiring cost,  $H$ , of 2 and 3 times the quarterly profit flow ( $p - w$ ) yields relative volatilities between

$$\frac{\sigma_f}{\sigma_p} = 3.086 \text{ and } 7.168,$$

respectively. Obviously, these numbers bracket the observed value of 5.9 reported in Shimer (2005) (and a hiring cost of 2.81 quarters of flow profits exactly matches it). In sum, the complementarity between employed searchers and vacancies can serve as a strong source of amplification in a matching model, one that matches the data given a conservative estimate of the cost of hiring.

The effect of hiring costs in these simple calculations depends on the assumption that employed workers receive offers at the same rate as unemployed workers. Nagypál (2005) studies a general version of the above model where offer arrival rates vary across workers due to workers making optimal search-effort decisions. She shows that a hiring cost equaling three quarters of flow profits also succeeds in generating the observed amount of amplification in her extended model.

## 7 The Beveridge Curve

As mentioned above, one of Shimer's (2005a) criticism of job-destruction rate shocks is that they induce positive co-moment in vacancies and unemployment. These movements are inconsistent with the commonly observed negative association between these two variables captured by the Beveridge curve. Figure 1 is useful for the purpose of illustrating and evaluating Shimer's assertion.

Since vacancies are represented on the vertical axis and unemployment is measured along the horizontal axis, the free-entry condition determines a

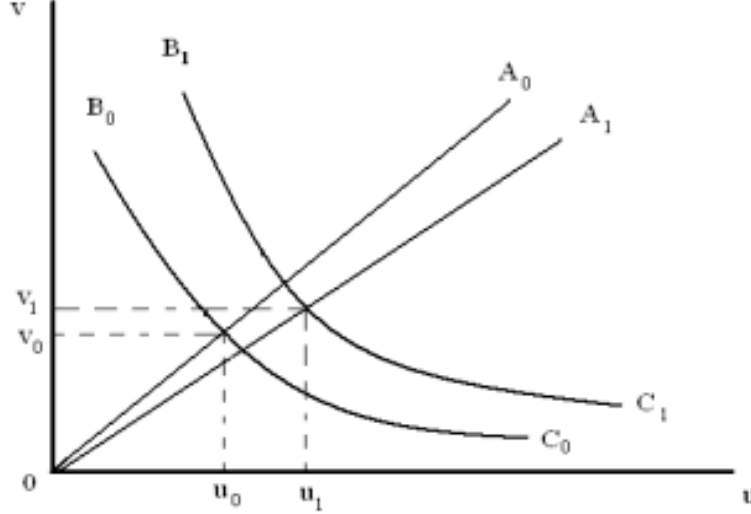


Figure 1: Destruction Shock: Without Search on the Job

ray, the vacancy-unemployment ratio. The two negatively sloped curves are the graphs of the unemployment steady-state condition

$$u = \frac{s}{s + f(\theta)} \quad (31)$$

for two different values of  $s$ . Since productivity is not an argument of this relationship but  $\theta$  increases with productivity, shocks to  $p$  identify the negatively sloped curve of Equation (31) for fixed  $s$ .

Consider a shock to  $s$  represented in Figure 1 by an increase in  $s$  from  $s_0$  to a larger value  $s_1 > s_0$ . This shock has two effects. First, the curve representing the steady-state condition shifts up and to the right along any ray. In Figure 1, let  $B_0C_0$  represent the curve when  $s = s_0$  and  $B_1C_1$  the curve when  $s = s_1$ . Second, the equilibrium vacancy-unemployment ratio falls because the job-destruction rate is a component of the rate at which future profits are discounted. Let the ray  $OA_0$  represent the equilibrium value when  $s = s_0$  and let  $OA_1$  designate the equilibrium when  $s = s_1$ . Thus, a positive job-destruction rate shock induces a movement in the equilibrium  $(v, u)$  pair from  $(v_0, u_0)$  to a point like  $(v_1, u_1)$ .

