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PASS-THROUGH OF EXCHANGE RATES AND PURCHASING POWER PARITY

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ABSTRACT

In this paper we develop and test two hypotheses about purchasing power parity (PPP) derived from the pricing behavior of profit-maximizing, exporting firms. The first is that changes in the price of traded goods relative to domestic substitutes, due to partial pass-through of exchange rates, will affect the PPP relation. The second is that PPP should hold on forward rather than spot exchange rates, due to hedging by firms. Using quarterly data for the United States, Canada, France, Germany, Japan and the United Kingdom, we find considerable support for the first but not the second hypothesis.

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1. Introduction

In this paper we develop and test a model of purchasing power parity (PPP) derived from the pricing behavior of profit-maximizing, exporting firms. It is well known that exporting firms facing a downward sloping demand curve will likely adjust their prices by less than the full change in the exchange rate. For example, as their currency appreciates, firms may lower their profit margins to absorb part of the exchange rate change, thereby passing through only part of the appreciation to the importer's price. This change in the price of traded goods relative to domestic substitutes, due to pass-through behavior, should be taken into account when measuring the parity between prices in the exporting and importing countries. This is the first hypothesis that we shall investigate.

The second hypothesis is that parity should hold between the prices in trading partners and their *forward rates* of foreign exchange, rather than their spot rates. From covered interest parity, the difference between the spot and forward rates equals the interest rate differential, so this second hypothesis implies that PPP equations of spot rates should include the interest rate differential as an explanatory variable. We will find considerable support for our first hypothesis in explaining deviations from PPP, but little support for the second hypothesis. One reason for this is that the interest rate differentials between most countries are stationary, or nearly so, so they cannot explain the nonstationary deviations from PPP. A significant portion of these deviations are, however, explained by the price of traded goods relative to domestic substitutes.

There is ample precedent in the literature for both the hypotheses that we test. The idea that partial pass-through of exchange rates may affect PPP is considered by Froot and Rogoff (1994), though they devote greater attention to a more common hypothesis: that deviations from PPP will arise due to the inclusion of nontraded goods in the wholesale or consumer price indexes. The implication of this hypothesis seems to be that we should correct the aggregate price indexes, possibly by including the relative price of traded goods as another variable in the PPP relation. Thus, the correction implied by the mismeasurement of the indexes (due to nontraded goods) is quite similar to the correction we propose to account for pass-through behavior, and in this sense the two hypotheses are similar. Nevertheless, we will argue that there are some subtle differences in the exact manner that these two hypotheses should be tested (see section 3.2).

The idea that the forward rate determines the price and/or output for exporters is also not new, and is an example of the "separation theorem" discussed by Ethier (1973), Baron (1976a) and Eldor and Zilcha (19897). We will derive this result from a model of a risk-averse, exporting firm, that must set the prices for its products before the exchange rate is known. The firm may set prices in either its own currency, or the currency of the importing country, and will optimally engage in transactions in the forward market. In either case, we show in section 2 that the optimal price for the firm is determined by the forward rate, even if only partial covering is optimal.

The optimal pricing relation for the firm can be estimated as a pass-through equation between forward rates and product prices, or alternatively, inverted to obtain a PPP relation between the product prices and the forward rate, as described in section 3. In section 4 we estimate the latter as a cointegrating relation, using quarterly data for the United States, Canada, France, Germany, Japan and the United Kingdom over 1974.1-1992.1. Applying the method of Johansen (1991), we find strong evidence of *multiple* cointegrating relations, so that not all variables need be included to obtain a stationary relation with the exchange rate. We find that the cointegrating vectors that include the relative traded goods price have significantly lower residuals, or smaller deviations from PPP, than the relations that only use the wholesale prices. In contrast, the interest rate differentials explain very little of the PPP deviations in most cases. Additional conclusions are given in section 5.

2. Theory

Numerous authors have examined the question of the currency in which exporting firms set their prices. Studies which have examined the optimal choice of invoicing strategy include those by Baron (1976b) and Giovannini (1988), with the result that the optimal choice is very sensitive to properties of the demand function. We shall consider invoicing in either the currency of the exporter or the importing country.

2.1. Invoicing in the Importing Country's Currency

In this section we suppose that the firm sets its price in the importing country's currency. The model we shall use is similar to Feenstra (1989), except that the firm is assumed to be risk averse. Buying one unit of importer's currency requires s_t units of the exporter's currency on the spot market, where s_t is stochastic. Since the exporter must set its price p_t in period t-1 (before this spot rate is known) the revenues received in its own currency are uncertain. This uncertainty can be covered by selling in period t-1 the amount y_t of the importer's currency on the forward market, at the price of ${}_{t-1}f_t$. The firm will experience a profit (or loss) on these forward contracts of $y_t({}_{t-1}f_t - s_t)$, which will offset the "translation exposure" from converting sales revenue to its own currency. The demand for imports is given by $x_t=x(p_t,q_t,I_t)$, where q_t is the (scalar) price of domestic import-competing goods, and I_t is consumer income or expenditure.[1] We assume that the firm is engaged in Bertrand competition with other firms, so it treats q_t as exogenous.

The exporter maximises expected utility of profits in its own currency:

$$\max_{p_t, y_t} \; E_{t-1}\{U[(s_t p_t - c^*_t)x(p_t,q_t,I_t) + y_t({}_{t-1}f_t - s_t)]\}, \tag{1}$$

where E_{t-1} denotes expected value using information available in period t-1; U is the firm's utility function; and c^*_t denotes marginal and average costs in the foreign currency. We will treat both costs and consumer income as stochastic, but independent of each other and of the spot rate. The firm will be forecasting the period t values of these variables using information available in t-1. The price q_t is chosen by competing firms in period t-1, using an analogous maximization problem to (1). This price will be fully determined by information available in period t-1, so we will treat it as nonstochastic in (1).

1 Note that the prices of domestic goods q_t could be a vector, but for convenience we shall treat it as a scalar aggregate. In addition, we assume that this competing good is an imperfect substitute for the product of the exporting firm.

Let $\pi^*_t = (s_t p_t - c^*_t)x(p_t, q_t, I_t) + y_t({}_{t-1}f_t - s_t)$ denote profits in the exporting country's currency, inclusive of the gain or loss from the forward transaction. Then $U(\pi^*_t)$ may be approximated by a second order Taylor expansion about expected profits as:

$$U(\pi^*_t) = U(E_{t-1}\pi^*_t) + U'(E_{t-1}\pi^*_t)(\pi^*_t - E_{t-1}\pi^*_t)$$
$$+ \frac{1}{2} U''(E_{t-1}\pi^*_t)(\pi^*_t - E_{t-1}\pi^*_t)^2. \tag{2}$$

Letting $e_t = E_{t-1}(s_t)$ denote the expected exchange rate, substituting (2) into equation (1) and using $E_{t-1}(\pi^*_t - E_{t-1}\pi^*_t) = 0$, we obtain:

$$\max_{p_t, y_t} U(E_{t-1}\pi^*_t) + \frac{1}{2} U''(E_{t-1}\pi^*_t)\text{var}(\pi^*_t)\ , \tag{3}$$

where, $$E_{t-1}\pi^*_t = (e_t p_t - E_{t-1}c^*_t)E_{t-1}x(p_t, q_t, I_t) + y_t({}_{t-1}f_t - e_t) \tag{4a}$$

and, $$\text{var}(\pi^*_t) = \sigma_s^2(p_t E_{t-1}x_t - y_t)^2. \tag{4b}$$

Assuming that $U''(E_{t-1}\pi^*_t)$ is constant[2] and taking the first-order condition of (3) with respect to y_t results in:

$$y_t = \frac{({}_{t-1}f_t - e_t)}{R_u \sigma_s^2} + p_t E_{t-1}x(p_t, q_t, I_t)\ , \tag{5}$$

where $R_u \equiv -[U''(E_{t-1}\pi^*_t)/U'(E_{t-1}\pi^*_t)]$ is the Arrow-Pratt measure of absolute risk-aversion. From (5), the optimal forward contract is decomposed into two terms: the first is a "speculative" purchase (or sale) that reflects the difference between the forward and expected future spot rate; while the second term is the sales revenue that the firm needs to convert to its own currency. If ${}_{t-1}f_t < e_t$, indicating that the exporter expects an appreciation of the importer's currency relative to the current forward rate, then the optimal speculative position is to buy forward contracts, so in that case the firm will not sell forward enough of the importer's currency to convert its total sales

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[2] Thus, our analysis is exact for a quadratic utility function U.

revenue. The speculative purchase or sale is also affected by the firm's attitude towards risk, as indicated by R_u. The relation between the forward and expected future spot rate is determined in general equilibrium (as in Hodrick, 1989, for example), and is related to the risk premium in the foreign exchange market. We will simply accept these rates as exogenous to the firm.

Before determining the optimal price p_t, it is useful to substitute (5) back into (4b) and rewrite the variance of profits as:

$$\mathrm{var}(\pi_t^*) = \frac{({}_{t-1}f_t - e_t)^2}{R_u^2 \sigma_s^2} . \tag{6}$$

This expression indicates that the uncertainty in profits is related solely to the speculative purchase or sale of forward contracts. Substituting (6) and the optimal choice for y_t in (5) into (3), the objective function can be rewritten as:

$$\max_{p_t} U\left((e_t p_t - E_{t-1} c_t^*) E_{t-1} x_t + ({}_{t-1}f_t - e_t) E_{t-1} x_t + \frac{({}_{t-1}f_t - e_t)^2}{R_u \sigma_s^2} \right) + \frac{1}{2} U''(E\pi_t^*) \frac{({}_{t-1}f_t - e_t)^2}{R_u^2 \sigma_s^2} . \tag{7}$$

Treating $U''(E_{t-1}\pi_t^*)$ as fixed, the coefficient of absolute risk-aversion R_u will still vary with changes in p_t. However, working out the algebra shows that the derivatives of R_u with respect to p_t - in the two places where it appears in (7) - cancel each other out. Thus, the only terms that matter are the first terms that appear in (7), which are simplified as:

$$(e_t p_t - E_{t-1} c_t^*) E_{t-1} x_t + ({}_{t-1}f_t - e_t) p_t E_{t-1} x_t = ({}_{t-1}f_t p_t - E_{t-1} c_t^*) E_{t-1} x_t .$$

In other words, the firm will seek to maximize profits evaluated at the hypothetical own-currency price obtained at the forward exchange rate.

