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SHORT-TERM AND LONG-TERM INTEREST RATES IN A MONETARY MODEL OF A SMALL OPEN ECONOMY

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ABSTRACT

This paper analyzes the effects of both anticipated and unanticipated monetary and fiscal disturbances, on the dynamic behavior of a monetary model of a small open economy. It focuses on the adjustment of the short-term and long-term interest rates and the divergence of their transitional paths, particularly in anticipation of these disturbances. The analysis demonstrates how anticipation of a future policy change can generate perverse short-run behavior. The essential reason for the divergence between the short and long rates is that the latter is dominated by long-term expectations, while the former is primarily determined by current influences.

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1. INTRODUCTION

Most economists would subscribe to the view that portfolio decisions are generally more flexible than real expenditure decisions. Consequently, it seems reasonable to assume that the former are based on short-term rates of interest, while the latter are more likely to depend upon long-term rates. Nevertheless, most existing macroeconomic models treat assets as being of common maturity in these two sets of decisions. Typically, this is assumed to be either extremely short (a short-term bill) or infinitely long (a perpetuity). Recently, several authors have recognized the fact that different agents in the economy are concerned with rates of return over different time horizons. Using standard domestic macro models they have shown how arbitrage between the long-term and short-term rates in efficient financial markets provides important linkages between the present and the future. The forward-looking information contained in the long rates turns out to have important implications for the effects of monetary and fiscal policy; see Blanchard (1981, 1983), Turnovsky and Miller (1984).

In this paper, we introduce the distinction between short-term and long-term interest rates in a standard monetary model of an open economy.[1/] Much of the current literature in this area emphasizes the informational content of the exchange rate. It is clear that a similar informational role is played by the long-term interest rate. The paper analyzes the effects of a variety of disturbances, both unanticipated and anticipated, and discusses the time paths followed by the short-term and long-term interest rates in response to these disturbances.

In particular, the divergence in the adjustment between the short-term and long-term rates in anticipation of such disturbances, is highlighted.

The paper is structured as follows. In Section 2, the model, (which for the most part is familiar), together with its solution, is outlined. Sections 3, 4 analyze two alternative disturbances, namely: (i) domestic monetary expansion; (ii) domestic fiscal expansion. The conclusions are summarized in Section 5, while an Appendix contains some of the technical details of the analysis.

2. THE MODEL

The model we employ is a variant of the standard Dornbusch (1976) model, embodying perfect foresight, see Gray and Turnovsky (1979). It consists of the following equations

$$Z = \beta_1\overline{Y} - \beta_2 R + \beta_3(E-P) + G \qquad 0 < \beta_1 < 1\ ,\ \beta_2 > 0\ ,\ \beta_3 > 0 \tag{1a}$$

$$M - P = \alpha_1\overline{Y} - \alpha_2 i \qquad \alpha_1 > 0\ ,\ \alpha_2 > 0 \tag{1b}$$

$$i = i^* + \dot{E} \tag{1c}$$

$$r = i - \dot{P} \tag{1d}$$

$$r = R - \frac{\dot{R}}{R} \tag{1e}$$

$$\dot{P} = \gamma(Z-\overline{Y}) \qquad \gamma > 0 \tag{1f}$$

where

Z = real aggregate demand for domestic output,

$\overline{Y}$ = supply of domestic output, assumed to be fixed at full employment, G = real domestic government expenditure,

R = domestic long-term real rate of interest,

r = domestic short-term real rate of interest,

i = domestic short-term nominal interest rate,

i* = foreign nominal (and real) interest rate, taken to be fixed,

E = exchange rate (expressed in terms of units of foreign currency per unit of domestic currency), expressed in logarithms,

P = domestic price level, expressed in logarithms,

M = domestic nominal money supply, expressed in logarithms.

Equation (1a) specifies the aggregate demand for domestic output to be a negative function of the domestic long-term real interest rate and a positive function of the relative price (E-P), where we assume that the foreign price level remains fixed at unity. It also depends positively upon the fixed level of output and upon real government expenditure.

The introduction of the long-term real interest rate R into the real expenditure function Z is a key part of the model. While this specification has by now been adopted by several authors, it nevertheless merits further comment; see also Blanchard (1981, 1984), Miller and Turnovsky (1984), Sachs and Wyplosz (1984). Several justifications for this can be given. First, to the extent that Z includes expenditures on investment goods, it depends upon "Tobin's q," which in turn is inversely related to the long-term real interest rate. Secondly, it reflects asset values, and their impact through the wealth effect on current consumption; see Blanchard (1981). Finally, Buiter and Miller (1983) suggest that for Britain, most government debt is held by institutional investors, such as pension funds, having long term horizon so that some notion of permanent real interest income, based on a long-term rate, provides a better approximation of the actual flow of disposable interest income to the ultimate wealth owning and spending units. This leads them to argue for the plausibility of the long-term rate as a determinant of private expenditure. It is also possible for consumption to depend upon the short-term real rate of interest r, as well. The inclusion of this variable, in addition to R, does not alter the substance of our analysis in any essential way.

Domestic money market equilibrium is specified by (1b), with the demand for money depending upon the short-term nominal interest rate. Note that we follow Dornbusch and do not introduce the distinction between the price of domestic output and overall domestic cost of living. To introduce this distinction adds little insight and merely complicates the analysis.[2/] Domestic and foreign bonds are assumed to be perfect substitutes on an uncovered basis, so that the uncovered interest parity condition (1c) applies, while the short-term real rate of interest is defined in (1d).

The long-term real rate of interest is defined to be the yield on a consol paying a constant (real) coupon flow of unity. If we denote such a yield by R, the price of the consol is 1/R. The instantaneous rate of return on consols is therefore

$$R + \frac{d(1/R)/dt}{1/R} = R - \dot{R}/R$$

We assume that the long-term bonds and the short-term bonds are perfect substitutes, so that their instantaneous real rates of return are equal, as in (1e).[3/] Integrating this equation, we obtain

$$R(t) = \frac{1}{\int_t^{\infty} e^{-\int_t^x r(t')dt'} dx} \qquad (1e')$$

This relationship shows explicitly how the current long-term rate embodies information about the future (expected) short rates.[4/] Finally, equation (1f) describes the rate of price adjustment in terms of a simple Phillips curve relationship.

The steady state of the economy is attained when $\dot{R} = \dot{P} = \dot{E} = 0$ and is described by

$$(1-\beta_1)\overline{Y} = -\beta_2\tilde{R} + \beta_3(\tilde{E}-\tilde{P}) + G \qquad (2a)$$

$$M - \tilde{P} = \alpha_1\overline{Y} - \alpha_2\tilde{R} \qquad (2b)$$

$$\tilde{R} = \tilde{r} = \tilde{i} = i^* \qquad (2c)$$

where tildes denote steady-state values. In equilibrium, the product market clears and the short-term and long-term real and nominal rates are all equal to the exogenously given world interest rate i^*. The following long-run equilibrium effects are immediately deduced

$$\frac{d\tilde{E}}{dM} = \frac{d\tilde{P}}{dM} = 1 \; ; \; \frac{d\tilde{R}}{dM} = 0 \qquad (3a)$$

$$\frac{d\tilde{P}}{dG} = \frac{d\tilde{R}}{dG} = 0 \; ; \; \frac{d\tilde{E}}{dG} = -\frac{1}{\beta_3} < 0 \qquad (3b)$$

An expansion in the domestic nominal money supply leads to long-run proportional changes in the exchange rate and domestic price level, leaving the long-run interest rates unchanged. An expansion in domestic government expenditure leaves the domestic price level and interest rate(s) unchanged. The exchange rate must appreciate, thereby lowering private demand, and accommodating the increased government expenditure, given the fixed output.

