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TESTING LONG RUN NEUTRALITY

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ABSTRACT

Propositions about long run neutrality are at the heart of most macroeconomic models. Yet, since the 1970's when Lucas and Sargent presented powerful critiques of traditional neutrality tests, empirical researchers have made little progress on testing these propositions. In this paper we show that, in spite of the Lucas-Sargent critique, long run neutrality can be tested without specifying a complete model of economic activity. This is possible when the variables are integrated. In this case, permanent shifts in the historical data can be uncovered using VAR methods, and neutrality can be tested when there is *a priori* knowledge of one of the structural impact multipliers or one of the structural long run multipliers. In most circumstances such *a priori* knowledge is available. We use this framework to test four long run neutrality propositions: (i) the neutrality of money, (ii) the superneutrality of money, (iii) a vertical long run Phillips curve, and (iv) the Fisher effect. In each application, our *a priori* knowledge consists of a range of plausible values for the relevant impact and long run multipliers. We find that the U.S. postwar data are consistent with the neutrality of money and a vertical long run Phillips curve, but find evidence against the superneutrality of money and the long run Fisher relation. The sign of the estimated effect of money growth on output depends on the particular identifying assumption used. For a wide range of plausible identifying restrictions, nominal interest rates are found to move less than one-for-one with inflation in the long run.

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1. Introduction

Key classical macroeconomic hypotheses specify that permanent changes in nominal variables have no effect on real economic variables in the long run. The simplest "long run neutrality" proposition specifies that a permanent change in the money stock has no long run consequences for the level of real output. Other classical hypotheses specify that a permanent change in the rate of inflation has no long run effect on unemployment (a vertical long run Phillips curve), or real interest rates (the long run Fisher relation). In this paper we provide an econometric framework for studying these classical propositions, and use the framework to investigate their relevance for the postwar U.S. experience.

Testing these propositions is a subtle matter. For example, Lucas (1972) and Sargent (1971) provide examples in which rational expectations together with short run nonneutrality makes it impossible to test long run neutrality using reduced form econometric methods. In their examples, realizations from the model do not contain the sustained changes in nominal variables necessary to directly test long run neutrality. In the context of these models, Lucas and Sargent argued that structural econometric methods were required to test the neutrality propositions. McCallum (1984) extended these arguments and showed that low frequency band spectral estimators calculated from reduced form models were also subject to the Lucas-Sargent critique. While these arguments stand on firm logical ground, structural econometric analysis has not yet yielded convincing evidence on the neutrality propositions. This undoubtedly reflects a lack of consensus among macroeconomists on the appropriate structural model to use for the investigation.

Rather than rely on the Lucas-Sargent framework, we investigate neutrality using reduced form econometric models. We have two primary objectives. The first is pedagogical: we use econometric models of the sort that motivated Lucas and Sargent to review some important econometric points concerning neutrality tests made by Geweke (1986), Stock and Watson (1988), Fisher and Seater (1990) and others. The objective is to show how tests for neutrality are affected by assumptions concerning (i) the order of integration (or required level of differencing) of the real and nominal variables, and (ii) the endogeneity of money. The second objective is more substantive: we summarize the reduced form information in the postwar U.S data on the neutrality propositions. The information is summarized in a way that highlights the tradeoff between what the data can say about the long run and short run interaction of the variables. For example, in our framework the estimated value of the long run elasticity of output with respect to money depends critically on what is assumed about one of three other elasticities: (i) the impact elasticity of output with respect to money, (ii) the impact elasticity of money with respect to output, or (iii) the long run elasticity of money with respect to output. We present results for a wide range of values for these elasticities.

To accomplish these two goals we begin, in Section 2, with the theoretical problem of testing for neutrality in economies that are consistent with the Lucas-Sargent conclusions. We show how the order of integration of the processes characterizing the real and nominal variables can be used to help construct tests of neutrality. Essentially, the idea is that if the processes are integrated then the data will exhibit permanent shifts, and tests for neutrality can be constructed by asking whether the permanent shifts in the real variables were caused by the permanent shifts in the nominal variables.

In section 3, we show that the endogeneity of money introduces identification problems familiar from the analysis of simultaneous equation models. When money is endogenous, long run correlation does not imply long run causation, and this makes the long run neutrality testing problem more difficult. Indeed, without additional identifying assumptions, long run neutrality cannot be tested. The objective of this section is present a set of alternative identifying assumptions for the model. Empirical support for the neutrality propositions can then be determined as these identifying assumptions are systematically altered.

Section 4 contains an empirical investigation of (i) the long run neutrality of money, (ii) the long run superneutrality of money, (iii) the slope of the long run Phillips curve, and (iv) the long run Fisher relation. Even with an unlimited amount of data, the identification problems discussed in Section 3, make it impossible to a carry out definitive test of the long run propositions. Instead, we show what the data say about the propositions across a wide range of observationally equivalent models. As a preview of our results we find that over a wide range of identifying assumptions, there is little evidence in the data against the hypothesis that money is neutral in the long run, or that inflation has no long run effect on the unemployment rate.

On the other hand, the data do not appear to be consistent with the hypothesis that, over the long run, money is superneutral or that nominal interest rates move one-for-one with inflation.. The estimated long run effect of changes in the growth rate of money on the level of output depends critically on the particular identifying restriction employed. Some restrictions suggest that output increases in response to exogenous increases in the money growth rate, while other restrictions suggest that output falls. In contrast, a wide range of plausible identifying restrictions suggests that, in the long run, nominal interest rates move less than one-for-one with inflation.

2. The Basic Macroeconometrics of Neutrality Tests

Early empirical researchers investigated long run neutrality by examining the coefficients in the distributed lag:

(1) $y_t = \sum \alpha_j m_{t-j} + \text{error} = \alpha(L)m_t + \text{error}$

where $\alpha(L)=\sum \alpha_j L^j$, and L is the lag operator. Since the sum of the α_j coefficients, $\alpha(1)$, expresses the "long run multiplier" associated with a permanent change in m, this appears to be a reasonable procedure for investigating long run neutrality. But, in models with short run nonneutrality and rational expectations, the approach can be very misguided.

The Lucas-Sargent Critique

A simple macroeconomic model can be used to replicate the Lucas (1972)-Sargent (1971) demonstration of the subtleties involved in testing for long run neutrality. The model consists of an aggregate supply schedule (2a); a monetary equilibrium condition (2b); and a monetary supply rule (2c).

(2a) $y_t = \theta(p_t - E_{t-1}p_t)$

(2b) $p_t = m_t - \delta y_t$

(2c) $m_t = \rho m_{t-1} + \epsilon_t$

where y_t is output; p_t is the price level; $E_{t-1}p_t$ is the expectation of p_t formed at t-1, and m_t is the money stock. The reduced form for output is:

$$\text{(3)} \quad y_t = \alpha_0 m_t + \alpha_1 m_{t-1} = \pi m_t - \rho\pi m_{t-1} = \pi(1-\rho L)m_t$$

with $\pi=\theta/(1+\delta\theta)$.

As in Lucas(1973), the model is constructed so that only surprises in the money stock are nonneutral: permanent changes in money have no long run effects on output. However, the reduced form suggests that a one unit permanent increase in money will increase output by $\alpha(1) = \pi(1-\rho)$. Moreover, as argued by McCallum, the reduced form implies that there is nonzero long run correlation between money and output, measured by the spectral density matrix of the variables at frequency zero. Yet, by construction, permanent increases in money have no effect on output.

On this basis, Lucas (1972) argues that a valid test of long run neutrality can only be conducted by determining the structure of monetary policy (ρ) and its interaction with the short run response to monetary shocks (π). While easy enough in this simple setting, this is a much more difficult matter in richer dynamic models, or models with a more sophisticated specification of monetary policy.

However, if $\rho=1$, there is a straightforward test of the long run neutrality proposition in this simple model. Generally, the model implies:

$$\text{(3')} \quad y_t = \pi\rho\Delta m_t + \pi(1-\rho)\, m_t$$

so that with $\rho=1$ there is a zero effect of the level of money under the neutrality restriction. Hence, one can simply examine whether the coefficient on the level of money is zero when m_t is included in a bivariate regression that also involves Δm_t as a regressor.

With permanent variations in the money stock, the reduced form of this simple model has the property that (i) the coefficient on m_t corresponds to the experiment of permanently changing the level of the money stock; and (ii) the coefficient on Δm_t captures the short run nonneutrality of monetary shocks. Equivalently, with $\rho=1$, the neutrality hypothesis implies that in the specification $y_t = \sum \alpha_j m_{t-j}$, the neutrality restriction is $\alpha(1)=0$. This restriction carries over to richer models.

Long Run Neutrality in a Prototypical Macroeconomic Model

Consider the following linear dynamic macroeconomic model:

(4a) $\gamma_y(L)y_t = \theta p_t + \phi_m(F)E_t m_t + \phi_\eta(F)E_t \eta_t$

(4b) $\gamma_p(L)p_t = -\delta y_t + \Psi_m(F)E_t m_t + \Psi_\eta(F)E_t \eta_t$

(4c) $\Delta m_t = \mu(L)\epsilon_t^m$

(4d) $\Delta \eta_t = \lambda(L)\epsilon_t^\eta$

where η_t is a real disturbance, $E_t m_s$ is the conditional expectation of m_s formed at date t, F is the forward operator defined so that $F^j[E_t m_\tau]=E_t m_{\tau+j}$, L is the lag operator, and $\gamma_y(.)$, $\phi_m(.)$, $\phi_\eta(.)$, $\gamma_p(.)$, $\Psi_m(.)$, $\Psi_\eta(.)$, $\mu(.)$, and $\lambda(.)$ are one sided polynomials in non-negative powers of their arguments.

We will argue by example using this model, but the general points that we make carry over to more general models. We chose this model because has three important properties. First, real disturbances are introduced in (4a), so that the joint (y,m) process is nonsingular. Second, we included gradual output and price adjustment through $\gamma_y(L)$ and $\gamma_p(L)$, to make it clear that the results apply equally well to Keynesian systems and Classical systems that have a long run neutrality property. Third, future expectations are incorporated because they are important determinants of economic activity and to make it clear that the results do not depend on backward looking behavior.[1]

In the appendix we derive the consequences of requiring monetary neutrality in a certainty stationary state where the levels of m_t and η_t are constant for all time. This imposes one restriction on the parameters in equations (4a) and (4b). We also derive the solution to the model with uncertainty given by (4a)-(4d). We summarize some key points of the solution here. To begin, we need to study the dynamics of the money process. This is facilitated by decomposing m_t into its stochastic "trend" and "stationary" component. To do this, write:

$$(5) \qquad \Delta m_t = \mu(1)\epsilon_t^m + [\mu(L)-\mu(1)]\epsilon_t^m,$$

so that

$$(6) \qquad m_t = \mu(1)\bar{m}_t + \mu^*(L)\epsilon_t^m + m_0$$

where $\mu(1)\bar{m}(t)=\mu(1)\sum_{s=1}^{t}\epsilon_s^m$ is the permanent (random walk) component of m_t, and $\mu^*(L)=(1-L)^{-1}[\mu(L)-\mu(1)]$, so that $\mu^*(L)\epsilon_t^m$ is the stationary component of m_t.[2] When the money supply is stationary, there is no permanent component since $\mu(1)=0$.

