NBER WORKING PAPER SERIES

MISMATCH

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Working Paper 11888
http://www.nber.org/papers/w11888

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 2005

I am grateful for comments by Steven Davis, Xavier Gabaix, Robert Hall, John Haltiwanger, Samuel Kortum, Robert Lucas, Derek Neal, Christopher Pissarides, Michael Pries, Ivan Werning, numerous seminar participants, and especially Ricardo Lagos on previous versions of this paper. I also thank C. Aspen Gorry for research assistance. This research is supported by grants from the National Science Foundation and the Sloan Foundation. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 11888
December 2005, Revised September 2006
JEL No. E24, J63, J64

ABSTRACT

This paper develops a dynamic model of mismatch. Workers and jobs are randomly assigned to labor markets. Each labor market clears at each instant but some labor markets have more workers than jobs, hence unemployment, and some have more jobs than workers, hence vacancies. As workers and jobs move between labor markets, some unemployed workers find vacant jobs and some employed workers lose or leave their job and become unemployed. The model is quantitatively consistent with the comovement of unemployment, job vacancies, and the rate at which unemployed workers find jobs over the business cycle. It can also address a variety of labor market phenomena, including duration dependence in the job finding probability and employer-to-employer transitions, and it helps explain the cyclical volatility of vacancies and unemployment.

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1 Introduction

Why do unemployed workers and job vacancies coexist? What determines the rate at which unemployed workers find jobs? This paper advances the proposition that at any point in time, the skills and geographical location of unemployed workers are poorly matched with the skill requirements and location of job openings. The rate at which unemployed workers find jobs depends on the rate at which they retrain or move to locations with available jobs, the rate at which jobs open in locations with available workers, and the rate at which employed workers vacate jobs in locations with suitable unemployed workers.

My main finding is that such a model of mismatch is quantitatively consistent with two robust features of labor markets: the negative correlation between unemployment and vacancies at business cycle frequencies (the *Beveridge curve*) and the positive correlation between the rate at which unemployed workers find jobs and the vacancy-unemployment (v-u) ratio (the *reduced-form matching function*). The model-generated Beveridge curve has a slope of approximately -1, quantitatively consistent with evidence from the United States. The model predicts that a ten percent increase in the v-u ratio should be associated with a two percent increase in the job finding rate. In particular, the elasticity of the model-generated reduced-form matching function is virtually constant. Empirically the elasticity is constant but closer to 0.3. I also use the model to explore employment-to-unemployment and job-to-job transitions and duration dependence in the job finding rate.

The view of unemployment and vacancies that I advance in this paper is conceptually distinct from the one that search theory has advocated since the pioneering work of McCall (1970), Mortensen (1970), and Lucas and Prescott (1974). According to search theory, unemployed workers have left their old job and are actively searching for a new employer. In contrast, this paper emphasizes that unemployed workers are attached to an occupation and a geographic location in which jobs are currently scarce. Mismatch is a theory of former steel workers remaining near a closed plant in the hope that it reopens. Search, particularly as articulated in Lucas and Prescott (1974),[1] is a theory of former steel workers moving to a new city to look for positions as nurses. These two theories are complementary and it is a priori reasonable to think that mismatch may be as important as search in understanding equilibrium unemployment.

Indeed, the mismatch view of unemployment and vacancies is not new.[2] Tobin (1972, p. 9) advances a theory of a "stochastic macro-equilibrium" in which "excess supplies in labor

[1] A potential drawback to Lucas and Prescott (1974) is that they do not have a notion of job vacancies; however, Rocheteau and Wright (2005) have introduced vacancies into a monetary search model based on the Lucas-Prescott framework.

[2] Padoa-Schioppa (1991) argues that there are four distinct meanings to the term mismatch. The notion of mismatch in this paper is closest to the second approach that he discusses.

markets take the form of unemployment, and excess demands the form of unfilled vacancies. At any moment, markets vary widely in excess demand or supply, and the economy as a whole shows both vacancies and unemployment."[3] Drèze and Bean (1990) discuss important subsequent developments, including conditions on the joint distribution of workers and jobs across labor markets which ensure that the aggregation of many small markets yields a constant elasticity of substitution Beveridge curve. But both of these papers link mismatch with disequilibrium, where the wage does not clear each labor market. This paper shows that a mismatch model is quantitatively consistent with macro-labor facts even in an environment where the welfare theorems hold. Section 2 discusses other related papers.

Section 3 develops a dynamic stochastic model of mismatch. There are many local labor markets, each of which represents a particular geographic location and a particular occupation. The wage clears each market at each instant, but there may be unemployed workers in one market and job vacancies in another. Workers and jobs randomly enter and exit markets, causing unemployed workers to find jobs and employed workers to lose jobs, sometimes moving directly to another job. There is one key economic decision, firms' option to create more jobs. I prove that the equilibrium is unique and maximizes the present discounted value of output net of job creation costs given the constraints imposed by market segmentation.

Section 4 considers the impact of aggregate productivity shocks on unemployment and vacancies. An increase in aggregate productivity induces firms to create more jobs, which raises the vacancy rate and reduces the unemployment rate, moving the economy along a downward-sloping Beveridge curve. I compare the theoretical relationship with evidence from the Job Openings and Labor Turnover Survey (JOLTS) and show that the theoretical and empirical Beveridge curves are nearly indistinguishable. Moreover, fluctuations in many other variables, including the turnover rate of jobs, induce movements along a downward sloping Beveridge curve in the mismatch model. In contrast, in Pissarides's (1985) matching model, fluctuations in the turnover rate induce a counterfactual positive co-movement of unemployment and vacancies (Abraham and Katz, 1986; Shimer, 2005a).

Section 5 performs comparative statics with respect to aggregate productivity. I find that the v-u ratio responds more than 4 times as much to productivity shocks in the mismatch model as in the matching model. Shimer (2005a) argues that the matching model only explains about ten percent of the volatility in vacancies and unemployment, so this helps to reconcile the theory and the data. I also examine the source of this additional volatility.

I then turn to the comparative static relationship between the rate at which unemployed workers find jobs and the v-u ratio. Not only is the reduced-form matching function in-

[3] Tobin (1972) cites a number of previous authors in developing these ideas including Lipsey (1960) and Holt (1970). Hansen (1970) proposes a similar model of mismatch.

creasing in the model, it is nearly indistinguishable from a Cobb-Douglas. An increase in productivity that raises the v-u ratio by 10 percent raises the job finding rate by about 2 percent. This is roughly consistent with U.S. data, where it is impossible to reject the hypothesis of a constant elasticity, although the elasticity is closer to 0.3. This last fact is usually interpreted by search theorists as evidence in favor of a Cobb-Douglas matching function (Petrongolo and Pissarides, 2001); this paper provides the first structural explanation for why the matching function appears to be Cobb-Douglas.

The comparative statics also show that higher productivity is associated with a lower separation rate into unemployment and a higher job-to-job transition rate, even though the total separation rate is acyclic. Conditional on an employment relationship ending, a worker is more likely to be able to switch employers immediately when jobs are more plentiful.

Section 6 calibrates the model parameters to match some steady state facts from the U.S. and then simulates the impact of aggregate productivity shocks. The simulations confirm the comparative statics. The mismatch model explains more than a quarter of the volatility in the job finding rate, more than a third of the volatility in the v-u ratio, and almost half the volatility in the separation rate in response to small productivity shocks. It is consistent with evidence on the Beveridge curve and reduced-form matching function

A careful examination of the job finding rate requires me to account for heterogeneity in the exit rate from unemployment, which I do in Section 7. The long-term unemployed are typically located in labor markets where jobs are particularly scarce, which makes their prospects for exiting unemployment unusually bleak. This dynamic sorting explains much of the empirical duration dependence in the job finding rate. The remainder presumably reflects unmodeled worker heterogeneity. I also find that accounting for duration dependence lowers the measured level of the job finding rate and slightly lowers the elasticity of the reduced-form matching function.

Section 8 takes a step towards relaxing the paper's strongest assumption, that all workers and jobs are equally likely to move. I introduce a parameter δ and assume that a worker never enters a labor market with more than δ excess workers and never exits one with more than δ excess jobs. Similarly firms never create jobs in a labor market with more than δ excess jobs and never destroy jobs in a market with more than δ excess workers. I find that my characterization of the Beveridge curve and the reduced-form matching function are qualitatively robust to any positive value of δ, although the quantitative fit of the model is slightly better when δ is large.

I conclude in Section 9.

2 Related Literature

2.1 Mismatch Models

A number of previous authors have developed formal models of mismatch as a source of unemployment. Many use an urn-ball structure, where workers (balls) are randomly assigned to jobs (urns); see Butters (1977) and Hall (1977) for early examples. The random assignment ensures that some jobs are unfilled, yielding vacancies, and some jobs are assigned multiple workers, only one of whom can be hired, yielding unemployment. Hall (2000) supposes that workers are randomly assigned to locations and then matched in pairs. One worker is necessarily unemployed in any location with an odd number of workers, linking the importance of matching (the number of workers per location) and the unemployment rate. Den Haan, Ramey, and Watson (2000) offer an alternative model of matching frictions based on workers and firms searching in different "channels;" however, they simply assume that the number of channels is a constant elasticity function of unemployment and vacancies.

Stock-flow matching models offer another sensible theory of mismatch (Taylor, 1995; Coles and Muthoo, 1998; Coles and Smith, 1998; Coles and Petrongolo, 2003). According to these models, only a small proportion of worker-job matches are feasible. When a worker loses her job, she looks among the available stock of vacancies to see if her skills are suitable for any of them. If so, she is immediately paired with a suitable vacancy, while otherwise she remains unemployed. Symmetrically, entering job vacancies search for a match within the stock of unemployed workers.

Perhaps the most similar models of mismatch are Lagos's (2000) model of the taxicab market and Sattinger's (2005) model of queuing. According to Lagos (2000), there are a fixed set of locations and two types of economic agents, drivers and passengers. The short side of the market is served within each location and drivers optimally relocate to the best possible location. Nevertheless, Lagos finds that empty taxis and unserved riders can coexist in equilibrium if prices are fixed exogenously, yielding an aggregate Beveridge curve. Sattinger (2005) assumes workers are randomly assigned to job queues and wait to be "served." A worker on a long queue experiences a longer unemployment spell. He shows that a combination of queuing and search is consistent with a downward sloping Beveridge curve. To generate mismatch, one must take one of the approaches adopted in these papers, either prices that do not clear markets or limited mobility of workers and jobs.

There are many small differences between these earlier approaches to mismatch and the model that I propose in this paper. For example, by making the notion of a labor market explicit, it is sensible to think about wages being determined by competition for labor within markets. The literature on urn-ball and stock-flow matching models has typically assumed that wages are either posted by firms as a recruiting device or bargained ex post by workers and firms. But the most important difference between this paper and the urn-ball and stock-flow literatures is one of emphasis. No previous paper has shown that a mismatch model is quantitatively consistent with the empirical comovement of unemployment, vacancies, and the job finding rate. Instead, the literature has focused on the theoretical shortcomings of the reduced-form matching function approach by arguing that mismatch models do not deliver a structural matching function. Indeed, this seems to be merely a matter of emphasis. In some preliminary work, I have found that the quantitative behavior of the model in this paper almost indistinguishable from a stock-flow matching model (Shimer, 2006).