Linearizing the system about the stationary equilibrium and substituting, the dynamics can be reduced to the following matrix equation in R, E, and P

$$\begin{bmatrix} \dot{R} \\ \dot{E} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} \tilde{R}(1-\gamma\beta_2) & \tilde{R}\gamma\beta_3 & -\tilde{R}[1/\alpha_2 + \gamma\beta_3] \\ 0 & 0 & 1/\alpha_2 \\ -\gamma\beta_2 & \gamma\beta_3 & -\gamma\beta_3 \end{bmatrix} \begin{bmatrix} R-\tilde{R} \\ E-\tilde{E} \\ P-\tilde{P} \end{bmatrix} \tag{4}$$

It can be shown that the three eigenvalues λ_1, λ_2, λ_3, of this system have the following properties

$$\lambda_1\lambda_2\lambda_3 = -\frac{\gamma\beta_3}{\alpha_2}\tilde{R} < 0$$

$$\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = -\gamma[\tilde{R}\beta_3 + \frac{\beta_3}{\alpha_2} + \tilde{R}\frac{\beta_3}{\alpha_2}] < 0$$

It then follows from these two relationships that there must be one negative and two positive roots, say $\lambda_1 < 0$, $\lambda_2 > 0$, $\lambda_3 > 0$. The system therefore possesses saddlepoint behavior. We assume that while the price level P always evolves continuously, in accordance with the Phillips curve, both the exchange rate and the long-term real rate can jump discontinuously in response to unanticipated disturbances. They are therefore "news" variables.

In Sections 3,4 below, we consider once-and-for-all unit increases in the following quantities:[5/]

(i) domestic nominal money supply,

(ii) domestic government expenditure.

These changes are assumed to be announced at time 0 for time $T \geq 0$, with the limiting case $T = 0$ describing an unanticipated shock.

In the Appendix, we derive the general solution to (4) on the assumption that the economy begins in an initial steady state. Given the assumptions we have made about the nature of the dynamic variables, these solutions are unique. They form the basis for our subsequent descriptions of the response of the economy to the various disturbances. Because the dynamics is third order, we are unable to give a simple two-dimensional illustration of the phase diagram in the three state variables R, P, and E. However, we do see from the solutions (A.3a')-(A.3c') that when $t \geq T$, that is <u>after</u> the announced disturbance has occurred, that R, E, and P follow the pairs of linear relationships, which ensure stable adjustment of the economy:

$$E - \tilde{E} = \frac{h_{23}}{\lambda_1} (P-\tilde{P}) \tag{5a}$$

$$R - \tilde{R} = \frac{-[\lambda_1 h_{13} + h_{12}h_{23}]}{(h_{11}-\lambda_1)\lambda_1} (P-\tilde{P}) \tag{5b}$$

where $\lambda_1 < 0$ and h_{ij} are the elements of the matrix of coefficients appearing in (4). It is seen that $h_{13} < 0$, $h_{12} > 0$, $h_{23} > 0$, while adding the restriction $1 > \gamma\beta_2$, ensures $h_{11} > 0$. Equation (5a) is indeed the locus of the stable arm of the saddlepoint in terms of the exchange rate and the domestic price level, familiar from the Dornbusch model, and is <u>negatively</u> sloped. Equation (5b) is an analogous relationship between the long-term real rate and the domestic price level. Under the conditions stated above, it is <u>positively</u> sloped.

3. DOMESTIC MONETARY EXPANSION

Insofar as possible, our treatment shall be graphical. Formal solutions for the time paths for this (and other) disturbances can be obtained from the general solution given in the Appendix. Following the monetary expansion, the economy follows a stable first order locus (the stable arm of the saddlepoint) and is easily illustrated. In the case of an announced increase, however, during the period after the announcement, but prior to the change, the economy follows an unstable third order locus. In this case it is difficult to illustrate the time paths precisely and the paths we have illustrated in a few instances are based on a consideration of plausible limiting cases.[6/]

Figure 1 describes the adjustment of the exchange rate, the long-term real rate, and the price level, in phase space. Figure 2 illustrates the paths for the short-term real and nominal, and long-term real, interest rates over time. For ease of comparison, the short-term real rate r appears in both graphs in Figures 2A, 2B. Unanticipated and anticipated monetary expansions are considered in turn, and illustrated separately in Figure 2.

As shown in Section 2, a permanent unit increase in M increases the exchange rate and the price level proportionately, while leaving the long-term rate R unchanged. Accordingly, if 0 depicts the original steady state in both Figures 1A, 1B, the new equilibria are at N and W, respectively. Thus the monetary expansion causes the stable locuses, corresponding to (5a) and (5b), and illustrated by XX', YY', to move to X_1X_1', Y_1Y_1', respectively.

A. Unanticipated Domestic Monetary Expansion

An unanticipated increase in the domestic money supply causes the exchange rate to jump instantaneously from 0 to L on X_1X_1'. Thereafter, the exchange rate begins to appreciate gradually, while the price level begins to increase, taking the system towards N. This figure illustrates the well known 'overshooting' of the exchange rate in the Dornbusch model.[7/] At the same time, the monetary expansion causes the long-term real rate to fall instantaneously from 0 to U. It then begins to climb gradually back towards its original level, as the price level begins to increase.

The time paths for the interest rates i, r, and R are illustrated in Figure 2A. The fact that following the initial depreciation, the exchange rate begins to immediately appreciate at A ($\dot{E} < 0$), means that

$$i(0) < i^*$$

so that the domestic short-term nominal interest rate is immediately driven below the world rate. With the price level rising and the nominal money stock now fixed at its new (higher) level, the real money stock begins to fall and the domestic short-term nominal rate rises back up begins to fall and the domestic short-term nominal rate rises back up towards the equilibrium world rate.

A further consequence of the steadily rising price subsequent to the monetary expansion ($\dot{P} > 0$) is that

$$r < i$$

so that the short-term real rate lies below the short-term nominal rate. At the same time, since the short-term rate is rising over time, and since the long-term real rate R is discounting the expected future time path of r, it follows that the current long-term real rate must always exceed the current short-term real rate. Another way of seeing this is from the arbitrage relationship (1e), which we may write as

$$\frac{\dot{R}}{R} = R - r \tag{1e'}$$

Since the long-term real rate rises continuously, following the initial drop, for the short-term and long-term rates of return to be equal, $R > r$, as illustrated.