When solved (see the Appendix), the model has two useful representations for y_t. The first is:

$$(7) \qquad y_t = \gamma_{ym}\mu(1)\bar{m}_t + \beta_m(L)\epsilon_t^m + \beta_\eta(L)\eta_t + \kappa$$

The coefficient γ_{ym} is a function of the parameters in the polynomials $\gamma_y(L)$, $\gamma_p(L)$, $\phi_m(F)$, $\Psi_m(F)$ and the parameters θ and δ. The restriction $\gamma_{ym}=0$ reflects the "neutrality" in the macroeconomic system. It is the neutrality restriction that arises in the stationary state of the certainty model and the long run neutrality restriction that arises in the model with uncertainty. That is, γ_{ym} shows the model's long run response of y_t to a one unit permanent increase in m_t. Equation (7) shows why neutrality tests depend critically on assumptions about the degree of integration in the m_t process. If m_t is not integrated, then $\mu(1)=0$, and the first first term on the right hand side of (9) vanishes. In this case γ_{ym} can only be determined by the Lucas-Sargent procedure of estimating the structural parameters and solving the model.

However, when $\mu(L)$ is invertible, so that m_t is integrated and $\mu(1)$ is nonzero, (7) can be rewritten as:

(8) $$y_t = \gamma_{ym} m_t + \tilde{\beta}_m(L)\Delta m_t + \beta_\eta(L)\eta_t + \kappa$$

where $\tilde{\beta}_m(L)$ is a function of the coefficients in $\beta_m(L)$ and $\mu^*(L)$ (see Appendix A). Equation (8) suggests that it is possible to investigate neutrality by estimating γ_{ym} in (8) as the coefficient on the level of m_t in the regression of y_t onto m_t and a distributed lag of Δm_t.

Since the error term in the regression (8), $\beta_\eta(L)\eta_t$, is serially correlated, the coefficient γ_{ym} should be estimated by generalized least squares. When η_t is integrated, as suggested by equation (4d), equation (8) can be rewritten as:

(9) $$\nu(L)\Delta y_t = \nu(1)\gamma_{ym}\Delta m_t + \dot{\beta}_m(L)\Delta^2 m_t + \epsilon_t^\eta$$

where $\nu(L)=\lambda(L)^{-1}\beta_\eta(L)$ and $\dot{\beta}_m(L)=\nu(L)\tilde{\beta}_m(L)$. This equation can be estimated by ordinary least squares, and the long run neutrality restriction tested by checking whether Δm_t enters the regression.

Equation (9) also shows that γ_{ym} is the spectral gain of Δy with respect to Δm, at frequency 0. Thus, estimates of γ_{ym} can be constructed nonparametrically using frequency domain methods. A general discussion of these "band spectral" regression techniques is in Engle (1974). Applications in the context of the neutrality propositions include Lucas (1980), Summers (1983) and Fisher and Seater (1990).

3. A More General Framework

While the analysis above highlighted several key features of the neutrality restriction it ignored two complications present in the data. First, it assumed that the money supply process was exogenous, when in fact there are potentially important feedbacks from output to money. Second, it assumed that money followed an I(1) process, when, at least over certain historical periods, money is arguably characterized by an I(2) process. In this section we generalize the framework of the previous section to allow for these complications.

Endogenous Money

Suppose that the central bank follows an operating procedure of the form:

$$\text{(10a)} \qquad \Delta m_t = \lambda_{my}\Delta y_t + \sum_{j=1}^{p}\alpha_{my}^{j}\Delta y_{t-j} + \sum_{j=1}^{p}\alpha_{mm}^{j}\Delta m_{t-j} + \epsilon_t^m$$

where λ_{my} indicates the contemporaneous effect of output on the money supply. When this money supply equation is added to the model in the last section, the reduced form equation for output becomes:

$$\text{(10b)} \qquad \Delta y_t = \lambda_{ym}\Delta m_t + \sum_{j=1}^{p}\alpha_{yy}^{j}\Delta y_{t-j} + \sum_{j=1}^{p}\alpha_{ym}^{j}\Delta m_{t-j} + \epsilon_t^\eta$$

where the parameter λ_{ym} measures the contemporaneous response of output to changes in the money supply.

A more convenient representation of the model is

$$\text{(11a)} \qquad \alpha_{mm}(L)\Delta m_t = \alpha_{my}(L)\Delta y_t + \epsilon_t^m$$

$$\text{(11b)} \qquad \alpha_{yy}(L)\Delta y_t = \alpha_{ym}(L)\Delta m_t + \epsilon_t^\eta$$

where $\alpha_{mm}(L)=1-\sum_{j=1}^{p}\alpha_{mm}^{j}L^j$, $\alpha_{my}(L)=\lambda_{my}+\sum_{j=1}^{p}\alpha_{my}^{j}L^j$, $\alpha_{yy}(L)=1-\sum_{j=1}^{p}\alpha_{yy}^{j}L^j$, and $\alpha_{ym}(L)=\lambda_{ym}+\sum_{j=1}^{p}\alpha_{ym}^{j}L^j$. In stacked form, the model becomes:

$$\text{(12)} \qquad \alpha(L)X_t = \epsilon_t,$$

where $\alpha(L)=\sum_{j=0}^{p}\alpha_j L^j$, and

$$X_t = \begin{bmatrix}\Delta m_t \\ \Delta y_t\end{bmatrix}, \quad \epsilon_t = \begin{bmatrix}\epsilon_t^m \\ \epsilon_t^\eta\end{bmatrix}, \quad \alpha_0 = \begin{bmatrix}1 & -\lambda_{my} \\ -\lambda_{ym} & 1\end{bmatrix}, \quad \text{and } \alpha_j = -\begin{bmatrix}\alpha_{mm}^j & \alpha_{my}^j \\ \alpha_{ym}^j & \alpha_{yy}^j\end{bmatrix}, \quad j=1,\ldots,p.$$

Using this notation the long run multipliers are $\gamma_{my}=\alpha_{my}(1)/\alpha_{yy}(1)$ and $\gamma_{ym}=\alpha_{ym}(1)/\alpha_{yy}(1)$.

The first, γ_{my}, is the long run response of m to a one unit permanent increase in y. The second, γ_{ym}, is the long run response of y to a one unit permanent increase in m. The long run neutrality restriction is $\gamma_{ym}=0$.

When the money supply is endogenous, the neutrality restriction is no longer testable. As noted by Geweke (1982), the model is econometrically unidentified in this situation. To see the source of the identification problem, notice that (12) is a standard linear simultaneous equations model with reduced form:

$$(13) \qquad X_t = \sum_{i=1}^{p}\Phi_i X_{t-i} + e_t$$

where $\Phi_i=-\alpha_0^{-1}\alpha_i$ and $e_t=\alpha_0^{-1}\epsilon_t$. The matrices α_i and Σ_ϵ are determined by the set of equations:

$$(14) \qquad \alpha_0^{-1}\alpha_i = -\Phi_i, \quad i=1,\ldots,p$$

$$(15) \qquad \alpha_0^{-1}\Sigma_\epsilon\alpha_0^{-1\prime} = \Sigma_e.$$

When there are no restrictions on coefficients on lags entering (12), equation (14) imposes no restrictions α_0; it serves to determine α_i as a function of α_0 and Φ_i. Equation (15) determines both α_0 and Σ_ϵ as a function of Σ_e. But Σ_e (a 2×2 symmetric matrix) has only three unique elements: only three unknown parameters in α_0 and Σ_ϵ can be identified. Thus, even if we maintain the assumption that ϵ_t^m and ϵ_t^η are uncorrelated, the four unknown parameters: $\sigma_{\epsilon m}$, $\sigma_{\epsilon\eta}$, λ_{my} and λ_{ym} cannot be identified. One additional restriction is required.

Where might this additional restriction come from? One approach is to assume that the model is recursive, so that either $\lambda_{my}=0$ or $\lambda_{ym}=0$. Geweke (1986), Stock and Watson (1988) and Fisher and Seater (1991) present tests for neutrality under the assumption that $\lambda_{ym}=0$, and Geweke (1986) also presents results under the assumption that $\lambda_{my}=0$. Alternatively, neutrality might be *assumed*, and the restriction $\gamma_{ym}=0$ used to identify the model. This assumption has been used by Gali (1991), King Plosser Stock and Watson (1991), Shapiro and Watson (1988) and others to disentangle the structural shocks ϵ_t^m and ϵ_t^η.

The approach that we take in the empirical section is more eclectic and potentially more informative. Rather than report results associated with a single identifying restriction, we use graphs to present a wide range of observationally equivalent estimated models. This allows the reader to specify a value for any one of the parameters λ_{ym}, λ_{my}, γ_{ym} or γ_{my} and find the implied estimates for the other three parameters. But, before moving the empirical work, the framework must be generalized to accommodate I(2) processes.

A Model with Money following an I(2) Process

Over certain sample periods, money supply growth rates are highly persistent. Indeed, in our empirical analysis, we cannot reject the the hypothesis that the money supply (M2) is generated by an I(2) process over the entire postwar period. Since the framework developed above depended on an I(1) process for money, a modification is necessary when money is I(2).

Conceptually, the modification is quite simple: merely replace m_t in the analysis above with Δm_t. The shocks ϵ_t^m are now interpreted as shocks that have a permanent effect on money growth, and the restriction tested is the superneutrality of money.

Neutrality can't be tested in a system in which output is I(1) and money is I(2). Intuitively this follows because neutrality concerns the relationship between shocks to the level of money and the level of output. When money is I(2), shocks affect the rate of growth of money, and there are no shocks to the level of money.

To see this formally, write equation (11b) as:

$$\text{(16)} \qquad \begin{aligned} \alpha_{yy}(L)\Delta y_t &= \alpha_{ym}(L)\Delta m_t + \epsilon_t^{\eta} \\ &= \alpha_{ym}(1)\Delta m_t + \alpha_{ym}^{*}(L)\Delta^2 m_t + \epsilon_t^{\eta} \end{aligned}$$

where $\alpha_{ym}^{*}(L)=(1-L)^{-1}[\alpha_{ym}(L)-\alpha_{ym}(1)]$. When money is I(1), the neutrality restriction is $\alpha_{ym}(1)=0$. But when money is I(2) and output is I(1), $\alpha_{ym}(1)=0$ by construction. (When $\alpha_{ym}(1)\neq 0$, output is I(2).) For a more detailed discussion of neutrality restrictions with possibly different orders of integration see Fisher and Seater (1991).

Summing Up

Equation (12) will serve as the basis for the empirical work that we present in the next section. Different definitions of X_t in equation (12) allow us to test different economic hypotheses. In particular, using $X_t=(\Delta m_t\ \Delta y_t)'$, with m_t assumed to follow an I(1) process, the model can be used to investigate the neutrality of money. Superneutrality can be investigated using $X_t=(\Delta^2 m_t\ \Delta y_t)'$, with m_t assumed to follow an I(2) process. In economies in which rate of inflation, π_t, and the unemployment rate, u_t, follow integrated processes, the framework can be used to investigate the slope of the long run Phillips curve. In particular, when π_t and u_t are I(1), equation (12) with $X_t=(\Delta\pi_t\ \Delta u_t)'$ can be used to investigate the effect of permanent shocks to inflation on the unemployment rate, and permanent shocks to the unemployment rate on inflation. Finally, in economies in which both the inflation rate and the real interest rate are I(1), the framework can be used to test for the long run Fisher effect. In this case, $X_t=(\Delta\pi_t, \Delta R_t)$, where R_t is the nominal interest rate, and the long run Fisher relation implies a unit long run response of R_t to π_t.

