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EXPECTATIONS AND THE FORWARD  
EXCHANGE RATE

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Expectations and the Forward Exchange Rate

ABSTRACT

This paper provides an empirical examination of the hypothesis that the forward exchange rate provides an "optimal" forecast of the future spot exchange rate, for five currencies relative to the dollar. This hypothesis provides a convenient norm for examining the erratic behavior of exchange rates; this erratic behavior represents an efficient market that is quickly incorporating new information into the current exchange rate. This hypothesis is analyzed using two distinct, but related, approaches. The first approach is based on a regression of spot rates on lagged forward rates. When using weekly data and a one month forward exchange rate, ordinary least squares regression analysis of market efficiency is incorrect. Econometric methods are proposed which allow for consistent (though not fully efficient) estimation of the parameters and their standard errors. This paper also presents a new approach for testing exchange market efficiency. This approach is based on a general time series process generating the spot and forward exchange rate. The hypothesis of efficiency implies a set of cross-equation restrictions imposed on the parameters of the time series model. This paper derives these restrictions, proposes a maximum likelihood method of estimating the constrained likelihood function, estimates the model and tests the validity of the restrictions with a likelihood ratio statistic.

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## 1 Introduction

The hypothesis of efficiency has been used in many studies of the foreign exchange market. This hypothesis implies that there are no unexploited profit opportunities. In the foreign exchange market this implies that the forward rate summarizes all relevant and available information useful for forecasting the future spot rate. Analyzing this aspect of efficiency requires an equilibrium model of pricing in the foreign exchange market. Consequently, any empirical test of efficiency is a joint test of efficiency and the equilibrium model.

This paper will focus on two different methods of testing market efficiency. Section 2 will examine the issue of efficiency from the point of view of the standard regression model. This procedure emphasizes the regression of the spot rate on the lagged forward rate. Several econometric difficulties arise and must be overcome when the forecast horizon (one month) exceeds the sampling interval (one week). The stochastic structure of the error term may be interpreted as new information reaching the foreign exchange market. To analyze the nature of efficiency with this procedure requires an empirical analysis of the properties of the error term. This will be done in Section 3.

Section 4 will present a new, and alternative, procedure for analyzing the hypothesis of efficiency. This procedure is based on a time series analysis of the spot and one month forward rate. It is assumed that the spot and forward rate can be described by a bivariate stochastic process. The structure of the stochastic process provides a convenient method for extracting forecasts of the future spot rate as a function of all available information (given the model being assumed). The hypothesis of

market efficiency states that, in general, the expected value of the future spot rate is the current forward rate. Therefore, the parameters of the original stochastic process are not free, but must be constrained. The restrictions are highly nonlinear. Section 4 provides a computationally feasible method for calculating the likelihood function subject to these restrictions. The restrictions are then tested for five currencies relative to the dollar. Section 5 concludes the paper.

## 2. Econometric Testing and Procedures

Conventional tests of foreign exchange market efficiency focus on the relation between spot and lagged forward rates. (Assume, for now, that one month is exactly four weeks.) If one wants to forecast  $\ln S_{t+4}$ , based on information up to (and including) time  $t$ , then the following regression is appropriate:

$$\ln S_{t+4} = x'_t \beta + u_{t+4} \quad (1)$$

where  $x_t$  represents variables relevant for forecasting  $\ln S_{t+4}$ , and where

$$E(u_{t+4} : \ln S_t, \ln S_{t-1}, \dots, x_t, x_{t-1}, \dots) = 0. \quad (2)$$

Conditions (1) and (2) imply  $E u_t u_{t-k} = 0$ , for all  $k \geq 4$ , so that  $u_t$  is a moving average of order 3 process, MA(3). The reason for this serial correlation of the errors is that the forecast horizon (four weeks) exceeds the sampling interval (one week). This data overlapping problem induces serial correlation since, loosely speaking, it takes four weeks to realize an error has been made.<sup>1</sup>

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<sup>1</sup>The four week forward rate set on Tuesday is for delivery four weeks hence, also a Tuesday. However, a one month forward rate set on Tuesday is for delivery four weeks and two days hence, on a Thursday. The extra two days induces additional serial correlation, so the error term will be MA(4). (See also Stockman (1978).) Notice that if one examines  $\ln S_{t+4} - \ln S_t$ , one would expect an MA(3) process, since the comparison is between spot rates on different Tuesdays.

Given that  $u_t$  can be written as an MA(3) process, one can write  $u_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \theta_3 \epsilon_{t-3} = \theta(L)\epsilon_t$ . Notice that equation (2) does not require that  $x_t$  be strictly exogenous (at all leads and lags). Obtaining estimates of the  $\beta$  vector by ordinary least squares is equivalent to minimizing the sum of squared prediction errors. To test the forecasting ability of the forward rate, equation (1) will be restricted by requiring  $x_t$  to consist of a constant and  $\ln F_t$ . Therefore, equation (1) may be rewritten as

$$\ln S_{t+4} = a + b \ln F_t + u_{t+4} \quad (1')$$

where

$$E(u_{t+4} : \ln S_t, \ln S_{t-1}, \dots, \ln F_t, \ln F_{t-1}, \dots) = 0. \quad (2')$$

For future reference let  $I_t = \{\ln S_t, \ln S_{t-1}, \dots, \ln F_t, \ln F_{t-1}, \dots\}$ .

Restricting the  $x_t$  vector in this way will impose restrictions on the data that can be rejected. These restrictions test if the forward rate summarizes all information (and that the risk premium, if it exists, is such that equation (1') holds).

There are several difficulties involved in estimating equation (1') efficiently as a regression equation. The problem arises since  $\ln F_t$  is not strictly exogenous (that is,  $\ln F_t$  is a lagged endogenous variable) and the errors are serially correlated. Because of these two facts, one must be careful in applying generalized least squares. It does not appear that there exists a simple and efficient two step GLS procedure with lagged endogenous variables and moving average error terms. It is, however, possible to obtain consistent, though not fully efficient, estimates of a

and  $b$  by using ordinary least squares.<sup>1</sup> It is not fully efficient since one does not use the information that  $u_t$  is MA(3); it is, however, more efficient than dropping three-fourths of the observations.

A sufficient condition for the forward rate to be an unbiased forecast of the future spot rate is  $a = 0$  and  $b = 1$  in equation (1').<sup>2</sup> Two of the studies using weekly data with a forward rate of maturity greater than one week (Stockman 1978 used a one-month forward rate and Hansen and Hodrick 1979 used a three month forward rate) constrained, a priori,  $b = 1$ .

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<sup>1</sup>The nature of the inconsistency in this case can be easily seen (see Hansen 1979). Let  $u_t$  have an MA representation:  $u_t = \theta(L)\epsilon_t$ , where  $\theta(L)$  is invertible, with inverse  $\theta(L)^{-1}$ . Multiplying equation (1') by  $\theta(L)^{-1}$  we have

$$\theta(L)^{-1} \ln S_{t+4} = a\theta(L)^{-1} + b\theta(L)^{-1} \ln F_t + \epsilon_{t+4}.$$

Now, it is the case that  $\epsilon_t$  is serially uncorrelated; however, in general it will be the case that

$$E[\theta(L)^{-1} \ln S_{t+4} : \theta(1)^{-1}, \theta(L)^{-1} \ln F_t] \neq a\theta(1)^{-1} + b\theta(L)^{-1} \ln F_t$$

since we cannot rule out possible correlation between  $\epsilon_{t+4}$  and  $\theta(L)^{-1} \ln F_t$ .

Hansen (1979) shows that the usual standard errors (as reported in most computer packages) are incorrect.

<sup>2</sup>If unbiasedness means  $E(S_{t+4} : I_t) = F_t$  and  $u_t$  is normally distributed in (3'), then it is easily shown that the "correct" null hypothesis is

$a = -\frac{1}{2} \sigma_u^2$  and  $b = 1$  (see Frenkel 1979). A difficulty is that to state the null hypothesis, one must make some distributional assumption on the residuals (normally, we make distributional assumptions when we want to test the null hypothesis (see Garber 1978)).

