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Piecemeal Oligopoly, Exchange Rate Uncertainty, and Trade Policy

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Resumen. Este artículo analiza el efecto sistemático de la volatilidad de la tasa de cambio, cuando un gobierno local debe evaluar políticas comerciales estratégicas lineales y cuadráticas. Este ejercicio se realiza para modelos de mercado Cournot y Bertran. El modelo prueba que tanto el esquema lineal como el cuadrático tienen el mismo efecto sobre el bienestar social de los paises, y que la volatilidad de la tasa de cambio doméstica lleva a los gobiernos a reducir los subsidios a las exportaciones o bajan los impuestos a las exportaciones, de acuerdo a la variable estratégica elegida por las firmas. La tasa de cambio extrangera tiene diferentes efectos dependiendo de si las firmas producen bajos rendimientos a escalas constantes o decrecientes.

Palabras clave: Comercio, Estructuras de Mercado y de Precios, Estrategia de Firma, Información e Incertidumbre.

Clasificación JEL: D43,D81, F13, L13.

Abstract. This paper examines the systematic effect of the exchange rate volatility, when a local government has to evaluate linear and quadratic strategic trade policies. The exercise is realized for both Cournot and Bertrand markets fashions. The model proves that the linear or quadratic scheme have the same effect on the countries' social welfare, and that the volatility of the domestic exchange rate leads governments to reduce export subsidies or to cut export taxes, according to the strategic variable chosen by firms. The foreign exchange rate volatility has different effects if firms produce under constant return or decreasing return of scale.

Keywords: Trade, Markets Structure and Pricing, Firm Strategy, and information and uncertainty.

JEL classification: D43,D81, F13, L13.

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1. Introduction

Under assumptions of imperfect competition in external markets and strategic behaviour by firms, prices are not equal to marginal costs, as the competitive markets analysis shows, and lucky firms in some industries may be able to earn returns higher than the opportunity costs of the resources they employ. Indeed, international economists have long recognized that governments can participate in external markets in order to transfer part of foreign firms' potential profits to domestic firms (Brander and Spencer, 1985).

Eaton and Grossman (1986) provided an integrative treatment of the effects of trade policy on national welfare under imperfect competition. They showed that the results are sensitive to the character of the competition. While export subsidies are desirable in a Cournot market (where competition is based on quantity of output), they are never desirable in a Bertrand market (competition based on price), where an export tax is more appropriate.

So under specific conditions, a public policy of taxing or subsidizing exports can be a strategic move that tilts the international competition in favour of domestic firms. This outcome increases national welfare as it has a deterrent effect on foreign competition. Yet few studies have attempted to test the effects of government intervention by constructing formal models of trade under uncertainty, both in the market demand and cost functions. This paper extends the new international trade theory by linking theoretical underpinnings with new insights from the real world.

In contrast to the few models where uncertainty is introduced through external shocks (Cooper and Riezman, 1989; Laussel, 1992; Qiu, 1995; and Caglayan, 2000), this model systematically adds uncertainty directly to the exchange rate. This is an important factor, and it better captures the reality of firms in the international market, which have to include this variable in decisions.[1]

Klemperer and Meyer (1986) describe the strategic trade policy as an endogenous instrument where firms identify the best type of competition. Its framework is constructed under uncertainty conditions, relayed through an external shock in demand. Klemperer and Meyer's main finding is that firms play a Cournot game if the total cost function is convex, but prefer a Bertrand game if the total cost function is concave.[2]

Qiu (1995) extends Brander and Spencer (1985) by incorporating the main results derived in Klemperer and Meyer. Qiu introduces a linear-quadratic trade policy to show that a linear export subsidy is strictly dominated by a non-linear subsidy scheme.

In contrast to the above studies, the present model introduces uncertainty via the exchange rates used by firms and governments. The main finding is that the export tax or the export subsidy decreases as uncertainty in the domestic

[1] The main motivation of the paper is to analyze the uncertainty of the exchange rate on the trade policy decisions, and not the general uncertainty in the demand function.

