

NBER WORKING PAPER SERIES

AN EMPIRICAL MODEL OF LABOR SUPPLY IN THE UNDERGROUND ECONOMY

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Working Paper No. 3392

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
June 1990

We have received many helpful comments from a large number of people. In particular, we would like to thank Joe Altonji, Orley Ashenfelter, Paul Beaudry, Dwayne Benjamin, John DiNardo, Ross Finnie, John Ham, Guy Lacroix, Bruce Meyer, Whitney Newey and in particular David Card for very helpful comments, and Louis Beausejour for the computer program used to compute the marginal tax rates. The excellent research assistance of Joelle Noreau for the construction of the data is gratefully acknowledged. Financial support was provided by the Social Sciences and Humanities Research Council of Canada and by the Fonds F.C.A.R. of Quebec. This paper is part of NBER's research program in Labor Studies. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

This paper uses micro data from a random survey carried out in the region of Quebec City, Canada, to estimate a model of labor supply in the underground economy. The model assumes that the individual's gross wage rate in the regular sector is parametric while his gross labor earnings in the underground sector are a concave function of hours of work. This distinction between the two sectors is used to generate a simple separation result between preferences and the magnitude of underground labor market activities. This result implies that the individual's labor supply in the underground economy is generally a negative function of his net wage rate in the regular sector. The separation result also implies a set of restrictions on the parameters of the reduced form of the model, which are imposed using minimum distance methods of estimation. Various generalized method of moments specification tests allow us to verify the validity of these restrictions.

According to our results, the marginal tax rates embodied in the Quebec tax-transfer system are an important determinant of the decision to participate in the underground sector.

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## 1. Introduction

Most empirical work pertaining to the effects of the tax-transfer system on individual choices in the labor market has focused on the work-leisure choice (e.g. Hausman 1985). However other important decisions may be affected as well. Thus, by driving a wedge between labor costs and net returns to workers, a tax on labor income may stimulate tax evasion activities. Source withholding or information reporting requirements severely limit opportunities of tax evasion in the regular sector. In contrast, income from underground activities is administratively hard to detect since no official records of these activities exist at all. As a consequence, the decision to evade taxes on labor income is typically a decision to take one or more jobs in the underground economy<sup>1</sup>.

While the theoretical literature on tax evasion with labor income has expanded considerably over the past decade,<sup>2</sup> empirical analyses are still uncommon. Studies on tax evasion based on official audit samples provide useful evidence on the impact of tax rates on tax evasion by type of income.<sup>3</sup> However, information provided by these samples is often limited to regular income underreporting. Moreover, they do not provide information on individual wage rates and hours of work. It is therefore impossible to estimate from those data sources distortions in the allocation of labor resources between the regular and the underground economies.

On the other hand, there has been recent efforts in some European countries to estimate labor supply functions in the underground economy based on interview data.<sup>4</sup> To our knowledge, only Isachsen, Samuelsen and Strom

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<sup>1</sup> In the sample used in this paper, only 17 percent of the individuals who evaded taxes on labor income underreported their regular labor income (if any) to the government authorities.

<sup>2</sup> For example, Baldry 1979, Pencavel 1979, Sandmo 1981, Cowell 1985, Watson 1985, and Fluet 1987.

<sup>3</sup> See, for example, Clotfelter 1983, Slemrod 1985 and Alexander and Feinstein 1987.

<sup>4</sup> See Pestieau (1985) and Ginsburgh, Perelman and Pestieau (1987) for Belgium, Isachsen, Samuelsen and Strom (1985) for Norway and Van Eck and Kazemier (1988) for the Netherlands.

(1985) derived their empirical model from a full-fledged choice-theoretic approach. It is estimated through maximizing a likelihood function which takes into account various corner solutions. They found that marginal tax rates positively influence labor supply in the underground sector while tax enforcement variables have a negative impact. However, they assume away any substitution between the regular and the underground economies, since gross regular labor income is an exogenous variable in their model.

In this paper, we develop and estimate a model of labor supply in the underground economy which allows for substitution between the regular and the underground sectors. For this purpose, we use a random survey of 2134 adults living in the Census Metropolitan Area of Quebec City, conducted in 1986 by two authors of this paper (see Fortin and Fréchette (1987)).

Our theoretical model adds the features of a standard tax evasion approach to the standard framework developed by Gronau (1977) to analyze the allocation of time among leisure, work in the regular market, and work at home. In our model, however, this latter variable becomes time allocated in the underground sector. One crucial feature of Gronau's model is that marginal returns from home production are a decreasing function of hours worked at home. For two main reasons, that feature of returns to home production is also likely to characterize labor earnings in the underground sector. First, the underground labor markets are typically likely to be small, to minimize the probability of detection by the tax authorities. Furthermore, the absence of formal warranties for goods and services produced in the underground economy limits the substitutability between these goods and services and those produced in the regular economy (Cowell and Gordon 1989). Thus, the underground wage is not likely to be parametric and the worker faces a demand curve instead. Labor earnings in the underground

economy will therefore typically be a concave function of the amount of hours worked<sup>5</sup>.

Another explanation for the concavity of the underground earnings function has been proposed by Usher (1986). He argues that the marginal cost of resources used by the agent to hide earnings to the tax authorities is likely to increase with the level of those earnings. Under this assumption, the underground earnings net of these costs will be a concave function of hours of work in the underground economy, even if gross hourly earnings are constant.

The remainder of the paper is the following. In Section 2, we describe how the data set was collected and its main characteristics. In Section 3, we present the model retained for the empirical analysis. In Section 4, the stochastic specification of the model is outlined and the econometric strategy is discussed. Participation and hours of work decisions are estimated using two-step methods to adjust for selectivity, while restrictions of the model are imposed on the estimated reduced forms using minimum distance techniques. In Section 5, the empirical results are presented and various specification tests are performed. We conclude the paper in Section 6.

## 2. The Data

The survey used in this paper was conducted during the Spring 1986. The questionnaires were distributed by a group of trained interviewers who stressed the anonymous and confidential character of the survey, as well as

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<sup>5</sup> Under the assumptions that a worker faces a linear demand, that there are constant returns to scale in production, and that there is no price discrimination, the underground earnings function will be globally concave (see Cowell and Gordon (1989) for a model which retains these assumptions). When the demand curve is non-linear but cuts each axis at some point, the earnings function will be at least locally concave. In this case, it is easy to show, using our framework, that the optimal solution for an individual working in both the regular and the irregular sector will always fall in a concave region of the earnings function. Note also that under perfect price discrimination, this function will be globally concave as long as the demand curve is negatively sloped.

its scientific purposes. The respondents were asked to put the filled-in questionnaire in a sealed envelope which was picked up a few days later by the interviewer.

The main sample is of the random cluster type and is restricted to household members over 18 years of age. The main sample was supplemented by a small quota sample to compensate for the difficulties encountered in reaching people in some areas or in some socio-economic groups through household visits. 63.8 percent of the sampled households had at least one individual answer the questionnaire. Moreover, in these respondent households, 81.1% of the members over 18 years of age answered the questionnaire.

Table 1 summarizes the socio-economic characteristics of the sample and the magnitude of labor market activities in the underground sector. These activities are defined as the set of all paid activities that are not reported to the tax authorities. A sample of 2107 persons (of whom 285 from the quota sample) is usable for our analysis. 183 persons (or 8.7% of the sample) report that they worked in the underground economy, for an average of 360 hours a year per person. Their average underground income was \$2,171 (Canadian) in 1985. Males, youth, and unmarried people are more likely to work in the underground sector than their counterparts. Furthermore, 28.6% of the people who report that being in school or on unemployment was their main labor market status during the year participated in the underground sector<sup>6</sup>. The participation rate rises to 35% in the case of welfare recipients.

The effect of education on the participation rate in the underground sector is more ambiguous: it reaches a maximum for the category "some college" and then goes down. This may reflect cohort effects as young people in Quebec are more educated than their elders. Table 1 also shows that the probability of participating and the number of hours worked in the underground economy are inversely related to labor income in the regular sector.

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<sup>6</sup> The labor market status reported in Table 1 is the activity among work, school, unemployment, housework, or retirement, on which the individual spent the most weeks during the year. Someone who worked during 27 weeks and was unemployed during 25 weeks is thus classified as a worker.