Stockman set  $b = 1$  so that he could estimate the moving average parameters efficiently and Hansen and Hodrick set  $b = 1$  to induce stationarity. However, constraining  $b = 1$  may bias the other coefficient.<sup>1</sup>

Much of the attention in the paper will be focussed on the residuals  $u_t$ . There are two reasons for this. The first, discussed earlier, is that if the forward rate is to be an efficient forecast of the future spot rate, then the residuals should be a pure forecast error. The second reason comes from considering the return from holding foreign currency. Decompose the total return from holding foreign currency,  $\ln S_{t+4} - \ln S_t$ , as follows:

$$\ln S_{t+4} - \ln S_t = (\ln S_{t+4} - E_t \ln S_{t+4}) + (E_t \ln S_{t+4} - \ln S_t) \quad (3)$$

where the notation  $E_t x_s = E(x_s : I_t)$  is used.<sup>2</sup> Many studies look at the total return (Poole 1967 and Bilson 1979). The second term on the right hand side,  $E_t \ln S_{t+4}$ , is a known quantity (to the economic agent), it is the expected change as of time  $t$  in the exchange rate. The first term,  $\ln S_{t+4} - E_t \ln S_{t+4}$ , is a random variable that represents the unanticipated return from holding

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<sup>1</sup>Consider a simple example (no serial correlation) where we project (linearly)  $y_t$  on a constant  $x_0$ ,  $x_{1t}$  and  $x_{2t}$ :  $E(y_t : x_0, x_{1t}, x_{2t}) = b_0 + b_1 x_{1t} + b_2 x_{2t}$ . Suppose we constrain  $b = 1$ , then  $E(y_t - x_{1t} : x_0, x_{2t}) = a_0 + a_2 x_{2t}$ . It can be shown that  $b_2 = \text{cov}(x_{2t}, y_t) / \text{var}(x_{2t})$  and  $a_2 = b_2 - \text{cov}(x_{2t}, x_{1t}) / \text{var}(x_{2t})$ .

<sup>2</sup>See Roll and Solnick (1977, p. 167) for a similar decomposition.

foreign currency, due to new information which affects the exchange rate. Since agents are compensated for bearing uncertainty, one is led to focus on that term. But equation (1') states that  $E_t \ln S_{t+4} = a + b \ln F_t$ ; hence  $\ln S_{t+4} - E_t \ln S_{t+4} = u_{t+4}$ .

It was stated earlier that  $u_t$  can be written as  $u_t = \theta(L)\varepsilon_t$ , where  $\theta(L)$  is a third degree polynomial in the lag operator  $L$ . Since one can write  $u_t = \theta(L)\varepsilon_t$ , equation (1') may be rewritten as

$$\ln S_{t+4} = a + b \ln F_t + \theta_3 \varepsilon_{t+1} + \theta_2 \varepsilon_{t+2} + \theta_1 \varepsilon_{t+3} + \varepsilon_{t+4}. \quad (1'')$$

Assuming the  $\varepsilon$ -shocks strike with equal weight over time, one would expect the  $\theta$ 's to be approximately equal to 1. Appendix A provides an interpretation of the  $\theta$ 's and the  $\varepsilon$ 's.

### 3 Econometric Results

Equation (1') is estimated for five currencies, with respect to the dollar, for the period April 1973 to May 1977.<sup>1</sup> Ordinary least squares estimates of  $a$  and  $b$  are reported in Table 1. In all cases  $b$  is less than one. However, only in the case of the Netherlands and Germany is  $b$  significantly less than one. In addition, the constant term is significantly less than zero for the Netherlands and Germany, but insignificantly different from zero for Canada, Switzerland and the U.K. These results indicate a significant risk premium for

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<sup>1</sup>I wish to thank Alan C. Stockman for providing me with this data. See the Data Appendix for a description of the data.



only the Netherlands and Germany.

Having obtained consistent estimates of  $a$  and  $b$  from Table 1, a consistent estimate of  $u_t$  is obtained. An MA(4) process was fit to the OLS residuals. The results are reported in Table 1.  $\theta_1$  and  $\theta_2$  are generally within one standard deviation of unity and  $\theta_3$  is generally within two standard deviations of unity. Switzerland is the exception, where  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  are all significantly less than 1.0.  $\theta_4$  is significantly less than 1.0, as expected, since  $\epsilon_t$  is picking up "the last two days of the month." These results conform to one's prior notion of  $\theta_j$ 's.

As stated previously,  $b$  has often been constrained to be unity (Hansen and Hocrick 1979 and Stockman 1978). If  $b$  is in fact unity, then this method will provide efficient estimates of  $a$  and the  $\theta$ 's. However, if the data rejects the assumption that  $b$  is unity, imposing this restriction will bias our results. Table 2 reports the estimates of  $a$  obtained from constraining  $b$  to be unity. In all cases,  $a$  is insignificantly different from 0. Estimates of  $\theta_j$  are also reported in Table 2 for the case in which  $b$  is constrained to be unity. In this case,  $a$  and  $\theta_j$  ( $j= 1, 2, 3, 4$ ) are jointly estimated in an efficient manner;  $\ln F_t$  was taken to the left hand side so that all right hand side variables ( $a$  constant) are strictly exogenous. The results are similar to those in Table 1, except that  $\theta_3$  is significantly different from unity for the Netherlands and Germany.

As stated previously, the time series behavior of the residuals is of great interest. In particular, does  $u_t$  follow an MA(4) process, or equivalently, is  $\epsilon_t$  white noise? To test this, a number of tests of  $\epsilon_t$  being white noise are reported. Durbin's periodogram test (Durbin 1969) provides a frequency domain test of the null hypothesis that  $\epsilon_t$  is white noise, against the alternative hypothesis that  $\epsilon_t$  is not white noise (PER in the tables). PER = 0 means that the actual cumulative periodogram does not differ

TABLE 1

$$\ln S_{t+4} = a + b \ln F_t + u_{t+4}$$

$$u_t \sim \text{MA}(4)$$

	Netherlands	Germany	Canada	Switzerland	United Kingdom
a	-0.230 (0.078)	-0.257 (0.075)	0.001 (0.000)	-0.084 (0.045)	0.013 (0.018)
b	0.757 (0.081)	0.719 (0.082)	0.915 (0.079)	0.913 (0.045)	0.979 (0.023)
$\theta_1$	1.012 (0.064)	0.996 (0.064)	1.102 (0.056)	0.628 (0.069)	1.023 (0.067)
$\theta_2$	1.076 (0.073)	0.964 (0.073)	1.070 (0.065)	0.682 (0.069)	0.939 (0.084)
$\theta_3$	0.860 (0.071)	0.839 (0.070)	1.002 (0.064)	0.629 (0.068)	0.747 (0.083)
$\theta_4$	0.378 (0.064)	0.441 (0.061)	0.629 (0.056)	0.226 (0.069)	0.336 (0.068)
$\sigma_u^2$	0.851	0.974	0.130	0.958	0.577
$\sigma_\varepsilon^2$	0.184	0.213	0.027	0.377	0.131
PER	0	0	0	0	0
Q(12)	10.7	7.4	13.8*	9.4	16.6*
Q(24)	23.7	20.1	30.0	19.0	30.8
Q(36)	30.9	30.7	34.0	41.5	34.4

NOTES: Standard errors are in parentheses.  $\sigma_u^2$  and  $\sigma_\varepsilon^2$  are in units of  $10^{-3}$ . Both  $\sigma_u^2$  and  $\sigma_\varepsilon^2$  are calculated with back forecasting. PER gives the number of times (or the periods) that the actual cumulative periodogram exceeds the expected cumulative periodogram, at the 10 percent level of significance. Q(k) tests the null hypothesis that the first k autocorrelations are equal to zero. Q(k) is distributed as  $\chi^2(k-q)$ , where q is the number of parameters estimated. An asterisk (\*) denotes significant at the 5 percent level of significance.

TABLE 2

$$\ln S_{t+4} = a + 1.0 \ln F_t + u_{t+4}$$

$$u_t \sim \text{MA}(4)$$

	Netherlands	Germany	Canada	Switzerland	United Kingdom
a	0.0036 (0.0044)	0.0024 (0.0048)	0.0007 (0.0018)	0.0024 (0.0044)	-0.0034 (0.0033)
$\theta_1$	0.9883 (0.0660)	0.9821 (0.0648)	1.1029 (0.0549)	0.6239 (0.0698)	1.0264 (0.0675)
$\theta_2$	1.0332 (0.0774)	0.9429 (0.0771)	1.0709 (0.0645)	0.6704 (0.0705)	0.9387 (0.0849)
$\theta_3$	0.8017 (0.0759)	0.7870 (0.0752)	1.0055 (0.0630)	0.6099 (0.0697)	0.7466 (0.0839)
$\theta_4$	0.3525 (0.0662)	0.4154 (0.0636)	0.6495 (0.0549)	0.2107 (0.0694)	0.3335 (0.0678)
$\sigma_u^2$	0.990	1.153	0.134	1.025	0.586
$\sigma_e^2$	0.231	0.278	0.029	0.418	0.134
PER	0	0	0	0	0
Q(12)	6.8	3.5	14.3*	9.7	16.3*
Q(24)	21.1	19.7	30.6	20.0	31.2
Q(36)	32.7	32.1	34.7	43.8	35.6

NOTES: See notes to Table 1.

significantly from the expected cumulative periodogram, at any frequency. The Box Pierce Q-statistic,  $Q(k)$ , tests the null hypothesis that the first  $k$  auto-correlations are equal to zero. It can be shown that  $Q(k)$  is distributed as  $\chi^2(k-q)$ , where  $q$  is the number of parameters estimated. So far, only a very broad alternative has been considered:  $\epsilon_t$  is not white noise. It is possible to test more restrictive alternative hypotheses.<sup>1</sup> One particular alternative that is considered is that  $u_t$  is MA(5).

