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## ABSTRACT

We propose and implement a Wald test of the international capital asset pricing model. Ex post asset returns are regressed on asset supplies. CAPM reequires that the matrix of coefficients from a regression of $n$ rates of return on $n$ asset supply shares be proportional to the covariance matrix of the residuals from those regressions. We test this restriction in the context of a model that aggregates all outside financial assets for each of ten countries. We do not find strong support for the restrictions of CAPM.

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## 1. Introduction

The capital asset pricing model (CAPM) is a popular description of investors' behavior, but one which has received mixed support empirically. As applied to international asset markets, it implies that demand for foreign bonds depends on real exchange rate risk and real rates of return. In this paper we propose a simple wald test of CAPM, and apply the test to a ten country asset pricing model.

Our test is closely related to a test of CAPM implemented by Frankel (1982,1983a,1983b,1984) in several recent papers. Unlike previous tests of the model, the Frankel procedure allows completely unrestricted movements in expected returns and in "betas" (the covariance of an asset's rate of return and the market rate of return). It also formulates a natural alternative hypothesis to CAPM, thus allowing a statistical test of the restrictions imposed by CAPM. The technique we employ retains almost all the desirable properties of the Frankel test but eliminates perhaps its biggest difficulty -- the fact that if the market consists of $n$ assets, $n^{2}$ parameters must be estimated with an extremely complex, nonlinear maximum likelihood procedure. We replace Frankel's likelihood ratio test with a Wald test. The Wald test requires estimation without imposing the restrictions of the CAPM model -which in this case implies that only least squares regressions are employed. Because estimation is easier, we can extend the empirical model to include larger collections of assets.

As a general model, one might estimate a system of equations in which the expected return of any asset is a linear function of the shares of all assets
in the market portfolio. CAPM constrains the matrix of coefficients from these regressions to be proportional to the variance-covariance matrix of the regression errors of the system. The Wald test looks to see if the matrix of coefficients that comes from the unconstrained estimation is proportional to the estimated covariance matrix of the residuals. This procedure requires only the output from OLS regressions. Furthermore, since there are closed-form expressions for OLS estimates and for the Wald statistic, our results are completely reproducible. The same cannot be claimed for Frankel's technique, which employs a "hill-climbing" method to find the maximum of the complicated restricted likelihood function. There, the estimated coefficients will depend on such things as the size of the steps taken in climbing the hill, and the degree of precision set for the estimates.

We apply the Wald test of CAPM to an international asset pricing model. There are ten assets which represent aggregate bonds held by the public in ten currencies of denomination. In some ways this model performs better than previous smaller-scale models. When we have a priori convictions about the signs of coefficients they usually turn out to be correct in the estimation. In a joint test of the significance of all the restrictions of CAPM, we find we cannot reject the model at the 5 percent level. However, the CAPM hypothesis imposes a very large number of restrictions, so, given the limited data set (141 monthly observations), the test seems to have low power. A test of a subset of the restrictions of the hypothesis strongly rejects the CAPM mode 1.

Section 2 of this paper reviews the CAPM hypothesis and discusses the relative merits of the likelihood ratio and the Wald tests. In Section 3, we
use the Wald method to test international CAPM. First we use the data set of Frankel and Engel (1984) so that we can directly compare our test to Frankel's. Then we use the wald test on a larger model over a longer time period. The concluding section discusses the limitations of these tests, and outlines the possibilities for future research. In the Appendix, we present the formulas for calculating the Wald statistic.

## 2. Testing CAPM

It is useful in developing our test to review the capital-asset-pricing model.1 A well-known relation that emerges from CAPM is that for any asset $i$,

$$
\begin{equation*}
E_{t}\left(r_{i t}-r_{0 t}\right)=\beta_{i t} E_{t}\left(r_{m t}-r_{0 t}\right) \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
& r_{i t}=\text { real return on asset } i \text { from time } t \text { to } t+1 \\
& r_{0 t}=\text { riskless real rate of return } \\
& r_{m t}=\text { real return on the market portfolio. }
\end{aligned}
$$

The coefficient $\beta_{i t}$ is, in general, a time-varying coefficient that is defined as

$$
\begin{equation*}
\beta_{i t}=\operatorname{Cov}_{t}\left(r_{i t}, r_{m t}\right) / \operatorname{Var}_{t}\left(r_{m t}\right) \tag{2}
\end{equation*}
$$

Using equation (2), we could rewrite (1) as the "security market line"

$$
\begin{equation*}
E_{t}\left(r_{i t}-r_{0 t}\right)=\rho \operatorname{Cov}_{t}\left(r_{i t}, r_{m t}\right) \tag{3}
\end{equation*}
$$

where

$$
\begin{equation*}
\rho=E_{t}\left(r_{m t}-r_{0 t}\right) / \operatorname{Var}_{t}\left(r_{m t}\right) \tag{4}
\end{equation*}
$$

The coefficient $\rho$ is known in the finance literature as the market price of risk -- it represents the tradeoff between the expected return on the market portfolio and the variance of return on that set of assets. A critical assumption is that this "price" is constant over time. In a representative investor model, this variable can be shown to be equal to the investor's coefficient of relative risk aversion, which might plausibly be constant. In reality, the variable could change over time if investors do not have constant relative risk aversion, or if there are significant income redistribution effects. However, the assumption of constancy of $\rho$ seems quite innocuous compared to some other common assumptions made to test CAPM (see Frankel (1982,1984) for a discussion of these).