The bottom part of Table 1 provides information on the industrial composition of jobs in the underground economy. It should first be noted that the probability of working in the underground economy is higher for workers with a regular sector job in primary industries or in the construction industry. Furthermore, the jobs in the underground sector are concentrated in the construction industry and in services, which include repairs. An interesting fact is that for most people working in both the regular and the underground sector, the jobs in the two economies are in different industries.

Further insight on the nature of the regular and of the underground economies is provided by the sample covariance matrix of hours, earnings, and wage (all expressed in logs) presented in Table 2. The table also presents the partial covariance matrix of the same variables, partialling out for standard control variables<sup>7</sup>. A first observation is that the variance of annual hours of work in the underground sector is approximately three times bigger than the same variance in the regular sector. This provides some support to the hypothesis that the underground sector allows for more "flexibility" than the regular sector in the choice of hours of work.

A second striking result that emerges from Table 2 is that in the underground sector, both the covariance and the partial covariance between hours of work and the wage rate is negative, while it is positive in the regular sector. Such a result is a little puzzling as it does not fit with a labor-leisure theoretic framework in any obvious manner<sup>8</sup>.

Any reasonable model of labor supply in the underground sector should be able to explain this puzzle. The approach we will follow uses the concavity of the function relating underground earnings to underground hours of work to explain such a result. To illustrate that point, consider an unobservable

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<sup>7</sup> In Table 2, the control variables used are a dummy variable for sex, number of years of education, number of years of experience in the regular market, and its square.

<sup>8</sup> The result is quite robust to the choice of control variables and to the subsample used. In particular, it holds over the subsample of people working in both the regular and the underground economies.

factor, such as the moral attitude toward work in the underground sector, that influences the choice of hours supplied in this sector. Assume further that this factor does not affect the underground earnings function. Due to the concavity of this function, the individuals who work more hours in the underground sector will receive, *ceteris paribus*, a lower level of average hourly earnings than those who work less hours. This fact will tend to generate a negative correlation between hours of work and the average wage rate in the underground sector.

In fact, Table 2 also provides some information on the concavity of the earning functions in both the regular and the underground sector. Recall that the regression coefficient of a variable  $Y$  on a variable  $X$ , controlling for a set of variables  $Z$  can be written as  $\sigma_{XY.Z}/\sigma_{XX.Z}$ , where  $\sigma_{ij.Z}$  is the partial covariance between  $i$  and  $j$ , partialling out for  $Z$ . From this result, one can easily compute estimates of the elasticity of earnings with respect to hours of work in each sector. In the regular sector, this elasticity ranges from 1.07 to 1.28, depending on whether the partial or the raw covariances are used. On the other hand, it ranges from .73 to .74 in the underground sector. This suggests that the earnings function is close to linear in the regular sector, at least when partial covariances are considered, while it is concave in the underground sector. Therefore, a model which retains these features seems to be appealing not only for theoretical but also for empirical reasons.

### 3. The Model

Following the basic model of tax evasion with labor income, the tax evader faces two possible states of nature: either he gets caught working in the underground sector (with probability  $P$ ) or does not (with probability  $1-P$ ). In this simplest version of the model,  $P$  is assumed to be exogenous. Getting caught entails a penalty proportional to the amount of tax evaded<sup>9</sup>. Assuming, for the moment, a proportional income tax at rate  $r$ , the penalty rate on underground income can be written as  $\lambda r$ ,  $\lambda$  being the penalty rate on evaded tax (with  $\lambda \geq 1$ ). The expected rate of tax penalty,  $P\lambda r$ , on evaded

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<sup>9</sup> This formulation was first proposed by Yitzhaki (1974).



income is assumed to be less than the rate of tax  $\tau$  on reported income, i.e.  $P\lambda < 1$ . The stochastic budget constraint faced by the individual is given by:

$$(1) \quad X = W_0(1-\tau)H_0 + f(H_1)(1-D\lambda\tau) + I$$

where:  $W_0$  : gross wage rate in the regular sector

$H_0$  : hours of work in the regular sector

$H_1$  : hours of work in the underground sector

$D$  : dummy variable =  $\begin{cases} 1 & \text{with probability } P \\ 0 & \text{with probability } 1-P \end{cases}$

$I$  : nonlabor income (including any lump-sum transfers)

$X$  : consumption (the *numéraire*).

Following our discussion above,  $f(H_1)$  represents an underground earnings function à la Gronau, with  $f(0)=0$ ,  $f'(H_1)>0$ , and  $f''(H_1)\leq 0$ .

We assume that the individual is an amoral expected utility maximizer, and that utility has consumption and total hours of work as arguments. Thus, the problem to be solved by the individual is:

$$(2) \quad \text{Max}_{(H_0, H_1)} E[U(H_0 + H_1, X)]$$

subject to (1) and to the non-negativity constraints on  $H_0$  and  $H_1$ . The utility function  $U(\cdot)$  is assumed to be decreasing with  $H_0+H_1$ , increasing with  $X$ , and to be a concave (but not necessarily strictly concave) function of its arguments.

Now, consider the following assumptions:

A1a: the "wage rate" in the underground sector is parametric with respect to hours, i.e.  $f'(H_1)=W_1$  for all  $H_1$ , and therefore  $f''(H_1)=0$ .

A1b: The underground earning function is strictly concave, that is  $f''(H_1)<0$ .

A2a:  $U(\cdot)$  is strictly concave, which implies that the individual is risk-averse.

**A2b:** The individual is risk-neutral with respect to consumption, i.e. there exists an affine transformation of the utility function such that  $U(H_0 + H_1, C) = V(H_0 + H_1)C$ .

The case where A1a and A2a are satisfied has been analyzed by several authors<sup>10</sup>. Unfortunately, this leads to few interesting comparative statics results, even with strong restrictions on preferences for consumption and leisure<sup>11</sup>. As noted by Cowell (1985, p. 21), the basic reason for these ambiguities is that "in reaction to any perturbation, the individual can substitute across two margins (risk/no risk and labor/leisure), so that in principle all sorts of behavior could be consistent with rational expected utility maximization"<sup>12</sup>. In this branch of the literature, risk aversion provides the "smoothness" required for interior solutions. Risk neutral agents would specialize in only one sector with probability one.

An alternative approach, which we employ in this paper, consists of assuming A1b and A2b; i.e., that the individual is risk-neutral and that the function  $f(H_1)$  representing his opportunities in the underground sector is strictly concave. Under the latter assumption, risk aversion is no longer required for the existence of interior solutions. This approach is closely related to those recently adopted by authors dissatisfied with the lack of predictive power of models based on risk aversion.<sup>13</sup> Under assumptions A1b and A2b, the first-order conditions for interior solutions imply:

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<sup>10</sup> See, for example, Sandmo (1981), Cowell (1985), and Fluet (1987).

<sup>11</sup> For a critical analysis of this approach, see Kesselman (1989).

<sup>12</sup> However, Fluet (1987) has shown that under the assumption that  $W_1 < W_0$ , there necessarily exists a threshold tax rate above which there exists a positive relationship between the tax rate and labor supply in the irregular sector. Fluet assumes that the degree of absolute risk aversion is decreasing in consumption and that there is an interior solution for  $H_0$  and  $H_1$ .

<sup>13</sup> Examples of models of tax evasion that do not assume risk aversion are Hansson (1985), Usher (1986), and Kesselman (1989). These models suppose that tax evasion activities are full-proof (i.e. no chance of being detected) but entail real or psychic costs.

$$(3) \quad W_0(1-\tau) = f'(H_1)(1-P\lambda\tau)$$

Equation (3) means that, at the optimum, the marginal earnings from an hour of work in the underground sector net of the expected marginal penalty is equal to the net wage rate in the regular sector. Note that  $P\lambda\tau$  is smaller than one for  $\tau \leq 1$ , since it was assumed that  $P\lambda < 1$ . (3) can be solved for the labor supply in the underground sector:

$$(3)' \quad H_1 = H_1 \left[ \frac{W_0(1-\tau)}{1-P\lambda\tau} \right], \quad \text{with } H_1'(\cdot) < 0$$

It is easy to verify that  $H_1$  is increasing with  $\tau$  while it is decreasing with  $P\lambda$  and  $W_0$ , which is what one should intuitively expect. Furthermore, even in the absence of any tax illusion, the elasticity of  $H_1$  with respect to the gross wage rate in the regular sector,  $W_0$ , will be higher (in absolute value) than the elasticity of  $H_1$  with respect to  $(1-\tau)$ . This is the case, since an increase in the tax rate also raises the expected penalty rate,  $P\lambda\tau$ , on underground earnings.