The results of these tests of white noise are reported in Table 1. In all cases, Durbin's periodogram test indicates that  $\epsilon_t$  is white noise (PER = 0 in the table). The Box-Pierce Q-statistic also indicates that  $\epsilon_t$  is white noise for the Netherlands, Germany and Switzerland. However, for Canada and the U.K, the Box-Pierce  $Q(12)$  statistic of 13.8 and 16.6 indicates that there is a departure from white noise, at the 5 percent level of significance for these currencies. (Note that  $Q(12)$  is insignificant at the  $Q(24)$  and  $Q(36)$  are insignificant at the 5 percent level.)

To test if  $u_t$  is MA(5), an MA(5) process was fit to the OLS residuals. The results are reported in Table 3. One finds that  $\theta_5$  is statistically significant for all currencies except Canada and the U.K. Recall that Canada and the U.K. had significant  $Q(12)$  statistics. This would seem to imply that although for these currencies there is serial correlation in  $u_t$  beyond an MA(4), it is not of a moving average form. To test this assertion, one can perform a likelihood ratio test of whether one needs to go to an MA(5) process for  $u_t$  (over an MA(4)). The results of this test are reported in Table 4 under

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<sup>1</sup>The power of a test relates to the alternative hypothesis. A test may be powerful against some alternatives but less powerful against other alternatives. Durbin's periodogram test has good power against the alternative of white noise, however, there are more powerful tests against, for example, an MA(5) alternative.

TABLE 3

$$\ln S_{t+4} = a + b \ln F_t + u_{t+4}$$

$$u_t \sim \text{MA}(5)$$

	Netherlands	Germany	Canada	Switzerland	United Kingdom
a	-0.230 (0.078)	-0.257 (0.075)	0.001 (0.000)	-0.084 (0.045)	0.013 (0.018)
b	0.757 (0.081)	0.719 (0.082)	0.915 (0.079)	0.913 (0.045)	0.979 (0.023)
$\theta_1$	0.999 (0.068)	1.009 (0.070)	1.100 (0.071)	0.677 (0.070)	1.033 (0.071)
$\theta_2$	1.050 (0.088)	1.008 (0.091)	1.067 (0.095)	0.773 (0.080)	0.974 (0.097)
$\theta_3$	0.928 (0.093)	0.912 (0.095)	0.996 (0.098)	0.778 (0.080)	0.819 (0.103)
$\theta_4$	0.578 (0.086)	0.547 (0.089)	0.620 (0.096)	0.364 (0.080)	0.422 (0.097)
$\theta_5$	0.222 (0.065)	0.122 (0.066)	0.006 (0.069)	0.171 (0.067)	0.086 (0.069)
$\sigma_u^2$	0.851	0.974	0.130	0.958	0.577
$\sigma_\varepsilon^2$	0.181	0.212	0.027	0.370	0.131
PER	0	0	0	0	0
Q(12)	3.8	4.4	13.7	5.5	14.2*
Q(24)	15.3	15.0	30.1	13.3	28.0
Q(36)	24.7	26.9	34.1	35.9	31.5

NOTE: See notes to Table 1.

TABLE 4  
HYPOTHESIS TESTING RESULTS

Country	Error	Variance	Theta
Netherlands	3.32	4.31	12.81
Germany	0.95	3.94	5.00
Canada	0.00	0.53	16.27
Switzerland	3.79	6.90	6.89
United Kingdom	0.00	0.58	4.53

NOTE: The column Error tests the hypothesis that  $u_t$  is MA(4) against MA(5). A likelihood ratio test is used, where  $2\ln\lambda$  is distributed  $\chi^2(1)$ , where

where  $\lambda = \frac{\sigma_2^2(\text{MA4})}{\sigma_\epsilon^2(\text{MA5})}^{(T/2)}$ . The critical values are 2.71 (10 percent),

3.84 (5 percent) and 6.63 (1 percent). The column Variance tests the hypothesis that the variance of  $\epsilon_t$  is equal in both periods ( $\sigma^2(\text{PER1}) = \sigma^2(\text{PER2})$ ) against the hypothesis that they are unequal. An F-statistic is calculated as  $F = s_1^2/s_2^2$ , F is distributed as F(99,97). Critical values, using F(100,100), for the two sided alternative are:  $0.719 < F < 1.39$  (10 percent) and  $0.629 < F < 1.59$  (2 percent). To test if the variance decreases over time, the critical values are 1.39 (5 percent) and 1.59 (1 percent). To test if the variance increases over time, the critical values are 0.719 (5 percent) and 0.629 (1 percent). The column Theta tests the hypothesis that  $\theta(\text{PER}) = \theta(\text{PER2})$ . A  $\chi^2$  statistic can be calculated to test this hypothesis (see Morrison 1967). The critical values for  $\chi^2(4)$  are 7.78 (10 percent), 9.49 (5 percent) and 13.3 (1 percent).

the column headed "Error." For the Netherlands and Switzerland an MA(5) yields significantly better results at the 10 percent level, but not significantly better results at the 5 percent level, as indicated by  $\chi^2$  values of 3.32 and 3.79, respectively. For Germany, Canada and the U.K. there is no significant improvement at the 10 percent level. In fact, for Canada and the U.K., the variance of  $\epsilon_t$  is unchanged, reflected in  $\chi^2$  values of 0.000, thereby confirming our earlier assertion.

As stated earlier, another issue of interest concerns the behavior of the system over time. For example, can one assert in any meaningful way that the system has become less noisy over time? Also, have economic agents reacted to new information in the same way over time? To answer these questions, the sample period was divided into two equal halves (non-overlapping) and equation (1') was reestimated for each half. The results are presented in Tables 5 and 6. An F-statistic was calculated to test the hypothesis that  $\sigma_\epsilon^2$  is the same in both periods. The results of this calculation are reported in Table 4 under the column headed "Variance." In all cases one can reject the hypothesis (at the 10 percent and 2 percent level) that the variances are equal, due to the extreme values of the F reported. For the Netherlands, Germany and Switzerland one can accept the alternative (at the 5 percent and 1 percent level) that the variance has increased over time, as reflected in the large values of F. On the other hand, for Canada and the U.K., one can accept the alternative (at the 5 percent and 1 percent level) that the variance has increased over time, as reflected in the small values of F.

The reaction of economic agents to the new information coming to the market is summarized by the  $\theta$ -vector. A  $\chi^2$  test can be used to test if the  $\theta$ 's change over time. The result of such a test is reported in Table 4 under column headed "Theta." For the Netherlands and Canada, one can reject, at the

TABLE 5

$$\ln S_{t+4} = a + b \ln F_t + u_{t+4}$$

$$u_t \sim MA(4)$$

(April 1973 - April 1975)

	Netherlands	Germany	Canada	Switzerland	United Kingdom
a	-0.231	-0.298	-0.001	-0.096	0.301
b	0.765	0.672	0.972	0.904	0.652
$\theta_1$	0.972 (0.093)	0.988 (0.090)	0.978 (0.093)	0.506 (0.091)	0.981 (0.097)
$\theta_2$	1.017 (0.106)	0.924 (0.103)	0.906 (0.101)	0.660 (0.063)	0.849 (0.110)
$\theta_3$	0.799 (0.102)	0.827 (0.098)	0.847 (0.097)	0.835 (0.050)	0.831 (0.109)
$\theta_4$	0.341 (0.093)	0.459 (0.086)	0.402 (0.090)	0.430 (0.086)	0.414 (0.096)
$\sigma_u^2$	1.211	1.473	0.067	1.583	0.352
$\sigma_\varepsilon^2$	0.302	0.339	0.018	0.642	0.083
PER	0	0	0	0	0
Q(12)	5.1	4.3	12.6	9.8	9.3
Q(24)	15.7	16.1	24.2	18.8	20.3
Q(36)	22.0	23.4	32.5	37.4	25.4

NOTES: See notes to Table 1.



