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### ABSTRACT

Educational subsidies are frequently justified as a method of altering the income distribution. It is thus natural to compare education to other tax-transfer schemes designed to achieve distributional objectives. While equity-efficiency trade-offs are frequently discussed, they are rarely explicitly treated. This paper creates a general equilibrium model of school attendance, labor supply, wage determination, and aggregate production, which is used to compare alternative redistribution devices in terms of both deadweight loss and distributional outcomes. A wage subsidy generally dominates tuition subsidies in ex ante (or "opportunity") calculations, but this reverses in ex post (or "realized") calculations. Both are generally superior to a negative income tax. With externalities in production, however, there is an unambiguous role for governmental subsidy of education, because it both raises GDP and creates a more equal income distribution.

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# 1 Introduction

Education occupies a central position in the policies of governments around the world and is almost always heavily subsidized. The underlying justification for governmental involvement takes a variety of forms, but increasingly it is suggested that expanded educational investments both strengthen the national economy and improve the societal distribution of income and welfare. Education, for example, had a prominent role in the United State's "War on Poverty," begun in the 1960s and the programs begun then continue through today. And the expansion of public colleges and universities over the past three decades has rested on distributional underpinnings. This paper takes seriously the potential for education to play a role in redistribution, and in that vein considers how well education compares to alternative approaches to redistribution. The ultimate objective is to compare alternative programs in terms of both aggregate output effects and redistributive effects.

By pointing to the high economic returns to additional education, many people readily accept a significant governmental role in the production and financing of education. But of course the appropriateness, or even beneficial influence, of governmental involvement requires more than merely finding high private and social returns to schooling. As a general rule, an active role for government is justified either by some market imperfection or by an alternative objective of government, such as redistributive motives, that extends past simple maximization of aggregate output.

Appeals to externalities such as improving the functioning of democratic government or reducing crime have provided traditional support for government's ensuring free and universal elementary and secondary schools. But such arguments are less convincing when considering governmental investments in higher education. It is difficult to imagine that marginal externalities of this sort are large, or anywhere near the 40 percent of higher education revenues that come from governmental appropriations (National Center for Education Statistics, 2000).<sup>1</sup>

Instead of relying on externality arguments, providing subsidies to higher education, especially through free or reduced tuition programs at public colleges and universities, is more frequently

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<sup>1</sup>An alternative externality argument could also follow from growth effects as highlighted by endogenous growth models. To address these issues, our subsequent analysis considers such production externalities – although this situation clearly stacks the case in favor of educational subsidies because of the potential efficiency gains.

justified either on distributional grounds or on capital market imperfections and the inability to borrow against human capital (e.g., Becker 1993[1964] or Garratt and Marshall 1994). Access to higher education is seen as a way of improving the distribution of income – particularly as related to parental income, race, or socio-economic status. Once put into a distributional context, however, it is natural to compare educational subsidies with alternative ways of distributing income.<sup>2</sup> Education may have unique features, since human capital investments have productive value, but the governmental interventions involving taxes to support governmental provision of higher education and price modifications through tuition reductions are still distortionary. Therefore, it is plausible that other redistributive tools could have lower efficiency costs.

Since the act of redistributing resources and income typically will introduce distortions into the economy, no consideration of the redistributive impact of a governmental program is complete without understanding any efficiency costs related to the program. For the most part, analyses of governmental transfer programs are partial equilibrium analyses that assume little aggregate distortion and thus concentrate largely on the impact to the recipients.<sup>3</sup> Throughout the world, however, educational subsidies and other transfer programs are large (Smeeding et al., 1993) and could have a noticeable impact on output and wages in the economy. This paper focuses on just the interaction of aggregate output and distributional outcomes.

The simple general equilibrium model of the economy developed here combines both tax and transfer programs and permits a full comparison of alternative transfer mechanisms. In the basic model, the only role of government is the redistribution of income. It accomplishes this task by raising funds with a (distortionary) linear income tax. It then redistributes income through three canonical transfer programs: lump sum redistribution, a wage subsidy, or a tuition subsidy to schooling. (When lump sum spending is combined with the income tax, this transfer device becomes

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<sup>2</sup>An earlier formulation of this problem can be found in Layard (1979, 1981).

<sup>3</sup>There are a few exceptions, although each focuses on different aspects of the economy than we do. Fair (1971) considers a model of the economy which incorporates the optimal distribution of income into the analysis. Thurow (1971) also highlights individual preferences over the distribution of income. Bishop (1979) compares alternative transfer programs in an aggregate, general equilibrium model. Gramlich and Wolkoff (1979) provide a methodology for assessing the utility gains from transfers but do not consider any general equilibrium impacts. And, Browning (1993) investigates how efficiency losses enter into the calculation of the costs of governmental redistribution.

a negative income tax program). Individuals make both schooling and labor supply decisions. Schooling has productive value, but no externalities are initially considered. In many ways, the education subsidies considered here look like the provision of higher education in the United States – where tuition is heavily subsidized and where there are few supply constraints.

A central methodological consideration is treatment of the trade-off between equity and efficiency. While policy discussions frequently suggest considerations of such a trade-off, it is difficult to find examples of analyses that deal explicitly with both equity and efficiency. Most analyses of public transfer programs discuss only the redistribution without mention of any efficiency losses, while other analyses of public programs with explicit outcome objectives discuss efficiency, or cost-benefit considerations, without any integrated treatment of distributional consequences. The one exception to this dichotomy is abstract analyses of maximizing social welfare functions that can include distributional arguments. But it is generally true that different social welfare functions – that meet standard preference axioms but that allow very different weights for efficiency and equity interests – provide minimal guidance, since they can suggest very different optimal policies depending on the specific functional form.

This analysis focuses directly on the equity-efficiency trade-off in a general equilibrium framework that makes efficiency issues central. Our approach describes in a very general way the locus of feasible results for each redistributive device in output-distribution space. If any device dominates the others in the sense of permitting greater equity for any given level of output, we know that it will be chosen with any social welfare function (that positively values both output and more equality). Of course, if such universal dominance is not found, choice of the optimal policy and redistributive device will revert to a dependence on the precise social welfare function that is applied.

In our analysis, the exact definition of distributional aspects of the economy proves to be decisive in identifying the optimal policy. Specifically, there are two distinct ways of calculating the distributional outcomes of policies: in an *ex ante* or in an *ex post* sense.<sup>4</sup> The former, which is cal-

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<sup>4</sup>Throughout this analysis we concentrate on outcomes measured in terms of utility, although it is possible to measure outcomes simply in terms of the distribution of income. Where this distinction is important, we note it in

culated before the outcomes of decisions are known, corresponds most closely to an “opportunity” standard, while the latter, which is calculated on outcomes observed after the results of decisions are realized, corresponds more closely to conventional distributional discussions based on current empirical information. In many ways, a criterion based on the *ex ante* distribution of utility seems to match most distributional discussions best, but it does not permit the empirical verification that considerations of the *ex post* distribution does.

In our base case, a wage subsidy can obtain any feasible level of aggregate utility along with more equality in the *ex ante* distribution of utility than is possible with the alternative subsidy schemes. This result does not prove to be sensitive to the underlying distribution of abilities in the economy or to reasonably wide variation in the fundamental parameters of the economy. On the other hand, depending on the underlying distribution of abilities in the economy, education subsidies can dominate when distributional calculations are based on *ex post* outcomes or simply on income rather than utility. Further, with the introduction of production externalities, the use of education subsidies becomes an efficient approach over most levels of governmental intervention, but this result is not particularly surprising because of the efficiency value of counteracting the externality.

## 2 The Basic Model

The model focuses on the role of schooling and transfers in an economy where society cares about both aggregate consumption and the distribution of individual welfare. The basic structure revolves around a one-period general equilibrium model of a competitive economy. The government provides schools and operates transfer programs, all of which must be paid for by either tuition or proportional taxes on income which are sufficient to balance the budget. Individuals make optimizing choices about school attendance and the labor-leisure trade-off based upon school costs and expected wages. The schooling decision involves uncertainty because individuals with different ability have different chances of successfully completing schooling. Because taxes can be raised only

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the analysis.

through distortionary taxes, the effects of alternative transfer policies on either the performance of the economy or the distribution of welfare are not obvious.

## 2.1 Agent Behavior

The model considers an economy with an uncountably infinite number of types of agents with differing ability,  $a$ . Ability has no direct labor market payoff but instead indicates ‘educational ability’, the chance of succeeding in schooling; completing school, however, does have a direct labor market payoff.<sup>5</sup> For simplicity, the population of agents is normalized to unity and the index of ability,  $a$ , is distributed on the interval  $[0, 1]$  according to the density function  $f(a)$ ,  $\int_0^1 f(a)da = 1$ . (More details on this will appear in later sections). An agent of type  $a$  faces the fully known probability  $P_a = 1 - a$  of being successful in school. (Note that higher  $a$  means a lower probability of success). Ex post there are only two skill levels of workers in the economy – educated workers who successfully completed school and uneducated workers who either did not attend school or did not successfully complete school. Each agent chooses at the beginning of his or her life whether to go to school or not, based on school tuition,  $t$ , and the expected wages from school attendance. In this one-period model, schooling is instantaneous, and there is no opportunity cost of attempting schooling, although there is the uncertainty of school completion. Agents also have perfect knowledge of the equilibrium wage structure: successfully completing school earns a wage of  $w^e$  and not successfully completing school earns a wage of  $w^u$ . Given a proportional income tax rate,  $\tau \in [0, 1]$ , and possible direct government transfers,  $m$  (described below), all agents maximize a utility function:

$$U(c, l) = c - \epsilon \frac{(L^i)^{1+\nu}}{1+\nu}$$

subject to a budget constraint:

$$c \leq w^i L^i (1 - \tau) + m - tI$$

where  $c$  is consumption,  $L^i$  is labor supply,  $\nu$  and  $\epsilon$  ( $\nu, \epsilon > 0$ ) are parameters related to the disutility of labor,  $i = e$  (educated) or  $u$  (uneducated),  $m$  is any lump-sum cash transfer from government,  $\tau$

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<sup>5</sup>Partial completion of schooling has been shown to have labor market returns, e.g., Kane and Rouse (1995). We employ this simplification for computational reasons, although it can clearly be relaxed within the spirit of the model.

is the tax rate,  $t$  is the tuition fee, and  $I$  is an indicator function that takes on the value 1 if the agent attends school and 0 otherwise.<sup>6</sup> Since the utility function is semi-linear, it allows us to focus on the effects of redistribution without the presence of any insurance incentives on the part of the agent.<sup>7</sup>

The optimal labor supply choice of the individual,  $L^i$ , is simply a function of the wage rate as:

$$L^i = \left( \frac{(1 - \tau)w^i}{\epsilon} \right)^{\frac{1}{\nu}}.$$

Labor supply is increasing in wages, and backward bending behavior of the supply function is ruled out. With  $\nu < 1$ , the labor supply function is convex; with  $\nu > 1$ , the labor supply function is concave. The marginal utility of leisure is independent of the lump sum transfer  $m$ , that is, direct transfers do not affect the supply of labor.