[2] Symeonis (2003) compares Bertrand and Cournot equilibriums in a differentiated duopoly with substitute goods and product R&D showing that a Cournot equilibrium can be more or less efficient than a Bertrand equilibrium depending on the extent of R&D spillover and the degree of product differentiation.

exchange rate increases. However, if the uncertainty is related to the foreign exchange rate, the export subsidy or the export tax levels could be set higher.

These results help clarify the debate as to whether public trade policy should be oriented toward subsidising or taxing exports. The answer depends on the type of competition, the level of efficiency and the uncertainty condition that firms face. Whenever the competition is in quantities, efficiency is high, and uncertainty is low, governments should offer subsidies to domestic exporters. When the competition is in price, efficiency is high, and uncertainty is low, government should impose taxes on exports. Such trade policy decisions are the best to maximize social welfare in the economies.

The elements of the model are as follows. There are three countries: the home country, a foreign country, and a third country that is the sole market for one differentiated product produced in both the domestic country and in the foreign one. In addition, there is only one firm in the domestic country and only one in the foreign country. The level of welfare in the domestic and foreign countries is determined by the profits of the respective firms, net of any government subsidy or tax. The focus of the analysis is on the domestic country's welfare.

The study restricts its domain to international competition by assuming that there is no demand for the differentiated good in the two producing countries and that the domestic and foreign firms compete only in the third country's market. This artificial assumption neglects the effects of trade policy on the domestic consumer but allows the model to focus on international trade effects as a criterion in the formulation of economic policy.

The goal is to characterise the Nash equilibrium for a two stage sequential game and to derive the optimum public policy for the domestic country with respect to an export tax or subsidy. In the first stage of the game, the domestic government sets trade policy. In the second stage, the domestic and foreign firms simultaneously choose their output or price levels for the third country's market, given the level of the domestic trade policy intervention. At the end of the period, the uncertainty in the exchange rates is resolved. Using backward induction to analyse the rational Nash equilibrium for the entire game, we consider the second stage of the game first, then consider the initial stage. The domestic government acts as a Stackelberg leader vis-à-vis both the domestic and the foreign firm in setting the subsidy or tax rates. Thus the firms set outputs or prices, taking the subsidy or tax rates as given.

This paper is divided into four sections. The model framework under uncertainty conditions is constructed in the next section. The international trade policy for the Cournot and the Bertrand market settings are examined in sections 2 and 3, respectively. The relations between the export tax or export subsidy, and changes in the variance of the exchange rate currencies are derived for each type of market. Finally, concluding remarks are made in section 4.

2. The formal framework

In subsequent sections the optimal public policy is characterised under the presence of oligopolistic competition, where the domestic and foreign firms are both incumbents and sell all of their outputs in the international market.

To simplify notation, we refer to the firm in the home country as the domestic firm and in the foreign country as the foreign firm. Variables relating to the domestic firm and the foreign firm are identified by subscripts d and f, respectively. Variables associated with the third country are identified by an asterisk. The domestic firm produces the quantity X_d and the foreign firm produces the quantity X_f.

The linear inverse demand function for differentiated products in the third country is written as

$$P_i^* = \alpha - \beta X_i - \zeta X_j, \tag{1}$$

where $i = d,\ f$; and $0 < \zeta < \beta$. The international price (P_i^*) is measured in terms of the third country's currency. The symbol ζ measures the degree of product differentiation. If $\zeta = 0$, varieties are completely differentiated, and each producer is then a monopolist with respect to its own brand. If $\zeta = \beta$, products are completely homogeneous or standardised. Hence the values of ζ, between 0 and β, describe all cases in which goods are imperfect substitutes (Martin 1993).

We assume that firms have the following cost structures

$$C_i\left(X_i\right) = C_{i1}X_i + \frac{1}{2}C_{i2}X_i^2\ , \tag{2}$$

C_{i1} and C_{i2} being parameters. When $C_{i2} > 0$ firms produce under decreasing returns, and if $C_{i2} = 0$ firms produce under constant returns. The study does not consider explicitly fixed cost levels since these do not effect the comparative static analysis. Also the increasing returns are not taken into account because the equilibrium of the model might present stability problems.