TABLE 6

$$\ln S_{t+4} = a + b \ln F_t + u_{t+4}$$

$$u_t \sim \text{MA}(4)$$

(May 1975 - May 1977)

	Netherlands	Germany	Canada	Switzerland	United Kingdom
a	-0.259	-0.192	0.002	-0.170	0.069
b	0.730	0.792	0.919	0.820	0.879
$\theta_1$	1.140 (0.083)	1.056 (0.093)	1.232 (0.067)	0.866 (0.098)	1.010 (0.098)
$\theta_2$	1.269 (0.077)	1.121 (0.102)	1.283 (0.064)	0.950 (0.101)	0.930 (0.121)
$\theta_3$	1.177 (0.065)	0.942 (0.099)	1.230 (0.045)	0.836 (0.099)	0.715 (0.119)
$\theta_4$	0.598 (0.078)	0.414 (0.091)	0.782 (0.060)	0.282 (0.098)	0.313 (0.097)
$\sigma_u^2$	0.487	0.451	0.194	0.325	0.495
$\sigma_\varepsilon^2$	0.070	0.086	0.034	0.093	0.142
PER	0	0	0	0	0
Q(12)	20.3*	10.1	8.5	10.1	8.5
Q(24)	26.0	17.1	15.9	18.2	23.3
Q(36)	29.2	19.8	22.1	24.9	28.2

NOTES: See notes to Table 1.

at the 5 percent level, the hypothesis that the  $\theta$ 's are the same in both periods. For Germany, Switzerland and the U.K. one cannot reject the hypothesis that the  $\theta$ 's are the same in both periods. As a general proposition, it appears that  $\theta(\text{period 2})$  is larger than  $\theta(\text{period 1})$ .

In summarizing the results in Tables 1-6, it is helpful to look for any patterns that emerge. The  $\theta$ -vector does appear to equal a vector of ones (except, of course, for  $\theta_4$ ). Switzerland is a striking exception since the  $\theta$ 's are significantly less than one. In addition, there is a tendency for  $\theta_j$  to decrease as  $j$  increases. Finally, although the slope is significantly less than one only for the Netherlands and Germany, the point estimates are always less than one. This might seem to indicate that in fact  $b < 1$ . Notice, however, that this is not an exact statistical statement since the five exchange rates are not independent. What this observation suggests is that pooling all five currencies might lead to more precise estimates. In fact, this is what Bilson (1979) finds in a slightly different context.

It is also possible to divide the sample into two groups: (1) Canada and U.K. (2) the Netherlands, Germany and Switzerland. For Canada and the U.K. the constant in equation (1') is negative, while for the others it is positive. For a given slope, this implies that the U.S. dollar is safer, with respect to the Canadian dollar and U.K. pound, but riskier with respect to the Dutch guilder, German mark and the Swiss franc. Canada and the U.K. were the only countries that showed any indication of serial correlation in the  $\epsilon_t$  (Q(12) was significant at the 5 percent level). This indicates that for these two countries  $\ln F_t$  did not summarize all information about the value of  $\ln S_{t+4}$ . (It should be noted that all other tests of randomness indicated a lack of serial correlation in  $\epsilon_t$ .) Finally,  $\sigma_\epsilon^2$  increased in the second period for Canada and the U.K., while it decreased for the others. How, then, is Canada and the

U.K. different from the Netherlands, Germany and Switzerland? A major difference would seem to be that the U.S. dollar appreciated against the Canadian dollar and U.K. pound for the period as a whole, while it depreciated against the other currencies. Not coincidentally, Canada and the U.K. had the highest rates of inflation for the whole period and each of the two subperiods.

To understand the change in the variance of the system ( $\sigma_e^2$ ), one should consider some of the institutional changes. In March of 1973, Germany and the Netherlands agreed to fix their rates within 2.5 percent but to float against the dollar. Late 1973 witnessed the dramatic increase in the price of oil. In December 1975 (at the Rambouillet meeting) and January 1976 (in Jamaica) it became "official" that we were in a period of floating exchange rates; previously, there was some hope of a return to fixed exchange rates. Finally, there was the IMF support of sterling in late 1976: in June of 1976, a \$5 billion stand-by credit and in December of 1976, a \$3.9 billion loan.

#### 4 A New Procedure

Equation (1') expressed  $E(\ln S_{t+4} : I_t)$  as a linear function of  $\ln F_t$ :  $E(\ln S_{t+4} : I_t) = a + b \ln F_t$ . This is quite a strong assumption, and one would like to test it. One possibility is to include variables  $x_{it}$  in the regression equation (1') and test if the coefficients are significant (see, for example, Hansen and Hodrick 1979); one would like a more efficient method. In addition, one would like to obtain a more efficient estimate of  $b$ .

Assume that  $\{s_t, f_t\}$  is a bivariate, linearly indeterministic,

covariance stationary stochastic process.<sup>1</sup> It will be assumed, in the empirical work, that  $s_t$  and  $f_t$  are the first differences of the logarithms of  $S_t$  and  $F_t$ . Under the assumption that  $\{s_t, f_t\}$  is invertible, one can express  $\{s_t, f_t\}$  as an infinite order bivariate autoregression. Assuming one can truncate the autoregression at lag  $M$ , one can write

$$s_t = \sum_{i=1}^M \alpha_i s_{t-i} + \sum_{i=1}^M \beta_i f_{t-i} + w_t \quad (4a)$$

$$f_t = \sum_{i=1}^M \gamma_i s_{t-i} + \sum_{i=1}^M \delta_i f_{t-i} + v_t \quad (4b)$$

$$w_t = s_t - E(s_t : s_{t-1}, s_{t-2}, \dots, f_{t-1}, f_{t-2}, \dots)$$

$$v_t = f_t - E(f_t : s_{t-1}, s_{t-2}, \dots, f_{t-1}, f_{t-2}, \dots)$$

$$E w_t w_{t-k} = \begin{cases} \sigma_w^2 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

$$E v_t v_{t-k} = \begin{cases} \sigma_v^2 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

$$E w_t v_{t-k} = \begin{cases} \sigma_{wv} & k = 0 \\ 0 & k \neq 0 \end{cases}$$

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See Sargent (1979b) for a good exposition of the general methodology. This assumption implies that the variances of  $s_t$  and  $f_t$  exist and are independent of  $t$ , the covariance between  $s_t$  and  $f_{t-k}$  exists and is a function of  $k$  only, and the mean values of  $s_t$  and  $f_t$  are zero.

To estimate  $(\alpha, \beta, \gamma, \delta)$ , rewrite equation (4) as a first order vector stochastic difference equation. Define the matrix A, and vectors  $x_t$  and  $a_t$  as

$$A = \begin{bmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_M & \beta_1 & \beta_2 & \dots & \beta_M \\ 1 & 0 & & 0 & 0 & 0 & & 0 \\ 0 & 1 & & 0 & 0 & 0 & & 0 \\ 0 & 0 & & 1 & 0 & 0 & & 0 \\ \gamma_1 & \gamma_2 & \dots & \gamma_M & \delta_1 & \delta_2 & & \delta_M \leftarrow \text{row } M + 1 \\ 0 & 0 & & 0 & 1 & 0 & & 0 \\ \cdot & & & & & & & \\ \cdot & & & & & & & \\ \cdot & & & & & & & \\ 0 & 0 & & 0 & 0 & 0 & & 1 \end{bmatrix}$$

$$x_t = \begin{bmatrix} s_t \\ s_{t-1} \\ \cdot \\ \cdot \\ \cdot \\ s_{t-M+1} \\ f_t \\ f_{t-1} \\ \cdot \\ \cdot \\ f_{t-M+1} \end{bmatrix} \quad a_t = \begin{bmatrix} w_t \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ v_t \leftarrow \text{row } M + 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

Then, one can rewrite equation (4) as

$$x_t = A x_{t-1} + a_t \quad (5)$$

Defining  $c = (1 \ 0 \ \dots \ 0)$  and  $d = (0 \ \dots \ 0 \ 1 \ 0 \ \dots \ 0)$ , rewrite (4), using (5), as

$$s_t = c A x_{t-1} + w_t \quad (6)$$

$$f_t = d A x_{t-1} + v_t.$$

Update equation (5) by + j to obtain

$$\begin{aligned} x_{t+j} &= A x_{t+j-1} + a_{t+j} \\ &= A[A x_{t+j-2} + a_{t+j-1}] + a_{t+j} \end{aligned} \quad (7)$$

$$\begin{aligned} &\cdot \\ &\cdot \\ &\cdot \\ &= A^{j+1} x_{t-1} + A^j a_t + \dots + a_{t+j}. \end{aligned}$$

Taking expectations of both sides of (7), conditional on information up to  $t - 1$ ,  $I_{t-1}$ , one obtains

$$E(s_{t+j} | I_{t-1}) = A^{j+1} x_{t-1}. \quad (8)$$

Recalling that  $s_t = c A x_{t-1} + w_t$ , one can calculate  $E_{t-1} S_t$  by pre-multiplying both sides of (8) by the (row) vector  $c$ :

$$E(S_t | I_{t-1}) = E(c x_{t+j} | I_{t-1}) = c A^{j+1} x_{t-1}. \quad (9)$$

The assumption that  $f_t$  summarizes all information implies that

$$E(s_{t+4} | I_{t-1}) = b E(f_t | I_{t-1}).$$

Equation (6) implies that

$$b E(f_t | I_{t-1}) = b(dA) x_{t-1}. \quad (10)$$

Therefore, by equating equations (9) and (10), the following set of cross-equation restrictions are obtained:

$$b(dA) = c A^5. \quad (11)$$

The assumption that the forward rate is an unbiased estimate of the future spot rate implies  $b = 1$ . Therefore, the restrictions embodied in

(11) become

$$dA = cA^5 \quad (12)$$

Under the hypothesis that  $\{w_t, v_t\}$  is bivariate normal, the likelihood function of a sample of size T of  $\{w_t, v_t\}$  can be written

$$L(\alpha, \beta, \gamma, \delta | \{s_t\}, \{f_t\}) = (2\pi)^{-T} |V|^{-T/2} \exp(-1/2 \sum_{t=1}^T e_t' V^{-1} e_t) \quad (13)$$

where

$$e_t = \begin{bmatrix} w_t \\ v_t \end{bmatrix} \quad V = E e_t e_t'$$

Maximizing (13) unconstrained is equivalent to estimating (4) by least-squares (to obtain efficient standard errors, one would use Zellner's unrelated regression).