The schooling decision can be understood by comparing the expected utility obtained from enrolling in school with the utility from not attending school but instead entering the labor market. Individuals attending school either successfully finish and become educated labor (e) or fail and are relegated to being uneducated labor (u), the same status as not attending school at all. The utility of an agent who is successful in school ( $U^e$ ) is,

$$U^e = [w^e(1 - \tau)]^{\frac{1+\nu}{\nu}} \epsilon^{-\frac{1}{\nu}} \left( \frac{\nu}{1 + \nu} \right) + m - t$$

while the utility of an agent who goes to school but fails ( $U^f$ ) is,

$$U^f = [w^u(1 - \tau)]^{\frac{1+\nu}{\nu}} \epsilon^{-\frac{1}{\nu}} \left( \frac{\nu}{1 + \nu} \right) + m - t$$

The indirect utility of agents will be convex in (after-tax) wages as long as  $\frac{1+\nu}{\nu} > 1$ , but it is linear in transfers. This implies that agents will not have any incentive to buy insurance against school failure. The expected utility of an agent of ability type  $a$  who attends school ( $EU^s$ ) is just the appropriately weighted average of these utilities:

$$EU^s = P_a U^e + (1 - P_a) U^f$$

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<sup>6</sup>Among others, Greenwood et. al. (1988) and Gomes, Greenwood, and Rebelo (1997) use this utility function in real business cycle applications. In international context, it can generate realistic cross country correlations in consumption. See Devereaux, Gregory and Smith (1992) and Leung (1995).

<sup>7</sup>The model can be easily extended to the case with concave utility function.



which gives

$$EU^s = m - t + \epsilon^{-\frac{1}{\nu}} \left( \frac{\nu}{1+\nu} \right) \left[ (1-a)(w^e)^{\frac{1+\nu}{\nu}} + a(w^u)^{\frac{1+\nu}{\nu}} \right] (1-\tau)^{\frac{1+\nu}{\nu}}$$

The expected utility of uneducated agent who never attend school is

$$EU^u = [w^u(1-\tau)]^{\frac{1+\nu}{\nu}} \epsilon^{-\frac{1}{\nu}} \left( \frac{\nu}{1+\nu} \right) + m.$$

Note that the lowest ex post utility in the economy is obtained by failures, since failure leaves an agent with the skills of an uneducated person but with having paid tuition in order to attempt schooling. Thus, those choosing not to attend school have certain utility of  $U^n = U^f + t$ . (Agents attending school are not allowed to default on tuition).

The agent goes to school if  $EU^s \geq EU^u$ . Thus, from the previous comparisons,

$$\begin{aligned} EU^s \geq EU^u &\Leftrightarrow P_a U^e + (1-P_a)U^f \geq U^f + t \\ &\Leftrightarrow (1-a) \geq \frac{t}{U^e - U^f} \end{aligned}$$

which yields a unique ability cutoff, which is also equivalent to the enrollment ratio,

$$a^* = 1 - \frac{t}{\epsilon^{-\frac{1}{\nu}} \left( \frac{\nu}{1+\nu} \right) (1-\tau)^{\frac{1+\nu}{\nu}} \left[ (w^e)^{\frac{1+\nu}{\nu}} - (w^u)^{\frac{1+\nu}{\nu}} \right]}. \quad (1)$$

Since there is a continuum of agents, the measure of the population who choose not to go to school will be  $1 - \int_0^{a^*} f(a)da$ . The measure who go to school and succeed is  $N^e = \int_0^{a^*} (1-a)f(a)da$  where  $(1-a)$  is again the probability of success. The uneducated population,  $N^u$ , is the sum of the measure who go to school and fail,  $N^f$ , and those who do not go to school at all,  $N^d$ ,  $N^u = 1 - N^e$ . These are the key measures of the labor force. Given this basic structure, a first-best approach would be to tax ability,  $a$ . Because ability is exogenously set for each individual, taxing it would not distort education or labor supply decisions. Thus, any redistribution could be accomplished without the efficiency loss that accompanies the income tax considered here. At the same time, it is reasonable to presume that the social planner cannot observe an individual's true ability and therefore cannot use ability taxes.

## 2.2 Wage Determination

The economy has only two types of workers in the economy: those who have successfully completed school and those who have not. In order to determine wages, it is assumed that all agents have access to an aggregate CES production function:

$$Y = A [\xi(E^e)^\rho + (1 - \xi)(E^u)^\rho]^{\frac{1}{\rho}}$$

where  $E^e = L^e N^{ew}$  is the effective units of educated labor, where  $N^{ew}$  is the amount of successfully educated agents who also participate in goods production and  $E^u = L^u N^u$  is the effective units of uneducated labor taking into account labor supply of each type of worker.<sup>8</sup> With an underlying competitive economy, wages are simply the marginal product for each type of worker:

$$w^e = A\xi \left[ \xi + (1 - \xi) \left( \frac{E^u}{E^e} \right)^\rho \right]^{\frac{1-\rho}{\rho}}$$
$$w^u = A(1 - \xi) \left[ \xi \left( \frac{E^e}{E^u} \right)^\rho + (1 - \xi) \right]^{\frac{1-\rho}{\rho}}$$

The degree of substitution between factors is defined by the parameter  $\rho$ . When  $\rho = 0$ , this is the Cobb-Douglas case. When  $\rho = 1$ ,  $E^e$  and  $E^u$  are perfect substitutes, and when  $\rho \rightarrow -\infty$  factors are perfect complements and the production function is Leontief. The elasticity of substitution is  $\sigma = 1/(1 - \rho)$ .

## 2.3 Government Transfers and Budget

This model abstracts from how the composition of government expenditures is determined and ignores any role of government other than the redistribution of income and welfare. The government must maintain a balanced budget and is restricted to the use of a proportional income tax to raise revenues. The level and form of this budget is determined by the type of redistribution. Three redistribution schemes are considered: tuition subsidies for education, a negative income tax, and a wage subsidy.

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<sup>8</sup>As discussed below,  $N^{ew}$  recognizes that educated workers are also needed to teach.

### 2.3.1 Education Subsidies

A significant portion of the discussion of higher education finance has concentrated on intergenerational equity and access. For example, Hansen and Weisbrod (1969) suggested that the implicit subsidies in the California public higher education system were skewed toward the wealthy; McPherson and Schapiro (1991), in their broad evaluation of higher education finance, focus on how public tuition subsidies interact with parental incomes. We on the other hand do not consider any intergenerational effects but instead ask the more fundamental question, 'What are the redistributive effects of education subsidies compared to no governmental intervention or to alternative redistributive programs?'

With education subsidies, the government taxes income at rate  $\tau$  and offers education at a subsidized tuition  $t$ , which is set by policy to be less than the cost of education per student,  $g$  (determined below). Since the population in the economy is normalized to 1, the budget constraint facing the government simply equates total expenditure on schools to tuition and tax payments:

$$N^r g = N^r t + \tau[w^e N^e L^e + w^u N^u L^u] \quad (2)$$

where  $N^r = \int_0^{a^*} f(a) da$  is the equilibrium enrollment. For individual agents in this economy, the government provides no cash transfers, so  $m = 0$ , but the tuition faced by anybody attending school is less than its production costs.

### 2.3.2 Negative Income Tax (NIT)

Reacting in part to the then-existent high marginal tax rates on welfare and transfers, Friedman (1962) and others have proposed transfers to the low income population through a negative income tax. With redistribution through a negative income tax (NIT), all individuals in society receive a lump sum transfer  $m$  which acts as the guaranteed income of an agent with no other income. Labor income is then taxed (or the transfer is reduced by some portion of labor income), but at a rate below 100 percent. This vision of fundamentally different transfer mechanisms than existing programs led, among other things, to a series of random-assignment experiments designed

to evaluate programmatic effects, although the clear focus was on changes in labor supply behavior (see Munnell 1986).

The combination of a linear income tax and lump-sum transfers  $m$  assumed here is a special case of the NIT. Again, with the normalization of the population to 1, the government budget constraint is

$$m = \tau[w^e N^e L^e + w^u N^u L^u] \quad (3)$$

While some NIT schemes propose different tax rates above and below break-even level for receiving positive net subsidies, our analysis considers a single marginal tax rate,  $\tau$ . Since all workers pay a proportional income tax, individual income is subsidized when  $m > \tau w^i L^i$  and taxed otherwise. Because of the special nature of this economy with just two different wage rates, the more educated cannot receive a net subsidy. In this case, education is not subsidized (i.e.,  $g = t$ ), which is equivalent to schools being provided privately.

### 2.3.3 Wage Subsidy

A final alternative is direct subsidization of the wages of the uneducated. Wage subsidies have been advocated by economists because of the ability to target them on identified populations. Various temporary and permanent forms of wage subsidies have been employed in the United States and in OECD countries, but their effectiveness is not fully understood (see Katz 1998). Much of the attention and discussion of currently available subsidies focuses on the employment effects, but here we focus entirely on the income redistributions aspects.

In our implementation, a tax  $\tau^e$  is levied on educated agents who earn  $w^e$ , while the uneducated receive a proportional wage subsidy of  $\tau^u$  on their wages of  $w^u$ . In other words transfers in terms of wage supplements go directly to those who fail and those who do not go to school. The government budget constraint is thus:

$$\tau^e w^e N^e L^e = \tau^u w^u N^u L^u. \quad (4)$$

Note that the budget constraint facing the individual agents is also altered to reflect the different tax (subsidy) rates on income for the educated and uneducated workers. The labor supply of the

educated and uneducated agents become:

$$L^e = \left( \frac{(1 - \tau^e)w^e}{\epsilon} \right)^{\frac{1}{\nu}}$$

$$L^u = \left( \frac{(1 + \tau^u)w^u}{\epsilon} \right)^{\frac{1}{\nu}}$$

The modified school selection constraint is:

$$a^* = 1 - \frac{t}{\epsilon^{-\frac{1}{\nu}} \left( \frac{\nu}{1+\nu} \right) \left[ (1 - \tau^e)^{\frac{1+\nu}{\nu}} (w^e)^{\frac{1+\nu}{\nu}} - (1 + \tau^u)^{\frac{1+\nu}{\nu}} (w^u)^{\frac{1+\nu}{\nu}} \right]} \quad (5)$$

In addition, it must be true in equilibrium that educated agents earn more than uneducated agents, i.e.,

$$w^e L^e (1 - \tau^e) > w^u L^u (1 + \tau^u) \quad (6)$$

Government expenditures, thus, consist entirely of work subsidies.