This model also provides a useful "separation theorem" for the choice of hours of work in the underground sector. The point is that, under risk neutrality assumption, hours worked in the underground sector do not depend on preferences (see eq. 3). This result will be very important for the rest of the paper: it means that the only way preferences can affect the choice of hours in the underground sector is through the marginal tax rate which depends indirectly on the number of hours worked in the regular sector. In other words, the separation result provides a number of overidentification restrictions that can help to identify the effect of the marginal tax rate on underground labor supply, and that can be tested.

Until now, it was assumed that the individual was working in both sectors at the optimum. Under the assumption that the individual works in the regular sector and that, following Cogan (1980), he must entail a fixed cost of  $C$  dollars to participate in the underground sector, the condition

that determines whether hours in the underground sector are positive or not is<sup>14</sup>:

$$(4) \quad H_1^* \gtrless H_1^C$$

where  $H_1^*$  denotes the desired hours of work in the underground sector and  $H_1^C$  denotes the reservation hours in the underground sector.  $H_1^C$  is the solution to the following equation:

$$(5) \quad f(H_1^C) - C = f'(H_1^C)H_1^C$$

Equation (5) means that, at the reservation hours  $H_1^C$ , the net expected earnings  $[f(H_1^C) - C](1 - P\lambda\tau)$  in the underground sector are just equal to what an individual working  $H_0 = H_1^C$  at the net critical wage rate given by  $W_0^C(1 - \tau) = f'(H_1^C)(1 - P\lambda\tau)$  would earn in the regular sector. Figure 1 illustrates how  $H_1^C$  is determined.

Figure 1 also illustrates the impact of a change in the tax rate on hours of work in the underground and in the regular sector, conditional on participating in both sectors. The effect of an increase in the tax rate from  $\tau_0$  to  $\tau_1$  on the hours worked in the underground sector is unambiguously positive (from A to B). On the other hand, its effect on total hours of work  $H_T$  (C to D) is ambiguous in general, since it depends on substitution and income effects which work in opposite directions<sup>15</sup>.

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<sup>14</sup> When the individual does not work in the regular sector, the decisions on whether to participate and on how many hours to work in the underground sector both depend on preferences. As a consequence, the separation result does not hold. It is beyond the scope of this paper to estimate a model taking account of this possibility. See Lemieux (1989) for a model that endogenizes the participation decisions in both sectors.

<sup>15</sup> Under the assumption that leisure is a normal good.

#### 4. Estimation Strategy

##### 4.1 Stochastic specification

The last step before the model can be directly confronted with the data consists in building a full stochastic specification and making some specific functional form assumptions. Given the presence of corner solutions, it is natural to formulate the model using a latent variables framework. To simplify the derivations, we will first consider the case where the gross wage rate in the regular sector and the marginal tax rate are assumed to be exogenous. We will later show how these assumptions can be relaxed.

The first fundamental stochastic equation of the model specifies the latent earnings function of the individual  $i$  in the underground sector:

$$(6) \quad Y_{1i}^* = f_i(H_{1i}^*) = \text{EXP}(\theta \ln(H_{1i}^*) + \beta_0 + x_{1i}'\beta_1 + \epsilon_{y1})$$

where:  $Y_{1i}^*$  : potential (or latent) earnings in the underground sector.

$x_{1i}$  : exogenous variables affecting the productivity in the underground sector.

$\epsilon_{y1}$  : a random error term which reflects the effects of unobservable variables affecting the productivity in the underground sector.

Equation (6) is a simple Cobb-Douglas earnings function that is strictly concave provided that  $0 < \theta < 1$ .<sup>16</sup>

The equation for  $H_{1i}^*$  is obtained from the first-order condition (3). Using (6), it can be rewritten as :

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<sup>16</sup> The Cobb-Douglas specification can be interpreted as a (first-order) approximation of any well-behaved earnings function. With this functional form, the reduced form of our model is linear in its unrestricted parameters. This greatly simplifies the computation of our minimum distance estimator.

$$(7) \quad f'_i(H_{1i}^*) = \theta H_{1i}^{*\theta-1} \text{EXP}(\theta \ln(H_{1i}^*) + \beta_0 + x'_{1i} \beta_1 + \epsilon_{y1}) = \frac{W_{0i}(1-r_i)e^{\epsilon_{hi}}}{1-P\lambda r_i}$$

where  $\epsilon_{hi}$  is an optimization error. An alternative interpretation of  $\epsilon_{hi}$  is that it reflects unobservable hours rationing in the regular and the underground sectors.<sup>17</sup>

Furthermore, with our Cobb-Douglas specification, the reservation hours equation (5) becomes:

$$(8) \quad (1-\theta) \text{EXP}(\theta \ln(H_{1i}^C) + \beta_0 + x'_{1i} \beta_1 + \epsilon_{y1}) = C_i$$

Finally, it is assumed that the fixed costs of working in the underground sector (in log) is a linear function of a vector of explanatory variables  $x_{c1}$  and of an error term  $\epsilon_{c1}$ :

$$(9) \quad C_i = \text{EXP}(\delta_0 + x'_{c1} \delta_1 + \epsilon_{c1})$$

Substituting (9) into (8) and taking the logs of (6), (7), and (8) yield the structural form of our model:

$$(6)' \quad \ln(Y_{1i}^*) = \theta \ln(H_{1i}^*) + \beta_0 + x'_{1i} \beta_1 + \epsilon_{y1}$$

$$(7)' \quad \ln(H_{1i}^*) = \frac{-\ln(W_{0i}(1-r_i))}{1-\theta} + \frac{\ln(1-P\lambda r_i)}{1-\theta} + \frac{\ln \theta}{1-\theta} + \frac{\beta_0}{1-\theta} + \frac{x'_{1i} \beta_1}{1-\theta} + \frac{\epsilon_{y1} + \epsilon_{hi}}{1-\theta}$$

$$(8)' \quad \ln(H_{1i}^C) = \frac{\delta_0 - \beta_0}{\theta} - \frac{\ln(1-\theta)}{\theta} + \frac{x'_{c1} \delta_1}{\theta} - \frac{x'_{1i} \beta_1}{\theta} + \frac{\epsilon_{c1} - \epsilon_{y1}}{\theta}$$

$$\text{where } \left\{ \begin{array}{l} \epsilon_{y1} \\ \epsilon_{hi} \\ \epsilon_{c1} \end{array} \middle| \begin{array}{l} x_{c1}, x_{1i} \end{array} \right\} \text{ has mean zero and variance } \Sigma.$$

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<sup>17</sup> As long as the level of this rationing is not correlated with the independent variables, this will not affect the consistency property of the estimators.

Moreover, the relationship between the observed and the latent variables is given by:

$$(9) \begin{cases} Y_{1i} = Y_{1i}^* & \text{and } H_{1i} = H_{1i}^* & \text{if } \ln H_{1i}^* > \ln (H_{1i}^c) , \\ Y_{1i} = 0 & \text{and } H_{1i} = 0 & \text{otherwise} \end{cases}$$

One problem with (7)' is that we do not have any direct information on the expected penalty rate  $P\lambda$  on evaded taxes. However, consider the simple case where  $P\lambda$  is a constant (and is the same for everybody). Then  $P\lambda$  can be treated as a parameter to be estimated in equation (7)'. The only complication occurs because  $P\lambda$  enters in (7)' in a non-linear fashion, through the expression  $\ln(1-P\lambda\tau_i)$ . We have therefore decided to use the following first-order approximation of  $\ln(1-P\lambda\tau_i)$ :<sup>18</sup>

$$(10) \quad \ln(1-P\lambda\tau_i) \approx \alpha_0 + \alpha_1 \ln(1-\tau_i)$$

$$\text{where: } \alpha_0 = \ln(1-P\lambda\bar{\tau}) - \frac{P\lambda(1-\bar{\tau})}{1-P\lambda\bar{\tau}} \ln(1-\bar{\tau})$$

with  $\bar{\tau}$  denoting the sample mean of  $\tau_i$ , and

$$\alpha_1 = \frac{P\lambda(1-\bar{\tau})}{1-P\lambda\bar{\tau}} \quad , \quad \text{with } \alpha_1 \in [0,1]$$

Using (10), it follows that (7)' can be rewritten as:

$$(7)'' \quad \ln(H_{1i}^*) = \frac{-\ln(W_{0i})}{1-\theta} - \frac{(1-\alpha_1)\ln(1-\tau_i)}{1-\theta} + \frac{\ln\theta}{1-\theta} + \frac{\beta_0 + \alpha_0}{1-\theta} + \frac{x'_{1i}\beta_1}{1-\theta} + \frac{\varepsilon_{y_i} + \varepsilon_{h_i}}{1-\theta}$$

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<sup>18</sup> Equation (10) is obtained by redefining  $\ln(1-P\lambda\tau_i)$  as  $\ln(1-P\lambda(1-\exp(x_i)))$ , with  $x_i = \ln(1-\tau_i)$ , and taking the first-order Taylor series expansion of this function of  $x_i$  around  $x_0 = \ln(1-\bar{\tau})$ .