Since there is no asset that is riskless in real terms, to implement (3) empirically, it is useful to rewrite expected returns relative to the return on asset 1 :

$$
\begin{align*}
& E_{t}\left(r_{i t}-r_{1 t}\right)=\rho \operatorname{Cov}_{t}\left(r_{i t}-r_{1 t}, r_{m t}\right)  \tag{5}\\
& =\rho \operatorname{Cov}\left(r_{i t}-r_{1 t}, r_{1 t}\right)+\rho \operatorname{Cov}_{t}\left(r_{i t}-r_{1 t}, r_{m t}-r_{1 t}\right)
\end{align*}
$$

The return on the market portfolio is a weighted average of returns on each of the individual assets:

$$
r_{m t}=\sum_{j=1}^{n} \lambda j t r_{j t}
$$

where

$$
\lambda_{j t}=\text { ratio of the value of outstanding shares of asset } j \text { to the }
$$

value of all assets.

Therefore, we can write

$$
\operatorname{Cov}_{t}\left(r_{i t}-r_{1 t}, r_{m t}-r_{1 t}\right)=
$$

(6)

$$
\sum_{j=2}^{n} \lambda_{j t} \operatorname{cov}\left(r_{i t}-r_{1 t}, r_{j t}-r_{7 t}\right)
$$

One can see that $\operatorname{Cov}_{t}\left(r_{i t}{ }^{-r} 1 t, r_{m t^{-r}} l_{t}\right)$ could vary over time either because supplies of assets (the $\lambda s$ ) change, or because the underlying stochastic process of returns is time-varying, meaning that $\operatorname{Cov}\left(r_{i t}, r_{j t}\right)$ is not constant. We assume only the $\lambda$ m move over time.

$$
\begin{align*}
& \text { Let } z_{i t} \equiv r_{i t}-r_{1 t} \text {, and then use (6) to recast (5) as } \\
& E_{t} z_{i t}=\rho \operatorname{Cov}\left(z_{i t}, r_{i t}\right)+\rho \sum_{j=2}^{n} \lambda_{j t} \operatorname{Cov}\left(z_{i t}, z_{j t}\right) . \tag{7}
\end{align*}
$$

In matrix form we have

$$
\begin{equation*}
E_{t} z_{t}=\rho \operatorname{Cov}\left(z_{t}, r_{1 t}\right)+\rho \Omega \lambda_{t} \tag{8}
\end{equation*}
$$

where

$$
\begin{aligned}
& z_{t}=\text { vector of asset returns relative to asset } 1, j=2, \ldots n \\
& \lambda_{t}=\text { vector of asset shares, } j=2, \ldots n \\
& \Omega=E_{t}\left(z_{t}-E_{t} z_{t}\right)\left(z_{t}-E_{t} z_{t}\right)^{\prime} .
\end{aligned}
$$

If expectations are rational

$$
z_{t}=E_{t} z_{t}+\epsilon_{t}
$$

where $\epsilon_{t}$ is a vector of forecast errors. So, (8) may be written as a system of regression equations

$$
\begin{equation*}
z_{t}=c+B \lambda_{t}+\epsilon_{t} . \tag{9}
\end{equation*}
$$

Equation (9) could be a general equation that says the return on an asset is related to the supplies of all assets in a linear way. Note, however, that the variance-covariance matrix of $\epsilon_{t}$ is $\Omega$. Therefore, the restriction that CAPM places on the general system (9) is that the matrix of regression coefficients $B$ be proportional to the covariance matrix of the residuals. Frankel's method of testing CAPM is to compare the likelihood of the system (9) without imposing any restrictions on $B$ to the likelihood obtained from estimating (9) under the restriction that $B$ be proportional to $\Omega$. The likelihood ratio test will fail to reject CAPM if the estimated likelihood with the restriction imposed is not significantly smaller than the likelihood from the unrestricted regressions.

The effect on the expected return of asset $i$ (relative to asset 1 ) of an increase in the share of asset $j$ is given by the coefficient $b_{i j}$. The CAPM hypothesis is that this multiplier is proportional to the covariance of the return on asset $i$ to the return on asset $j$ (each expressed relative to the return on asset 1 ), with the constant of proportionality being the market price of risk. That is, ${ }^{2}$

$$
\begin{equation*}
b_{i j} / \omega_{i j}=\rho, \forall i, j \tag{10}
\end{equation*}
$$

where

$$
\begin{aligned}
& b_{i j}=i j^{\text {th }} \text { element of } B \\
& \omega_{i j}=i j^{\text {th }} \text { element of } \Omega .
\end{aligned}
$$

Since the unconstrained estimation of (9) provides estimates of $B$ and $\Omega$, by
taking the ratios of corresponding elements of the matrices we obtain $(n-1)^{2}$ separate estimates of $\rho$. We can then construct a Wald statistic that tests the proposition that all of the ratios are equal.