2.2 Search and Matching Models

The issues this paper examines have traditionally been the realm of search models, especially Pissarides's (1985) matching model and its variants. Under appropriate restrictions on the reduced-form matching function and on the nature of shocks, the matching model is quantitatively capable of describing the Beveridge curve (Abraham and Katz, 1986; Blanchard and Diamond, 1989) and the relationship between the v-u ratio and the rate at which unemployed workers find jobs (Pissarides, 1986; Blanchard and Diamond, 1989).

Despite these successes, the matching model has two significant shortcomings. The first is the matching function itself. It is intended to represent "heterogeneities, frictions, and information imperfections" and to capture "the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicit" (Pissarides, 2000, pp. 3–4). But Lagos (2000) emphasizes that if the matching function is a reduced-form relationship, one should be concerned about whether it is invariant to policy changes. Addressing this issue requires an explicit model of heterogeneity that gives rise to an empirically successful reduced-form matching function.

The second is wage determination. In the matching model, workers and firms are typically in a bilateral monopoly situation, and so competitive theories of wage determination are inapplicable. Wages are instead set via bargaining. Some recent research has emphasized that the details of the bargaining protocol are quantitatively critical to the ability of the model to replicate business cycle fluctuations in unemployment and vacancies (Shimer, 2005a; Hall, 2005; Hall and Milgrom, 2005). The model I develop in this paper circumvents both of these issues. There is no matching function and wages are set competitively.

3 A Model of Mismatch

3.1 Economic Agents

There are a M workers and a large number of firms. All agents are risk-neutral, infinitely-lived, and discount future income at rate r. Time is continuous.

3.2 Stocks

I start by looking at the state of the economy at any moment in time t. Section 3.3 describes the flow of workers and jobs and shows that this is consistent with the stocks described here. At any point in time, each worker is assigned to one of L labor markets. These assignments are independent across workers, so the distribution of workers across labor markets is a multinomial random variable. Each firm may have zero, one, or more jobs. Let $\mathsf{N}(t)$ denote the total number of jobs; later this will be determined endogenously. Each job is assigned to one labor market. Again, these assignments are independent across jobs and independent of the number of workers assigned to the labor market. Thus the distribution of jobs across labor markets is an independent multinomial random variable.

Let $M \equiv \mathsf{M}/\mathsf{L}$ and $N(t) \equiv \mathsf{N}(t)/\mathsf{L}$. In the remainder of this paper, I focus on the limit as $\mathsf{L} \to \infty$ with $M > 0$ an exogenous parameter and $N(t) \geq 0$ an endogenous variable. In this limit, the number of workers and jobs in a labor market are independent Poisson random variables. In a standard abuse of the law of large numbers, I assume that the fraction of labor markets with i workers and j jobs is deterministic, hence equal to

$$\pi(i, j; N(t)) \equiv \frac{e^{-(M+N(t))} M^i N(t)^j}{i!j!} \tag{1}$$

if $(i, j) \in \{0, 1, 2, \ldots\}^2$ and $\pi(i, j; N(t)) = 0$ otherwise. To conserve on notation, I suppress the dependence of π on the parameter M.

The cross-sectional distribution of workers and jobs is critical for what follows. It will prove useful to describe how changes in M and N affect this probability:

Lemma 1 $\frac{\partial \pi(i, j; N)}{\partial M} = \pi(i-1, j; N) - \pi(i, j; N)$ *and* $\frac{\partial \pi(i, j; N)}{\partial N} = \pi(i, j-1; N) - \pi(i, j; N)$

Proof. The results follow directly from differentiating π in equation (1). ■

Workers and jobs must match in pairs in order to produce market output. One worker and one job in the same labor market can jointly produce $p(t)$ units of the numeraire homogeneous consumption good. A single worker (an unemployed worker) produces $z < p(t)$ units of the same good at home, while a single job (a vacancy) produces nothing. Workers and jobs are indivisible. These stark assumptions give a concrete notion of unemployment and vacancies.

There is perfect competition within each labor market so unemployed workers and vacant jobs cannot coexist in the same market. Let i denote the number of workers in some labor market and j denote the number of jobs. If $i > j$, $i - j$ workers are unemployed but all workers are indifferent about being unemployed; the wage is driven down to the value of home production, z. If $i < j$, $j - i$ jobs are vacant but all firms are indifferent about their jobs being vacant; the wage is driven up to the marginal product of labor, $p(t)$. If $i = j$, there is neither unemployment nor vacancies in the market and the wage is not determined. I assume that if $i = j$, the wage is equal to workers' reservation wage, z. The quantitative results are scarcely affected if I instead assume the wage is $p(t)$ when $i = j$.

The number of unemployed workers per labor market is equal to the difference between the number of workers i and the number of jobs j, summed across labor markets with more workers than jobs, and similarly for the number of vacancies per labor market:

$$U(N) = \sum_{i=1}^{\infty} \sum_{j=0}^{i} (i - j)\pi(i, j; N) \quad \text{and} \quad V(N) = \sum_{j=1}^{\infty} \sum_{i=0}^{j} (j - i)\pi(i, j; N). \tag{2}$$

The v-u ratio is $V(N)/U(N)$ and the unemployment and vacancy rates are

$$u(N) \equiv U(N)/M \quad \text{and } v(N) \equiv V(N)/N. \tag{3}$$

It is also useful to define the share of markets with unemployed workers,

$$S(N) = \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} \pi(i, j; N). \tag{4}$$

I again suppress these variables' dependence on the parameter M.

Perfect competition within labor markets is a stark assumption and implies that wages only taking on two possible values at any point in time. However, the movement of workers and jobs across markets, which I discuss next, ensures that the expected present value of wages differs continuously across markets depending on the current value of i and j. If workers and firms can commit to long-term contracts, wage payments may be much smoother than is suggested by this spot-market model of wages.

3.3 Flows

Each worker's human capital is shocked according to a Poisson process with arrival rate q. The arrival of this shock is exogenous, independent of the worker's current employment status or wage. When the "quit" shock hits, the worker must leave her labor market and move to a random new one, independent of conditions in the new labor market. This means that the arrival rate of workers into a labor market is qM. This random inflow and outflow of workers implies a Poisson distribution of workers across labor market at each instant.

Symmetrically, each job is destroyed according to a Poisson process with arrival rate l. When this "layoff" shock occurs, the job leaves the labor market and disappears. Conversely, a firm may create a new job by paying a fixed cost $k > 0$. When it does so, the job is randomly assigned to a labor market. Again, both the entry and exit of jobs is independent of conditions in the local labor market, although the decision to create a job depends on aggregate labor market conditions.

I assume a pair remains matched until either a quit or layoff hits the match, at rate $q + l$, consistent with a small unmodeled turnover cost.

3.4 Aggregate Shock

I focus on a single type of aggregate shock, fluctuations in aggregate productivity $p(t)$, but indicate throughout the paper where the results extend to fluctuations in other parameters. Assume $p(t) = p_{y(t)} = e^{y(t)} + (1 - e^{y(t)})(z + (r + l)k)$, where $y(t)$ is a jump variable lying on a discrete grid:

$$y \in Y \equiv \{-\nu\Delta, -(\nu - 1)\Delta, \dots, 0, \dots, (\nu - 1)\Delta, \nu\Delta\}.$$

$\Delta > 0$ is the step size and $2\nu + 1 \geq 3$ is the number of grid points. A shock hits y according to a Poisson process with arrival rate λ. The new value y' is either one grid point above or below y:

$$y' = \begin{cases} y + \Delta \\ y - \Delta \end{cases} \text{with probability} \begin{cases} \frac{1}{2}\left(1 - \frac{y}{\nu\Delta}\right) \\ \frac{1}{2}\left(1 + \frac{y}{\nu\Delta}\right) \end{cases}.$$

Note that although the step size is constant, the probability that $y' = y + \Delta$ is smaller when y is larger, falling from 1 at $y = -\nu\Delta$ to 0 at $y = \nu\Delta$. Shimer (2005a) shows that one can represent the stochastic process for y as

$$dy = -\gamma y dt + \sigma dx,$$

where $\gamma \equiv \lambda/\nu$ measures the speed of mean reversion and $\sigma \equiv \sqrt{\lambda}\Delta$ is the instantaneous standard deviation. This is similar to an Ornstein-Uhlenbeck process, except that the innovations in y are not Gaussian, since y is constrained to lie on a discrete grid.[4]

[4]Suppose one changes the three parameters of the stochastic process, the step size, arrival rate of shocks, and number of steps, from (Δ, λ, ν) to $\left(\Delta\sqrt{\varepsilon}, \frac{\lambda}{\varepsilon}, \frac{\nu}{\varepsilon}\right)$ for any $\varepsilon > 0$. This does not change either γ or σ, but

Note that by construction $p_y > z + (r + l)k$, so output exceeds the sum of the value of leisure and the "user cost of capital," the price of capital multiplied by the sum of the interest and depreciation rates. This ensures that the economy never shuts down. To save on notation, let $\mathbb{E}_p X_{p'}$ denote the expected value of an arbitrary state-contingent variable X following the next aggregate shock, conditional on the current state p.

3.5 Equilibrium

Firms create jobs whenever doing so is profitable. Let $J_p(N)$ denote the expected value of a job when productivity is p and there are N jobs in the average market. If the sample paths of N were differentiable, we could express this using a standard Hamilton-Jacobi-Bellman (HJB) equation:

$$rJ_p(N) = (p - z)S(N) - lJ_p(N) + J_p'(N)\dot{N} + \lambda(\mathbb{E}_p J_{p'}(N) - J_p(N)). \tag{5}$$

The left hand side is the flow value of a job. The current payoff is the difference between output and home production income multiplied by the probability that the job is in a market without vacancies.[5] If the job is located in a market with vacancies, either it is vacant and produces nothing or it is filled and pays a wage equal to labor productivity and so again yields no profit. The second term on the right hand side accounts for the chance the job exits. The final two terms deal with aggregate changes. The number of jobs increases at rate $\dot{N}$ and the shock can change productivity from p to p' at any time.

Free entry implies that no new jobs are created, $\dot{N} = -lN$, whenever $J_p(N)$ is smaller than the cost of creating a job k. Conversely, if ever $J_p(N)$ exceeded k, the number of jobs would jump up instantaneously until the point where $J_p(N)$ is driven down to k; for this reason, the sample paths of N are typically not continuous. The process stops because an increase in N reduces the share of markets with excess workers, $S(N)$, which in turn reduces the expected value of a job. This ensures that $J_p(N) \leq k$ for all p and N.