One fundamental econometric problem that our estimation method will try to address is the endogeneity of some of the regressors in equation (6)' and (7)". Concerning the underground earnings equation (6)', it should be clear that, as equation (7)" shows, underground hours will be correlated with the error term  $\varepsilon_{y_i}$ . That problem can be easily resolved by substituting (7)" into (6)', which yields<sup>19</sup>:

$$(6)'' \ln(Y_{1i}) = \frac{-\theta \ln(W_{0i})}{1-\theta} - \frac{\theta(1-\alpha_1)\ln(1-\tau_1)}{1-\theta} + \frac{\beta_0 + \theta\alpha_0}{1-\theta} + \frac{\theta \ln(\theta)}{1-\theta} + \frac{x'_{1i}\beta_1}{1-\theta} + \frac{\varepsilon_{y_i} + \theta\varepsilon_{h_i}}{1-\theta}$$

A second problem is that the regular wage rate might also be endogenous. To illustrate this point, consider the following Mincer-type specification for the regular wage:

$$(11) \ln(W_{0i}) = \gamma_0 + x'_{0i}\gamma_1 + \varepsilon_{w_i}$$

where  $x_{0i}$  is a standard set of human capital variables such as education, experience, etc.. The problem is that  $\varepsilon_{w_i}$  might very well be correlated with  $\varepsilon_{y_i}$ <sup>20</sup>. For example, if, after controlling for observable characteristics, the individuals who are more productive in the regular sector are also more productive in the underground sector, the OLS estimates of (6)" or (7)" will be inconsistent. The standard remedy for such problem is to construct an instrument for the regular wage. In the context of equations (6)", (7)" and (11), the structural parameters  $\alpha$ ,  $\theta$ , and  $\beta$  will be identified if some observable variables (instrumental variables) affect the regular wage without

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<sup>19</sup> Note that the  $\alpha$ s, the  $\beta$ s and  $\theta$  can still be identified in (6)" since  $x_{1i}$  is associated with the same vector of parameters  $\beta_1$  in both (6)' and (7)".

<sup>20</sup> Alternatively, (11) could be replaced by a hedonic wage equation in which the regular wage would depend on the number of hours worked in the regular sector (see Rosen 1976). Consistent estimates of  $\alpha$ ,  $\theta$  and  $\beta$  could still be obtained using I.V. methods. In other words, the model does not crucially rely on the linearity of the labor income function in the regular sector.



affecting the underground earnings function. In other words, it is necessary to have at least one variable in  $x_{0i}$  not included in  $x_{1i}$ <sup>21</sup>.

Several variables potentially satisfy that condition. First, the union status in the regular sector would be an ideal instrumental variable if union workers were not also more productive than their non-unionized counterparts. The point is that the union wage differential would then raise the regular sector wage without affecting the the underground earnings function, since it is uncorrelated with  $\epsilon_{yi}$ . The union status would thus fulfill the two conditions for being a good instrumental variable. The question of whether union workers are more productive than their nonunion counterparts is, however, still an unresolved issue in the literature (see Lewis (1986)).

Second, dummies for the industry in the regular sector could also be a good set of instrumental variables because of inter-industry wage differentials unexplained by differences in workers' productivity. In that sense, industry dummies would play a role very similar to the union status in the identification of the model.

One could argue, however, that the industry in which an individual works in the regular sector affects the number of hours he works in the underground sector because of the different constraints on hours worked for each industry in the regular markets. Under that interpretation, the underground sector acts as a labor market opportunity of the last resort for people who cannot work as much as they would like in the regular sector. We attempt to handle that problem by including in the underground hours equation a measure of "excess employment" in the regular sector industry estimated from aggregate time series employment data in the Province of Quebec<sup>22</sup>.

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<sup>21</sup> Note that the structural parameters could still be identified from the cross-equation restrictions between (6)" and (7)" even in the absence of exclusion restrictions.

<sup>22</sup> This excess employment variable is constructed by computing the industry employment for the regular job in excess of usual employment (employment predicted by estimating an AR1 process with quadratic trend).

One last variable that is potentially endogenous is the marginal tax rate  $-\ln(1-\tau_1)$ . It can be argued that this variable is much more likely to be exogenous in an underground than in a regular labor supply equation. The point is simply that, although the tax rate directly depends on labor earnings in the regular sector because of the non-proportionality of the tax and transfer system, it does not directly depend on labor earnings in the underground sector. On the other hand, the tax rate may still be endogenous in an underground hours equation to the extent that the hours of work in the two sectors are jointly determined in our model.

Given these considerations, we will present specifications which assume that the marginal tax rate is either exogenous or endogenous. In the empirical model, this variable takes into account the marginal tax rates associated with the payroll taxes and the federal and provincial income taxes, as well as the tax-back rates embodied in the social transfer system. In the case where the marginal tax rate is endogenous, it is instrumented by a "predicted" tax rate, based on a measure of the predicted gross income in the regular sector<sup>23</sup>.

Finally, the order condition for the coefficients associated with the fixed costs equation to be identified is that at least one exogenous regressor affects the number of hours worked in the underground sector but not the fixed costs<sup>24</sup>. In the empirical section, we will present results for fixed costs  $C$  and reservation underground hours under various assumptions concerning the choice of the regressor excluded from  $x_{C1}$ .

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<sup>23</sup> More precisely, we estimate a reduced form equation for regular labor income, using as all the exogenous variables of the model as explanatory variables, and we construct a measure of predicted labor income. The marginal tax rate is then computed at that predicted labor income. This "predicted" tax rate is not equal to the expectation of the tax rate conditional on the exogenous  $x$ 's, but it can be used as an instrument for the actual tax rate, provided that the  $x$ 's are stochastically independent from the error terms in (6) and (7)".

<sup>24</sup> In other words, some variables included in  $x_{11}$  are not included in  $x_{C1}$ . Alternatively, identification could be achieved by imposing some restrictions on the covariance structure of hours and participation.

## 4.2 Econometric Methods

Ideally, the generalized Tobit model given by (6)", (7)", (8)', (9), and (11) should be estimated using a full information maximum likelihood approach. However, given the complexity of such an approach, we relied on a method which provides less efficient though consistent estimates of the parameters. The minimum distance approach allows us to take into account the cross-equation restrictions imposed by the model on the reduced form equations for underground earnings and hours. Under this approach, the participation and hours decisions are not estimated simultaneously but using a two-step method generalizing Heckman's (1976) approach.

### The Minimum Distance Approach

In the first step, the decision on whether to participate or not in the underground sector is estimated from a discrete choice model based in the system of equations (7)", (8)', and (9). (Equation (11) is also considered when  $W_0$  is assumed to be endogenous). Assuming that the error terms are bivariate normally distributed, the appropriate discrete choice model is a probit model. A two-stage procedure (see Maddala 1983) is used in the specifications which assume that the regular wage and the tax rate are endogenous.

The estimates from the probit are used to construct the inverse Mill's ratio which allows us to take sample selectivity biases into account in the regular hours and earnings equations. However, the information contained in the participation equation is not sufficient to identify the structural parameters of the model. An appropriate generalization of the second step of the Heckman (1976) method is necessary to estimate the structural parameters of the model. For our particular application, we need an estimation procedure which enables us to impose all the cross-equation restrictions contained in the model. Estimating the structural parameters using the minimum distance (MD) method (Malinvaud (1980), Chamberlain (1982, 1984)) is

particularly convenient in this regard, because of the non-linear nature of the cross-equation restrictions involved in our model.<sup>25</sup>

Let us specify the restrictions on the parameters  $\pi$  of the reduced form by the conditions that  $\pi = f(\eta)$ , where  $\eta$  is the vector of parameters of the structural form. The MD estimator  $\hat{\eta}$  of the model minimizes:

$$[\hat{\pi} - f(\hat{\eta})]' V(\hat{\pi})^{-1} [\hat{\pi} - f(\hat{\eta})]$$

where  $\hat{\pi}$  denotes the vector of OLS (corrected for selectivity) estimates of the reduced form with an estimated covariance matrix given by  $V(\hat{\pi})$ .  $\hat{\eta}$  is consistent and  $\sqrt{N}(\hat{\eta} - \eta)$  is asymptotically normally distributed with mean zero and variance-covariance matrix  $(F'V(\pi)^{-1}F)^{-1}$ , where  $F = \partial f/\partial \eta$  and  $N$  is the number of observations. Moreover the minimand from optimal minimum distance is asymptotically distributed  $\chi^2(q)$ , where  $q$  is the number of restrictions imposed by the model. It is thus very simple to perform a specification (or goodness-of-fit) test of our model in this context.