The alternative transfer schemes considered here operate in very different ways. The wage subsidy is in some sense the most targeted of the three, because only those succeeding in school pay taxes and only those failing in school or never attending receive the transfer. For the other two subsidies, taxes are proportionate to the realized wages and the chosen labor supply, but the transfers are more diffuse. With the education subsidy, all people attending school (regardless of success) receive the transfer. For the NIT, transfers benefit everyone equally.

## 2.4 School Costs (g)

The social cost of education has been treated as a fixed material cost with no direct consideration of the opportunity cost of human capital employed by the education sector and unavailable for direct production. Clearly, however, the largest element of the production cost of schools is skilled labor, making it appropriate to consider how school costs vary with the wage rates and demands for schooling that are central to this analysis.

The simplest approach to defining school costs assumes that it takes  $b$  teaching hours to educate a student, whether he or she will graduate or not. Further, it is assumed that a teacher can only teach  $n_b$  students simultaneously (i.e., that schooling is produced by a simple fixed coefficient

technology). Underlying this development is an implicit perspective that there is no choice over quality of schooling and that all educated workers are equally productive in teaching or in goods production. In equilibrium, all skilled workers must receive the same utility from teaching or from goods production. Thus, the teacher is only willing to work the same amount of time as any skilled worker is willing to work  $L^e$ , and they also face the same tax rate as other educated workers.

For the model economy with a population equal to unity and with an equilibrium enrollment ratio of  $N^e$ , the number of teachers demanded is  $N^t \equiv (N^e \cdot b) / (n_b \cdot L^e)$ , which is the total teaching hours needed divided by the number of student classroom hours each teacher can provide.<sup>9</sup> Hence, the teacher-student ratio in this model is  $N^t/N^e = b/(n_b \cdot L^e)$ , which is endogenous because  $L^e$  is endogenous. It is also obvious that we only need to consider the ratio of the parameters  $b/n_b$ , rather than their levels separately.

Because some educated citizens are needed to teach, we have to modify the consideration of workers in the economy. Specifically, we have  $N^e = N^{ew} + N^t$  rather than  $N^e$  in (2), (3) and (4). The total “number,” or measure, of educated workers for goods production  $N^{ew}$  is equal to the total number of successful students  $N^e$ , net of the number of teachers  $N^t$ , or  $N^{ew} = N^e - N^t$ . We consider only cases where  $N^{ew}$  is positive. Since only workers directly contribute to goods production, the social cost of education ( $g$ ) is measured by the working hours of the teachers times the wage rate of educated workers.<sup>10</sup>

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<sup>9</sup>Notice this formulation implicitly assumes that the teachers themselves need to be students first. This calculation is somewhat awkward in a static model but understandable if the static model is perceived as being a steady state of a dynamic economy.

<sup>10</sup>The formulation with endogenous schooling costs yields some sharply different conclusions than a formulation with fixed schooling costs. For example, with general productivity improvements, wages of all types of workers will increase proportionally. If the (social) cost of education is exogenous, the school enrollment ratio ( $a^*$ ) unambiguously increases, because the enrollment ratio will depend on the relative level of the exogenous cost of education to the level of productivity. However, if the social cost of education is endogenous, an increase in the skilled/educated worker’s wage also increases the opportunity cost of being a teacher. In fact, under the particular formulation employed here, the level of productivity will have no effect on the equilibrium enrollment ratio. With endogenous school costs, the model also generates the prediction that, as the working hours of skilled workers decrease, the teacher-student ratio will increase. This seems consistent with the historical experience internationally, although similar results could be generated by other models of schooling demand.

Formally,  $g = (\text{Total wage bill for teachers})/(\text{Total number of students}) = (N^t * L^e * w^e) / a^* = (b/n_b) * w^e$ .

### 3 Measuring Performance

Our criteria for performance of the economy consider both the aggregate output and utility of individuals and the distribution of individual welfare. Most other analyses of transfers concentrate on one or the other dimension of outcomes without focusing on their interaction.

Aggregate distributional issues are seldom explicitly considered, but there are several consistent ways to formulate the problem to incorporate such distribution. For example, if distributional elements enter each individual's utility function, distribution would automatically be taken care of when social utility is calculated as the aggregation of individual welfare. Alternatively, society's concerns about distributional issues could be introduced directly at the level of the social welfare function – by explicitly identifying weights on distributional outcomes. We follow a different approach. For each tax rate and distributional mechanism we trace out the feasible surface for combinations of aggregate outcomes and the distribution of welfare. This equity-efficiency locus then permits a social planner to maximize overall welfare by selecting both a transfer mechanism and a size of government. Even within this analytical framework, however, a variety of natural alternatives exist. Here we describe the computations for education subsidies or a negative income tax. The straightforward modification for the multiple tax rates in the wage subsidy case is not explicitly described but is easy to derive.

#### 3.1 Aggregate Expected Utility (AEU)

We consider a social planner who maximizes the sum of the expected utility levels of all agents,  $S(\tau, t)$ . This simple utilitarian welfare function, which aggregates the utility of agents who are successful in school, who fail and become uneducated, and who do not go to school at all, is simply:

$$\begin{aligned} S(\tau, t) &= \int_0^{a^*} [(1-a)U^e + aU^f] f(a)da + \int_{a^*}^1 U^u f(a)da \\ &= U^e N^e + U^f N^f + U^u N^u, \end{aligned}$$

where  $a^*$  is the ability of the marginal student enrolled in college,  $U^e$  is the utility of successful agents,  $U^f$  is the expected utility of those who fail, and  $U^u$  is that for agents who do not go to

school at all.<sup>11</sup>

In an economy with education subsidies, the planner maximizes this function subject to equations (1) and (2), while in the economy with a negative income tax (lump-sum cash transfers), the planner maximizes  $S(\tau, t)$  subject to (1) and (3) since agents bear all the costs of education. In the case of wage subsidy, the planner maximizes social welfare subject to (5) and (4)

### 3.2 Measurement of Inequality

The effects of the redistribution schemes can be viewed in two separate ways – ex ante and ex post – with resulting differences in interpretation. The ex ante calculations can be directly interpreted as the degree of equality of opportunity faced by the population. The ex post calculation on the other hand indicates the degree of contemporaneous inequality and is, in a political economy sense, likely to be very relevant for policy decisions about redistribution. In our simplified economy, alternative ways of aggregating the utility distribution make little difference, and therefore the Gini coefficient is selected as the summary measure.<sup>12</sup> Gini coefficients are computed based on income as well as utility. The virtue of calculations based on utility is that they capture the gains in leisure of the agents in the economy, and we emphasize these.

The computation with either after-tax income or utility levels is particularly straightforward in the ex post problem. There are three income classes in the economy. The highest wage earners are those who are successful in schooling with a net income of  $(1 - \tau)w^e L^e - t + m$ . The second highest wage earners are those who do not go to school and who have a net income of  $(1 - \tau)w^u L^u + m$ , while the poorest are those who fail in school and have a net income lower by the amount of tuition paid,  $(1 - \tau)w^u L^u - t + m$ . The discrete nature of the underlying incomes and utilities implies that the Lorenz curve (relating the cumulative population distribution to the cumulative income distribution) has three linear segments whose length reflects the proportions of the population in each group. The Gini coefficient is easily calculated from the area between the Lorenz curve and

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<sup>11</sup>Note that at this point there is no need to distinguish between teachers and educated workers involved in production, since they must have the same utility. The weights in  $S$  reflect the total number of agents successfully completing school.

<sup>12</sup>See Lambert (1990) and Cowell (2000) for more details.



the 45 degree line representing an equal income (or utility) distribution. The larger the area, the more inequality that exists, and the larger the value of the Gini coefficient.

The nature of the ex post distribution also points out the conceptual superiority of the ex ante calculations. The people who try school but fail clearly have a higher expected utility at the time of the decision – otherwise they would not have attended school. While the realized outcome may differ, they are better off than those not attending in the sense that they have better opportunities.<sup>13</sup>

The ex ante calculations, however, vary directly with ability level. For low ability people (who do not attempt further schooling), ex ante and ex post utility are the same. People with higher ability (who attempt schooling) will always have ex ante at least as high as these low income people. In fact, even among those who enroll in colleges, ex ante utility rises with ability as the probability of successfully completing schooling rises with ability. In the computations, the Lorenz curve for ex ante utility is approximated through discretization.

## 4 Base Case Outcomes

The intuition behind the mechanics of the model is as follows: Wages between educated and uneducated workers are unequal due to a skill premium arising from successfully completing schooling. Wages are determined by the marginal products derived from an aggregate production function. Governments's only function is redistributing income, which is accomplished by first raising revenues with a distortionary tax that directly affects labor-leisure choices. The form of subsidy employed has direct implications for the amount of schooling attempted and completed and thus for wages in the economy. The feedback through distorted decisions has implications for both aggregate outcomes and the distribution of welfare.

Comparisons among the alternative policy regimes requires fixing a number of key parameters and underlying distributions. Unfortunately, the key parameters have not been estimated very precisely. We begin with a base case benchmarked to prevailing estimates of the central elasticity

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<sup>13</sup>Part of the difference in ex post calculations comes from the fact that there is no labor market return to either ability or partial schooling. If either of these existed, it could close if not overcome the gap introduced by the "wasted" tuition payments.

parameters. Subsequent sections investigate the sensitivity of the results both to parameter choices and to more fundamental specification issues including the underlying ability distribution and the presence of growth externalities.

## 4.1 Fundamental parameter values

The decision of parameter values begins with the preference side. The key elements affecting individual choice are the underlying elasticity of labor supply ( $1/\nu$ ) and the elasticity of substitution between educated and uneducated labor ( $\sigma = 1/(1 - \rho)$ ). Despite considerable empirical analysis employing both experimental and econometric approaches, a surprisingly wide range of estimates for labor supply elasticities exists (Pencavel 1986; Killingsworth and Heckman 1986; Blundell and MaCurdy 1999). As a base case, we use an uncompensated wage elasticity of  $1/3$ , which falls between the (generally lower) elasticity estimates for males and the (generally higher) estimates for married females (Blundell and MaCurdy 1999).

The substitution between different classes of labor has received less attention. An early estimate by Johnson (1972) places the elasticity of substitution between college and high school workers at 1.3, although sensitivity analysis yields a very broad range. Katz and Murphy (1992) provide a series of alternative estimates that depend on the time series of relative demand shifts, but their point estimate in direct estimation is 1.41. Katz and Autor (1999) review and evaluate alternative estimation approaches and results and show a considerably larger range of estimates. Our base case estimates use  $\sigma = 1.3$ .