To be precise, the equilibrium is characterized by a sequence of targets N_p^*. If $N(t) < N_p^*$, firms instantaneously create $N_p^* - N(t)$ jobs. If $N(t) = N_p^*$, gross job creation and destruction are equal. If $N(t) > N_p^*$, no jobs are created. We can write the HJB equation (5) as

$$rJ_p(N) = \begin{cases} (p - z)S(N) - lJ_p(N) - J_p'(N)lN + \lambda(\mathbb{E}_p J_{p'}(N) - J_p(N)) & \text{if } N \geq N_p^* \\ rk & \text{if } N < N_p^* \end{cases} \tag{6}$$

as $\varepsilon \to 0$, y converges to an Ornstein-Uhlenbeck process.

[5] Knowing my job is located in a particular market, the probability there are i workers and j jobs in that market is $\pi(i, j - 1; N)$. For this reason, the relevant probability is $S(N)$, the share of markets with $j < i$.

In addition, evaluating the evaluating the HJB at $N = N_p^*$, where $\dot{N} = 0$, gives value matching and smooth pasting conditions,

$$J_p(N_p^*) = k \text{ and } J_p'(N_p^*) = 0. \tag{7}$$

This is a standard irreversible investment problem (see, for example , Pindyck, 1988), which yields the following characterization of equilibrium:

Proposition 1 *There is a unique equilibrium. In it, the targets N_p^* are increasing.*

Proof. See Appendix A. ■

The proof is constructive and so also provides a computational algorithm for N_p^*.

3.6 Social Planner's Solution

We can alternatively imagine a social planner who decides on gross job creation in order to maximize the presented discounted value of output net of job creation costs. A version of the first and second welfare theorems holds in this model:

Proposition 2 *The equilibrium maximizes the present value of net output.*

Proof. See Appendix A. ■

Intuitively, there is only one margin to get correct in this economy, the amount of entry. A job is valuable whenever it employs a worker who would otherwise be unemployed, i.e. whenever it is located in a market without vacancies. In this event, the job needs to recoup its full marginal product. Otherwise it should get nothing. Competition in the labor market ensures this happens. Note that the tie-breaking assumption that the wage is equal to z when the number of workers and jobs are equal is important for this result.

3.7 Discussion

This model is deliberately parsimonious. The only economic decision is one by firms, which must decide at each instant whether to create new jobs.[6] In particular, the movement between labor markets is exogenous and random. While the reader may be accustomed to models in which mobility is endogenous, there are advantages to the approach I adopt here.

On a theoretical level, it introduces relatively few free parameters and stresses that the main results are a consequence of limited mobility and aggregation. There is also

[6]This is also the only economic decision in Chapter 1 of Pissarides (2000) and in Shimer (2005a).

evidence that mobility at business cycle frequencies is primarily for idiosyncratic reasons. Kambourov and Manovskii (2004) show that gross occupational mobility is 10 to 15 percent per year at the one digit level while net mobility is only 1 to 3 percent. Blanchard and Katz (1992) argue that for 5 to 7 years after an adverse shock to regional employment, the impact is primarily on local unemployment rather than on net migration.

In addition, there are substantial unmodeled costs to switching occupations or moving to a new location. Kennan and Walker (2006) estimate that the cost for a 20-year-old of moving to a random state is \$274,000, in 2005 dollars. This reflects the fact that in their data set, relative wages have little impact on migration decisions. In the numerical work that follows, no worker could increase her lifetime income by more than 5 percent if she moved to a random new location. If mobility is endogenous but mobility costs, including retraining costs, the loss of human capital, etc., exceed this amount, the analysis in this paper is applicable. Finally, I evaluate the robustness of my results to alternate mobility assumptions in Section 8.

4 The Beveridge Curve

This section evaluates the ability of the model to explain the comovement of unemployment and vacancies, the Beveridge curve.

4.1 Theory

Recall from equation (3) that the unemployment and vacancy rates depend only on the exogenous number of workers per market M and the endogenous number of jobs per market N. Productivity shocks therefore affect unemployment and vacancies through their impact on the number of jobs per market. The following proposition shows how:

Proposition 3 *The unemployment rate u is increasing in the number of workers per labor market M and decreasing in the number of jobs per labor market N:*

$$\frac{\partial u}{\partial \log M} = \frac{N}{M} \sum_{i=2}^{\infty} \sum_{j=0}^{i-2} \pi(i,j) \quad \textit{and} \quad \frac{\partial u}{\partial \log N} = -\frac{N}{M} \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} \pi(i,j). \tag{8}$$

The vacancy rate v is decreasing in M and increasing in N:

$$\frac{\partial v}{\partial \log M} = -\frac{M}{N} \sum_{j=1}^{\infty} \sum_{i=0}^{j-1} \pi(i,j) \quad \textit{and} \quad \frac{\partial v}{\partial \log N} = \frac{M}{N} \sum_{j=2}^{\infty} \sum_{i=0}^{j-2} \pi(i,j). \tag{9}$$

Proof. See Appendix A. ■

This has a number of implications. First, productivity shocks cause movements along a downward-sloping v-u locus. Higher productivity raises the number of jobs per labor market N and thus reduces the unemployment rate and raises the vacancy rate.

Second, a proportional increase in both M and N reduces both the unemployment and vacancy rates.[7] Doubling M and N is equivalent to merging randomly selected pairs of labor markets. If both markets have unemployment, this merger does not affect the unemployment or vacancy rates, and similarly if both markets have vacancies. But merging a market with unemployment and a market with vacancies reduces the unemployment and vacancy rate in both. This comparative static suggests that the mismatch construction may be useful in other markets where the coexistence of unemployment and vacancies is more or less common. If matching is a more severe problem, as might be the case in marriage or housing markets, M and N should be modeled as relatively small numbers. If it is a less severe problem, as in commodity markets, M and N may be thought of as very large numbers.

Finally, Proposition 3 implies that data on the unemployment and vacancy rates pin down the number of workers and jobs per labor market:

Proposition 4 *For any $u \in (0,1)$ and $v \in (0,1)$, there is a unique $M \in (0,\infty)$ and $N \in (0,\infty)$ solving equation (3).*

Proof. See Appendix A. ■

4.2 Measurement

Since December 2000, the Bureau of Labor Statistics (BLS) has measured job vacancies using the JOLTS. This is the most reliable time series for vacancies in the U.S.. According to the BLS, "A job opening requires that 1) a specific position exists, 2) work could start within 30 days, and 3) the employer is actively recruiting from outside of the establishment to fill the position. Included are full-time, part-time, permanent, temporary, and short-term openings. Active recruiting means that the establishment is engaged in current efforts to fill the opening, such as advertising in newspapers or on the Internet, posting help-wanted signs, accepting applications, or using similar methods."[8] I measure the vacancy rate as the ratio of vacancies to vacancies plus employment.

The Bureau of Labor Statistics (BLS) uses the Current Population Survey (CPS) to measure the unemployment rate each month. The CPS measures employment and unemployment using a household questionnaire designed to determine whether an individual is working or,

[7]A proportional increase in M and N raises u by $\frac{\partial u}{\partial \log M} + \frac{\partial u}{\partial \log N} = -\frac{N}{M}\sum_{i=1}^{\infty} \pi(i, i-1)$ times the percentage change in M and N, and similarly for v.

[8]See BLS news release, July 30, 2002, available at `http://www.bls.gov/jlt/jlt_nr1.pdf`

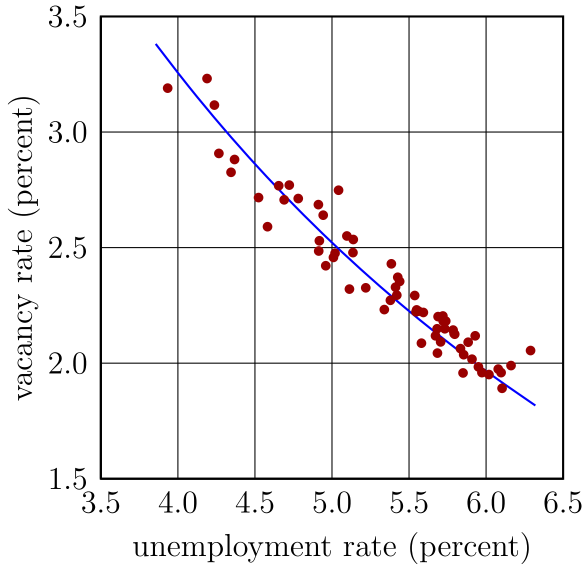

Figure 1: The brown dots show U.S. monthly data from December 2000 to April 2006. The unemployment rate is measured by the BLS from the CPS. The vacancy rate is measured by the BLS from the JOLTS. The solid blue line shows the model-generated Beveridge curve with $M = 244.2$ and $N \in [233, 243]$.

if she is not working, available for and actively seeking work. The ratio of unemployment to the sum of unemployment and employment is the unemployment rate. The brown dots in Figure 1 show the strong negative correlation between unemployment and vacancies over this time period, the empirical Beveridge curve.

From December 2000 to April 2006, the unemployment and vacancy rates averaged 5.4 percent and 2.3 percent, respectively. Using Proposition 4, these two numbers uniquely determine $M = 244.2$ and $N = 236.3$.[9] Productivity shocks affect the unemployment and vacancy rates by changing the number of jobs per market. As N varies between 233 and 243, the unemployment and vacancy rates in equation (3) trace out the blue line in Figure 1. The fit of the model to the data is remarkable.

A shortcoming of JOLTS is that it only covers one recession and subsequent expansion. Moreover, the recovery was unusual in that employment growth proceeded much slower than normal. While unfortunately no ideal measure of job vacancies exists over a longer time period, the Conference Board Help Wanted Index provides a crude one since 1951 (see

[9]To get a sense of whether these magnitudes are reasonable, observe that there are about 134 million workers in the U.S. according to the Current Employment Statistics (CES). Dividing by 244.2 gives about 550,000 labor markets. The Occupational Employment Statistics (OES) counts about 800 occupations, while there are 362 metropolitan statistical areas (regions with at least one urbanized area of 50,000 or more inhabitants) and 560 micropolitan statistical areas (regions with an urban area of 10,000 to 50,000 inhabitants). Together this gives a total of about 740,000 occupations and geographic areas. Although the sharp theoretical distinction between labor markets is less obvious in the data, this back-of-the-envelope calculation suggests that 244.2 workers per labor market is plausible.

Abraham, 1987, for a discussion of this dataset). The business cycle frequency correlation between unemployment and the Help Wanted Index is about -0.9 and the two variables are equally volatile (Abraham and Katz, 1986; Blanchard and Diamond, 1989; Shimer, 2005a). In Section 6 I show that the model is consistent with this.

The fact that the level of the model-generated Beveridge curve fits the JOLTS data reflects how I chose the number of workers per labor market M. But the fact that the slope and curvature of the model-generated Beveridge curve also fits the data comes from the structure of the model. The model cannot generate a different Beveridge curve in response to fluctuations in productivity p. Indeed, aggregate fluctuations in any parameter except M would also affect unemployment and vacancies only through the number of jobs per market N and therefore would lead to the same comovement of unemployment and vacancies. In other words, the shape of the Beveridge curve does not depend on the source of shocks.

