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THE MUNDELL-FLEMING MODEL: A QUARTER CENTURY LATER

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ABSTRACT

The Mundell-Fleming model of international macroeconomics originated in the writings of Robert A. Mundell and J. Marcus Fleming in the early 1960s. The key contribution of the model has been a systematic analysis of the role played by international capital mobility in determining the effectiveness of macroeconomic policies under alternative exchange rate regimes. During the ensuing quarter century, the model was extended in various directions and is still the main "work horse" of traditional open-economy macroeconomics.

This paper develops an exposition that integrates the various facets of the model and incorporates its extensions into a unified analytical framework. Attention is given to the distinction between short-run and long-run effects of policies, the implication of debt and tax financing of government expenditures, the role of the exchange rate regime in this regard, and debt revaluation and trade-balance revaluation effects associated with exchange rate changes. The resulting integration clarifies the key economic mechanisms operating in the Mundell-Fleming model and helps to identify its limitations. Among these is the neglect of intertemporal budget constraints and of the consequences of forward-looking behavior consistent with this constraint. The formulation in the paper casts the model in a manner that facilitates comparisons with more modern approaches. In so doing, the exposition provides a bridge between the traditional and the more modern approaches to international macroeconomics.

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I. Introduction

This paper is an exposition of the Mundell-Fleming model of international macroeconomics. The foundations of the model were laid a quarter century ago in the classic writings of Robert A. Mundell (1960, 1961a, 1961b, 1963, 1964, collected in 1968) and J. Marcus Fleming (1962). The key contribution of this model has been a systematic analysis of the role played by international capital mobility in determining the effectiveness of macroeconomic policies under alternative exchange rate regimes. The analysis extended the simple version of the Keynesian income-expenditure model developed by Machlup (1943) and Metzler (1942) as well as the policy-oriented model developed by Meade (1951) to economies open to international trade in both goods and financial assets. Over the years the model has been extended in various directions and is still the main "work horse" of traditional open-economy macroeconomics. Noteworthy among such extensions are: a stock (portfolio) specification of capital mobility by McKinnon (1969), Branson (1970), Floyd (1969) and Frenkel and Rodriguez (1975); an analysis of debt-revaluation effects induced by exchange rate changes by Boyer (1977) and Rodriguez (1979); a long-run analysis by Rodriguez (1979); and an analysis of expectations and exchange rate dynamics by Kouri (1976) and Dornbusch (1976). A recent critical evaluation of the model is provided by Purvis (1985). 1/

1/ Expositions of the model for alternative exchange rate regimes and for different degrees of international capital mobility are presented in Swoboda and Dornbusch (1973) and Mussa (1979). The diagrammatic analysis used in this paper builds in part on these two expositions. Recent surveys of various open-economy-macroeconomic issues, discussed in the context of this model, are contained in Frenkel and Mussa (1985) and Kenen (1985). In addition, Marston (1985) surveys applications of the model to the analysis of stabilization policies, and Obstfeld and Stockman (1985) contains a survey of exchange rate dynamics in this and other models. The most comprehensive treatment of the Mundell-Fleming model to date is provided by Dornbusch (1980).

The purpose of the present paper is to provide an exposition which integrates the various facets of the model into a unified analytical framework. Our specification of the model incorporates the various extensions. Special focus is given to the distinction between short-run and long-run consequences of policies, the implications of debt and tax finance of government budget, and the role of the exchange rate regime in this regard. The resulting integration clarifies the key economic mechanisms operating in the Mundell-Fleming model and helps identify its limitations. Our formulation casts the model in a way which facilitates possible comparisons with more modern approaches. In so doing the exposition provides a bridge between the traditional and the more modern approaches to international macroeconomics.

The specification of the model is sufficiently general to permit an analysis of a wide variety of macroeconomic policies. To conserve on space, however, we choose to illustrate the working of the model by focusing on the instrument of fiscal policy.

The organization of the paper is as follows. Section II outlines the analytical framework. Section III deals with the operation of the economic system under a fixed exchange rate regime. In this context we first analyse the small-country case and then proceed to analyse the two-country model of the interdependent world economy. Section IV contains a parallel analysis appropriate for the flexible exchange rate regime. Section V is an integrative summary and an overview of the Mundell-Fleming model. To facilitate the exposition, the main analysis is carried out diagrammatically. The Appendices that follow the text contain algebraic derivations and a formal treatment of exchange-rate expectations.

II. The Analytical Framework

Consider a two-country model of the world economy. The two countries are referred to as the home (domestic) country and the foreign country. Each country produces a distinct commodity: the domestic economy produces good x and the foreign economy produces good m. The domestic level of output is denoted by Y and the foreign level of output by Y^*. In specifying the behavioral functions it is convenient to focus on the domestic economy. Accordingly the budget constraint is

$$(1) \qquad Z_t + M_t - B_t^p = P_t(Y_t - T_t) + M_{t-1} - R_{t-1}B_{t-1}^p$$

where B_t^p denotes the domestic-currency value of private sector's one-period debt issued in period t, and R_t denotes one plus the rate of interest. The right-hand side of equation (1) states that in each period, t, the resources available to individuals are composed of disposable income, $P_t(Y_t-T_t)$--where the GDP deflator is P_t, domestic output is Y_t and taxes are T_t--and the net value of assets carried over from period t-1. The latter consist of money, M_{t-1}, net of debt commitment $R_{t-1}B_{t-1}^p$ (where the latter includes principal plus interest payments). For subsequent use we denote these assets by A_{t-1} where

$$(2) \qquad A_{t-1} = M_{t-1} - R_{t-1}B_{t-1}^p$$

The left-hand side of equation (1) indicates the uses of these resources including nominal spending, Z_t, money holding, M_t, and bond holding, $-B_t^p$.

In conformity with the original Mundell-Fleming formulation the GDP deflator, P_t, is assumed to be fixed and is normalized to unity. In that case nominal spending also equals real spending E_t. Due to the absence of changes in prices we identify the real rate of interest, $r_t = R_t - 1$, with the corresponding nominal rate of interest (we return to this issue later where we analyze the implications of exchange rate changes).

Assuming that the various demand functions depend on the available resources and on the rate of interest, we express the spending and the money-demand function as

$$(3) \qquad E_t = E(Y_t - T_t + A_{t-1}, r_t)$$

$$(4) \qquad M_t = M(Y_t - T_t + A_{t-1}, r_t)$$

In specifying these functions we assume for simplicity that the marginal propensities to spend and to hoard out of disposable income are the same as the corresponding propensities to spend and hoard out of assets. A similar specification underlies the demand for bonds which is omitted due to the budget constraint. We assume that desired spending and money holdings depend positively on available resources and negatively on the rate of interest.

The domestic private sector is assumed to allocate its spending between domestic goods, C_{xt}, and foreign goods, C_{mt}. The real value of domestic spending, E_t, is $C_{xt} + p_{mt}C_{mt}$, where p_{mt} denotes the <u>relative</u> price of good m in terms of good x. This relative price is assumed to be equalized across countries through international trade. The relative share of domestic spending on good m (the foreign good) is denoted by $\beta_m = p_{mt}C_{mt}/E_t$.

The level of real government spending in period t, measured in terms of own GDP, is denoted by G_t. Analogously to the private sector, the

Government also allocates its spending between the two goods. Domestic government spending on importables (good m) is $\beta_m^g G_t/p_{mt}$.

A similar set of demand functions and government spending patterns characterize the foreign economy whose variables are denoted by an asterisk and its fixed GDP deflator, P*, is normalized to unity. Analogously to the domestic economy the relative share of foreign private spending on good x (the good produced by the home country) is denoted by $\beta_x^* = C_{xt}^*/p_{mt}E_t^*$; correspondingly, the foreign government spending share on good x is β_x^{g*}.

The relative price of good m in terms of good x, p_{mt}, which is assumed to be equal across countries, can be written as $p_{mt} = e_t P_t^*/P_t$ where e_t is the nominal exchange rate expressing the price of the foreign currency in terms of the domestic currency. The specification of the equilibrium in the world economy depends on the exchange rate regime. We start with the analysis of equilibrium under a fixed exchange rate regime.

III. Capital Mobility with Fixed Exchange Rates

Equilibrium in the world economy necessitates that the markets for goods, money and bonds clear. Under a fixed exchange rate, domestic and foreign money (in their role as assets) are perfect substitutes. Therefore, money-market equilibrium can be specified by a single equilibrium relation stating that the world demand for money equals the world supply. Likewise, the assumptions that bonds are internationally tradable assets and that domestic and foreign bonds are perfect substitutes imply that in equilibrium the rate of return on domestic bonds, r_t, equals the corresponding rate on foreign bonds, r_{ft}, and that bond-market equilibrium can also be specified by a single equation pertaining to the unified world bond market. These considerations imply that the world economy can be characterized by four markets: the markets for domestic output, foreign output, world money, and world bonds. By Walras's Law the bond market can be omitted from the equilibrium specification of the two-county model of the world economy. Accordingly, the equilibrium conditions are

$$(5) \qquad (1-\beta_m)E(Y_t - T_t + A_{t-1}, r_t) + (1-\beta_m^g)G + \beta_x^* \bar{e} E^*(Y_t^* + A_{t-1}^*, r_t) = Y_t$$

$$(6) \qquad \beta_m E(Y_t - T_t + A_{t-1}, r_t) + \beta_m^g G + (1-\beta_x^*)\bar{e}E^*(Y_t^* + A_{t-1}^*, r_t) = \bar{e}Y_t^*$$

$$(7) \qquad M(Y_t - T_t + A_{t-1}, r_t) + \bar{e}M^*(Y_t^* + A_{t-1}^*, \bar{r}_t) = M$$

where $\bar{e}$ denotes the fixed exchange rate expressing the price of foreign currency in terms of domestic currency. To focus on the effects of the domestic government policy, we assume in what follows that foreign government spending and taxes are zero. The (predetermined) value of foreign assets is measured in foreign-currency units so that $A_{t-1}^* = M_{t-1}^* + R_{t-1}B_{t-1}^p/\bar{e}$. Due to the assumed fixity of the GDP deflators, $\bar{e}$ also measures the relative price of importables in terms of exportables. The world supply of money, measured in terms of domestic goods (whose domestic-currency price is unity) is denoted by M. In specifying equation (7) we assume that the government does not finance its spending through money creation. This permits a focus on the pure effects of fiscal policies.

The specification of the equilibrium system (5) - (7) embodies the arbitrage condition by which $r_t = r_{ft}$ so that the yields on domestic and foreign bonds are equal. This equality justifies the use of the same rate of interest in the behavioral functions of the domestic and the foreign economies. The system (5) - (7) determines the short-run equilibrium values of domestic output, Y_t, foreign output, Y_t^*, and the world rate of interest, r_t, for given (predetermined) values of domestic and foreign net assets, A_{t-1} and A_{t-1}^*, and for given levels of government spending, G_t, and taxes, T_t.

The international distribution of the given world money supply associated with the short-run equilibrium is determined endogenously according to the demands. Thus

$$(8) \qquad M_t = M(Y_t - T_t + A_{t-1}, r_t)$$

$$(9) \qquad M_t^* = M^*(Y_t^* + A_{t-1}^*, r_t)$$

This equilibrium distribution of the world money supply obtains through international asset swaps.

This formulation of the short-run equilibrium system reveals the significant role played by international capital mobility. In the absence of such mobility, the short-run equilibrium would have determined the levels of domestic and foreign output from the goods-market equilibrium conditions. Associated with these levels of outputs there would be equilibrium monetary <u>flows</u>. These flows cease in the long run in which a stationary equilibrium distribution of the world money supply obtains. In contrast, the equilibrium system (5) - (7) shows that with perfect capital mobility equilibrium in the world money-market obtains through instantaneous asset swaps involving exchanges of money for bonds. These instantaneous <u>stock</u> adjustments are reflected in equation (7).

1. Fiscal policies in a small country

To illustrate the effects of fiscal policies under a regime of fixed exchange rates with perfect capital mobility, it is convenient to begin with an analysis of a small country facing a given world rate of interest, $\bar{r}_f$, and a given world demand for its goods, $\overline{D}^* = \beta_x^* E^*$. Under these circumstances the equilibrium condition for the small economy reduces to

$$(5') \qquad (1-\beta_m)E(Y_t - T_t + A_{t-1}, \bar{r}_f) + (1-\beta_m^g)G + \bar{e}\,\overline{D}^* = Y_t$$

This equilibrium condition determines the short-run value of output for the given (predetermined) value of assets and for given levels of government spending and taxes. The money supply, M_t, associated with this equilibrium is obtained from the money-market equilibrium condition (8'). Accordingly,

$$(8') \qquad M(Y_t - T_t + A_{t-1}, \bar{r}_f) = M_t$$

This quantity of money is endogenously determined through instantaneous asset swaps at the prevailing world rate of interest.

To analyze the effects of fiscal policies we differentiate equation (5'). Thus

$$(10) \qquad \frac{dY_t}{dG} = \frac{1-a^g}{s+a} \qquad \text{for } dT_t = 0$$

and

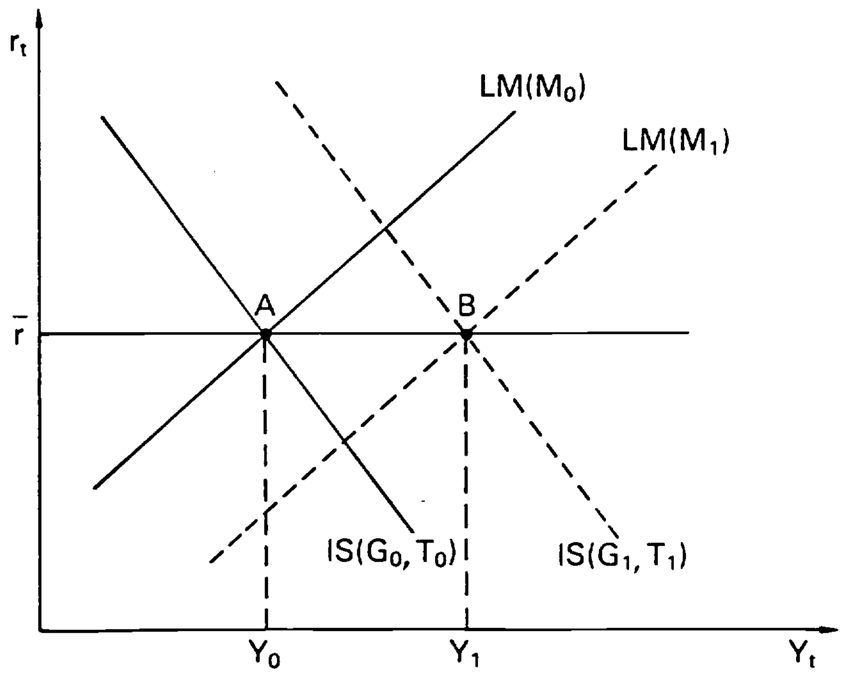
$$(11) \qquad \frac{dy_t}{dG} = 1 - \frac{a^g}{s+a} \qquad \text{for } dT_t = dG$$

where s and a denote, respectively, the domestic marginal propensities to save and to import out of income (or assets) and where $a^g = \beta_m^g$ is the government marginal propensity to import, and $1/(s+a)$ is the small-country foreign-trade multiplier. Equations (10) and (11) correspond, respectively, to a bond-financed and to a tax-financed rise in government spending. As is evident if all of government spending falls on domestic goods (so that $a^g = 0$), then the fiscal expansion which is financed by government borrowing raises output by the full extent of the foreign trade-multiplier, while the balanced-budget fiscal expansion yields the closed-economy balanced-budget multiplier of unity. If, on the other hand, all of government spending falls on imported goods (so that $a^g = 1$), then the bond-financed multiplier is zero whereas the balanced-budget multiplier is negative and equal to $(s+a-1)/(s+a)$.

The changes in output induce changes in the demand for money. The induced changes in money holding can be found by differentiating equation (8') and using (10) and (11). Accordingly, the debt-financed unit rise in government spending raises money holdings by $(1-a^g)M_y/(s+a)$ units where M_y denotes the effect of a rise in income on money demand (the inverse of the marginal income velocity). Likewise, the balanced-budget rise in government spending lowers money holdings by $a^gM_y/(s+a)$.

This analysis is summarized by Figure 1 in which the IS schedule portrays the goods market equilibrium condition (5'). It is negatively sloped since both a rise in the rate of interest and a rise in output create an excess supply of goods. The initial equilibrium obtains at point A at which the rate of interest equals the exogenously given world rate, $\bar{r}_f$, and the level of output is Y_0. As indicated, the schedule IS

Figure 1: The Short-Run Effects of Fiscal Policy Under Fixed Exchange Rates: The Small-Country Case



is drawn for given levels of government spending and taxes, G_0 and T_0. The LM schedule passing through point A portrays the money-market equilibrium condition (8'). It is positively sloped since a rise in income raises the demand for money while a rise in the rate of interest lowers money demand. As indicated, the LM schedule is drawn for a given level of (the endogenously determined) money stock, M_0.

A unit rise in government spending creates an excess demand for domestic product (at the prevailing level of output). If it is bond financed then the excess demand is $1-a^g$ units, and if it is tax financed then the excess demand is of $s+a-a^g$ units (which, depending on the relative magnitudes of the parameters, may be negative). The excess demand is reflected by a horizontal shift of the IS schedule from $IS(G_0)$ to $IS(G_1)$. As drawn, the IS schedule shifts to the right, reflecting the positive excess demand at the prevailing level of output. The new equilibrium obtains at point B at which the level of output rises to Y_1. This higher level of output raises the demand for money which is met instantaneously through an international swap of bonds for money that is effected through the world capital markets. The endogenous rise in the quantity of money from M_0 to M_1 is reflected in the corresponding rightwards displacement of the LM schedule from $LM(M_0)$ to $LM(M_1)$.