Finally, in terms of key assumptions, we begin with a symmetric distribution of abilities (symmetric around 0.5). (The details can be found in the appendix). While it is common to find estimates of IQ or other measures of ability to be normally distributed, we know of no analysis that addresses the functional form for scholastic ability – the ability to complete schooling – as used here. As with the other key parameters, however, we subsequently investigate the sensitivity of the results to this distributional specification.<sup>14</sup>

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<sup>14</sup>The details of the distribution can be found in the appendix. The rest of the base calibration is as follows. We set  $\xi$  to be 0.5 so that the wage ratio between skilled and unskilled worker is around 1.4, i.e.,  $w^e/w^u \approx 1.4$ . Notice also

## 4.2 Base results

With the balanced governmental budget, the tax rate simply indexes the size of each program. Table 1 shows how the different subsidy schemes promote very different patterns of attendance in school. With no taxes or transfers (i.e., the no government case), 41 percent of the population attends school and successfully completes, another 21 percent attends school but fail, and the remaining 38 percent never attends. Given the structure of the economy, all of the people attending school have higher ability than the most able person not attending school, but the failed and successful groups will each have people of overlapping ability. As the tax rate and subsidy increase, the programs have very different attendance patterns.<sup>15</sup> With greater education subsidies, the net tuition to the student falls, and a larger proportion of students attends school. The largest impact of this, however, is on the failure rate. While successful completers go from 41 percent of the population in the no subsidy case to 50 percent in the case of a 12 percent tax to support education, the proportion attending but failing school rises from 21 percent to 45 percent. This response reflects the high levels of tuition subsidy. At a 6 percent tax rate, tuition to students is only 45 percent of school costs; at a 12 percent tax rate, tuition is just 10 percent of school costs.<sup>16</sup> The school attendance behavior is always individually optimal given the tuition costs and wage structure that results, but many more now fall into the lowest utility group (the failed students).

The pattern is very different for the two other subsidy schemes. For each, increasing levels of subsidy lead to lower school attendance and a smaller educated work force, with the declines

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that under the current formulation, the tuition-skilled worker wage ratio is related to the zero-tax teacher-student ratio,  $t/w^e = g/w^e = b/n^b$ . We set  $b/n^b$  equals to 0.05, which is close to the empirical calculation of  $t/w^e$ . Lastly, we set the productivity parameter  $A$  in the production function to be 0.4 so that the equilibrium enrollment ratio with zero tax and zero subsidy is about 60%. To see this, notice that in the symmetric distribution case,  $a^* = 1 - (t/w^e) * \left\{ \epsilon^{-1/\gamma} \gamma (1 + \gamma)^{-1} (w^e)^{1/\gamma} \left[ 1 - (w^e/w^u)^{(1+\gamma)/\gamma} \right] \right\}^{-1}$ . It is clear that the ratios  $t/w^e$  and  $w^e/w^u$  will be invariant to the productivity parameter  $A$ . Thus, the enrollment ratio  $a^*$  depends on the wage for skilled worker  $w^e$ , which in turns depends on the productivity parameter. The cases for other distributions are similar. To our knowledge, there does not exist a reliable estimate for the scale parameter in the labor supply function,  $\epsilon$ , although this is not a key parameter for our analysis. In the benchmark case,  $\epsilon$  is set to 3 so that the working hours for both educated and uneducated workers are in between 30 to 40 percent of their total time endowment.

<sup>15</sup>Our simulations have tax rates for schooling going from 0 to 12 percent (the point where virtually everybody attends). The upper bound is obviously far beyond current expenditures on college.

<sup>16</sup>The cost of schooling does decline slightly with higher subsidies for tuition because the wages of educated workers and thus teachers are driven down, making schooling cheaper given the linear cost function. But the cost decline is not significant for the results.

Tax rate ( $\tau$ )	School successes			Nonattendees			School Failures		
	E	W	N	E	W	N	E	W	N
.00	.41	.41	.41	.38	.38	.38	.21	.21	.21
.03	.44	.39	.40	.31	.42	.39	.25	.19	.20
.06	.46	.38	.40	.23	.46	.40	.31	.17	.20
.09	.48	.36	.40	.14	.49	.41	.37	.15	.19
.12	.50	.34	.39	.05	.52	.42	.45	.13	.19

Table 1: Distribution of School Attendance and Success by Subsidy Scheme and Tax Rate for Baseline Equilibrium (proportions of population): E=education subsidy; W=wage subsidy; N=negative income tax

Tax Rate ( $\tau$ )	Education Subsidy	Wage Subsidy	Negative Income Tax
.00	1.42	1.42	1.42
.03	1.33	1.49	1.43
.06	1.26	1.57	1.45
.09	1.21	1.65	1.46
.12	1.19	1.75	1.48

Table 2: Relative Wages of Educated and Uneducated Workers by Subsidy Scheme and Tax Rate for Baseline Equilibrium

being most dramatic for the direct wage subsidy. Because the earnings disadvantage of not being educated are leveled out, high wage subsidies work against investment in schooling and lead to substantial changes in attendance.

The distortions in the economy introduced by the taxes and subsidies have direct implications for wage distributions in the economy. Table 2 shows the relative wages of educated to uneducated workers. In the competitive economy with no government ( $\tau = 0$ ), educated workers have gross wage rates that are forty-two percent above those of uneducated workers. Education subsidies induce more people to go to school, and the increased proportion of educated workers squeezes their relative wages. The other two subsidy schemes, however, work in just the opposite direction, with the most dramatic impact coming for the wage subsidy where the relative earnings of the educated grow to 1.75 with a 12 percent tax rate.

These outcomes, nonetheless, do not show the complete picture of the effects on the economy. First, they neglect any consideration of how the distortions influence aggregate production and

welfare. Second, they must be combined with the transfer programs in order to understand the full impact on individual welfare, since the pre-tax wages and outcomes ignore the direct transfers. Third, at any tax rate, the different subsidy schemes introduce different amounts of distortion into the economy, implying that better comparisons would involve subsidy schemes at levels of equal aggregate distortion.

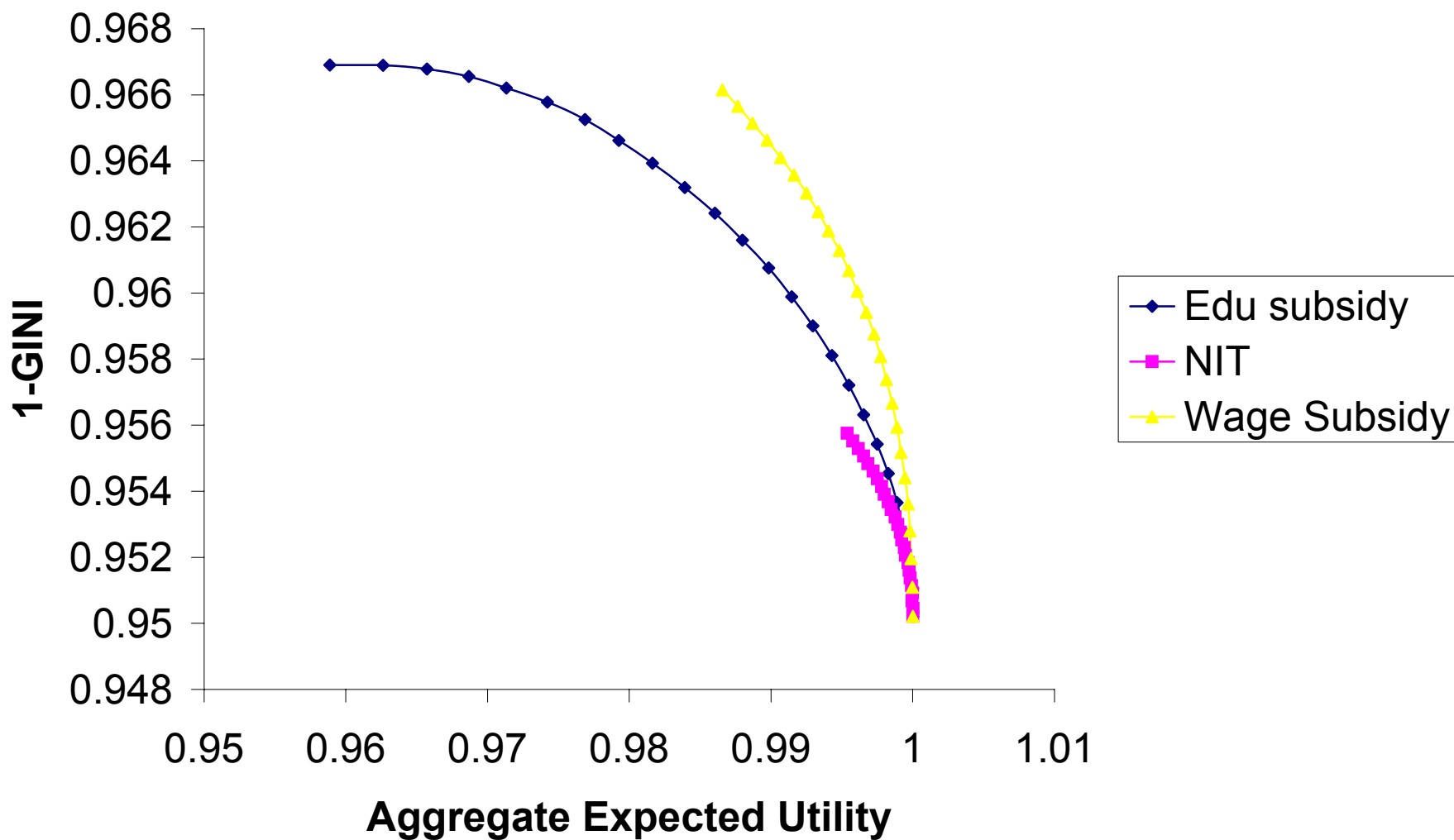
Figure 1 provides a direct comparison of each of the subsidy schemes in terms of their effects on aggregate expected utility and on the *ex ante* distribution of welfare. This figure highlights the trade-off of efficiency – measured by losses in AEU – and of equity – measured by 1-Gini defined in terms of *ex ante* utility. Figure 1 plots each of the subsidies for tax rates between zero to 12 percent. The competitive economy with a zero tax rate yields the largest aggregate utility, and increases in the tax rate decrease aggregate utility in each of the subsidy schemes. But, with higher taxes more redistribution occurs, and the distribution of utility becomes more equal (as seen by increasing values of 1-Gini). Thus, if society values both aggregate output and more equality, movement up and to the right represents improvement in overall societal welfare.

Figure 1, providing the locus of feasible aggregate output (in utility terms) and opportunity distributions under each scheme, indicates that a wage subsidy is a superior subsidy scheme to education tuition subsidies, which in turn are superior to negative income tax. Under any social welfare function, the wage subsidy can provide higher welfare, because it can achieve any feasible level of aggregate expected utility with more equality than either of other two subsidy schemes.