The restrictions implied by equations(11) and (12) are highly non-linear. First, consider estimation strategies for the restriction implied by  $b = 1$ . Sargent (1979b) proposes two alternative estimation strategies. The first method requires estimating row one of A, equation (4a), by least squares. Then, the (M+1) st row of A, equation (4b), is calculated using an iterative procedure. Form a preliminary estimate of A, call it  $A_0$ , by setting row M + 1 to a row of zeroes, and all other rows to their known (or consistent) values. Calculate the (M+1)st row of A, at iteration  $i + 1$ , as

$$(\text{row } M+1)_{i+1} = dA_{i+1} = c A_i^5 \quad (14)$$

where  $A_i$  is the estimate of A on the  $i$ 'th iteration. At each step in forming  $A_i$ , all rows (except the (M+1)st) are kept equal to the corresponding row of  $A_0$ . If this procedure converges, it will find an A that satisfies (12). The condition for convergence is that the roots of A be

less than one in modulus. Since the elements of row 1 are consistently estimated by least squares, the (M+1)st row will be consistently estimated as a function of the first row of A.

Define the solution to the iteration on (14) as the (set) function  $\Phi$ :

$$(\gamma, \delta) = \Phi(\alpha, \beta) \quad (15)$$

$\Phi$  maps the  $\alpha$ 's and  $\beta$ 's into a set of  $\gamma$ 's and  $\delta$ 's that satisfy restriction (12). Hence, one (consistent) estimator of  $\gamma, \delta$  is  $\Phi(\alpha, \beta)$ .

Estimating equation (4), with restriction (11) imposed, is analogous. As before, begin by estimating the first row of A, equation (4a), by least squares. An initial guess for b is obtained from Table 1. Then, the (M+1)st row of A, equation (4b), is calculated using an iterative procedure. In a manner similar to before, one can obtain as a solution to the iteration procedure the set function  $\tilde{\Phi}$ :  $\tilde{\Phi}(\alpha, \beta, b) = (\gamma, \delta)$ .  $\tilde{\Phi}$  maps the  $\alpha$ 's,  $\beta$ 's and b into a set of  $\gamma$ 's and  $\delta$ 's that satisfy restriction (11).

Under the restriction (12) (equivalently (15)), the likelihood function in (13),  $L(\alpha, \beta, \gamma, \sigma | \{s_t\}, \{f_t\})$ , becomes a function only of the  $\alpha$ 's and  $\beta$ 's. As Wilson (1973) argues, maximum likelihood estimates with an unknown V, are obtained by minimizing  $|\hat{V}|$ , with respect to the  $\alpha$ 's and  $\beta$ 's, where

$$|\hat{V}| = \frac{1}{T} \left| \sum_{t=1}^T e_t(\alpha, \beta) e_t(\alpha, \beta)' \right|$$

and the  $e_t(\alpha, \beta)$ , the residuals from (4), are functions of the  $\alpha$ 's and  $\beta$ 's only, since they were calculated from (4) with (12) (equivalently (15) imposed).

A derivative free nonlinear minimization routine can be used to estimate (2.8) under the restriction (2.16) or (2.15). The IMSL subroutine



ZXMIN, which uses a quasi-Newton method, was used. Generally, 600 iterations were required to obtain three significant digits. The least squares estimates of  $\alpha$  and  $\beta$  were used as starting values.

Tables 7-11 report three estimates of equation (4) under various assumptions for the five currencies vis-a-vis the U.S. dollar. The tables report estimates of the bivariate autoregression (4) unconstrained, the maximum likelihood estimates that impose (12) ( $b=1$ ) and the maximum likelihood estimates that impose (13) ( $b$  free). Also reported are the row sums of the  $\alpha$ 's,  $\beta$ 's,  $\gamma$ 's and  $\delta$ 's. The likelihood ratio statistic, which is distributed  $\chi^2(8)$ , and the marginal significance level, which is the probability that a random variable that is distributed  $\chi^2(8)$  attains a value greater than or equal to the test statistic, are also reported.

According to the likelihood ratio statistic, the hypothesis is generally rejected. Only for Germany, with  $b$  free, is the marginal significance level greater than 0.02. The assumption that  $\{s_t, f_t\}$  is stationary is equivalent to the assumption that the characteristic roots of  $A$  are all less than one in modulus (Sargent (1979a), p. 273). The roots of  $A$  were calculated for all three sets of estimates and all were found to be less than one in modulus.

There appears to be a number of regularities that can be found in these results, some of which may be related to the cause for rejection. One regularity concerns the sign pattern of the coefficients. In all cases but two (Canada and the U.K.), the sum of the  $\alpha_i$ 's and the sum of the  $\delta_i$ 's are negative, while the sum of the  $\beta_i$ 's and the sum of the  $\gamma_i$ 's are positive. Recall that the  $\alpha$ 's are the coefficients on lagged  $s_t$ 's and the  $\beta$ 's are the coefficients on lagged  $f_t$ 's in the  $s_t$  equation, while the  $\gamma$ 's are the coefficients on lagged  $s_t$ 's and the  $\delta$ 's are the coefficients on

TABLE 7

NETHERLANDS: ESTIMATES OF BIVARIATE AUTOREGRESSION  
UNRESTRICTED AND RESTRICTED

j	1	2	3	4	Row Sum
<u>Unrestricted Estimates</u>					
$\alpha_j$	0.01913	-0.04513	-0.01726	-0.00722	-0.05049
$\beta_j$	0.26103	0.08057	0.06862	-0.02894	0.38128
$\gamma_j$	0.82962	0.49743	0.20908	0.00174	1.53787
$\delta_j$	-0.51371	-0.35106	-0.14112	0.01910	-0.98679

$$\hat{V} = \begin{pmatrix} 1.303 \cdot 10^{-4} & 8.345 \cdot 10^{-5} \\ & 9.713 \cdot 10^{-5} \end{pmatrix} \quad |\hat{V}| = 5.692 \cdot 10^{-9}$$

<u>Maximum Likelihood Estimates</u>					
b=1					
$\alpha_j$	-0.56550	-0.59600	-0.40654	-0.40114	-1.96918
$\beta_j$	0.68043	0.56195	0.44296	0.16306	1.84840
$\gamma_j$	0.29193	0.07700	0.02809	0.04198	0.43900
$\delta_j$	-0.17457	-0.05475	-0.04048	-0.01706	-0.25686

$$\hat{V} = \begin{pmatrix} 1.541 \cdot 10^{-4} & 1.061 \cdot 10^{-5} \\ & 1.159 \cdot 10^{-4} \end{pmatrix} \quad |\hat{V}| = 6.596 \cdot 10^{-9}$$

Likelihood ratio statistic = 29.775  
Marginal significance level = 0.00023

<u>Maximum Likelihood Estimates</u>					
b free					
$\alpha_j$	-0.65142	-0.65475	-0.29087	-0.18303	-1.78007
$\beta_j$	0.82390	0.55780	0.25424	0.03786	1.67380
$\gamma_j$	0.32057	0.10130	0.01639	0.06781	0.50607
$\delta_j$	-0.11494	-0.08055	-0.07529	-0.01403	-0.28481

$$b = 0.44848$$

$$\hat{V} = \begin{pmatrix} 1.577 \cdot 10^{-4} & 1.054 \cdot 10^{-4} \\ & 1.116 \cdot 10^{-4} \end{pmatrix} \quad |\hat{V}| = 6.492 \cdot 10^{-9}$$