The foregoing analysis determined the <u>short-run</u> consequences of an expansionary fiscal policy. The instantaneous asset swap induced by the requirement of asset-market equilibrium alters the size of the economy's external debt. Specifically, if initially the economy was in a long-run equilibrium (so that $B_t^p = B_{t-1}^p = B^p$, $M_t = M_{t-1} = M$, $A_t = A_{t-1} = A$ and $Y_t = Y_{t-1} = Y$), then the fiscal expansion which raises short-run money holdings as well as the size of the external debt, raises the debt-service requirement and (in view of the positive rate of interest) lowers the value of net assets $M_t-(1+\overline{r}_f)B_t^p$ carried over to the subsequent period. This change sets in motion a dynamic process that is completed only when the economy reaches its new long-run equilibrium. We turn next to determine the long-run consequences of government spending.

The long-run equilibrium conditions can be summarized by the system (12) - (14):

$$(12) \qquad E(Y - T + M - (1+\overline{r}_f)B^p, \overline{r}_f) = Y - \overline{r}_f B^p - T$$

$$(13) \qquad (1-\beta_m)E(Y - T + M - (1+\overline{r}_f)B^p, \overline{r}_f) + (1-\beta_m^g)G + \overline{e}\,\overline{D}^* = Y$$

$$(14) \qquad M(Y - T + M - (1+\overline{r}_f)B^p, \overline{r}_f) = M$$

where the omission of the time subscripts indicates that in the long run the various variables do not vary over time. Equation (12) is obtained from the budget constraint (1) by using the spending function from equation (3) and by imposing the requirement that in the long run $M_t = M_{t-1}$ and $B_t^p = B_{t-1}^p$. This equation states that in the long run private-sector spending equals disposable income, so that private-sector savings are zero. Equation (13) is obtained from (5') and (8') together with the long-run stationarity requirement. This equation is the long-run market clearing condition for domestic output. Finally, equation (14), which is the long-run counterpart to equation (8'), is the condition for long-run money-market equilibrium.

Up to this point we have not incorporated explicitly the government budget constraint. In the absence of money creation the long-run government budget constraint states that government outlays on purchases, G, and debt service, $\bar{r}_f B^g$ (where B^g denotes government debt), must equal taxes, T. Accordingly,

$$(15) \qquad G + \bar{r}_f B^g = T$$

Substituting this constraint into equation (12) yields

$$(12') \qquad E(Y - G + M - B^p - \bar{r}_f(B^p + B^g), \bar{r}_f) + G = Y - \bar{r}_f(B^p + B^g)$$

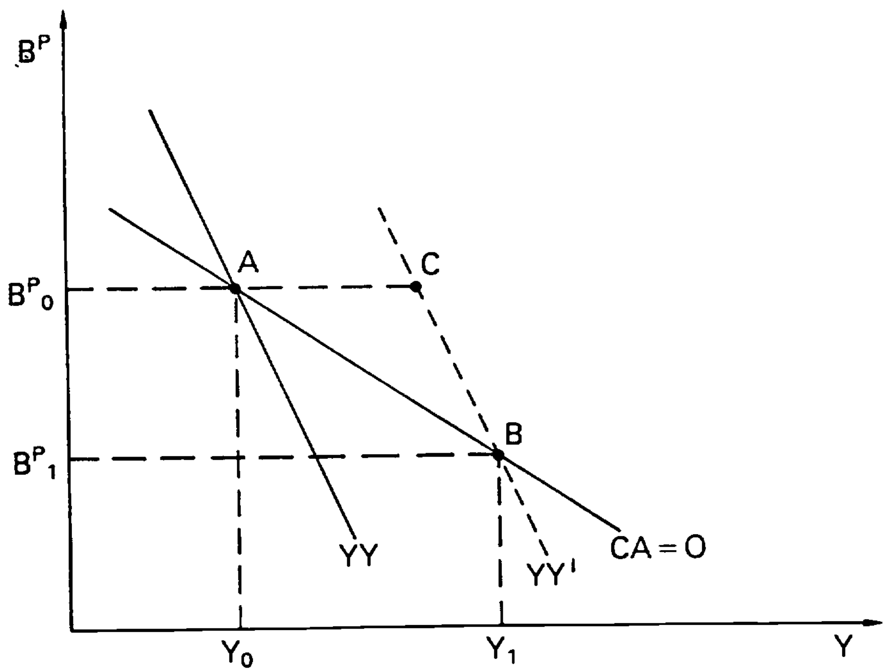
Equation (12') states that in the long run the sum of private-sector and government spending equals GNP. This equality implies that in the long run the current account of the balance of payments is balanced.

Using equations (12), (14) and (15) we obtain the combinations of output and debt that satisfy the long-run requirement of current account balance as well as money-market equilibrium. These combinations are portrayed along the CA = 0 schedule in Figure 2. Likewise, using equations (13) - (15) we obtain the combinations of output and debt that incorporate the requirements of goods and money-market equilibrium. These combinations are portrayed along the YY schedule in Figure 2. The slopes of these schedules are

$$(16) \qquad \frac{dB^p}{dY} = -\frac{(s-M_y)}{(1-s) - \bar{r}_f(s-M_y)} \qquad \text{along the CA = 0 schedule}$$

$$(17) \qquad \frac{dB^p}{dY} = -\frac{(s-M_y)+a}{(1+\bar{r}_f)(1-s-a)} \qquad \text{along the YY schedule}$$

Figure 2: The Long-Run Effects of A Unit Debt-Financed Rise in Government Spending Under Fixed Exchange Rates: The Small-Country Case



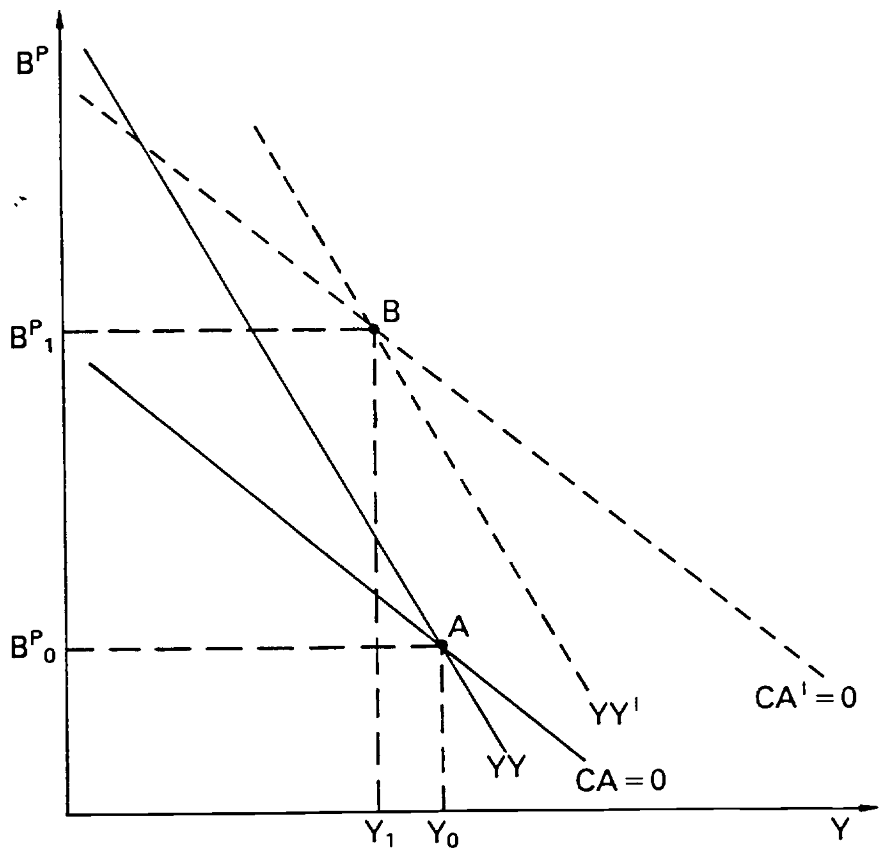
In these equations, the term M_y is the marginal propensity to hoard (the inverse of the marginal income velocity) and $s-M_y$ represents the marginal propensity to save in the form of bonds. As is evident the numerators in equations (16) - (17) are positive. The denominator of equation (17) is positive since $1-s-a > 0$ and the denominator of equation (16) is positive on the assumption that $(1-s) > \bar{r}_f\ (s-M_y)$. The latter assumption is a stability condition ensuring that the perpetual rise in consumption $(1-s)$ made possible by a unit rise in debt exceeds the perpetual return on the saving in bonds $\bar{r}_f(s-M_y)$ made possible by the initial unit rise in debt. If this inequality does not hold then consumption and debt rise over time and do not converge to a long-run stationary equilibrium. The foregoing discussion implies that the slopes of both the CA = 0 and the YY schedules are negative. Further, since the numerator of (17) exceeds the one in (16) and the denominator of (17) is smaller than the one in (16), the YY schedule in Figure 2 is steeper than the CA = 0 schedule. The initial long-run equilibrium is indicated by point A in Figure 2 in which the levels of output and private-sector debt are Y_0 and B_0^p.

Consider the long-run effects of a debt-financed rise in government spending. As is evident by inspection of the system (12) - (14), as long as taxes remain unchanged, the CA = 0 (which is derived from equations (12) and (14)) remains intact. On the other hand the rise in government spending influences the YY schedule which is derived from equations (13) - (14). Specifically, to maintain goods-market equilibrium (for any given value of private-sector debt, B^p) a unit rise in government spending must be offset by $(1-a^g)/(s+a)$ units rise in output. Thus, as long as some portion of government spending falls on domestic goods so that $a^g < 1$, the YY schedule in Figure 2 shifts to the right. The new equilibrium is indicated by point B at which the level of output rises from Y_0 to Y_1 and private-sector debt falls to B_1^p. The new equilibrium is associated with a rise in money holdings, representing the cumulative surpluses in the balance of payments during the transition period.

A comparison between the short-run multiplier shown in equation (10) and the corresponding long-run multiplier (shown in equation (A-7) of the Appendix) reveals that the latter exceeds the former. In terms of Figure 2, in the short run the output-effect of the debt-financed rise in government spending is indicated by the point C whereas the corresponding long-run equilibrium is indicated by point B.

Consider next the effects of a tax-financed rise in government spending. Such a balanced-budget rise in spending alters the positions of both the CA = 0 and the YY schedules. Using equations (12) and (14) together with the balanced-budget assumption that $dG = dT$ it can be shown that a unit rise in government spending induces a unit rightward shift of the CA = 0 schedule. By keeping the value of Y-T intact and holding B^p constant, such a shift maintains the equality between private-sector spending and disposable income and it also satisfies the money-market equilibrium condition. Likewise using equations (13) and (14) together with the balanced-budget assumption, it is shown in the Appendix that as long as the government import propensity, a^g, is positive, the YY schedule shifts to the right by less than one unit. The resulting new long-run equilibrium is indicated by point B in Figure 3. For the case drawn, the long-run level of output falls from Y_0 to Y_1 and private-sector debt rises

Figure 3: The Long-Run Effects of a Unit Balanced-Budget Rise in Government Spending Under Fixed Exchange Rates: The Small-Country Case



from B_0^p to B_1^p. Since government debt remains unchanged, the rise in private-sector debt corresponds to an equal rise in the economy's external-debt position. In general, however, depending on the parameters, domestic output may either rise or fall in the long run.

The size of the long-run multiplier of the balanced-budget rise in government spending depends on the government import propensity. At the limit, if all government spending falls on domestic output so that $a^g = 0$, the long-run balanced-budget multiplier is unity. In this case the YY schedule in Figure 3 shifts to the right by one unit, the long-run level of output rises by one unit, and private-sector debt (and the economy's external debt) remains unchanged. At the other limit, if all government spending falls on foreign goods so that $a^g = 1$, the long-run balanced budget multiplier is negative. In that case the rise in the economy's external debt is maximized.

The comparison between the short-run balanced-budget multiplier shown in equation (11) with the corresponding long-run multiplier (shown in equation (A-10) of the Appendix) highlights the contrasts between the two. If the government propensity to spend on domestic goods $(1-a^g)$ equals the corresponding private-sector propensity (1-s-a), then the short-run multiplier is zero while the long-run multiplier is negative. On the other hand, if the government propensity $(1-a^g)$ exceeds the private-sector propensity (1-s-a), both the short and the long-run balanced budgets are negative, but the absolute value of the long-run multiplier exceeds the corresponding short-run multiplier. Finally, if government spending falls entirely on domestically produced goods (so that $a^g = 0$), then the short-run and the long-run multipliers are equal to each other and both are unity.

2. Fiscal policies in a two-country world

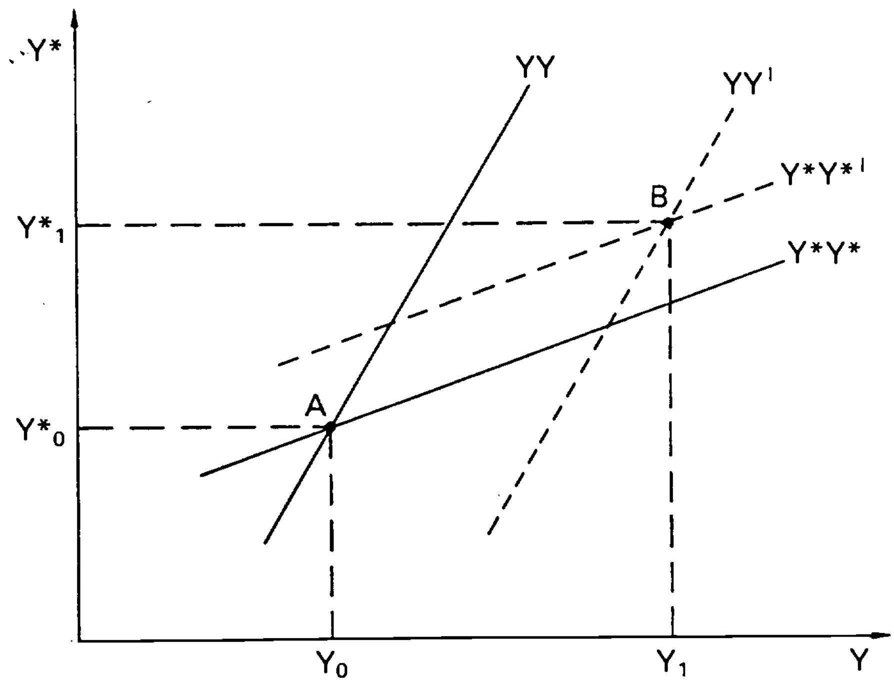
In this section we return to the two-country model outlined in equations (5) - (7) and analyze the short-run effects of a debt and tax-financed rise in government spending on the equilibrium levels of domestic and foreign outputs as well as on the equilibrium world rate of interest. The endogeneity of the last two variables distinguishes this analysis from the one conducted for the small country case. To conserve on space we do not analyze here the long-run effects; the formal system applicable to the long-run equilibrium of the two-country world is presented in Appendix I.3.

The analysis is carried out diagrammatically with the aid of Figures 4 and 5. In these figures the YY schedule portrays combinations of domestic and foreign levels of output which yield equality between the levels of production of domestic output and the world demand for it. Likewise, the Y*Y* schedule portrays combinations of output that yield equality between the level of production of foreign output and the world demand for it. The two schedules incorporate the requirement of equilibrium in the world money market. It is shown in the Appendix that the slopes of these schedules are

$$(18) \qquad \frac{dY_t^*}{dY_t} = \frac{1}{\bar{e}} \frac{(s+a)(M_r + \bar{e}M_r^*) + M_y H_r}{a^*(M_r + \bar{e}M_r^*) - M_{y^*}^* H_r} \qquad \text{along the YY schedule}$$

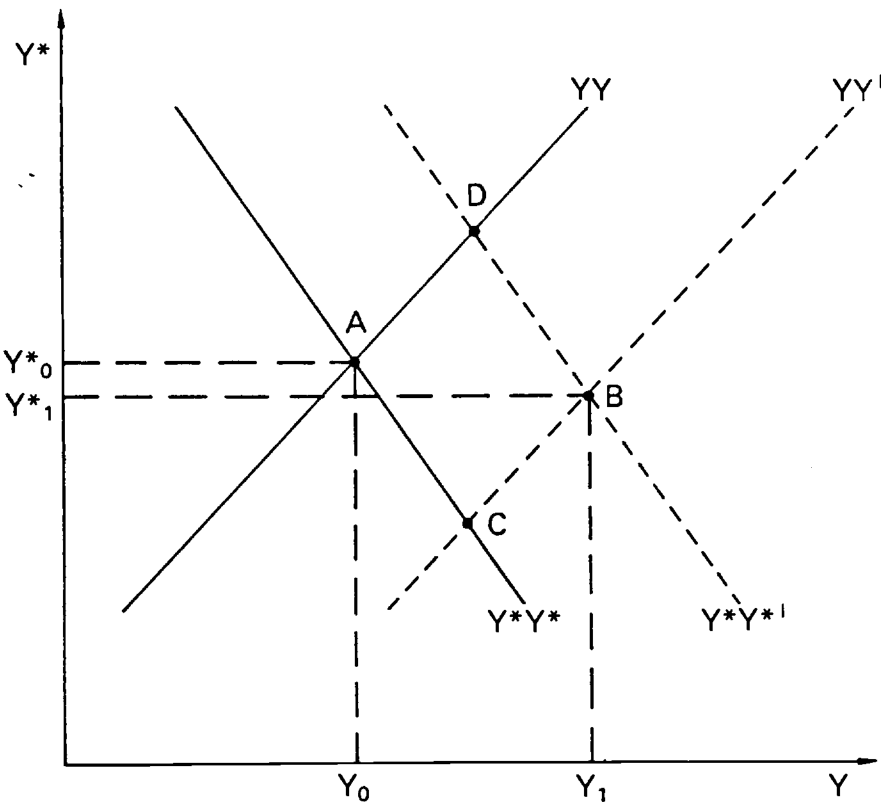
$$(19) \qquad \frac{dY_t^*}{dY_t} = \frac{1}{\bar{e}} \frac{a(M_r + \bar{e}M_r^*) - M_y F_r}{(s^*+a^*)(M_r + \bar{e}M_r^*) + M_{y^*}^* F_r} \qquad \text{along the Y*Y* schedule}$$

Figure 4: A Unit Debt-Financed Rise in Government Spending Under Fixed Exchange Rates: The Two-Country Case



Data: $a^* (M_r + \bar{e} \ M^*_r) - M^*_{y^*} H_r < 0$
$a (M_r + \bar{e} \ M^*_r) - M_y F_r < 0$

Figure 5: A Unit Debt-Financed Rise in Government Spending Under Fixed Exchange Rates: The Two-Country Case



Data: $a^* (M_r + \bar{e}\ M^*_r) - M^*_{y^*} H_r < 0$

$a (M_r + \bar{e}\ M^*_r) - M_y F_r > 0$

where H_r and F_r denote the partial (negative) effect of the rate of interest on the world demand for domestic and foreign outputs, respectively, and where E_r, M_r, E_r^* and M_r^* denote the partial (negative) effects of the rate of interest on domestic and foreign spending and money demand. As may be seen the slopes of the two schedules may be positive or negative. To gain intuition we note that in the special case for which spending does not depend on the rate of interest (so that $H_r = F_r = 0$) both schedules must be positively sloped. If on the other hand the rate of interest exerts a strong negative effect on world spending then the excess supply induced by a rise in one country's output may have to be eliminated by a fall in the other country's output. Even though this fall in foreign output lowers directly the foreign demand for the first country's exports, it also induces a decline in the world rate of interest which indirectly stimulates spending and may more than offset the direct reduction in demand. In that case market clearance for each country's output implies that domestic and foreign outputs are negatively related.