The domestic export subsidy is a function of export quantity. In particular, the trade policy has the same mathematical expression used by Qiu (1995), in that is a linear-quadratic scheme. This equation is

$$S\left(X_d\right) = S_1 X_d + \frac{1}{2}S_2 X_d^2\ , \tag{3}$$

where S_1 and S_2 are parameters. If $\mathrm{S}_1 \neq 0$ and $\mathrm{S}_2 = 0$ the domestic government pre-commit to a linear scheme, but if $\mathrm{S}_1 = 0$ and $\mathrm{S}_2 \neq 0$ the government pre-commit to a non-linear scheme. To simplify the model, the assumption is made that the foreign government is passive, i.e. it does not apply any policy against the domestic government.

The profit functions under uncertainty for the domestic and foreign firms are

$$\tilde{\pi}^d = \tilde{e}_d P_d^* X_d - C\left(X_d\right) + S\left(X_d\right), \tag{4}$$

$$\tilde{\pi}^f = \tilde{e}_f P_f^* X_f - C\left(X_f\right)\ , \tag{5}$$

The tilde above the domestic and foreign exchange rates ($\tilde{e}_d$, $\tilde{e}_f$) refers to uncertain conditions.[3] The same conditions are applied to the profit outcomes ($\tilde{\pi}^d$, $\tilde{\pi}^f$). Firms maximize the certainty equivalence, as it is used in the Markowitz model of mean-variance analysis of portfolio selection. Then profits are a linear combination of the expected value and the standard deviation of profits. The last term is defined as the product of the relative measure of risk preference (γ_i) and the standard deviation of profits.[4] Thus the equation is

$$\pi_i^c = E\left[\tilde{\pi}_i\right] - \gamma_i SD\left[\tilde{\pi}_i\right]. \tag{6}$$

The certainty equivalence device guarantees that firm i can solve its profit maximization problem by setting the price or the quantity. Therefore firms choose the optimal export quantity or export price when they maximize the certainty equivalence of their profits, depending on the conjecture that each firm makes on the others' choice. The certainty equivalence of the domestic or foreign firms' profits are

$$\pi_c^d = \mathbf{a}_i P^* X_i - C\left(X_i\right) + S\left(X_i\right) \ , \tag{7}$$

where $\mathbf{a}_i$ is the certainty equivalence of the domestic or the foreign exchange rates, and they are defined in terms of the expected value μ_e and the standard deviation σ_e of the respective exchange rates. It is also important to notice that firms only export if $\mathbf{a}_i > 0$. In particular,

$$\mathbf{a}_i = \mu_{e_i} - \gamma_i \sigma_{e_i} > 0 \ . \tag{8}$$

The distributions of the random variables are log-normal such that

$$\mu_{e_i} = e^{\mu_i + \frac{1}{2}\sigma_i^2},$$

$$\sigma_{e_i}^2 = e^{2\mu_i + \sigma_i^2}\left(e^{\sigma_i^2} - 1\right).$$

Given that all production is exported, then domestic government wishes to maximise the local value added, which is defined as the sum of the profit of the domestic country's firm, excluding any cost from the trade policy. The social welfare function is written as

$$\tilde{W} = \tilde{\pi}^d - S\left(X_d\right) \ ,$$

[3]Exchange rates are exogenous, given that the model is constructed under partial equilibrium framework. An endogenous exchange rate should be analyzed in a general equilibrium model.