B. Anticipated Monetary Expansion

Suppose that at time 0 the domestic monetary authorities announce an expansion in the money supply to take effect at time T say. The announcement of this event causes the exchange rate to depreciate to L', while the long-term real rate of interest falls to U'. The combination of the fall in the long-term real rate coupled with the devaluation of the exchange rate, leads to an increase in real demand Z, causing the domestic price level to begin rising.

Given that the money supply remains fixed prior to time T and that the price level is constrained to move continuously, the real money stock remains fixed at the time of announcement, $t = 0$. Hence, the short-term nominal interest rate i also remains fixed at time 0. The interest rate parity condition (1c) therefore implies that following the initial jump in the exchange rate, the rate of exchange depreciation is immediately zero ($\dot{E} = 0$). As the price level begins to rise, however, in anticipation of the monetary expansion, the domestic real money stock begins to fall and the exchange rate starts to increase again to equilibrate the money market; in E-P space, the economy moves along the locus L'M' in Figure 1A. At time T, when the anticipated monetary expansion takes place, E and P begin to move along the stable locus X_1X_1'.

The behavior of the long-term real rate is given in Figure 1B and is more difficult to establish. The arbitrage condition indicates that the behavior of R depends upon the difference between the short-term and long-term real rates of interest. On the one hand, the announcement of the monetary expansion causes the long-term real rate to fall. At the same time, the short-term real rate also falls, by virtue of the short-run rise in the rate of inflation and the fact that the nominal interest rate remains fixed. A linear approximation to the difference $R - r$, and therefore to the initial rate of change $\dot{R}$, expressed in terms of the initial discrete change in R and E is given by

$$d\dot{R}(0) = \tilde{R}(1 - \gamma\beta_2)dR(0) + \tilde{R}\gamma\beta_3 dE(0) \qquad (6)$$

In principle, it appears that either the long-term real rate effect, or the short-term real rate effect (which operates via the inflation rate) may dominate, in which case R(0) may either begin to rise or fall ($\dot{R}(0) \gtrless 0$). Much depends upon the flexibility of the domestic price level, as described by γ. However, if we assume that the price level moves sufficiently slowly for the fall in the long-term real rate to dominate, then R will continue to fall following its initial fall. Thus the real rate follows a time path such as illustrated by U'V' in Figure 1B. Following the monetary expansion at time T, the long-term real rate begins to rise, along with the price level, as R and P move along the locus Y_1Y_1'.

We turn now to the time paths of interest rates, illustrated in Figure 2B. As noted, at the time of announcement, the real money supply remains fixed, so that for money market equilibrium to prevail, the demand for money, and hence the short-term nominal interest rate, remains fixed instantaneously. As the price level rises during the period prior to the monetary expansion (the interval 0, T), the real money stock falls and the short-term nominal interest rate rises along AA'. At the time of the monetary expansion, the real money stock M - P rises, so that the short-term nominal interest rate falls. Thereafter, with the nominal money supply fixed at the new level and the price level continuing to rise, the real money stock falls continuously and the short-term nominal interest rate must again be rising, the short-term nominal rate must fall to a point such as B, which lies below the equilibrium level, i*, which it then approaches along the path BB'.

The initial positive inflation rate, generated by the announcement, means that on impact the short-term real rate of interest falls, as already noted. With prices rising throughout the adjustment, this also means that the short-term real rate is always less than the short-term nominal rate; see Figure 2B. The adjustment of the short-term rate during the period (0, T) depends upon whether the exchange rate increases faster than the price level. We have drawn the time path on the assumption that this is the case, so that r rises along with the nominal rate i. However, we cannot rule out the contrary behavior.

At the time of the monetary expansion (time T), the inflation rate moves continuously. This is seen from equation (1a) and (1f), where with E, P, R constrained to adjustment continuously everywhere (other than for possible jumps in the latter two at the announcement date), the same must apply to the rate of inflation $\dot{P}$. It then follows that the fall in the short-term real rate at time T must equal that of the short-term nominal rate at that time; i.e., the distances A'B, C'D in Figure 2B must be equal.

In the second part of Figure 2B we have plotted the short-term and long-term real rates. We have commented how at time 0, the long-term rate drops by an amount which is likely to exceed the fall in the short-term real rate. This means that for the arbitrage condition to hold, the long-term real rate must continue to fall (at least initially). If the short-term real rate r, beging driven primarily by the rise in the short-term nominal rate, begins to rise, then the long-term rate will lie below it and will continue to fall, in order to generate the capital gains to ensure the equality between the real rates of return on the short-term and long-term securities.

The long-term real rate falls along FF' and therefore is less than the short-term real rate, until F', when the monetary expansion takes place. At that time, the stable locus is reached and R begins to rise. At the same instance, the downward jump in the short-term rate occurs, so that DD' lies below F'G as R and r approach their common equilibrium level i*.

Looking at Figures 2B together, we see that the various interest rates being considered exhibit diverse behavior during the various phases of the adjustment. The short-run monetary contraction generated by the rising price level, which immediately follows the announcement, causes the nominal interest rate to begin rising in the short run. The short-term real rate will initially fall, but will likely then begin rising and follow the nominal rate. The long-term real rate, on the other hand, also falls initially, but in contrast to the short-term real rate, will likely continue falling, as it discounts the expected future downward jump in the short-term real rate which will take place at time T. Prior to the monetary expansion at time T, the long-term real rate lies below the short-term real rate; after the policy change is introduced, this relationship is reversed.

4. DOMESTIC FISCAL EXPANSION

A permanent unit increase in domestic government expenditure leads to a long-run appreciation of the exchange rate, while leaving the steady-state domestic price level and real rate of interest unchanged. This means that the stable locus in E-P space shifts down from XX' to X_1X_1', while YY' remains fixed in R-P space.

A. Unanticipated Increase in Government Expenditure

The effect of an unanticipated fiscal expansion on the economy is very simple. All it does is to cause the exchange rate to appreciate instantaneously to its new steady state level at N in Figure 3A, with the domestic price level and real (and nominal) interest rates remaining unchanged. The transitional dynamics generates.

B. Anticipated Increase in Government Expenditure

The response of the economy to an announced fiscal expansion is very different. At the time of the announcement, the exchange rate immediately appreciates to L in Figure 3A. The immediate response in the long-term real rate, R(0), however, is not clear. It is shown in the Appendix that in the short run, the long-term real rate of interest will rise if the interest elasticity of the demand for money, η say, evaluated at the long-run equilibrium is greater than unity (in magnitude); it will fall otherwise.

An intuitive explanation for this behavior runs as follows. A large value of the equilibrium rate of interest $\tilde{R}$, and hence for a given value of α_2, a large value of the interest elasticity of the demand for money, η, will tend to generate a low level of real private expenditure. This in turn generates a high rate of deflation, thereby leading to a high level of the short-term real rate. Now the long-term real interest rate at any moment of time is a discounted average of all the expected future short-term real rates. As we shall see below, following an announced fiscal expansion, the short-term real rate initially rises above its steady-state equilibrium and subsequently falls below this level. Thus if the elasticity η is large and r is relatively large, the positive movements in r, which occur during the initial phases of the adjustment, will dominate and initially, the long-term real rate will rise. On the other hand, if η is small so thatr is relatively low, the negative movements in r, which occur during the latter phases of the adjustment, will cominate and the initial long-term real rate will fall.