Even though the two schedules may be positively or negatively sloped, it may be verified (and is shown in the Appendix) that the YY schedule must be steeper than the Y*Y* schedule. This restriction leaves four possible configurations of the schedules. The common characteristic of these configurations is that starting from an initial equilibrium, if there is a rightwards shift of the YY schedule which exceeds the rightwards shift of the Y*Y* schedule, then the new equilibrium must be associated with a higher level of domestic output.

Two cases capturing the general pattern of world-output allocations are shown in Figures 4 and 5. The other possible configurations which are omitted do not yield different qualitative results concerning the effects of fiscal policies. In both figures the initial equilibrium is indicated by point A at which the domestic level of output is Y_0 and the foreign level is Y_0^*.

A debt-financed rise in government spending raises the demand for domestic output and induces a rightwards shift of the YY schedule from YY to YY'. On the other hand the direction of the change in the position of the Y*Y* schedule depends on the relative magnitudes of the two conflicting effects influencing world demand for foreign output. On the one hand the rise in the domestic government spending raises the demand for foreign goods but on the other hand the induced rise in the world rate of interest lowers the demand. If the Y*Y* schedule is positively sloped, as in Figure 4, then the rise in the domestic government spending induces a leftwards (upwards) shift of the Y*Y* schedule. The opposite holds if the Y*Y* schedule is negatively sloped as in Figure 5. The formal expressions indicating the magnitudes of the displacements of the schedules are provided in the Appendix.

The new equilibrium obtains at point B at which domestic output rises from Y_0 to Y_1. In the case shown in Figure 4 (for which the interest-rate effect on the world demand for foreign output is relatively weak) foreign output rises. On the other hand in the case shown in Figure 5 (for which the interest-rate effect on the world demand for foreign output is relatively strong) foreign output may rise or fall depending on the magnitude of the parameters, especially the composition of government spending. For example, if government spending falls entirely on domestic output (so that $a^g = 0$), the Y*Y* schedule does not shift and the new equilibrium obtains at a point like point C in Figure 5 at which foreign output falls. In the other extreme, if government spending falls entirely on foreign goods (so that $a^g = 1$) then the YY schedule does not shift and the new equilibrium obtains at a point like point D at which foreign output rises.

It is shown in the Appendix that, independent of the direction of output changes, the debt-financed rise in government spending must raise the world rate of interest. The expressions reported in the Appendix also reveal that if the (negative) interest-rate effect on the world demand for domestic output is relatively strong, then domestic output might fall. The balance of payments effects of the debt-financed rise in government spending are not clear cut, reflecting "transfer-problem criteria" familiar from the theory of international transfers. But, if the behavioral parameters of the domestic and foreign private sectors are equal to each other, then the balance of payments must improve and the domestic money holdings are raised.

A tax-financed rise in government spending also alters the positions of the various schedules as shown in the Appendix where we also provide the formal expressions for the various multipliers. In general, in addition to the considerations highlighted in the debt-financed case, the effect of a tax-financed fiscal spending also reflects the effects of the reduction in domestic disposable income on aggregate demand. This effect may more than offset the influence of government spending on domestic output. The effect on foreign output is also modified. If the interest-rate effect on world demand for foreign output is relatively weak (the case underlying Figure 4), then the shift from a debt to a tax-finance mitigates the expansion in foreign output. If on the other hand the interest-rate effect on the demand for foreign output is relatively strong (the case underlying Figure 5) then the shift from debt to tax finance exerts expansionary effects on foreign output.

It is shown in the Appendix that the direction of the change in the rate of interest induced by the tax-financed rise in government spending depends on a "transfer-problem criterion" indicating whether the redistribution of world disposable income consequent on the fiscal policy raises or lowers the world demand for money. Accordingly, the rate of interest rises if the domestic-country ratio, s/M_y, exceeds the corresponding foreign-country ratio, s^*/M^*_{y*}, and vice versa. Independent, however, of the change in the rate of interest, the tax-financed rise in government spending must deteriorate the domestic-country balance of payments and reduce its money holdings.

IV. Capital Mobility with Flexible Exchange Rates

In this section we assume that the world economy operates under a flexible exchange-rate regime. With this assumption national moneys become nontradable assets whose relative price (the exchange rate, e) is assumed to be determined freely in the world market for foreign exchange. We continue to assume that in each country, the GDP deflators, P and P*, are fixed and equal to unity. Under such circumstances the nominal exchange rates represent the terms of trade and the nominal rates of interest in each country equal the corresponding (GDP-based) real rates. Further, as was traditionally postulated in the early literature on modeling macroeconomic policies in the world economy, we start the analysis by assuming that exchange rate expectations are static. Under such circumstances the international mobility of capital brings about equality among national (GDP-based) real rates of interest. We return to the issue of exchange rate expectations in a subsequent section.

Equilibrium in the world economy requires that world demand for each country's ouput equals the corresponding supply and that in each country the demand for cash balances equals the supply. Accordingly, the system characterizing the equilibrium in the two-country world economy is:

$$(20) \qquad (1-\beta_m)E(Y_t - T_t + A_{t-1}, r_t) + (1-\beta_m^g)G + e_t\beta_x^* E^*(Y_t^* + A_{t-1}^*, r_t) = Y_t$$

$$(21) \qquad \beta_m E(Y_t - T_t + A_{t-1}, r_t) + \beta_m^g G + e_t(1-\beta_x^*)E^*(Y_t^* + A_{t-1}^*, r_t) = e_t Y_t^*$$

$$(22) \qquad M(Y_t - T_t + A_{t-1}, r_t) = M$$

$$(23) \qquad M^*(Y_t^* + A_{t-1}, r_t) = M^*$$

Equations (20) - (21) are the goods-market equilibrium conditions (analogous to equations (5) - (6)), and equations (22) - (23) are the domestic and foreign money-market equilibrium conditions where M and M* denote the supplies of domestic and foreign money. In contrast with the fixed exchange rate system in which each country's money supply was determined endogenously, here it is determined <u>exogenously</u> by the monetary authorities. We also note that by Walras's Law the world market equilibrium condition for bonds has been left out.

Finally, it is noteworthy that the value of securities may be expressed in terms of domestic or foreign currency units. Accordingly, the domestic-currency value of private-sector debt, B_t^p, can be expressed in units of foreign currency to yield $B_{ft}^p = B_t^p/e_t$. Arbitrage ensures that the expected rates of return on securities of different currency denomination are equalized. Accordingly, if r_t and r_{ft} are, respectively, the rates of interest on domestic and foreign-currency denominated bonds then $1+r_t = (\tilde{e}_{t+1}/e_t)(1+r_{ft})$ where $\tilde{e}_{t+1}$ denotes the expected future exchange rate. By equating r_t to r_{ft} the system (20) - (22) embodies the assumption of static exchange rate expectations and perfect capital mobility. In Appendix II.3 we return to the issue of exchange rate expectations.

1. Fiscal policies in a small country

Analogously with our procedure in the analysis of fiscal policies under fixed exchange rates we start the analysis of flexible exchange rates with an examination of the effects of fiscal policies in a small country facing a given world rate of interest, $\bar{r}_f$, and a given foreign demand for its goods, $\bar{D}^*$. The equilibrium conditions for the small country state that world demand for its output equals domestic GDP and that the domestic demand for money equals the supply. In contrast with the situation prevailing under a fixed exchange rate regime where the monetary authorities, committed to peg the exchange rate, do not control the domestic money supply, under a flexible exchange rate regime the supply of money is a policy instrument controlled by the monetary authorities.

The goods and money-markets equilibrium conditions are

$$(20') \qquad (1-\beta_m)E(Y_t - T_t + A_{t-1}, \bar{r}_f) + (1-\beta_m^g)G + e_t\bar{D}^* = Y_t$$

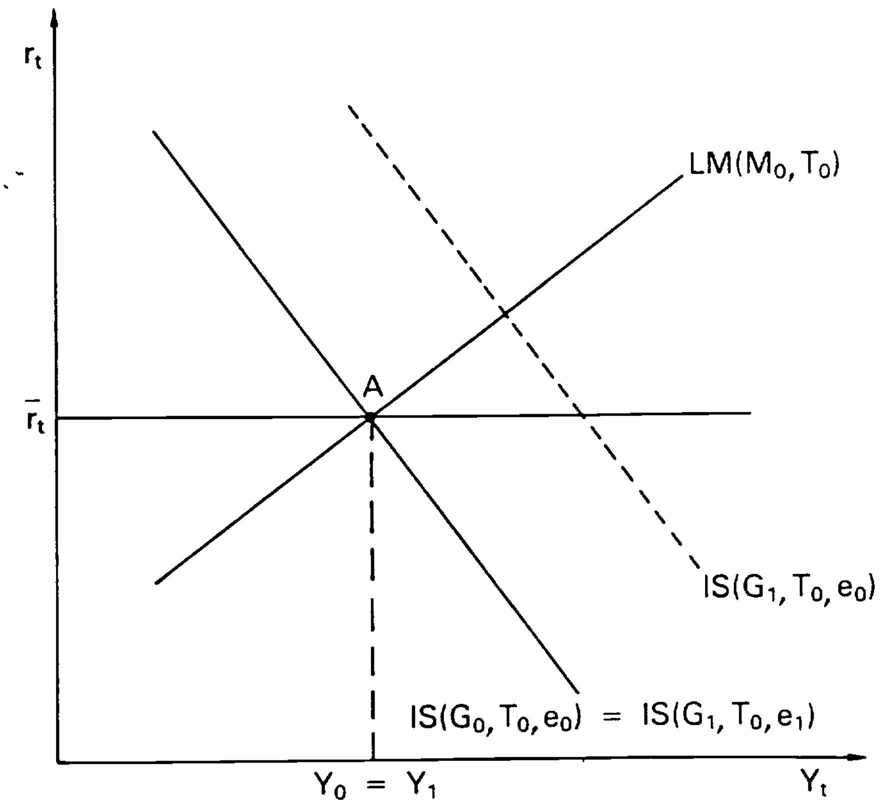
$$(22') \qquad M(Y_t - T_t + A_{t-1}, \bar{r}_f) = M$$

where $A_{t-1} = M_{t-1} - (1+\bar{r}_f)e_t B^p_{f,t-1}$.

As indicated, the valuation of the foreign-currency denominated debt commitment, $(1+\bar{r}_f)B^p_{f,t-1}$, employs the current exchange rate, e_t. These equilibrium conditions determine the short-run values of output and the exchange rate and for comparison we recall that under the fixed exchange rate regime the money supply rather than the exchange rate was endogenously determined.

The equilibrium of the system is exhibited in Figure 6. The downwards sloping IS schedule shows the goods-market equilibrium condition (20'). It is drawn for given values of government spending, taxes, and the exchange rate (representing the terms of trade). The upwards sloping LM schedule portrays the money-market equilibrium condition (22'). It is drawn for given values of the money supply, the exchange rate and taxes. The initial equilibrium obtains at point A at which the rate of interest equals the world rate, $\bar{r}_f$, and the level of output is Y_0. The endogenously-determined exchange rate associated with this equilibrium is e_0. It is relevant to note that in this system if the initial debt $B^p_{f,t-1}$ is zero, the LM schedule does not depend on the exchange rate and the level of output is determined exclusively by the money-market equilibrium condition whereas (given the equilibrium level of output) the equilibrium exchange rate is determined by the goods-market equilibrium condition. This case underlies Figure 6. Again a comparison with the fixed exchange-rate system is relevant. There, the equilibrium money stock is determined by the money-market equilibrium condition whereas the equilibrium level of output is determined by the goods-market equilibrium condition.

Figure 6: The Short-Run Effects of A Unit Debt-Financed Rise in Government Spending Under Flexible Exchange Rates: The Small-Country Case

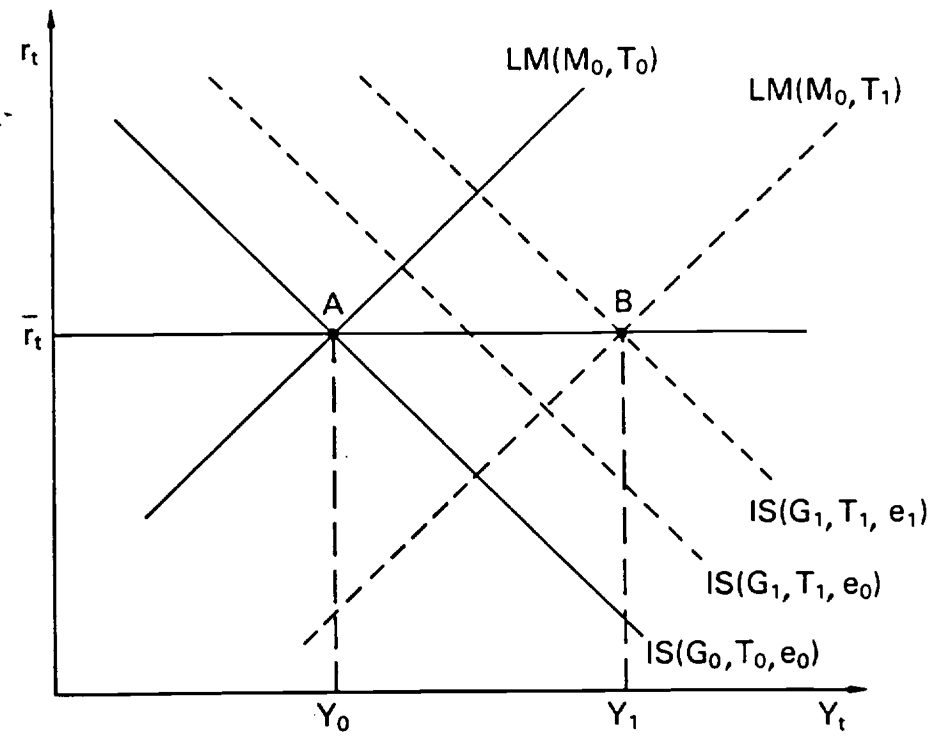


Data: $B^P_{f,-1} = 0$

Consider the effects of a debt-financed unit rise in government spending from G_0 to G_1 and suppose that the initial debt commitment is zero. At the prevailing levels of output and the exchange rate, this rise in spending creates an excess demand for domestic output and induces a rightwards shift of the IS schedule by $(1-a^g)/(s+a)$ units. This shift is shown in Figure 6 by the displacement of the IS schedule from the initial position indicated by $IS(G_0, T_0, e_0)$ to the position indicated by $IS(G_1, T_0, e_0)$. Since with zero initial debt the LM schedule is unaffected by the rise in government spending, it is clear that given the world rate of interest the level of output that clears the money market must remain at Y_0 corresponding to the initial equilibrium indicated by point A. To restore the initial equilibrium in the goods market the exchange rate must fall (that is, the domestic currency must appreciate). The induced improvement in the terms of trade lowers the world demand for domestic output and induces a leftwards shift of the IS schedule. The goods market clears when the exchange rate falls to e_1 so that the IS schedule indicated by $IS(G_1, T_0, e_1)$ also goes through point A. We conclude that under flexible exchange rates with zero initial debt a debt-financed fiscal policy loses its potency to alter the level of economic activity; its full effects are absorbed by changes in the exchange rate (the terms of trade).