[4]See, e.g. Newbery and Stiglitz (1981) for a discussion on this concept. As has been shown in Newbry and Stiglitz, the formulation bellow does not need any approximation if the utility function is of a particular type and the random variable follow a normal distribution. Also some foundations are presented by Jenseen (1972). Papers that used this theoretical approach before (and where the random variable is exclusively the exchange rate) were Hooper and Kohlhagen (1978), Cushman (1985), Viaene and de Vries (1992), and recently Lahiri and Mesa (2006).

which is simplified as

$$\tilde{W} = \tilde{e}_d P_d^* X_d - C(X_d). \tag{9}$$

The certainty equivalence of the social welfare expression is[5]

$$W_d^c = \mathbf{a}_d P_d^* X_d - C(X_d). \tag{10}$$

This completes the basic description of the framework of analysis. In the next two sections it is carry out the comparative static of the exchange rate uncertainty effects on the trade policy decisions.

3. Cournot Duopoly Market

Firstly firms set quantities in the second stage of the sub-game. As usual, under Cournot conditions each firm takes as given its rival's sales and finds its best response. After that, the two international trade policy schemes are introduced. Finally, the relation between trade policy and exchange rate volatility is assessed for each policy scheme.

3.1. *Basic Results*

Firms set their outputs to maximize their profits. The first order conditions for each firms are

$$\frac{d\left(\pi_c^d\right)}{dX_d} = \mathbf{a}_d P_d^* + S_1 - C_{d1} - \left(\mathbf{a}_d\beta + C_{d2} - S_2\right) X_d = 0\ , \tag{11}$$

$$\frac{d\left(\pi_c^f\right)}{dX_f} = \mathbf{a}_f P_f^* - C_{f1} - \left(\mathbf{a}_f\beta + C_{f2}\right) X_f = 0 \tag{12}$$

In the Cournot-Nash equilibrium we have

$$X_d^* = \frac{\left(\alpha + \frac{S_1}{\mathbf{a}_d} - \frac{C_{d1}}{\mathbf{a}_f}\right)\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right) - \alpha\zeta}{\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right)\left(2\beta + \frac{C_{d2}}{\mathbf{a}_d} - \frac{S_2}{\mathbf{a}_d}\right) - \zeta^2}, \tag{13}$$

$$X_f^* = \frac{\left(\alpha - \frac{C_{f1}}{\mathbf{a}_f}\right)\left(2\beta + \frac{C_{d2}}{\mathbf{a}_d} - \frac{S_2}{\mathbf{a}_d}\right) - \zeta\left(\alpha - \frac{C_{d1}}{\mathbf{a}_d} + \frac{S_1}{\mathbf{a}_d}\right)}{\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right)\left(2\beta + \frac{C_{d2}}{\mathbf{a}_d} - \frac{S_2}{\mathbf{a}_d}\right) - \zeta^2}. \tag{14}$$

The domestic firm's output is positively related to the market size (α); and negatively related to its respective marginal costs ($C_{d1}/\mathbf{a}_d$ and $C_{d2}/\mathbf{a}_d$). The foreign firm's output is positively related to the market size (α), and the linear domestic firm's marginal cost, ($C_{d1}/\mathbf{a}_d$); and negatively related to the foreign firm's marginal cost ($C_{f1}/\mathbf{a}_f$, and $C_{f2}/\mathbf{a}f$).

[5]Since different relative risk preferences between the government (γ_g) and the domestic firm (γ_d) becomes cumbersome to obtain any clear cut results, it is assumes that both are equals ($\gamma_d = \gamma_g$). This assumption is justified in the sense that the optimal public policy is taken under some private sector pressure, so that government risk aversion coefficient might be assimilated to the private firm.

3.2. Optimal Public Policy

The first stage of the game, when the domestic government determines optimal public policy is now analyzed. The aim of this subsection is to compare the optimal linear scheme to the optimal quadratic subsidy scheme. The relation between the optimum trade policy level and the volatility of the exchange rate currencies is analysed as well. It is important to notice that only one country applies trade policy and the other one is passive and does not retaliate to the domestic policy.