The Wald-test is equivalent (to the first order) to the likelihood-ratio test in large samples, but does not require calculation of the restricted estimates. Estimation of (9) unrestricted requires only $n-1$ ordinary least squares regressions, each with n-1 right-hand-size variables. Calculation of the Wald statistic, which is discussed in the Appendix, requires some relatively simple matrix manipulations involving the data and the estimated residuals from the regressions. The price for this increased simplicity is that, if the null hypothesis is true, the unconstrained estimates of $B$ are less efficient than the estimates of $B$ with the constraint imposed.

## 3. Testing International CAPM

A hypothesis of considerable interest in international finance is that international investors are risk neutral and have rational expectations. If this hypothesis were true, speculators would drive the expected real returns on all assets into equality (see Engel (1984)). If anticipated real returns differ among assets, there may be a lack of efficient speculation, but there may alternatively be a risk premium separating their returns.

The capital asset pricing model outlined in Section 2 provides a framework for assessing the possibility of risk premia driving a wedge between anticipated real returns on assets from different countries. This section takes the approach of aggregating the obligations of each of the countries
included in the study and considering the real returns on those assets. If the expected interest rates do differ on these assets, a risk premium is a likely suspect if the restrictions of the CAPM model cannot be rejected. Otherwise, we might need to explore the possibility that expectations are not rational or that asset markets are not fully efficient.

We wish to have as our measure of asset supplies the amount of debt held by the public denominated in each currency. The measured asset stocks for each country are essentially the cumulated value of that country's government deficits. We must correct this figure, however, to allow for foreign exchange intervention by the country's central bank, intervention into that country's currency by other central banks, and debt originally issued in foreign currencies.

The real returns for each asset are calculated as $[(1+i)(1+d) /(1+\pi)]-1$, where $i$ is the interest rate, $d$ is the rate of depreciation of the currency relative to the dollar, and $\pi$ is a dollar inflation index. $\pi$ is calculated as a weighted average of inflation rates of CPIs in all the countries (converted into dollars) with the weights equalling GNP shares.

The first set of regressions uses the data for six countries that were used by Frankel and Engel (1984). That data set includes asset supplies for the U.S., Germany, Japan, Canada, France and England from June 1973 to August 1980. The interest rate used in calculating real returns is the return on one month Eurocurrency assets.

Table 1 reports the OLS regressions that correspond to equation (9). In these regressions, dollars are taken as the numeraire asset. A cursory glance at these estimates suggest that the relation between asset supplies and
returns is weak. Few of the coefficients are significantly different from zero. Furthermore, four of the five diagonal coefficients, which we expect to be positive, are negative. The only positive diagonal coefficient -- for the yen -- is not significant.

Even though the unrestricted model does not fit the data well, it is still possible that we could be unable to reject the restriction that all p's are equal. Table 2 presents estimates of the covariance matrix $\Omega$. A quick check reveals that there are, in fact, wide variations in the $b_{i j} / \omega_{j}$. Again, this does not necessarily invalidate the restriction because the coefficients could be estimated with a low degree of precision.

The wald statistic for the test that all $\rho^{\prime} s$ are equal is $\chi^{2}(24)=45.36$. This rejects the null hypothesis at the 1 percent level.

The estimated $\rho$ 's are not very close to 2 , so, not surprisingly, the test of the hypothesis that all the $\rho$ 's are equal to 2 can easily be rejected at the 1 percent level. The statistic in this case is $\chi^{2}(25)=45.45$. This corresponds to the test in Frankel and Engel. They also reject the null hypothesis and report a likelihood ratio test statistic of $\chi^{2}(25)=59.0$. Thus, even with a limited number of observations the Wald statistic and the LR statistic are quite close.

The next set of regressions uses asset supplies and returns from ten countries from April 1973 to December 1984. This collection includes the original six countries plus Belgium, Italy, the Netherlands and Switzerland. The asset supply data comes from Giavazzi and Giovannini (1986). The interest rates are government bond yields which, along with the exchange rate and price data, are collected from International Financial Statistics.

The regressions corresponding to equation (9) are reported in Table 3 , and Table 4 contains estimates of $\Omega$. We tested each by equation for autocorrelation of the error terms using the Breusch (1978) and Godfrey (1978) test, allowing for serial correlation up to a twelfth order AR or MA process. Only in the case of the Canadian regression did we find even remote evidence of correlated residuals, so we made no corrections for this problem.