In contrast, while the matching model is able to produce a negative correlation between unemployment and vacancies, doing so is not trivial. For example, Mortensen and Pissarides (1994) report a theoretical correlation between unemployment and vacancies of -0.26. Merz (1995) reports the correlation is -0.15 if search intensity is exogenous and 0.32 if it moves endogenously over the business cycle. Shimer (2005a) finds that shocks to aggregate productivity induce a strong negative correlation between unemployment and vacancies and a judicious choice of the matching function yields the correct slope of the Beveridge curve as well. But even then, adding realistic fluctuations in the separation rate to the model induces a positive correlation between unemployment and vacancies.

5 Comparative Statics

This section performs comparative statics in a deterministic version of the model. In the absence of aggregate shocks, $\lambda = 0$, the HJB equation (6) reduces to

$$(r + l)k = (p - z)S(\mathcal{N}_p^*). \tag{10}$$

The user cost of capital must equal the profit from having a job in a market with unemployed workers times the share of markets with unemployed workers. Throughout this section, I assume this equation holds at every point in time and examine the effect of the level of productivity on unemployment, vacancies, the job finding rate for unemployed workers, and the separation rate into unemployment for employed workers. The results are useful because they are simple but also accurately foreshadow the simulations of the stochastic model which I report in Section 6.

5.1 Volatility of the v-u Ratio

I start by examining how a permanent productivity shock affects the v-u ratio:

Proposition 5 *The responsiveness of the v-u ratio to productivity is*

$$\frac{\partial \log(V(\mathcal{N}_p^*)/U(\mathcal{N}_p^*))}{\partial \log p} = -\left(\frac{V'(\mathcal{N}_p^*)}{V(\mathcal{N}_p^*)} - \frac{U'(\mathcal{N}_p^*)}{U(\mathcal{N}_p^*)}\right)\frac{U'(\mathcal{N}_p^*)}{U''(\mathcal{N}_p^*)}\frac{p}{p-z} > 0. \tag{11}$$

Proof. See Appendix A. ■

Equation (23) implies $U'(N) = -S(N)$; since $N - V(N) = M - U(N)$, $V'(N) = 1 - S(N)$; and equation (18) provides the formula for $S'(N) = -U''(N)$. Thus we can compute equation (11) analytically. The equation shows that the responsiveness of the v-u ratio to a permanent productivity shock depends on the number of workers and jobs per labor market and on $\frac{p}{p-z}$ but not on other details of the model. Let $M = 244.2$ and $N = 236.3$, values consistent with the mean unemployment and vacancy rates in recent years. Then equation (11) implies that a one percent increase in productivity will raise the v-u ratio by $4.25\frac{p}{p-z}$. By contrast, Shimer (2005a, p. 36) argues that in a matching model with wages determined by Nash bargaining, the elasticity of the v-u ratio with respect to labor productivity is less than one-fourth as large, about $1.03\frac{p}{p-z}$, although the exact value depends on some other parameters, especially workers' bargaining power.

Why is the v-u ratio so much more responsive in the mismatch model than in a matching model? Part of the reason has to do with the nature of job creation costs. Shimer (2005a) assumes that firms must pay a flow cost to maintain a vacancy, while here the cost is sunk. This irreversibility is qualitatively important for the coexistence of unemployment and vacancies, since otherwise firms would close vacancies in labor markets without enough workers. If I make vacancy creation irreversible in a simple matching model, the elasticity of the v-u ratio with respect to labor productivity rises to $1.9\frac{p}{p-z}$.[10] This explains almost half of the difference between models.

The other half of the explanation is more intimately tied to the structure of mismatch.

[10] This is isomorphic to introducing a capital cost $(r + s)k$ into the simple matching model, which Mortensen and Nagypal (2005) argue significantly raises the responsiveness of the v-u ratio. To be precise, I introduce a sunk cost of creating a job and set the flow cost of a vacancy to zero. The elasticity of the v-u ratio with respect to net labor productivity $p - z$ is

$$\frac{r + s + \left(\frac{1-\beta}{\theta} + \beta\right)f(\theta)}{(r+s)(1-\eta(\theta)) + \beta f(\theta)},$$

where $r = 0.012$ is the quarterly interest rate, $s = 0.102$ is the separation rate, $\beta = 0.72$ is worker's bargaining power, θ is the v-u ratio, $f(\theta) = 1.35$ is workers' job finding rate, and $\eta(\theta) = 0.28$ is the elasticity of f, as in Shimer (2005a). I also set $\theta = 0.412$, consistent with a 5.4 percent unemployment rate and a 2.3 percent vacancy rate. This gives an elasticity $1.90\frac{p}{p-z}$.

For expositional simplicity, I focus in this paragraph on the case when home production income, z, is zero. Shimer (2005a) argues that in the matching model, a one percent increase in labor productivity leads to an almost one percent increase in wages in all jobs with little change in profitability and hence in job creation. In the mismatch model, a one percent increase in labor productivity raises the wage by one percent in markets with vacancies but does not affect the wage in markets with unemployment, where it is fixed at z. In addition, some markets shift from having excess workers to having excess jobs, with an associated large wage increase. The responsiveness of the number of jobs to productivity is determined by this last channel. At the benchmark values of M and N, a one percent increase in the number of jobs reduces the share of markets with vacancies by 6.5 percent. Equivalently, to offset a one percent increase in productivity in the mismatch model, we require roughly a 0.15 percent increase in the number of jobs, significantly more than in the matching model. Part of the greater volatility of the mismatch model therefore comes directly from its central feature, the distinction between markets with unemployment and markets with vacancies.

5.2 Job Finding and Separation Rates

Next I examine the determinants of the transition rates from unemployment to employment, the job finding rate, and from employment to unemployment, the separation rate. If an employed worker quits her labor market, an unemployed worker may take her old job (an unemployment-to-employment or UE transition) and she may fail to find a job in her new labor market (an employment-to-unemployment or EU transition). If an unemployed worker quits his labor market, he may find a job in his new labor market (UE transition). If a filled job leaves the labor market, its old employee may be left jobless (EU transition). But whenever a new job enters a labor market, it may hire a worker (UE transition). These events may also lead an employed worker to switch employers.

Let $\rho_q^{UE}(N)$ denote the probability that a quit leads to a UE transition. This occurs if either the quitting worker is employed in a labor market with unemployed workers or if the worker is unemployed and moves to a labor market with vacant jobs:

$$\rho_q^{UE}(N) \equiv \frac{1}{M} \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} j\pi(i,j;N) + u(N) \sum_{i=0}^{\infty} \sum_{j=i+1}^{\infty} \pi(i,j;N). \tag{12}$$

The first term is the fraction of workers who are employed in labor markets with unemployed workers. This is equal to j workers in every labor market with $i > j$. The second term is the product of the fraction of workers who are unemployed and the fraction of labor markets with vacancies, $j > i$.

Equation (26) in Appendix A implies $\sum_{i=0}^{\infty}\sum_{j=0}^{i}\pi(i,j;N)-\frac{1}{M}\sum_{i=1}^{\infty}\sum_{j=0}^{i-1}i\pi(i,j;N)=0$. Add this to the right hand side of equation (12) and simplify using equation (3):

$$\rho_q^{UE}(N)=(1-u(N))\sum_{i=0}^{\infty}\sum_{j=0}^{i}\pi(i,j;N). \tag{13}$$

This is the product of the employment rate and the fraction of labor markets without vacancies. Equivalently, the probability a quit shock leads to an unemployed worker becoming employed is the same as the probability it leads to an employed worker becoming unemployed: $\rho_q^{UE}(N)=\rho_q^{EU}(N)$. This result is sensible. Since the quit rate does not affect the unemployment rate, the probability that a quit shock leads to a UE transition must be the same as the probability that it leads to an EU transition.

I similarly let $\rho_n^{UE}(N)$ denote the probability that a job entering a labor market causes a UE transition. This occurs whenever the job enters a market with unemployed workers, so $\rho_n^{UE}(N)=S(N)$. Conversely, the probability that a job leaving a market causes an EU transition is equal to fraction of jobs in markets without excess jobs $i\geq j$:

$$\rho_l^{EU}(N)=\frac{1}{N}\sum_{j=1}^{\infty}\sum_{i=j}^{\infty}j\pi(i,j;N)=\sum_{j=0}^{\infty}\sum_{i=j+1}^{\infty}\pi(i,j;N)=\rho_n^{UE}(N). \tag{14}$$

The second equation uses the same logic as going from equation (12) to equation (13) while the third equation reorders the sum to prove that $\rho_l^{EU}(N)=\rho_n^{UE}(N)=S(N)$.

Putting these together, we get the instantaneous transition rate from unemployment to employment in steady state, i.e. the job finding rate for unemployed workers:

$$f(N)=\frac{qM\rho_q^{UE}(N)+lN\rho_n^{UE}(N)}{U(N)}. \tag{15}$$

There are qM quit shocks per labor market, each leading to a UE transition with probability $\rho_q^{UE}(N)$. Similarly, jobs enter to offset exits, at rate lN, leading to a UE transition with probability $\rho_n^{UE}(N)$. This gives the total rate at which unemployed workers find jobs in an average labor market. Dividing by the total number of unemployed workers per labor market gives the instantaneous job finding rate for unemployed workers.

We can similarly define the separation rate to unemployment as

$$s(N)=\frac{qM\rho_q^{EU}(N)+lN\rho_l^{EU}(N)}{M-U(N)}. \tag{16}$$

The fact that $s(N)(M-U(N))=f(N)U(N)$ is again consistent with steady state. Note

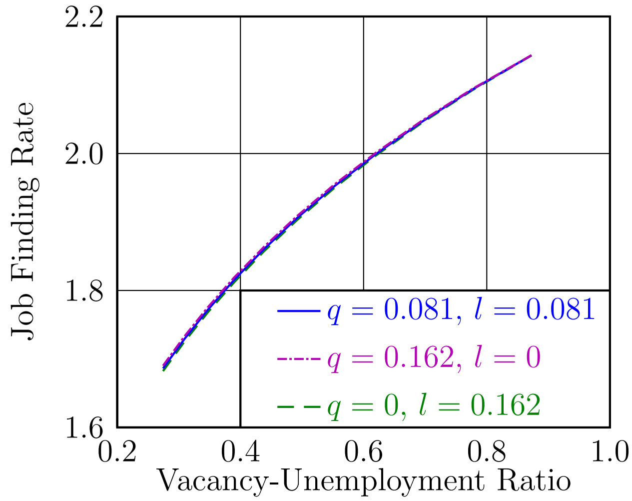

Figure 2: Comparative statics of f and V/U with respect to changes in $N \in [233, 243]$. $M = 244.2$ throughout.

that a proportional increase in q and l causes a proportional increase in the job finding and separation rates but does not affect the curvature of either function. Finally, a q or l shock that does not cause a separation must cause a job-to-job transition and so the job-to-job transition rate is $q + l - s(N)$.