3.2.1. CASE 1: Linear Subsidy Scheme

In this case the government pre-commits to offer a linear subsidy scheme, i.e. $S_2 = 0$, and $S_1 \neq 0$. The reduced social welfare is found by substituting the Nash equilibrium outputs (13) and (14) into (10). The optimum public trade policy is obtained by setting $dW_{c1}/dS_1 = 0$. The process yields[6]

$$S_1^* = \mathbf{a}_d \zeta^2 \frac{\left[\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right)\left(\alpha - \frac{C_{d1}}{\mathbf{a}_d}\right) - \zeta\left(\alpha - \frac{C_{f1}}{\mathbf{a}_f}\right)\right]}{\left[\left(2\beta + \frac{C_{d2}}{\mathbf{a}_d}\right)\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right) - 2\zeta^2\right]\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right)}. \tag{15}$$

In the oligipoly theory an export subsidy shifts the domestic firm's downward sloping reaction curve and this intersects the foreign firm's reaction function at a higher domestic output and lower foreign output. Therefore the subsidy policy raises domestic welfare in the Cournot equilibrium by transferring industry profit to the domestic firm.

I.- Relation between optimal linear subsidy (S_1^*), and changes in the variance of the domestic exchange rate (σ_d^2).

a) Under constant returns ($C_{d2} = 0$, and $C_{f2} = 0$):

$$\frac{dS_1^*}{d\sigma_d^2} = \zeta^2 \frac{\left[(2\beta - \zeta)\,\alpha + \zeta \frac{C_{f1}}{\mathbf{a}_f}\right]}{4\beta\,(2\beta^2 - \zeta^2)} \mathbf{a}_{d\sigma_d^2} < 0. \tag{16}$$

Since the derivative of the certainty equivalence of the domestic exchange rate $\mathbf{a}_{d\sigma_1^2}$ is negative, equation (16) is negative. That is, the level of optimal intervention decreases as the variance in the domestic exchange rate increases.

b) Under decreasing returns ($C_{d2} > 0$, and $C_{f2} > 0$):

$$\frac{dS_1^*}{d\sigma_d^2} = \zeta^2 \frac{\Omega_1}{\left[\left(2\beta + \frac{C_{d2}}{\mathbf{a}_d}\right)\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right) - 2\zeta^2\right]^2\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right)} \mathbf{a}_{d\sigma_d^2}\,, \tag{17}$$

[6] In case that the foreign government retaliates against the domestic policy the expected optimal public policies in both countries would be lower than in unilateral policy decision.

where

$$\begin{aligned}\Omega_1 &= \left(2\alpha - \frac{C_{d1}}{\mathbf{a}_d}\right)\frac{C_{d2}}{\mathbf{a}_d}\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right)^2 + 2\zeta^3\alpha \\ &+2\alpha\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right)\left[\beta\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right) - \zeta\left(\beta + \zeta + \frac{C_{d2}}{\mathbf{a}_d}\right)\right] \\ &+2\zeta\frac{C_{f1}}{\mathbf{a}_f}\left[\left(\beta + \frac{C_{d2}}{\mathbf{a}_d}\right)\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right) - \zeta^2\right].\end{aligned} \quad (18)$$

Since $\Omega_1 > 0$ and $\mathbf{a}_{d\sigma_d^2} < 0$, therefore $dS_1/d\sigma_d^2 < 0$.

Thus, under either constant or decreasing returns optimal export subsidy is decreasing in σ_1^2. This result is similar to that in De Meza (1986), who concluded that countries with low costs will set the high subsidies. Here a higher variance effectively increased unit costs and therefore reduces the optimal level of subsidy. If the variance were sufficiently high the optimal policy would be to revoke the trade policy. This last outmost situation could occur if and only if the value of $\mathbf{a}_d$ was quite small.