We now turn to an investigation of these neutrality propositions using data for the postwar U.S. economy.

4. Evidence on the Neutrality Propositions in the Postwar U.S. Economy

The neutrality propositions that we will test involve linkages between (i) real output and the nominal money stock; (ii) real output and the money growth rate; (iii) unemployment and inflation; and (iv) nominal interest rates and inflation. We use gross national product for output; money is M2; unemployment is the civilian unemployment rate; price inflation is calculated from the consumer price index; and the nominal interest rate is the yield on three month Treasury Bills.[3] Figure 1 plots the data, and descriptive statistics are given in Table 1.

Unit Root Properties:

Since the unit root properties of the data play a key role in the analysis, Table 1 also presents statistics describing these properties of the data. We use two measures: (i) augmented

Dickey-Fuller (ADF) "t-statistics" and (ii) 95% confidence intervals for the largest autoregressive root. (These were constructed from the ADF statistics using Stock's (1991) procedure.)

The ADF statistics indicate that unit roots cannot be rejected at the 5% level for any of the series: from this perspective, output y_t), money (m_t), money growth (Δm_t), inflation (π_t), unemployment (u_t) and nominal interest rates (R_t) all can be taken to possess the nonstationarity necessary for testing long run neutrality. Moreover, a unit root cannot be rejected for $r_t = R_t - \pi_t$, consistent with the hypothesis that R_t and π_t are not cointegrated.

However, the confidence intervals are very wide, suggesting a large amount of uncertainty about the unit root properties of the data. For example, the real GNP data are consistent with the hypothesis that the process is I(1), but are also consistent with the hypothesis that the data are trend stationary with an autoregressive root of .89. The money supply data are consistent with the trend stationary, I(1) and I(2) hypotheses. The results in Table 1 suggest that it is reasonable to carry an empirical investigation of the neutrality propositions predicated on integrated processes, but that the results from the investigation have to be carefully interpreted.

Identification and Estimation:

Our empirical investigation is centered around the four economic interpretations of equation (12) that were listed at the end of the last section. For each interpretation we have estimated the model using the following identifying assumptions:

(i) α_0 has 1's on the diagonal,

(ii) Σ_ϵ is diagonal,

and, writing $X_t = (x_t^1 \ x_t^2)$, one of the following:

(iii.a) the impact elasticity x^1 with respect to x^2 is known (e.g λ_{my} is known in the money-output system),

(iii.b) the impact elasticity of x^2 with respect to x^1 is known (e.g λ_{ym} is known in the money-output system),

(iii.c) the long run elasticity of x^1 with respect to x^2 is known (e.g γ_{my} is known in the money-output system), (iii.d) the long run elasticity of x^2 with respect to x^1 is known (e.g γ_{ym} is known in the money-output system).

The models are estimated using simultaneous equation methods. The details are provided in the appendix, but the basic strategy is quite simple. For example for the money-output system, when we fix the value of λ_{my}, the maximum likelihood estimator of the money supply equation (11a) is the ordinary least squares estimator (OLS). However, the output equation (11b) cannot be estimated by OLS since it contains Δm_t, which is potentially correlated with the error term. The maximum likelihood estimator of this equation is constructed by instrumental variables, using the residual from the estimated money supply equation together with lags of Δm_t and Δy_t as instruments. The residual is a valid instrument because of assumption (ii). In the appendix we show how a similar procedure can be used when assumptions (iii.b)-(iii.d) are maintained. Formulae for the standard errors of the estimators are also provided in the appendix.

We will report results for a wide range of values of the parameters in assumptions (iii.a)-(iii.d). All of the models included six lags of the relevant variables. The sample period was 1949:1-1990:4 for the models that did not include the unemployment rate; when the unemployment rate was included in the model, the sample period was 1950:1-1990:4. Data prior to the initial periods were used as lags in the regressions. The robustness of the results to choice of lag length and sample period will be discussed below.

Reporting the Results for the Estimated Models:

With our approach to identification, literally hundreds of models are estimated, and there is a tremendous amount of information that can potentially be reported. In reporting the results we proceed in four steps.

First, we present some summary information on the estimated reduced form VAR, namely the the covariance matrix of (i) the VAR forecast errors ($e_t=\Phi(L)X_t$), and (ii) the shocks to the stochastic trends ($\Delta\tau_t=\Phi(1)^{-1}e_t$). For example, in panel A of Table 2, we see that money and output are slightly positively related in the short run (the correlation between the forecast errors is .08) and negatively related in the long run (the correlation between the stochastic trend innovations is -.25).

Second, we present information on the links between the individual behavioral parameters (λ_{my}, λ_{ym} and γ_{my}) and the neutrality hypothesis (γ_{ym}=0). This information focuses on two questions:

(1) For what values of the behavioral parameters is it possible to reject the neutrality hypothesis at the 5% level?

(2) Under the neutrality hypothesis, what are the estimates of the behavioral parameters and their associated standard errors?

For example, panel A of Table 2 shows the answers to these questions for the money-output system. We find that neutrality cannot be rejected at the 5% level for any value of λ_{my} less than 1.4. Thus for example, the common identifying assumption of contemporaneous exogeneity (λ_{my}=0) would not lead to a rejection of the neutrality hypothesis. This identifying assumption is challenged by those who see the central bank responding the changes in output to accomplish interest rate smoothing (e.g. Goodfriend (1987)). Alternative identifying restrictions would lead to a rejection of the neutrality hypothesis only if the central bank was aggressively accommodative (λ_{my}>1.4). Panel A of Table 2 also shows the results of tests of the neutrality hypothesis for a range of values of (i) the short run impact of money on output (λ_{ym}) and (ii) the long run impact of output on money (γ_{my}).

The answer to the second question is also provided in Table 2. For example, Panel A of Table 2 shows that when long run neutrality is maintained (γ_{ym}=0), the point estimate of λ_{my} is .22 with a standard error of .20. If the estimated value of λ_{my} was nonsensical when γ_{ym}=0 was maintained, then this would be evidence against the neutrality hypothesis. Since λ_{my}=.22 strikes us a plausible value, this experiment provides no evidence against the neutrality hypothesis.

The third step in presenting the results is Figure 2. This figure traces out the relationship between the behavioral parameters and the neutrality hypothesis. For example, Panel A of

Figure 2 presents the point estimates of γ_{ym} and 95% confidence intervals for a wide range of values of λ_{my}. Analogous results are reported in Panels B and C for a range of values of λ_{ym} and γ_{my}, respectivley. These figures show precisely how the outcome of the neutrality test depends on a particular identifying assumption. Panels A-C also how provide a complete description of the relation between the estimates of the four parameters: given a value for any one of the parameters, the point estimates for the other three parameters can be determined from the figure.[4]

In Panel D of Figure 2, we show the joint 95% confidence region for λ_{my} and λ_{ym} under the maintained assumption of long run neutrality. In line with our discussion above, this figure allows the reader to carry out a "specification test" of the neutrality hypothesis: if the reader believes that the true value of the pair $(\lambda_{my},\lambda_{ym})$ lies outside the 95% confidence region, then the model with the long run neutrality hypothesis imposed is rejected at the 5% level.

Finally, Table 3 provides information on the robustness of selected empirical results to the sample period and number of lags included in the VAR.

Evidence on the Various Neutrality Hypotheses:

We now review the evidence on the four neutrality hypotheses: (i) the long run neutrality of money, (ii) the long run superneutrality of money, (iii) the long run Phillips curve slope, and (iv) the Fisherian theory.

The Neutrality of Money: We find little evidence in the psotwar U.S. data against the long run neutrality hypothesis. In particular (i) a wide range of plausible identifying restrictions do not lead to rejection of the neutrality hypothesis (Panel A of Table 2 and Panels A-C of Figure 2), and (ii) a plausible fitted model obtains when long run neutrality is imposed (Panel A of Table 2 and Panel D of Figure 2).

Superneutrality of Money: Evidence on the superneutrality of money is contained in Panel B of Tables 2 and 3, and in Figure 3. These summarize results from a VAR with $X_t=(\Delta^2 m_t, \Delta y_t)$. The figures are read the same way as previously, except that now the experiment involves the effects of changes in the rate of growth of money, so that the parameters are $\lambda_{\Delta m,y}$, $\lambda_{y,\Delta m}$, $\gamma_{\Delta m,y}$ and $\gamma_{y,\Delta m}$.

There are two substantive conclusions to be drawn from the Tables and Figure. First, it is easy to find evidence against the superneutrality of money. For example, when money is contemporaneously exogenous ($\lambda_{\Delta m,y}=0$), the estimated long run effect of money growth on output is quite large ($\hat{\gamma}_{y,\Delta m}=3.80$) and superneutrality is rejected at the 5% level. Thus, a 1% permanent increase in the money growth rate is predicted to increase the flow of output by 3.8% per year in perpetuity. Our sense is that even those who believe that the Tobin (1965) effect is empirically important do not believe that it is this large. Panel of Figure 3 shows that the estimated value of $\gamma_{y,\Delta m}$ falls sharply as $\lambda_{\Delta m,y}$ is increased: $\hat{\gamma}_{y,\Delta m}=0$ when $\lambda_{\Delta m,y}=.30$.

The second substantive conclusion is that the particular identifying assumption that is employed has a large effect on the sign and the magnitude of the estimated value of $\lambda_{y,\Delta m}$. When $\lambda_{y,\Delta m}=0$ or $\lambda_{y,\Delta m}=0$, the estimated value of $\gamma_{y,\Delta m}$ is positive. On the other hand, if money growth is assumed to be exogenous in the long run, so that $\lambda_{\Delta m,y}=0$, the point estimate of $\gamma_{y,\Delta m}$ is negative, consistent with the predictions of cash-in-advance models in which sustained inflation is a tax on investment activity (Stockman [1981]) or on labor supply (Aschauer and Greenwood [1983] or Cooley and Hansen [1989]).[5]

The Long Run Phillips Curve: In the 1950-1990 sample, the estimated long run correlation between inflation and unemployment is -.38 (Table 2, panel C). Thus, the data are consistent with a long run "statistical" or reduced form relation of the sort uncovered by Phillips (1958). However, an identifying assumption is required to assess whether this correlation arises from a causal relationship from inflation to unemployment ($\gamma_{u\pi}\neq0$) – the causal link that Solow (1970) and Gordon (1970) had in mind – or from unemployment to inflation ($\gamma_{\pi u}\neq0$). Our results provide little evidence against the hypothesis that $\gamma_{u\pi}=0$. For example, from Table 3, both $\lambda_{u\pi}=0$ and $\lambda_{\pi u}=0$ lead to small *positive* values of $\gamma_{u\pi}$ (.03 and .06, respectively); while $\gamma_{\pi u}=0$ leads to a small negative value of $\gamma_{u\pi}$ (-.17). None of these estimates are significantly different from 0 at the 5% level.