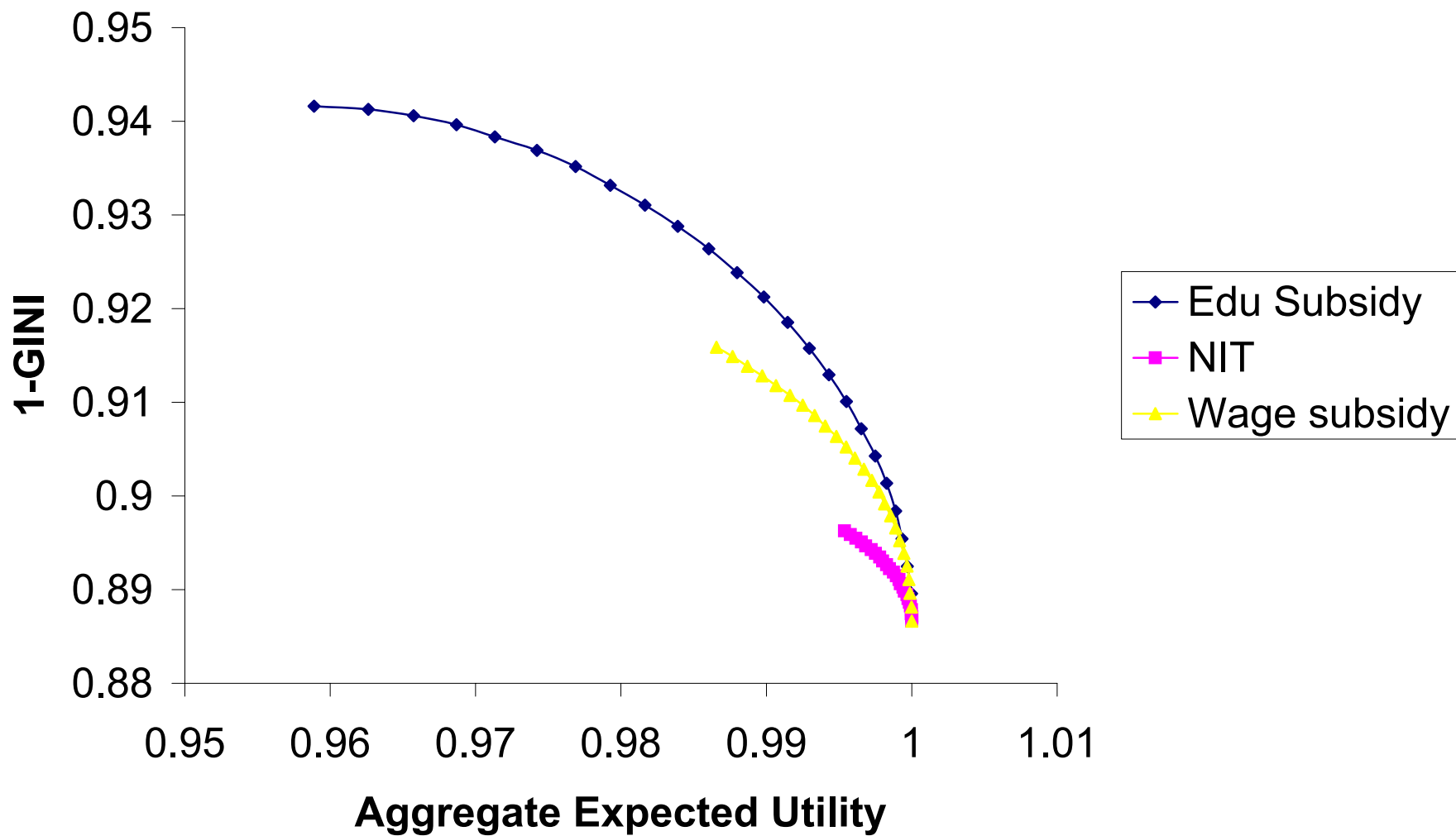
As noted, however, other measures of equity are possible, and Figure 2 displays the same trade-offs for the *ex post* calculations of the utility distribution. The most obvious comparison of these distributions is that the ordering of subsidy schemes actually changes with *ex post* inequality measures. The distribution of utility now depends on the realizations of school success. Those who fail school have lower incomes and utilities than those who did not attempt more schooling, and the *ex post* calculations focus not on opportunities but on realized outcomes.

In an *ex ante* comparison, people of higher ability always are better off than those with lower abilities, because they have increased probabilities of successfully completing schooling and thereby

**Figure 1: AEU and 1-Gini (ex-ante utility)**



**Figure 2: AEU and 1-Gini (ex-post utility)**



obtaining higher wages. In an *ex post* sense, however, this is not the case because all failures begin with higher ability than the group that did not attempt schooling. Thus, not only does the amount of redistribution change but the character of the subsidies also changes. We return to a more detailed comparison of the distributional aspects in the next section.

The figures vividly illustrate one additional important feature: the tax rate is a very imperfect index of the impact of governmental interventions. Importantly, the varying distributional schemes have very different distortionary effects at any given tax rate, so the typical practice of comparing the redistribution from alternative transfer mechanisms by choosing a common tax rate will yield very misleading comparisons. At a 12 percent tax rate, the economy employing a negative income tax loses only 0.3 percent of aggregate expected utility, compared to losses of 0.8 percent for the wage subsidy, and 3.2 percent for the education subsidy. Put the other way, an education subsidy program with a 3 percent tax rate, a wage subsidy program with a 6 percent tax rate, and an NIT program with a 10 percent tax rate each has an equivalent deadweight loss (but they will have very different implications on inequality). The combined general equilibrium effects of tax and subsidy programs illustrate the importance of programmatic detail in determining the welfare implications of governmental interventions, although most conventional program analyses miss this.

### **4.3 *Ex ante v. ex post***

By focusing on why wage subsidies and education tuition subsidies change place in the orderings, it is possible to understand better how these subsidies work. The results here may seem counter-intuitive at first sight. It is often asserted that education subsidy will improve the “equalization of opportunities” and thus would seem to be a good policy from the *ex ante* point of view. On the other side, a wage subsidy seems to target on those who have not attended or could not finish the college and thus would be a good policy from the *ex post* point of view. The results in this paper reverse these assertions and therefore deserve more discussion. While the stated assertions contain some truth, they miss a general equilibrium perspective and a framework for “fair” comparison across regimes. For instance, while an education subsidy indeed induces more agents to receive



college education (Table 1), it also change the composition of the labor force and hence changes the relative wage ratio (Table 2). In addition, the marginal failure rate increases as the subsidy rate increases, so that the expected impact for each tax dollar will decrease as the scale of education subsidies is enlarged and the efficiency of the system thus decreases. Thus, while an education subsidy might be able to generate a higher level of equality, the adverse effect on the efficiency cannot be ignored.

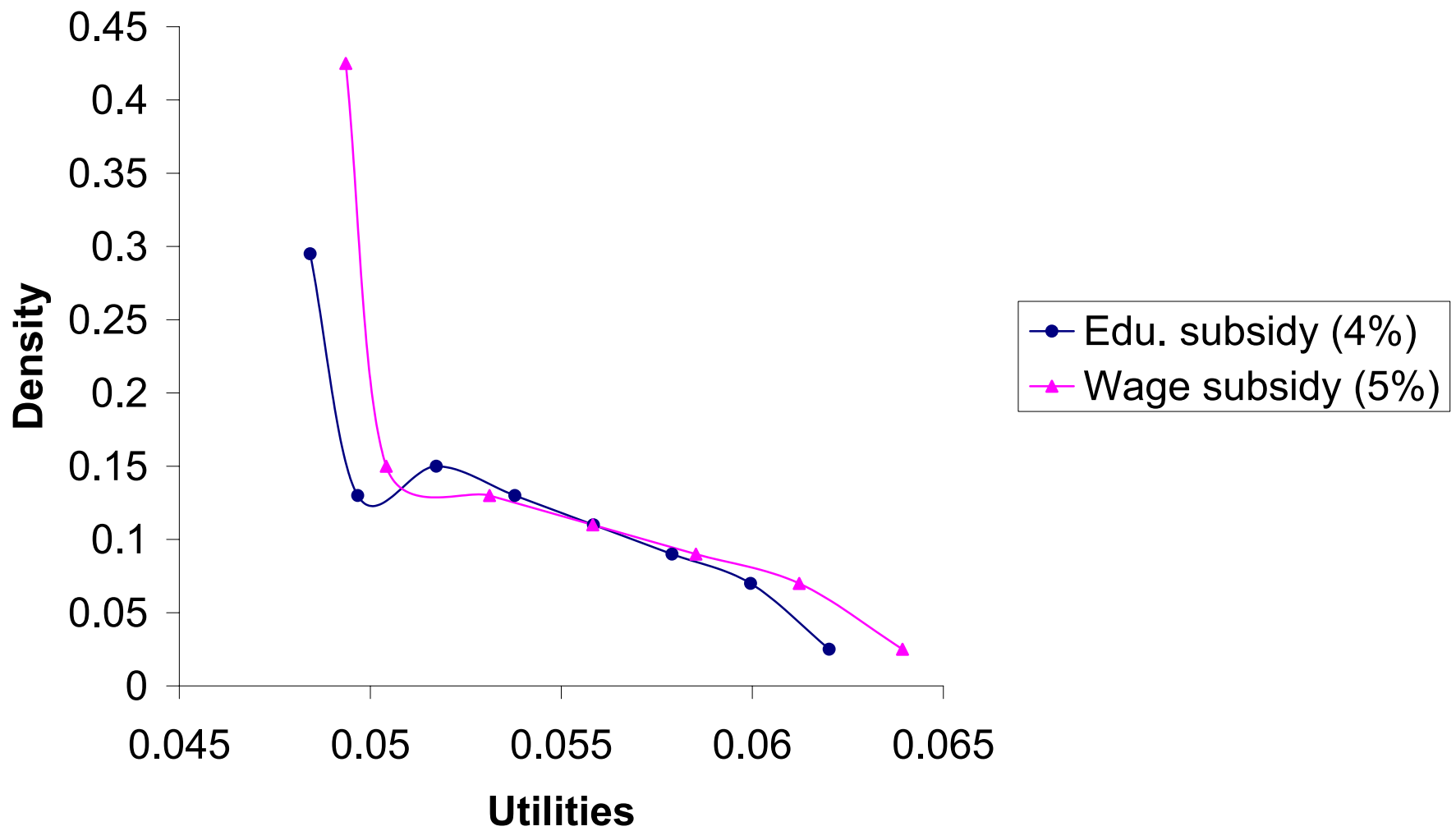
The full comparison, however, of the differences between the ex ante and ex post results requires going behind the summary outcomes depicted in Figures 1 and 2. Each point on the outcome loci of each regime represents a full underlying distribution of utilities, and it is instructive to compare the underlying distribution across the wage subsidy and tuition subsidy regimes. (Because the negative income tax is always dominated, we drop consideration of it in these comparisons). Our criteria of “dominance of regimes” is that, at a given level of efficiency, a higher level of equality can be reached. And, since as noted different regimes carry different implications to the distribution of utility even at the same tax rate, a natural candidate is to compare a point on one regime’s loci to another point on the other regime’s loci with both higher efficiency (as measured by AEU) and higher equality (as measured by 1-Gini).