Letting $-\eta_t \equiv \partial\ln(E_{t-1}x_t)/\partial\ln p_t$ denote the elasticity of demand, the first-order condition for (7) is simply:

$$p_t\left(1 - \frac{1}{\eta_t}\right) = \left(\frac{E_{t-1}c^*_t}{{}_{t-1}f_t}\right) \tag{8}$$

This is a conventional monopolistic pricing formula, with the exporter's marginal costs $E_{t-1}c^*_t$ converted to the importing country's currency using the *forward rate* ${}_{t-1}f_t$. Thus, even when the revenue received from export sales is only partially covered by forward contracts, it is the forward rate that determines the optimal price. This is an illustration of the "separation theorem" discussed by Ethier (1973), Baron (1976a) and Eldor and Zilcha (1987). We next examine whether this same result holds with invoicing in the exporter's own currency.

2.2. Invoicing in the Exporting Country's Currency

The maximization problem confronting an exporting firm which sets price in its own currency is similar to that above, except that now profit is maximized by choosing p^*_t and y_t:

$$\max_{p^*_t, y_t} U(E_{t-1}\pi^*_t) + \frac{1}{2}U''(E_{t-1}\pi^*_t)\mathrm{var}(\pi^*_t), \tag{9}$$

where $\pi^*_t = (p^*_t - c^*_t)x(p^*_t/s_t, q_t, I_t) + y_t({}_{t-1}f_t - s_t)$ again denotes profits in the exporting country's currency, and p^*_t/s_t is the random price in the importing country. As before, the firm sets price before the exchange rate is known but, unlike the case where the exporter sets price in the domestic currency, revenues are uncertain due to random fluctuations in import price and demand. This means that the terms $E_{t-1}\pi^*_t$ and $\mathrm{var}(\pi^*_t)$ take on the form:

$$E_{t-1}\pi^*_t = (p^*_t - E_{t-1}c^*_t)E_{t-1}x_t + y_t({}_{t-1}f_t - e_t), \tag{10a}$$

$$\mathrm{var}(\pi^*_t) = (p^*_t - E_{t-1}c^*_t)\mathrm{var}(x_t) + y_t^2\sigma_s^2 - 2y_t(p^*_t - E_{t-1}c^*_t)\mathrm{cov}(x_t,s_t). \tag{10b}$$

Treating $U''(E_{t-1}\pi^*_t)$ as constant, the first-order condition for (9) with respect to y_t is:

$$y_t = \frac{({}_{t-1}f_t - e_t)}{R_u\sigma_s^2} + (p^*_t - E_{t-1}c^*_t)\left[\frac{\mathrm{cov}(x_t, s_t)}{\sigma_s^2}\right]. \tag{11}$$

Thus, the desirability of forward covering depends on both the relation between the forward and expected future spot rate - which is the same speculative effect obtained in (5) - and on the covariance between the future spot rate and product demand. The latter term enters because changes in the exchange rate will affect the product price in the importer's currency, and therefore product demand, and the exporter will want to hedge against this "operating exposure." A depreciation of the importer's currency lowers the spot rate s_t, which raises the price $p_t = p^*_t/s_t$, and and lowers expected demand and profits. It follows that $\mathrm{cov}(x_t, s_t) > 0$, so the firm will hedge by *selling* forward contracts in the importer's currency. Then a depreciation of that currency results in greater profits earned on the forward contract, which offset the loss in profits on its reduced sales.

Substituting (11) back into (10b), we can rewrite the variance of profits as:

$$\mathrm{var}(\pi^*_t) = \frac{({}_{t-1}f_t - e_t)^2}{R_u^2\sigma_s^2} + \frac{(p^*_t - E_{t-1}c^*_t)^2}{\sigma_s^2}|V_t|, \tag{12a}$$

where,

$$V_t = \begin{bmatrix} \sigma_s^2 & \mathrm{cov}(x_t, s_t) \\ \mathrm{cov}(x_t, s_t) & \mathrm{var}(x_t) \end{bmatrix}. \tag{12b}$$

The first term on the right of (12a) reflects the uncertainly in profits due to the speculative purchase or sale of forward contracts. The second term depends on $|V_t|$, which reflects the correlation between the changes in the spot rate and product demand, since:

$$|V| = \sigma_s^2\mathrm{var}(x_t)\left(1 - \frac{\mathrm{cov}(x_t, s_t)^2}{\sigma_s^2\mathrm{var}(x_t)}\right).$$

The magnitude of this term depends on the functional form of demand, as the following example makes clear.

Let the demand function be given by:

$$x(p_t, q_t, I_t) = \left(\frac{\alpha}{p_t} - \frac{\beta}{q_t}\right) I_t, \quad \alpha, \beta > 0 \ . \tag{13}$$

This functional form, while not familiar, has very conventional properties:

(a) Decreasing function in own price, $x_p < 0$;

(b) Increasing in the price of the domestic import-competing good, $x_q > 0$;

(c) If the price of imported good is sufficiently higher than domestic good, then demand for imported good will be zero: $x_t > 0$ only for $p_t < q_t(\alpha/\beta)$.

Substituting $p_t = p^*_t/s_t$ into (13), and keeping p^*_t fixed, it is immediate that changes in demand are *perfectly correlated* with changes in the spot rate s_t. In this case, the firm can entirely eliminate the uncertainty in its profits by selling foward contracts in the importer's currency. Formally, it is readily verified that for the demand function (13), $|V_t|=0$.

More generally, we would expect that for other functional forms of demand, the firm would still be able to eliminate the uncertainty in its profits arising from fluctuating price and demand if it had available a complete set of put and call options on the foreign currency. Then for any possible change in the spot rate, the exporter could calculate the corresponding change in expected demand and profits, and make the appropriate forward sale to offset the fluctuation in profits. In that case, the remaining variation in profits would consist of only the first term in (12a), reflecting the speculative holding of forward contracts. Thus, while we will focus on the special case of the demand function in (13), we expect that similar results would hold for more general demand functions, with a complete set of exchange rate options.

Using $|V_t|=0$ in (12a), computing $\text{cov}(x_t, s_t)$ from (13) and using this in (11) and (10a), the objective function (7) can be rewritten as:

$$\max_{p_t^*} U\left((p_t^*-E_{t-1}c_t^*)\left(E_{t-1}x_t + ({}_{t-1}f_t-e_t)\frac{\alpha I_{t-1}}{p_t^*}\right) + \frac{({}_{t-1}f_t-e_t)^2}{R_u\sigma_s^2}\right)$$

$$+ \frac{1}{2} U''(E_{t-1}\pi_t^*)\frac{({}_{t-1}f_t-e_t)^2}{R_u^2\sigma_s^2}. \qquad (14)$$

Again, the coefficient of absolute risk-aversion R_u will vary with changes in p_t^*, but the derivatives of R_u with respect to p_t^* in the two places where it appears in (14) cancel each other out. Thus, the only terms that matter are the first terms that appear in (14), which are simplified using (13) as:

$$(p_t^*-E_{t-1}c_t^*)[E_{t-1}x_t + ({}_{t-1}f_t-e_t)\alpha(I_{t-1}/p_t^*)]$$

$$= (p_t^*-E_{t-1}c_t^*)[\alpha({}_{t-1}f_t/p_t^*) - (\beta/q_t)]E_{t-1}I_t = (p_t^*-E_{t-1}c_t^*)[E_{t-1}x(p_t^*/{}_{t-1}f_t,q_t,I_t)].$$

In other words, the firm will seek to maximize profits evaluated at the hypothetical import price obtained at the forward exchange rate.

Letting $\eta_t^* \equiv -\partial\ln[E_{t-1}x(p_t^*/{}_{t-1}f_t,q_t,I_t)]/\partial\ln(p_t^*)$ denote the elasticity evaluated at this forward rate, the first-order condition for p* is simply:

$$p_t^*\left(1 - \frac{1}{\eta_t^*}\right) = E_{t-1}c_t^*. \qquad (15)$$

Notice the similarity with equation (8) for the exporter invoicing in the domestic currency. Again, a conventional monopolistic pricing formula is obtained, but with the forward exchange rate in the elasticity to compute a hypothetical price in the importer's currency. This foward rate is used despite the fact that the firm may be only partially covering its operating exposure.