We also tested for heteroskedasticity using the White (1980) test. Here we did find evidence of a problem -- five of the tests were significant at the 5 percent level. However, no statistic was significant at the 1 percent level. A general correction for heteroskedasticity in this model might be quite difficult because, under the null hypothesis, the coefficient matrix would vary over time if the variance matrix did. OLS regressions do not yield consistent estimates of the residuals. Given the somewhat weak evidence for heteroskedasticity, we attempt no correction, while recognizing this may render our test statistics inconsistent.

The results reported in Table 3 are somewhat more encouraging than those from the smaller model. Six of the nine diagonal elements have the postulated positive sign. None of the three that are negative is significantly different from zero. Moreover, we cannot reject the restrictions of the CAPM hypothesis. The statistic for equality of all the $\rho$ 's is $\chi^{2}(80)=100.98$. The statistic for the test that all the $\rho^{\prime}$ s are 2 is $\chi^{2}(81)=101.04$. Neither can reject at the 5 percent level of significance. This is the opposite conclusions from the one reached using only six assets.

There is, however, evidence that the test has low power. We are testing a very large number of restrictions. Tables 3 and 4 reveal that the point
estimates of the $\rho$ 's vary greatly. We must conclude that they are estimated with low precision. For example, we are also unable to reject (at the 5 percent level) the hypothesis that all $\rho$ 's are zero. This would imply that the risk-neutral uncovered interest parity formulation holds.

One way to increase the power of the test is to test only a subset of the restrictions. When we test for equality only of the diagonal $\rho$ 's, we test only eight restrictions. This hypothesis is easily rejected at the 1 percent level.

## 4. Conclusion

The evidence presented in Section 3 does not provide strong support for international CAPM. However, it does apply a methodology that might be useful in future work in this area. There are still several weaknesses of tests of this nature, several of which are discussed by Frankel and Engel. In our conclusion, we will concentrate on a problem that they did not discuss in great detail.

This type of test of CAPM requires that the vector of asset shares contain correctly measured shares of all assets available to investors. Aggregation of assets is only strictly correct if the assets lumped together are perfect substitutes in investors' portfolios. There are several reasons why the data set used here, though very carefully constructed, may fall short of the ideal.

To begin with, the aggregation of all obligations of a government into a single asset is clearly inappropriate. For example, long-term bonds and
short-term bonds certainly have different risk characteristics.
The vector of assets described here contains no measure of real assets. For example, Frankel (1984) uses measures of the value of the housing stock and other tangible assets, and the value of corporate equities in his study using U.S. data. This, too, suffers from too much aggregation. All stocks are collected together as a single asset, when, in fact, CAPM was originally formulated as an explanation of how risk and return characteristics of stocks differ.

Clearly, a good test of an international version of CAPM will require many right-hand-side variables. We have claimed as an advantage for the wald test the simplicity of its estimation technique and the ease with which results can be reproduced. We also believe that it has a large advantage in terms of the size of the model that can be handled. There are some problems of dimensionality with the wald test. It requires, if there are $n$ assets, estimation of $n-1$ regressions, each with $n-1$ regressands. Furthermore, the calculation of the wald statistic requires inversion of a matrix with $(n-1)^{2}-1$ rows and columns. Although these might require a large amount of calculation, there are very efficient programs written for OLS estimation and matrix inversion. On the other hand, the constrained likelihood estimation could be very time consuming and expensive. At each iteration on the parameter estimation a $n(n-1) / 2 \times n(n-1) / 2$ matrix must be inverted (as opposed to a one-time inversion of a slightly larger matrix in calculating the wald statistic). If $n$ is very large, the maximization routine may require many iterations to achieve convergence. Furthermore, the matrix inversion technique imbedded in most maximization routines is unlikely to be as
efficient as one that can be called separately for the one-time inversion required for the wald statistic.

We believe there is still promise in testing international CAPM. Much work is left to be done in assembling good data sets. The principle of the Frankel technique seems correct, but, because of dimensionality problems, and the difficulty of estimating the constrained likelihood function, there does not seem to be much hope of applying his method directly to large systems. The Wald test, though, offers a more feasible alternative.

Appendix
The Wald Statistic ${ }^{3}$

If $\theta$ is a vector of parameters, and we wish to test the hypotheses

$$
h_{i}(\theta)=0 \quad i=1, \ldots, p
$$

the Wald statistic provides a test of the closeness to zero of the vector

$$
h(\hat{\theta})=\left\{h_{1}(\hat{\theta}), h_{2}(\hat{\theta}) \ldots h_{p}(\hat{\theta})\right\}^{\prime},
$$

where the " represents the estimated values of parameters.
If there are $s$ parameters, let $H_{A}$ be the sxp matrix
$\theta$

$$
H_{\hat{\theta}} \equiv\left[\partial h_{j}(\theta) / \partial \theta_{i}\right]
$$