The appropriate reduced form of the model, conditional on working in the underground sector, depends on whether the regular wage (and the tax rate) is assumed to be either exogenous or endogenous. In the former case, the reduced form simply consists of equations (6)" and (7)". In the latter case, the regular wage equation (11) has to be substituted into (6)" and (7)". The reduced form of the model thus becomes equation (11) and the modified versions of (6)" and (7)"<sup>26</sup>.

When the regular wage is endogenous, the sample used to estimate the model is unbalanced since  $W_{0i}$  is observed over the whole sample of people working in the regular sector, while  $Y_{1i}$  and  $H_{1i}$  are observed only for those

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<sup>25</sup> In our context, this method is equivalent to the least squares approach developed by Amemiya (1978, 1979).

<sup>26</sup> Note that there could be a fourth reduced form equation for the tax rate. The true equation for the tax rate is, however, a complicated non-linear equation (step-function) of regular labor income. We avoided those complications by simply instrumenting the tax rate with the "predicted" tax rate in the reduced form equations.

who work also in the underground sector. One advantage of the MD estimator is that it is straightforward to implement in the unbalanced sample we are using, since the reduced form coefficients are sufficient statistics for the structural parameters. The advantage of using all the observations on the regular wage is that the estimates of the structural parameters will be more efficient than if the regular wage equation was only estimated on the sample of people working in the underground sector.

It is also well known that the residuals in a regression that has been adjusted for selectivity are heteroskedastic. Once again, it is straightforward to incorporate that feature in a MD framework. Chamberlain (1982, 1984) has shown that the optimal minimum distance estimator is more efficient than three stage least-squares in the presence of heteroskedasticity.

## 5. Results

### Participation Decision

The results of the discrete decision of working in the irregular sector are shown in Table 3. We have shown in section 4 that the individual  $i$  will work in the irregular sector whenever  $\ln(H_{1i}^*)$  is larger than  $\ln(H_{1i}^C)$ . It is not possible, however, to identify all the structural parameters associated with the participation decision from the estimates of a simple probit model<sup>27</sup>. The estimates presented in Table 3 should thus be interpreted as reduced form parameters.

The first column of Table 3 presents estimates from a model in which both regular wage and marginal tax rate are assumed to be exogenous. The estimated coefficient associated with the gross regular wage ( $\ln(W_0)$ ) is negative (-0.59) and statistically significant while the coefficient associated with the marginal tax rate ( $-\log(1-r)$ ) is positive (0.21) and significant. The sign of the two coefficients is thus consistent with the

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<sup>27</sup> The probit coefficients are function of all the structural parameters included in equations (7)", (8)', and possibly (11). They can only be identified when an hours equation is estimated as well.

prediction of the model. Note, however, that the effect of the tax rate on the probability of participating is only a third as large as the effect of the regular wage. This result is still consistent with our model, provided that  $P\lambda$  is large enough (the results on column 1 of Table 1 imply an estimate of .759 for  $P\lambda$ ). Two other complementary explanations for the result are that there is some tax illusion, or that the tax rate is measured with error, perhaps in part because participation rate to the welfare programs is lower than 100%. As is well known, in the presence of measurement errors the coefficient of the tax rate will be asymptotically biased toward zero.

The effect of the remaining variables is generally consistent with the results presented in Table 1 with one notable exception: the age of the individual increases the probability of working in the irregular sector. Note also that the estimated coefficients are not affected very much by the inclusion of industry dummies in column 2.<sup>28</sup>

The simple probit estimates presented in column 1 and 2 are inconsistent when the regular wage and the tax rate are endogenous. In columns 3 and 4, we present the two-stage probit version of the simple probits of column 1 and 2 which are consistent even when the regular wage rate and the implicit marginal tax rate are endogenous. Excluded instrumental variables in column 3 are the union status, the "predicted" tax rate and the industry dummies. For the model presented in column 4, the industry dummies are directly included in the probit equation instead of being used as instruments<sup>29</sup>. The two-stage procedure invariably increases the magnitude of the estimated coefficients associated with the regular wage and the marginal tax rate. The estimated standard errors increase even more in percentage, so that the

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<sup>28</sup> To avoid perfect collinearity among the regressors, only I-2 industry dummies are included on the right hand side of the equation (I is the number of industries). This follows from the fact that a constant and the excess employment in the industry, which is a linear combination of the industry dummies, are already included. This also explains why the coefficients associated with the excess employment variable become much larger (in absolute terms) when the industry dummies are included on the right-hand side of the equation.

<sup>29</sup> In column 5, the union status is also used as an instrumental variable for the sake of comparison with the specification used in Table 4.

estimated coefficients are not statistically different from zero anymore. For similar reasons, Hausman-type specification tests do not reject the specifications of column 1 and 2, which suggests that the regular wage and the implicit marginal tax rate are exogenous<sup>30</sup>.

In the probit model, the standard error of the error term is always normalized to one. As a consequence, we cannot directly test the hypothesis that the concavity coefficient  $\theta$  belongs to the  $]0,1[$  interval, as the Cobb-Douglas specification would imply. Furthermore, the simple probit estimates and the two-stage probit estimates are not strictly comparable to each other since they are normalized by different factors<sup>31</sup>. Later in the section we will present some estimates that do not depend on these implicit normalizations by exploiting the cross-equation restrictions between the participation equation and the irregular hours (and earnings) equation.

#### Minimum Distance Estimates

In Table 4, we present estimates of the two central equations of our model: the irregular earnings equation (6)" and the irregular hours equation (7)". The MD technique enables us to estimate those two equations independently or jointly, when the regular wage (and the tax rate) are either endogenous or exogenous.

Column 1 presents single-equation estimates of the irregular earnings function in which the regular wage and the tax rate are not instrumented. In that particular case, MD is equivalent to estimating the irregular earnings equation using Heckman's two-step procedure. One problem with the results is that the estimated value of  $\theta$  (-.350) is inconsistent with the Cobb-Douglas

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<sup>30</sup> The value of the specification test statistics are  $.187 - \chi^2(2)$  and  $.574 - \chi^2(1)$  respectively.

<sup>31</sup> The problem is that the two-stage estimates are normalized by a factor that includes the variance of  $W - \hat{W}$ , where  $W$  is the variable that is being instrumented while  $\hat{W}$  is the instrument that is used (predicted value for  $W$  from the first-stage). This is hardly surprising since it is well known that applying the standard OLS formula to the second stage of standard two-stage least squares produces inconsistent estimates of the standard error of the estimated coefficients.

functional form for irregular earnings. Column 2 presents estimates of the same earnings equation when the regular wage and the tax rate are instrumented by the set of industry dummies and by the "predicted" tax rate. The estimated value of  $\theta$  is now positive and equal to .486. Furthermore, one cannot reject the null hypothesis that the model of column 1 is misspecified (Hausman test statistic of 7.79 -  $\chi^2(2)$ ).

Column 3 and 4 present similar specifications for the irregular hours equation. The results presented in column 3 are thus similar to the estimates of the irregular hours equation that we would obtain using Heckman's two-step procedure. Note that the estimated value of  $\theta$  is negative (-1.286) as it was the case in column 1. Column 4 shows that, once again,  $\theta$  becomes positive (.384) when the regular wage and the tax rate are instrumented.  $\theta$  is estimated very imprecisely, however, and the null hypothesis that the model of column 3 is misspecified can now be rejected (Hausman test statistic of 0.49 -  $\chi^2(2)$ ). Note also that for all the specifications considered, the estimated effect of the marginal tax rate has the wrong sign but is never statistically significant.