The diagram in Figure 1 illustrates that a positive shock to the destruction



rate can induce positive comovements in vacancies and unemployment, but it does so only if the shift in the curve representing the free-entry condition is small relative to the shift in the steady-state condition. The size of this shift is determined, in turn, by the magnitude of the wage pressure from changes in the job-finding rate, as discussed above. Moreover, when a job-destruction shock is accompanied by a drop in productivity, the shift in the curve representing free-entry is larger than if a job-destruction shock is considered in isolation.

However, if the feedback from the job-finding rate to the wage is small and/or search by employed workers is extensive, then the counterfactual positive correlation implied by the model is reversed. Given the strategic bargaining solution suggested by Hall and Milgrom, the downward shift in the  $OA$  ray is larger than in the standard model because there is less feedback on the wage in response to the initial decrease in the vacancy-unemployment ratio induced by an increase in the job-destruction rate. When employed workers search, the free-entry condition is no longer represented by a ray. Instead, it is an increasing relationship with a flatter slope. Indeed, in the benchmark case considered in which employed workers search at the same intensity as the unemployed, the job-finding rate depends on the level of vacancies. Since the free-entry condition in this case can be represented as a horizontal line in Figure 1, an increase in the job destruction rate always decreases vacancies and increases unemployment.

To assess quantitatively the consequence of the correlation between vacancies and unemployment implied by the two-shock model and search on the job studied in the previous section, notice that

$$\Delta \ln u = \frac{f}{s+f} (\Delta \ln s - \Delta \ln f) = \tag{32}$$

$$= \frac{f}{s+f} \left( \Delta \ln s - \frac{\eta}{1-\eta+\eta\hat{c}} (\hat{a}\Delta \ln p - \hat{b}\Delta \ln s) \right)$$

$$= 1.890\Delta \ln s - 2.682\Delta \ln p$$

$$\Delta \ln v = \frac{1}{1-\eta+\eta\hat{c}} (\hat{a}\Delta \ln p - \hat{b}\Delta \ln s) \tag{33}$$

$$= 5.495 \ln p - 1.964 \ln s$$

hold as linear approximation where the parameter values for  $\hat{a}$ ,  $\hat{b}$ , and  $\hat{c}$  are those derived using the equations of (30) when the hiring cost parameter,  $H$ , is set at the value that generates the observed job-finding rate volatility,

$2.81 \times (p - w)$ . Notice that vacancies and unemployment respond in opposite directions to both productivity and job-destruction shocks. Indeed, the volatility of unemployment and vacancies are and

$$\begin{aligned}\sigma_u &= 0.176 \\ \sigma_v &= 0.225\end{aligned}$$

while their correlation is

$$\rho_{vu} = -0.986.$$

In short, the model implies greater correlations than observed in the data. Furthermore, the slope of the implied Beveridge curve is  $\rho_{vu}\sigma_v/\sigma_u = -0.986 \times 0.225/0.176 = -1.263$ , close to a rectangular hyperbola.

## 8 Conclusion

Shimer (2005a) argues that the Mortensen-Pissarides equilibrium search model of unemployment with shocks to productivity explains less than 10% of the volatility in the job-finding rate. Some of the recent papers inspired by his critique are reviewed and commented on here and compared within a unified framework that highlights the importance of parameter choices. Given the wide range of specifications and parametrizations used in the literature, it is important to evaluate the claims of different authors within one framework. Overall, we find that the literature has overemphasized the need to introduce wage rigidity into the model. Indeed, we show that an extended version of the model that accounts for job-destruction shocks and job-to-job worker flows can explain both the volatility of vacancies and of unemployment as well as the quantitative properties of the Beveridge curve inferred from U.S. data.