Furthermore, let $B$. represent the estimated information matrix of the $\theta$
parameters. Then, if there are $T$ observations, the Wald statistic, given by

$$
W=T[h(\hat{\theta})]^{\prime}\left[\begin{array}{c}
\left.H_{\hat{\prime}}^{\prime} B_{\hat{\theta}}^{-1} H_{\hat{\theta}}\right]^{-1}[h(\hat{\theta})], ~
\end{array}\right.
$$

is asymptotically distributed $\boldsymbol{\chi}^{\boldsymbol{\theta}}(\mathrm{r})$.
In our case, we want to test the hypothesis that the $b_{i j} / \omega_{i j}$ are equal for all $i$ and $j$. If there are $n$ assets, there are $n-1$ regressions, and $(n-1)^{2}$ coefficients in the $B$ matrix. This implies that there are $(n-1)^{2}-1$ independent constraints to be tested. There are many different ways to write these constraints (for example $b_{11} / \omega_{11}=b_{12} / \omega_{12}, b_{12} / \omega_{12}=b_{13} / \omega_{13}$, etc., or $b_{12} / \omega_{12}=b_{11} / \omega_{11}, b_{13} / \omega_{13}=b_{11} / \omega_{11}$, etc.) but the calculated value of the Wald statistic, in this instance, is independent of the way the constraint is written. We choose to express the constraints relative to the last element -that is,

$$
b_{i j} / \omega_{i j}-b_{n-1, n-1} / \omega_{n-1, n-1}=0
$$

In the tests we report in Section 3 , the elements in our $h(\hat{\theta})$ are obtained by
reading across $B$ in rows - (with $n=6$ ) the first element is $b_{11} / \omega_{11}-b_{55} / \omega_{55}$, then $b_{12} / \omega_{12}-b_{55} / \omega_{55}, b_{13} / \omega_{13}-b_{55} / \omega_{55} \ldots b_{54} / \omega_{54}-b_{55} / \omega_{55}$.

Each row of $H_{n}$ contains the derivative of every constraint with respect to one parameter. The first $(n-1)^{2}$ rows of $H$ contain the derivatives of constraints with respect to the elements of the $B$ matrix. Again, the parameters are taken from $B$ row by row. There are only $n(n-1) / 2$ independent parameters in $\Omega$. The next $n(n-1) / 2$ rows of $H_{\theta}$ are comprised of the derivatives of the constraints with respect to the elements of $\Omega$. If $\Omega$ is expressed in lower triangular form, the elements of $\Omega$ are taken row by row (i.e., with $n=6, \omega_{11}, \omega_{21}, \omega_{22}, \omega_{31}, \ldots \omega_{54}, \omega_{55}$ ). It is usefut to write the matrix H as

$$
H=\left[\begin{array}{l}
H_{2} \\
H_{2}
\end{array}\right]
$$

where $H_{1}$ is $(n-1)^{2} \times\left[(n-1)^{2}-1\right]$ and $H_{2}$ is $[n(n-1) / 2] \times\left[(n-1)^{2}-1\right]$.
The inverse of the information matrix is given by

$$
B_{\hat{\theta}}^{-1}=\left[\begin{array}{ll}
v_{11} & 0 \\
0 & v_{22}
\end{array}\right]
$$

In this matrix, if $\hat{\Sigma}$ is the estimated covariance matrix of the right-hand-side variables, then

$$
v_{11}=\hat{\Omega} \times \hat{\Sigma}^{-1}
$$

The matrix $V_{22}$ is the covariance matrix of the estimates of $\Omega$. The element corresponding to the covariance of $\omega_{\mathrm{pq}}$ and $\omega_{r s}$ is given by $\hat{\omega}_{\mathrm{pr}} \hat{\omega}_{\mathrm{qs}}+\hat{\omega}_{\mathrm{ps}} \hat{\omega}_{\mathrm{qr}}$ (see Rothenberg (1973), pp. 87-88).

From above, the expression for the Wald statistic can be rewritten as

$$
T[h(\hat{\theta})] \cdot\left[H_{1}^{\prime} V_{11} H_{1}+H_{2}^{\prime} V_{22} H_{2}\right]^{-1}[h(\hat{\theta})]
$$

Notice that the statistic requires only the estimated coefficients, the covariance matrix of the residuals, and the covariance matrix of the asset
shares. All calculations for the test statistic, as well as for the estimation, involve only simple linear algebra.

If one had a prior belief that the coefficient of relative risk aversion were some number, the test would be altered slightly for example, Frankel and Engel impose the condition $\rho=2$, and test the restriction $B=2 \Omega$. The equivalent set of restrictions for the Wald test are

$$
b_{i j} / \omega_{i j}=2 \forall i, j
$$

Section 3 reports this test as well. There are now $(n-1)^{2}$ independent restrictions, so $h(\hat{\theta})$ is a $(n-1)^{2}$ vector and $H_{\hat{\theta}}$ is $\left[(n-1)^{2}+(n(n-1) / 2)\right] x$ $(n-1)^{2}$.