The three final columns of Table 4 show the results obtained when the irregular earnings and the irregular hours equations are estimated simultaneously, i.e. when the cross-equation restrictions implied by the model are imposed on those two equations. Column 5 presents the uninstrumented version of the model. The estimated value of  $\theta$  is negative (-.456) while the  $\chi^2$  goodness-of-fit statistic rejects the null hypothesis that the cross-equation restrictions embodied in equation (6)" and (7)" are valid (35.94 -  $\chi^2(10)$ ). Both of those results might simply reflect the fact that the estimated coefficients are inconsistent because the regular wage and the tax rate are endogenous. The evidence that follows supports that conjecture.

Column 6 presents estimates from a model in which industry dummies, the union status, and the "predicted" tax rate are used to instrument the regular wage and the implicit marginal tax rate. In column 7, the union status is included on the right hand side of the irregular earnings and hours equations instead. In both cases, the estimated value of  $\theta$  is increased while the goodness-of-fit test statistic is essentially equal to its critical value at



a 95% significance level ( $36.04 - \chi^2(24)=36.42$  and  $35.97 - \chi^2(23)=35.17$ ). Note that for the models of column 6 and 7, the goodness-of-fit statistic is a joint test of the validity of the overidentification restrictions (variables only included in the regular wage equation) and of the cross-equation restrictions between the irregular earnings and the irregular hours equation. One reason why the goodness-of-fit test does not tend to reject the model when the regular wage and the tax rate are instrumented is that the estimated value of  $\theta$  in column 2 and 4 (regular wage and tax rate instrumented) are much closer to each other than in column 1 and 3 (regular wage and tax rate not instrumented). More formally, the null hypothesis that the estimates of  $\theta$  and  $\beta$  implied by the irregular earnings equation are different from those implied by the irregular hours equation is tested using a Lagrange multiplier type test. For the model presented in column 7, the value of the LM test statistic is  $1.52 - \chi^2(10)$ , which suggests that the restrictions embodied in equation (6)" and (7)" are valid when the endogeneity of the regular wage and of the implicit tax rate is properly treated in the estimation procedure<sup>32</sup>.

This suggests that the goodness of fit statistic in column 7 (and 6) mostly reflect the large value of the overidentification test statistic for the irregular earnings equation (25.36). Finally, the null hypothesis that the regular wage and the tax rate are exogenous is strongly rejected (Hausman test statistic of  $64.39 - \chi^2(2)$ ). We conclude from the results presented in Table 4 that our model gives a relatively accurate representation of the covariance structure of irregular earnings, irregular hours, and of the regular wage. Furthermore, the number of hours worked in the irregular sector is very responsive to changes in the wage in the regular sector. Thus the estimates of the gross regular wage elasticity vary between -1.62 and -3.02 (see the penultimate row of Table 4), when only the results consistent with our Cobb-Douglas specification are retained.

One disappointing feature of the results is the absence of a systematic effect of the marginal tax rate on the number of hours worked in the irregular sector (last row of Table 4). We have already mentioned many

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<sup>32</sup> The test is not very powerful, however, since the instrumented version of the hours equation is very imprecisely estimated.

reasons why the tax rate should have a smaller effect on irregular hours than the gross regular wage. One further possibility is that people who face higher tax rates and thus higher expected penalties if they get caught, *ceteris paribus*, are more likely to underreport (in the questionnaire) the number of hours they work in the irregular sector<sup>33</sup>. On the other hand, they might still be willing to admit that they did some work in the irregular sector.

We thus conclude that it is still valuable to use the estimated effect of the marginal tax rate on the probability of working in the irregular sector to infer how irregular hours respond to changes in the implicit marginal tax rate. Two identifying assumptions are required to do this. First, the MD estimates of  $\beta$ , the vector of coefficients associated to the exogenous variables  $x_{11}$ , must be consistent. Second, as discussed in section 4, at least one exogenous variable included in  $x_{11}$  must not influence the fixed cost of working in the irregular sector. It is then possible to identify  $\theta$  and  $\alpha_1$  from the probit coefficients and the MD estimates of  $\beta$ <sup>34</sup>. From those estimates, it is easy to construct measures of the elasticity of irregular hours with respect to the regular wage and the marginal tax rate which are implied by the probit estimates of the participation decision.

#### Estimates of Elasticities Based on the Participation Decision

Table 5 presents estimates of the elasticity of irregular hours with respect to the regular wage that are implied by the probit estimates of the

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<sup>33</sup> Greenberg and Halsey (1983) have documented a similar phenomenon in the context of the SIME-DIME negative income tax experiments.

<sup>34</sup> In general, the estimated value of  $\theta$  and  $\alpha_1$  obtained with that procedure are different from the MD estimates of  $\theta$  and  $\alpha_1$ . Many consistent estimates of the same parameters can be computed because the system of participation, irregular earnings and irregular hours equations is overidentified.

participation decision.<sup>35</sup> Similar elasticities of irregular hours with respect to the marginal tax rate are also presented. The elasticities are computed under various assumptions regarding which variables should be excluded from the fixed costs function. Compared to the reduced form probit estimates presented in Table 3, one advantage of the structural estimates is that they are not sensitive to the different normalization used for the simple probit and the two-stage probit. One drawback of the structural estimates is that they have to rely on relatively arbitrary exclusion restrictions.

The estimates of the elasticity of irregular hours with respect to the regular wage presented in Table 5 are of the same order of magnitude as the estimates obtained with MD when the regular wage and the tax rate are instrumented. On the other hand, the elasticity of irregular hours with respect to the tax rate is now always positive. This reflects the fact that the tax rate always has a positive and significant effect on the probability of working in the irregular sector. Significant estimates (at the 10 percent level) of this elasticity range between 0.62 and 0.96, depending on the specification considered. It should be noted that the estimates are substantially larger when the regular wage and the implicit marginal tax rate are instrumented.

Table 6 provides estimates of fixed costs of working and of reservation hours in the irregular sector that are implied by the uninstrumented estimates of Table 3 and 4. Fixed costs are of the order of an average of several hundred dollars, while the average individual working in the irregular sector earns more than two thousand dollars from irregular labor market activities.

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<sup>35</sup> Call that estimate of theta  $\hat{\theta}_p$  while the estimate obtained from the irregular hours (or earnings) is  $\hat{\theta}_h$ . For simplicity, assume that  $x_{1i}$  is a single variable and that it does not affect fixed costs. Finally, ignore the tax rate for the moment. The irregular hours and the latent participation equations can be written as:

$$\ln(H_{1i}^*) = a_w \ln(W_{0i}) + a_x x_{1i} + \mu_{hi}$$

$$(\ln(H_{1i}^*) - \ln(H_{1i}^c)) / \sigma = b_w \ln(W_{0i}) + b_x x_{1i} + \mu_{pi} / \sigma$$

where:  $\sigma^2 = \text{Var}(\mu_{pi})$ ,  $a_w = -1/(1-\theta)$ ,  $a_x = \beta/(1-\theta)$ ,  $b_w = -1/\sigma(1-\theta)$ ,  $b_x = \beta/\sigma\theta(1-\theta)$ . It then follows that  $\hat{\theta}_h = (a_w + 1)/a_w$  and that  $\hat{\theta}_p = a_x b_w / (a_x b_w - b_x)$ .  $\alpha_1$  can be estimated in a similar way.

### Conclusion

In this paper, a model of labor supply in the underground economy was developed and estimated. The theoretical model relied on the empirical observation that earnings in the underground sector seems to be a concave function of hours worked in the underground sector. By contrast, evidence suggests that the gross wage rate in the regular sector is parametric. Those two features of the model were shown to generate a separation result between preferences and the number of hours worked in the underground sector. The discrete decision of working in the underground sector was also modelled by introducing some fixed money costs of participating in the underground economy.

The model was estimated over a micro data set collected in the region of Quebec City, Canada. Conditional on working in the underground sector, cross-equation restrictions on the reduced form parameters were imposed using a Minimum Distance procedure. The results obtained were generally consistent with the model. In particular, we find a negative and often significant effect of the gross wage in the regular sector on both the decision of participating and the number of hours worked in the underground sector. We also find that the marginal tax rate has a positive and often significant effect on the probability of working in the underground sector.