By contrast, the conventional view in the late 1960's and early 1970's was that there was a much more favorable tradeoff between inflation and unemploymnet. For example, in discussing Gordon's famous (1970) test of an accelerationist Phillips curve model, Solow calculated that there was a a one-for-one long run tradeoff implied by the study. This calculation was sufficiently conventional that it led to no sharp discussion among the participants at the Brookings panel. Essentially the same tradeoff was suggested by the 1969 *Economic Report of the President*, which provided a graph of inflation and unemployment between 1954 and 1968.[6] An interesting question for future research is why the convential estimates from the late 60's are so much larger than the estimates we obtain. Panel C in Table 3 suggests that sample period alone cannot be the answer: estimates from the 1950-1972 period do not differ significantly from the full sample results.

The Fisherian Theory of Inflation and Interest Rates: The strongest evidence against the classical neutrality hypotheses emerges with respect to the Fisherian link between long run components of inflation and nominal interest rates. It is difficult to find any reasonable identifying restrictions that make the Fisherian theory look plausible in the postwar U.S. data. To begin, there is a positive correlation between the stochastic trends in the inflation and nominal interest rates (ρ=.56 in Table 2, panel D). But this correlation is sufficiently small that if inflation is assumed to be exogenous in the long run ($\gamma_{\pi R}$=0) then the long run multiplier from inflation to nominal rates ($\gamma_{R\pi}$) is only .34 (with a standard error of .12 – see Table 3, panel D). Further, identifying assumptions that rely on short run information make the neutrality of real rates similarly incredible: for example, Table 2 shows that the short run effect of interest rates on inflation ($\lambda_{\pi R}$) must be less than -5.0 for the hypothesis $\gamma_{R\pi}$=1 to be consistent with the data at the 5% level.

One way of describing the puzzle is that the VAR model implies substantial volatility in trend inflation: the estimated standard deviation of the inflation trend is much larger (1.25) than that of nominal rates (0.75). Thus, to reconcile the data with $\gamma_{R\pi}$=1, a large negative effect of nominal interest rates on inflation is required. (The estimated value of $\gamma_{\pi R}$=-19, when $\gamma_{r\pi}$=1 is imposed [Table 2, panel D].)

While these results reflect the conventional finding that nominal interest rates do not adjust fully to sustained inflation in the postwar U.S. data, they appear even more puzzling since long run neutrality fails for such a large range of identifying assumptions. One possible explanation is that the failure depends on the particular specification of the bivariate model that we employ. One candidate source of potential misspecification is potential cointegration between nominal rates and inflation. This is discussed in some detail in a very interesting paper by Mishkin (1992).[7] In some companion research on long run inflation trends in the United States, we are using multivariate model to investigate some alternative sources of potential misspecification.

5. Concluding Remarks

In this paper we have investigated four long run neutrality propositions using bivariate models and forty years of quarterly observations. We conclude that the postwar U.S. data does contain some evidence against the long run superneutrality of money and the long run Fisher relation. On the other hand, the data contain little evidence against the long run neutrality of money and a zero long run elasticity of the unemployment rate with respect to permanent changes in the rate of inflation.

These conclusions are tempered by three important caveats. First, the results are predicated on specific assumptions concerning the degree of integration of the data, and with forty years of data the degree of integration is necessarily uncertain. Second, even were the degree of integration were known, only limited "long run" information is contained in data that span forty years. This suggests that a useful extension of this work is to carry out similar analyses on long annual series. Third, the analysis has been carried out using bivariate models. If there are more than two important sources macroeconomic shocks, then bivariate models may be subject to significant omitted variable bias. Thus another extension of this work is to expand the set of variables under study, so that the vector of innovations spans the space of structural macroeconomic shocks. Unfortunately, the identification problem becomes much difficult in this case since the number of necessary identifying restrictions increases with the square of the variables in the model.

In spite of these caveats, we think that the work presented here makes two important contributions. First, it shows that when data are integrated, traditional critiques of neutrality tests can potentially be overcome. This allows long run neutrality to be tested without a complete specification of the economic environment. In our bivariate framework, we need specify only one parameter: one of the stuctural impact multipliers or one of the structural long run multipliers. The second contribution of the paper is to show how the neutrality propositions fare across a wide range of assumptions about these structural impact and long run multipliers.

1. Alternative interpretations of the model are also possible. For example, ϵ_t^η could be interpreted as a money demand shock and ϵ_t^m as a money supply shock. Alternatively, ϵ_t^η could be interpreted as a real shock and ϵ_t^m as a money demand shock. This interpretation is consistent with a model in which the money supply authority completely accomodates money demand shocks and there are no other exogenous shocks to money. Below we will modify the model to allow feedback from output to the money supply so that money can respond to both ϵ_t^m and ϵ_t^η.

2. If we write $\mu^*(L)=\sum\mu_j^* L^j$, then $\mu_j^*=\sum_{i=j+1}^{\infty}\mu_i$. The coefficients of $\mu^*(L)$ will be absolutely summable if the coefficients of $\mu(L)$ are 1-summable, i.e. if $\sum_{i=1}^{p} i|\mu_i|<\infty$.

3. Data sources: Output: Citibase series GNP82 (real GNP). Money: The monthly Citibase M2 series (FM2) was used for 1959-1989; the earlier M1 data were formed by splicing the M2 series reported in Banking and Monetary Statistics, 1941-1970, Board of Governors of the Federal Reserve System to the Citibase data in January 1959. Inflation: Log first differences of Citibase series PUNEW (CPI-U: All Items). Unemployment Rate: Citibase Series LHUR (Unemployment rate: all workers, 16 years & over (%,sa)). Interest Rate: Citibase series FYGM3 (yield on three month U.S. Treasury Bills). Monthly series were averaged to form the quarterly data.

4. Graphs like the one shown here were suggested by Jim Stock during work on the Stock and Watson (1988) project.

5. We have also carried out the neutrality and superneutrality analysis for M1 as well as M2. The results are similar to those for M2. We chose to report the results for M2 instead of M1 because unit root tests for M1 suggest that the series is trend stationary in growth rates. Such a process would make it difficult to interpret either the neutrality tests or the superneutrality tests as we have developed them. Stock and Watson (1988) carry out neutrality tests after first linearly detrending M1.

6. See McCallum (1989, page 180) for a replication and discussion of this graph.

7. Mishkin assumes that real rates $R-\pi_t$ are I(0), and investigates the "strength" of the Fisher relation over different sample periods. He finds that the data are consistent with the Fisher relation over periods when π_t behaves like an I(1) process so that $R_t-\pi_t$ is a cointegrating relation; he also finds that the data are not consistent with the Fisher relation when π_t behaves like and I(0) process. In contrast, out results are predicated on the assumption that π_t and R_t are is I(1) and not cointegrated over the entire sample. As the results in Table 1 make clear, our maintained assumption cannot be rejected by the data, but neither can Mishkin's.

Appendix

The Long Run Neutrality Restriction in (4.a) and (4.b)

In a certainty model with all variables constant through time, long run neutrality is defined as the requirement that money does not effect output. The implied restrictions on (4a) and (4b) can be determined by solving for the steady-state values of y and p using the driving processes:

(4.c') $m_t = m$

(4.d') $\eta_t = \eta.$

This yields

(A.1) $y = \gamma_{ym} m + \gamma_{y\eta} \eta$

(A.2) $p = \gamma_{pm} m + \gamma_{p\eta} \eta$

where $\gamma_{ym}=D[\gamma_p(1)\phi_m(1)+\theta\Psi_m(1)]$, $\gamma_{y\eta}=D[\gamma_p(1)\phi_\eta(1)+\theta\Psi_\eta(1)]$, $\gamma_{pm}=D[\gamma_y(1)\gamma_m(1)-\delta\phi_m(1)]$, $\gamma_{m\eta}=D[\gamma_y(1)\gamma_\eta(1)-\delta\phi_\eta(1)]$ and $D=[\gamma_y(1)\gamma_p(1)+\theta\delta]^{-1}$. Thus, the restriction on the parameters in (4.a) and (4.b) implied by long run neutrality is $\gamma_{ym}=0$. (Another sensible restriction is the long run homogeneity restriction $\gamma_{pm}=1$.)

In a stochastic setting, we can work out the restriction implied by the analogous restriction that a permanent change in the level m_t has no lasting effect on y_t. To do this we solve the model using the forcing processes (4.c) and (4.d). To begin, rewrite equations (4c) and (4d) as:

(A.3) $m_t = \mu(1)\bar{m}_t + \mu^*(L)\epsilon_t^m + m_0$

(A.4) $\eta_t = \lambda(1)\bar{\eta}_t + \lambda^*(L)\epsilon_t^\eta + \eta_0$

where $\bar{m}_t=\sum_{s=1}^t \epsilon_s^m$, $\bar{\eta}_t=\sum_{s=1}^t \epsilon_s^\eta$, $\mu^*(L)=(1-L)^{-1}[\mu(L)-\mu(1)]$ and $\lambda^*(L)=(1-L)^{-1}[\lambda(L)-\lambda(1)]$.
Equation (A.3) implies

$$E_t m_{t+k} = \mu(1)\bar{m}_t + [L^{-k}\mu^*(L)]_+\epsilon_t^m + m_0$$

for k≥0. Thus,

$$\phi_m(F)[E_t m_t] = \phi_m(1)\mu(1)\bar{m}_t + [\phi_m(L^{-1})\mu^*(L)]_+ \epsilon_t^m + \phi_m(1)m_0,$$

where the $[.]_+$ is the annihilator operator defined as $[\sum_{i=-k}^{j} a_i z^i]_+ = \sum_{i=0}^{j} a_i z^i$.

Similarly,

$$\phi_\eta(F)[E_t \eta_t] = \phi_\eta(1)\lambda(1)\bar{\eta}_t + [\phi_\eta(L^{-1})\lambda^*(L)]_+ \epsilon_t^\eta + \phi_\eta(1)\eta_0,$$
$$\Psi_m(F)[E_t m_t] = \Psi_m(1)\mu(1)\bar{m}_t + [\Psi_m(L^{-1})\mu^*(L)]_+ \epsilon_t^m + \Psi_m(1)m_0,$$
$$\Psi_\eta(F)[E_t \eta_t] = \Psi_\eta(1)\lambda(1)\bar{\eta}_t + [\Psi_\eta(L^{-1})\lambda^*(L)]_+ \epsilon_t^\eta + \Psi_\eta(1)\eta_0.$$

Substituting these expressions into (4a) and (4b) yields:

$$\text{(A.5)} \quad \gamma_y(L)y_t = \theta p_t + \phi_m(1)\mu(1)\bar{m}_t + \phi_\eta(1)\lambda(1)\bar{\eta}_t + \bar{\phi}_m(L)\epsilon_t^m + \bar{\phi}_\eta(L)\epsilon_t^\eta + \beta_y$$
$$\text{(A.6)} \quad \gamma_p(L)p_t = -\delta y_t + \Psi_m(1)\mu(1)\bar{m}_t + \Psi_\eta(1)\lambda(1)\bar{\eta}_t + \bar{\Psi}_m(L)\epsilon_t^m + \bar{\Psi}_\eta(L)\epsilon_t^\eta + \beta_p$$

where $\bar{\phi}_m(L)=[\phi_m(L^{-1})\mu^*(L)]_+$, $\bar{\phi}_\eta(L)=[\phi_\eta(L^{-1})\lambda^*(L)]_+$, $\bar{\Psi}_m(L)=[\Psi_m(L^{-1})\mu^*(L)]_+$, $\bar{\Psi}_\eta(L)=[\Psi_\eta(L^{-1})\lambda^*(L)]_+$, $\beta_y=\phi_m(1)m_0+\phi_\eta(1)\eta_0$, and $\beta_p=\Psi_m(1)m_0+\Psi_\eta(1)\eta_0$.