Consider next the effects of a tax-financed unit rise in government spending from G_0 to G_1, shown in Figure 7. In that case, at the prevailing levels of output and the exchange rate, the excess demand for domestic output induces a rightwards displacement of the IS schedule by $1-a^g/(s+a)$ units to the position indicated by $IS(G_1, T_1, e_0)$. In addition,

Figure 7: The Short-Run Effects of
A Unit Tax-Financed Rise in
Government Spending Under
Flexible Exchange Rates:
The Small-Country Case



Data: $B^P_{f,-1} = 0$

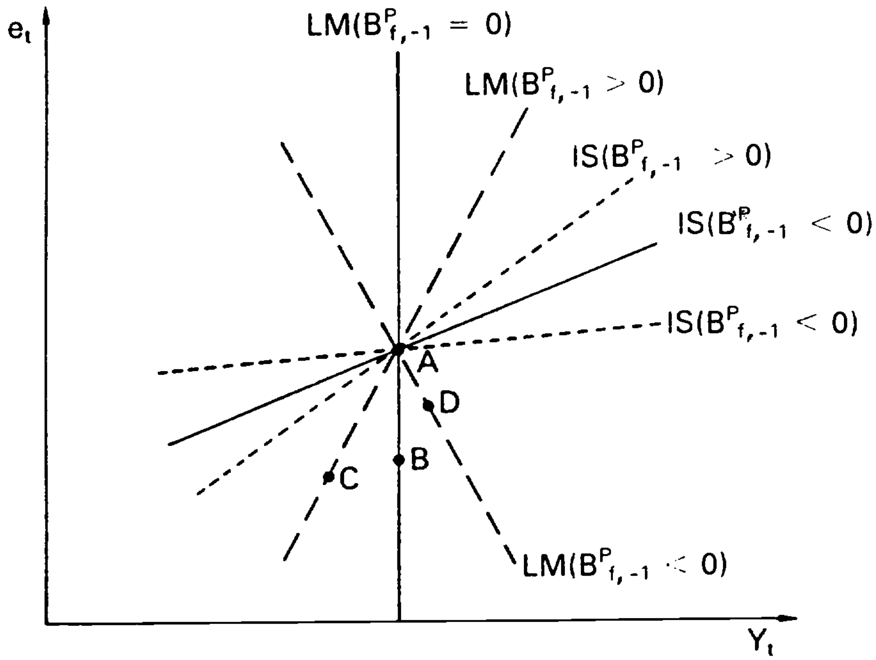
the unit rise in taxes lowers disposable income by one unit and reduces the demand for money. To maintain money-market equilibrium at the given world rate of interest the level of output must rise by one unit so as to restore the initial level of disposable income. Thus, the LM schedule shifts to the right from its initial position indicated by LM(M_0, T_0) to the position indicated by LM(M_0, T_1). With a zero level of initial debt (the case assumed in the figure), the LM schedule does not depend on the value of the exchange rate and the new equilibrium obtains at point B where the level of output rises by one unit from Y_0 to Y_1. Since at the initial exchange rate the horizontal displacement of the IS schedule is less than unity (as long as government spending falls in part on imported goods) it follows that at the level of output which clears the money-market there is an excess supply of goods. This excess supply is eliminated through a rise in the exchange rate (that is, a depreciation of the domestic currency) from e_0 to e_1. This deterioration in the terms of trade raises the world demand for domestic output and induces a rightwards shift of the IS schedule to the position indicated by IS(G_1, T_1, e_1). We conclude that under flexible exchange rates with zero initial debt the tax-financed rise in government spending regains its full potency in effecting the level of economic activity.

Up to this point we have assumed that the initial debt position was zero. As a result, the only channel through which the exchange rate influenced the system was through altering the domestic-currency value of the exogenously given foreign demand, $\overline{D}^*$. In general, however, with a non-zero level of initial debt, $B^p_{f,t-1}$ (denominated in units of foreign currency), the change in the exchange rate also alters the domestic currency value of the initial debt and, thereby, of the initial assets, A_{t-1}. The revaluation of the debt commitment constitutes an additional channel through which the exchange rate influences the economic system. As a result, the demand for money and thereby the LM schedule also depend on the exchange rate.

To appreciate the role played by debt-revaluation effects we examine in Figure 8 the implications of a non-zero level of initial debt. The various IS and LM schedules shown in the Figure correspond to alternative assumptions concerning the level of initial debt $B^p_{f,t-1}$, and the rest of the arguments governing the position of the schedules are suppressed for simplicity. The initial equilibrium is shown by point A and the solid schedules along which $B^p_{f,t-1} = 0$ corresponds to the cases analyzed in Figures 6 and 7. With a positive value of initial debt a rise in the exchange rate lowers the value of assets and lowers the demand for money. Restoration of money-market equilibrium requires a compensating rise in output. As a result the LM in that case is positively sloped. By a similar reasoning a negative value of initial debt corresponds to a negatively sloped LM schedule. The level of initial debt also influences the slope of the IS schedule. As shown in the Figure, using similar considerations, the IS schedule is steeper than the benchmark schedule (around point A) if $B^p_{f,t-1} > 0$, and vice versa.

We can now use this Figure to illustrate the possible implications of the initial debt position. For example, a debt-financed fiscal expansion induces a rightward shift of the IS schedule and leaves the LM schedule intact. The short-run equilibrium of the system is changed from point A to point B if the level of initial debt is zero, to point C

Figure 8: The Short-Run Effects of A Unit Debt-Financed Rise in Government Spending Under Flexible Exchange Rates: The Debt-Revaluation Effect



if the level of initial debt is positive, and to point D if this level is negative. Thus, the debt revaluation effects critically determine whether a debt-financed rise in government spending is contractionary or expansionary.

Using the system (20') and (22'), the changes in the level of output are

$$(24) \qquad \frac{dY_t}{dG} = \frac{(1-a^g)(1+\bar{r}_f)B^p_{f,t-1}}{(1+\bar{r}_f)B^p_{f,t-1} - \bar{D}^*} \qquad \text{for } dT_t = 0$$

$$(25) \qquad \frac{dY_t}{dG} = 1 - \frac{a^g(1+\bar{r}_f)B^p_{f,t-1}}{(1+\bar{r}_f)B^p_{f,t-1} - \bar{D}^*} \qquad \text{for } dT_t = dG$$

Likewise, the induced changes in the exchange rates are

$$(26) \qquad \frac{de_t}{dG} = \frac{1-a^g}{(1+\bar{r}_f)B^p_{f,t-1} - \bar{D}^*} \qquad \text{for } dT_t = 0$$

and

$$(27) \qquad \frac{de_t}{dG} = \frac{-a^g}{(1+\bar{r}_f)B^p_{f,t-1} - \bar{D}^*} \qquad \text{for } dT_t = dG$$

These results highlight the role played by the debt-revaluation effect of exchange rate changes. Specifically, as is evident from equations (24) - (25) a rise in government spending may be contractionary if the initial debt commitment is positive. If, however, the private sector is initially a net creditor then, independent of its means of finance, government spending must be expansionary. In the benchmark case shown in Figures 6 and 7, the initial debt position is zero, a tax finance is expansionary (yielding the conventional balanced-budget multiplier of unity), and a debt finance is not. The key mechanism responsible for this result is the high degree of capital mobility underlying the fixity of the rate of interest faced by the small country. With the given rate of interest and with a given money supply, there is in the short run a unique value of disposable income that clears the money market as long as the initial debt commitment is zero. Hence, in this case, a rise in taxes is expansionary and a rise in government spending is neutral.

A comparison between the exchange rate effects of government spending also reveals the critical importance of the means of finance and of the debt-revaluation effect. In general, for the given money supply, the direction of the change in the exchange rate induced by a rise in government spending depends on whether the government finances its spending through taxes or through debt issue. If the initial debt commitment falls short of the (exogenously given) foreign demand for domestic output, then a debt-financed rise in government spending appreciates the currency while a tax-financed rise in government spending depreciates the currency. The opposite holds if the initial debt commitment exceeds exports.

The foregoing analysis determined the short-run effects of government spending. We proceed to analyze the long-run effects of these policies. The long-run equilibrium conditions are shown in equations (28) - (30) below. These equations are the counterpart to the long-run fixed exchange rate system (12) - (14). Accordingly,

$$(28) \qquad E(Y - T + M - (1+\bar{r}_f)eB_f^p, \bar{r}_f) = Y - \bar{r}_f eB_f^p - T$$

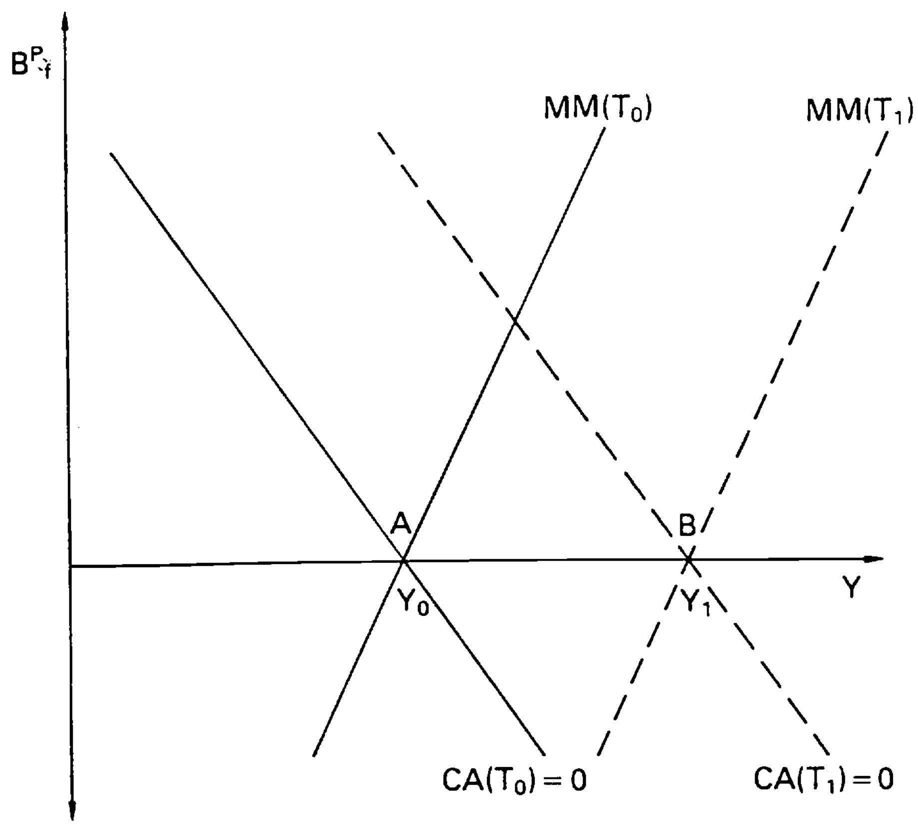
$$(29) \qquad (1-\beta_m)E(Y - T + M - (1+\bar{r}_f)eB_f^p,\ \bar{r}_f) + (1-\beta_m^g)G + e\bar{D}^* = Y$$

$$(30) \qquad M(Y - T + M - (1+\bar{r}_f)eB_f^p,\ \bar{r}_f) = M$$

To set the stage for the analysis, consider first the bench-mark case for which the initial equilibrium was associated with a zero private-sector debt. For this case the long run is analyzed in Figure 9. The CA = 0 schedule portrays combinations of private-sector debt and output which yield equality between spending and income, and thereby satisfying equation (28). In view of the government budget constraint shown in equation (15), this equality between private-sector income and spending also implies current account balance. The MM schedule portrays combinations of debt and output which yield money market equilibrium, and thereby satisfy equation (30). Around zero private-sector debt, both of these schedules are independent of the exchange rate. The slope of the CA = 0 schedule is $-s/e(1-s(1+\bar{r}_f))$. Analogously to the previous discussion of the long-run equilibrium under fixed exchange rates, this slope is assumed negative for stability. The slope of the MM schedule is $1/(1+\bar{r}_f)e$. It indicates that a unit rise in long-run private sector debt raises debt commitment (principal plus debt service) by $(1+\bar{r}_f)e$ and lowers the demand for money. To offset the reduction in disposable resources and restore the demand for money to its initial level, output must be raised by $(1+\bar{r}_f)e$ units.

The initial long-run equilibrium is shown by point A at which the level of private-sector debt is assumed to be zero and the level of output is Y_0. As is evident from equations (28) and (30), changes in the levels of government spending and government debt do not alter the CA = 0

Figure 9: The Long-Run Effects of
A Unit Rise in Government Spending
Under Flexible Exchange Rates:
The Small-Country Case



schedule and the MM schedule. It follows that with zero private-sector debt a debt-financed rise in government spending does not alter the long-run equilibrium value of private sector debt indicated by point A in Figure 9. In this long-run equilibrium the level of output remains unchanged and the currency appreciates to the level shown in the short-run analysis of Figure 6.

A rise in taxes alters both the CA = 0 and the MM schedules. As is evident from equations (28) and (30) a rise in output which keeps disposable income unchanged (at the given zero level of private-sector debt) maintains the initial current account balance as well as money-market equilibrium intact. Thus, a tax-financed unit rise in government spending induces a unit rightwards displacement of both the CA = 0 and the MM schedules and yields a new long-run equilibrium at point B. At this point private-sector debt remains at its initial zero level. Also, the level of output rises to Y_1 and the currency depreciates to e_1 as shown in the short-run analysis of Figure 7.

The above discussion shows that under flexible exchange rates with zero initial private-sector debt the long-run and the short-run effects of fiscal policies coincide. This characteristic is in contrast to the one obtained for fixed exchange rates where the long-run effects of fiscal policies differ from the corresponding short-run effects. In interpreting these results we note that due to the non-tradability of national monies under a flexible exchange rate regime, the mechanism of adjustment to fiscal policies does not permit instantaneous changes in the composition of assets through swaps of interest bearing assets for national money in the world capital markets. As a result the only mechanism by which private-sector debt can change is through savings. Since with zero initial private-sector debt both debt-financed and tax-financed government spending do not alter disposable income (as seen from equations (24) - (25)), it follows that these policies do not affect private-sector saving. Hence, if the initial position was that of a long-run equilibrium with zero savings and zero debt, the instantaneous short-run equilibrium following the rise in government spending is also characterized by zero savings. This implies that the economy converges immediately to its new long-run equilibrium.

The foregoing analysis of the long-run consequences of government spending abstracted from the debt-revaluation effect arising from exchange rate changes. In general, if in the initial equilibrium the level of private-sector debt differs from zero, then the debt-revaluation effect breaks the coincidence between the short and the long-run fiscal policy multipliers. Using the system (28) - (30), the long-run effects of a debt-financed rise in government spending are

$$(31) \qquad \frac{dY}{dG} = 0 \qquad \text{for } dT = 0$$

$$(32) \qquad \frac{dB_f^p}{dG} = \frac{(1-a^g)B_f^p}{e\bar{D}^*} \qquad \text{for } dT = 0$$

$$(33) \qquad \frac{de}{dG} = -\frac{(1-a^g)}{\bar{D}^*} \qquad \text{for } dT = 0$$

Likewise, the long-run effects of a balanced-budget rise in government spending are

$$(34) \qquad \frac{dY}{dG} = 1 \qquad \text{for } dT = dG$$

$$(35) \qquad \frac{dB_f^p}{dG} = -\frac{a^g B_f^p}{e\bar{D}*} \qquad \text{for } dT = dG$$

$$(36) \qquad \frac{de}{dG} = \frac{a^g}{\bar{D}*} \qquad \text{for } dT = dG$$

These results show that, independent of the debt-revaluation effects, a rise in government spending does not alter the long-run level of output if it is debt-financed while the same rise in government spending raises the long-run level of output by a unit multiplier if it is tax financed. Thus, in both cases the long-run level of disposable income, Y-T, is independent of government spending. The results also show that if government spending is debt financed then, in the long run, if initial private sector debt was positive, then it rises while the currency appreciates. The opposite holds for the case in which government spending are tax financed.

In comparing the extent of the long-run changes in private-sector debt with the corresponding changes in the exchange rate we note that the <u>value</u> of debt, eB_f^p (measured in units of domestic output) remains unchanged. This invariance facilitates the interpretation of the long-run multipliers. Accordingly, consider the long-run equilibrium system (28) - (30) and suppose that government spending is debt financed. In that case as is evident from the money-market equilibrium condition (30), the equilibrium level of output does not change as long as the money supply, taxes, and the value of the debt commitment are given. Since, however, the rise in government spending creates an excess demand for domestic output, it is seen from equation (29) that the currency must appreciate (that is, e must fall) so as to lower the value of foreign demand, $e\overline{D}^*$, and thereby maintain equilibrium output unchanged. Obviously, since e falls, (the absolute value of) private sector debt, B_f^p, must rise by the same proportion so as to maintain the product eB_f^p unchanged. Finally, these changes ensure that the zero saving condition (28) is also satisfied. A similar interpretation can be given to the effects of a tax-financed rise in government spending except that in this case the level of output rises in line with the rise in taxes so as to keep disposable income unchanged.

A comparison between these long-run effects and the corresponding short-run effects shown in equations (24) - (25) reveals that the relative magnitudes of these multipliers depend on the initial debt position. For example, if the initial debt commitment is positive but smaller than export earnings, then the short-run multiplier of tax-finance is positive and larger than unity. In this case the long-run multipliers are more moderate than the corresponding short-run multipliers. If, however, the initial debt commitment exceeds export earnings, then the short-run debt-finance multiplier is positive (in contrast with the long-run multiplier) and the short-run tax-finance multiplier is smaller than unity, and could even be negative (in contrast with the unitary long-run balanced-budget multiplier).

2. Fiscal policies in a two-country world

In this section we extend the analysis of the small-country case to the two-country model outlined in equations (20) - (23). To develop a diagrammatic apparatus useful for the analysis of fiscal policies we proceed in three steps. First, we trace the combinations of domestic and foreign output levels which clear each country's goods market, incorporating the conditions of market clearing in the two national money markets (which under flexible exchange rates are the two non-tradable assets). Second, we trace the combinations of domestic and foreign output levels which bring about money-market equilibrium in each country and, at the same time, yield equality between the domestic and the foreign rates of interest, thereby conforming with the assumption of perfect capital mobility. Finally, in the third step, we find the unique combination of domestic and foreign levels of output which satisfy simultaneously the considerations underlying the first two steps.