To illustrate, we compare the distribution of ex ante utility of four percent tax rate under the education subsidy regime with the five percent tax rate under the wage subsidy in the benchmark case. The latter achieves both higher level of efficiency and equality than the former. Figure 3 displays the distribution of ex ante utilities and the corresponding density of agents. Expected ex ante utility of individual agents increases along the horizontal axis while the height of the curves indicates the density of agents at each utility level.<sup>17</sup> The shape of the two distributions is similar. In fact, for people receiving the middle range of utility (between 0.05 and 0.06), the density of agents under the two regimes is similar. The distinctive difference appears at the ends of the distribution. The bottom group under both regimes is comprised of low ability individuals for whom attending school is not optimal. This population expects to receive a direct subsidy under the wage subsidy

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<sup>17</sup>The underlying continuous distribution of agents is discretized for these calculations and the density represents the “size” of each group.

### Figure 3: Ex-ante Utility Distribution

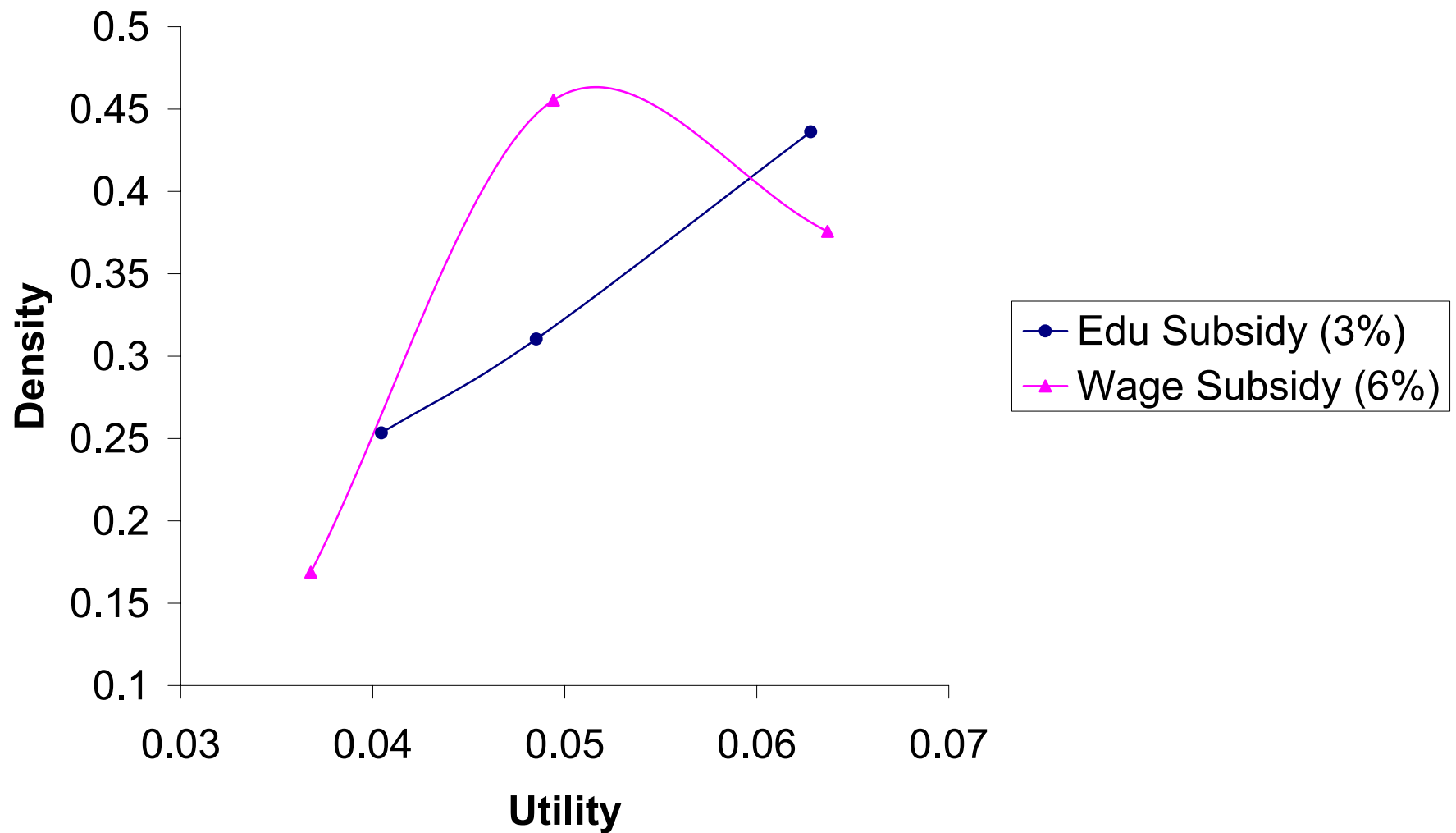


regime, whereas it expects only indirect effects (through relative wage improvement) under tuition subsidies, i.e., the lowest ability people see none of the tuition subsidy since they do not attempt further schooling. Interestingly, the top ability group also achieves a higher level of utility under the wage regime than under the education subsidy regime. Because the group of agents who successfully complete schooling is relatively small under the wage subsidy regime, the relative wage effects more than compensate for the higher tuitions (compared to the education subsidy case). Summing up these illustrative comparisons, the expected utility improvements to the lowest ability people under the wage subsidy are the dominant force leading to compression of the ex ante distribution of utility (for an overall level of aggregate utility).

Now consider the ex post case. We compare the distribution of ex post utility of a three percent tax rate under the education subsidy regime with the six percent tax rate under the wage subsidy regime. The former achieves both higher efficiency and higher ex post equality than the former. Figure 4 displays the distribution of the ex post utility under both regimes. Again, the main points are transparent from the figure. After schooling decisions are made and success in school is considered, there are only three groups of people in terms of utility outcomes. The bottom group in terms of realized utility is comprised entirely of school failures, and their utility is improved under the education subsidy regime because the wasted tuition is less than that under the wage subsidy. The top group, made up of school completers, has slightly higher realized utility under the wage subsidy but its representation in the population (38 percent) is significantly less than under the education regime (about 44 percent of the population; see Table 1).

In sum, the wage subsidy scheme has its most concentrated impact on those who face the worst opportunities – those with low ability for whom there is not sufficient expectation of gaining from further schooling. It proves superior in equalizing ex ante utilities (for any given level of governmentally induced inefficiency). The tuition subsidy proves better after schooling outcomes are realized, because it subsidizes school failures and brings up their ex post utility (even though these people have higher ability and higher ex ante utility than individuals who do not attempt schooling). In other words, behind the observed opportunity loci for the different subsidy schemes

**Figure 4: Ex-Post Utility Distribution**



Ability Distribution	Elasticity of Substitution for Education			
	$\sigma = 0.8$	$\sigma = 1.0$	$\sigma = 1.3$	$\sigma = 2.0$
	<i>Labor supply elasticity (<math>1/\nu</math>) = 1/3</i>			
Uniform	$W \succ E \succ N$	$W \succ E \succ N$	$W \succ E \succ N$	$W \succ N \approx E$
Symmetric	$W \succ E \succ N$	$W \succ E \succ N$	$\mathbf{W} \succ \mathbf{E} \succ \mathbf{N}$	$W \succ E \succ N$
Skewed	$W \succ N \succ E$	$W \succ N \succ E$	$W \succ N \succ E$	$W \succ N \succ E$
	<i>Labor supply elasticity (<math>1/\nu</math>) = 1/6</i>			
Uniform	$W \succ E \succ N$	$W \succ E \succ N$	$W \succ E \succ N$	$W \succ N \succ E$
Symmetric	$W \succ E \succ N$	$W \succ E \succ N$	$W \succ E \succ N$	$W \succ E \succ N$
Skewed	$W \succ E \approx N$	$W \succ E \succ N$	$W \succ E \succ N$	$W \succ N \approx E$

Table 3: Social Welfare Comparisons of Subsidy Regimes with Ex Ante Utility Calculations under Alternative Model Parameters: W=wage subsidy; N=negative income tax; E=education subsidy

lie very different patterns of subsidy.

## 5 Sensitivity Analysis

As mentioned, the key parameters for our simplified economy have not been estimated with much precision. We therefore consider the effects of different subsidies assuming different labor elasticities, different substitution across education classes, and different distributions of individual abilities. For labor elasticities ( $1/\nu$ ), we employ values of  $1/3$  and of  $1/6$ , values which appear to bound the bulk of existing estimates for men and women in the United States economy. For the elasticity of substitution between educated and uneducated labor, we consider  $\sigma = 0.8, 1.0, 1.3,$  and  $2.0$ , although estimates below one do not appear consistent with the time series evidence on wage changes (Katz and Autor 1999). An elasticity of  $1.0$  corresponds to the Cobb-Douglas production function case. Finally, in addition to the symmetric ability distribution, we consider two linearized distributions for  $a \in [0, 1]$ . The first is uniform over the interval  $[0, 1]$ , while the second is skewed toward low ability people with the peak at  $a = 0.75$ .

Table 3 (ex ante utility distributions) and table 4 (ex post utility distributions) provide a symbolic summary of the loci of aggregate utility and 1-Gini under the different subsidy schemes (comparable to Figures 1 and 2). For this,  $X \succ Y$  signifies that the plot for subsidy scheme  $X$  at different tax rates always lies above that for subsidy scheme  $Y$ .  $X \approx Y$  means that neither dominates over the entire distribution. In such a situation, the  $X$  and  $Y$  plots generally lie close to

Ability Distribution	Elasticity of Substitution for Education			
	$\sigma = 0.8$	$\sigma = 1.0$	$\sigma = 1.3$	$\sigma = 2.0$
<i>Labor supply elasticity (<math>1/\nu</math>) = 1/3</i>				
Uniform	$E \succ W \succ N$	$E \approx W \succ N$	$W \succ E \succ N$	$W \succ N \succ E$
Symmetric	$E \succ W \succ N$	$E \succ W \succ N$	<b><math>E \succ W \succ N</math></b>	$E \approx W \succ N$
Skewed	$E \succ W \succ N$	$W \succ E \succ N$	$W \succ E \succ N$	$W \succ E \succ N$
<i>Labor supply elasticity (<math>1/\nu</math>) = 1/6</i>				
Uniform	$E \succ W \succ N$	$E \succ W \succ N$	$E \succ W \succ N$	$W \succ E \succ N$
Symmetric	$E \succ W \succ N$	$E \succ W \succ N$	$E \succ W \succ N$	$E \succ W \succ N$
Skewed	$E \approx W \succ N$	$E \approx W \succ N$	$W \succ E \succ N$	$W \succ E \succ N$

Table 4: Social Welfare Comparisons of Subsidy Regimes with Ex Post Utility Calculations under Alternative Model Parameters: W=wage subsidy; N=negative income tax; E=education subsidy

each other and intersect one or more times at different tax rates.<sup>18</sup> The bold elements of the tables corresponds to the base case described above.