3. Empirical Model

3.1 Functional Form

In order to convert (8) and (15) into equations that can be estimated, it is very convenient to again use the demand function (13). This function implies a log-linear relationship between the chosen price and its determining variables. For the case where the exporter invoices in the importer's currency and its own currency, respectively, we obtain:

$$\ln p_t = \gamma_0 + \gamma_1(\ln E_{t-1}c_t^* - \ln_{t-1}f_t) + (1-\gamma_1)\ln q_t\,, \tag{8'}$$

and,

$$\ln p_t^* = \gamma_0 + \gamma_1 E_{t-1}\ln c_t^* + (1-\gamma_1)(\ln q_t + \ln_{t-1}f_t)\,, \tag{15'}$$

where $\gamma_0 = \frac{1}{2}\ln(\alpha/\beta)$ and $\gamma_1 = 1/2$. An appreciation of the exporter's currency in the forward market will lower ${}_{t-1}f_t$, raise the price of that product in the importer's currency in (8'), and lower the price received by the exporter in (15'). Both the log-linear form of these expressions, and the coefficients of $\gamma_1 = 1/2$, follow from the special form of demand in (11). For more general demand functions we can still obtain a log-linear form for these pass-through equations, but with other values for $\gamma_1 \neq 1/2$, as we shall allow. In general, the pass-through equations must be homogeneous of degree one in the right-hand side variables, so the coefficients sum to unity as shown above.[3]

Pass-through equations of the form (8') or (15') have been recently estimated on disaggregate data (e.g. Knetter, 1989, 1993; Feenstra, 1989; Marston, 1990), though using the spot rather than forward rate. The variables in these equations are sometimes found to be *cointegrated*, meaning that (some or all) of the variables are integrated of order one, I(1), but a linear combination is found to be stationary, or I(0). This will be the case for the equations estimated in this paper. Then without loss of generality, the coefficient of any variable can be normalized at unity, and it can be treated as the "dependent" variable for expositional puposes. We

[3] The existence of demand function yielding log-linear pass-through equation is discussed in Feenstra (1989), where the homogeneity properties are also established.

will find it convenient to invert the pass-through equations to obtain a purchasing power parity (PPP) fomulation.

Considering first the case where the exporter invoices in the importing country's currency, we can move the forward rate to the left of (8') to obtain:

$$\ln_{t-1}f_t = \left(\frac{\gamma_0}{\gamma_1}\right) + (E_{t-1}\ln c_t^* - \ln q_t) - \left(\frac{1}{\gamma_1}\right)(\ln p_t - \ln q_t) \quad . \tag{16}$$

This is interpreted as a PPP equation applied to the forward rate. The variable $(E_{t-1}\ln c_t^* - \ln q_{t-1})$ equals the exporter's costs relative to competing prices in the importing country, and its coefficient of unity accords with the conventional PPP equation: an increase in relative prices of the exporting country will depreciate its exchange rate, or raise ${}_{t-1}f_t$. The variable $(\ln p_t - \ln q_t)$ is the import price relative to the domestic price, and it while it does not normally appear in a PPP equation, our derivation from the optimal pricing behaviour of the exporting firm shows that this variable is relevant. The coefficient of this relative import price equals the inverse of the pass-through elasticity in (8').

To express (16) in terms of the sport rate, we can use the covered interest parity condition:

$$\ln_{t-1}f_t - \ln s_{t-1} = \ln[(1+i_{t-1}^*)/(1+i_{t-1})] \approx (i_{t-1}^* - i_{t-1}) \, , \tag{17}$$

where i_{t-1}^* (i_{t-1}) denotes the period t-1 nominal interest rate in the exporting (importing) country. Substituting this into (16), and combining variables, we obtain:

$$\ln s_{t-1} = \left(\frac{\gamma_0}{\gamma_1}\right) + (E_{t-1}\ln c_t^* - \ln q_t) - \left(\frac{1}{\gamma_1}\right)(\ln p_t - \ln q_t) + (i_{t-1} - i_{t-1}^*). \tag{18}$$

Thus, the PPP equation for the spot exchange rate includes the interest rate differential as an explanatory variable, with a coefficient of unity.

To estimate (18), we need to determine the forecasted value $E_{t-1}\ln c_t^*$. We will assume that $\ln c_t^*$ is integrated of order one, I(1), and verify that this property holds empirically. This means that $\ln c_t^* = \ln c_{t-1}^* + \varepsilon_t$, where ε_t is a stationary variable. It is quite possible that $E_{t-1}(\varepsilon_t) \neq 0$, because ε_t may be autocorrelated and depend on lagged differences of $\ln c_t^*$, which are observable in period t-1.[4] However, $E_{t-1}(\varepsilon_t)$ will be stationary in general. Thus, we can replace $E_{t-1}\ln c_t^*$ by $\ln c_{t-1}^*$, and add a stationary error onto the right of (18). We will also assume that $\ln p_t$ and $\ln q_t$ in (18) are I(1), while verifying that this property holds empirically. This allows us to replace these variables by their lagged values in (18), and add other stationary errors onto the right. Then updating the subscript on all variables from t-1 to t yields:

$$\ln s_t = \left(\frac{\gamma_0}{\gamma_1}\right) + (\ln c_t^* - \ln q_t) - \left(\frac{1}{\gamma_1}\right)(\ln p_t - \ln q_t) + (i_t - i_t^*) + u_t . \tag{18'}$$

We expect that the spot exchange rate in also I(1), but the error u_t is stationary by construction; thus, (18') represents a cointegrating relation between the spot rate and the various prices. We will find that the interest differential is stationary, or nearly so, for most countries.

We next compare the results in (18) to those obtained when the exporting firm prices in its own currency. Moving the forward rate onto the left of (15'), and using the covered interest parity condition (17), we obtain:

$$\ln s_{t-1} = \frac{-\gamma_0}{(1-\gamma_1)} + (E_{t-1}\ln c_t^* - \ln q_t) + \frac{1}{(1-\gamma_1)}(\ln p_t^* - E_{t-1}\ln c_t^*) + (i_{t-1} - i_{t-1}^*). \tag{19}$$

This equation differs from (18) in that the *exporter's price relative to marginal cost* appears on the right, rather than the relative import price. Otherwise, (18) and (19) are the same in that the variable $(E_{t-1}\ln c_t^* - \ln q_t)$, and the interest rate differential, still appear with coefficients of unity. We can write (19) in a stochastic form by replacing $E_{t-1}\ln c_t^*$ by $\ln c_{t-1}^*$ plus a random error, and similarly replacing $\ln p_t$ and $\ln q_t$ by their lagged values pus errors, to obtain:

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[4] For example, if $\ln c_t^*$ follows that time series process $\ln c_t^* = \ln c_{t-1}^* + \varepsilon_t$ with $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$, and u_t uncorrelated over time, then $E_{t-1}(\ln c_t^*) = \ln c_{t-1}^* + E_{t-1}(\varepsilon_{t-1}) = \ln c_{t-1}^* + \rho(\ln c_{t-1}^* - \ln c_{t-2}^*)$.

$$\ln s_t = \frac{-\gamma_0}{(1-\gamma_1)} + (\ln c^*_t - \ln q_t) + \frac{1}{(1-\gamma_1)}(\ln p^*_t - \ln c^*_t) + (i_t - i^*_t) + v_t \,. \qquad (19')$$

While (18') and (19') provide us with estimating equations when the exporter sets prices in the importer's currency, and its own currency, respectively, the price indexes available in aggregate data would always be a combination of these two cases. To see how this affects the estimation, suppose that a fraction λ of products are priced in the importer's currency , using (18'), and the remaining $(1-\lambda)$ are priced in the exporter's currency using (19'). The import price index P_t is constructed as:

$$\ln P_t \equiv \lambda \ln p_t + (1-\lambda)(\ln p^*_t - \ln s_t), \qquad (20a)$$

where p_t is the nonstochastic price chosen by the exporter in the importing country's currency, whereas $(\ln p^*_t - \ln s_t)$ is the stochastic price of those imports whose price is set in the exporter's currency. Similarly, the export price index is constructed as:

$$\ln P^*_t \equiv \lambda(\ln p_t + \ln s_t) + (1-\lambda)\ln p^*_t \,, \qquad (20b)$$

where $(\ln p_t + \ln s_t)$ is the stochastic price received by the exporter on the fraction λ of products, while $\ln p^*_t$ is nonstochastic.

Summing $\lambda\gamma_1$ times (18') and $-(1-\gamma_1)(1-\lambda)$ times (19'), and using (20), the following relation between the spot rate, prices and interest differential is obtained:

$$\ln s_t = \frac{\gamma_0}{(\lambda+\gamma_1-1)} + (\ln c^*_t - \ln q_t) + (i_t - i^*_t)$$
$$- \frac{1}{(\lambda+\gamma_1-1)}\left[\lambda(\ln P_t - \ln q_t) + (1-\lambda)(\ln P^*_t - \ln c^*_t)\right] + \frac{1}{(\lambda+\gamma_1-1)} w_t \,, \qquad (21)$$

where $w_t=[\lambda\gamma_1 u_t-(1-\lambda)(1-\gamma_1)v_t]$, and $\lambda \neq (1-\gamma_1)$ is assumed. Thus, the spot rate depends on a *weighted average* of the relative import and export price indexes, which we shall refer to as the "relative traded goods price." The weights used reflect the proportion of traded goods prices in the importer's and exporter's currency, respectively. Note that this proportion λ also affects the coefficient of the average traded goods price, in a nonlinear fashion. In particular, when $\lambda=1$ we obtain the coefficient on the relative import price in (18'), and when $\lambda=0$ we obtain the coefficient on the relative export price in (19').

3.2 Data and Identification

We will consider the PPP relation between the U.S. and five major trading partners - Canada, the Federal Republic of Germany, France, Japan and the United Kingdom (U.K.). Aggregate quarterly data from 1974:1 to 1992:1 comprise the sample period (see the Appendix for sources). Variables of equation (21) for the five countries are as follows. The dependent variable ($\ln s_t$) is the average quarterly spot rate of foreign currency per U.S. dollar. Period averages seem more reasonable than using end-of-quarter figures since exporters ship goods throughout the period. For the foreign marginal costs ($\ln c_t^*$) and the U.S. domestic price ($\ln q_t$) we use the wholesale price indices (WPI) of these countries.[5]

The US import price variable ($\ln P_t$) is a Divisia index which *excludes* the following categories of imports: (1) food, feeds, and beverages; (2) petroleum products; and (3) automobiles. These sectors were omitted because they did not conform to the imperfectly competitive model of section 2 (as with food), due to cartel behavior (as with oil), or due to import quotas (as with autos). As a rough approximation, one finds that the combined importance of the excluded goods account for an average 45% of total US imports, leaving 55% to be explained by the model. The export price indexes of the foreign countries ($\ln P_t^*$) is the aggregate export price index reported by these countries. The last variable of equation (21) is the interest rate differential, which was the difference between the 90-day interest rate for the U.S. and foreign countries.[6]

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[5] The wholesale prices are preferred to consumer prices to the extent that the former exclude nontraded services. For France the WPI was not available, so we used the consumer price index instead.