Using the domestic money-market equilibrium condition (22) we can express the domestic money-market clearing rate of interest, r_t, as a positive function of disposable resources, $Y_t - T_t + A_{t-1}$, and as a negative function of the domestic money stock, M, that is, $r_t = r(Y_t - T_t + A_{t-1}, M)$. Applying a similar procedure to the foreign country, we can express the foreign money-market clearing rate of interest, r_t^*, as a function of foreign disposable resources and money stock that is, $r_t^* = r^*(Y_t^* + A_{t-1}^*, M^*)$, where $A_{t-1}^* = M_{t-1}^* + R_{t-1}B_{t-1}^p/e_t$. By substituting these money-market clearing rates of interest into the goods market equilibrium conditions (20) - (21), we obtain the reduced-form equilibrium conditions (37) - (38).

$$(37) \qquad (1-\beta_m)\tilde{E}(Y_t - T_t + A_{t-1}, M) + (1-\beta_m^g)\, G + e_t\beta_x^*\tilde{E}^*(Y_t^* + A_{t-1}^*, M^*) = Y_t$$

$$(38) \qquad \beta_m\tilde{E}(Y_t - T_t + A_{t-1}, M) + \beta_m^g G + e_t(1-\beta_x^*)\tilde{E}^*(Y_t^* + A_{t-1}^*, M^*) = e_tY_t^*$$

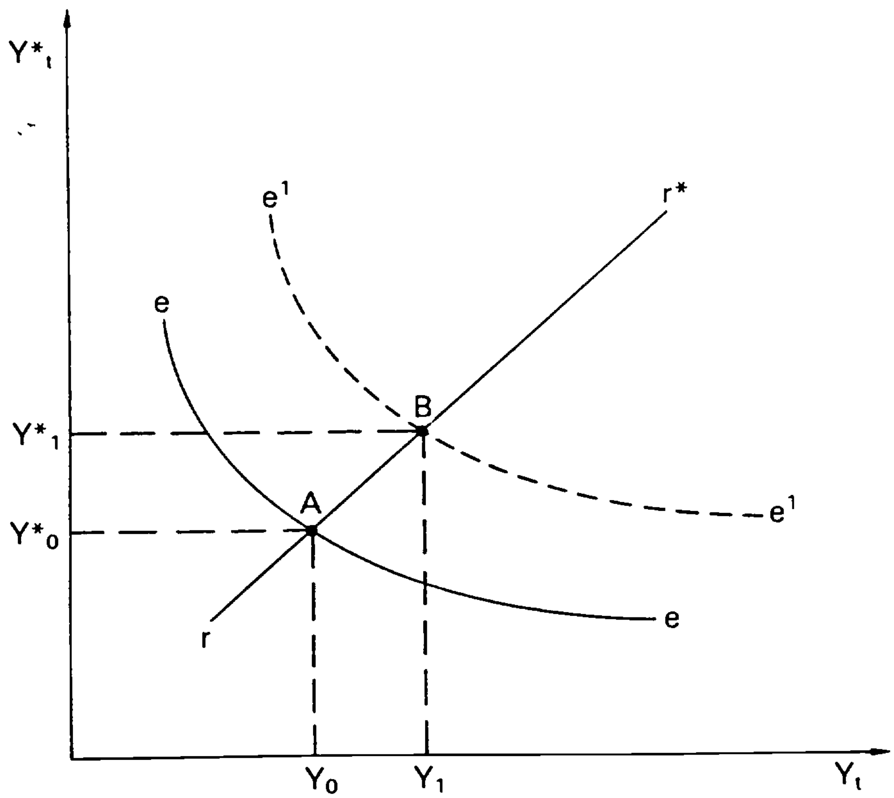
where a tilde (~) indicates a reduced-form function incorporating the money-market equilibrium conditions. For each and every value of the exchange rate, e_t, equations (37) - (38) yield the equilibrium combination of domestic and foreign output which clear the world markets for both goods. The schedule ee in Figure 10 traces these equilibrium output levels for alternative values of the exchange rate. The detailed derivation of this schedule is provided in the Appendix where it is shown that around balanced-trade equilibria with a zero initial private-sector debt (so that exchange rate changes do not exert revaluation effects) this schedule is negatively sloped. In general the ee schedule is negatively sloped if a rise in the exchange rate (a deterioration in the terms of trade) raises the world demand for domestic output and lowers the world demand for foreign output, allowing for the proper adjustments in each country's rate of interest so as to clear the national money market.

So far we have not yet incorporated the constraints imposed by the perfect international mobility of capital. To incorporate this constraint the two national money-market clearing rates of interest, r_t and r_t^*, must equal each other. This equally implies that

$$(39) \qquad r(Y_t - T_t + A_{t-1}, M) = r^*(Y_t^* + A_{t-1}^*, M^*).$$

The combinations of domestic and foreign output levels conforming with the perfect capital-mobility requirement are portrayed by the rr* schedule in Figure 10. With a zero level of initial debt (so that the debt revaluation effects induced by exchange rate changes are absent) this schedule is positively sloped since a rise in domestic output raises the demand for domestic money and raises the domestic rate of interest;

Figure 10: A Debt-Financed Unit Rise in Government Spending Under Flexible Exchange Rates: The Two-Country Case



Data: $IM_0 = IM_0^*$

$B^P_{f,-1} = 0$

international interest rate equalization is restored through a rise in foreign output which raises the foreign demand for money and the foreign rate of interest.

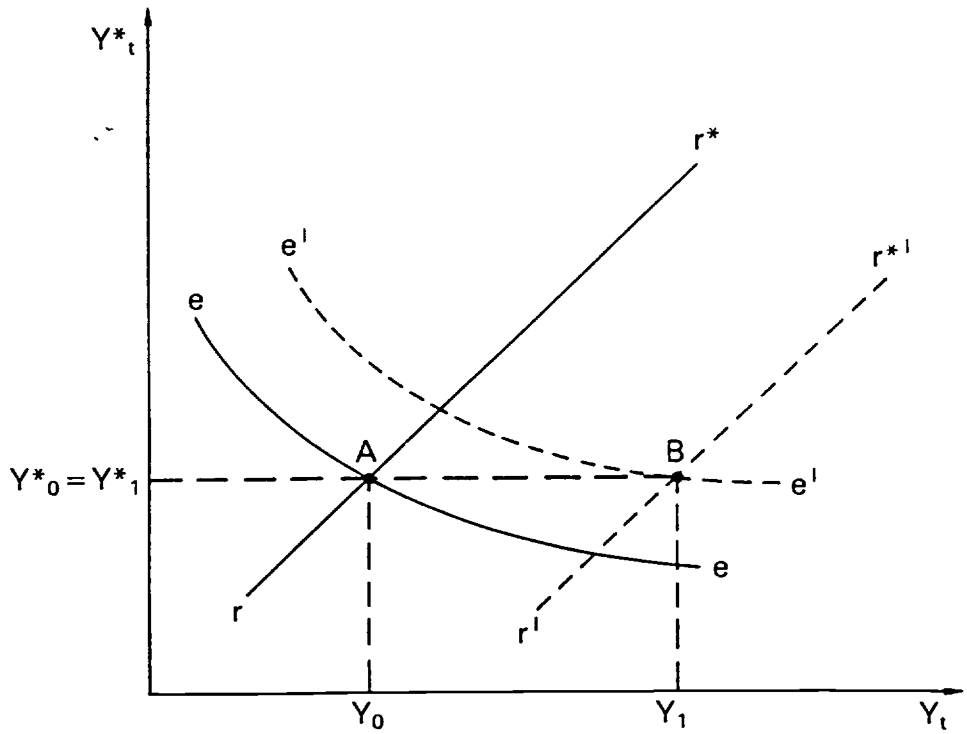
The short-run equilibrium is indicated by point A in Figure 10. At this point both goods markets clear, both national money markets clear and the rates of interest are equalized internationally. The levels of output corresponding to this equilibrium are Y_0 and Y_0^*.

A debt-financed unit rise in government spending alters the position of the goods market equilibrium schedule ee but does not impact on the capital-market equilibrium schedule, rr*. It is shown in the Appendix that for an initial trade-balance equilibrium with zero debt the ee schedule shifts to the right by $1/\tilde{s}$ units. The new equilibrium is indicated by point B in Figure 10. Thus (in the absence of revaluation effects), in the new short-run equilibrium both the domestic and the foreign levels of output rise from Y_0 and Y_0^* to Y_1 and Y_1^*, respectively.

For the given supply of money and for the higher level of output (which raises the demand for money), money-market equilibrium obtains at a higher rate of interest (which restores money demand to its initial level). Finally, it is shown in the Appendix that the exchange rate effects of the debt-financed rise in government spending are not clear cut, reflecting "transfer-problem criteria." These criteria reflect the relative pressures on the rates of interest in the domestic and foreign money markets induced by the changes in world demand for domestic and foreign outputs. If these pressures tend to raise the domestic rate of interest above the foreign rate, then the domestic currency must appreciate so as to lower the demand for domestic output and reduce the upward pressure on the domestic rate of interest. The opposite follows in the converse circumstances. But, if the behavioral parameters of the two private sectors are equal to each other, then the domestic currency must appreciate.

A tax-financed unit rise in government spending alters the position of both the ee and the rr^* schedules. As is evident by inspection of equations (37) - (39) both schedules shift to the right by one unit. This case is illustrated in Figure 11 where the initial equilibrium is indicated by point A and the new short-run equilibrium by point B. At the new equilibrium the domestic level of output rises by one unit so that disposable income remains unchanged. With unchanged levels of disposable income the demand for money is not altered and the initial equilibrium rate of interest remains intact. As a result the initial equilibrium in the foreign economy is not disturbed and the foreign level of output remains unchanged. Finally, in order to eliminate the excess supply in the domestic-goods market arising from the rise in domestic output and the unchanged level of disposable income, the currency must depreciate so as to raise the domestic-currency value of the given foreign demand. It follows that in the absence of revaluation effects the flexible exchange rate regime permits a full insulation of the foreign economy from the consequences of the domestic tax-financed fiscal policies. The more general results allowing for revaluation effects are provided in the Appendix. Analogously to the procedure adopted in the fixed exchange rate case, we do not analyze explicitly the long-run equilibrium of the two-country world under the flexible exchange rate regime. The formal equilibrium system applicable for such an analysis is presented in Appendix II.2.

Figure 11: A Tax-Financed Unit Rise in Government Spending Under Flexible Exchange Rates: The Two-Country Case



Data: $IM_0 = IM_0^*$
$B^P_{f,-1} = 0$

V. Summary and Overview

In this paper we analyzed the effects of government spending under fixed and flexible exchange rate regimes in an exposition of the Mundell-Fleming model. Throughout we have assumed that the world capital markets are highly integrated so that capital is perfectly mobile internationally. To conserve on space we have focused on the pure effects of fiscal policies and assumed that there is no active monetary policy. In particular we abstracted from money-financed government spending. Accordingly, we analyzed the predictions of the Mundell-Fleming model concerning short- and long-run consequences of debt-financed and of tax-financed changes in government spending. In this context we focused on the effects of fiscal policies on the levels of output, debt and the rate of interest under the two alternative exchange rate regimes. In addition, for the fixed exchange rate regime we examined the induced changes in the money supply and, for the flexible exchange rate regime we determined the induced change in the exchange rate.

The short- and long-run effects of a unit debt-financed and tax-financed rise in government spending for a small country facing a fixed world rate of interest are summarized in Table 1. This table shows the various multipliers applicable to the fixed as well as to the flexible exchange-rate regimes. The output multipliers under the fixed exchange rate regime are the typical simple text-book version of the foreign trade multipliers. These results are of course expected since the rate of interest is exogenously given to the small country. The fixity of the rate of interest implies that the typical crowding-out mechanism induced by changes in the rate of interest are not present.

Table 1. The Short- and Long-Run Effects of a Unit Rise in Government Spending Under Fixed and Flexible Exchange Rates: The Small-Country Case

Fixed Exchange Rates	Debt-Financed		Tax-Financed	
	Short-Run	Long-Run	Short-Run	Long-Run
Effects On:				
Y	$\frac{1-a^g}{s+a}$	$\frac{1-a^g}{\Delta}\left(1-s-\bar{r}_f(sM_y)\right)$	$1-\frac{a^g}{s+a}$	$1-\frac{a^g}{\Delta}\left(1-s-\bar{r}_f(s-M_y)\right)$
$B^p_{f,t-1}$	0	$-\frac{1-a^g}{\Delta}(s-M_y)$	0	$\frac{a^g}{\Delta}(s-M_y)$
M	$\frac{(1-a^g)M_y}{s+a}$	$\frac{1-a^g}{\Delta}M_y$	$\frac{-a^g}{s+a}M_y$	$-\frac{a^g}{\Delta}M_y$
Flexible Exchange Rates				
Effects On:				
Y	$\frac{(1-a^g)R_fB^p_{f,t-1}}{R_fB^p_{f,t-1}-\bar{D}^*}$	0	$1-\frac{a^gR_fB^p_{f,t-1}}{R_fB^p_{f,t-1}-\bar{D}^*}$	1
$B^p_{f,t-1}$	0	$\frac{(1-a^g)B^p_f}{e\bar{D}^*}$	0	$-\frac{a^gG^p_f}{e\bar{D}^*}$
e	$\frac{(1-a_g)}{R_fB^p_{f,t-1}-\bar{D}^*}$	$-\frac{(1-a^g)}{\bar{D}^*}$	$\frac{-a^g}{R_fB^p_{f,t-1}-\bar{D}^*}$	$\frac{a^g}{\bar{D}^*}$

Note: $\bar{D}^*$ denotes export earnings measured in units of foreign currency, $R_f = 1+\bar{r}_f$ and $\Delta = a-\bar{r}_f(s-M_y)$. $\Delta > 0$ under the assumption that a rise in income worsens the current account of the balance of payments. The term $1-s-\bar{r}_f(s-M_y) > 0$ for stability.

Under flexible exchange rates the short-run output multipliers of fiscal policies depend crucially on the debt-revaluation effect induced by exchange rate changes. Indeed, in the absence of such an effect (as would be the case if the initial debt position is zero), fiscal policies lose their capacity to alter disposable income. Accordingly, with debt finance the output multiplier is zero and with tax finance the corresponding multiplier is unity. In general, however, the signs and magnitudes of the short-run output multipliers depend on the size of the initial debt. In contrast, these considerations do not influence the long-run output multipliers. As seen in the table, with perfect capital mobility and flexible exchange rates, the long-run value of disposable income cannot be affected by fiscal policies.

One of the important points underscored by the results reported in Table 1 is the critical dependence of the direction of change in the key variables on the means of fiscal finance. Specifically, a shift from a debt finance to a tax finance reverses the signs of the multipliers of B_f^p, M and e.

For example, a tax-financed rise in government spending under a fixed exchange rate regime induces a balance of payments deficit and reduces both the short- and the long-run money holdings. On the other hand a similar rise in government spending which is debt-financed induces a surplus in the balance of payments and raises money holdings in the short run as well as in the long run. Likewise, under a flexible exchange rate regime the tax-financed rise in government spending depreciates the long-run value of the currency whereas the debt-financed rise in government spending appreciates the long-run value of the currency.

As indicated earlier, a similar reversal in the direction of the change in the exchange rate also pertains to the short run but whether the currency depreciates or appreciates in the short run depends on the size of the debt which in turn governs the debt-revaluation effect.

To study the characteristics of the international transmission mechanism our exposition of the Mundell-Fleming model was extended to a two-country model of the world economy. The new channel of transmission is the world rate of interest which is determined in the unified world capital market. Table 2 summarizes the short-run effects of fiscal policies under the two alternative exchange rate regimes. To avoid a tedious taxonomy the summary results for the flexible exchange rates reported in the table are confined to the case in which the twin revaluation effects—debt revaluation and trade balance revaluation--induced by exchange rate changes are absent; accordingly it is assumed that the initial debt is zero and that the initial equilibrium obtains with a balanced trade.

As shown, independent of the exchange rate regime, a debt-financed rise in government spending raises the world rate of interest. Under the flexible exchange rate regime the debt-financed rise in government spending stimulates demand for both domestic and foreign goods and results in an expansion of both outputs. Thus, in this case, the international transmission of the rise in goververnment spending, measured by co-movements of domestic and foreign outputs, is positive. On the other hand, under a fixed exchange rate regime the rise in the world rate of interst may offset the direct effect of government spending on aggregate demand and may result in lower levels of output. But, if the (negative)

Table 2. The Direction of the Short-Run Effects of a Rise in Government Spending Under Fixed and Flexible Exchange Rates: The Two-Country World

Fixed Exchange Rates	Debt-Financed	Tax-Financed
Effects On:		
Y	+ (for small H_r)	+ (for $a^g < a$)
Y^*	+ (for small F_r)	+
r	+	+ (for $A > 0$) - (for $A < 0$)
M	+ (for $B+C < 0$) - (for $B+C > 0$)	-

Flexible Exchange Rates		
Effects On:		
Y	+	+
Y^*	+	0
r	+	0
e	+ (for $\tilde{B} > 0$) - (for $\tilde{B} < 0$)	+

Note: The signs indicated in the flexible exchange-rate part of the Table are applicable to the case of an initial equilibrium with balanced trade and zero initial debt. H_r and F_r denote, respectively, the negative effect of the rate of interest on the world demand for domestic and foreign goods, $A = s/M_y - s^*/M^*_{y*}$, $B = \bar{e}(M_y/M_r)[a^* + s^*(1-a^g)] - (M^*_{y*}/M^*_r)(a+sa^g)$, and $\tilde{B} = e_t(M_y/M_r)[\tilde{a}^* + \tilde{s}^*(1-a^g)] - (M^*_{y*}/M^*_r)(\tilde{a}+\tilde{s}a^g)$, correspond, respectively to the fixed and flexible exchange rate regimes, and where $C = M_y M^*_{y*} M_r M^*_r [F_r(1-a^g) - H_r a^g]$.

interest-rate effect on aggregate demand is relatively weak, then both domestic and foreign outputs rise, thereby resulting in a positive international transmission. Finally, we note that there is no presumption about the direction of change in money holdings (under fixed exchange rates) and in the exchange rate (under flexible exchange rates) in response to the debt-financed fiscal expansion. As indicated, depending on the relative magnitudes of the domestic and foreign saving and import propensities and the domestic and foreign sensitivities of money demand with respect to changes in the rate of interest and income, the balance of payments may be in a deficit or in a surplus and the currency may depreciate or appreciate.