The most important aspect of these sensitivity analyses is that the prior results are not affected much by considering a broad range of parameters for the economy. When evaluated on an ex ante basis, the wage subsidy regime proves superior. On an ex post basis, however, we find ambiguity about whether tuition subsidies are superior.

The distribution of abilities clearly influences the ex post results. The tuition subsidies have their strongest influence when abilities follow a symmetric distribution that peaks in the middle. Since the cutoff enrollment rate in the no tax case is reasonably close to the center of the distribution (62 percent), changes in incentives for school attendance have large effects on the population induced to continue schooling. With the uniform and skewed distributions, the impact on enrollment is less, and tuition subsidies cannot have the same influence on the distribution of outcomes. In fact, negative income taxes are uniformly ranked at the bottom except when the ability distribution is skewed toward low ability, at which point they tend to dominate tuition subsidies in an ex ante sense. Additionally, not surprisingly, tuition subsidies look better when there are lower elasticities of substitution and thus when the relative importance of education increases. Nonetheless, the overall conclusions are remarkably insensitive to the specific parameters used.

<sup>18</sup>In some cases, tax rates greater than 0.12 for the wage subsidy are required to compare the different regimes. Specifically, the tuition subsidy with a 12 percent tax rate often yields very large relative inefficiency but more equality than a wage subsidy at 12 percent tax rates, even though a wage subsidy still dominates by going to a larger governmental intervention.

## 6 Production Externalities of Education

A strong motivation for educational investments by society has traditionally been the presumption that there are significant externalities associated with education. The usual arguments about externalities, however, apply best to elementary and secondary education and less well to higher education (Hanushek 1996; Poterba 1996). This view is also supported by Acemoglu and Angrist (1999). The one exception is consideration of how human capital might affect national growth rates – through, say, the development of ideas or the diffusion of technologies. In such a case, while competitive labor markets might exist, they will not be Pareto optimal, and government action might be called for on pure efficiency grounds. Here we consider such a case in a simple extension of the basic model similar to that of Romer (1986). This particular form is not only simple but also incorporates an externality that is compatible with perfectly competitive markets.

While all other aspects of our model of the economy remain the same, we now assume that all agents have access to an aggregate production function which of the CES form:

$$Y = A [\xi(E^e)^\rho + (1 - \xi)(E^u)^\rho]^{\frac{1}{\rho}} (\overline{E^e})^\chi$$

where  $\overline{E^e}$  is the “average effective units of educated labor”, the externality part, and  $\chi$  ( $\chi > 0$ ) is a parameter indexing the strength of the externality. Since individuals take the  $\overline{E^e}$  part as given, this economy is compatible with perfectly competitive markets but individual schooling decisions will tend to yield less than optimal education in the economy. Wages are simply the marginal products:

$$w^e = A\xi \left[ \xi + (1 - \xi) \left( \frac{E^u}{E^e} \right)^\rho \right]^{\frac{1-\rho}{\rho}} (\overline{E^e})^\chi,$$

$$w^u = A(1 - \xi) \left[ \xi \left( \frac{E^e}{E^u} \right)^\rho + (1 - \xi) \right]^{\frac{1-\rho}{\rho}} (\overline{E^e})^\chi.$$

In equilibrium,  $\overline{E^e} = E^e$ .

The presence of the production externality changes the situation, because a subsidy to education now acts like a Pigouvian tax that enhances aggregate performance of the economy. At low levels of taxes and education subsidies, *both* aggregate expected utility and equality (1-Gini) improve,

Ability Distribution	Elasticity of Substitution for Education			
	$\sigma = 0.8$	$\sigma = 1.0$	$\sigma = 1.3$	$\sigma = 2.0$
<i>Labor supply elasticity <math>(1/\nu) = 1/3</math></i>				
Uniform	$\tau > .08$	$\tau > .08$	$\tau > .08$	$\tau > .09$
Symmetric	E	E	E	E
Skewed	$\tau > .06$	$\tau > .06$	$\tau > .06$	$\tau > .07$
<i>Labor supply elasticity <math>(1/\nu) = 1/6</math></i>				
Uniform	E	E	E	E
Symmetric	E	E	E	E
Skewed	$\tau > .06$	E	E	E

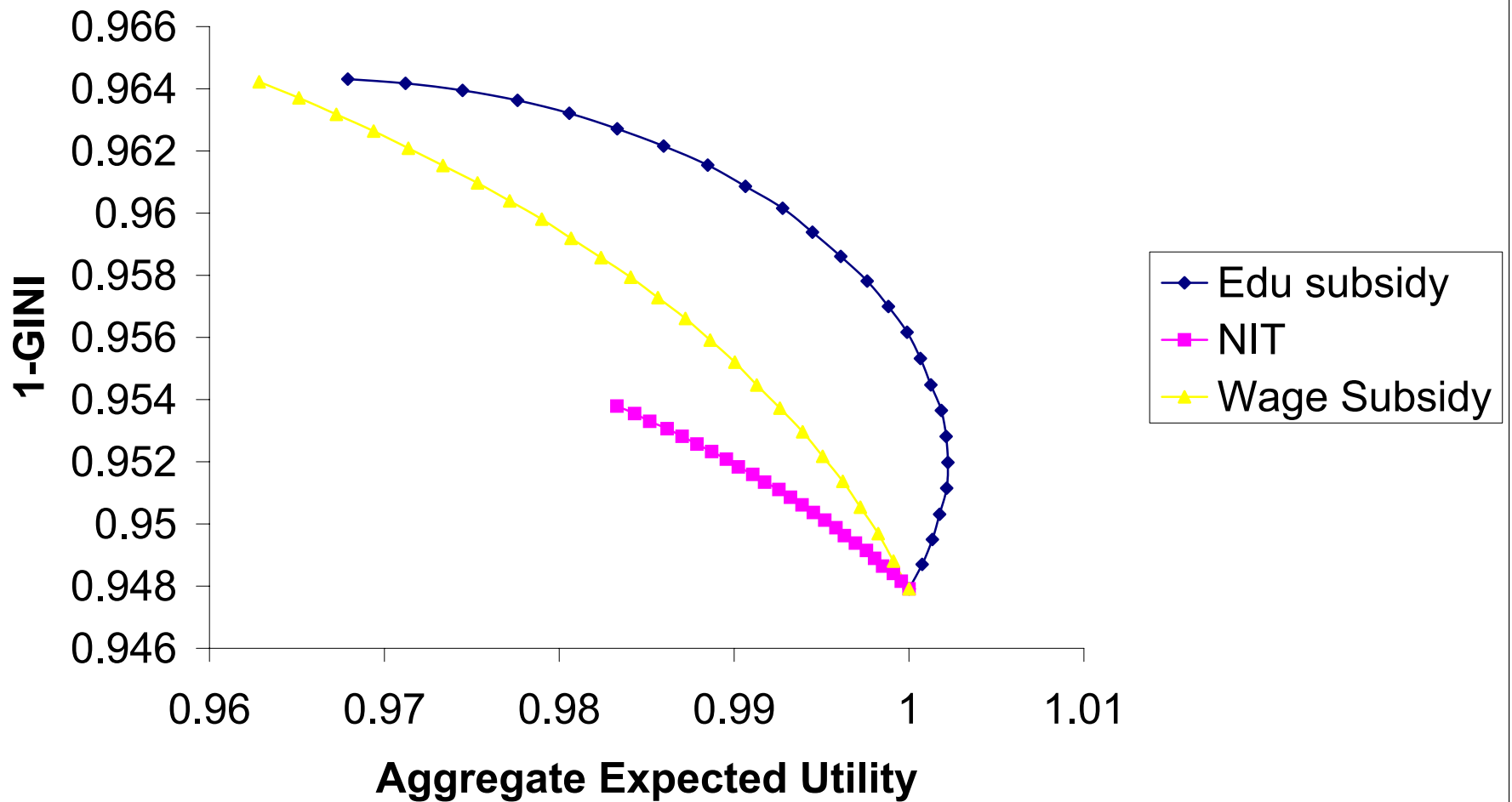
Table 5: Education Tax Rate above which Wage Subsidies Dominate Education Subsidies based on Ex Ante Utility Calculations (E implies education subsidies always dominate)

making education subsidies the clearly superior transfer mechanism. At higher levels of education subsidies, however, the inefficiency from tax distortions sometimes overcomes the efficiency gains from correcting the externality. The typical situation can be readily seen from considering the base case employed previously with the addition of the externality where  $\chi = .1$ . Figures 5 (ex ante) and 6 (ex post) illustrate the output and distributional patterns from the subsidy schemes operated at differing tax rates (for the base case defined previously except for the externalities). For  $\tau \leq 0.04$ , AEU and (1-Gini) increase under the tuition subsidy. For  $.04 \leq \tau \leq .085$  applied to education subsidies, aggregate utility (AEU) is decreasing, but it still remains above that obtainable without governmental intervention or with the other transfer devices.