Section 3 also considers a test of the first-type, but only on the diagonal elements:

$$
b_{i i} / \omega_{i i}=b_{n-1, n-1} / \omega_{n-1, n-1} \quad \forall i
$$

This examines orily a subset of the restrictions imposed by CAPM, paying attention to effects of increased value of outstanding shares of an asset on its own return, but ignoring cross-effects.

## Footnotes

1. This discussion draws on Frankel's (1983b) presentation of his CAPM test.
2. There is a possibility that $\omega_{i j}=0, i \neq j$, so that the ratio $b_{i j} / \omega_{i j}$ is undefined. Under most plausible assumptions of the underlying forces that determine the forecast errors, this would be an event with probability zero. Nonetheless, this may be a justification for testing equality only of the ratio of the diagonal elements, as is done in Section 3. One might consider writing the constraints in such a way as to avoid possible division by zero, such as $b_{11} \omega_{12}=b_{12} \omega_{11}, b_{12} \omega_{13}=b_{13} \omega_{12}$, etc. However, unlike the test for equality of the $\rho$ 's, the calculated test statistic will depend on the way the constraints are written. The statistic for the test $b_{11} \omega_{12}=b_{12} \omega_{11}, b_{12} \omega_{13}=$ $b_{13} w_{12}$, etc. will not be the same as the one for $b_{11} \omega_{12}=b_{12} \omega_{11}, b_{11} \omega_{13}=$ $\mathrm{b}_{13} \omega_{11}$, etc.
3. The discussion of the Wald test draws on Silvey (1975).
4. See footnote 2 for further discussion.

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Table 1: Unconstrained Asset Demand Functions, OLS

Dependent variable: $r_{t+1}-r_{t+1}^{\$}$, real rate of return on national currency relative to the dollar.

Independent variable: $\lambda_{t}$, share of asset supplies in the world portfolio.
Sample: June 1973 to August 1980 (87 observations)

| National Currency | Constant | $\lambda_{t}^{C D}$ | $\lambda_{t}^{F F}$ | $\lambda_{t}^{D M}$ | $\lambda_{t}^{J Y}$ | $\lambda_{t}^{B P}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Canadian dollar | $\begin{gathered} 0.125 \\ (0.060) \end{gathered}$ | $\begin{aligned} & -1.466 \\ & (0.692) \end{aligned}$ | $\begin{aligned} & -0.020 \\ & (0.322) \end{aligned}$ | $\begin{gathered} 0.384 \\ (0.243) \end{gathered}$ | $\begin{aligned} & -0.120 \\ & (0.082) \end{aligned}$ | $\begin{gathered} 0.150 \\ (0.087) \end{gathered}$ |
| French franc | $\begin{gathered} 0.014 \\ (0.138) \end{gathered}$ | $\begin{gathered} 1.770 \\ (1.584) \end{gathered}$ | $\begin{aligned} & -1.132 \\ & (0.737) \end{aligned}$ | $\begin{aligned} & -0.710 \\ & (0.557) \end{aligned}$ | $\begin{gathered} 0.311 \\ (0.188) \end{gathered}$ | $\begin{aligned} & -0.159 \\ & (0.199) \end{aligned}$ |
| Deutsche mark | $\begin{gathered} 0.153 \\ (0.145) \end{gathered}$ | $\begin{gathered} 1.324 \\ (1.669) \end{gathered}$ | $\begin{aligned} & -0.818 \\ & (0.776) \end{aligned}$ | $\begin{aligned} & -1.773 \\ & (0.587) \end{aligned}$ | $\begin{gathered} 0.361 \\ (0.198) \end{gathered}$ | $\begin{aligned} & -0.211 \\ & (0.210) \end{aligned}$ |
| Japanese yen | $\begin{gathered} 0.289 \\ (0.130) \end{gathered}$ | $\begin{gathered} 0.319 \\ (1.494) \end{gathered}$ | $\begin{aligned} & -1.309 \\ & (0.695) \end{aligned}$ | $\begin{aligned} & -2.213 \\ & (0.525) \end{aligned}$ | $\begin{gathered} 0.271 \\ (0.177) \end{gathered}$ | $\begin{aligned} & -0.141 \\ & (0.188) \end{aligned}$ |
| British pound | $\begin{gathered} 0.028 \\ (0.121) \end{gathered}$ | $\begin{gathered} 1.772 \\ (1.389) \end{gathered}$ | $\begin{aligned} & -0.938 \\ & (0.646) \end{aligned}$ | $\begin{aligned} & -0.993 \\ & (0.488) \end{aligned}$ | $\begin{gathered} 0.419 \\ (0.165) \end{gathered}$ | $\begin{gathered} -0.182 \\ (0.175) \end{gathered}$ |

Note: Standards errors are in parentheses.