The results in Table 2 also highlight the significant implication of alternative means of budgetary finance. Indeed, in contrast with debt finance, a tax-financed rise in government spending under a flexible exchange rate regime leaves the world rate of interest unchanged, raises domestic output, and depreciates the currency. The reduction in the domestic private-sector demand for foreign output, induced by the depreciation of the currency, precisely offsets the increased demand induced by the rise in government spending. As a result, foreign output remains intact and the flexible exchange rate regime fully insulates the foreign economy from the domestic tax-financed fiscal policy. In this case the analysis of the two-country world economy reduces to the one carried out for the small-country case. Therefore, the long-run multipliers for the two countries operating under flexible exchange rates coincide with the short-run multipliers, the domestic short- and long-run output multipliers are unity and the corresponding foreign output multipliers are zero.

In contrast with the flexible exchange rate regime in which the currency depreciates to the extent needed to maintain world demand for (and thereby the equilibrium level of) foreign output unchanged, the fixed exchange rate regime does not contain this insulating mechanism. As a result, the tax-financed rise in the domestic government spending raises the world demand for (and thereby the equilibrium level of) foreign output. On the other hand, depending on the relative magnitude of the domestic-government import propensity, the domestic level of output may rise or fall. If, however, the government import propensity does not exceed the corresponding private-sector propensity, then domestic output rises and the international transmission, measured by co-movements of domestic and foreign outputs, is positive. Finally, since at the prevailing rate of interest domestic disposable income falls and foreign disposable income rises, these changes in disposable incomes alter the world demand for money and necessitate equilibrating changes in the world rate of interest. As shown in Table 2, the change in the world demand for money (at the prevailing rate of interest) reflects a "transfer-problem criterion." If the ratio of the domestic saving to hoarding propensities, s/M_y, exceeds the corresponding foreign ratio, s^*/M^*_{y*}, then the international redistribution of disposable income raises the world demand for money and necessitates a rise in the world rate of interest. The opposite holds if s/M_y falls short of s^*/M^*_{y*}. Independent, however, of the direction of the change in the interest rate, the tax-financed rise in government spending must worsen the balance of payments and lower the short-run equilibrium money holdings.

Throughout the exposition of the model it was assumed that expectations are static. Since under a flexible exchange rate the actual exchange rates do change, the assumption that exchange rate expectations are static result in expectational errors during the period of transition towards the long-run equilibrium. The incorporation of a consistent expectations scheme into the Mundell-Fleming model introduces an additional mechanism governing the short-run behavior. Aspects of this mechanism are examined in the Appendix.

We conclude this summary with an overview of the Mundell-Fleming model. A key characteristic of the formulation of the income-expenditure framework underlying the Mundell-Fleming model is the lack of solid microeconomic foundations underlying the behavior of the private and public sectors, and the absence of an explicit rationale for the holdings of zero interest-bearing money in the presence of safe interest-bearing bonds. The latter issue is of relevance in view of the central role played by monetary flows in the international adjustment mechanism. Furthermore, no attention was given to the intertemporal budget constraints and the behavior of both the private and the public sectors was not forward-looking in a consistent manner. As a result, there is no mechanism ensuring that the patterns of spending, debt accumulation and money hoarding, which are the key elements governing the equilibrium dynamics of the economic system, are consistent with the relevant economic constraints. The implication of this shortcoming is that in determining the level and composition of spending, saving and asset holdings, the private sector does not incorporate explicitly the intertemporal consequences of government policies.

To illustrate the significance of this issue consider a debt-financed rise in current government spending. A proper formulation of the government's intertemporal budget constraint must recognize that to service the debt and maintain its solvency the government must accompany this current fiscal expansion by either cutting down future spending or by raising future (ordinary or inflationary) taxes. Furthermore, a proper specification of the private sector's behavior must allow for the fact that the forward-looking individuals may recognize the future consequences of current government policies and incorporate these expected consequences into their current as well as planned future spending, saving and asset holdings.

The Mundell-Fleming model presented in this paper assumes that producer prices are given and outputs are demand-determined. In this framework nominal exchange rate changes amount to changes in the terms of trade. As a result, the key characteristics of the economic system are drastically different across alternative exchange rate regimes. More recent theoretical research has relaxed the fixed-price assumption and has allowed for complete price flexibility. With this flexibility prices are always at their market clearing equilibrium levels. Accordingly, changes in the terms of trade induced by equilibrium changes in prices trigger an adjustment mechanism that is analogous to the one triggered by nominal exchange rate changes in the Mundell-Fleming model.

The neglect of the intertemporal budget constraints and of the consequences of forward-looking behavior consistent with these constraints are among the main limitations of the model. Recognition of these limitations provide both, the rationale for and the bridge to the growing body of newer theoretical developments aiming to rectify these shortcomings. This newer literature develops models that are derived from optimizing behavior consistent with the relevant temporal and intertemporal economic constraints. The resulting macroeconomic model which is grounded upon solid microeconomic foundations is capable of dealing with new issues in a consistent manner. Among these issues are the effects of various time patterns of government spending and taxes. The newer literature thus distinguishes between temporary and permanent as well as between current and future policies. Likewise, it is capable of analyzing the macroeconomic consequences of alternative specifications of the tax structure. It can, therefore, distinguish between the effects of different types of taxes (such as income taxes, value-added taxes and international capital flow taxes) used to finance the budget. An illustration of this literature is contained in Frenkel and Razin (1987). An important feature of the modern approach is that, being grounded on microeconomic foundations, it is capable of dealing explicitly with the welfare consequences of economic policies. This feature reflects the basic attribute of the macroeconomic model: the economic behavior underlying this model is derived from, and is consistent with, the principles of individual utility maximization. Therefore, in contrast with the traditional approach, the intertemporal optimizing approach provides for a framework suitable for the normative evaluation of international macroeconomic policies.

Appendix I. Fixed Exchange Rates

1. Long-run equilibrium: the small-country case

The long-run equilibrium conditions are specified by equations (12) - (15) of the text. Substituting the government budget constraint (15) into equations (12) - (14) yields

$$\text{(A-1)} \qquad E(Y-G+M-B^p - \bar{r}_f(B^p + B^g), \bar{r}_f) + G = Y - \bar{r}_f(B^p+B^g)$$

$$\text{(A-2)} \qquad (1-\beta_m)E(Y-G+M-B^p - \bar{r}_f(B^p+B^g), \bar{r}_f) + (1-\beta_m^g)G + \bar{e}\,\overline{D^*} = Y$$

$$\text{(A-3)} \qquad M(Y-G+M-B^p - \bar{r}_f(B^p+B^g), \bar{r}_f) = M$$

Equations (A-1) and (A-3) yield the combinations of output and private sector debt underlying the CA = 0 schedule, and equations (A-2) and (A-3) yield the combinations of these variables underlying the YY schedule. To obtain the slope of the CA = 0 schedule we differentiate equations (A-1) and (A-3) and obtain

$$\text{(A-4)} \qquad \begin{bmatrix} -s & s(1+\bar{r}_f)-1 \\ M_y & -(1+\bar{r}_f)M_y \end{bmatrix} \begin{bmatrix} dY \\ dB^p \end{bmatrix} = \begin{bmatrix} -(1-s) \\ 1-M_y \end{bmatrix} dM$$

where $s = 1-E_y$ and $a = \beta_m E_y$. Solving (A-4) for dY/dM and dividing the resultant solutions by each other yields the expression for dB^p/dY along the CA = 0 schedule. This expression is reported in equation (16) of the text.

Likewise differentiating equations (A-2) - (A-3) yields

$$\text{(A-5)} \quad \begin{bmatrix} -(s+a) & -(1+\bar{r}_f)(1-s-a) \\ M_y & -(1+\bar{r}_f)M_y \end{bmatrix} \begin{bmatrix} dY \\ dB^p \end{bmatrix} = \begin{bmatrix} -(1-s-a) \\ 1-M_y \end{bmatrix} dM$$

Following a similar procedure we obtain the expression for dB^p/dY along the YY schedule. This expression is reported in equation (17) of the text.

To obtain the horizontal displacements of the CA = 0 schedule following a balanced-budget rise in government spending we differentiate equations (A-1) and (A-3) holding B^g and B^p constant. Accordingly, equation (A-1) implies that $(1-s)(dY-dG+dM) = dY-dG$ and equation (A-3) implies that $dM = M_y(dY-dG)/(1-M_y)$. Substituting the latter expression into the former reveals that $dY/dG = 1$. Thus, a unit balanced-budget rise in government spending induces a unit rightwards shift of the CA = 0 schedule.

Analogously, to obtain the horizontal shift of the YY schedule we differentiate equations (A-2) - (A-3) holding B^g and B^p constant. Equation (A-2) implies that $(1-s-a)(dY-dG+dM) + (1-a^g)dG = dY$ where $a^g = \beta^g_m$, and equation (A-3) implies that $dM = M_y(dY-dG)/(1-M_y)$. Substituting the latter into the former shows that the horizontal shift of the YY schedule is

$$1 - \frac{(1-M_y)a^g}{s+a - M_y}$$

Thus, in contrast with the unit rightwards displacement of the CA = 0 schedule, the unit balanced-budget rise in government spending shifts the YY schedule to the right by less than one unit. These results underly the diagrammatic analysis in Figures 2 and 3.

The long-run effects of fiscal policies are obtained by differentiating the system (12) - (14) of the text and solving for the endogenous variables. Accordingly,

$$\text{(A-6)} \quad \begin{bmatrix} -s & s(1+\bar{r}_f)-1 & 1-s \\ -(s+a) & -(1+\bar{r}_f)(1-s-a) & 1-s-a \\ M_y & -(1+\bar{r}_f)M_y & -(1-M_y) \end{bmatrix} \begin{bmatrix} dY \\ dB^p \\ dM \end{bmatrix} = \begin{bmatrix} 0 \\ -(1-a^g) \\ 0 \end{bmatrix} dG + \begin{bmatrix} -s \\ 1-s-a \\ M_y \end{bmatrix} dT$$

Using this system the long-run effects of a debt-financed rise in government spending (that is, dT = 0) are

$$\text{(A-7)} \quad \frac{dY}{dG} = \frac{1-a^g}{\Delta}[1-s-\bar{r}_f(s-M_y)] > 0 \qquad \text{for } dT_t = 0$$

$$\text{(A-8)} \quad \frac{dB^p}{dG} = -\frac{1-a^g}{\Delta}(s-M_y) < 0 \qquad \text{for } dT_t = 0$$

$$\text{(A-9)} \quad \frac{dM}{dG} = \frac{1-a^g}{\Delta}M_y > 0 \qquad \text{for } dT_t = 0$$

where $\Delta = a-\bar{r}_f(s-M_y) > 0$ under the assumption that a rise in income worsens the current account of the balance of payments. Correspondingly, the long-run effects of a balanced-budget rise in government spending (that is, dG = dT) are

(A-10) $$\frac{dY}{dG} = 1 - \frac{a^g}{\Delta}\left((1-s-\bar{r}_f(s-M_y)\right) \gtrless 0 \qquad \text{for } dG = dT_t$$

(A-11) $$\frac{dB^p}{dG} = \frac{a^g}{\Delta}(s-M_y) > 0 \qquad \text{for } dG = dT_t$$

(A-12) $$\frac{dM}{dG} = -\frac{a^g}{\Delta}M_y < 0 \qquad \text{for } dG = dT_t$$

2. Short-run equilibrium: the two-country world

In this part of the Appendix we analyze the short-run equilibrium of the system (5) - (7) in the text. This system determines the short-run equilibrium values of Y_t, Y_t^* and r_t. The YY and Y^*Y^* schedules in Figure 4 show combinations of Y_t and Y_t^* which clear the markets for domestic and foreign output, respectively. Both of these schedules incorporate the world money-market equilibrium condition (7) of the text. To derive the slope of the YY schedule we differentiate equations (5) and (7) of the text. This yields

(A-13) $$\begin{bmatrix} -(s+a) & \bar{e}a* \\ M_y & \bar{e}M^*_{y*} \end{bmatrix} \begin{bmatrix} dY_t \\ dY_t^* \end{bmatrix} = -\begin{bmatrix} H_r \\ (M_r+\bar{e}M_r^*) \end{bmatrix} dr_t$$

where H_r denotes the partial (negative) effect a change in the rate of interest on the world demand for domestic output, that is, $H_r = (1-\beta_m)E_r + \bar{e}\beta_x^*E_r^*$, and where E_r, M_r, E_r^* and M_r^* denote the partial (negative) effects of the rate of interest on domestic and foreign spending and money demand. To eliminate r_t from the goods-market equilibrium schedule we solve (A-13) for dY_t/dr_t and for dY_t^*/dr_t, and divide the solutions by each other. This yields

$$\text{(A-14)} \qquad \frac{dY_t^*}{dY_t} = \frac{1}{\bar{e}} \frac{(s+a)(M_r+\bar{e}M_r^*) + M_y H_r}{a^*(M_r+\bar{e}M_r^*) - M_{y^*}^* H_r} \qquad \text{along the YY schedule}$$

Analogously, differentiating equations (13) - (14) of the text yields

$$\text{(A-15)} \qquad \begin{bmatrix} a & -\bar{e}(s^*+a^*) \\ M_y & \bar{e}M_{y^*}^* \end{bmatrix} \begin{bmatrix} dY_t \\ dY_t^* \end{bmatrix} = - \begin{bmatrix} F_r \\ M_r+\bar{e}M_r^* \end{bmatrix} dr_t$$

where $F_r = \beta_m E_r + \bar{e}(1-\beta_x^*)E_r^*$ denotes the partial (negative) effect of the rate of interest on the world demand for foreign output. Applying a similar procedure as before, the slope of the Y^*Y^* schedule is

$$\text{(A-16)} \qquad \frac{dY_t^*}{dY_t} = \frac{1}{\bar{e}} \frac{a(M_r+\bar{e}M_r^*) - M_y F_r}{(s^*+a^*)(M_r+\bar{e}M_r^*) + M_{y^*}^* F_r} \qquad \text{along the } Y^*Y^* \text{ schedule}$$

A comparison of the slopes in (A-14) and (A-16) shows that there are various possible configurations of the relative slopes of the YY and Y^*Y^* schedules. However, two configurations are ruled out: if both schedules are positively sloped then the slope of the Y^*Y^* cannot exceed the slope of the YY schedule. This can be verified by noting that in the numerator of (A-14) the negative quantity $a(M_r+eM_r^*)$ is augmented by additional negative quantities whereas the same negative quantity in the numerator of (A-16) is augmented by an additional positive quantity. A similar comparison of the denominators of (A-14) and (A-16) shows that the negative quantity $a^*(M_r+eM_r^*)$ is augmented by additional negative quantities in (A-16) and by a positive quantity in (A-14). Likewise, if both schedules are negatively sloped then, by subtracting one slope from the other it can be verified that the Y^*Y^* schedule cannot be steeper than the YY schedule. These considerations imply that for all situations in which there is a rightwards shift of the YY schedule exceeding the rightwards shift of the Y^*Y^* schedule, the new equilibrium must be associated with a higher level of domestic output.

A rise in the domestic government spending alters the position of both schedules. To determine the horizontal shift of the YY schedule we use equations (5) and (7) of the text; holding Y^* constant and solving for dY/dG after eliminating the expression for dr/dG. A similar procedure is applied to determine the horizontal shift of the Y^*Y^* schedule from equations (6) and (7). Accordingly, the horizontal shifts of the schedules induced by a debt-financed rise in government spending are

$$\text{(A-17)} \qquad \frac{dY}{dG} = \frac{1-a^g}{s+a+\dfrac{M_yH_r}{M_r+\bar{e}M_r^*}} > 0 \qquad \text{for the YY schedule}$$

$$\text{(A-18)} \qquad \frac{dY}{dG} = \frac{-a^g}{a-\dfrac{M_yF_r}{M_r+\bar{e}M_r^*}} > 0 \qquad \text{for the } Y^*Y^* \text{ schedule}$$

The corresponding shifts for the tax-financed rise in government spending are

$$\text{(A-19)} \qquad \frac{dY}{dG} = 1-\frac{a^g}{s+a+\dfrac{M_yH_r}{M_r+\bar{e}M_r^*}} > 0 \qquad \text{for the YY schedule}$$

$$\text{(A-20)} \qquad \frac{dY}{dG} = 1-\frac{a^g}{a-\dfrac{M_yF_r}{M_r+\bar{e}M_r^*}} > 0 \qquad \text{for the } Y^*Y^* \text{ schedule}$$

Comparisons of (A-17) with (A-18) and of (A-19) with (A-20) reveal the difference between the shifts of the YY and the Y^*Y^* schedules.