Likelihood ratio statistic = 26.503  
Marginal significance level = 0.00086

TABLE 8

GERMANY: ESTIMATES OF BIVARIATE AUTOREGRESSION  
UNRESTRICTED AND RESTRICTED

j	1	2	3	4	Row Sum
<u>Unrestricted Estimates</u>					
$\alpha_j$	-0.06783	-0.15063	-0.06351	-0.00627	-0.28824
$\beta_j$	0.30407	0.22275	0.15479	-0.02938	0.65223
$\gamma_j$	0.70532	0.43141	0.27823	0.00165	1.41661
$\delta_j$	-0.44493	-0.31663	-0.16010	0.03287	-0.88879

$$\hat{V} = \begin{pmatrix} 1.573 \cdot 10^{-4} & 9.831 \cdot 10^{-5} \\ & 1.154 \cdot 10^{-4} \end{pmatrix} \quad |\hat{V}| = 8.488 \cdot 10^{-9}$$

<u>Maximum Likelihood Estimates</u>					
b=1					
$\alpha_j$	-0.56630	-0.62942	-0.49703	-0.39033	-2.08308
$\beta_j$	0.70566	0.66585	0.54469	0.14558	2.06178
$\gamma_j$	0.23349	0.04501	-0.00001	0.00329	0.28178
$\delta_j$	-0.12809	-0.01655	-0.00303	-0.00123	-0.14890

$$\hat{V} = \begin{pmatrix} 1.732 \cdot 10^{-4} & 1.179 \cdot 10^{-4} \\ & 1.362 \cdot 10^{-4} \end{pmatrix} \quad |\hat{V}| = 9.680 \cdot 10^{-9}$$

Likelihood ratio statistic = 26.545  
Marginal significance level = 0.00085

<u>Maximum Likelihood Estimates</u>					
b free					
$\alpha_j$	-0.48035	-0.63593	-0.54612	-0.38979	-2.05219
$\beta_j$	0.69258	0.65924	0.52844	0.15190	2.03216
$\gamma_j$	0.31682	0.07705	-0.00284	-0.01249	.37854
$\delta_j$	-0.18501	-0.02871	0.01123	0.00487	- .19762

b = 0.66335

$$\hat{V} = \begin{pmatrix} 1.700 \cdot 10^{-4} & 1.130 \cdot 10^{-5} \\ & 1.296 \cdot 10^{-4} \end{pmatrix} \quad |\hat{V}| = 9.280 \cdot 10^{-9}$$

Likelihood ratio statistic = 18.019  
Marginal significance level = 0.02108

TABLE 9

CANADA: ESTIMATES OF BIVARIATE AUTOREGRESSION  
UNRESTRICTED AND RESTRICTED

j	1	2	3	4	Row Sum
<u>Unrestricted Estimates</u>					
$\alpha_j$	0.14948	0.07733	0.05981	-0.18817	0.09845
$\beta_j$	0.07164	-0.07935	0.02268	0.09671	0.11168
$\gamma_j$	0.82945	0.30516	0.15284	-0.02884	1.25861
$\delta_j$	-0.45814	-0.26949	-0.11163	0.04070	-0.79856

$$\hat{V} = \begin{pmatrix} 2.199 \cdot 10^{-5} & 1.114 \cdot 10^{-5} \\ & 1.240 \cdot 10^{-5} \end{pmatrix} \quad |\hat{V}| = 1.499 \cdot 10^{-10}$$

<u>Maximum Likelihood Estimates</u>					
b=1					
$\alpha_j$	-0.46541	-0.06324	0.25763	-0.11134	-0.38236
$\beta_j$	0.39378	-0.11220	-0.06007	-0.02458	0.19693
$\gamma_j$	0.26696	-0.01351	-0.07107	0.02913	0.21151
$\delta_j$	-0.11470	0.01886	0.01490	0.00643	-0.07451

$$\hat{V} = \begin{pmatrix} 2.668 \cdot 10^{-5} & 1.508 \cdot 10^{-5} \\ & 1.644 \cdot 10^{-5} \end{pmatrix} \quad |\hat{V}| = 2.144 \cdot 10^{-10}$$

Likelihood ratio statistic - 72.246  
Marginal significance level = 0.0001

<u>Maximum Likelihood Estimates</u>					
b free					
$\alpha_j$	-0.47036	-0.11258	0.20141	-0.11788	-0.49941
$\beta_j$	0.37960	-0.07881	-0.07815	0.01220	0.23484
$\gamma_j$	0.29606	-0.00506	-0.06488	0.03605	0.26217
$\delta_j$	-0.15633	0.02658	0.02424	-0.00374	-0.10925

b = 0.87230

$$\hat{V} = \begin{pmatrix} 2.704 \cdot 10^{-5} & 1.511 \cdot 10^{-5} \\ & 1.616 \cdot 10^{-5} \end{pmatrix} \quad |\hat{V}| = 2.085 \cdot 10^{-10}$$

Likelihood ratio statistic = 66.696  
Marginal significance level = 0.0001

TABLE 10

SWITZERLAND: ESTIMATES OF BIVARIATE AUTOREGRESSION  
UNRESTRICTED AND RESTRICTED

j	1	2	3	4	Row Sum
<u>Unrestricted Estimates</u>					
$\alpha_j$	-0.09582	0.00814	-0.01271	-0.01044	-0.11083
$\beta_j$	0.09477	0.08734	0.10093	-0.07744	0.20560
$\gamma_j$	0.82700	0.80629	0.34803	0.00003	1.98135
$\delta_j$	-0.83059	-0.50919	-0.16071	-0.08883	-1.58932

$$\hat{V} = \begin{pmatrix} 2.158 \cdot 10^{-4} & 1.193 \cdot 10^{-4} \\ & 1.696 \cdot 10^{-4} \end{pmatrix} \quad |\hat{V}| = 2.237 \cdot 10^{-8}$$

<u>Maximum Likelihood Estimates</u>					
b=1					
$\alpha_j$	-0.41943	-0.17910	0.06281	-0.16359	-0.69931
$\beta_j$	0.36243	0.09490	0.04269	0.09038	0.08255
$\gamma_j$	0.20600	0.03741	-0.03392	0.04641	0.25590
$\delta_j$	-0.14204	-0.01189	-0.00320	-0.02565	-0.18278

$$\hat{V} = \begin{pmatrix} 2.419 \cdot 10^{-4} & 1.492 \cdot 10^{-4} \\ & 2.390 \cdot 10^{-4} \end{pmatrix} \quad |\hat{V}| = 3.555 \cdot 10^{-8}$$

Likelihood ratio statistic - 93.570  
Marginal significance level = 0.0001

<u>Maximum Likelihood Estimates</u>					
b free					
$\alpha_j$	-0.62217	-0.50351	-0.09868	-0.13233	-1.35669
$\beta_j$	0.50449	0.36383	0.12815	0.13226	1.12873
$\gamma_j$	0.41899	0.23797	-0.00424	0.08930	0.74202
$\delta_j$	-0.40328	-0.12797	-0.01568	-0.08925	-0.63618

b = 0.69214

$$\hat{V} = \begin{pmatrix} 2.649 \cdot 10^{-4} & 1.498 \cdot 10^{-4} \\ & 1.993 \cdot 10^{-4} \end{pmatrix} \quad |\hat{V}| = 3.035 \cdot 10^{-8}$$

Likelihood ratio statistic = 61.599  
Marginal significance level = 0.0001

TABLE 11

UNITED KINGDOM: ESTIMATES OF BIVARIATE AUTOREGRESSION  
UNRESTRICTED AND RESTRICTED

j	1	2	3	4	Row Sum
<u>Unrestricted Estimates</u>					
$\alpha_j$	-0.02734	0.11613	0.04996	-0.00269	0.13606
$\beta_j$	0.07475	0.02602	-0.13810	0.32083	0.28350
$\gamma_j$	0.69528	0.47395	0.26807	-0.00221	1.43509
$\delta_j$	-0.50338	-0.34317	-0.22520	0.31747	-0.75428

$$\hat{V} = \begin{pmatrix} 1.055 \cdot 10^{-4} & 6.942 \cdot 10^{-5} \\ & 7.122 \cdot 10^{-5} \end{pmatrix} \quad |\hat{V}| = 2.695 \cdot 10^{-9}$$

<u>Maximum Likelihood Estimates</u>					
b=1					
$\alpha_j$	-0.63784	-0.44721	-0.03676	-0.12006	-1.24187
$\beta_j$	0.52810	0.28584	0.09796	0.10231	1.01421
$\gamma_j$	0.25553	0.14629	-0.02008	0.04839	0.43013
$\delta_j$	-0.22631	-0.06691	-0.00974	-0.04123	-0.34419

$$\hat{V} = \begin{pmatrix} 2.682 \cdot 10^{-4} & 1.657 \cdot 10^{-4} \\ & 2.224 \cdot 10^{-4} \end{pmatrix} \quad |\hat{V}| = 3.217 \cdot 10^{-8}$$