Solving (A.5) and (A.6) for y_t yields an equation of the form:

$$\text{(A.7)} \qquad y_t = \gamma_{ym}\mu(1)\bar{m}_t + \gamma_{y\eta}\lambda(1)\bar{\eta}_t + \beta_m(L)\epsilon_t^m + \theta_\eta(L)\epsilon_t^\eta + \beta,$$

where the coefficients γ_{ym} and $\gamma_{y\eta}$ are the same as those appearing in the steady-state solution to the model, equations (A.1) and (A.2). When $(1-L)^{-1}\lambda(L)$ is invertible, $\gamma_{y\eta}\lambda(1)\bar{\eta}_t+\theta_\eta(L)\epsilon_t^\eta$ can be written as a distributed lag of η_t, say $\beta_\eta(L)\epsilon_t^\eta$, yielding equation (7) in the text.

Equation (8) follows directly from (7) when $\mu(L)$ is invertible, since $\bar{m}_t=\mu(1)^{-1}(m_t+\mu^*(L)\epsilon_t^m + m_0)$ and $\epsilon_t^m=\mu(L)^{-1}\Delta m_t$.

Estimation Methods

Under each alternative identifying restriction, the Gaussian maximum likelihood estimates can be constructed using standard regression and instrumental variable calculations. When λ_{my} is assumed known, equation (10a) can be estimated by ordinary least squares by regressing $\Delta m_t - \lambda_{my}\Delta y_t$ onto $\{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^{p}$. Equation (10b) can't be estimated by OLS because Δm_t, one of the regressors, is potentially correlated with ϵ_t^{η}. Instrumental variables must be used. The appropriate instruments are $\{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^{p}$ together with the residual from the estimated (10a). This residual is a valid instrument because of the assumption that ϵ_t^{η} and ϵ_t^{m} are uncorrelated. When λ_{ym} is assumed known, rather than λ_{my}, this process was reversed.

When a value for γ_{my} is used to identify the model, a similar procedure can be used. First, rewrite (10a) as:

$$\text{(A.6)} \qquad \Delta m_t = \alpha_{my}(1)\Delta y_t + \beta_{mm}\Delta m_{t-1} + \sum_{j=0}^{p-1}\tilde{\alpha}_{my}^{j}\Delta^2 y_{t-j} + \sum_{j=1}^{p-1}\tilde{\alpha}_{mm}^{j}\Delta^2 m_{t-j} + \epsilon_t^{m},$$

where $\beta_{mm} = \sum_{j=1}^{p}\alpha_{mm}^{j}$. Equation ((A.6) replaces the regressors $(\Delta y_t, \Delta y_{t-1}, \ldots, \Delta y_{t-p}, \Delta m_{t-1}, \ldots, \Delta m_{t-p})$ in (10a) with the equivalent set of regressors $(\Delta y_t, \Delta m_{t-1}, \Delta^2 y_t, \Delta^2 y_{t-1}, \ldots, \Delta^2 y_{t-p+1}, \Delta^2 m_{t-1}, \ldots, \Delta^2 m_{t-p+1})$. In (A.6), the long run multiplier is $\gamma_{my} = \alpha_{my}(1)/(1-\beta_{mm})$, so that $\alpha_{my}(1) = \gamma_{my} - \beta_{mm}\gamma_{my}$. Making this substitution, (A.6) can be written as:

$$\text{(A.7)} \quad \Delta m_t - \gamma_{my}\Delta y_t = \beta_{mm}(\Delta m_{t-1} - \gamma_{my}\Delta y_t) + \sum_{j=0}^{p-1}\tilde{\alpha}_{my}^{j}\Delta^2 y_{t-j} + \sum_{j=1}^{p-1}\tilde{\alpha}_{mm}^{j}\Delta^2 m_{t-j} + \epsilon_t^{m}.$$

Equation (A.7) can be estimated by instrumental variables by regressing $\Delta m_t - \gamma_{my}\Delta y_t$ onto $(\Delta m_{t-1} - \gamma_{my}\Delta y_t, \Delta^2 y_t, \Delta^2 y_{t-1}, \ldots, \Delta^2 y_{t-p+1}, \Delta^2 m_{t-1}, \ldots, \Delta^2 m_{t-p+1})$ using $\{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^{p}$ as instruments. (Instrumental variables is required because of the potential correlation between Δy_t and the error term.) Equation (10b) can now be estimated by instrumental variables using the residual from the estimated (A.7) together with $\{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^{p}$. When a value for γ_{ym} is used to identify the model, this process was reversed.

Two complications arise in the calculation of standard errors for the estimated models. The first is that the long run multipliers, γ_{ym} and γ_{my}, are nonlinear functions of the regression coefficients. Their standard errors are calculated from standard formula derived from delta method arguments. The second complication arises because one of the equations is estimated using instruments that are residuals from another equation. This introduces the kind of "generated regressor" problems discussed in Pagan (1984). To see the problem in our context, notice that all of the models under consideration can be written as:

(A.8) $\quad y_t^1 = x_t^{1\prime}\delta_1 + \epsilon_t^1$

(A.9) $\quad y_t^2 = x_t^{2\prime}\delta_2 + \epsilon_t^2$.

Where, for example, when λ_{my} is assumed known, $y_t^1=\Delta m_t-\lambda_{my}\Delta y_t$, x_t^1 represents the set of regressors $\{\Delta y_{t-i},\Delta m_{t-i}\}_{i=1}^p$, $y_t^2=\Delta y_t$, and x_t^2 represents the set of regressors $[\Delta m_t, \{\Delta y_{t-i},\Delta m_{t-i}\}_{i=1}^p]$. Alternatively, when γ_{my} is assumed known, $y_t^1=\Delta m_t-\gamma_{my}\Delta y_t$, x_t^1 represents the set of regressors $[\Delta m_{t-1}-\gamma_{my}\Delta y_t, \Delta^2 y_t, \{\Delta^2 y_{t-i}, \Delta^2 m_{t-i}\}_{i=1}^{p-1}]$, $y_t^2=\Delta y_t$, and x_t^2 represents the set of regressors $[\Delta m_t, \Delta y_{t-i},\Delta m_{t-i}\}_{i=1}^p]$.

Equations (A.8) and (A.9) allow us to discuss estimation of all the models in a unified way. First, (A.8) is estimated using $z_t=\{\Delta y_{t-i},\Delta m_{t-i}\}_{i=1}^p$ as instruments. Next, equation (A.9) is estimated using $\hat{u}_t=(\hat{\epsilon}_t^1\ z_t')$ as instruments, where $\hat{\epsilon}_t^1$ is the estimated residuals from (A.8). If ϵ_t^1 rather than $\hat{\epsilon}_t^1$ was used as an instrument, standard errors could be calculated using standard formulae. However, when $\hat{\epsilon}_t^1$, an estimate of ϵ_t^1 is used, a potential problem arises. This problem will only effect the estimates in (A.9) since $\hat{\epsilon}_t^1$ is not used as an instrument in (A.8).

To explain the problem, some additional notation will prove helpful. Stack the observations for each equation so that the model can be written as:

(A.10) $\quad Y_1 = X_1\delta_1 + \epsilon_1$

(A.11) $\quad Y_2 = X_2\delta_2 + \epsilon_2$

where Y_1 is T×1, etc. Denote the matrix of instruments for the first equation by Z, the matrix of instruments for the second equation by $\hat{U}=[\hat{\epsilon}_1 \; Z]$, and let $U=[\epsilon_1 \; Z]$. Since $\hat{\epsilon}_1=\epsilon_1-X_1(\hat{\delta}_1-\delta_1)$, $\hat{U}=U-[X_1(\hat{\delta}_1-\delta_1) \quad 0]$. Let $V_1=\sigma^2_{\epsilon 1}\text{plim}[T(Z'X_1)^{-1}(Z'Z)(X_1'Z)]$ denote the asymptotic covariance matrix of $T^{1/2}(\hat{\delta}_1-\delta_1)$.

Now write,

$$\text{(A.12)} \qquad T^{1/2}(\hat{\delta}_2-\delta_2) = (T^{-1}\hat{U}'X_2)^{-1}(T^{-1/2}\hat{U}'\epsilon_2)$$

$$= (T^{-1}\hat{U}'X_2)^{-1}(T^{-1/2}U'\epsilon_2) - (T^{-1}\hat{U}'X_2)^{-1}\begin{bmatrix} T^{1/2}(\hat{\delta}_1-\delta_1)'(T^{-1}X_1'\epsilon_2) \\ 0 \end{bmatrix}$$

It is straightforward to verify that $\text{plim}\ T^{-1}\hat{U}'\hat{U}=\text{plim}\ T^{-1}U'U$ and that $\text{plim}\ T^{-1}\hat{U}'X_2=\text{plim}\ T^{-1}U'X_2$. Thus, the first term on the right hand side of (A.12) is standard: it is asymptotically equivalent to the expression for $T^{1/2}(\hat{\delta}_2-\delta_2)$ that would obtain if U rather than $\hat{U}$ were used as instruments. This expression converges in distribution to a random variable distributed as $N(0,\sigma^2_{\epsilon 2}\text{plim}[T(\hat{U}'X_2)^{-1}(\hat{U}'\hat{U})(X_2'\hat{U})^{-1}]$, which is the usual expression for the asymptotic distribution of the IV estimator.

Potential problems arise because of the second term on the right hand side of (A.12). Since $T^{1/2}(\hat{\delta}_1-\delta_1)$ converges in distribution, the second term can only be disregarded asymptotically when $\text{plim}\ T^{-1}X_1'\epsilon_2=0$, that is, when the regressors in (A.8) are uncorrelated with the error terms in (A.9). In our context, this will occur when λ_{my} and λ_{ym} are assumed known, since in this case x^1_t contains only lagged variables. However, when γ_{my} or γ_{ym} are assumed known, x^1_t will contain the contemporaneous value of Δy_t or Δm_t, and thus x^1_t and ϵ^2_t will be correlated. In this case the covariance matrix of $\hat{\delta}_2$ must be modified to account for the second term on the right hand side of (A.12).