To compute the short-run multipliers of fiscal policies we differentiate the system (5) - (7) of the text. Thus

$$\text{(A-21)} \quad \begin{bmatrix} -(s+a) & \bar{e}a^* & H_r \\ a & -\bar{e}(s^*+a^*) & F_r \\ M_y & \bar{e}M^*_{y^*} & M_r+\bar{e}M^*_r \end{bmatrix} \begin{bmatrix} dY_t \\ dY^*_t \\ dr_t \end{bmatrix} = -\begin{bmatrix} 1-a^g \\ a^g \\ 0 \end{bmatrix} dG + \begin{bmatrix} 1-s-a \\ a \\ M_y \end{bmatrix} dT_t$$

With a debt-financed rise in government spending $dT_t = 0$ and thus the short-run effects are

$$\text{(A-22)} \quad \frac{dY_t}{dG} = \frac{1}{\Delta}\left((s^*(1-a^g)+a^*)(M_r+\bar{e}M^*_r) + M^*_{y^*}(F_r(1-a^g) - a^gH_r)\right) \quad \text{for } dT_t = 0$$

$$\text{(A-23)} \quad \frac{dY^*_t}{dG} = \frac{1}{\bar{e}\Delta}\left((sa^g+a)(M_r+\bar{e}M^*_r) - M_y(F_r(1-a^g) - a^gH_r)\right) \quad \text{for } dT_t = 0$$

$$\text{(A-24)} \quad \frac{dr_t}{dG} = -\frac{1}{\Delta}\left((s^*(1-a^g)+a^*)M_y + (sa^g+a)M^*_{y^*}\right) > 0 \quad \text{for } dT_t = 0$$

where $\Delta = s\left((s^*+a^*)(M_r+\bar{e}M^*_r) + M^*_{y^*}F_r\right) + a\left(s^*(M_r+\bar{e}M^*_r) + M^*_{y^*}(F_r+H_r)\right)$

$$+ M_y\left(s^*H_r + a^*(F_r+H_r)\right) < 0$$

Differentiating the domestic demand for money function (equation (8) of the text) and using (A-22) and (A-24) yields the short-run change in the domestic money holdings, that is, the balance of payments:

$$\text{(A-25)} \qquad \frac{dM_t}{dG} = \frac{1}{M_r M_r^* \Delta} \Big(\frac{\bar{e}M_y}{M_r} [a^*+s^*(a-a^g)] - \frac{M^*_{y*}}{M^*_r} (a+sa^g)$$

$$+ M_y M^*_{y*} \; M_r M^*_r [F_r(1-a^g) - H_r a^g] \Big) \qquad \text{for } dT_t = 0$$

With a balanced-budget rise in government spending $dG = dT_t = dT$. Accordingly, the solutions of (A-21) are

$$\text{(A-26)} \qquad \frac{dY_t}{dG} = \frac{1}{\Delta} \Big\{ s\big((s^*+a^*)(M_r+\bar{e}M^*_r) + M^*_{y*}F_r\big)$$

$$+ (a-a^g)\big(s^*(M_r+\bar{e}M^*_r) + M^*_{y*}(F_r+H_r)\big)$$

$$+ M_y\big(s^*H_r + a^*(F_r+H_r)\big)\Big\} \qquad \text{for } dG = dT_t$$

$$\text{(A-27)} \qquad \frac{dY^*_t}{dG} = \frac{a^g}{\bar{e}\Delta} \big(M_y(F_r+H_r) + s(M_r+\bar{e}M^*_r)\big) > 0 \qquad \text{for } dG = dT_t$$

$$\text{(A-28)} \qquad \frac{dr_t}{dG} = \frac{a^g}{\Delta} (s^*M_y - sM^*_{y*}) \qquad \text{for } dG = dT_t$$

Differentiating the domestic money demand function and using (A-26) and (A-28) yields

$$\text{(A-29)} \qquad \frac{dM_t}{dG} = -\frac{a^g}{\Delta} \big((sM_r M^*_{y*} + s^*\bar{e}M^*_r M_y) + M_y M^*_{y*}(F_r+H_r)\big) < 0$$

$$\text{for } dG = dT_t$$

3. Long-run equilibrium: the two-country world

The long-run equilibrium of the system is specified by equations (A-30) - (A-36) where the first five equations are the long-run counterpart to the short-run conditions (5) - (9) of the text and the last two equations are the zero-savings requirements for each country implying (once the government budget constraint is incorporated) current account balances. By employing a common rate of interest, this long-run system embodies the assumption of perfect capital mobility.

$$(1-\beta_m)\ E(Y-T+M - (1+r)B^p,\ r) + (1-\beta_m^g)G + \beta_x^*\bar{e}E^*\ (Y^*+M^* + (1+r)B^p/\bar{e},\ r) = Y \tag{A-30}$$

$$\beta_m E(Y-T+M - (1+r)B^p,\ r) + \beta_m^g G + (1-\beta_x^*)\ \bar{e}E^*(Y^*+M^* + (1+r)B^p/\bar{e},\ r) = Y^* \tag{A-31}$$

$$M(Y-T+M - (1+r)B^p,\ r) + \bar{e}M^*(Y^*+M^* + (1+r)B^p/\bar{e},\ r) = \bar{M} \tag{A-32}$$

$$M(Y-T+M - (1+r)B^p,\ r) = M \tag{A-33}$$

$$M^*(Y^*+M^* + (1+r)B^p/\bar{e},\ r) = M^* \tag{A-34}$$

$$E(Y-T+M - (1+r)B^p,\ r) = Y - rB^p - T \tag{A-35}$$

$$E^*(Y^*+M^* + (1+r)B^p/\bar{e},\ r) = Y^* + rB^p/\bar{e} \tag{A-36}$$

By Walras's Law one of the seven equations can be omitted and the remaining six equations can be used to solve for the long-run equilibrium values of Y, Y^*, B^p, M, M^* and r as functions of the policy variables.

Appendix II. Flexible Exchange Rates

1. Short-run equilibrium: the two-country world

In this part of the Appendix we analyze the short-run equilibrium of the two-country model under flexible exchange rates. Using the domestic money market equilibrium condition (22) of the text the domestic market clearing rate of interest is

$$\text{(A-37)} \qquad r_t = r(Y_t - T_t + A_{t-1}, M)$$

where a rise in disposable resources raises the equilibrium rate of interest while a rise in the money supply lowers the rate of interest. Similarly, using the foreign money-market clearing condition (23) of the text but not imposing yet an equality between the foreign rate of interest, r_t^*, and the domestic rate, r_t, yields

$$\text{(A-38)} \qquad r_t^* = r^*(Y_t^* + A_{t-1}^*, M^*)$$

Substituting (A-37) into the domestic expenditure function (3) of the text and substituting (A-38) into the corresponding foreign expenditure function yields

$$\text{(A-39)} \qquad E_t = \tilde{E}(Y_t - T_t + A_{t-1}, M)$$

$$\text{(A-40)} \qquad E_t^* = \tilde{E}^*(Y_t^* + A_{t-1}^*, M^*)$$

Equations (A-39) - (A-40) are the reduced-form expenditure functions which incorporate the conditions of money-market equilibrium. A rise in disposable resources exerts two conflicting influences on the reduced-form expenditure function. On the one hand it stimulates spending directly but on the other hand, by raising the equilibrium rate of interest, it discourages spending. Formally, $\tilde{E}_y = E_y - (E_r/M_r)M_y$. In what follows we assume that the direct effect dominates so that $\tilde{E}_y > 0$. For subsequent use we note that the reduced-form saving propensity $\tilde{s} = 1-\tilde{E}_y$ exceeds $M_y[1+(E_r/M_r)]$. This follows from the assumption that bonds are normal goods (so that $1-E_y-M_y > 0$) together with the former expression linking $\tilde{E}_y$ with E_y.

Substituting the reduced-form expenditure functions (A-39) - (A-40) into the good-markets clearing conditions yields

$$\text{(A-41)} \qquad (1-\beta_m)\tilde{E}(Y_t-T_t + A_{t-1}, M) + (1-\beta_m^g)G + e_t\beta_x^*\tilde{E}^*(Y_t^* + A_{t-1}^*, M^*) = Y_t$$

$$\text{(A-42)} \qquad \beta_m\tilde{E}(Y_t-T_t + A_{t-1}, M) + \beta_m^g G + e_t(1-\beta_x^*)\tilde{E}^*(Y_t^* + A_{t-1}^*, M^*) = e_tY_t^*$$

where we recall that $A_{t-1} = M_{t-1} - (1+r_{t-1})B_{t-1}^p$ and $A_{t-1}^* = M_{t-1}^* + (1+r_{t-1})B_{t-1}^p/e_t$. Thus, while A_{t-1} is predetermined, the value of A_{t-1}^* depends on the prevailing exchange rate. Equations (A-41) - (A-42) are the reduced-form good-markets clearing conditions. These conditions link the equilibrium values of domestic output, foreign output, and the exchange rate. In the first step of the analysis we derive the ee schedule of the text which portrays alternative combinations of Y and Y^* satisfying equations (A-41) - (A-42) for alternative values of the exchange rate (which is treated as a parameter). The slope of this schedule is obtained by differentiating equations (A-41) - (A-42) and solving for dY_t^*/dY_t. Accordingly,

$$\text{(A-43)} \quad \begin{bmatrix} -s(\tilde{s}+\tilde{a}) & e_t\tilde{a}^* \\ \tilde{a} & -e_t(\tilde{s}^*+\tilde{a}^*) \end{bmatrix} \begin{bmatrix} dY_t \\ dY_t^* \end{bmatrix} = \begin{bmatrix} -IM_t^*+\tilde{a}^*H \\ IM_t+(1-\tilde{s}^*-\tilde{a}^*)H \end{bmatrix} de_t - \begin{bmatrix} 1-a^g \\ a^g \end{bmatrix} dG + \begin{bmatrix} 1-\tilde{s}-\tilde{a} \\ \tilde{a} \end{bmatrix} dT_t$$

where $H = (1+r_{t-1})B^p_{t-1}/e_t$ denotes the debt commitment of the home country, the reduced-form saving and import propensities are designated by a tilde (~), and where $IM_t^* = \beta_x^* \tilde{E}^*$ and $IM_t = Y^* - (1-\beta_x^*)\tilde{E}^*$ are, respectively, the foreign and the domestic values of imports expressed in units of foreign goods. For given fiscal policies we obtain

$$\text{(A-44)} \quad \frac{dY_t}{de_t} = \frac{\tilde{s}^*IM_t^* + \tilde{a}^*(IM_t^* - IM_t) - \tilde{a}^*H}{\Delta}$$

$$\text{(A-45)} \quad \frac{dY_t^*}{de_t} = - \frac{\tilde{s}IM_t - \tilde{a}(IM_t^* - IM_t) + [\tilde{s}(1-\tilde{s}^*-\tilde{a}^*) + \tilde{a}(1-\tilde{s}^*)]H}{e_t\Delta}$$

where $\Delta = \tilde{s}\tilde{s}^* + \tilde{s}\tilde{a}^* + \tilde{s}^*\tilde{a} > 0.$

To obtain the slope of the ee schedule we divide (A-45) by (A-44) yielding

$$\text{(A-46)} \quad \frac{dY_t^*}{dY_t} = - \frac{\tilde{s}IM_t - \tilde{a}(IM_t^* - IM_t) + [\tilde{s}(1-\tilde{s}^*-\tilde{a}^*) + \tilde{a}(1-\tilde{s}^*)]H}{e_t[\tilde{s}^*IM_t^* + \tilde{a}^*(IM_t^* - IM_t) - \tilde{a}^*H]}$$

along the ee schedule.

Around a trade-balance equilibrium with zero initial debt (that is, $IM_t = IM_t^*$ and $H = 0$) this slope is negative and is equal to $-\tilde{s}/e_t\tilde{s}^*$. With the negatively sloped ee schedule a downwards movement along the schedule (that is a rise in Y_t and a fall in Y_t^*) is associated with higher values of e_t.

To determine the effects of changes in government spending we compute the horizontal shift of the ee schedule by setting $dY_t^* = dT_t = 0$ in the system (A-43) and solving for dY_t/dG. This yields

$$\text{(A-47)} \qquad \frac{dY_t}{dG} = \frac{IM_t + a^g(IM_t^* - IM_t) + [(1-\tilde{s}^*)(1-a^g) - \tilde{a}^*]H}{\tilde{s}IM_t - \tilde{a}(IM_t^* - IM_t) + [\tilde{s}(1-\tilde{s}^*-\tilde{a}^*) + \tilde{a}(1-\tilde{s}^*)H}$$

for the ee schedule.

Thus, around trade-balance equilibrium and zero initial debt the schedule shifts to the right by $1/\tilde{s}$.

By setting $dY_t^* = dG = 0$ and following a similar procedure, the horizontal shift of the ee schedule induced by a unit rise in taxes is

$$\text{(A-48)} \qquad \frac{dY_t}{dT_t} = -\frac{\tilde{a}(IM_t^* - IM_t) + (1-\tilde{s})IM_t + [(1-\tilde{s})(1-\tilde{s}^*-\tilde{a}^*(1-\tilde{s}^*)]H}{-\tilde{a}(IM_t^* - IM_t) + \tilde{s}IM_t + [\tilde{s}(1-\tilde{s}^*-\tilde{a}^*) + \tilde{a}(1-\tilde{s}^*)]H}$$

for the ee schedule.

Thus, around trade-balance equilibrium and zero initial debt schedule shifts to the left by $(1-\tilde{s})/\tilde{s}$ units.

By combining the results in (A-47) - (A-48) we obtain the effect of a balanced budget unit rise in government spending. Accordingly,

$$\text{(A-49)} \qquad \frac{dY_t}{dG} = \frac{\tilde{s}IM_t + (a^g-\tilde{a})(IM_t^* - IM_t) + [\tilde{s}(1-\tilde{s}^*-\tilde{a}^*) + (1-\tilde{s}^*)(\tilde{a}-a^g)]H}{\tilde{s}IM_t - \tilde{a}(IM_t^* - IM_t) + [\tilde{s}(1-\tilde{s}^*-\tilde{a}^*) + \tilde{a}(1-\tilde{s}^*)]H}$$

for the ee schedule with $dG = dT_t$.

Thus, around trade-balance equilibrium with zero initial debt, a balanced-budget unit rise in government spending shifts the ee schedule to the right by one unit.

In the second step of the diagramatic analysis we assume that H = 0 and we derive the rr* schedule portraying combinations of Y and Y^* along which the money-market clearing rates of interest (under the assumption of static exchange rate expectations) are equal across countries so that

$$\text{(A-50)} \qquad r(Y_t - T_t + A_{t-1}, M) = r^*(Y^* + A^*_{t-1}, M^*).$$

The slope of this schedule is r_y/r^*_{y*} which can also be expressed in terms of the characteristics of the demands for money according to

$$\text{(A-51)} \qquad \frac{dY^*_t}{dY_t} = \frac{M_y}{M^*_{y*}} \frac{M^*_{r*}}{M_r} > 0 \qquad \text{along the rr* schedule.}$$

Obviously, around $r = r^*$, $M^*_{r*} = M^*_r$. As is evident, the level of government spending does not influence the rr* schedule whereas a unit rise in taxes shifts the schedule to the right by one unit.

Formally, the effects of fiscal policies can be obtained by differentiating the system (A-41) - (A-42) and (A-50). Thus,

$$\text{(A-52)} \begin{bmatrix} -(\tilde{s}+\tilde{a}) & e_t\tilde{a}^* & IM^*_t-\tilde{a}^*H \\ \tilde{a} & -e_t(\tilde{s}^*+\tilde{a}^*) & -IM_t-(1-\tilde{s}^*-\tilde{a}^*)H \\ M_y/M_r & -M^*_{y*}/M^*_r & HM^*_{y*}/e_tM^*_r \end{bmatrix} \begin{bmatrix} dY_t \\ dY^*_t \\ de_t \end{bmatrix} = - \begin{bmatrix} 1-a^g \\ a^g \\ 0 \end{bmatrix} dG + \begin{bmatrix} 1-\tilde{s}-\tilde{a} \\ a \\ M_y/M_r \end{bmatrix} dT_t$$

Solving (A-52) the short-run effects of a debt-financed rise in government spending are:

$$\text{(A-53)} \qquad \frac{dY_t}{dG} = \frac{M^*_{y^*}}{\Delta M^*_r}\left(IM_t(1-a^g) + IM^*_t a^g + (1-a^g)H\right) \qquad \text{for } dT_t = 0$$

$$\text{(A-54)} \qquad \frac{dY^*_t}{dG} = \frac{M_y}{\Delta M_r}\left(IM_t + a^g(IM^*_t - IM_t)\right)$$

$$+ \frac{1}{\Delta}\left(\frac{M^*_{y^*}}{M^*_r}(\tilde{a} + \tilde{s}a^g) + \frac{M_y}{M_r}[(1-a^g)(1-\tilde{s}^*) - \tilde{a}^*]\right)H$$

$$\text{for } dT_t = 0$$

$$\text{(A-55)} \qquad \frac{de_t}{dG} = \frac{1}{\Delta}\left(\frac{M^*_{y^*}}{M^*_r}(\tilde{a} + \tilde{s}a^g) - \frac{e_t M_y}{M_r}[\tilde{a}^* + \tilde{s}^*(1-a^g)]\right) \qquad \text{for } dT_t = 0$$

where $$\Delta = \frac{M^*_{y^*}}{M^*_r}\left((\tilde{s}+\tilde{a})IM_t - \tilde{a}IM^*_t\right) + \frac{e_t M_y}{M_r}\left((\tilde{s}^*+\tilde{a}^*)IM^*_t - a^*IM_t\right)$$

$$+ \left((\tilde{s}+\tilde{a})\frac{M^*_{y^*}}{M^*_r} - e_t\tilde{a}^*\frac{M_y}{M_r}\right)H.$$

Thus, with an initial balanced trade and with zero initial debt, $\Delta < 0$.