Likelihood ratio statistic = 500.886  
Marginal significance level = 0.0001

<u>Maximum Likelihood Estimates</u>					
b free					
$\alpha_j$	-0.5770	-0.1623	0.0426	-0.0297	-0.7264
$\beta_j$	0.5073	0.0729	-0.0625	-0.0607	0.4570
$\gamma_j$	0.0936	0.0080	-0.0041	0.0030	0.1005
$\delta_j$	-0.0263	-0.0237	0.0069	0.0062	-0.0369

b = 0.8697

$$\hat{V} = \begin{pmatrix} 1.301 \cdot 10^{-4} & 9.136 \cdot 10^{-5} \\ & 9.283 \cdot 10^{-5} \end{pmatrix} \quad |\hat{V}| = 3.733 \cdot 10^{-9}$$

Likelihood ratio statistic = 65.814  
Marginal significance level

lagged  $f_t$ 's in the  $f_t$  equation. In other words, the "own" lagged coefficients are negative while the "cross" lagged coefficients are positive. This indicates that if  $s_t$  is high this period,  $s$  will be lower next period and  $f$  will be higher next period. For the unrestricted estimates, the sum of the  $\alpha$ 's and  $\beta$ 's (the parameters of the  $s_t$  equation) and the sum of the  $\gamma$ 's and  $\delta$ 's (the parameters of the  $f_t$  equation) are both positive. This fact, in conjunction with the previous facts, indicate that the positive "cross" effects dominate the negative "own" effects. However, when one examines the restricted ( $b=1$ ) estimates, one observes a difference. In this case, the sum of the parameters of the  $s_t$  equation is negative, while the sum of the parameters of the  $f_t$  equation is positive. That is, for the restricted estimates, the negative "own" effects dominate the positive "cross" effects for the  $s_t$  equation, but not for the  $f_t$  equation. Finally, both the sum of the parameters of the  $f_t$  equation decrease in going from the unrestricted to the restricted estimates.

## 5. Conclusions

Many studies of the foreign exchange market assume that the market is efficient. This implies that there are no unexploited profit opportunities. In terms of the foreign exchange market, this means that the forward rate summarizes all relevant, and available, information about the future spot rate. If one desires to test such an assumption, one requires an equilibrium model of pricing in the foreign exchange market which includes specifying an information set. Consequently, any empirical test of market efficiency is a joint test of market efficiency and the equilibrium model being used. Therefore, a rejection of the empirical test may reflect a rejection of market efficiency, or a rejection of the model being used, or both.

The first section of this paper examined the regression  $\ln S_{t+4} = a + b \ln F_t + u_{t+4}$ . The results were mixed, but generally favorable. For Canada, Switzerland and the U.K. the hypothesis that  $a = 0$ ,  $b = 1$  could not be rejected; for the Netherlands and Germany, the hypothesis could be rejected. More importantly, for the Netherlands, Germany and Switzerland, the residuals behaved in a random fashion, indicating that the forward rate does summarize all available information. Canada and the U.K., on the other hand, had a significant  $Q(12)$  statistic, indicating a departure from randomness. When the restriction  $b=1$  was imposed, the constant term was insignificantly different from zero and the residuals behaved in a similar fashion to when  $b$  was free.

The second half of this paper examined a bivariate autoregression for  $(\ln S_t - \ln S_{t-1}, \ln F_t - \ln F_{t-1})$ ; the results were unfavorable. It is worth noting that the null hypothesis is a single point in the parameter space. Consequently, when this single point is rejected, what model is to be accepted as an alternative: it takes a model to reject a model. What was shown is that the model proposed is not compatible with the data; not that the hypothesis of market efficiency is to be rejected. What are some possible reasons for this rejection?

The reason could be purely econometric: For example, was  $\{s_t, f_t\}$  a linearly indeterministic stationary stochastic process? The simple evidence indicates that  $s_t$  and  $f_t$  were stationary. However, Section 3 showed that  $\sigma_e^2$  was not constant over the sample period and so  $\{s_t, f_t\}$  might not have been stationary: the variance might have changed over time.



The other reason questions the theory used in developing the hypotheses. As Michael Jensen stated, in a slightly different context (1978, p. 95): ". . . as our econometric sophistication increases, we are beginning to find inconsistencies that our cruder data and techniques missed in the past," and "the eventual resolution of these anomalies will result in more precise and more general theories of market efficiency and equilibrium models of the determination of asset prices under uncertainty" (Jensen 1978, p.96). The theory also relies on the assumption of a constant risk premium. If the risk premium was not constant--perhaps it followed some (low order) stochastic process or was a function of other variables--then the model was misspecified. In addition, Harris and Purvis (1978) construct a model in which a distinction is drawn between permanent and transitory shocks. Consider, for example, a permanent shock such as the oil crisis. People initially interpreted this as a temporary shock, and only over time was it perceived to be a permanent shock. It is only over time, while people accumulated new information, that the exchange rate (gradually) attained its new level. Consequently, looking at a time series we observe serial correlation. However, this serial correlation simply reflects people eliminating their initial confusion of whether the oil shock was permanent or transitory; it does not represent irrational behavior.

Finally, much of the theory is taken directly from the theory of efficient markets in finance (see Fama 1969). There are a number of differences that need to be considered. The foreign exchange market is much less regulated than the U.S. stock market. However, there is much more direct government intervention in terms of manipulating exchange

rates than in the U.S. stock market. Finally, there is a stronger presumption that the equilibrium stock price is constant than that the equilibrium exchange rate is constant.

Data Appendix

Five exchange rates (with respect to the dollar) will be examined. These currencies are (1) the Dutch guilder (the Netherlands), (2) the German mark, (3) the Canadian dollar, (4) the Swiss franc and (5) the U.K. pound. The period to be analyzed is April 24, 1973 to May 5, 1977. The ending date was arbitrary, while the starting date reflected the fact that there appeared to be a structural change in the international monetary system in March 1973, when the EEC agreement to stabilize the dollar value of their currencies within a 2.25 percent band was abandoned (see Frenkel 1978, Frenkel-Levich 1977, and Levich 1977 for evidence on this observation). Weekly observations were obtained (from the International Monetary Market Yearbook) on bid spot and one month forward rates on the New York foreign exchange market. Forward rates are (generally) observed on Tuesdays, while spot rates are observed on Thursdays. The reason for this staggering of observations is that one month is equal to (approximately) four weeks and two days. See Hakkio (1979) for a more detailed description of the data and some summary statistics for the whole period and its two subperiods.

Appendix A

This appendix attempts to provide an interpretation of the  $\theta$ -vector and  $\varepsilon_t$ . Recall that  $s_t = \ln S_t - \ln S_{t-1}$ ,  $f_t = \ln F_t - \ln F_{t-1}$ , and  $\ln S_{t+4} = a + b \ln F_t + u_{t+4}$ , where  $u_{t+4} = \theta(L) \varepsilon_{t+4}$ .<sup>1</sup>

$$\begin{aligned} E_t \ln S_{t+4} &= a + b \ln F_t \\ E_t \ln S_{t+3} &= a + b \ln F_{t-1} + \theta_3 \varepsilon_t \\ E_t s_{t+4} &= b f_t - \theta_3 \varepsilon_t \end{aligned} \tag{A1}$$

Therefore, market efficiency implies

$$E_{t-1} s_{t+4} = b E_{t-1} f_t \tag{A2}$$

Assume that  $\{s_t, f_t\}$  is a bivariate, linearly indeterministic, covariance stationary stochastic process. Therefore, using the Wold decomposition theorem, one can write  $\{s_t, f_t\}$  as an infinite order moving average process:

$$\begin{aligned} s_t &= \alpha(L)w_t + \beta(L)v_t \\ f_t &= \gamma(L)w_t + \delta(L)v_t \end{aligned} \tag{A3}$$

where  $\alpha(L), \beta(L), \gamma(L)$  and  $\delta(L)$  are one-sided polynomials in the lag operator  $L$ ,

$$\begin{aligned} w_t &= s_t - E(s_t : s_{t-1}, s_{t-2}, \dots, f_{t-1}, f_{t-2}, \dots) \\ v_t &= f_t - E(f_t : s_{t-1}, s_{t-2}, \dots, f_{t-1}, f_{t-2}, \dots) \end{aligned}$$

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<sup>1</sup>What is required is a transformation to induce stationarity. This is one such transformation, an alternative would be to consider  $\ln S_{t+4} - \ln S_t$  and  $\ln F_t - \ln S_t$ . However, such a transformation means one cannot estimate  $b$ .