The necessary modification is as follows. It is straightforward to verify that $T^{1/2}(\hat{\delta}_1-\delta_1)$ and $T^{-1/2}U'\epsilon_2$ are asymptotically independent under the maintained assumption that $E(\epsilon_2|\epsilon_1)=0$; thus, the two terms on the right hand of (A.12) are asymptotically uncorrelated. A straightforward calculation demonstrates that $T^{1/2}(\hat{\delta}_2-\delta_2)$ converges to random variable with a $N(0,V_2)$ distribution where

$$V_2=\sigma_{\epsilon 2}^2 \mathrm{plim}[T(\hat{U}'X_2)^{-1}(\hat{U}'\hat{U})(X_2'\hat{U})^{-1}] + \mathrm{plim}[T(\hat{U}'X_2)^{-1}D\ (X_2'\hat{U})^{-1}],$$

where D is a matrix with all elements equal to zero, except that $D_{11}=(\epsilon_2'X_1)TV_1(X_1'\epsilon_2)$, where $TV_1=\sigma_{\epsilon 1}^2(Z'X_1)^{-1}(Z'Z)(X_1'Z)^{-1}$. Similarly, it is straightforward to show that the asymptotic covariance between $T^{1/2}(\hat{\delta}_1-\delta_1)$ and $T^{1/2}(\hat{\delta}_2-\delta_2)=-\mathrm{plim}[V_1(T^{-1}X_1'\epsilon_2)\quad 0][T^{-1}X_2'\hat{U}]$.

An alternative to this approach is the "augmented" 3SLS (A3SLS) estimator in Hausman, Newey and Taylor (1987). This approach considers the estimation problem as a GMM problem with moment conditions $E(z_t\epsilon_t^1)=0$, $E(z_t\epsilon_t^2)=0$ and $E(\epsilon_t^1\epsilon_t^2)=0$. The A3SLS approach is more general than the one we have employed, and when the errors terms are non-normal, may produce more efficient estimates. However, it does require systems estimation, which is computationally demanding. Since we estimate the model hundreds of times (using different identifying restrictions) we chose to use the equation-by-equation method outlined above.

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Table 1
Descriptive Statistics

A. Sample Mean and Standard Deviation

Variable	Mean	Standard Devation
Δy_t	3.11	4.36
Δm_t	6.66	3.51
$\Delta^2 m_t$	0.02	2.47
u_t	5.69	1.66
π_t	4.07	3.59
Δu_t	-0.01	0.42
$\Delta \pi_t$	0.06	2.27
R_t	5.23	3.15
ΔR_t	0.03	0.81

B. Unit Root Statistics

Variable	ADF $\hat{\tau}^{\tau}$	ADF $\hat{\tau}^{\mu}$	----- 95% Confidence Int. for ρ ----- Detrended Data	Demeaned Data
y_t	-2.53	--	(.89 1.02)	--
m_t	-2.40	--	(.90 1.03)	--
Δm_t	-2.76	-2.90	(.86 1.02)	(.84 1.01)
π_t	-3.27	-2.86	(.81 1.02)	(.84 1.02)
u_t	-3.35	-2.34	(.81 1.01)	(.89 1.02)
R_t	-3.08	-1.87	(.84 1.02)	(.92 1.02)
r_t	-3.34	-2.94	(.82 1.02)	(.85 1.01)

Notes: The regressions used to calculate the ADF statistics included six lagged differences of the variable. All regressions were carried out over 1949:1-1990:4 using quarterly data except those involving u_t, which began in 1950:1. The variables y_t, m_t are the logarithms of output and money multiplied by 400, so that their first differences represents rates of growth at annual rates; similarly, π_t represents price inflation at an annual rate. The 95% confidence intervals were based on that ADF statistics using the procedure developed in Stock (1991).

Table 2

Short Run and Long Run Second Moments

A. Neutrality of Money

$X_t = (\Delta m_t \quad \Delta y_t)'$

Sample Period: 1949:1-1990:4

---- VAR Results ----

VAR Forecast Errors: σ_y=3.80, σ_m=2.10, cor(y,m)=0.08

Shocks to Stochastic Trends: σ_y=6.04, σ_m=12.39, cor(y,m)=-0.25

---- Structural Model Results ----

	λ_{my}	λ_{ym}	γ_{my}
γ_{ym}=0 in 95% conf. int.	≤1.40	≥-4.61	---
Estimates imposing γ_{ym}=0	0.22 (.20)	-0.59 (.68)	-0.51 (.80)
(Std. Errrors in Parentheses)			

B. Superneutrality of Money

$X_t = (\Delta^2 m_t \quad \Delta y_t)'$

Sample Period: 1949:1-1990:4

---- VAR Results ----

VAR Forecast Errors: σ_y=3.77, $\sigma_{\Delta m}$=2.14, cor(y,Δm)= 0.07

Shocks to Stochastic Trends: σ_y=5.88, $\sigma_{\Delta m}$=0.92, cor(y,Δm)=-0.15

---- Structural Model Results ----

	$\lambda_{\Delta m,y}$	$\lambda_{y,\Delta m}$	$\gamma_{\Delta m,y}$
$\gamma_{y,\Delta m}$=0 in 95% conf. int.	≤-.25,(.08,.53)	(-1.43,-.26),≥1.02	≤.07
Estimates imposing $\gamma_{y,\Delta m}$=0	0.30 (.10)	-0.83 (.29)	-0.02 (.04)

Table 2
(continued)

C. Long Run Phillips Curve
$X_t=(\Delta\pi_t \quad \Delta u_t)'$
Sample Period: 1950:1-1990:4

---- VAR Results ----

VAR Forecast Errors:	σ_u=0.31, σ_π=1.91, cor(u,π)= -0.07
Shocks to Stochastic Trends:	σ_u=0.53, σ_π=1.16, cor(u,π)= -0.38

---- Structural Model Results ----

	$\lambda_{\pi u}$	$\lambda_{u\pi}$	$\gamma_{\pi u}$
$\gamma_{u\pi}$=0 in 95% conf. int.	≤2.34	(-.07,.07)	---
Estimates imposing $\gamma_{u\pi}$=0 (Std. Errrors in Parentheses)	0.30 (1.06)	-.02 (.03)	-.81 (.50)

	$\lambda_{\pi u}$	$\lambda_{u\pi}$	$\gamma_{u\pi}$
$\gamma_{\pi u}$=0 in 95% conf. int.	≥0.66	≤-0.02	≤0.04
Estimates imposing $\gamma_{\pi u}$=0	2.92 (1.44)	-.09 (.04)	-0.17 (.11)

D. Long Run Fisher Effect
$X_t=(\Delta\pi_t \quad \Delta R_t)'$
Sample Period: 1949:1-1990:4

---- VAR Results ----

VAR Forecast Errors:	σ_R=0.68, σ_π=1.91, cor(R,π)= 0.16
Shocks to Stochastic Trends:	σ_R=0.77, σ_π=1.25, cor(R,π)= 0.56

---- Structural Model Results ----

	$\lambda_{\pi R}$	$\lambda_{R\pi}$	$\gamma_{\pi R}$
$\gamma_{R\pi}$=1 in 95% conf. int.	≤-5.0	≥0.55	≤2.37,≥102.1
Estimates imposing $\gamma_{R\pi}$=1	-14.02(15.13)	1.02 (.53)	-19.2 (69.2)

Notes: All results are based on VAR's with six lags. The results for shocks to the stochastic trends are calculated from the long run covariance matrix implied by the estimated VAR (i.e. the spectral density matrix of the variables at frequency 0).

Table 3

Robustness to Sample Period and Lag Length

A. Neutrality of Money

$X_t = (\Delta m_t \quad \Delta y_t)'$

Sample Period	Lag Length	---------- Estimates of γ_{ym} when -------		
		$\lambda_{my}=0$	$\lambda_{ym}=0$	$\gamma_{my}=1$
1949 - 1990	6	0.23 (0.21)	0.17 (0.19)	-0.32 (0.22)
1949 - 1972	6	0.15 (0.24)	0.13 (0.24)	-0.18 (0.33)
1973 - 1990	6	0.77 (0.47)	0.65 (0.37)	-0.25 (0.31)
1949 - 1990	4	0.24 (0.17)	0.20 (0.15)	-0.31 (0.21)
1949 - 1990	8	0.12 (0.19)	0.07 (0.17)	-0.34 (0.20)

B. Superneutrality of Money

$X_t = (\Delta^2 m_t \quad \Delta y_t)'$

Sample Period	Lag Length	---------- Estimates of $\gamma_{y,\Delta m}$ when ------		
		$\lambda_{\Delta m,y}=0$	$\lambda_{y,\Delta m}=0$	$\gamma_{\Delta m,y}=0$
1949 - 1990	6	3.80 (1.74)	3.12 (1.36)	-0.95 (1.57)
1949 - 1972	6	3.50 (1.66)	3.32 (1.49)	1.67 (1.99)
1973 - 1990	6	4.02 (4.57)	2.65 (2.62)	-4.11 (1.14)
1949 - 1990	4	1.81 (0.90)	1.31 (0.63)	-1.55 (0.97)
1949 - 1990	8	3.94 (1.81)	3.43 (1.53)	0.10 (1.66)

C. Long Run Phillips Curve

$X_t = (\Delta \pi_t \quad \Delta u_t)'$

Sample Period	Lag Length	---------- Estimates of $\gamma_{u\pi}$ when -------		
		$\lambda_{\pi u}=0$	$\lambda_{u\pi}=0$	$\gamma_{\pi u}=0$
1950 - 1990	6	0.03 (0.09)	0.06 (0.09)	-0.17 (0.11)
1950 - 1972	6	-0.04 (0.10)	-0.03 (0.09)	-0.07 (0.14)
1973 - 1990	6	0.29 (0.35)	0.51 (0.56)	-0.21 (0.16)
1950 - 1990	4	-0.03 (0.06)	-0.00 (0.05)	-0.18 (0.07)
1950 - 1990	8	0.08 (0.09)	0.12 (0.09)	-0.11 (0.10)

D. Long Run Fisher Effect

$X_t = (\Delta \pi_t \quad \Delta R_t)'$

Sample Period	Lag Length	---------- Estimates of $\gamma_{R\pi}$ when -------		
		$\lambda_{\pi R}=0$	$\lambda_{R\pi}=0$	$\gamma_{\pi R}=0$
1949 - 1990	6	0.18 (0.09)	0.08 (0.08)	0.34 (0.12)
1949 - 1972	6	0.04 (0.06)	0.03 (0.05)	0.07 (0.09)
1973 - 1990	6	0.40 (0.16)	0.23 (0.18)	0.53 (0.20)
1949 - 1990	4	0.15 (0.07)	0.07 (0.06)	0.28 (0.09)
1949 - 1990	8	0.26 (0.09)	0.14 (0.08)	0.39 (0.13)