On the other hand, the estimates of the effect of the marginal tax rate on hours of work, as estimated from our minimum distance approach, have the wrong sign but are never significant. One possible explanation is that individuals who face higher marginal tax rates are more likely to underreport (in the questionnaire) their hours of work in the underground sector, since they are also likely to face higher penalty rate on their underground labor income. However, one can argue that they may still be ready to declare that they participated in the underground sector. Our results suggest that this is the case since the estimates of the tax rate elasticity of hours worked in the underground sector, as obtained from the participation equations, are positive. Significant estimates (at the 10 percent level) of the latter elasticity range between 0.62 and 0.96, depending on the specification considered. All-in-all, our results suggest that the marginal tax rates embodied in the Quebec tax-transfer system are an important determinant of the labor supply in the underground economy.

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TABLE 1: SAMPLE STATISTICS

Characteristics	Sample size	Percentage of total sample	<u>Work in the irregular sector</u>		
			Particip. <sup>1/</sup> rate	Annual <sup>2/</sup> hours	Annual <sup>2/</sup> earnings
<u>Total</u>	2107	100.0	8.7	360	2171
<u>Sex</u>					
Male	1026	48.7	10.0	338	2627
Female	1081	51.3	7.5	391	1564
<u>Age</u>					
18-24	323	15.3	23.8	349	2137
25-39	907	43.1	8.5	370	2055
40-59	708	33.6	4.0	378	2658
60+	169	8.0	1.2	120	500
<u>Marital Status</u>					
Married	1496	71.0	4.2	354	2296
Head of one-parent family	94	4.5	16.0	522	1872
Living with parents	268	12.7	20.9	310	2060
Single (or other)	249	11.8	20.1	382	2236
<u>Schooling completed</u>					
Less than high school	763	36.2	5.5	538	2790
High school	669	31.8	9.4	374	2442
College	320	15.2	15.6	236	1639
University	355	16.9	8.2	306	1664
<u>Labor market status</u>					
Student	241	11.4	28.6	332	1976
Retired	107	5.1	1.9	120	500
Housekeeper	370	17.6	6.8	581	2216
Unemployed	84	4.0	28.6	310	2459
Worker	1305	61.9	4.9	369	2068
<u>Social assistance claimant</u>	103	4.9	35.0	468	3491
<u>Regular income</u>					
0-10000	842	40.0	14.7	428	2274
10000-20000	425	20.2	7.3	236	1975
20000-30000	339	16.1	4.4	282	2167
30000-40000	222	10.5	3.2	98	1729
40000+	279	13.2	2.5	94	1575



TABLE 1 (continuation)

<u>Industry on the regular job</u>					
Primary	29	2.0	17.2	572	2706
Manufacturing	118	8.2	7.6	262	2351
Construction	65	4.5	10.8	332	6311
Transportation	103	7.2	6.8	44	350
Trade	207	14.4	8.2	243	1308
Finance, insurance	152	10.6	2.0	217	2462
Services	454	31.5	8.6	249	1370
Public administ.	312	21.7	5.5	154	1674
<u>Industry on the irregular job</u>					
Unclassified	--	--	1.9	250	1414
Construction	--	--	2.8	337	2147
Transportation	--	--	0.2	270	1343
Trade	--	--	0.5	289	1494
Finance	--	--	0.6	403	3065
Services	--	--	2.7	477	2748

---

Notes: 1/ Everybody reporting an irregular job.  
 2/ Only people with non-missing irregular hours and earnings.

TABLE 2: COVARIANCE MATRIX OF HOURS, EARNINGS, AND WAGE.<sup>1</sup> (in logs)

		regular			underground		
		hours	wage	earnings	hours	wage	earnings
r e g u l a r	hours	0.432 (0.340)					
	wage	0.122 (0.026)	0.350 (0.178)				
	earnings	0.554 (0.366)	0.472 (0.204)	1.026 (0.571)			
u n d e r g r o u n d	hours	-0.110 (-0.131)	0.233 (0.114)	0.123 (-0.017)	1.224 (1.150)		
	wage	-0.107 (-0.094)	0.177 (0.110)	0.070 (0.016)	-0.321 (-0.312)	0.601 (0.484)	
	earnings	-0.217 (-0.225)	0.410 (0.224)	0.193 (-0.001)	0.902 (0.838)	0.280 (0.172)	1.182 (1.010)

<sup>1</sup> Partial covariances, i.e. covariances controlling for sex, education, experience and experience squared, are in parentheses. The subsample used to compute the covariances among the variables related to the regular (resp. irregular) sector includes all the persons who worked in the regular (resp. irregular) sector. In the case of the covariances among the variables of both the regular and the irregular sector, the subsample retained includes all the persons who worked in both sectors.

TABLE 3: PARTICIPATION EQUATIONS

Estimation Method:	(1)	(2)	(3)	(4)	(5)
	Probit	Probit	Two-Stage <sup>1</sup> Probit	Two-Stage <sup>2</sup> Probit	Two-Stage <sup>3</sup> Probit
Age	0.125 (0.063)	0.133 (0.064)	0.130 (0.075)	0.167 (0.085)	0.129 (0.063)
Age Squared (/100)	-0.179 (0.091)	-0.192 (0.091)	-0.187 (0.102)	-0.231 (0.112)	-0.185 (0.089)
Sex Dummy (1=women)	-0.481 (0.129)	-0.451 (0.132)	-0.501 (0.162)	-0.555 (0.193)	-0.497 (0.132)
Experience in Regular Mkt	-0.024 (0.029)	-0.017 (0.030)	-0.028 (0.035)	-0.004 (0.042)	-0.029 (0.028)
Exp. Squared (/100)	0.048 (0.087)	0.034 (0.088)	0.068 (0.091)	0.032 (0.096)	0.068 (0.079)
Education	0.044 (0.023)	0.051 (0.024)	0.050 (0.042)	0.091 (0.065)	0.048 (0.039)
Gross Regular Wage	-0.593 (0.140)	-0.617 (0.148)	-0.659 (0.476)	-1.212 (0.897)	-0.633 (0.485)
Marg. Tax Rate	0.212 (0.090)	0.210 (0.090)	0.332 (0.304)	0.364 (0.315)	0.331 (0.185)
Marital Status (1=married)	-0.559 (0.142)	-0.618 (0.146)	-0.535 (0.182)	-0.502 (0.215)	-0.539 (0.141)
Excess Empl. in Industry	-3.705 (3.407)	-489.557 (329.228)	-3.290 (3.843)	-491.480 (387.169)	-3.241 (2.934)
Industry Dummies:	No	Yes	No	Yes	
Log-Likelihood:	-292.6	-283.7	-302.8	-293.7	-302.8
Observations:	1390	1390	1390	1390	1390

(Consistent estimates of standard errors are in parentheses).

<sup>1</sup> Excluded instrumental variables: union status, industry dummies and "predicted" marginal tax rate. Standard errors adjusted for the two-stage procedure.

<sup>2</sup> Excluded instrumental variables: union status and "predicted" implicit marginal tax rate. Standard errors adjusted for the two-stage procedure.

<sup>3</sup> Union status included among the regressors.

TABLE 4: MINIMUM DISTANCE ESTIMATION<sup>1</sup>

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Equations							
Fitted:	$Y_1$	$Y_1, W_0$	$H_1$	$H_1, W_0$	$Y_1, H_1$	$Y_1, H_1, W_0$	$Y_1, H_1, W_0$
Excluded							
Instruments:	---	$\tau, D$	---	$\tau, D$	---	$\tau, D, U$	$\tau, D$
Curvature	-0.350	0.486	-1.286	0.384	-0.456	0.655	0.670
Parameter( $\theta$ )	(0.394)	(0.281)	(1.482)	(1.536)	(0.419)	(0.088)	(0.103)
$\alpha_1$	0.707	1.553	1.208	1.226	1.042	1.203	1.185
	(0.463)	(0.681)	(0.276)	(0.599)	(0.062)	(0.120)	(0.135)
Irreg. Mkt							
Productivity ( $\beta$ )							
Age	0.018	-0.002	-0.117	-0.040	0.122	0.040	0.042
	(0.173)	(0.079)	(0.327)	(0.374)	(0.171)	(0.044)	(0.043)
Sex Dummy	-0.444	-0.132	0.260	0.059	-0.884	-0.233	-0.234
(female=1)	(0.423)	(0.145)	(0.863)	(0.902)	(0.436)	(0.081)	(0.078)
Experience	0.090	0.065	0.200	0.092	0.063	0.047	0.046
in Reg. Mkt	(0.080)	(0.033)	(0.172)	(0.201)	(0.079)	(0.018)	(0.018)
Education	-0.153	0.011	-0.091	0.042	-0.096	0.025	0.027
	(0.087)	(0.041)	(0.158)	(0.122)	(0.078)	(0.021)	(0.021)
Union	-0.343	-0.074	-0.382	0.058	-0.748	---	0.023
Status	(0.396)	(0.189)	(0.847)	(0.609)	(0.476)		(0.085)
Marital Stat.	-0.459	-0.287	0.111	0.290	-0.995	-0.199	-0.190
(married=1)	(0.506)	(0.232)	(1.021)	(0.967)	(0.588)	(0.113)	(0.115)
Excess Empl.	-15.222	-8.353	-17.027	-4.975	-22.420	-5.397	-5.306
in Industry	(9.246)	(4.330)	(17.339)	(15.990)	(9.877)	(1.962)	(1.934)
Regular Wage ( $\gamma$ )							
Sex Dummy	---	-0.141	---	-0.145	---	-0.147	-0.146
(female=1)		(0.022)		(0.022)		(0.022)	(0.022)
Experience	---	0.030	---	0.030	---	0.030	0.030
in Reg. Mkt		(0.005)		(0.005)		(0.005)	(0.005)
Education	---	0.071	---	0.071	---	0.071	0.071
		(0.004)		(0.004)		(0.004)	(0.004)
Union	---	0.140	---	0.143	---	0.148	0.136
Status		(0.023)		(0.023)		(0.028)	(0.023)
Goodn.-of-Fit:	0	25.39	0	2.03	35.94	36.02	35.95
D. of Freedom:	(0)	(6)	(0)	(6)	(10)	(24)	(23)