$$E w_t w_{t-k} = \begin{cases} \sigma_w^2 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

$$E v_t v_{t-k} = \begin{cases} \sigma_v^2 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

$$E w_t v_{t-k} = \begin{cases} \sigma_{wv} & k = 0 \\ 0 & k \neq 0 \end{cases}$$

$$\alpha(0) = \delta(0) = 1 \text{ and } \beta(0) = \gamma(0) = 0.$$

Using the Wiener-Kolmogorov prediction formulas, one obtains

$$E_{t-1} s_{t+4} = \left[ \frac{\alpha(L)}{L^5} \right]_+ w_{t-1} + \left[ \frac{\beta(L)}{L^5} \right]_+ v_{t-1} \tag{A4}$$

Using equation (A3), one can calculate  $bE_{t-1} f_t$ :

$$bE_{t-1} f_t = b \left[ \frac{\gamma(L)}{L} \right]_+ w_{t-1} + b \left[ \frac{\delta(L)}{L} \right]_+ v_{t-1} \tag{A5}$$

Therefore, market efficiency - equation (A2) - implies the following set of cross equation restrictions:

$$\left[ \frac{\alpha(L)}{L^5} \right]_+ = b \left[ \frac{\gamma(L)}{L} \right]_+$$

$$\left[ \frac{\beta(L)}{L^5} \right]_+ = b \left[ \frac{\delta(L)}{L} \right]_+ \tag{A6}$$

One would next like to use equations (A3) and (A6) and equation (1') of the text to express the  $\theta$ 's as a function of the  $\alpha$ 's,  $\beta$ 's,  $\gamma$ 's,  $\delta$ 's,  $\sigma_w^2$ ,  $\sigma_v^2$  and  $\sigma_{wv}$  and to express  $\varepsilon_t$  as a function of these same variables plus the history of  $w_t$  and  $v_t$ . However, it seems that this is not possible. Define  $z_{t+4}$  to be the forecast error:  $z_{t+4} = s_{t+4} - E_{t-1} s_{t+4}$ . It is easy to show that  $z_{t+4}$  can be written, using (A6), as

$$\begin{aligned} z_{t+4} = s_{t+4} - E_{t-1} s_{t+4} &= w_{t+4} + (\alpha_1 w_{t+3} + \beta_1 v_{t+3}) + (\alpha_2 w_{t+2} + \beta_2 v_{t+2}) \\ &\quad + (\alpha_3 w_{t+1} + \beta_3 v_{t+1}) + (\alpha_4 w_t + \beta_4 v_t) \end{aligned} \quad (A7)$$

Therefore,  $z_{t+4}$  can be expressed as an MA(4) process, by virtue of it being a 5-step ahead predictor. Using (1'),  $z_{t+4}$  can also be written as

$$\begin{aligned} z_{t+4} = s_{t+4} - E_{t-1} s_{t+4} &= (\ln S_{t+4} - E_{t-1} \ln S_{t+4}) - (\ln S_{t+3} - E_{t-1} \ln S_{t+3}) \\ &= u_{t+4} - u_{t+3} + b(f_t - E_{t-1} f_t) \\ &= \varepsilon_{t+4} + (\theta_1 - 1)\varepsilon_{t+3} + (\theta_2 - \theta_1)\varepsilon_{t+2} + (\theta_3 - \theta_2)\varepsilon_{t+1} \\ &\quad + \theta_3 \varepsilon_t + b(f_t - E_{t-1} f_t) \end{aligned} \quad (A8)$$

Equation (1') provides no expression for  $(f_t - E_{t-1} f_t)$ . Using (A3), equation (A8) can be rewritten as

$$z_{t+4} = \varepsilon_{t+4} + \phi_1 \varepsilon_{t+3} + \phi_2 \varepsilon_{t+2} + \phi_3 \varepsilon_{t+1} + \phi_4 \varepsilon_t + b v_t \quad (A8')$$

It is reasonable to expect that  $\varepsilon_t$  and  $v_t$  are at least contemporaneously correlated. To determine  $\phi_1, \phi_2, \phi_3, \phi_4$  (and hence  $\theta_1, \theta_2, \theta_3$ ), and  $\sigma_\varepsilon^2$  one would normally next calculate the autocovariogram of  $z_{t+4}$  based on equation (A7) and equation (A8'); however,  $v_t$  appears in both (A7) and (A8'). Proceeding with

this procedure, one obtains  $c(\tau) = E z_t z_{t-\tau}$ , based on (A7):

$$\begin{aligned}
 c(0) &= (1 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2) \sigma_w^2 + (\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_4^2) \sigma_v^2 \\
 &\quad + 2(\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3 + \alpha_4\beta_4) \sigma_{wv} \\
 c(1) &= (\alpha_1 + \alpha_2\alpha_1 + \alpha_3\alpha_2 + \alpha_4\alpha_3) \sigma_w^2 + (\beta_2\beta_1 + \beta_3\beta_2 + \beta_4\beta_3) \sigma_v^2 \\
 &\quad + (\alpha_2\beta_1 + \alpha_3\beta_2 + \alpha_4\beta_3 + \beta_1 + \beta_2\alpha_1 + \beta_3\alpha_2 + \beta_4\alpha_3) \sigma_{wv} \\
 c(2) &= (\alpha_2 + \alpha_3\alpha_1 + \alpha_4\alpha_2) \sigma_w^2 + (\beta_3\beta_1 + \beta_4\beta_2) \sigma_v^2 \\
 &\quad + (\alpha_3\beta_1 + \alpha_4\beta_2 + \beta_2 + \beta_3\alpha_1 + \beta_4\alpha_2) \sigma_{wv} \\
 c(3) &= (\alpha_3 + \alpha_4\alpha_1) \sigma_w^2 + \beta_4\beta_1 \sigma_v^2 + (\alpha_4\beta_1 + \beta_3 + \beta_4\alpha_1) \sigma_{wv} \\
 c(4) &= \alpha_4 \sigma_w^2 + \beta_4 \sigma_{wv}
 \end{aligned} \tag{A9}$$

For equation (A8'), one obtains (assuming  $v_t$  and  $\varepsilon_t$  are only contemporaneously correlated):

$$\begin{aligned}
 c(0) &= (1 + \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) \sigma_\varepsilon^2 + b^2 \sigma_v^2 + \phi_4 b \sigma_{\varepsilon v} \\
 c(1) &= (\phi_1 + \phi_1\phi_2 + \phi_2\phi_3 + \phi_3\phi_4) \sigma_\varepsilon^2 + \phi_3 b \sigma_{\varepsilon v} \\
 c(2) &= (\phi_2 + \phi_1\phi_3 + \phi_2\phi_4) \sigma_\varepsilon^2 + \phi_2 b \sigma_{\varepsilon v} \\
 c(3) &= (\phi_3 + \phi_1\phi_1) \sigma_\varepsilon^2 + \phi_1 b \sigma_{\varepsilon v} \\
 c(4) &= \phi_4 \sigma_\varepsilon^2 + b \sigma_{\varepsilon v}
 \end{aligned} \tag{A10}$$

Equating equations (A9) and (A10) yields five nonlinear equations in the unknown parameters  $P^u = \{\theta_1, \theta_2, \theta_3, \sigma_\varepsilon^2, \sigma_{\varepsilon v}, b\}$  (the set  $\{\phi_1, \phi_2, \phi_3, \phi_4\}$  is a

transformation of the three parameters  $\theta_1, \theta_2, \theta_3$ ); the known parameters are  $P^k = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4, \sigma_w^2, \sigma_v^2, \sigma_{wv}\}$ . Therefore, if a solution exists, and if it is unique, it can be written as

$$P^u = F(P^k) \tag{A11}$$

Since  $z_{t+4}$  is an MA(4) process, one can write  $z_{t+4} = \Psi(L)\eta_{t+4}$ , where  $\Psi(L)$  is a fourth degree polynomial in L. Assuming  $\Psi(L)$  is invertible, and equating (A7) to  $\Psi(L)\eta_{t+4}$ , one obtains

$$\eta_{t+4} = \frac{\alpha'(L)}{\Psi(L)} w_{t+4} + \frac{\beta'(L)}{\Psi(L)} v_{t+4} \tag{A12}$$

where the definitions of  $\alpha'(L)$  and  $\beta'(L)$  are obvious from (A7). Therefore,  $\eta_{t+4}$  is an infinite order distributed lag of  $w_{t+4}$  and  $v_{t+4}$ . One would have preferred being able to write  $z_t$  as a function solely of the  $\varepsilon_t$  in equation (A10). In that case, one could then write  $\varepsilon_{t+4}$  as an infinite order distributed lag of  $w_{t+4}$  and  $v_{t+4}$ , which would provide the desired interpretation of  $\varepsilon_{t+4}$ . However, in light of equation (A8), this does not seem to be possible.



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