## 9 Appendix

### 9.1 Proof to Proposition 1

By substitution from Equation (5) and (6), Equation (4) implies that an equilibrium surplus value function is a fixed point of the map

$$(TV)_p = \Gamma^{-1} \left( \frac{p - z + \lambda E_p V_{p'}}{r + s + \lambda} \right)$$

from the set of real-valued functions of  $p$  to itself where  $\Gamma(V)$  is the real-valued function defined by

$$\Gamma(V) \equiv V + \frac{\beta c \theta(V)}{(1 - \beta)(r + s + \lambda)}$$

and  $\theta(V)$  is the function implicitly defined by the free-entry condition

$$\frac{c\theta}{f(\theta)} = (1 - \beta)V.$$

Because  $\theta(V)$  is continuous, increasing, and convex and  $\theta(0) = 0$  under hypothesis (ii),  $\Gamma(V)$  has these same properties.

To prove uniqueness, we show that the mapping  $T$  satisfies Blackwell's sufficient conditions for a contraction. Since  $\Gamma^{-1}(\cdot)$  is increasing and  $E_p(V_{p'} + k) \geq E_p(V_{p'})$  for all  $k \geq 0$ ,  $T$  is increasing. Hence,

$$\begin{aligned} (T(V + k))_p &= \Gamma^{-1} \left( \frac{p - z + \lambda E_p(V_{p'} + k)}{r + s + \lambda} \right) = \Gamma^{-1} \left( \frac{p - z + \lambda E_p V_{p'} + \lambda k}{r + s + \lambda} \right) \\ &\leq \Gamma^{-1} \left( \frac{p - z + \lambda E_p V_{p'}}{r + s + \lambda} \right) + \frac{1}{\Gamma'(V)} \left( \frac{\lambda k}{r + s + \lambda} \right) \\ &\leq (TV)_p + \beta k \end{aligned}$$

for any positive constant  $k$  and  $\beta = \lambda / (r + s + \lambda) < 1$  where the first inequality follows from the concavity  $\Gamma^{-1}(\cdot)$  (recall that  $\Gamma(\cdot)$  is convex) and the second is implied by the fact that  $d\Gamma^{-1}(x)/dx = 1/\Gamma'(y) \leq 1$ .

If condition (i) holds and  $p'$  is stochastically increasing in  $p$ , then  $T$  maps the set of continuous and increasing function of  $p$  into itself. Hence, the fact that  $T$  is a contraction implies that its fixed point is increasing in  $p$ . All the

other equilibrium outcomes can be expressed as increasing functions of  $p$  and  $V_p$ . Finally, the assertion that  $V_p$  is strictly increasing is implied by the fact that  $T$  transforms any increasing functions into the set of strictly increasing functions.

## 9.2 Proof to Proposition 2

The claim is an immediate implication of Equation (9) in the case of  $\lambda \rightarrow 0$ . As the specification in Equation (14) implies,

$$p' - z = (p - z)e^\Delta,$$

it follows that

$$\lim_{\Delta \rightarrow 0} E_p \phi_{p'} = \phi_p$$

for any real-valued integrable function  $\phi$  of  $p$ . The free-entry condition (4) and Equation (6) imply that the Bellman equation can be written as

$$V_p = \frac{p - z - \frac{\beta c \theta_p}{1 - \beta} + \lambda E_p V_{p'}}{r + s + \lambda}.$$

It follows that,

$$\begin{aligned} \lim_{\Delta \rightarrow 0} V_p &= \frac{p - z - \frac{\beta c}{1 - \beta} \lim_{\Delta \rightarrow 0} E_p \theta_{p'} + \lambda \lim_{\Delta \rightarrow 0} E_p V_{p'}}{r + s + \lambda} \\ &= \frac{p - z - \frac{\beta c}{1 - \beta} \theta_p + \lambda \lim_{\Delta \rightarrow 0} V_p}{r + s + \lambda} = \frac{p - z - \frac{\beta c}{1 - \beta} \theta_p}{r + s}. \end{aligned}$$

Hence, the free-entry condition can be approximated by

$$\frac{c \theta_p}{f(\theta_p)} = (1 - \beta) V_p = \frac{(1 - \beta)(p - z) - \beta c \theta_p}{r + s}.$$

By differentiating this expression with respect to  $\ln p$ , one obtains Equation (10).

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