Figure 1

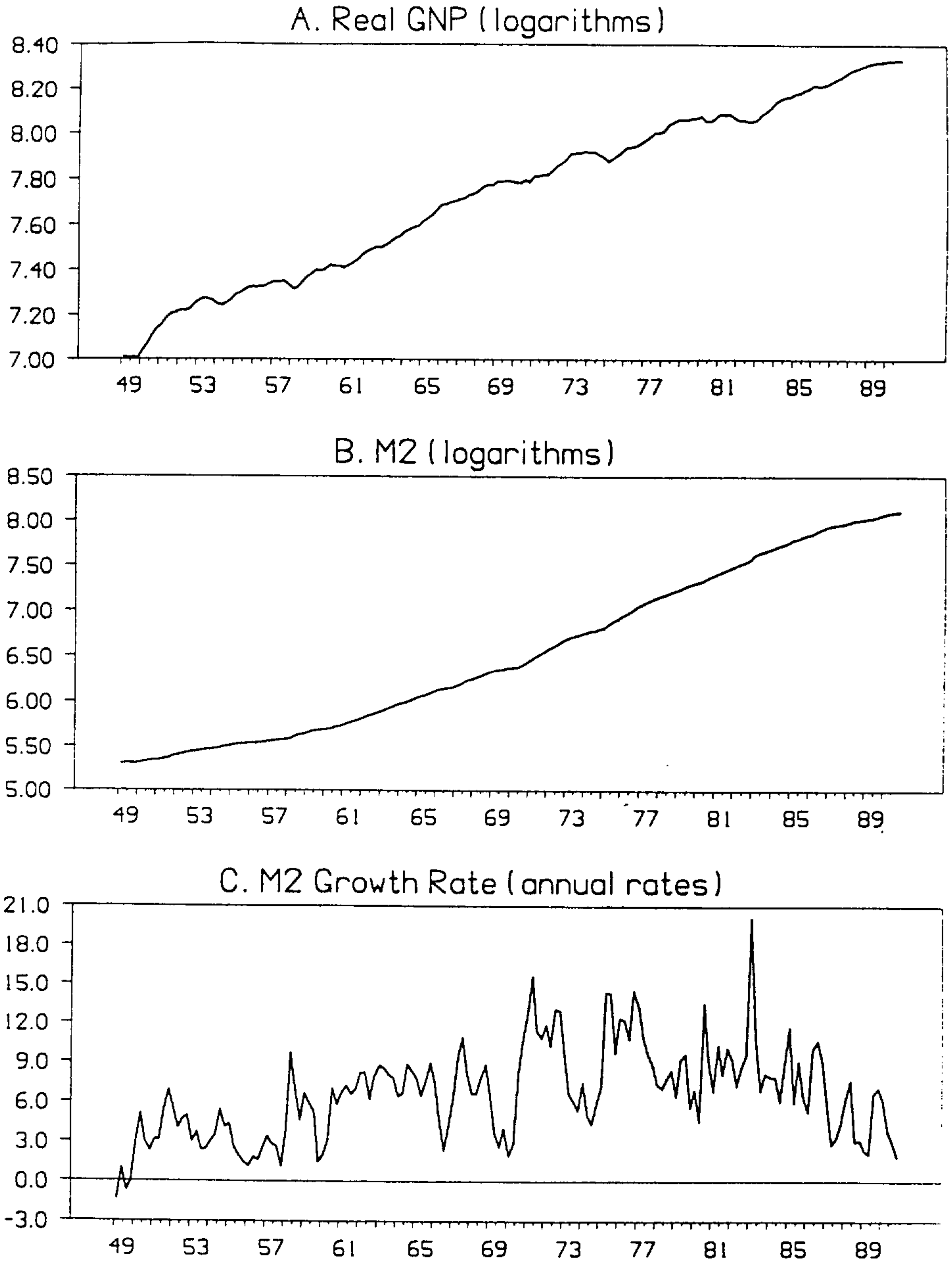
A. Real GNP (logarithms)
8.40
8.20
8.00
7.80
7.60
7.40
7.20
7.00
49
53
57
61
65
69
73
77
81
85
89
B. M2 (logarithms)
8.50
8.00
7.50
7.00
6.50
6.00
5.50
5.00
49
53
57
61
65
69
73
77
81
85
89
C. M2 Growth Rate (annual rates)
21.0
18.0
15.0
12.0
9.0
6.0
3.0
0.0
-3.0
49
53
57
61
65
69
73
77
81
85
89

Figure 1 (Continued)