I start by exploring the behavior of a "reduced-form matching function," the comovement of $f(N)$ with the v-u ratio $V(N)/U(N)$. Fix $M = 244.2$ and $q = l = 0.081$ (per quarter) and let N vary between 233 and 243; if $N = 236.3$, the unemployment and vacancy rates are 5.4 percent and 2.3 percent, respectively, and the separation rate $s(N)$ is 0.105 per quarter, consistent with average values reported in Shimer (2005a). The solid blue line in Figure 2 shows the resulting relationship between the job finding rate f and the v-u ratio as N varies. When jobs are plentiful, vacancies are high, unemployment is low, and unemployed workers are likely to move rapidly into jobs. In addition, Figure 2 shows that the relationship is insensitive to the composition of the total separation rate, $s \equiv q + l$, between quits and layoffs. The dashed lines with $q = 0.162$ and $l = 0$ or with $q = 0$ and $l = 0.162$ are distinguishable from the solid line. I conclude that the ratio of the job finding rate to $q + l$ essentially depends only on the number of workers and jobs per labor market.

A striking feature of the relationship between f and V/U in Figure 2 is that it is nearly isoelastic. Fix $q = l = 0.081$ and let N vary between 233 and 243. As the v-u ratio falls, the elasticity of the job finding rate with respect to the v-u ratio declines slightly from 0.211 to 0.202. If $q = 0$ and $l = 0.162$, the decline is even smaller, from 0.212 to 0.205.[11] Although this decrease indicates that the relationship is not exactly a Cobb-Douglas, if the model were the data generating process, it would be virtually impossible to reject the hypothesis of a

[11] As $M = N \to \infty$, the elasticity converges to $\frac{1}{2} - \frac{1}{\pi} \approx 0.18$, where $\pi \approx 3.14$, regardless of l and q.

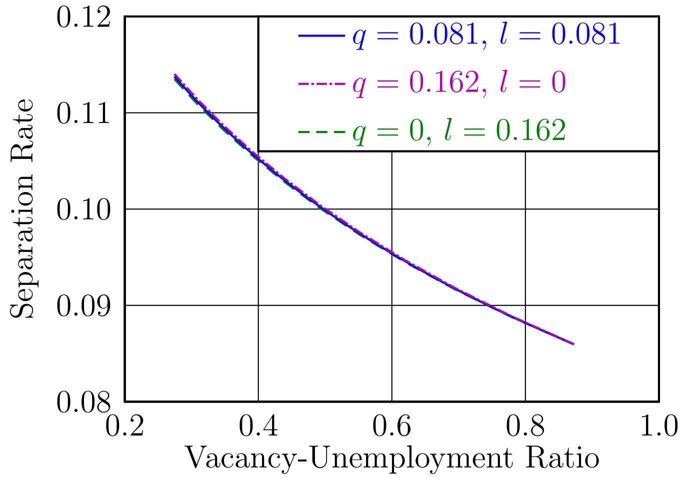

Figure 3: Comparative statics of s and V/U with respect to changes in $N \in [233, 243]$. $M = 244.2$ throughout.

Cobb-Douglas empirically.

Petrongolo and Pissarides (2001) survey a large literature that explores the behavior of empirical matching functions. Most papers find that the number of matches is a constant returns to scale function of unemployment and vacancies, or equivalently that matches per unemployed worker (the job finding rate) is a function of the v-u ratio. The literature also typically cannot reject a Cobb-Douglas relationship between these variables. On page 393, Petrongolo and Pissarides (2001) conclude that "a plausible range for the empirical elasticity" of the job finding rate with respect to the v-u ratio is 0.3 to 0.5, while this model implies a slightly lower elasticity, around 0.2. The aggregation of distinct labor markets may provide an explanation for the empirical evidence on matching functions.

The model also implies that when productivity is higher, the separation rate is lower. This is not because q or l shocks are less likely; by construction, the incidence of these shocks is constant. Instead, when productivity is higher, there are more jobs per labor market which implies that when an employed worker quits her labor market or loses her job, she is more likely to immediately find a new job. This suggests that the model will be able to explain some of the observed countercyclicality of the separation rate (Blanchard and Diamond, 1990). Conversely, the model also predicts that the job-to-job transition rate is procyclical, qualitatively consistent with evidence in Fallick and Fleischman (2004).

6 Simulations of the Stochastic Model

I now move beyond comparative statics to explore the comovement of unemployment, vacancies, the job finding rate, the separation rate, and labor productivity. Table 1 shows the

Summary Statistics, quarterly U.S. data, 1951 to 2003

		U	V	V/U	f	s	p
Standard Deviation		0.190	0.202	0.382	0.118	0.075	0.020
Quarterly Autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
Correlation Matrix	U	1	−0.894	−0.971	−0.949	0.709	−0.408
	V	—	1	0.975	0.897	−0.684	0.364
	V/U	—	—	1	0.948	−0.715	0.396
	f	—	—	—	1	−0.574	0.396
	s	—	—	—	—	1	−0.524
	p	—	—	—	—	—	1

Table 1: Seasonally adjusted unemployment u is constructed by the BLS from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index v is constructed by the Conference Board. The job finding rate f and separation rate s are constructed from seasonally adjusted employment, unemployment, and short-term unemployment, all computed by the BLS from the CPS. See Shimer (2005a) for details. u, v, f, and s are quarterly averages of monthly series. Average labor productivity p is seasonally adjusted real average output per person in the non-farm business sector, constructed by the Bureau of Labor Statistics (BLS) from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

empirical behavior of these variables in the U.S. economy from 1951 to 2003. This simply replicates Table 1 in Shimer (2005a); I refer the reader to that paper for details on the construction of the variables.

6.1 Calibration Procedure

This model is parameterized by 9 numbers: the number of workers per labor market M, the quit and layoff rates q and l, the discount rate r, the value of leisure z, the cost of creating a job k, and the three parameters of the stochastic process for productivity, the number of steps ν, the arrival rate of shocks λ, and the step size Δ. I choose these parameters to match certain facts and then explore the model's behavior along other dimensions.

I fix $M = 244.2$ to match the location of the Beveridge curve in Figure 1 and set $q = l = 0.081$ to match the average quarterly separation rate in a deterministic steady state with $N = 236.3$. The comparative statics suggest that the decomposition of $q + l$ is unimportant for the results and unreported results confirm this.[12] I set the quarterly interest rate to $r = 0.012$. I let $\nu = 1000$, $\lambda = 86.6$, and $\Delta = 0.00580276$. This implies a mean reversion parameter of $\gamma = 0.0866$ and a standard deviation of $\sigma = 0.054$ for the latent variable

[12]This irrelevance result breaks down if l is extremely close to zero. In that case the number of jobs decreases only very slowly, exacerbating the irreversibility of investment.

y. I choose these values to match the standard deviation and autocorrelation of detrended productivity, the first two numbers in the last column of Table 1. If I change ν, λ, and Δ without altering γ and σ, the results are again scarcely affected. I set the value of home production at $z = 0.4$ for comparability with Shimer (2005a); like in a matching model, this parameter is critical for the volatility of all the variables (Hagedorn and Manovskii, 2005).[13] Finally, I fix $k = 4.07848$ so that in the deterministic steady state with $p = 1$, there are indeed 236.3 jobs per labor market.

To characterize the equilibrium, I first compute the targets N_p^* for each of the $2\nu + 1$ states following the procedure in the proof of Proposition 1. I then choose an initial value for $p(0)$ and $N(0)$ and select the timing of the first shock t, an exponentially-distributed random variable with mean $1/\lambda$. I compute the number of unemployed workers who finds jobs and the number of employed workers who lose jobs during the interval $[0, t]$. These are slightly more complicated than in steady state because if $N(0) > N^*_{p(0)}$, there are time intervals when no new jobs are created. Similarly, if $N(0) < N^*_{p(0)}$, $N^*_{p(0)} - N(0)$ jobs immediately enter and $U(N(0)) - U(N^*_{p(0)})$ workers find work. I next compute the number of jobs at time t: if $N(0) \leq e^{lt} N^*_{p(0)}$, $N(t) = N^*_{p(0)}$; otherwise, $N(t) = e^{-lt} N(0)$ as the number of jobs decays with exits. Finally, I choose the next value of $p(t)$ as described in Section 3.4 and repeat.

At the end of each month (1/3 of a period), I record unemployment, vacancies, cumulative matches and separations, and productivity. I measure the job finding rate f for unemployed workers as the ratio of the number of matches during a month to the number of unemployed workers at the start of the month. I similarly measure the separation rate s as the number of separations divided by the number of employed workers at the start of the month. I throw away the first 25,000 years of data to remove the effect of initial conditions. Every subsequent 53 years of model-generated data gives one sample. I take quarterly averages of monthly data and express all variables as log deviation from an HP filter with parameter 10^5, the same low frequency filter that I use on U.S. data. I create 20,000 samples and report model moments and the cross-sample standard deviation of those moments.

6.2 Results

Table 2 summarizes the model generated data. The last column shows the driving force, labor productivity. By construction, I match the standard deviation and quarterly autocorrelation in U.S. data. The remaining numbers are driven by the structure of the model.

The first two columns show unemployment and vacancies. As I stressed in Section 4,

[13] In the deterministic steady state with $N = 236.3$, about 64.7 percent of employed workers are in markets with $j \leq i$ and hence are paid z. This implies the labor share is $0.647 \cdot 0.4 + 0.353 \cdot 1 = 0.61$, a bit lower than the usual value of 2/3.

Model Generated Data (and standard errors)

		U	V	V/U	f	s	p
Standard Deviation		0.059 (0.008)	0.084 (0.011)	0.143 (0.019)	0.031 (0.004)	0.032 (0.004)	0.020 (0.003)
Quarterly Autocorrelation		0.878 (0.030)	0.878 (0.030)	0.878 (0.030)	0.791 (0.050)	0.884 (0.029)	0.878 (0.030)
Correlation Matrix	U	1	-0.999 (0.001)	-1.000 (0.000)	-0.927 (0.019)	0.994 (0.001)	-0.999 (0.000)
	V	—	1	1.000 (0.000)	0.927 (0.019)	-0.992 (0.002)	0.996 (0.002)
	V/U	—	—	1	0.927 (0.019)	-0.993 (0.001)	0.997 (0.001)
	f	—	—	—	1	-0.939 (0.015)	0.925 (0.018)
	s	—	—	—	—	1	-0.995 (0.001)
	p	—	—	—	—	—	1

Table 2: Results from simulations of the benchmark model. See the text for details.

both of these variables only depend on the contemporaneous number of jobs. Thus the model generates a nearly-perfect negative correlation between them, stronger than the empirical correlation of -0.89. The model also explains 42 percent of the observed volatility in vacancies and 31 percent of the observed volatility in unemployment. The theoretical autocorrelations of the two variables are about equal, consistent with the empirical evidence. This last observation is notable since the equal persistence of unemployment and vacancies is a puzzle for matching models where unemployment is a state variable and vacancies are a jump variable (Shimer, 2005a; Fujita, 2003; Fujita and Ramey, 2005).

The third column shows that volatility of the v-u ratio is 7.03 times as large as the volatility of labor productivity. Recall that Proposition 5 suggested that a one percent increase in labor productivity would raise the v-u ratio by $4.25\frac{p}{p-z}$. Evaluating at $p = 1$ and $z = 0.4$, the predicted elasticity was 7.08, indistinguishable from the results in the full stochastic model.