[6] For France the 90-day rate was not available, so an overnight rate was used instead.

Note that there is one respect in which our data do not closely correspond to the theoretical model of section 2, and that is in the use of the U.S. *multilateral* import price index, and foreign *multilateral* export indexes, which include trade to and from all trading partners. This contrasts with the theoretical price indexes in (20), which are the *bilateral* price indexes between the importing and exporting country. Since bilateral indexes between the U.S. and its trading partners are not generally available, the multilateral indexes are used instead. For Canadian trade with the U.S., however, we felt that the multilateral U.S. import index would be a particularly poor measure of bilateral prices, because the three omitted sectors (along with forestry products) are among the largest Canadian export products. As an alternative, we rely on the fact the the U.S. is the principal destination market for Canadian exports. The multilateral Canadian export price index ($\ln P^*_t$) is available, and then a measure of the U.S. import prices from Canada is constructed by simply converting this index to U.S. dollars using the (period average) spot rate. That is, for Canadian-US. trade we construct the U.S. import index $\ln P_t$ from the Canadian export index $\ln P^*_t$ using the identity:

$$\ln P_t \equiv \ln P^*_t - \ln s_t\,, \tag{22}$$

which follows directly from (20). This identity has implications for the identification of the coefficients in (21), as we shall discuss below.

Data on the currency of invoice for exports is taken from Page (1981, Table 1). For the years 1979 or 1980, she lists the following percentages of *total* country exports that are invoiced in the country's own currency:[7] Canada, 15%; France, 62.4%; Germany, 82.3%; Japan, 32.7%; United Kingdom, 76%. For *total* United States imports, 85% are invoiced in dollars. These percentages correspond to $(1-\lambda)$ and λ in our theoretical model, or the fraction of trade priced in the exporter's and importers's currency, respectively. Comparing Canada and the U.S., these percentage of trade invoiced in each country's currency happen to sum to unity, but this does not

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[7] The figure for Canada is an estimate.

hold in the other cases because the pecentages are for multialteral trade. In order to scale these percentages so that they sum to unity, we simply divide by their sum, obtaining:

$$
\begin{array}{ll}
\text{Canada} & (1-\lambda)=0.15 \\
\text{France} & (1-\lambda)=0.624/(0.624+0.85)=0.42 \\
\text{Germany} & (1-\lambda)=0.823/(0.823+0.85)=0.49 \\
\text{Japan} & (1-\lambda)=0.327/(0.327+0.85)=0.28. \\
\text{U.K.} & (1-\lambda)=0.760/(0.760+0.85)=0.47
\end{array}
\tag{23}
$$

With these values for λ, the average traded goods price $[\lambda(\ln P_t-\ln q_t)+(1-\lambda)(\ln P^*_t-\ln c^*_t)]$ that appears in (21) is computed. These values of λ should be regarded a estimates, however, and it will be important to determine how sensitive our results are to other choices.[8] To determine the sensitivity to the estimating equation to the percentages used for invoicing currencies, let λ continue to represent the true percentage invoiced in the importer's currency, as in (20). However, suppose that the relative traded goods price is constructed using the weight λ' as:

$$
\begin{aligned}
&[\lambda'(\ln P_t-\ln q_t)+(1-\lambda')(\ln P^*_t-\ln c^*_t)] = \\
&\qquad (\lambda-\lambda')(\ln s_t-\ln q_t-\ln c^*_t)+\lambda(\ln p_t-\ln q_t)+(1-\lambda)(\ln p^*_t-\ln c^*_t),
\end{aligned}
\tag{24}
$$

where the equality follows from (20). Then again summing $\lambda\gamma_1$ times (18') and $-(1-\gamma_1)(1-\lambda)$ times (19'), and making use of (24), we obtain:

$$
\begin{aligned}
\ln s_t &= \frac{\gamma_0}{(\lambda'+\gamma_1-1)} + (\ln c^*_t-\ln q_t) + \left(\frac{\lambda+\gamma_1-1}{\lambda'+\gamma_1-1}\right)(i_t-i^*_t) \\
&\quad - \frac{1}{(\lambda'+\gamma_1-1)}\left[\lambda'(\ln P_t-\ln q_t)+(1-\lambda')(\ln P^*_t-\ln c^*_t)\right] + \left(\frac{1}{\lambda'+\gamma_1-1}\right)w_t .
\end{aligned}
\tag{25}
$$

8 For example, using *bilateral* data for a sample of products in 1973, Magee (1974, Table 2) finds that 28% of Japanese exports to the U.S. are invoiced in yen, which is the same figure that we calculate in (23). However, he also finds that between 60% or 81% (depending on the calculation) of German exports to the U.S. are invoiced in marks, which is higher than in (23). Grassman (1973,1976) provides evidence on the currency of invoice for exports from Sweden and Denmark.

Comparing (21) and (25) it is evident that mispecification of the weight λ - using λ' instead - will affect the coefficient of the interest rate differential and the relative traded goods price, though the coefficient of the relative wholesale prices $(\ln c^*_t - \ln q_t)$ is not affected. Another way of stating this result is that the coefficients of (21) are not identified without knowledge of λ, so that we could not estimate this parameter along with the other coefficients. The reason for this identification problem is that the ratio of the bilateral export and import prices is exactly equal to the spot rate, or $\ln s_t \equiv \ln P^*_t - \ln P_t$ as in (22). Adding any multiple of this identity to (21), we obtain other linear combinations of the variables that would have the same residuals, and in this sense, explain the data equally well. We expect this identification problem to be most severe on the Canadian-U.S. data, where the identity (22) holds by construction. However, even for the multilateral indexes used for other countries the relation (22) holds approximately, so that it may still be difficult to identify λ. We will avoid this identification problem by using the data in (23) for λ, but will also experiment with other values in our sensitivity analysis.

The identity (22) also has implications for testing the comon hypothesis that the wholesale prices are mismeasured due to the inclusion of nontraded goods. Rewriting (22), we obtain;

$$\ln s_t \equiv (\ln c^*_t - \ln q_t) - (\ln P_t - \ln q_t) + (\ln P^*_t - \ln c^*_t) \ . \qquad (22')$$

This can be interpreted as saying that the PPP equation defined over wholesale prices should be corrected by including the relative import and export prices, in which case PPP holds by construction when the bilateral price indexes are used. Note that there is an important difference between (22') and our specification (21) or (25), in that the relative traded goods prices appear with *opposite sign* in (22'), but with the *same sign* in (21) or (25), for $0 \leq \lambda \leq 1$. This means that the when estimating (21) or (25), there is no possibility of simply obtaining the identity (22'). Evidently, if one wanted to test whether deviations from PPP were caused by the mismeasurement of the wholesale prices (due to the inclusion of nontraded goods), a hypothesis that does not simply yield an identity like (22') would need to be developed.

4. Estimation

4.1. Testing for Unit Roots

The first task in estimation is to determine whether the variables are stationary or not. We use the augmented Dickey-Fuller (ADF) test, under which the null hypothesis is that the variable has a unit root (Dickey and Fuller, 1991). This hypothesis is tested by regressing the difference of a variable on a constant, its own lagged value, lagged differences, and possibly a time trend.[9] Under the null hypothesis, the coefficient of the lagged value should be insignificantly different from zero. The ADF test statistic is just the ratio of the coefficient to its standard error (though this does not have the conventional t-distribution). These test statistics are reported in Table 1, where the first row for each variable tests the null hypothesis of a unit root in the *level*, and the second row tests the null hypothesis of a unit root in the *first-difference*. We also included a time trend in the test when its coefficient was significant at the 10% level, as indicated by "t" in the table.

Looking first at the results for the spot exchange rates (relative to the dollar), we cannot reject the null hypothesis of a unit root in the levels, but do reject the hypothesis of a unit root in the first-differences. As expected, we conclude that these spot rates are I(1). The wholesale price indexes are also found to be I(1) for Canada, Germany and Japan. For France, where the CPI is used, we cannot reject the hypothesis of a unit root in either the levels or first-differences so it appears that this variable is I(2).[10] The economic interpretation is that the *inflation rates* for France appears to have a unit root, which is surprising. For Britian, we find that the wholesale price index is stationary. For the U.S. WPI (not shown in Table 1), the ADF test statistic is -1.48 for the level, and -2.82 for the first-difference, so we conclude this variable is I(1).

Turning to the relative traded goods price, $[\lambda(\ln P_t - \ln q_t) + (1-\lambda)(\ln P^*_t - \ln c^*_t)]$, we conclude that this variable has a unit root for all countries except Canada, though in that case we might find

[9] We included three quarterly lags of the differences, which were sufficient to eliminate serial correlation in the errors, except for the Canada WPI where four lags were used.

[10] The hypothesis that the second-difference has a unit root is strongly rejected. Using monthly data, Pippenger (1993) also finds evidence that the WPI of some countries are I(2).

the same result at a significance level weaker than 10%. In contrast, the interest rate differential is found to be stationary for Canada and France, and possibly also for Japan and the U.K. at a weaker significance level. The interest rate differential between Germany and the U.S. stands out as failing to reject a unit root in the levels, but soundly rejecting a unit root in the first differences, so that it is I(1). Summing up, most variables in (21) are found to have a unit-root, except for the interest differential which is stationary for some countries. Aside from the French CPI, we therefore need to allow for both I(0) and I(1) variables in the estimation.