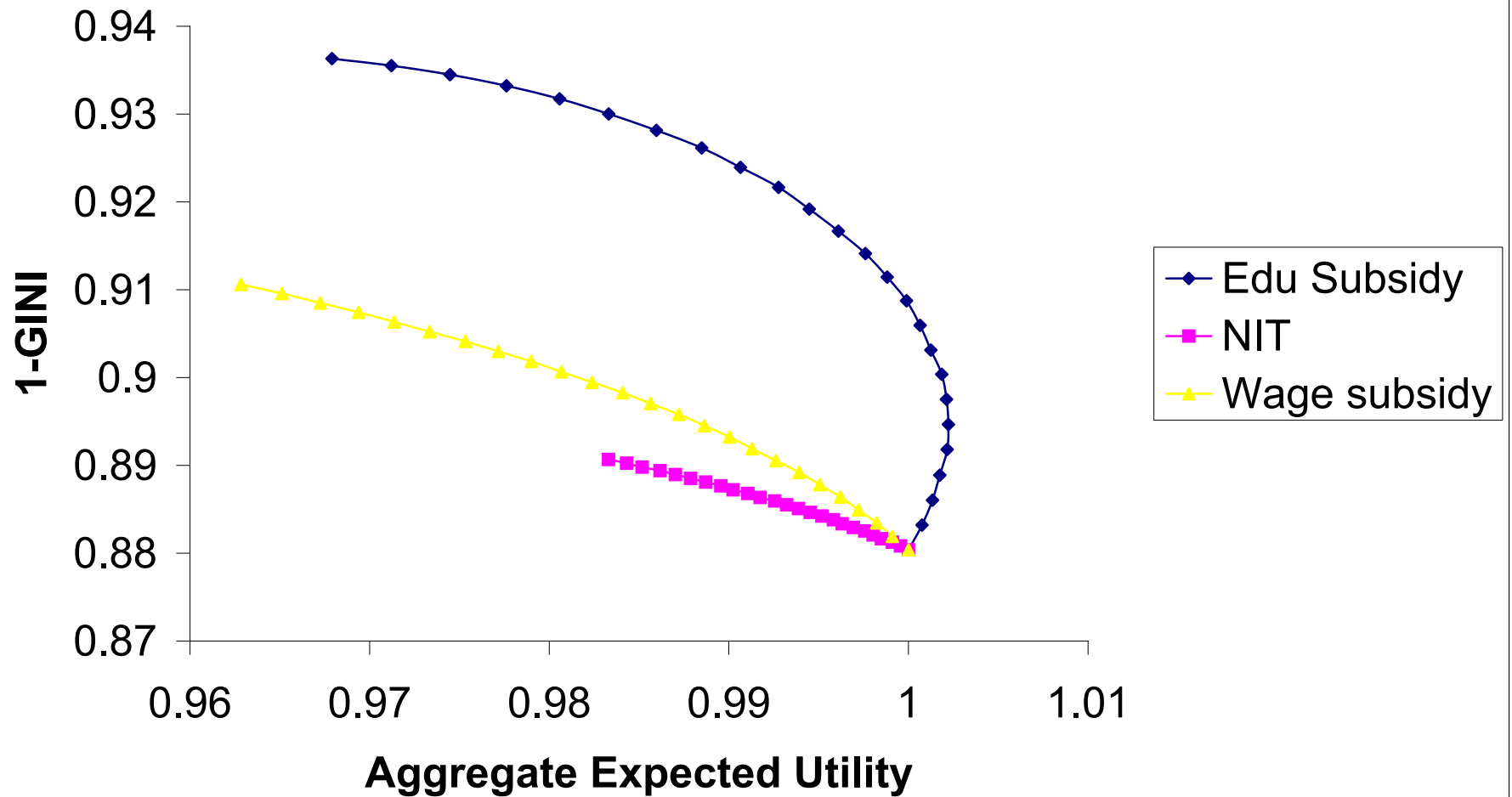
Table 5 displays the array of sensitivity results corresponding to those provided before. The general picture is that, under the education subsidy regime, only very high tax rates (above 7.5 percent) ever yield an alternative program that dominates educational subsidies. If we think of this as a model of higher education, however, total spending on institutions of higher education in 1999 was just 2.8 percent of GDP (National Center for Education Statistics, 2000). The education and wage subsidies always dominate the negative income tax in meeting societal output and distributional goals, i.e., the locus of feasible pairs is always lowest for an NIT. For ex post utility calculations, educational subsidies *always* dominate the others across all parameter values and distributions considered. Therefore, the externalities considered here could fully justify redistribution through education subsidies, although the driving force is its ability to correct an existing distortion



**Figure 5: AEU and 1-Gini with Externality  
(ex-ante utility)**



**Figure 6: AEU and 1-Gini with Externality  
(ex-post utility)**



from the externality.

## 7 Conclusions

This paper develops a methodology for evaluating transfer mechanisms that might be expected to have both output and distributional effects. The specific focus is the potential redistributive aspects of education tuition subsidies and how they compare with those of cash transfers through either a negative income tax or a wage subsidy to low wage workers. The comparisons incorporate both deadweight losses and redistribution within a one-period general equilibrium model of the economy. Workers differ by ability, where ability indexes the probability of successfully completing schooling. Individuals decide whether or not to pursue more schooling, and, based on the outcomes of that, choose labor supply levels. The government's only role is redistributing income. It provides transfers to individuals that are financed by a (distorting) linear income tax, and it must maintain a balanced budget.

The overall results of the comparison of transfer mechanisms are very illuminating. In the simple world considered here, if where there were no interest in distributional outcomes, the social optimum would be no governmental taxation or spending. With no externalities, individual schooling and labor supply choices will lead to maximum social welfare, defined by aggregate expected utility for individuals. With distributional motives which are supported by levying a distortionary income tax, however, the consideration of best governmental programs becomes more interesting.

Without externalities, the results are somewhat ambiguous. For our base case that relies on the best estimates of key parameters, wage subsidies dominate the alternative transfer mechanisms in the sense that any level of aggregate expected utility achieved by a negative income tax or an education subsidy can be achieved by a program of wage subsidies that also ensures more ex ante utility. This result breaks down, however, when utility is calculated on an ex post basis. With this metric, alternative descriptions of the production relationships, behavioral parameters, and ability distribution can yield superiority of tuition subsidies.

If there are production externalities related to the aggregate education level in the economy,

however, the education subsidies serve a dual purpose – redistributing income while potentially moving the economy toward Pareto superior outcomes. Thus, a rationalization for the heavy subsidies for higher education can be generally derived when externalities are involved. While the beneficial effects of education subsidies on aggregate output are not surprising when externalities of the growth variety are considered, educational subsidies also dominate in terms of the distribution of income and utility.

In order to focus on the key comparisons among alternative governmental transfer programs, this paper does not consider a series of issues that might also be important in evaluating governmental policies toward education and redistribution. Many have argued (e.g., Becker 1993[1964]; Garratt and Marshall 1994) that capital market imperfections inhibit individual ability to invest in human capital. The inability to use human capital as collateral for loans is a central element of such considerations. Moving from the one period model to a dynamic model of the economy could capture better the investment nature of education and the role of families. A multiperiod model would also permit direct investigation of the intergenerational transmission of income and its distributional implications. From a different direction, others question the potential inefficiency from governmental supply, particularly when it tends to be monopolistic (Hanushek 1986). Thus, they tend to concentrate on the cost and quality of schooling. Here we abstract from such supply issues and assume homogeneous and efficient provision of schools (and other governmental redistribution), although potential governmental failure could influence the results. Yet another obvious extension is to incorporate political economy features of the choice of mechanisms. For example, as the analysis of the details of distributional outcomes in figures 3 and 4 demonstrated, different private interests may well lead individuals to have differing views about the appropriate transfer mechanism. These views may support collective choice for nonoptimal transfer devices.

Finally, we believe that this paper provides important methodological improvements over prior analyses of transfer programs. The magnitude of both educational programs and direct transfer programs indicate their potential for significant general equilibrium effects. This paper provides a framework for considering both the redistributive and efficiency effects of alternative programs

and for evaluating the full impacts of governmental activities.

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## A Details of different distribution of ability

In this section, we will provide all the details about the different distribution of ability used in the text. To start with, the density function for symmetric distribution used in this paper is

$$f(a) = \begin{cases} 0.5 + 2a & a < 0.5 \\ 2.5 - 2a & a \geq 0.5 \end{cases} ,$$

$\forall a \in [0, 1]$ . Notice that the density function is continuous at every point. Recall that the formula for enrollment and total number of successful agents are  $N^r = \int_0^{a^*} f(a)da$  and  $N^e = \int_0^{a^*} (1-a)f(a)da$ . In the case of symmetric distribution, it is easy to check that

$$N^r = \begin{cases} 0.5a^* + (a^*)^2 & a^* < 0.5 \\ -0.5 + 2.5a^* - (a^*)^2 & a^* \geq 0.5 \end{cases} ,$$

$$N^e = \begin{cases} \frac{1}{2}a^* + \frac{4}{3}(a^*)^2 - \frac{2}{3}(a^*)^3 & a^* < 0.5 \\ -\frac{5}{12} + \frac{5}{2}a^* - \frac{9}{4}(a^*)^2 + \frac{2}{3}(a^*)^3 & a^* \geq 0.5 \end{cases} .$$

The other cases are analogous. The density function for uniform distribution is

$$f(a) = 1,$$

$\forall a \in [0, 1]$ . The density function is continuous at every point. Recall that the formula for enrollment and total number of successful agents are  $N^r = \int_0^{a^*} f(a)da$  and  $N^e = \int_0^{a^*} (1-a)f(a)da$ . In the case of uniform distribution, it is easy to check that

$$N^r = a^* ,$$

$$N^e = a^* - \frac{(a^*)^2}{2} .$$

The density function for skewed distribution used in this paper is

$$f(a) = \begin{cases} \frac{1}{2} + \frac{4}{3}a & a < 0.75 \\ \frac{9}{2} - 4a & a \geq 0.75 \end{cases} ,$$

$\forall a \in [0, 1]$ . Notice that the density function is continuous at every point. Recall that the formula for enrollment and total number of successful agents are  $N^r = \int_0^{a^*} f(a)da$  and  $N^e = \int_0^{a^*} (1-a)f(a)da$ . In the case of skewed distribution, it is easy to check that

$$N^r = \begin{cases} \frac{1}{2}a^* + \frac{2}{3}(a^*)^2 & a^* < 0.75 \\ -\frac{3}{2} + \frac{9}{2}a^* - 2(a^*)^2 & a^* \geq 0.75 \end{cases} ,$$

$$N^e = \begin{cases} \frac{1}{2}a^* + \frac{5}{12}(a^*)^2 - \frac{4}{9}(a^*)^3 & a^* < 0.75 \\ -\frac{9}{8} + \frac{9}{2}a^* - \frac{17}{4}(a^*)^2 + \frac{4}{3}(a^*)^3 & a^* \geq 0.75 \end{cases} .$$



## B Calculation of Gini coefficients

The measure of agents who are successful is simply  $p_1 = a^* - \frac{a^{*2}}{2}$  while those who go to school and fail is  $p_2 = \frac{a^{*2}}{2}$ . Those who do not go to school consists of  $1 - p_1 - p_2$ . This gives mean (after-tax) income  $\bar{y}$  as

$$[p_1((1 - \tau)w^e L^e - t + m)] + [p_2((1 - \tau)w^u L^u - t + m)] \\ + [(1 - p_1 - p_2)((1 - \tau)w^u L^u + m)]$$

The area underneath the Lorenz curve is computed by summing the integral of the density of the three after-tax income classes, where the Lorenz curve is represented by the following piece-wise continuous function:

$$Z_1(p) = \frac{[(1 - \tau)w^u L^u - t + m]}{\bar{y}} p \text{ for } p \in [0, p_2] \\ Z_2(p) = \frac{[(1 - \tau)w^u L^u - t + m]p_2 + (p - p_2)[(1 - \tau)w^u L^u + m]}{\bar{y}} \text{ for } p \in [p_2, p_1 + p_2]$$

and

$$Z_3(p) = \frac{[(1 - \tau)w^u L^u - t + m]p_2 + [(1 - \tau)w^u L^u + m]p_1}{\bar{y}} \\ + \frac{(p - p_1 - p_2)[(1 - \tau)w^e L^e - t + m]}{\bar{y}} \text{ for } p \in [p_1 + p_2, 1]$$

The sum of the integral of the piecewise continuous function gives the area underneath the Lorenz curve. The Gini coefficient is simply  $1 - (2 \times \text{area under Lorenz curve})$ .<sup>19</sup>

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<sup>19</sup>Again, straightforward modifications of the formulae are required for the wage subsidy case. In the case where the Gini is based on utility, the welfare of the economy measured by  $S(\tau, t)$  is used as a normalization.