Differentiating the money-market equilibrium condition (equation (8) of the text) and using (A-53), we obtain the equilibrium change in the rate of interest:

$$\text{(A-56)} \qquad \frac{dr_t}{dG} = -\frac{M_y M^*_{y^*}}{M_r M^*_r \Delta}\left(IM_t + a^g(IM^*_t - IM_t) + (1-a^g)H\right) \qquad \text{for } dT_t = 0$$

Likewise, the short-run efects of a tax-financed rise in government spending are

$$\text{(A-57)} \qquad \frac{dY_t}{dG} = \frac{1}{\Delta}\left\{ \frac{M^*_{y*}}{M^*_r} \left(\tilde{s}IM_t + (\tilde{a}-a^g)(IM_t - IM^*_t)\right) \right.$$

$$+ \frac{e_tM_y}{M_r} \left(\tilde{s}*IM^*_t + \tilde{a}*(IM^*_t - IM_t)\right)$$

$$\left. + \left(\frac{M^*_{y*}}{M^*_r} (\tilde{s}+\tilde{a}-a^g) - \frac{e_tM_y}{M_r} \tilde{a}*\right)H \right\} \qquad \text{for } dG = dT_t$$

$$\text{(A-58)} \qquad \frac{dY^*_t}{dG} = \frac{a^g}{\Delta} \left\{ \frac{M_y}{M_r} (IM^*_t - IM_t) - \left(\frac{M_y}{M_r} (1-\tilde{s}*) - \frac{M^*_{y*}}{M^*_r} \tilde{s}\right)H \right\} \qquad \text{for } dG = dT_t$$

$$\text{(A-59)} \qquad \frac{de_t}{dG} = \frac{a^g}{\Delta} \left(\frac{e_tM_y}{M_r} \tilde{s}* + \frac{M^*_{y*}}{M^*_r} \tilde{s}\right) \qquad \text{for } dG = dT_t$$

Using the money market equilibrium condition together with (A-57) yields

$$\text{(A-60)} \qquad \frac{dr_t}{dG} = -\frac{M_y}{M_r\Delta} \left\{ a^g(IM^*_t - IM_t) + \left(\frac{M^*_{y*}}{M^*_r} (\tilde{s}+\tilde{a}-a^g) + \frac{e_tM_y}{M_r} \tilde{a}*\right)H \right\}$$

$$\text{for } dG = dT_t$$

2. Long-run equilibrium: the two-country world

The long-run equilibrium of the system is characterized by equations (A-61) - (A-65) where the first three equations are the long-run counterparts to equations (A-41), (A-42) and (A-50), and the last two equations are the requirements of zero savings in both countries implying (once the government budget constraint is incorporated) current account balances. Embodied in the system are the requirements of money-market equilibria and perfect capital mobility.

$$\text{(A-61)} \qquad (1-\beta_m)\tilde{E}(Y-T+M-(1+r)B^P,\ M) + (1-\beta_m^g)G$$
$$+ e\beta_x^*\tilde{E}^*(Y^*+M^*+\frac{(1+r)}{e}B^P,\ M^*) = Y$$

$$\text{(A-62)} \qquad \beta_m\tilde{E}(Y-T+M-(1+r)B^P,\ M) + \beta_m^g G$$
$$+ e(1-\beta_x^*)\tilde{E}^*(Y^*+M^* + \frac{(1+r)}{e}B^P,\ M^*) = eY^*$$

$$\text{(A-63)} \qquad r(Y-T+M-(1+r)B^P,\ M) = r^*(Y^*+M^*+\frac{(1+r)}{e}B^P,\ M^*)$$

$$\text{(A-64)} \qquad \tilde{E}(Y-T+M-(1+r)B^P,\ M) = Y - rB^P - T$$

$$\text{(A-65)} \qquad \tilde{E}^*(Y^*+M^*+\frac{(1+r)}{e}B^P,\ M^*) = Y^* + \frac{eB^P}{e}$$

This system which determines the long-run equilibrium values of Y, Y^*, e, B^P and r, can be used to analyze the effects of government spending and taxes on these endogenous variables.

3. Exchange rate expectations

Up to this point we have assumed that the expectations concerning the evolution of the exchange rate are static. This assumption implied that the rates of interest on securities denominated in different currencies are equalized. Since, however, the actual exchange rate does change overtime, it is useful to extend the analysis and allow for exchange rate expectations that are not static. Specifically, in this part of the Appendix we assume that expectations are rational in the sense of being self fulfilling. We continue to assume that the GDP deflators are fixed. To illustrate the main implication of exchange rate expectations we consider a stripped-down version of the small-country flexible exchange-rate model and, for expository convenience, we present the analysis using a continuous-time version of the model.

The budget constraint can be written as

$$(A-66) \qquad E_t + \dot{M}_t - e_t\dot{B}^P_{ft} = Y_t - T_t - \bar{r}_f e_t B^P_{ft}$$

where a dot over a variable represents a time derivative. The spending and money-demand functions (the counterparts to equations (3) - (4) of the text) are

$$(A-67) \qquad E_t = E(Y_t - T_t - \bar{r}_f e_t B^P_{ft}, M_t - e_t B^P_{ft}, \bar{r}_f)$$

$$(A-68) \qquad M_t = M(Y_t - T_t - \bar{r}_f e_t B^P_{ft}, M_t - e_t B^P_{ft}, \bar{r}_f + \frac{\dot{e}_t}{e_t})$$

where the demand for money is expressed as a negative function of the expected depreciation of the currency, $\dot{e}_t/e_t$. In what follows we simplify the exposition by assuming that the world rate of interest, $\bar{r}_f$, is very low (zero), and that the effect of assets $(M_t - e_t B^P_{ft})$ on spending is negligible. With these simplifications the goods and money-market equilibrium conditions (the counterparts to equations (20') and (22') of the text) are

$$(A-69) \qquad (1-\beta_m)E(Y_t - T_t) + (1-\beta^g_m)G + e_t\overline{D}^* = Y_t$$

$$(A-70) \qquad M(Y_t - T_t, M - e_t B^P_{ft}, \frac{\dot{e}_t}{e_t}) = M$$

Equation (A-69) implies that the level of output which clears the goods market depends positively on the level of the exchange rate and on government spending, and negatively on taxes. This dependence can be expressed as

$$\text{(A-71)} \qquad Y_t = Y(e_t, G, T_t)$$

where $\partial Y_t/\partial e_t = \overline{D}^*/(s+a)$, $\partial Y_t/\partial G = (1-a^g)/(s+a)$ and $\partial Y_t/\partial T_t = -(1-s-a)/(s+a)$ are the conventional foreign trade multipliers. Substituting the functional relation (A-71) into the money-market equilibrium condition and solving for the (actual and expected) percentage change in the exchange rate yields

$$\text{(A-72)} \qquad \frac{\dot{e}_t}{e_t} = f(e_t, B^p_{ft}, G, T_t, M)$$

where
$$\partial f/\partial e = \left(-M_y\overline{D}^*/(s+a) + M_A B^p_{ft}\right)/M_r$$
$$\partial f/\partial B^p_{ft} = e_t M_A/M_r$$
$$\partial f/\partial G = -(1-a^g)/(s+a)M_r$$
$$\partial f/\partial T_t = (1-s-a)/(s+a)M_r$$

where M_A and M_r denote, respectively, the derivatives of the demand for money with respect to assets $(M - e_t B^p_{ft})$ and the rate of interest. The former is positive and the latter is negative. The interpretation of the dependence of the percentage change in the exchange rate, representing the money-market clearing interest rate, on the various variables follows. A rise in the exchange rate raises the goods-market clearing level of output and raises the demand for money. To restore money-market equilibrium the rate of interest must rise, that is, $\dot{e}_t/e_t$ must rise.

On the other hand, the rise in e raises the domestic-currency value of the debt B^p_{ft}. If the private sector is a net creditor, the depreciation of the currency raises the domestic-currency value of assets and raises the demand for money. This in turn also contributes to the rise in the rate of interest. If, however, the private sector is a net debtor then the value of assets falls, the demand for money is reduced, thereby contributing to a downward pressure on the rate of interest. The net effect on the rate of interest depends, therefore, on the net debtor position of the private sector; if, however, B^p_{ft} is zero, then the rate of interest must rise so that $\partial f/\partial e_t > 0$. Analogous interpretations apply to the other derivatives where it is evident that $\partial f/\partial B^p_{ft} < 0$, $\partial f/\partial G \geq 0$ and $\partial f/\partial T_t < 0$.

Equation (A-72) constitutes the first differential equation of the model governing the evolution of the exchange rate over time. The second variable whose evolution over time characterizes the dynamics of the system is the stock of private-sector debt. Substituting the goods-market equilibrium condition (A-71) into the budget constraint (A-66), using the fact that in the absence of monetary policy $\dot{M}_t = 0$ we can solve for the dynamics of private-sector debt. Accordingly,

$$\text{(A-73)} \qquad \dot{B}^p_{ft} = \frac{1}{e_t} h(e_t, G, T_t)$$

$$= \frac{1}{e_t} \left\{ E_t[Y(e_t, G, T_t) - T_t] - Y(e_t, G, T_t) + T_t \right\}$$

Equation (A-73) expresses the rate of change of private-sector debt as the difference between private-sector spending and disposable income.

The previous discussion implies that $\partial h/\partial e_t = -\overline{D}^* s/(s+a) < 0$, $\partial h/\partial G = -(1-a^g)s/(s+a) < 0$ and $\partial h/\partial T_t = s/(s+a) > 0$.

In interpreting these expressions we note that the function h represents the negative savings of the private sector. Accordingly, a unit rise in e_t or G raises savings by the saving propensity times the corresponding multiplier. Analogously, a unit rise in taxes which lowers disposable income, lowers savings by the saving propensity times the corresponding disposable-income multiplier.

The equilibrium of the system is exhibited in Figure A-1. The positively sloped $\dot{e}_t = 0$ schedule shows combinations of the exchange rate and private-sector debt which maintain an unchanged exchange rate. The schedule represents equation (A-72) for $\dot{e}_t = 0$. Its slope is positive around a zero level of private sector debt and its position depends on the policy variables G, T_t and M. Likewise, the $\dot{B}^p_{ft} = 0$ locus represents equation (A-73) for $\dot{B}^p_{ft} = 0$. It is horizontal since, as specified, the rate of change of private-sector debt does not depend on the value of debt. The arrows around the schedules indicate the directions in which the variables tend to move, and the solid curve shows the unique saddle path converging towards a stationary state. As customary in this type of analysis we associate this saddle path with the equilibrium path. The long-run equilibrium of the system is shown by point A in Figure A-1 where, for convenience, we show a case in which the long-run value of private-sector debt is zero.

The effects of a unit debt-financed rise in government spending from G_0 to G_1 are shown in Figure A-2. Starting from an initial long-run equilibrium at point A, the rise in G shifts the $\dot{B}^p_{ft} = 0$ schedule from

Figure A-1: The Equilibrium Exchange-Rate Dynamics and Debt Accumulation

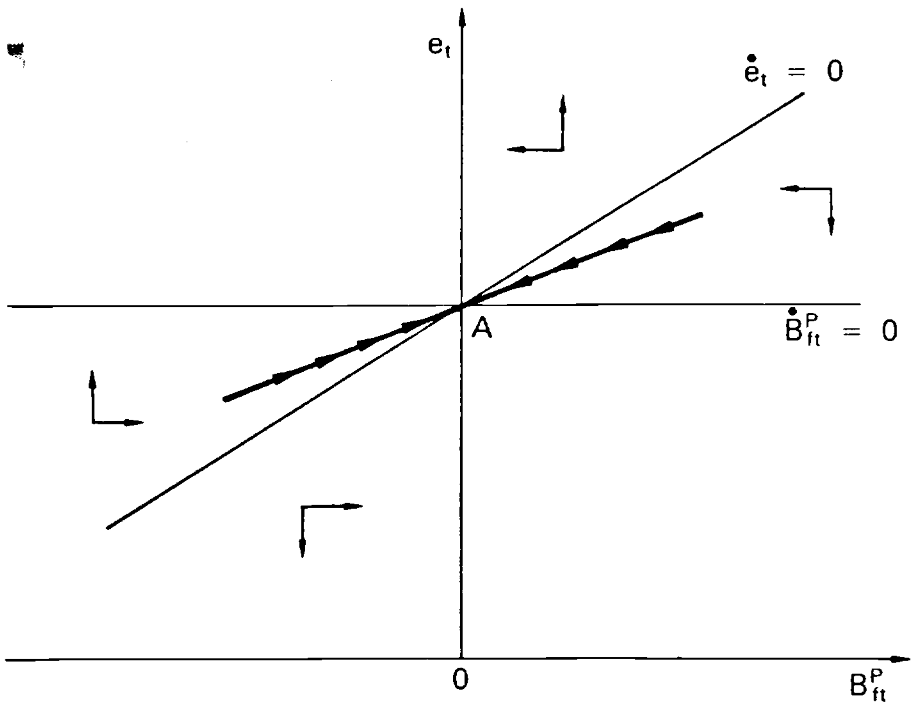
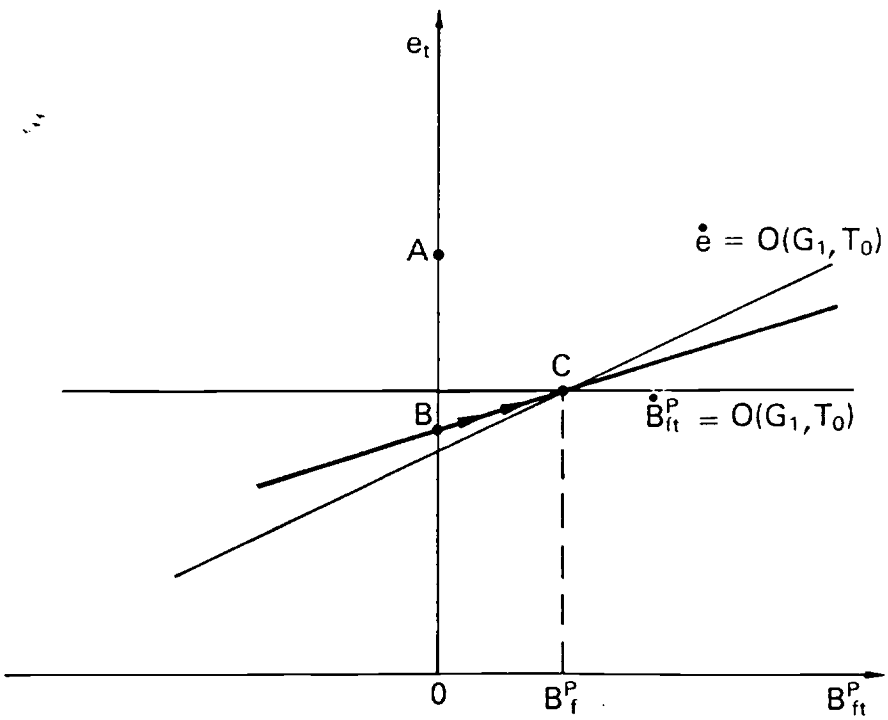


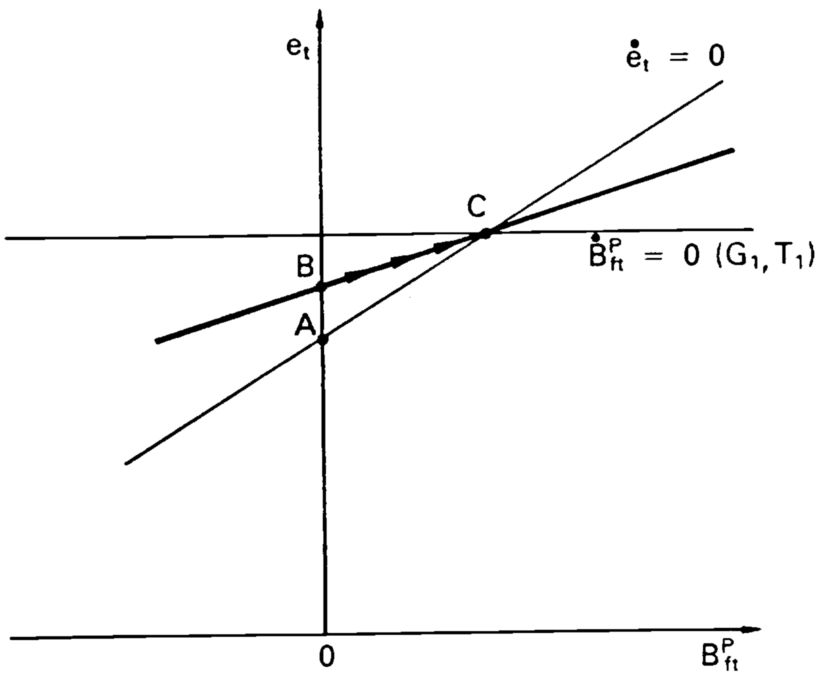
Figure A-2: The Effects of A Debt-Financed Rise in Government Spending on the Paths of the Exchange Rate and Private-Sector Debt



point A downwards by $-(1-a^g)/\overline{D}^*$ and it also shifts the $\dot{e} = 0$ schedule from point A downwards by $-(1-a^g)/M_y\overline{D}^*$. For $M_y < 1$ the vertical displacement of the $\dot{e} = 0$ schedule exceeds the corresponding displacement of the $\dot{B}^p_{ft} = 0$ schedule and the new long-run equilibrium obtains at point C at which the domestic currency has appreciated and private-sector debt has risen. The short-run equilibrium obtains at point B along the new saddle path and transition towards the long run follows along the path connecting points B and C. As is evident the initial appreciation of the currency overshoots the long-run appreciation.

The effects of a unit tax-financed rise in government spending are shown in Figure A-3. With $dG = dT$, the $\dot{B}^p_{ft} = 0$ schedule shifts upwards by $a^g/\overline{D}^*$ while the $\dot{e} = 0$ schedule shifts vertically by $(s+a-a^g)/M_y\overline{D}^*$. The benchmark case shown in Figure A-3 corresponds to the situation in which the private sector and the government have the same marginal propensities to spend on domestic goods (that is, $s+a = a^g$). In that case the $\dot{e} = 0$ remains intact, the short-run equilibrium is at point B and the long-run equilibrium is at point C. As seen in this case the domestic currency depreciates and the short-run depreciation undershoots the long-run depreciation. These results are sensitive to alternative assumptions, concerning the relative magnitudes of $(s+a)$ and a^g.

Figure A-3: The Effects of A Tax-Financed Rise in Government Spending on the Paths of the Exchange Rate and Private-Sector Debt



Data: $s + a = a^g$

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