The fourth column shows that the model produces 26 percent of the observed volatility in the job finding rate. The correlation between the detrended job finding rate and the detrended v-u ratio is 0.93, only slightly lower than the 0.95 in the data. I also estimate a Cobb-Douglas reduced-form matching function in the data, regressing the detrended job finding rate on the detrended v-u ratio in each 212 quarter sample. The resulting elasticity estimate averages 0.202 (standard error 0.005). When I add a quadratic term in the v-u ratio to test for a constant elasticity, I reject the null at the five percent level about 1.2 percent of the time. These findings are all consistent with the comparative statics shown in Figure 2.

The fifth column shows that the model generates 43 percent of the observed volatility in the separation rate into unemployment even though both q and l are constant. The flip side of this is that the model produces a strongly procyclical job-to-job transition rate, consistent with the facts reported in Fallick and Fleischman (2004).[14]

Because the model has only one shock, most of the correlations are close to one in absolute value. Moreover, a one shock model probably should not be able to explain all the volatility in vacancies and unemployment; there must be other shocks in the data, e.g. to the cost of investment goods k (Fisher, 2006). Hall (2005), Mortensen and Nagypal (2005), and Rudanko (2005) propose evaluating one shock models by examining the standard deviation of the projection of the detrended v-u ratio on detrended productivity. By this metric, the projection in the data is 0.151 and in the model it is barely smaller, 0.143. Similarly, the target for the separation rate should be just 0.039, compared to 0.032 in the model. By this metric, the mismatch model explains almost all of the volatility in these variables.

7 Duration Dependence

A distinctive feature of the mismatch model is that not all unemployed workers are equally likely to find a job. Even in steady state, the job finding rate for any particular unemployed worker may differ substantially from the average job finding rate in equation (15), since it depends on the number of workers i and the number of jobs j in her labor market. This gives rise to duration dependence in the job finding rate: if an econometrician observes a worker who has been unemployed for a long time but cannot observe local labor market conditions, he should infer that the worker is probably in a labor market in which jobs are scarce and workers plentiful. The worker's job finding rate is correspondingly low. Conversely, a newly unemployed worker's job finding rate is higher than the average job finding rate f.

In addition to being empirically relevant, duration dependence affects the reduced-form matching function depicted in Figure 2. At the start of the month, the average unemployed worker finds a job at rate f; however, conditional on staying unemployed, that worker's job finding probability falls by the end of the month. Equivalently, the full month probability of finding a job is less than $1-e^{-f}$, the probability of finding (at least) one job during a month if jobs arrive at a Poisson rate f. Since empirically I measure the fraction of unemployed workers who find a job during a month, this could represent an important distinction between the model and data. This section therefore also explores the implications of duration dependence for the measured job finding probability.

[14] I do not report the job-to-job transition rate here because Fallick and Fleischman's (2004) series is only available since 1994, a relatively tranquil period. A previous version of the paper showed that the model slightly underpredicts the number of job-to-job transitions.

7.1 Cross Section

I start by examining duration dependence in unemployment rates in a deterministic steady state. I simulate 200 million unemployment spells to recover the full month job finding probabilities numerically. In half the spells, I start with a "job leaver," a worker who quit her labor market and moved to one in which there were more workers than jobs, $i > j$. In the other half of the spells, I start with a "job loser," a worker whose job left a labor market that previously had i workers and $j \leq i$ jobs. In both cases, I simulate the evolution of the worker's local labor market, stochastic changes in the number of workers and jobs coming from entry and exit, until the worker finds a job either because a new job enters, an employed worker leaves, or our unemployed worker quits for a labor market with vacancies.

I assume that whenever a job is available, each unemployed worker is equally likely to be hired, independent of unemployment duration. For example, if at some point our unemployed worker is in a labor market with i workers and $j < i$ jobs and a new job enters, I assume that she gets the job with probability $1/(i-j)$.[15]

I use the usual value for the number of workers per labor market, $M = 244.2$. I set the quit and layoff rates at $q = l = 0.027$ and think of a time period as a month rather than a quarter. For now I fix $N = 236.3$, giving an unemployment rate of 5.4 percent. With these values, the instantaneous job finding rate, f in equation (15), is 61.0 percent. If the job finding rate were constant during a month, the full month probability of finding at least one job would be $1 - e^{-f} = 45.6$ percent. Figure 4 shows the theoretical monthly probability of finding a job—the fraction of workers who find a job during the next month—as a function of the current duration of an unemployment spell and the reason for unemployment. The job finding probability for job losers is slightly higher than for job leavers, reflecting slight differences in initial conditions for the two groups.

I next compute the average job finding probability for an unemployed worker, a weighted average of the job finding probability in Figure 4, with weights corresponding to the fraction of spells that do not end before a particular duration. 39.8 percent of job leavers and 40.2 percent of job losers find a job in given month. In U.S. data, I measure the job finding probability as

$$\frac{\sum_{i=1}^{\mathcal{U}_t} F_t^i}{\mathcal{U}_t} = 1 - \frac{\mathcal{U}_{t+1} - \mathcal{U}_{t+1}^s}{\mathcal{U}_t},$$

where F_t^i is the job finding probability for worker i, $\mathcal{U}_t$ is the number of unemployed in month

[15] The results in this section are sensitive to this assumption. If the most recently unemployed worker is always the first to get a job, the model generates significantly more duration dependence in the exit rate from unemployment. Conversely, if the unemployed queue for a job (Sattinger, 2005), duration dependence is inverted, with the long-term unemployed more likely to find a job than the short-term unemployed. Another approach would be to model heterogeneous workers, eliminating this ambiguity.

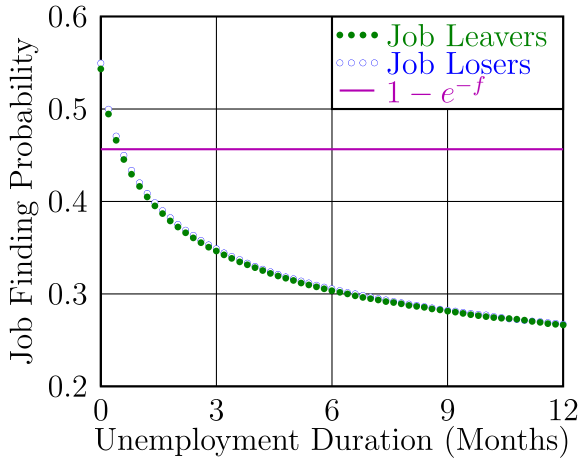

Figure 4: Monthly job finding probability as a function of unemployment duration. The number of workers per labor market is $M = 244.2$, the number of jobs per labor market is $N = 236.3$. The quit and layoff rates are $q = l = 0.027$ per month.

t, and $\mathcal{U}_t^s$ is the number of short-term unemployed, workers unemployed 0 to 4 weeks; see Shimer (2005b) for details. Since December 2000, this has averaged 39.7 percent. The model matches this number because of my choice of $q = l = 0.027$ and is scarcely affected by changes in q and l leaving $q + l$ constant.

Figure 4 shows that the job finding probability of both job losers and job leavers declines sharply during an unemployment spell. I summarize this decline in a single number by looking at a weighted average of the job finding probability, where weights correspond to unemployment duration. Shimer (2005b) shows that I can measure this empirically using time series on unemployment $\mathcal{U}_t$ and mean unemployment duration $\bar{d}_t$:

$$\frac{\sum_{i=1}^{\mathcal{U}_t} d_t^i F_t^i}{\sum_{i=1}^{\mathcal{U}_t} d_t^i} = 1 - \frac{(\bar{d}_{t+1} - 1)\mathcal{U}_{t+1}}{\bar{d}_t \mathcal{U}_t},$$

where d_t^i is worker i's unemployment duration. This averaged 24.3 percent in the U.S. since December 2000, while in the model it is 33.0 percent for job leavers and 33.2 percent for job losers. In other words, the model explains a good fraction of observed duration dependence. Presumably the rest is due to unmodeled heterogeneity among workers within labor markets.

7.2 Comparative Statics

I now explore how time aggregation and duration dependence affect the theoretical relationship between the job finding rate and the v-u ratio. I let N vary from 233 to 243 with M, q, and l fixed. At each value of N I compute the v-u ratio and simulate the fraction of unemployed workers who find a job within a month, $F(N)$. The solid blue circles in Figure 5 show

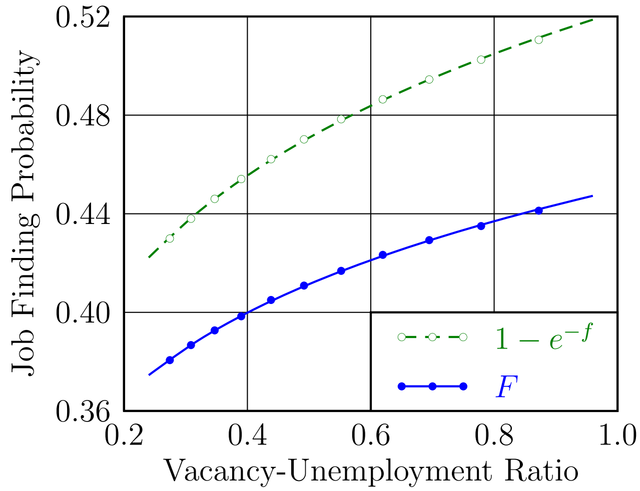

Figure 5: Theoretical monthly job finding probability as a function of the v-u ratio. The number of workers per labor market is fixed at $M = 244.2$ and the quit and layoff rates at $q = l = 0.027$. The entry rate of jobs varies so N takes values between 233 to 243.

the results. There is again an increasing relationship between the v-u ratio and the measured job finding probability. The solid blue line depicts a Cobb-Douglas function through these points. Again the fit is remarkable, although the elasticity is lower than before, just 0.13.

The hollow green circles in Figure 5 show the relationship between the v-u ratio and $1 - e^{-f(N)}$, where $f(N)$ is defined in equation (15). This is a full month measure of the job finding probability but ignores duration dependence. This is systematically about 15 percent higher than $F(N)$ but the quality of the Cobb-Douglas fit (dashed green line) and the elasticity (0.15) are similar. I conclude that accounting for time aggregation lowers the level of the theoretical job finding probability but does not affect the main conclusion that the model generates a Cobb-Douglas reduced-form matching function.

8 Mobility

In this section, I take a step towards relaxing the assumption that all workers and all jobs are equally likely to leave their labor market. I focus throughout on steady states. Rather than model such mobility costs explicitly, I consider an ad hoc structure that suggests how mobility costs might affect my results. Recall that a worker's payoff is monotonically increasing in the number of jobs in her labor market and monotonically decreasing in the number of workers. Thus the workers who are most motivated to quit their labor market are those where $i - j$ is largest. Conversely, jobs are most profitable in those labor markets. With an explicit cost of job mobility, workers will use a cutoff rule to decide when to exit a labor market. Conversely, they may choose never to exit a labor market where $i - j$ is sufficiently small. Firms behavior is likely to be similar.