Table 2: Variance-Covariance Matrix of Residuals (from regressions reported in Table 1)

| $\omega_{i j}$ | $C D$ | $F F$ | $D M$ | JY | BP |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Canadian dollar | $.1686 \times 10^{-3}$ |  |  |  |  |
| French franc | $.6552 \times 10^{-4}$ | $.8825 \times 10^{-4}$ |  |  |  |
| Deutsche mark | $.9881 \times 10^{-4}$ | $.7777 \times 10^{-3}$ | $.9804 \times 10^{-3}$ |  |  |
| Japanese yen | $-.6849 \times 10^{-3}$ | $.4156 \times 10^{-3}$ | $.3699 \times 10^{-3}$ | $.7848 \times 10^{-3}$ |  |
| British pound | $.6004 \times 10^{-4}$ | $.4261 \times 10^{-3}$ | $.4387 \times 10^{-3}$ | $.3119 \times 10^{-3}$ | $.6791 \times 10^{-3}$ |



| National Currency | Constant | $\lambda_{t}^{B F}$ | $\lambda_{t}^{\text {CD }}$ | $\lambda_{t}^{F F}$ | $\lambda_{t}^{\text {DM }}$ | $\lambda_{t}^{\text {IL }}$ | $\lambda_{t}^{J Y}$ | $\lambda_{t}^{\text {DG }}$ | $\lambda_{t}^{\text {SF }}$ | $\lambda_{t}^{B P}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Belgian franc | $\begin{gathered} 0.06 \\ (0.14) \end{gathered}$ | $\begin{gathered} 2.42 \\ (1.69) \end{gathered}$ | $\begin{aligned} & -2.74 \\ & (1.54) \end{aligned}$ | $\begin{aligned} & -0.18 \\ & (0.78) \end{aligned}$ | $\begin{aligned} & -1.61 \\ & (0.64) \end{aligned}$ | $\begin{aligned} & -0.54 \\ & (0.42) \end{aligned}$ | $\begin{aligned} & -0.20 \\ & (0.23) \end{aligned}$ | $\begin{gathered} 8.51 \\ (2.97) \end{gathered}$ | $\begin{gathered} 1.94 \\ (2.12) \end{gathered}$ | $\begin{gathered} 0.09 \\ (0.22) \end{gathered}$ |
| Canadian dollar | $\begin{aligned} & -0.13 \\ & (0.06) \end{aligned}$ | $\begin{aligned} & -0.71 \\ & (0.68) \end{aligned}$ | $\begin{gathered} 1.09 \\ (0.62) \end{gathered}$ | $\begin{gathered} 0.34 \\ (0.32) \end{gathered}$ | $\begin{gathered} 0.27 \\ (0.26) \end{gathered}$ | $\begin{aligned} & -0.11 \\ & (0.17) \end{aligned}$ | $\begin{gathered} 0.25 \\ (0.10) \end{gathered}$ | $\begin{gathered} 2.40 \\ (1.20) \end{gathered}$ | $\begin{aligned} & -0.85 \\ & (0.85) \end{aligned}$ | $\begin{aligned} & -0.09 \\ & (0.09) \end{aligned}$ |
| French franc | $\begin{gathered} 0.12 \\ (0.14) \end{gathered}$ | $\begin{gathered} 1.78 \\ (1.68) \end{gathered}$ | $\begin{aligned} & -2.44 \\ & (1.52) \end{aligned}$ | $\begin{aligned} & -0.12 \\ & (0.77) \end{aligned}$ | $\begin{aligned} & -1.37 \\ & (0.63) \end{aligned}$ | $\begin{aligned} & -0.57 \\ & (0.42) \end{aligned}$ | $\begin{aligned} & -0.19 \\ & (0.24) \end{aligned}$ | $\begin{gathered} 5.27 \\ (2.95) \end{gathered}$ | $\begin{gathered} 1.59 \\ (2.10) \end{gathered}$ | $\begin{gathered} 0.17 \\ (0.26) \end{gathered}$ |
| Deutsche mark | $\begin{gathered} 0.07 \\ (0.14) \end{gathered}$ | $\begin{gathered} 2.53 \\ (1.71) \end{gathered}$ | $\begin{aligned} & -2.83 \\ & (1.55) \end{aligned}$ | $\begin{aligned} & -0.49 \\ & (0.78) \end{aligned}$ | $\begin{gathered} -1.21 \\ (0.65) \end{gathered}$ | $\begin{aligned} & -0.48 \\ & (0.43) \end{aligned}$ | $\begin{aligned} & -0.29 \\ & (0.24) \end{aligned}$ | $\begin{gathered} 8.01 \\ (3.00) \end{gathered}$ | $\begin{gathered} 1.01 \\ (2.14) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.22) \end{gathered}$ |