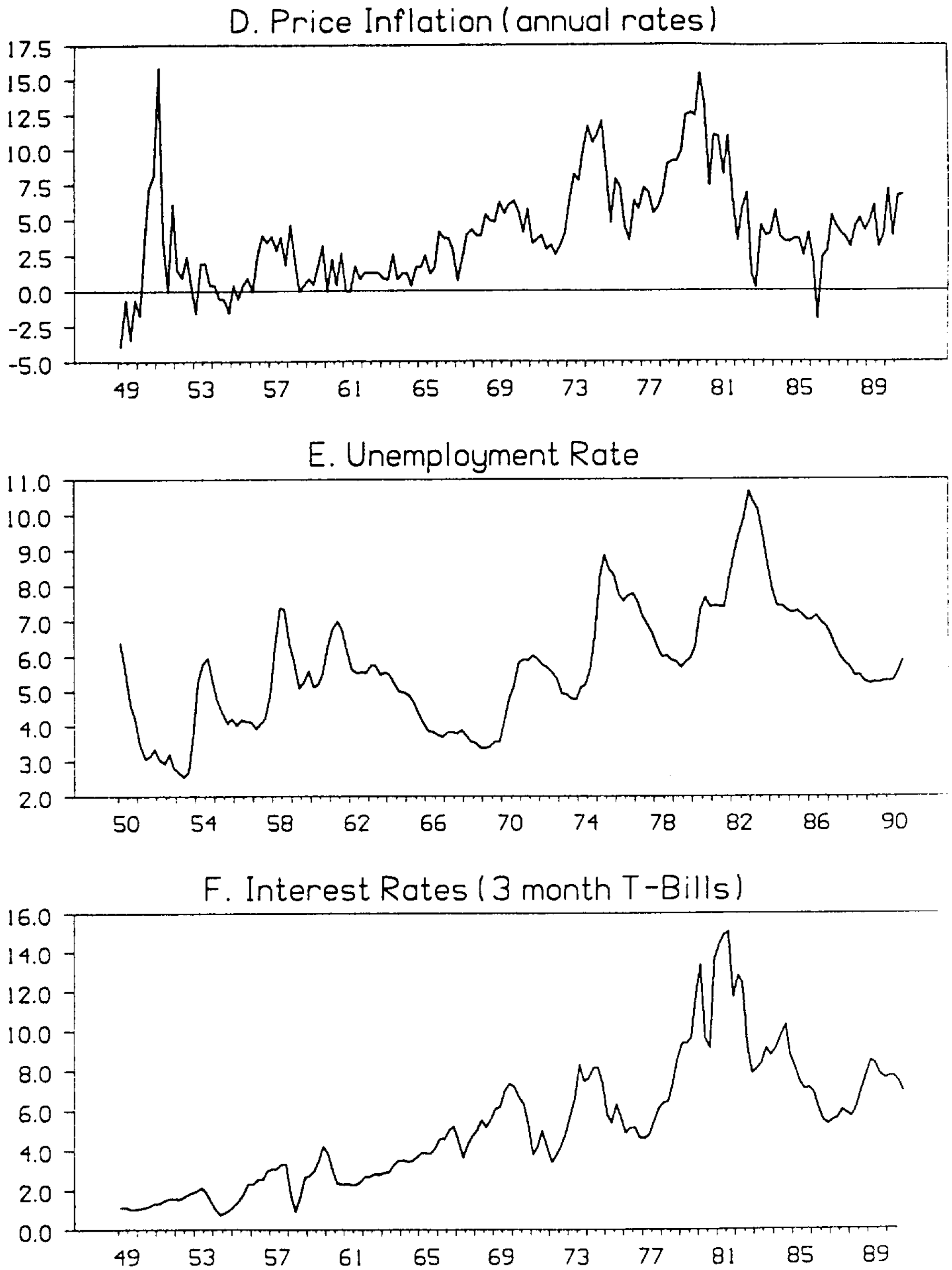

Figure 2: Money and Output

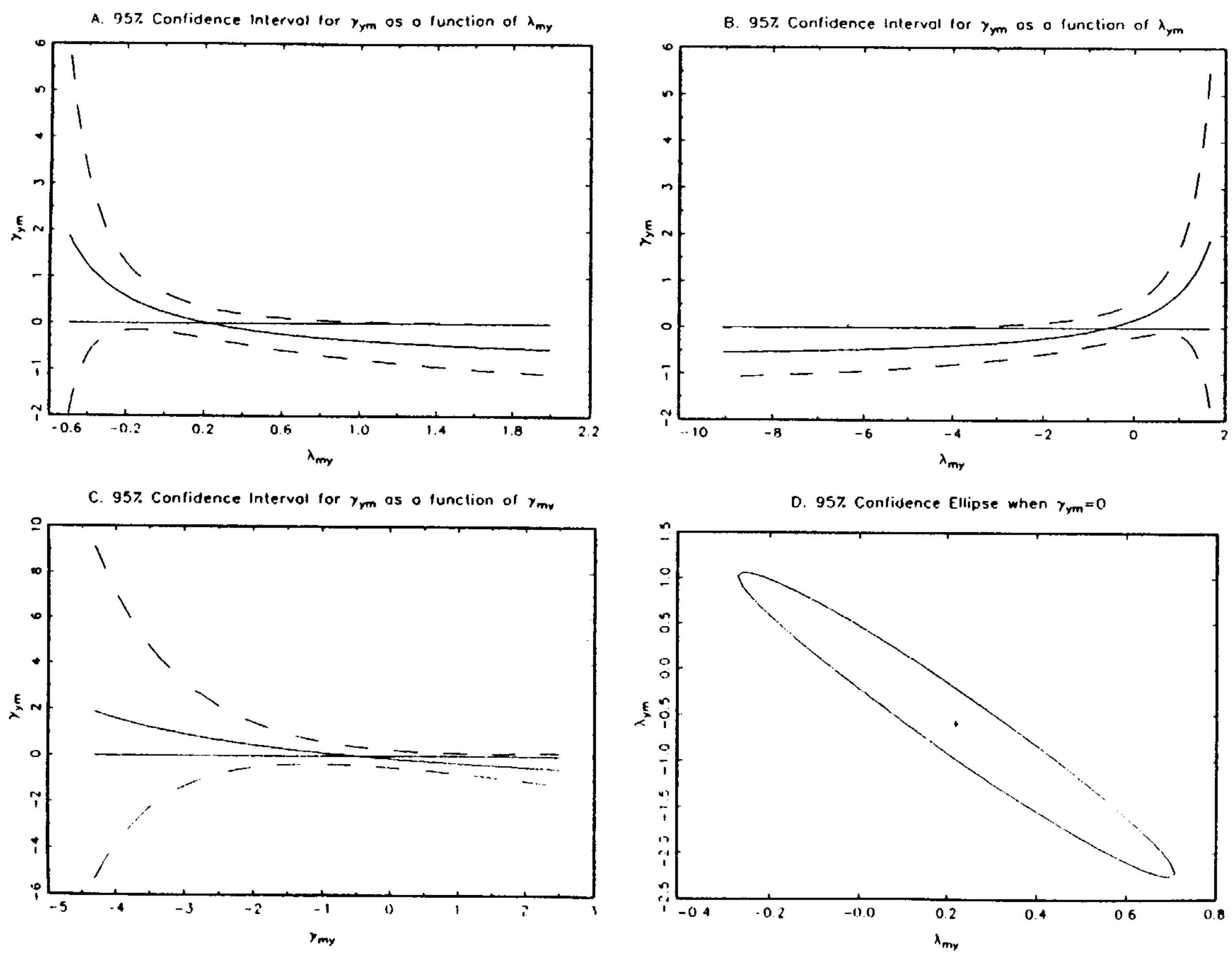

Figure 3: Money Growth and Output

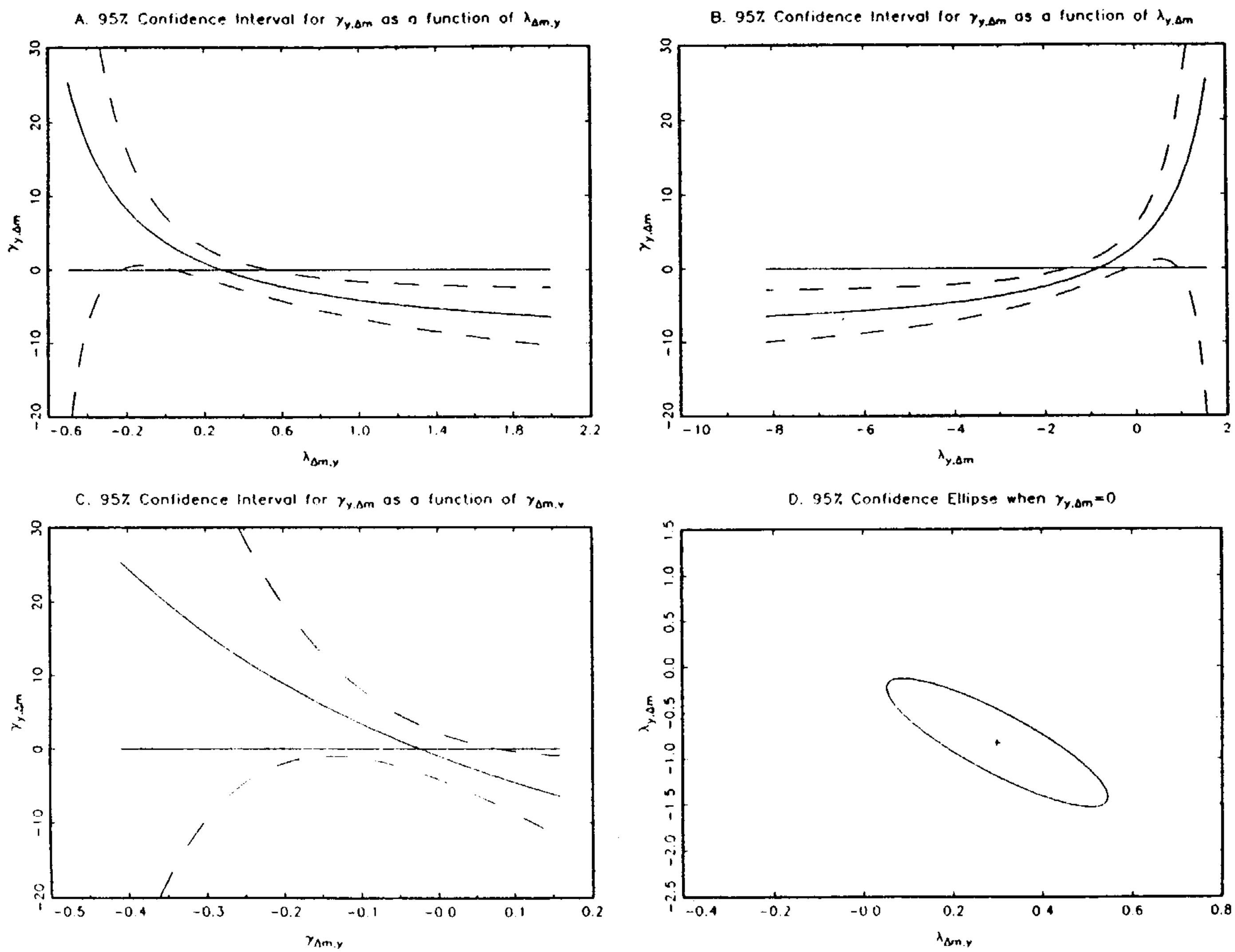

Figure 4: Inflation and Unemployment

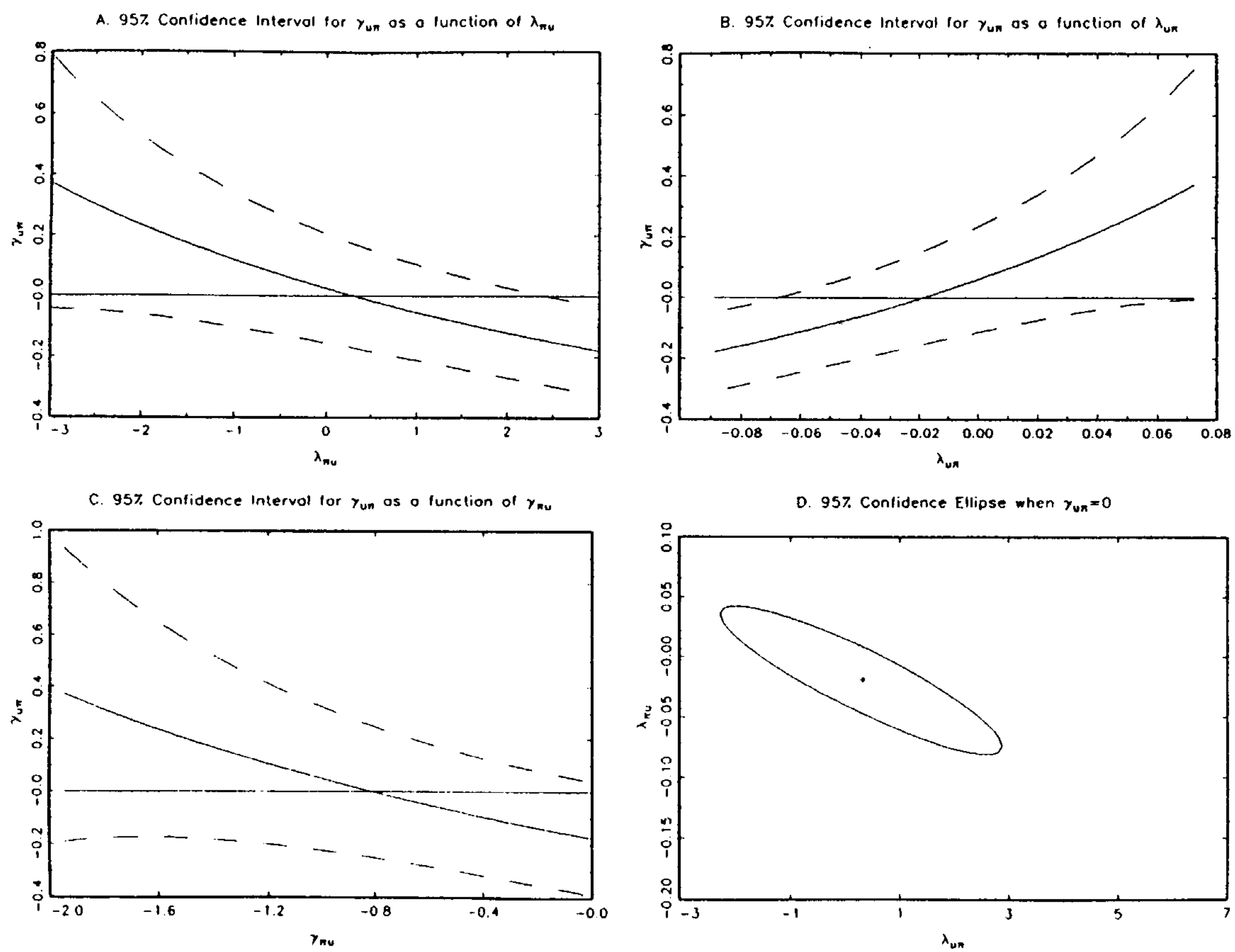

Figure 5: Unemployment and Inflation

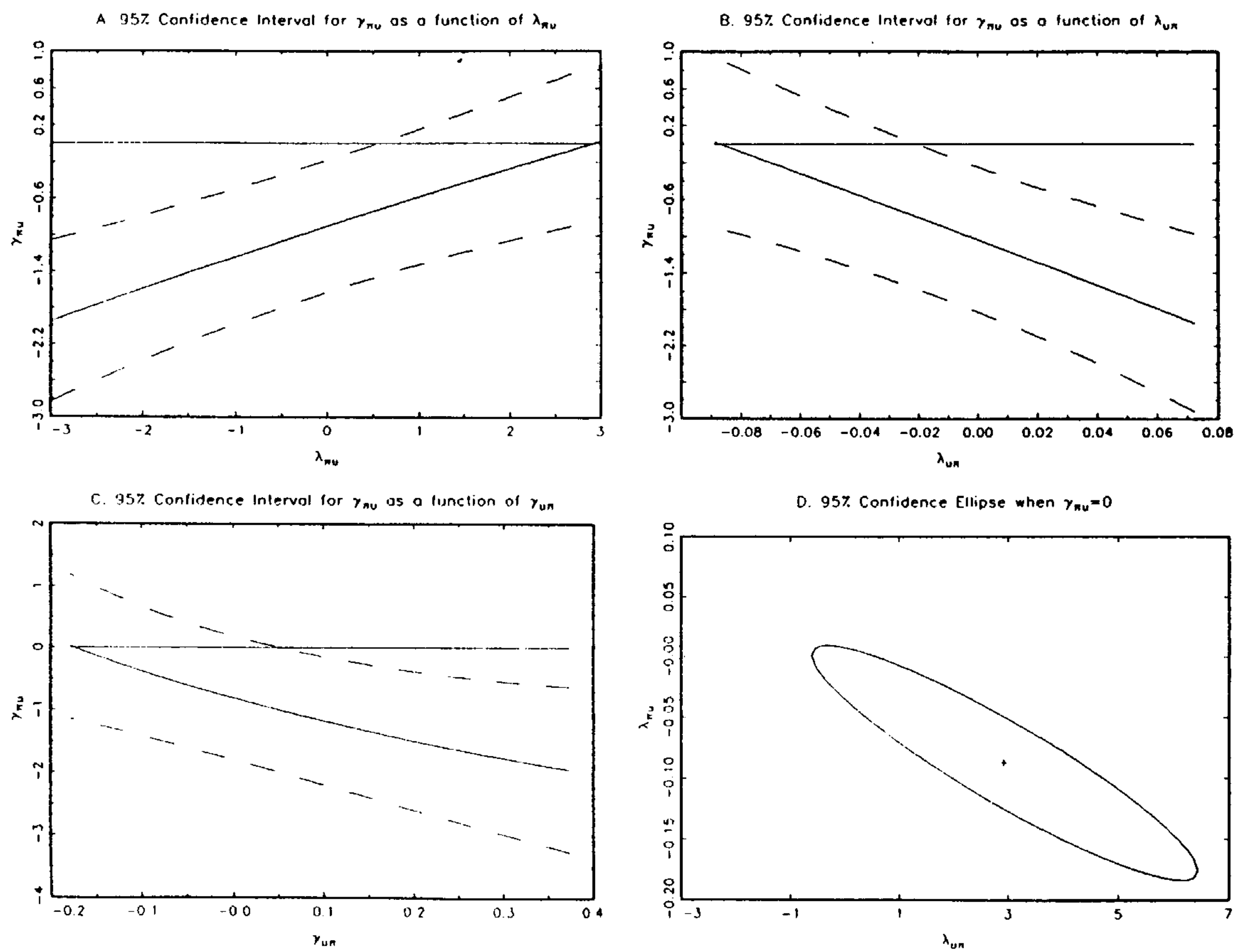

Figure 6: Inflation and Nominal Rates