I capture this with a new parameter, $\delta \in \{1, 2, \ldots\}$, and assume that in every labor market, the difference between the number of workers i and the number of jobs j is bounded by δ, so $|i - j| \leq \delta$. In a market with $|i - j| < \delta$, each worker quits at rate q, each job leaves at rate l, new workers enter at rate m, and new jobs enter at rate n. But jobs never leave and workers never enter a market with $i - j = \delta$, although the exit rate of workers and entry rate of jobs are unchanged. Similarly, workers never leave and jobs never enter a market with $j - i = \delta$. The benchmark model corresponds to $\delta \to \infty$, where $m = qM$ and $n = lN$.

The primitives are the average number of workers and jobs per labor market, M and N, and the quit and layoff rates, q and l. I determine the inflow rates m and n endogenously. It is straightforward to show that the distribution of workers and jobs across labor markets is a modest generalization of equation (1):

$$\pi_\delta(i, j) \equiv \frac{\bar{\pi}(m/q)^i (n/l)^j}{i!j!} \text{ if } |i - j| \leq \delta, \tag{17}$$

and $\pi_\delta(i, j) = 0$ otherwise, where the constant $\bar{\pi}$ ensures that this is a proper density and I suppress the dependence of the density on N. To verify equation (17), note that if $-\delta \leq i - j < \delta$, the rate at which labor markets switch from state (i, j) to $(i + 1, j)$ and the rate at which they switch back are equal:

$$m\pi_\delta(i, j) = q(i + 1)\pi_\delta(i + 1, j),$$

and similarly for other transition rates.

The inflow rates m and n must ensure that we have the correct number of workers and jobs per labor market:

$$M = \sum_{i=1}^{\infty} \sum_{j=i-\delta}^{i+\delta} i\pi_\delta(i, j) \quad \text{and} \quad N = \sum_{j=1}^{\infty} \sum_{i=j-\delta}^{j+\delta} j\pi_\delta(i, j)$$

An increase in the number of jobs per labor market N, for example, raises the inflow rate of jobs n and reduces the inflow rate of workers m. Intuitively, there are fewer labor markets where jobs can settle, with $j - i < \delta$, and more labor markets where jobs can exit, with $i - j < \delta$. Therefore the rate at which jobs leave is higher and the few labor markets that can absorb new jobs get them faster. The logic for how N affects m is similar.

I next examine how the Beveridge curve depends on δ. When $\delta = 1$, we require $M = 8.953$ workers and $N = 8.663$ jobs to deliver the same unemployment and vacancy rates as in the benchmark model with $M = 244.2$ and $N = 236.3$. The small value of δ mitigates

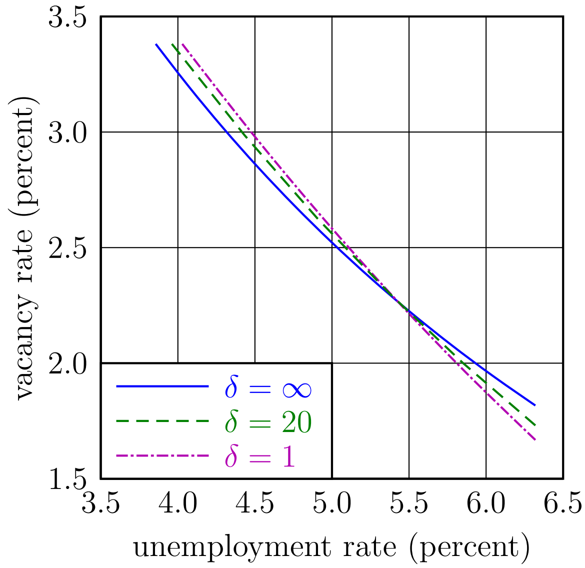

Figure 6: Model-generated Beveridge curve with different values of δ.

frictions and so implies that we need small labor markets to explain the average level of unemployment and vacancies. As N varies between about 8.47 and 8.94, with M, q, and l fixed, we trace out a downward-sloping Beveridge curve. I show this in Figure 6 as a dash-dot purple line. At higher values of δ, the model-generated Beveridge curve is flatter. At $\delta = 20$ and $M = 121.2$, we get the dashed green line. With $\delta = 50$ and $M = 222.3$, the Beveridge curve is indistinguishable from the solid blue line, which represents the benchmark model.

There are two ways to interpret Figure 6. On the one hand, the Beveridge curve is quite insensitive to the choice of δ and so seems to be a robust aggregation phenomenon. On the other hand, the fit of the model to the JOLTS data shown in Figure 1 is better when δ is larger. The root mean squared error in the vacancy rate falls from 0.14 percentage points when $\delta = 1$, to 0.11 when $\delta = 20$, to 0.09 when δ is infinite. The fit is always good, but it is better when mobility is less directed.

I can also examine the reduced-form matching function in this version of the model. The basic idea of how unemployed workers find jobs is unchanged. They can move to a labor market with jobs, wait for an employed worker to exit their labor market, or wait for a job to enter. For each value of δ, I fix $q+l$ at a level which ensures a common job finding rate across models when the unemployment rate is 5.4 percent and the vacancy rate is 2.3 percent. I then vary the number of jobs N and compute the v-u ratio and the job finding rate.

Figure 7 shows the results. Again, the basic shape of the reduced-form matching function is similar across models. When $\delta = 1$, the average elasticity of the job finding rate with respect to the v-u ratio is about 0.23, a better fit than with $\delta = \infty$, although the function is more convex than a Cobb-Douglas. For higher values of δ, the average elasticity falls but

Figure 7: Model-generated reduced-form matching function with different values of δ

the function is closer to a Cobb-Douglas. At $\delta = 20$, for example, the elasticity is around 0.17. For still higher values of δ, the elasticity increases again, converging to 0.21 in the limit. Again, there are two ways to interpret these findings. Figure 7 shows that the results are qualitatively insensitive to the value of δ, but some of the desirable model properties, in particular the constant elasticity reduced-form matching function, depend on δ being large.

9 Conclusions

This paper develops a mismatch model of unemployment, vacancies, and labor market transitions. It provides a coherent framework for exploring a variety of facts, including the comovement of unemployment, vacancies, the job finding rate, the separation rate, and the job-to-job transition rate. The model is deliberately simple and mechanical in order to highlight the major forces in a model of mismatch and stress that the findings are a consequence of aggregation. They are robust to the exact pattern of mismatch across markets and so I can allow workers to stay away from depressed labor markets without substantially affecting the conclusions. It therefore seems likely that aggregating other models of mismatch, e.g. the stock-flow matching model or the queuing model, would yield similar results. Preliminary results in Shimer (2006) confirm this hypothesis.

The matching model (Pissarides, 1985) is an important alternative explanation for these facts. It seems plausible that mismatch and search frictions are complementary to each other and both empirically relevant; however, there are some important differences between the two frameworks. While the matching model can deliver a Beveridge curve with the right slope, the mismatch model must deliver such a Beveridge curve. Moreover, the Beveridge curve in the mismatch model is unaffected by cyclical fluctuations in the separation rate. Indeed, mismatch provides a natural explanation for countercyclicality in the separation rate and procyclicality in the job-to-job transition rate: when an employment relationship ends, it is easier for a worker to move immediately into a new job when jobs are more plentiful. I find that small productivity shocks can explain much of the observed cyclicality in these variables. Similarly, the mismatch model explains much of the volatility in vacancies and unemployment and suggests why the two variables have similar persistence. It also predicts that the job finding rate should decline with unemployment duration even if workers are homogeneous, generating a good fraction of the observed duration dependence. All of these findings are problematic in the matching model.

I also found that the mismatch model generates a systematic relationship between the job finding rate and the v-u ratio. It predicts that a one percent increase in the v-u ratio should raise the job finding rate by about 0.2 percent, about two-thirds of the empirically relevant value. To my knowledge, this is the first theoretical explanation for the empirical relationship between these variables. In contrast, whether the matching function is Cobb-Douglas is exogenous in the matching model. However, given all the differences discussed in the previous paragraph, the mismatch model does not provide a microfoundation for writing down an aggregate matching function.

At a microeconomic level, some of the model's predictions are stark. For example, at any point in time, two wages are paid, $p(t)$ to workers in markets with vacancies and z to all other workers. A small change in labor market conditions can cause a dramatic change in wages. Note, however, that the continual reallocation of workers and jobs across labor markets means that the expected value of a worker varies smoothly with the number of workers and jobs in her labor market. If workers were risk averse and workers and firms could commit to long term contracts, firms would insure workers against sharp fluctuations in labor market conditions, making wages a much smoother function of the state.

Heterogeneity of workers within a market leads to a similar result. Suppose workers differ in their productivity x, where a type x worker produces $xp(t)$ units of output when employed at time t. If there are excess workers in a market, competition within the labor market ensures that the least productive workers are unemployed, the marginal employed worker receives her value of leisure z, and the remaining workers are paid the difference between their productivity and the productivity of the marginal worker, so firms are indifferent about whom to hire among the employed workers. The entry of an additional job increases employment by one, thereby reducing the productivity of the marginal worker. For firms to remain indifferent about whom to hire, all workers must get a small wage increase, so wages respond smoothly to local labor market conditions. This extension yields some other rich predictions. For example, wage cuts help to forecast future job loss since they imply that a worker is falling closer to being the marginal worker in her labor market.

There are other predictions of the mismatch model that I have not explored here. For example, the mismatch model predicts procyclical real wages since more workers are in labor markets with excess jobs during booms. It likewise predicts a stable link between local unemployment rates and wages, Blanchflower and Oswald's (1995) "wage curve." The mismatch model also provides a coherent theory of jobs and hence a model of job flows distinct from worker flows. In principle this means that the model could simultaneously be used to address facts about labor market flows and facts about job creation and job destruction (Davis, Haltiwanger, and Schuh, 1996). Preliminary work suggests that a simple feature of the labor market, the fact that the vacancy rate is less than the unemployment rate, may explain why job flows are systematically smaller than workers flows: it is easier to find a worker than to find a job.

A Omitted Proofs

Proof of Proposition 1. I start by constructing the unique equilibrium with increasing thresholds. The last paragraph proves that there is no other equilibrium. Start with the smallest value $p = p_{-\nu\Delta}$ with associated target $N^*_{p_{-\nu\Delta}}$. Following an aggregate shock, productivity increases by one step with certainty and so the target number of job increases to $N^*_{p_{-(\nu-1)\Delta}} > N^*_{p_{-\nu\Delta}}$. If $N = N^*_{p_{-\nu\Delta}}$, the value of a job is k both before and after the shock, $J_{p_{-\nu\Delta}}(N^*_{p_{-\nu\Delta}}) = J_{p_{-(\nu-1)\Delta}}(N^*_{p_{-\nu\Delta}}) = k$. In other words, evaluating equation (6) at $p = p_{-\nu\Delta}$ and $N = N^*_{p_{-\nu\Delta}}$ and using the smooth pasting condition in equation (7) gives

$$rk = (p_{-\nu\Delta} - z)S(N^*_{p_{-\nu\Delta}}) - lk.$$

This uniquely defines $N^*_{p_{-\nu\Delta}}$ since S is a decreasing function:

$$S'(N) = \sum_{i=1}^{\infty}\sum_{j=1}^{i-1}\pi(i, j-1; N) - \sum_{i=1}^{\infty}\sum_{j=0}^{i-1}\pi(i, j; N) = -\sum_{i=1}^{\infty}\pi(i, i-1; N). \tag{18}$$

The first equality uses Lemma 1 and the second eliminates common terms.