4.2. Estimation of Cointegrating Relations

The parameters of (21) are estimated using the method of Johansen (1991), which imbeds this cointegrating relation within a vector-autoregression (VAR) system of all five variables, denoted by the column vector $X_t \equiv [\ln s_t, \ln c^*_t, \ln q_t, (i_t - i^*_t), \lambda(\ln P_t - \ln q_t) + (1-\lambda)(\ln P^*_t - \ln c^*_t)]'$. The VAR specifies that the difference of each variable depends on a constant, lagged differences, and lagged cointegration relations that are linear combinations of the five variables:

$$\Delta X_t = \mu + \sum_{k=1}^{K} \theta_k \Delta X_{t-k} + \sum_{i=1}^{N} \alpha_i \beta_i' X_{t-1} + \varepsilon_t , \tag{26}$$

where μ is a (5x1) vector of constants, K is the number of lagged differences, θ_k is a (5x5) matrix of estimated coefficients for each lag k, N in the number of cointegrating relations, and α_i and β_i are (5x1) vectors of estimated coefficients. In particular, β_i is the cointegrating vector such that the cointegrating relation $\beta_i' X_{t-1}$ is stationary. The cointegrating relation can be interpeted as an error-correction term, which adjusts the change in each variable in the VAR according to the error from the long-run equilibrium. It is quite possible that there are ***multiple*** cointegrating relations, which we have indexed by i. Johansen (1991) derives the maximum likelihood estimates of coefficients in (26), and shows how to test for the number of cointegrating vectors β_i. In comparison with single-equation method for estimating cointegrating vectors, such as Engel and Granger (1987), the Johansen method is thought to be more powerful by using the full VAR system, and also has the advantage that the standard errors of the cointegrating vector(s) are normally distributed.

We first check for the number of cointegrating vectors. With five variables in our system, the maximum number of cointegrating vectors obtained from the estimation is also five. By construction, these vectors are independent. Since any linear combination of the cointegrating relations is also stationary, finding five such relations would mean that all of the variables in the system are stationary. We have already found that this is not the case for the unit root tests. A reduction in the number of cointegrating vectors is tested by a likelihood ratio test, where the null hypothesis is that the number is at most N. In Table 2 we report the results of these likelihood ratio tests for each value of N.[11]

The results in Table 2 show that we cannot reject the hypothesis that the number of cointegrating vectors is at most 3 for France and Germany, but do reject the hypothesis that the number is two or less. Thus, for these countries we conclude that there are three cointegrating relations. For Canada we find two such relations at the 5% level, though it is quite likely that at the 10% level we would conclude there are three cointegrating vectors. For Japan and the U.K. we find evidence of four cointegrating vectors at the 5% level, though minor changes in the VAR for Japan results in a smaller number of such relations.[12] While there is obviously some difference across the countries, we will proceed by treating the number of cointegrating relations in each case as three.

Johansen and Juselius (1990) suggest that the first cointegrating vector - which is associated with a certain maximum eigenvalue - is of special significance in that it is the "most

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[11] All empirical results in this section were computed on Econometric Views version 1.0, which also provides the critical values for Table 1. For all countries, three quarterly lags of the differenced variables were included, along with a constant term in each cointegrating relation, but not in the VAR system.

[12] The system (26) was estimated both with and without a constant term in the VAR, and the Schwartz criterion indicated that the constant was generally not needed. For Japan, the Schartz criterion was ambiguous, and if the constant term is instead included then the number of cointegrating vectors is reduced to two. This occurs because of a change in the critical values for the LR test, with minimal change to the cointegration estimates.

correlated with the stationary part of the model" (p. 192). For all five countries, we find that the first cointegrating vector provides quite reasonable estimates of equation (21). In the first row for each country in Table 3 we report this cointegrating vector, where we have normalized the coefficient of the spot rate at unity, and expressed the other coefficients as appearing on the right-hand side of (21).

The spot rates are measured as each country's currency per U.S. dollar, so the expected coefficients on the wholesesale prices are unity for the country's own WPI, and negative one on the U.S. WPI. The expected signs are obtained in most cases, but the magnitudes of the coefficients differ considerably from unity. The coefficient of the relative tradede goods price can be of either sign, and is highly significant for all countries. The interest rate differential is measured as the U.S. relative to the foreign country's rate, and has a predicted coefficient of unity. A point estimate of this magnitude is obtained for Japan, and a larger and highly significant estimate is obtained for Germany, but for the other three countries the estimates obtained are insignificantly different from zero. Thus, the relative trade goods price appears to be an important determinant of the exchange rates in the PPP equation, but this result is not generally obtained for the interest rate differentials.

For all countries except Germany, the second unnormalized cointegrating vector (not reported in Table 3) has its largest coefficient on the interest rate differential, and smallest coefficient on the spot exchange rate, where these differ by at least ten times. This result is not surprising since, for most countries, the results in Table 1 showed that interest rate differential was stationary, or nearly so. For the purposes of this paper, this second cointegrating vector is not of special interest in itself, but by taking a linear combination of the first and second vectors we can solve for a cointegrating relation with a zero coefficient on the interest rate differential. This vector is shown in the second row for each country (except Germany), again normalized as in (21).

If we continue with this method, a third cointegrating relation can be obtained by using a linear combination of the three estimated vectors, to obtain zero coefficients on the interest rate differential and the relative traded goods price. This results is a stationary relation between the spot rate and the foreign and U.S. WPI. Cheung and Lai (1993) have found that a stationary relation of this type holds for a number of countries, and we obtain reasonable estimates for Canada, France and Japan, which are reported in the third row for these countries.

For Germany. the same exercise of eliminating the interest rate differential leads to nonsensical results: the cointegrating vector obtained has coefficients of about 50 on the German WPI and -25 on the U.S. WPI, with a very small coefficient on the relative traded goods price. If we use these coefficients to measure the residual in equation (21), these deviations from PPP are much greater than the fluctuations in the mark/dollar rate itself. The same result is obtained if we consider eliminating the relative traded goods price instead. Thus, while the cointegrating relation obtained by eliminating the interest rate differential (or relative traded goods price) is in principle stationary, the results obtained in this case are not meaningful.

To obtain a second cointegrating vector for Germany, we make use of the fact that the coefficients on the German and U.S. WPI in Table 3 are quite similar, but with opposite sign. Thus, we consider imposing the "symmetry" restriction that these variables have the same coefficient (with opposite sign) in the PPP equation. We use a linear combination of the first two estimated vectors to obtain such a cointegrating relation, which is reported in the second row for Germany. We see that the resulting coefficient of the WPI is 1.26, with a standard error of 0.17, so it is insignificantly different from unity. A third cointegrating vector is then obtained by using a linear combination of the three estimated vectors, to obtain a relation where the WPI have coefficients of 1 and -1.

Finally, for the U.K. the second vector reported in Table 3 was obtained by eliminating the interest rate differential, but also eliminating the relative traded goods price leads to results that are not meaningful: the resulting deviations from PPP, measured by the residual in (21), are larger than the fluctuations in the pound/dollar rate. Instead, to obtain a third cointegrating vector we again impose the "symmetry" restriction, and take a linear combination of the three estimated vectors to obtain a zero coefficient on the interest rate differential and equal coefficients (with opposite sign) on the wholesale prices. This cointegrating relation is reported in the third row for the U.K. The fourth row for both the U.K. and Germany is an OLS regression of the spot rate on the wholesale prices, which is discussed below.

Summing up, the first row of estimates in Table 5 is the first cointegrating vector, while the second the third rows of estimates are obtained as linear combinations of the three estimated vectors.[13] The developed theory on cointegration offers little guidance on how to interpret multiple vectors. The particular linear combinations we have chosen appear to be meaningful in terms of the economic interpretation, and the coefficient values obtained. It is noteworthy that the estimates in each cointegrating vector are consistent even as variables are omitted, because the omitted linear combinations of variables are stationary.

To obtain additional insight into the cointegrating relations, we substitute the coefficients from Table 3 into (21) to calculate the residuals, which measure the deviations from PPP. These deviations are shown in Figures 1-5. In each Figure, the bold line shows the deviations from PPP as measured using only the data on the spot rate and wholesale prices. Thus, for Canada, France and Japan, the bold line - labelled "PPP3" - is the residual from the third cointegrating vector reported in Table 3. In contrast, for Britian and Germany we did not report a cointegrating vector between just the spot rate and wholesale prices, and instead use the residual from an OLS regression between these variables (also shown in Table 3) to calculate the bold lines labelled "PPP0." In Table 3 we also report the standard deviations of these calculated residuals.

The dashed lines labelled PPPi in each figure are the residuals calculated from the i'th cointegrating vector reported in Table 3. In all cases, these dashed lines show less variation than the bold lines, and have lower standard deviations, as reported in Table 3. In addition, the dashed lines are quite close. The similarity of PPP1 and PPP2 indicates that the interest rate differential is not important in explaining deviations from PPP (except possibly for Germany). For all countries,

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13 The standard errors reported in Table 3 are provided by Econometric Views, but should be interpreted with caution. In particular, the standard errors on the Nth cointegrating vector in Table 3 (N=1,2,3) are computed under the hypothesis that there are only N cointegrating vectors in the system, whereas the correct standard errors would be computed under the assumption that there are 3 cointegrating vectors. The authors are presently in communication with David Lilean to revise the presentation of the output in EViews, so that the desired standard errors can be obtained.

it is the relative traded goods price that is primarily responsible for explaining the deviation from PPP. By comparing the standard deviation of PPP0 or PPP3 with that of PPP1 or PPP2 in Table 3, we see that about one-half of the deviations from PPP is explained by the relative traded goods price for Japan and the U.K., and nearly two-thirds for Canada and Germany. In contrast, for France less than one-quarter of the deviations from PPP is explained.