Consider the case where the interest elasticity of the demand for money is greater than unity. The initial rise in the long-term real rate, together with the appreciation of the exchange rate means that at the time of the announcement, demand for domestic output falls, causing the domestic price level to begin falling. In the other case where the elasticity exceeds unity, the fall in the long-term real rate offsets the effect of exchange rate on demand, and in general we are unable to determine which influence dominates. However, if T is taken to be sufficiently small, the exchange rate effect dominates and the domestic price level begins to fall as illustrated in Figure 3B.

With the real money stock fixed instantaneously, the short-term nominal interest rate remains fixed, so that following its initial jump, the rate of exchange depreciation is zero initially. As the price level begins to fall, the real money stock begins to rise and the short-term nominal interest rate falls in order to equilibrate the money market. The appreciation of the exchange rate, together with the falling domestic price level means that E and P move in the direction of LM in Figure 3A. At time T, when the anticipated fiscal expansion occurs, the increase in the demand for output so generated causes the domestic price level to begin rising. The exchange rate, however, will continue to fall as long as the price level has not been restored to its initial equilibrium level. This is necessary because until this occurs, there will be an increase in the real money stock and for money market equilibrium to be maintained there must be an appreciating exchange rate.

The behavior of the long-term real rate is illustrated in Figure 3B. If the interest elasticity $\eta > 1$, so that R rises initially, then R may either begin to continue rising, or it may immediately begin to start falling; i.e., $\dot{R}(0) \gtrless 0$.[8/] We have drawn it falling and initially following the path U'V'. This seems the most plausible case since we know that eventually R must return to its initial level. On the other hand, if the interest elasticity $\eta < 1$ and R initially falls, then it will definitely initially continue to fall following the initial jump; i.e., $\dot{R}(0) < 0$. A path such as U"V" will be followed. During the transition, following the announcement but prior to the fiscal expansion, the long-term real rate will have fallen below its equilibrium. At time T, when the expansion occurs, the knowledge that the short-term real rate of interest is below its equilibrium and will therefore rise in the future, causes the long-term real rate to begin rising at time T.[9/]

We now consider the time paths of the interest rates, depicted in Figure 4. The short-term nominal rate remains fixed at the time of announcement. With the price level falling during the perior prior to the expansion (0, T), the real money stock rises and the short-term nominal interest rate falls along the path AA'. At the time of the fiscal expansion, the real domestic money stock remains unchanged, so that the path for the nominal interest is continuous at that point.

However, with the price level now beginning to rise, the real money stock now begins to fall and the nominal interest rate begins to rise back up towards the equilibrium world rate.

The initial deflation at time 0, generated by the announcement, means that the short-term real rate immediately rises above the short-term nominal rate. With prices falling throughout the entire period (0, T), this means that r always lies above i during that phase. Whether the short-term rate is actually rising or falling depends upon whether the exchange rate falls faster than the price level. We have drawn the time path on the assumption that this is the case, so that r falls with i. At the time of the fiscal expansion the price level begins to rise. The short-term real rate therefore immediately falls below the short-term nominal rate. It remains there during its subsequent adjustment to equilibrium as prices continue to rise during this phase.

The short-term and long-term real rates are drawn in Figures 4B and 4C for the two cases where the latter rises in the short run and falls in the short run, respectively. Consider Figure 4B first. With R falling during period (0, T), after its initial increase, it follows from the arbitrage condition (le) that the short-term real rate must exceed the long-term real rate. When R begins to increase following the fiscal expansion, the short-term real rate drops below it and remains below during the subsequent transition.

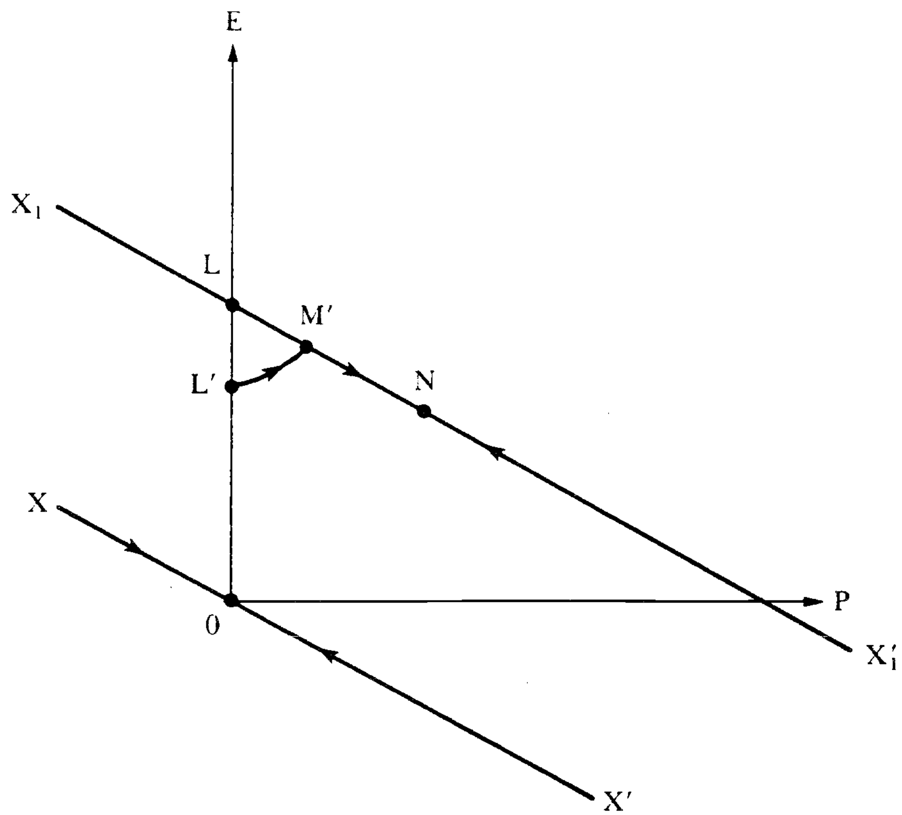
Figure 4C illustrates the case where R initially falls. While we have seen that in the short run R will continue to fall, it is reasonable to suppose that the declining price level, together with the falling

R, will tend to reduce the rate of deflation. As a consequence, the divergence between the short-term and long-term real rates is eliminated and this causes R to begin rising as it approaches the locus YY'. In that case, the time paths for the short-term and long-term real rates will intersect at the point where R reaches its minimum level.