II.- Relation between the linear subsidy (S_1^*), and changes in the variance of the foreign exchange rate (σ_f^2).

a) Under constant returns ($C_{d2} = 0$, and $C_{f2} = 0$):

$$\frac{dS_1^*}{d\sigma_f^2} = -\frac{\zeta^3 \frac{C_{f1}}{\mathbf{a}_f}\mathbf{a}_d}{4\beta\left(2\beta^2 - \zeta^2\right)}\frac{\mathbf{a}_{f\sigma_f^2}}{\mathbf{a}_f} > 0. \quad (19)$$

Since $\mathbf{a}_{f\sigma_f^2} < 0$, then $dS_1^*/d\sigma_f^2$ is positive. That is, higher variance of the foreign exchange rate increases optimal subsidy. This is because a higher σ_f^2 effectively raises foreign costs and therefore reduces the relative costs of the domestic firm.

b) Under decreasing returns ($C_{d2} > 0$, and $C_{f2} > 0$):

The derivative of the export subsidy with respect to σ_f^2 has an ambiguous sign. Here we shall examine if dW_{c1}/dS_1 increases or decreases with σ_f^2. If this derivative is positive, the optimal value of S_1^* increases with σ_f^2 as well. The derivative is

$$\begin{aligned}\frac{d\frac{dW_{c1}}{dS_1}}{d\sigma_f^2} &= \left\{-\frac{\zeta^2\left(\frac{C_{f2}}{\mathbf{a}_f}\left(\alpha + \zeta\frac{C_{f1}}{\mathbf{a}_f}\right) + \frac{C_{f1}}{\mathbf{a}_f}\right)}{\left(\left(2\beta + \frac{C_{d2}}{\mathbf{a}_d}\right)\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right) - \zeta^2\right)^2}\right. \\ &\left.+\frac{S_1}{\mathbf{a}_d}\frac{2\frac{C_{f2}}{\mathbf{a}_f}}{\left(2\beta + \frac{C_{d2}}{\mathbf{a}_d}\right)\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right) - \zeta^2}\right\}\frac{\mathbf{a}_{f\sigma_f^2}}{\mathbf{a}_f}.\end{aligned} \quad (20)$$

The right-hand side of equation (20) has two terms with different signs. Since $\mathbf{a}_{f\sigma_f^2}$ is negative, the first term is positive and the second term is negative. The positive term will dominate if the market size (α) and the level of the foreign firm's marginal costs ($C_{f1}/\mathbf{a}_f$) are high enough. Otherwise the subsidy could decrease.

The intuition behind the results under decreasing returns could be understood as follows. If the values of α and $C_{f1}/\mathbf{a}_f$ are small, the domestic government does not need to offer the same subsidy level to the domestic firm when σ_f^2 is large, since the trade policy becomes costly for the economy in terms of welfare. Conversely, if α and $C_{f1}/\mathbf{a}_f$ are high enough, it is optimal for the government to increase the subsidy, since this subsidy allows the domestic firm to raise its market share in the third market and as well to increase the welfare in the economy.

As a general rule, high volatility of the foreign exchange rate raises the domestic profit under both constant and decreasing returns if α and $C_{f1}/\mathbf{a}_f$ are large enough.

3.2.2. CASE 2: Quadratic Subsidy Scheme

This case is derived when the government pre-commits to offer a quadratic subsidy, i.e. $S_1 = 0$, and $S_2 \neq 0$. The optimum public trade policy is obtained by setting $dW_{c1}/dS_2 = 0$. The result is,

$$S_2^* = \frac{\mathbf{a}_d \zeta^2}{2\beta + \frac{C_{f2}}{\mathbf{a}_f}} > 0, \tag{21}$$

which is always positive, and this means that the government should offer a subsidy to the domestic firm.

I.-Relation between the quadratic subsidy scheme (S_2^*), and changes in the variance of the domestic exchange rate (σ_d^2).

a) Under constant returns:

$$\frac{dS_2^*}{d\sigma_d^2} = \frac{\zeta^2}{2\beta} \mathbf{a}_{d\sigma_d^2} < 0. \tag{22}$$

Since $\mathbf{a}_{d\sigma_d^2}$ is negative, these is always a negative relation between the optimal quadratic scheme subsidy and σ_d^2.

b) Under decreasing returns:

$$\frac{dS_2^*}{d\sigma_d^2} = \frac{\zeta^2}{2\beta + \frac{C_{f2}}{\mathbf{a}_f}} \mathbf{a}_{d\sigma_d^2} < 0. \tag{23}$$

So, for both constant and decreasing returns the derivatives of the quadratic subsidy scheme with respect to σ_d^2 are negative. The volatility of the domestic exchange rate makes the incumbent domestic firm less competitive in the international market; therefore the domestic government should cut the export subsidy in order to maximize the national welfare.