The final column of Table 3 reports the log likelihood value for the 5-equation system in (26), obtained under the hypothesis of 3 cointegrating vectors. These likelihood values will serve as a benchmark as we consider alternative values of λ.

4.3. Sensitivity of Results

In section 3.2, we argued that there was a problem in identifying the parameter λ, which we avoided by using data on the currency of invoice for exports from the various countries. The source of this identification problem was the identity (22), stating that the ratio of the bilateral price indexes equals the spot exchange rate. Summing any multiple of this identity and the estimating equation (21), we obtain an alternative estimating equation with different coefficient values, but the same residuals. A particular example of this is provided by (25), which shows how the coefficient values are affected when an incorrect value of λ (denoted by λ') is used. We expect this identification problem to be most severe on our Canadian-U.S. data, because the identity (22) holds for these Canadian export and U.S. import price indexes by construction. For the other countries, multilateral price indexes were used and these satisfy (22) only approximately.

In Table 4, we report the results from estimating the cointegrating vectors with alternative values of λ. Only the first cointegrating vector is reported, along with the standard deviation of the residuals from that vector, and the log likelihood of the system (26). For Canada, the results are in accordance with our discussion of the identification problem. Namely, the log likelihood for the system is constant for alternative values of λ, though the coefficient values (and the residuals obtained) vary somewhat. The use of bilateral export and import price indexes in this case means that alternative vales of λ cannot be distinguished by the data.

Jumping over the results for France, the results for the other three countries suggest that the use of multilateral indexes allows λ to be identified. For all three countries, the likelihood values obtain an interior maximum in the range $\lambda \in (0,1)$. For Japan and the United Kingdom, the likelihood value obtained for the λ value in Table 3 is higher than for any of the alternative values in Table 4. Thus, for these two countries the currencies of invoice reported in Page (1981) provided quite reasonable estimates for λ. For Germany, $\lambda=0.30$ in Table 4 provides a higher likelihood value than $\lambda=0.51$ in Table 3, which supports the hypothesis that Germany invoices higher percentage of its exports in its own currency than the 49% value computed in (23).[14] In any case, the interior maximum for the likelihood function suggests that with multilateral price indexes, λ can be identified and estimated.

Turning to the results for France in Table 4, we obtain an unusual result: the standard deviation of the residuals obtained by various values of λ are *inversely related* to the likelihood values obtained. Thus, the lowest standard deviation is obtained by letting $\lambda=0$, so that the relative traded goods price is simply measured by the French export prices relative to the French CPI. This is because the French export price index is strongly correlated with the franc/dollar movements (with a simple correlation of 0.96), so that small deviations from PPP are obtained when this variable is included. However, using this variable in the 5-equation system (26) results in the *lowest* likelihood value. The reason is that movements in the French export prices relative to the CPI are *not* well-explained within the VAR system: the standard error of the equation in (26) explaining $\Delta[\lambda(\ln P_t - \ln q_t) + (1-\lambda)(\ln P_t^* - \ln c_t^*)]$ for France is 0.038 when $\lambda=0$ is used, but 0.021 when $\lambda=0.58$ is used as in Table 3. These observations make the point that minimizing the deviations from PPP is not the same as maximizing the likelihood value of the system (26), and presumably, the latter is of greatest interest.

[14] As reported in note 8, Magee (1974) did find a higher percentage of German exports to the U.S. invoiced in marks.

5. Conclusions

Froot and Rogoff (1994) have recently surveyed the evidence on PPP, and considered various explanations, including "pricing to market" or pass-through behavior, to explain its failure. They conclude that "Pricing to market is an interesting and important issue. Because, however, it ifundamentally derives from short-term rigidities, it seems unlikely to explain the medium and long-term deviations from PPP that we have been focusing on here" (p. 43). In contrast to this finding, we have found that pass-through behavior appears to explain a significant portion of the deviations from PPP observed during the floating rate period since 1974. The variable we have used to measure pass-through behavior is a weighted average of import relative to domestic prices, and export prices relative to costs of production. Either of these relative traded goods prices are often treated as the dependent variable in conventional pass-through equations, and by inverting these equations, we obtain a PPP formulation where the weighted average appears (along with wholesale or consumer prices) as a determinant of exchange rates. We have found this weighted average is significantly correlated with exchange rate movements, and in some cases, can explain more than one-half of the observed deviations from PPP.

By inverting the passthrough equation, we have certainly not developed an equilibrium theory of exchange rates, but rather, have only provided one explanation for the observed deviations from PPP. It is possible that our approach can be used as a building block towards an equilibrium theory. In particular, our PPP equation (21) could be integrated into a monetary model of exchange rate determination, such as presented by Woo (1985) and West (1987). West (1987) emphasizes that *stochastic deviations* from PPP (and shocks to money demand) play a crucial role in his failure to reject the monetary model, and states that "It is therefore of interest in future work to model these shocks as functions at least in part of observable economic variables" (p. 72). Our PPP equation, obtained from the pass-through behavior of optimizing firms, can be considered a first step along these lines.

Appendix: Data Sources

Sources for import price index construction are the Citibase tape file and various issues of *Survey of Current Business*. Imports are given in both current and constant (1980) US dollars on a quarterly basis from 1967.1 to 1992.1 Implicit prices are derived by dividing current by constant dollar imports. A Divisia import price index was constructed from the following categories: (1) Industrial supplies and materials excluding petroleum, (a) Durable and (b) Nondurable goods; (2) Capital goods, except autos; (3) Consumer Goods, (a) Durable and (b) Nondurable goods; (5) Other Goods, (a) Durable and (b) Nondurable goods.

The rest of the series are from the DX-Online Database (Melbourne, Victoria) containing United Nations International Financial Statistics, quarterly data from 1974.1 to 1992.1. Exchange rates are line RF (period averages). Domestic price indices are line 63 (generally wholesale or comparable price index) except for France which was line 64, the consumer price index. For France the price relatives were matched with the US line 64. Interest rates are line 60C (90-day Treasury-Bill rate). For France, line 60B had to be used (Call Money Rate-Average) and for Japan, line 60L (Deposit Rate [end of period]). The export price indices were from line 74..D for each country. The Canadian export price index 74..D showed a sharp increase during 1979 due to the rise in oil prices. We would not expect this global shock to influence to Canadian/U.S. exchange rate as specified by the PPP equation. To omit this effect, the Canadian export price index was held constant at its 1979:1 value until 1980:1, and then it resumed its growth relative to 1980:1.

REFERENCES

Baron, David P., 1976a, "Flexible Exchange Rates, Forward Markets, and the Level of Trade," American Economic Review 66, March, 253-266.

Baron, David P., 1976b, "Fluctuating Exchange Rates and the Pricing of Exports," Economic Inquiry 14, September, 425-438.

Dickey, D. A. and W. A. Fuller, 1991, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," Econometrica, 49, 1057-1072.

Cheung, Yin-Wong and Kon S. Lai, 1993, Long-run Purchasing Power Parity During the Recent Float, Journal of International Economics, 34, 181-192.

Engle, Robert F. and C. W. J. Granger, 1987, "Co-integration and Error Correction: Representation, Estimation, and Testing," Econometrica, 55(2), March, 251-276.

Eldor, Rafael and Itzhak Zilcha, 1987, "Discriminating Monopoly, Forward Markets and International Trade," International Economic Review, 28(2), 459-468.

Ethier, Wilfred, 1973, "International Trade and the Forward Exchange Market," American Economic Review 63(3), June, 494-503.

Feenstra, Robert C., 1989 "Symmetric Pass-Through of Tariffs and Exchange Rates Under Imperfect Competition: An Empirical Test. Journal of International Economics, 27, 25-45.

Froot, Kenneth A. and Kenneth Rogoff, 1994, "Perspectives on PPP and Long-Run Real Exchange Rates," Harvard and Princeton Universities, mimeo. Forthcoming in Gene Grossman and Kenneth Rogoff, eds., Handbook of International Economics. North-Holland, Amsterdam.

Giovannini, Alberto, 1988, Exchange Rates and Traded Goods Prices. Journal of International Economics, 24, 45-68.

Grassman, Sven, 1973, "A Fundamental Symmetry in International Payment Patterns," Jounal of International Economics 3(2), May, 105-116.

Grassman, Sven, 1976, "Currency Distribution and Forward Cover in Foreign Trade: Sweden Revisited," Journal of International Economics 6(2), May, 215-221.

Hodrick, Robert J., 1989, "Risk, Uncertainty, and Exchange Rates", Journal of Monetary Economics, 23, 433-459.

Johansen, Soren, 1991 "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, Econometrica, 59, 1551-1580.

Johansen, Soren, and Katriana Juselius, 1990, "Maximum Likelihood Estimation and Inference on Cointegration-with Applications to the Demand for Money," Oxford Bulletin of Economics and Statistics, 52, 169-210.

Knetter, Michael M., 1989, "Price discrimination by U.S. and German exporters," American Economic Review, 79(1), March, 198-210.

Knetter, Michael M., 1993, "International comparisons of pricing-to-market behavior," American Economic Review, 83(3), June, 473-486.

Magee, Stephen, 1974, "U.S. Import Prices in the Currency-Contract Period," Brooking Papers on Economic Activity, 1, 117-164.

Marston, Richard, 1990, "Pricing to Market in Japanese Manufacturing, Journal of International Economics, 29, 217-236.

Page, S. A. B., 1981, "The Choice of Invoicing Currency in Merchandise Trade," National Institute Economic Review, 98, 60-72.

Pippenger, Michael K., 1993, "Cointegration Tests of Purchasing Power Parity: the Case of Swiss Exchange Rates," Journal of International Money and Finance, 12, 46-61.

West, Kenneth D., 1987, "A Standard Monetary Model and the Variability of the Deutschemark-Dollar Exchange Rate," Journal of International Economics, 23(1/2), August, 57-76.