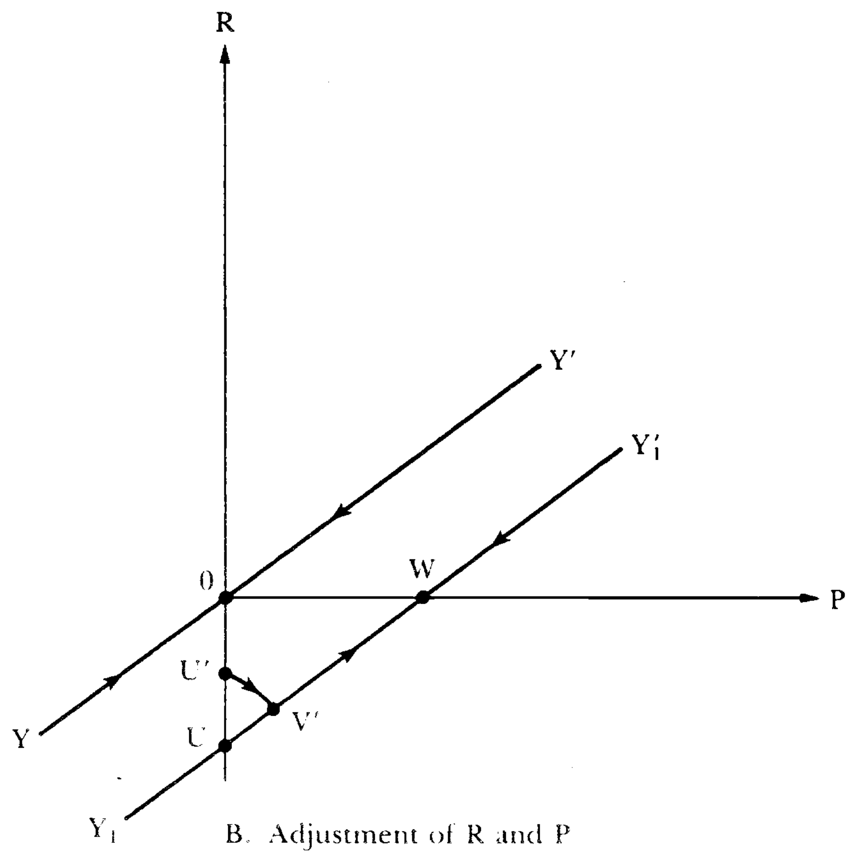
As in the case of an anticipated monetary expansion, Figure 4 illustrates the divergent behavior of the various rates of interest, in response to an anticipated fiscal expansion, particularly during the initial phase. The short-term nominal rate follows the smoothest path, gradually declining initially before eventually increasing back to its equilibrium level. The short-term real rate undergoes two jumps; an upward jump at the announcement date, a downward jump when the expansion actually occurs. During the early phase it declines gradually, while following the downward jump at time T, it gradually increases towards its equilibrium level. The long-term rate undergoes only jump, namely at the initial time of announcement. Thereafter, it will tend to decline before subsequently increasing back up to its equilibrium level. Like i it may also always lie below the equilibrium world rate, but this need not be the case.

5. CONCLUSIONS

In this paper we have analyzed the effects of monetary and fiscal disturbances, both unanticipated and anticipated, on the dynamic behavior of a monetary model of a small open economy. Attention has focused on the adjustment of the short-term and long-term interest rates and the divergence of their transitional time paths, particularly in anticipation of these disturbances. We have shown how the anticipation of a future policy change can generate perverse short-run behavior. Thus, for example, the announcement of a future fiscal expansion may cause the long-term rate to rise initially. In this case, initial contractionary effects in the economy are generated, though these are eventually reversed when the fiscal expansion occurs. It is also possible for the initial responses of the short-term and long-term rates to be in opposite directions, thereby twisting the yield curve in the short run. The essential reason for the difference in behavior of the short and long rates is the fact that the long rate is dominated by anticipated future events, while the short run is primarily determined by current influences.