I now proceed by induction. Suppose that for some $y > -\nu\Delta$, $y \in Y$, I have shown that the targets $N^*_{p_{y'}}$ are increasing and I have computed $J_{p_{y'}}(N^*_{p_{y-\Delta}})$ for all $y' < y$, $y' \in Y$. For

$N \in [N^*_{p_{y-\Delta}}, N^*_{p_y}]$ and $y' < y$, equation (6) implies

$$\begin{aligned} rJ_{p_{y'}}(N) &= (p_{y'} - z)S(N) - lJ_{p_{y'}}(N) - J'_{p_{y'}}(N)lN \\ &\quad + \frac{\lambda}{2}\left(1 + \frac{y'}{\nu\Delta}\right)\left(J_{p_{y'-\Delta}}(N) - J_{p_{y'}}(N)\right) + \frac{\lambda}{2}\left(1 - \frac{y'}{\nu\Delta}\right)\left(J_{p_{y'+\Delta}}(N) - J_{p_{y'}}(N)\right). \end{aligned} \tag{19}$$

In addition, $J_{p_y}(N) = k$ for $N \in [N^*_{p_{y-\Delta}}, N^*_{p_y}]$. This is a system of $\nu + y/\Delta$ differential equations in N with the same number of terminal conditions from the previous induction steps and so we can compute $J_{p_{y'}}(N)$, $N \in [N^*_{p_{y-\Delta}}, N^*_{p_y}]$ for all $y' < y$, $y' \in Y$. The only catch is that we do not yet know $N^*_{p_y}$. To compute it, evaluate equation (6) at p_y and $N = N^*_{p_y}$ and simplify with equation (7):

$$rk = (p_y - z)S(N^*_{p_y}) - lk + \frac{\lambda}{2}\left(1 + \frac{y}{\nu\Delta}\right)\left(J_{p_{y-\Delta}}(N^*_{p_y}) - k\right), \tag{20}$$

where I use $J_{p_{y+\Delta}}(N^*_{p_y}) = k$ to eliminate the term coming from a positive shock. This uniquely defines $N^*_{p_y}$ since both S and $J_{p_{y-\Delta}}$ are decreasing.

To complete the induction argument, suppose equation (20) defines $N^*_{p_y} \le N^*_{p_{y-\Delta}}$. Then

$$(p_y - z)S(N^*_{p_y}) = (r + l)k < (p_{y-\Delta} - z)S(N^*_{p_{y-\Delta}}). \tag{21}$$

The equality uses $J_{p_{y-\Delta}}(N^*_{p_y}) = k$ whenever $N^*_{p_y} \le N^*_{p_{y-\Delta}}$. The inequality uses equation (19) evaluated at $y' = y - \Delta$ and $N = N^*_{p_{y-\Delta}}$, but drops the capital gain terms; those are all negative-valued since $N^*_{p_y} \le N^*_{p_{y-\Delta}}$ (by assumption in this paragraph) and $N^*_{p_{y-2\Delta}} < N^*_{p_{y-\Delta}}$ (from the induction assumption). Since $p_y > p_{y-\Delta}$, equation (21) implies $S(N^*_{p_y}) < S(N^*_{p_{y-\Delta}})$ or equivalently $N^*_{p_y} > N^*_{p_{y-\Delta}}$, a contradiction.

Finally, suppose there were an equilibrium with $N^*_{p_y} \le N^*_{p_{y-\Delta}}$ for some $y \in Y$. Focus on the largest such y, so either $N^*_{p_y} < N^*_{p_{y+\Delta}}$ or $y = \nu\Delta$, in which case productivity can only decline from p_y. Analogous to the reasoning behind equation (21), we find

$$(p_y - z)S(N^*_{p_y}) = (r + l)k \le (p_{y-\Delta} - z)S(N^*_{p_{y-\Delta}}),$$

since a productivity shock when $p = p_y$ and $N = N^*_{p_y}$ does not affect the value of a job (the threshold goes up), while a productivity shock when $p = p_{y-\Delta}$ and $N = N^*_{p_{y-\Delta}}$ may reduce the value of a job. The inequalities imply $N^*_{p_y} > N^*_{p_{y-\Delta}}$, a contradiction. ■

Proof of Proposition 2. Let $W_p(N)$ denote the present value of net output and express it recursively:

$$rW_p(N) = \max_{g \geq 0} \; p(M - U(N)) + zU(N) - kg + W'_p(N)(g - lN) + \lambda\big(\mathbb{E}_p W_{p'}(N) - W_p(N)\big). \quad (22)$$

Here g is the gross increase in the number of jobs per labor market and $U(N)$ is the number of unemployed workers per labor market, given by equation (3). The flow value of the planner, $rW_p(N)$, can be divided into three terms. First is current net output, p for each of the $M - U(N)$ employed workers, z for each of the $U(N)$ unemployed workers, and $-k$ for each job created. Second is the future increases in in $W_p(N)$ coming from any net increase in the number of jobs, the difference between gross job creation and deprecation, $g - lN$. Third is the possibility of an aggregate shock, with arrival rate λ, at which point the planner anticipates a capital gain $\mathbb{E}_p\big(W_{p'}(N) - W_p(N)\big)$.

Next observe that

$$U'(N) = \sum_{i=1}^{\infty} \sum_{j=1}^{i} (i-j)\pi(i, j-1; N) - \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} (i-j)\pi(i, j; N) = -S(N). \quad (23)$$

The first equation follows from Lemma 1, while the the second eliminates common terms from the double sum. Use this to write the envelope condition from equation (22) as

$$rW'_p(N) = (p - z)S(N) - lW'_p(N) + W''_p(N)(g_p(N) - lN) + \lambda\big(\mathbb{E}_p W'_{p'}(N) - W'_p(N)\big). \quad (24)$$

The first order condition for the gross amount of job creation conditional on the current state (p, N) is

$$g_p(N) \geq 0, \quad W'_p(N) \leq k, \quad \text{and} \quad g_p(N)(W'_p(N) - k) = 0.$$

Substituting this into equation (24) gives expressions analogous to equations (6) and (7) with $W'_p(N) = J_p(N)$. ■

Proof of Proposition 3. I start with the response of u to M:

$$\frac{\partial u}{\partial \log M} = M \frac{\partial (U/M)}{\partial M} = \frac{1}{M}\left(M \frac{\partial U}{\partial M} - U\right) \quad (25)$$

Observe that

$$\sum_{i=0}^{\infty} \sum_{j=0}^{i-1} i\pi(i, j; N) = M \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} \pi(i-1, j; N) = M \sum_{i'=0}^{\infty} \sum_{j=0}^{i'} \pi(i', j; N) = M \frac{\partial U}{\partial M}. \quad (26)$$

The first equality uses the definition of $\pi(i, j; N)$ in equation (1), the second equality reindexes using $i' = i - 1$, and the third uses $\frac{\partial U}{\partial M} = \sum_{i=0}^{\infty} \sum_{j=0}^{i} \pi(i,j;N)$, with proof analogous to equation (23). Substituting this into equation (25) and replacing U using equation (3) gives

$$\frac{\partial u}{\partial \log M} = \frac{1}{M} \sum_{i=0}^{\infty} \sum_{j=0}^{i-1} j\pi(i,j;N).$$

Next, a logic similar to equation (26) establishes

$$\sum_{i=0}^{\infty} \sum_{j=0}^{i-1} j\pi(i,j;N) = N \sum_{i=0}^{\infty} \sum_{j=1}^{i-1} \pi(i,j-1;N) = N \sum_{i=0}^{\infty} \sum_{j=0}^{i-2} \pi(i,j';N). \tag{27}$$

Substitute this into the previous equation to get $\partial u/\partial \log M$ in equation (8).

To compute the response of u to N, plug equation (23) into $\frac{\partial u}{\partial \log N} = N \frac{\partial (U/M)}{\partial N} = \frac{N}{M} \frac{\partial U}{\partial N}$ to get the desired result. The partial derivatives of v are computed symmetrically. ■

Proof of Proposition 4. Consider the locus of pairs (M,N) that deliver a particular unemployment rate u_0. Equation (8) implies that this locus satisfies

$$\left.\frac{\partial \log N}{\partial \log M}\right|_{u=u_0} = \frac{\sum_{i=2}^{\infty} \sum_{j=0}^{i-2} \pi(i,j;N)}{\sum_{i=1}^{\infty} \sum_{j=0}^{i-1} \pi(i,j;N)} < 1 \tag{28}$$

That is, if (M_1,N_1) and (M_2,N_2) with $M_1 < M_2$ both yield the same unemployment rate u_0, $M_2/M_1 > N_2/N_1$. Similarly, equation (9) implies

$$\left.\frac{\partial \log N}{\partial \log M}\right|_{v=v_0} = \frac{\sum_{j=1}^{\infty} \sum_{i=0}^{j-1} \pi(i,j;N)}{\sum_{j=2}^{\infty} \sum_{i=0}^{j-2} \pi(i,j;N)} > 1,$$

so if (M_1,N_1) and (M_2,N_2) with $M_1 < M_2$ both yield the same vacancy rate v_0, $M_2/M_1 < N_2/N_1$. This proves that there is at most one pair (M,N) associated with each pair (u,v). The proof of existence is standard. ■

Proof of Proposition 5. The chain rule implies

$$\frac{\partial \log(V(\mathcal{N}_p^*)/U(\mathcal{N}_p^*))}{\partial \log p} = \left(\frac{V'(\mathcal{N}_p^*)}{V(\mathcal{N}_p^*)} - \frac{U'(\mathcal{N}_p^*)}{U(\mathcal{N}_p^*)} \right) \frac{\partial \mathcal{N}_p^*}{\partial \log p},$$

while implicit differentiation of equation (10) implies

$$\frac{S'(\mathcal{N}_p^*)}{S(\mathcal{N}_p^*)} \frac{\partial \mathcal{N}_p^*}{\partial \log p} + \frac{p}{p-z} = 0.$$

Recall from equation (23) that $U'(N) = -S(N)$ and so combining equations gives the desired expression for $\frac{\partial \log(V(\mathcal{N}_p^*)/U(\mathcal{N}_p^*))}{\partial \log p}$. To see that this is positive, note that $U(N)$ and $V(N)$ are positive. Since the share of markets with unemployed workers $S(N) \in (0, 1)$, $U'(N) < 0$. Also since $V(N) = N - M + U(N)$, $V'(N) = 1 - S(N) > 0$. Finally, equation (18) implies $U''(N) = -S'(N) > 0$. Combining inequalities yields the result. ■

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