Woo, Wing T., 1985, "The Monetary Approach to Exchange Rate Determination Under Rational Expectations," Journal of International Economics 18, 1-16.

Table 1

Unit Root Tests: Augmented Dickey-Fuller Statistic

Variable	Canada	France	Germany	Japan	U.K.
Spot exchange rate					
Level	-2.11	-1.59	-1.58	-2.54[t]	-2.23
First-difference	-3.20[a]	-3.44[a]	-2.97[a]	-3..60[a]	-3.39[a]
Wholesale price index					
Level	-1.72	-1.71	-1.25	-1.98	-3.40[t,b]
First-difference	-3.27[t,b]	-1.26	-2.59[a]	-3.36[a]	-2.78[b]
Rel. traded goods price					
Level	-1.73	-1.52	-2.02	-1.81	-1.91
First-difference	-2.31	-3.33[a]	-3.22[a]	-3.07[a]	-3.46[a]
Interest rate differential					
Level	-3.09[a]	-2.80[a]	-0.71[t]	-2.00	-2.51
First-difference	-4.71[a]	-4.24[a]	-4.27[t,b]	-4.65[a]	-5.70[a]

Notes

All series are quarterly from 1974:1 to 1992:1, excluding 1992:1 for the French relative traded goods price and 1974:1-1975:2 for the German interest rate differential. The consumer price index rather than wholesale price index is used for France. The ADF tests were run with 3 quarterly lags of the differences, except for the Canadian WPI which included 4 quarterly lags.

[a] Rejection of the null hypothesis of nonstationarity at the 10 per cent level, where the critical value is -2.59 with three lagged differences but no time trend.

[b] Rejection of the null hypothesis of nonstationarity at the 10 per cent level, where the critical value is -3.27 with three lagged differences and a time trend included.

[t] Time trend was significant at the 10% level, and was included in the ADF test.

Table 2

Likelihood Ratio Tests for the Number of Cointegrating Vectors

Hypothesised Number	5% Critical Value	Canada	France	Germany	Japan	U.K.
None	76.1	101.5[a]	114.4[a]	112.9[a]	111.5[a]	125.3[a]
≤ 1	53.1	54.6[b]	69.7[a]	62.1[b]	69.9[a]	77.6[a]
≤ 2	34.9	31.0	34.9[b]	36.8[b]	36.0[b]	42.7[a]
≤ 3	19.7	15.8	15.9	18.9	20.1[b]	21.0[b]
≤ 4	9.2	7.1	2.8	7.4	8.0	4.5

Notes

[a] Significant at the 1 percent level.

[b] Significant at the 5 percent level.

Table 3 - Cointegrating Relations

Country	Interest Differential	Relative Traded Goods Price	Country WPI	US WPI	Standard Dev. of Residual	Log Likelihood
Canada (λ=0.85)	0.17 (0.18)	-0.48 (0.031)	0.30 (0.062)	-0.56 (0.055)	0.021	975.2
		-0.46 (0.049)	0.35 (0.096)	-0.60 (0.068)	0.024	
			1.23 (0.18)	-1.12 (0.23)	0.064	
France (λ=0.58)	-0.18 (0.61)	-1.11 (0.20)	3.19 (0.55)	-3.97 (0.61)	0.120	856.8
		-1.13 (0.19)	3.08 (0.53)	-3.85 (0.60)	0.117	
			4.11 (0.88)	-4.58 (1.12)	0.156	
Germany (λ=0.51)	2.95 (0.38)	-1.10 (0.087)	2.28 (0.41)	-1.81 (0.27)	0.058	891.9
	3.05 (0.58)	-1.13 (0.12)	1.26 (0.17)	-1.26 (0.17)	0.070	
	2.72 (0.40)	-1.14 (0.11)	1.00	-1.00	0.056	
OLS			3.46 (0.58)	-2.46 (0.38)	0.148	
Japan (λ=0.72)	1.09 (0.42)	-1.50 (0.13)	0.88 (0.17)	-1.24 (0.052)	0.053	869.1
		-1.57 (0.11)	1.02 (0.11)	-1.18 (0.045)	0.046	
			2.67 (0.20)	-1.83 (0.083)	0.086	
United Kingdom (λ=0.53)	0.18 (0.38)	-2.59 (0.14)	-0.31 (0.11)	0.087 (0.15)	0.078	841.2
		-2.59 (0.14)	-0.34 (0.077)	0.11 (0.13)	0.075	
		-2.76 (0.21)	-0.48 (1.31)	0.48 (1.31)	0.106	
OLS			1.16 (0.31)	-0.93 (0.37)	0.141	

Notes: The sample period for all countries is 1974:1-1992:1, excluding 1992:1 for France and 1974:1-1975:2 for Germany. All cointegrating regressions also include a constant term.

Table 4 - Sensitivity of Cointegrating Relations

Country	λ	Interest Differ-ential	Relative Traded Goods Price	Country WPI	U.S. WPI	Standard Dev. of Residual	Log Likeli-hood
Canada	0.99	0.16 (0.17)	-0.45 (0.027)	0.34 (0.057)	-0.59 (0.051)	0.020	975.2
	0.70	0.18 (0.19)	-0.52 (0.036)	0.30 (0.062)	-0.53 (0.060)	0.023	975.2
	0.0	0.29 (0.30)	-0.82 (0.088)	-0.19 (0.15)	-0.26 (0.11)	0.036	975.2
France	0.80	0.80 (0.79)	-1.17 (0.26)	3.51 (0.49)	-4.34 (0.55)	0.139	870.9
	0.40	-0.52 (0.50)	-1.13 (0.15)	2.57 (0.52)	-3.28 (0.59)	0.100	859.5
	0.0	-0.20 (0.26)	-1.17 (0.050)	0.71 (0.24)	-1.12 (0.27)	0.049	843.4
Germany	0.70	3.44 (0.45)	-1.28 (0.14)	2.52 (0.53)	-2.11 (0.32)	0.071	882.0
	0.30	2.57 (0.33)	-0.97 (0.057)	2.11 (0.36)	-1.57 (0.24)	0.048	894.4
	0.0	1.89 (0.32)	-0.87 (0.038)	1.63 (0.36)	-1.11 (0.24)	0.038	879.6
Japan	0.90	0.97 (0.45)	-2.06 (0.19)	1.23 (0.12)	-1.53 (0.033)	0.063	866.6
	0.50	1.79 (0.73)	-1.11 (0.20)	0.65 (0.29)	-1.03 (0.14)	0.063	861.6
	0.0	-8.02 (6.05)	-1.31 (0.45)	1.38 (1.43)	0.07 (0.62)	0.199	827.0
United Kingdom	0.70	-1.48 (0.82)	-3.14 (0.14)	-0.15 (0.20)	-0.44 (0.23)	0.097	834.9
	0.30	-0.64 (0.63)	-2.45 (0.23)	-0.91 (0.19)	0.95 (0.24)	0.105	829.0
	0.0	-2.46 (1.36)	-2.38 (0.39)	-1.86 (0.47)	2.30 (0.60)	0.162	806.1

Notes: The sample period for all countries is 1974:1-1992:1, excluding 1992:1 for France and 1974:1-1975:2 for Germany. All cointegrating regressions also include a constant term.