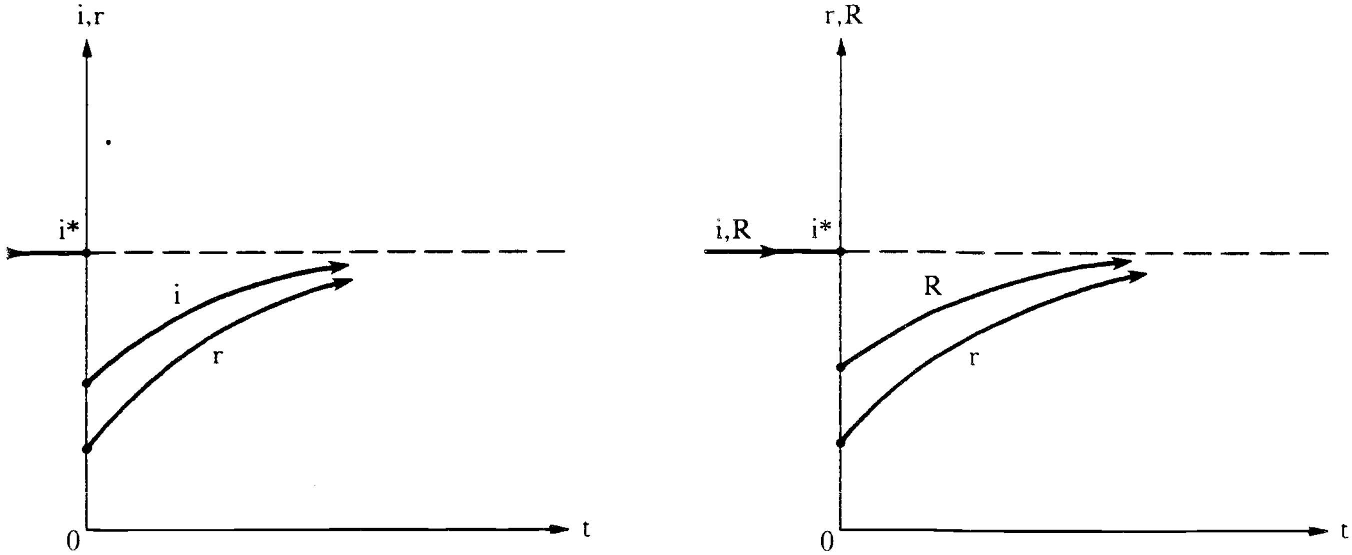


A. Adjustment of E and P

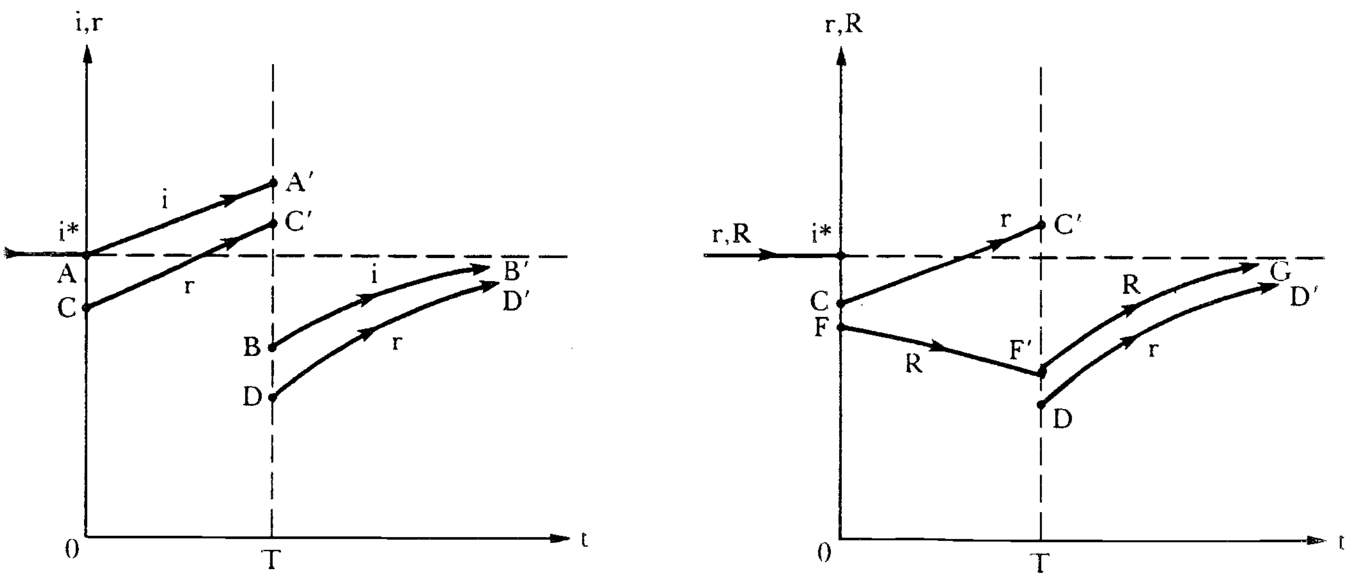


B. Adjustment of R and P

Figure 1: Response to Domestic Monetary Expansion

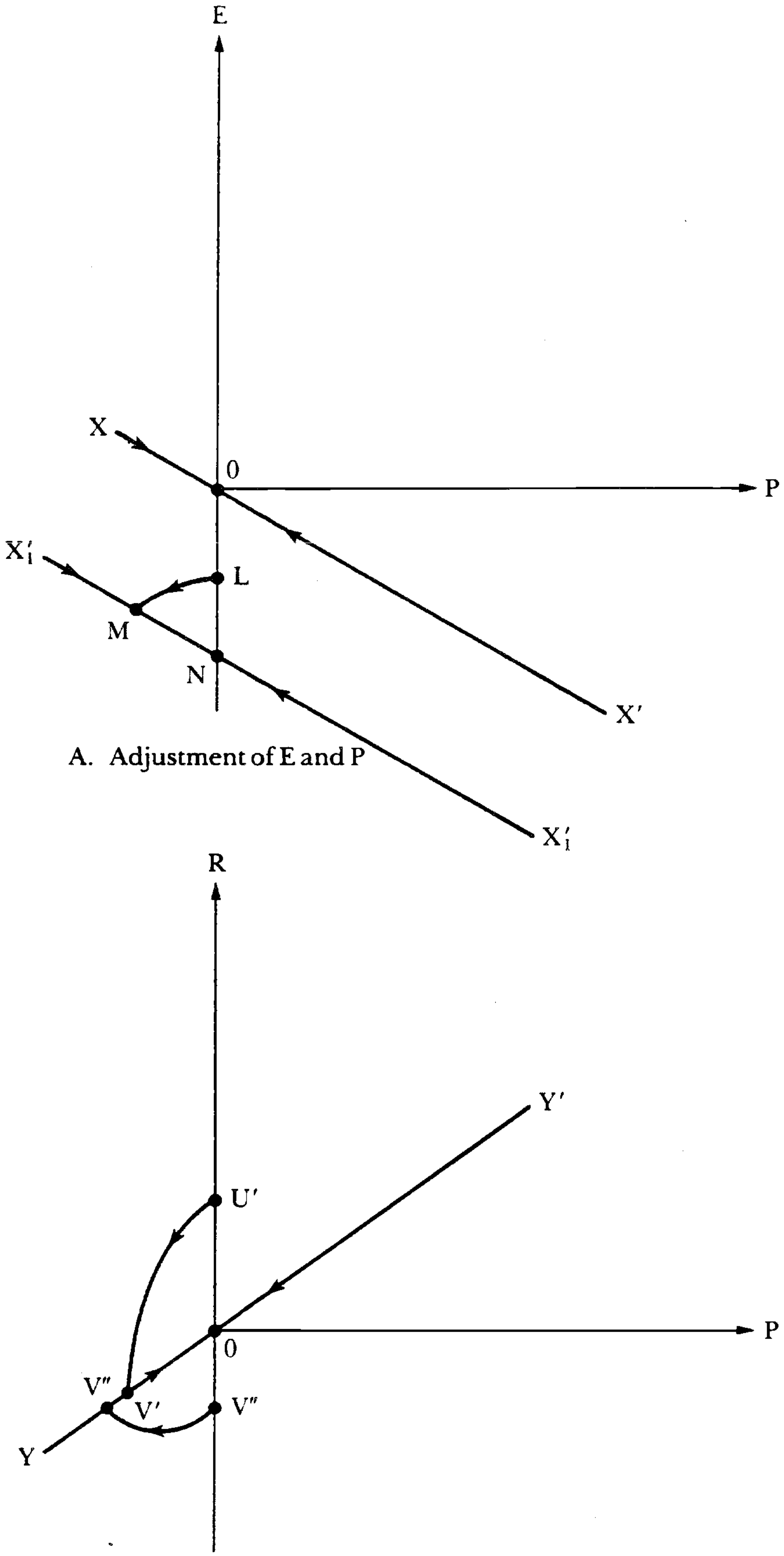


A. Unanticipated Monetary Disturbance



B. Anticipated Monetary Disturbance

Figure 2: Response of Interest Rates to Domestic Monetary Increase



A. Adjustment of E and P

B. Adjustment of R and P

Figure 3: Response to Anticipated Domestic Fiscal Expansion

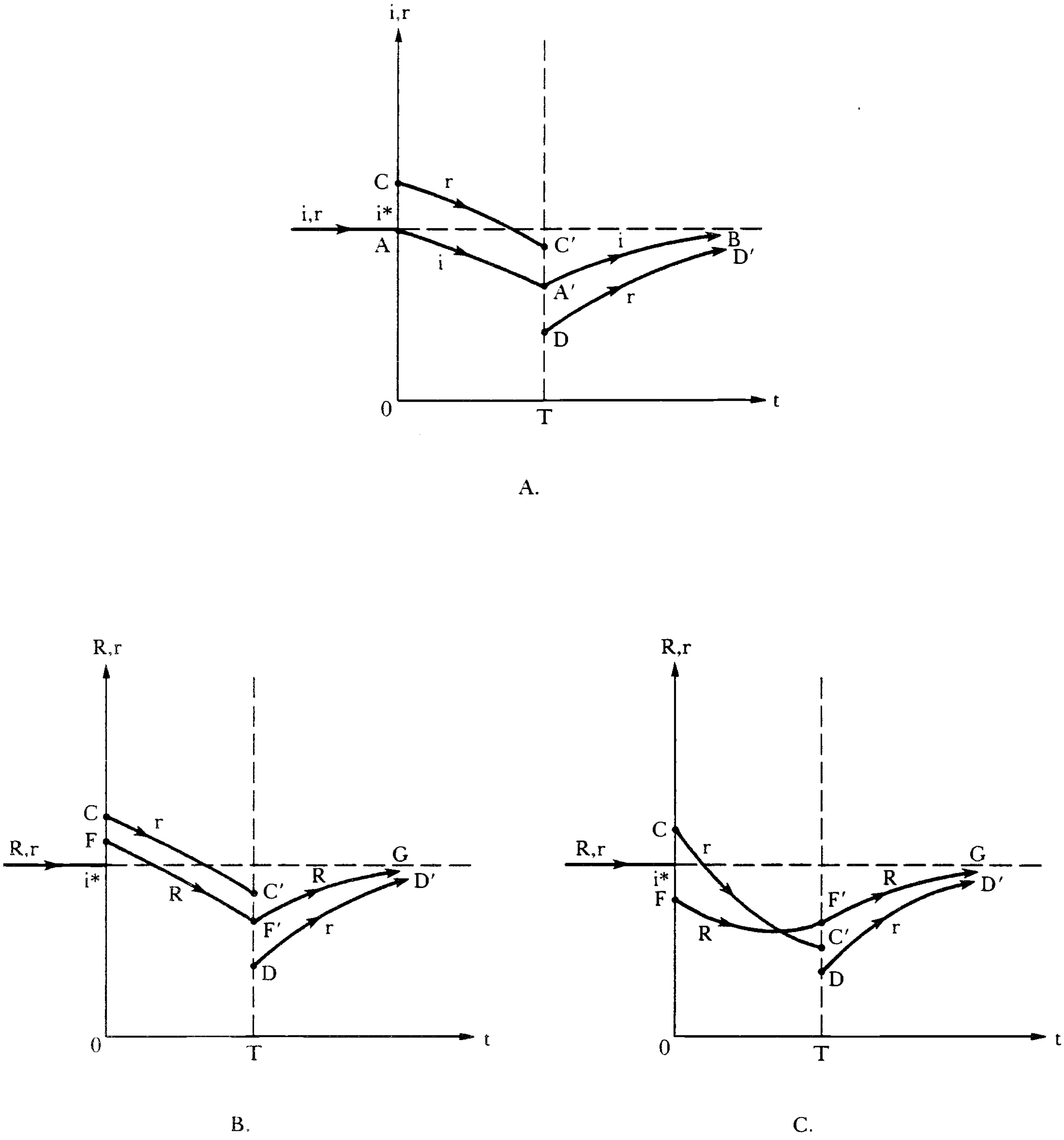


Figure 4: Response of Interest Rates to Anticipated Domestic Fiscal Expansion

FOOTNOTES

1/ The distinction between short-term and long-term interest rates in the context of an open economy is briefly considered by Sachs and Wyplosz (1984) in their simulation model.

2/ Most importantly, this would introduce the distinction between alternative measures of real interest rates.

3/ This relationship is based on the assumption of risk neutrality. To take account adequately of risk averse agents would require a full stochastic model, going beyond the scope of this paper.

4/ Equation (1e') assumes that the future short-term rates are known exactly. If this is not the case, r(t') would be replaced by r*(t',t), the prediction of the future short-term rate for time t' formed at time t. One can define an analogous relationship between the short-term and long-term nominal rates. However, since the latter does not play any role in the model, this is not required.

5/ In an expanded version of this paper, we also analyze the effects of a once-and-for-all unit increase in the foreign interest rate.

6/ For example, the limiting cases $\beta_3 \rightarrow \infty$, (purchasing power parity) and $\beta_3 \rightarrow 0$, (zero substitutability between domestic and foreign goods in demand) provide some insight into the behavior of the economy in these special cases.

7/ From equation (5a), we see

$$\frac{dE(0)}{dM} = 1 - \frac{1}{\alpha_2 \lambda_1} > 1$$

This is identical to the expression obtained by Gray and Turnovsky (1979) for the amount of exchange rate overshooting in the Dornbusch model.