| Italian lira | $\begin{gathered} 0.20 \\ (0.12) \end{gathered}$ | $\begin{gathered} 2.64 \\ (1.47) \end{gathered}$ | $\begin{aligned} & -2.35 \\ & (1.33) \end{aligned}$ | $\begin{aligned} & -0.09 \\ & (0.67) \end{aligned}$ | $\begin{aligned} & -1.56 \\ & (0.55) \end{aligned}$ | $\begin{aligned} & -0.55 \\ & (0.37) \end{aligned}$ | $\begin{aligned} & -0.33 \\ & (0.21) \end{aligned}$ | $\begin{gathered} 1.87 \\ (2.58) \end{gathered}$ | $\begin{gathered} 3.07 \\ (1.84) \end{gathered}$ | $\begin{gathered} 0.29 \\ (0.20) \end{gathered}$ |
| Japanese yen | $\begin{gathered} 0.01 \\ (0.14) \end{gathered}$ | $\begin{aligned} & -0.86 \\ & (1.67) \end{aligned}$ | $\begin{aligned} & -0.14 \\ & (1.52) \end{aligned}$ | $\begin{aligned} & -0.36 \\ & (0.77) \end{aligned}$ | $\begin{aligned} & -0.75 \\ & (0.63) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.42) \end{gathered}$ | $\begin{gathered} 0.08 \\ (0.24) \end{gathered}$ | $\begin{aligned} & -0.60 \\ & (2.94) \end{aligned}$ | $\begin{gathered} 3.44 \\ (2.09) \end{gathered}$ | $\begin{gathered} 0.24 \\ (0.22) \end{gathered}$ |
| Dutch guilder | $\begin{gathered} 0.10 \\ (0.14) \end{gathered}$ | $\begin{gathered} 2.54 \\ (1.65) \end{gathered}$ | $\begin{aligned} & -3.18 \\ & (1.50) \end{aligned}$ | $\begin{aligned} & -0.39 \\ & (0.76) \end{aligned}$ | $\begin{aligned} & -1.45 \\ & (0.62) \end{aligned}$ | $\begin{aligned} & -0.66 \\ & (0.41) \end{aligned}$ | $\begin{aligned} & -0.28 \\ & (0.23) \end{aligned}$ | $\begin{gathered} 8.65 \\ (2.90) \end{gathered}$ | $\begin{gathered} 1.55 \\ (2.07) \end{gathered}$ | $\begin{gathered} 0.66 \\ (0.22) \end{gathered}$ |
| Swiss franc | $\begin{gathered} 0.19 \\ (0.15) \end{gathered}$ | $\begin{gathered} 3.76 \\ (1.86) \end{gathered}$ | $\begin{aligned} & -3.82 \\ & (1.69) \end{aligned}$ | $\begin{aligned} & -0.79 \\ & (0.85) \end{aligned}$ | $\begin{aligned} & --2.26 \\ & (0.70) \end{aligned}$ | $\begin{aligned} & -0.41 \\ & (0.47) \end{aligned}$ | $\begin{aligned} & -0.56 \\ & (0.26) \end{aligned}$ | $\begin{gathered} 6.77 \\ (3.27) \end{gathered}$ | $\begin{gathered} 6.07 \\ (2.33) \end{gathered}$ | $\begin{aligned} & -0.04 \\ & (0.25) \end{aligned}$ |
| British pound | $\begin{gathered} 0.19 \\ (0.12) \end{gathered}$ | $\begin{gathered} 1.88 \\ (1.50) \end{gathered}$ | $\begin{aligned} & -1.88 \\ & (1.36) \end{aligned}$ | $\begin{aligned} & -0.36 \\ & (0.69) \end{aligned}$ | $\begin{gathered} -1.04 \\ (0.57) \end{gathered}$ | $\begin{aligned} & -0.86 \\ & (0.37) \end{aligned}$ | $\begin{aligned} & -0.25 \\ & (0.21) \end{aligned}$ | $\begin{gathered} 2.64 \\ (2.63) \end{gathered}$ | $\begin{gathered} 0.64 \\ (1.87) \end{gathered}$ | $\begin{gathered} 0.33 \\ (0.20) \end{gathered}$ |

Note: Standards errors are in parentheses.

Table 4: Variance-Covariance Matrix of Residuals (from regressions reported in Table 1 )

| National <br> Currency | BF | CD | FF | DM | IL | JY | DG | SF | BP |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Belgian franc | .975 |  |  |  |  |  |  |  |  |
| Canadian dollar | .118 | .157 |  |  |  |  |  |  |  |
| French franc | .827 | .100 | .957 |  |  |  |  |  |  |
| Deutsche mark | .932 | .101 | .831 | .993 |  |  |  |  |  |
| Italian lira | .579 | .072 | .633 | .592 | .731 |  |  |  |  |
| Japanese yen | .537 | .065 | .548 | .543 | .411 | .949 |  |  |  |
| Dutch guilder | .896 | .107 | .803 | .916 | .587 | .527 | .927 |  |  |
| Swiss franc | .859 | .117 | .805 | .908 | .592 | .611 | .853 | 1.174 |  |
| British pound | .491 | .095 | .456 | .471 | .364 | .413 | .487 | .482 | .760 |

Note: All numbers have been multiplied by 1000.