<sup>1</sup> Other regressors included in  $\beta$  and  $\gamma$  are: constant, age squared, experience squared. Age, marriage and industry dummies (D) are also included in  $\gamma$ . Other instruments used are the union status (U) and  $\tau = -\ln(1-\hat{\tau})$  where  $\hat{\tau}$  is the "predicted" marginal tax rate. Consistent estimates of standard errors are in parentheses.

TABLE 4 (continuation)

		Elasticity of Hours in the Irregular Sector						
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
Equations Fitted:		$Y_1$	$Y_1, W_0$	$H_1$	$H_1, W_0$	$Y_1, H_1$	$Y_1, H_1, W_0$	$Y_1, H_1, W_0$
Excluded Instruments:		---	$\tau, D$	---	$\tau, D$	---	$\tau, D, U$	$\tau, D$
Gross Reg. Wage Elasticity		-0.741 (0.216)	-1.947 (1.065)	-0.437 (0.284)	-1.624 (4.049)	-0.687 (0.198)	-2.897 (0.737)	-3.029 (0.948)
Tax Rate Elasticity:		0.217 (0.383)	-1.076 (0.792)	-0.091 (0.115)	-0.367 (0.432)	-0.029 (0.044)	-0.589 (0.298)	-0.560 (0.319)

**TABLE 5**  
**ELASTICITIES BASED ON THE ESTIMATES OF THE PARTICIPATION DECISION\***

Variable excluded from fixed costs	Regular Wage and Taxes not Instrumented <sup>1</sup>			Regular Wage and Taxes Instrumented <sup>2</sup>		
	Theta	Wage Elast.	Tax Elast.	Theta	Wage Elast.	Tax Elast.
	(5)	(6)	(7)	(8)	(9)	(10)
Marriage Dummy	0.421	1.727 (0.734)	0.617 (0.248)	0.404	1.678 (0.244)	0.845 (0.551)
Excess Employment	0.712	3.472 (20.629)	1.241 (2.174)	0.759	4.149 (4.762)	2.090 (2.434)
Sex Dummy	0.429	1.751 (0.646)	0.626 (0.222)	0.475	1.905 (0.258)	0.960 (0.560)

\* Consistent standard errors are in parentheses.

**TABLE 6**  
**FIXED COSTS AND CRITICAL HOURS AT THE  
MEAN OF THE SAMPLE RETAINED<sup>3</sup>**

Variable excluded from fixed costs	Whole Sample		Irregular Sector	
	Fixed Costs	Crit. Hours	Fixed Costs	Crit. Hours
	(1)	(2)	(3)	(4)
Marriage Dummy	876.4	1171.1	674.7	494.5
Excess Employment	789.5	3583.9	213.5	532.1
Sex Dummy	761.4	1216.3	545.0	495.8

<sup>1</sup> Based on calculations from Table 3, column 1, and Table 4, column 5.

<sup>2</sup> Based on calculations from Table 3, column 3, and Table 4, column 7.

<sup>3</sup> Based on calculations from Table 3, column 1, and Table 4, column 5.

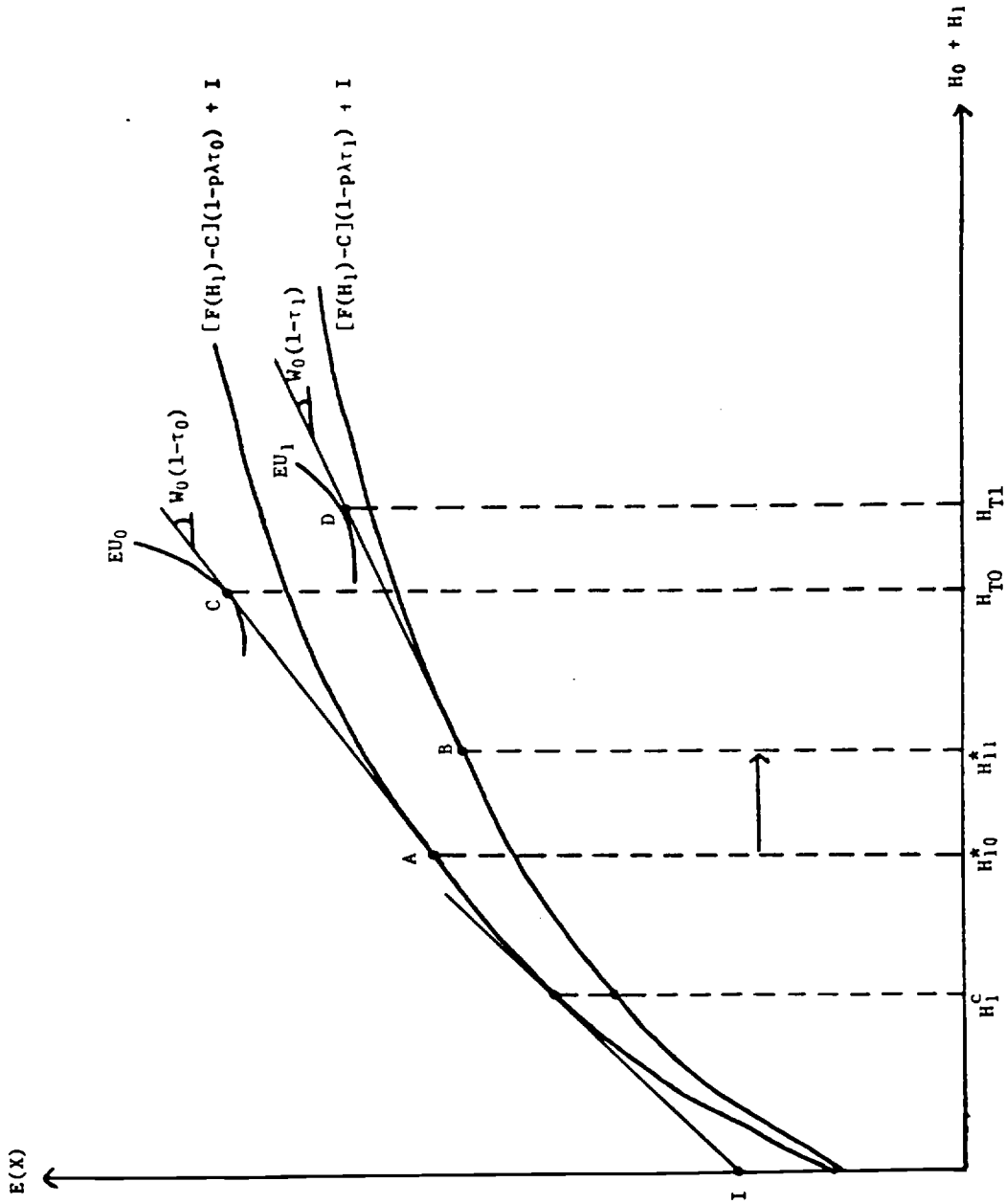


Figure 1: Impact of an increase in the marginal tax rate on hours worked in the underground sector and on total hours worked.