8/ The behavior of $\dot{R}(0)$ can be seen from equation (6) and noting that while R(0) may either rise or fall, E(0) definitely falls.

9/ In considering the partial phase diagrams illustrated in Figure 3, it is interesting to note that if the fiscal expansion is unannounced, the adjustment is instantaneous. On the other hand, the adjustment to an anticipated disturbance takes an infinite time, even if the lead time T is small but strictly positive. In this case, the adjustments in R and P would be small, while E would jump close to its new equilibrium level. But in a strict formal sense, it would still take an infinite time to reach the new equilibrium.

APPENDIX

1. General Solution to Model

We shall write the dynamic system (4) as

$$\begin{bmatrix} \dot{R} \\ \dot{E} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ 0 & 0 & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} R-\tilde{R} \\ E-\tilde{E} \\ P-\tilde{P} \end{bmatrix} \qquad (A.1)$$

where the h_{ij} are identified by the corresponding elements in the matrix in (4). We assume that at time 0 the system is in steady state with $R = \tilde{R}_1$, $E = \tilde{E}_1$, $P = \tilde{P}_1$. At time 0 a change is anticipated to take effect at time T. The new steady state is given by $\tilde{R}_2$, $\tilde{E}_2$, $\tilde{P}_2$.

The solution to (A.1) is as follows:

$0 < t \leq T$:

$$R - \tilde{R}_1 = - \sum_{i=1}^{3} \left[\frac{\lambda_i h_{13} + h_{12}h_{23}}{(h_{11}-\lambda_i)\lambda_i} \right] C_i e^{\lambda_i t} \qquad (A.2a)$$

$$E - \tilde{E}_1 = h_{23} \sum_{i=1}^{3} \frac{C_i}{\lambda_i} e^{\lambda_i t} \qquad (A.2b)$$

$$P - \tilde{P}_1 = \sum_{i=1}^{3} C_i e^{\lambda_i t} \qquad (A.2c)$$

$t \geq T$:

$$R - \tilde{R}_2 = - \sum_{i=1}^{3} \left[\frac{\lambda_i h_{13} + h_{12}h_{23}}{(h_{11}-\lambda_i)\lambda_i} \right] C_i' e^{\lambda_i t} \qquad (A.3a)$$

$$E - \tilde{E}_2 = h_{23} \sum_{i=1}^{3} \frac{C_i'}{\lambda_i} e^{\lambda_i t} \qquad (A.3b)$$

$$P - \tilde{P}_2 = \sum_{i=1}^{3} C_i' e^{\lambda_i t} \qquad (A.3c)$$

where $\lambda_1 < 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ are the eigenvalues. There are six arbitrary constants to be determined: C_i, C_i', $i = 1, 2, 3$. They are determined as follows:

(i) In order for the system to remain bounded as $t \to \infty$, we require

$$C_2' = C_3' = 0 \tag{A.4a}$$

(ii) We assume that P evolves continuously from its initial given condition, so that

$$C_1 + C_2 + C_3 = 0 \tag{A.4b}$$

(iii) The time paths for R, E, and P are assumed to be continuous for $t > 0$. In particular, at $t = T$, the solutions for (A.2) and (A.3) must coincide. Thus

$$(C_1 - C_1')e^{\lambda_1 T} + C_2 e^{\lambda_2 T} + C_3 e^{\lambda_3 T} = \tilde{P}_2 - \tilde{P}_1 \tag{A.4c}$$

$$\frac{h_{23}}{\lambda_1}(C_1 - C_1')e^{\lambda_1 T} + \frac{h_{23}}{\lambda_2} C_2 e^{\lambda_2 T} + \frac{h_{23}}{\lambda_3} C_3 e^{\lambda_3 T} = \tilde{E}_2 - \tilde{E}_1$$

$$-\frac{[\lambda_1 h_{13} + h_{12}h_{23}](C_1 - C_1')e^{\lambda_1 T}}{(h_{11} - \lambda_1)\lambda_1} - \frac{[\lambda_2 h_{13} + h_{12}h_{23}]C_2 e^{\lambda_2 T}}{(h_{11} - \lambda_2)\lambda_2} \tag{A.4d}$$

$$\frac{-[\lambda_3 h_{13} + h_{12}h_{23}]C_3 e^{\lambda_3 T}}{(h_{11} - \lambda_3)\lambda_3} = \tilde{R}_2 - \tilde{R}_1 \tag{A.4e}$$

Thus the general solution for the system is as follows:

$\underline{0 < t \leq T:}$

$$R - \tilde{R}_1 = -\sum_{i=1}^{3} \left[\frac{\lambda_i h_{13} + h_{12}h_{23}}{(h_{11}-\lambda_i)\lambda_i}\right] C_i e^{\lambda_i t} \tag{A.2a}$$

$$E - \tilde{E}_1 = h_{23} \sum_{i=1}^{3} \frac{C_i}{\lambda_i} e^{\lambda_i t} \tag{A.2b}$$

$$P - \tilde{P}_1 = \sum_{i=1}^{3} C_i e^{\lambda_i t} \tag{A.2c}$$

$\underline{t \geq T:}$

$$R - \tilde{R}_2 = \frac{-[\lambda_1 h_{13} + h_{12}h_{23}]}{(h_{11}-\lambda_1)\lambda_1} C_1' e^{\lambda_1 t} \tag{A.3a'}$$

$$E - \tilde{E}_2 = \frac{h_{23}C_1'}{\lambda_1} e^{\lambda_1 t} \tag{A.3b'}$$

$$P - \tilde{P}_2 = C_1' e^{\lambda_1 t} \tag{A.3c'}$$

where C_1, C_2, C_3, C_1' are determined by (A.4a)-(A.4d). The values of these constants depend upon the change in the steady state. This in turn depends upon the particular disturbance to the system and in each case can be obtained from equations (3) of the text.

2. Determination of Sgn R(0) Following Announcement of Fiscal Expansion

To begin, note that λ_1, λ_2, λ_3, being eigenvalues, satisfy the characteristic equation of the dynamic system (4), namely

$$\lambda^3 - [\tilde{R}(1-\gamma\beta_2) - \gamma\beta_3]\lambda^2 - \gamma[\tilde{R}\beta_3 + \beta_3/\alpha_2 + \tilde{R}\beta_2/\alpha_2]\lambda + \gamma\beta_3\tilde{R}/\alpha_2 = 0 \tag{A.5}$$

Using the standard properties of roots of polynomials, we have

$$\lambda_1\lambda_2\lambda_3 = -\gamma\beta_3\tilde{R}/\alpha_2 \tag{A.6a}$$

$$\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = \gamma[\tilde{R}\beta_3 + \beta_3/\alpha_2 + \tilde{R}\beta_2/\alpha_2] \qquad \text{(A.6b)}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = \tilde{R}(1-\gamma\beta_2) - \gamma\beta_3 \qquad \text{(A.6c)}$$

These relationships can be easily expressed in terms of the h_{ij}. We define the quantity

$$\phi(\lambda) \equiv -\lambda^2 - \gamma\lambda[\tilde{R}\beta_2 + \beta_3] + \gamma\beta_3/\alpha_2 \qquad \text{(A.7)}$$

so that (A.5) may be written as

$$(\tilde{R} - \lambda)\phi(\lambda) + \tilde{R}\gamma\beta_2\lambda(\tilde{R} - 1/\alpha_2) = 0 \qquad \text{(A.8)}$$

In the case of a fiscal expansion,

$$\tilde{R}_2 = \tilde{R}_1 \ (= \tilde{R} \text{ say}),\ \tilde{P}_2 - \tilde{P}_1 = 0,\ \tilde{E}_2 - \tilde{E}_1 = -1/\beta_3$$

Solving equations (A.4b)-(A.4e) for C_1, C_2, C_3, C_1', and substituting back into (A.2a), we may express the solution as

$$R(0) - \tilde{R} = \frac{(h_{23}h_{32} + \lambda_1\lambda_3)(h_{23}h_{32} + \lambda_1\lambda_2)}{\beta_2 h_{23}h_{31}\lambda_1}\left[\frac{e^{-\lambda_3 T} - e^{-\lambda_2 T}}{\lambda_2 - \lambda_3}\right] \qquad \text{(A.9)}$$

where $\lambda_1 < 0$, $\lambda_2 > 0$, $\lambda_3 > 0$. The term $[e^{-\lambda_3 T} - e^{-\lambda_2 T}]/(\lambda_2-\lambda_3) > 0$ and thus

$$\text{sgn}[R(0) - \tilde{R}] = \text{sgn}\{(h_{23}h_{32} + \lambda_1\lambda_3)(h_{23}h_{32} + \lambda_1\lambda_2)\} \qquad \text{(A.10)}$$

Multiplying out the expression on the right hand side of (A.10) and using (A.6a)-(A.6c), we find

$$(h_{23}h_{32} + \lambda_1\lambda_3)(h_{23}h_{32} + \lambda_1\lambda_2) = \frac{\gamma\beta_3}{\alpha_2}\phi_1$$

where

$$\phi_1 \equiv \phi(\lambda_1)$$

Thus

$$\text{sgn}[R(0) - \tilde{R}] = \text{sgn}\ \phi_1 \tag{A.11}$$

Now setting $\lambda = \lambda_1$ in (A.8) yields

$$\phi_1 = \frac{\tilde{R}\gamma\beta_2\lambda_1(1 - \alpha_2\tilde{R})}{\alpha_2(\tilde{R}-\lambda_1)} \tag{A.12}$$

so that

$$\text{sgn}\ \phi_1 = \text{sgn}\ [\alpha_2\tilde{R} - 1] \tag{A.13}$$

Since the money supply is measured in logarithms $-\alpha_2$ is the semi-elasticity of demand, so that $-\alpha_2 R$ measures the interest elasticity of demand, evaluated at steady state equilibrium. Letting $\gamma = \alpha_2\tilde{R}$, and combining (A.11) and (A.12) we obtain

$$\text{sgn}[R(0) - \tilde{R}] = \text{sgn}(\gamma-1) \tag{A.14}$$

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