Figure 1: Cointegrating Residuals for Canada

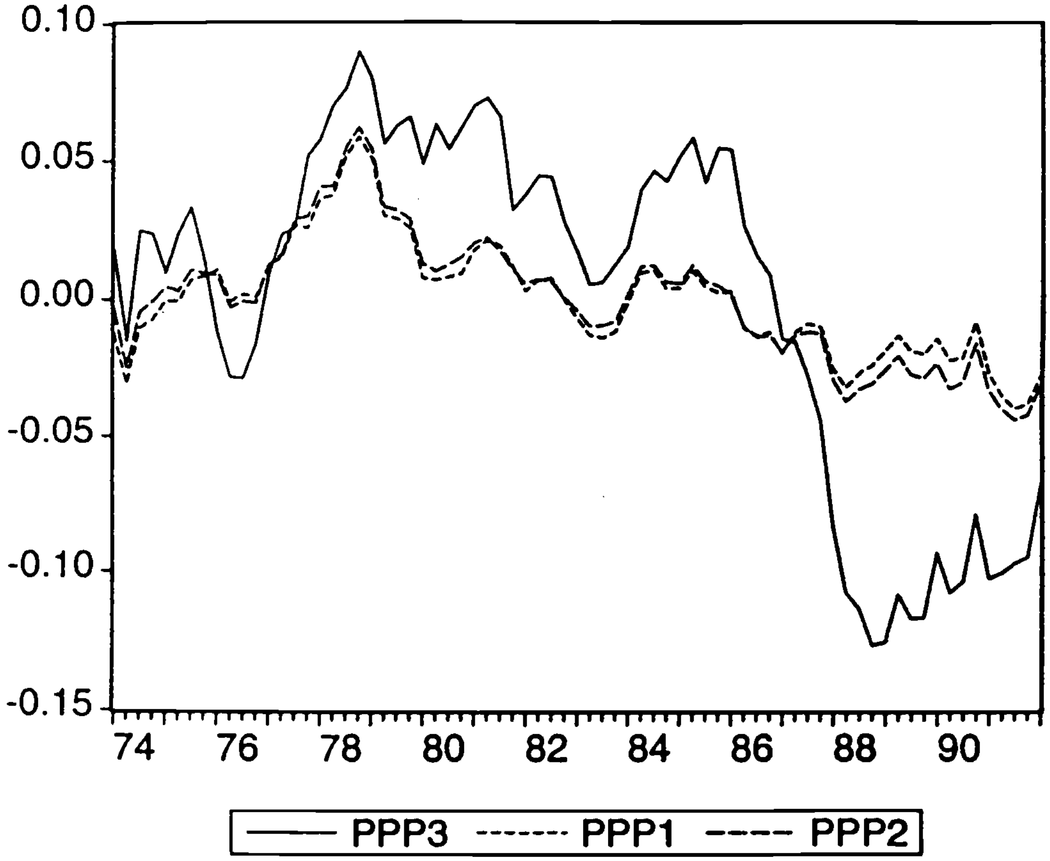

Figure 2: Cointegrating Residuals for France

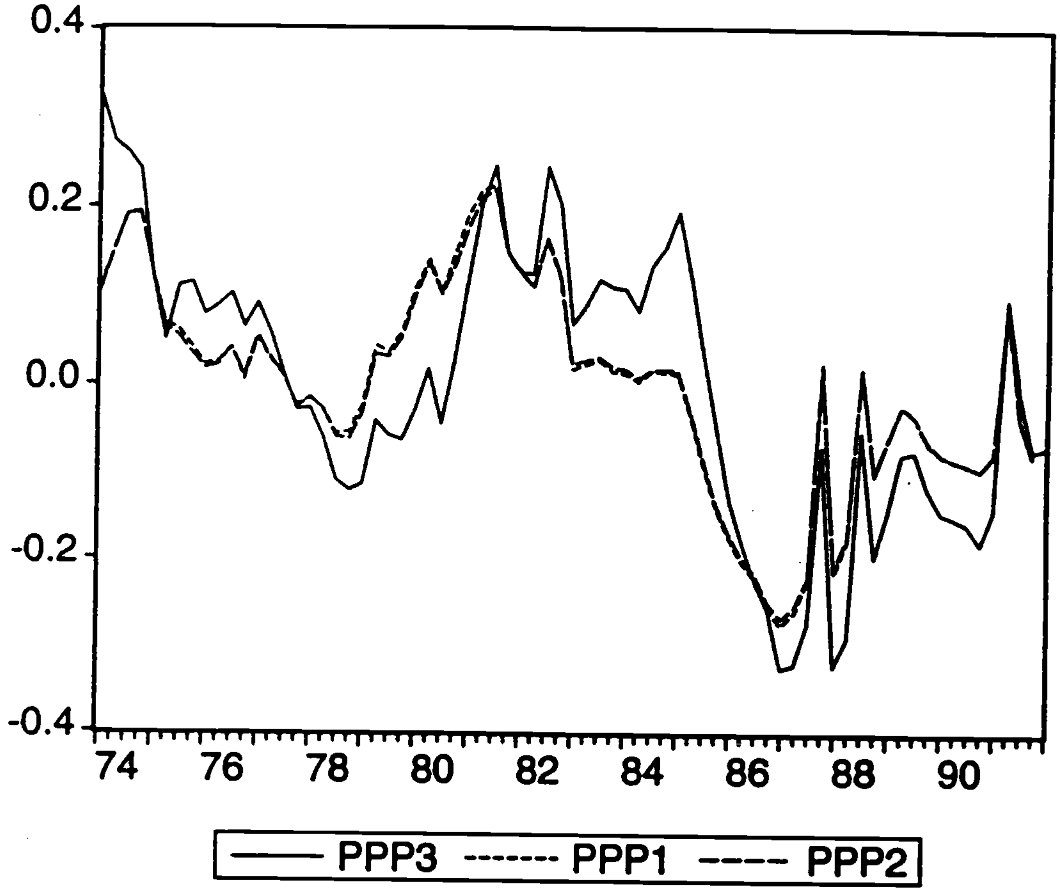

Figure 3: Cointegration Residuals for Germany

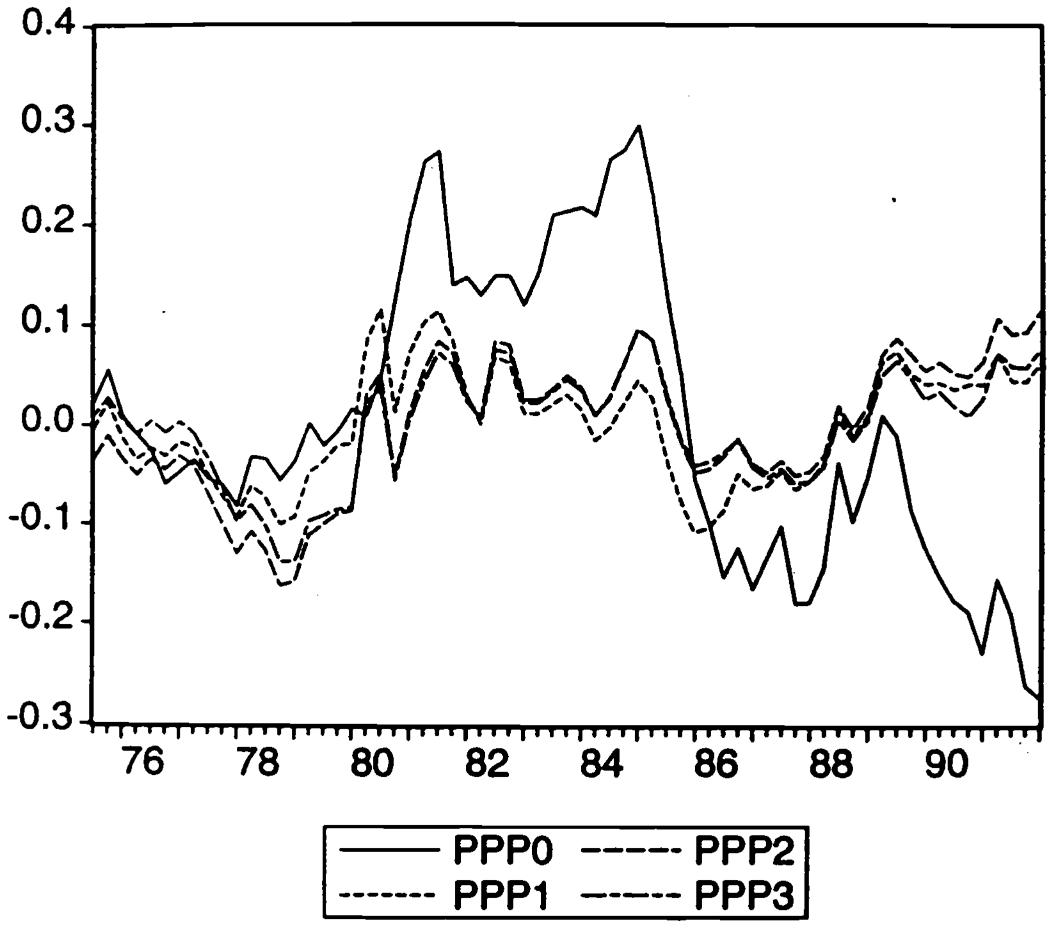

Figure 4: Cointegrating Residuals for Japan

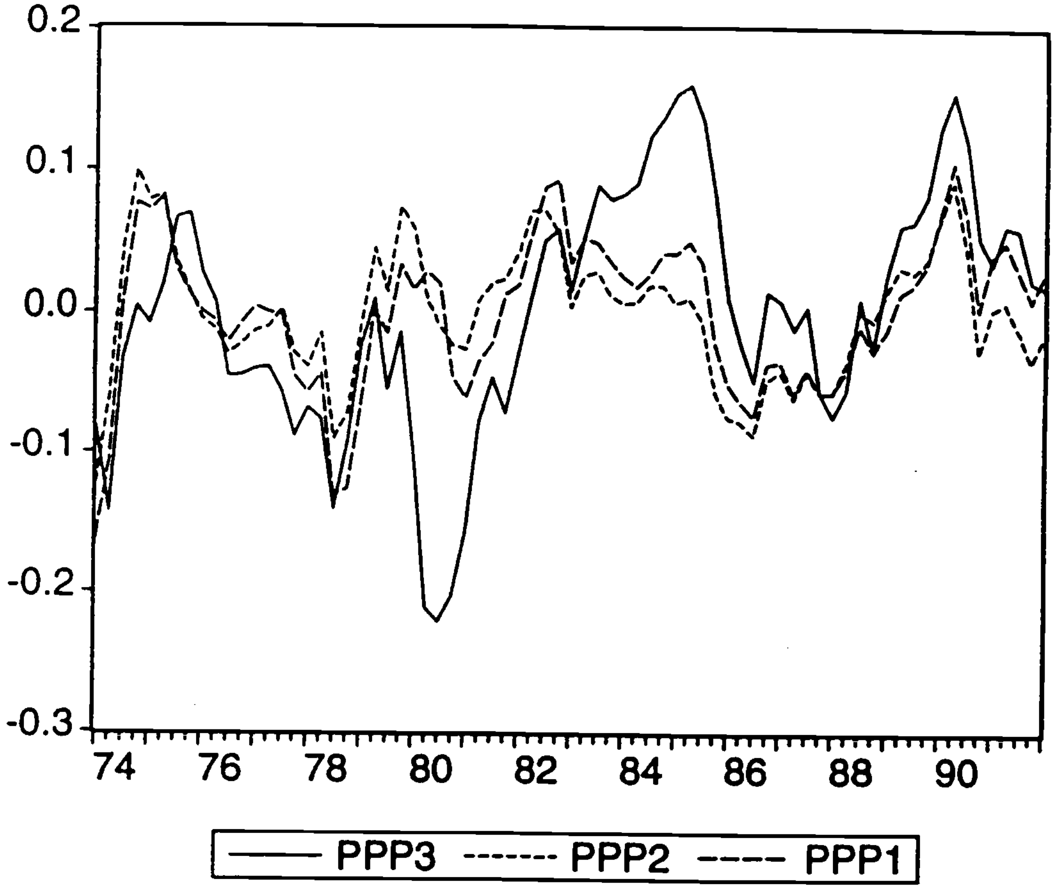

Figure 5: Cointegrating Residuals for the U.K.