II.- Relation between the quadratic subsidy scheme (S_2^*), and changes in the variance of the foreign exchange rate (σ_f^2).

a) Under constant returns:

$$\frac{dS_2^*}{d\sigma_f^2} = 0. \tag{24}$$

When the government offers a quadratic scheme, the domestic trade policy is not affected by σ_f^2.

b) Under decreasing returns:

$$\frac{dS_2^*}{d\sigma_f^2} = \frac{\mathbf{a}_d \zeta^2 \frac{C_{f2}}{\mathbf{a}_f}}{\left(2\beta + \frac{C_{f2}}{\mathbf{a}_f}\right)^2} \frac{\mathbf{a}_{f\sigma_f^2}}{\mathbf{a}_f} < 0. \tag{25}$$

Note that comparative static effects of a change in σ_f^2 gives quite different results for S_2^* than for S_1^*.

The main theoretical results in this section could be summarised in the following proposition:

If firms play a Cournot game, the optimum public policy is to offer an export subsidy.

1) *When σ_d^2 increases, the optimum export subsidy should be reduced under any scheme subsidy.*

2) *When σ_f^2 increases, we have the following situations:*

   a) *under a linear scheme, the subsidy will increase if the firms produce under constant returns, and under decreasing returns it will increase if and only if the α is large and $C_{f1}/\mathbf{a}_f$ are high.*

   b) *under a quadratic scheme and constant returns the trade policy is not affected, but under increasing marginal cost the export subsidy always decreases.*

4. Bertrand Duopoly Market

This strategy is adopted when firms set prices in the second-stage of the game. In the first stage, the two optimum trade policies schemes are introduced, and after that the relation between the trade policy and the effects of variance changes are analyzed for each type of scheme.[7]

4.1. Basic Results

To derive optimum subsidy in the Bertrand duopoly fashion, the same types of functions introduced in the Cournot case are used here. The external linear

[7] Once again, similar policy effects can be obtanied if we change γ_d (γ_f) instead of σ_1^2 (σ_2^2).

demand functions for the differentiated competing products from each firm are defined as

$$X_d = A + \theta P_f^* - BP_d^* \ , \tag{26}$$

$$X_f = A + \theta P_d^* - BP_f^*. \tag{27}$$

Equations (26) and (27) are derived by solving (1) in terms of P_d^* and P_f^*. The parameters of the direct demand functions link with those of inverse demand functions used in equation (1) are

$$A = \frac{\alpha}{\beta + \varsigma}, \ \theta = \frac{\varsigma}{\beta^2 - \varsigma^2}; \ B = \frac{\beta}{\beta^2 - \varsigma^2}. \tag{28}$$

The first order conditions for the two firms in a Bertrand-Nash equilibrium are

$$\begin{aligned} \frac{d\left(\pi_c^d\right)}{dP_d^*} &= \mathbf{a}_d \left(A + \theta P_f^* - 2BP_d^*\right) + B\left[C_{d1} - S_1\right. \\ &\quad \left. + \left(C_{d2} - S_2\right)\left(A + \theta P_f^* - BP_d^*\right)\right] = 0 \ , \end{aligned} \tag{29}$$

$$\begin{aligned} \frac{d\left(\pi_c^f\right)}{dP_f^*} &= \mathbf{a}_f \left(A + \theta P_d^* - 2BP_f^*\right) + B\left[C_{d1}\right. \\ &\quad \left. + C_{f2}\left(A + \theta P_d^* - BP_f^*\right)\right] = 0 \ . \end{aligned} \tag{30}$$

The Bertrand-Nash equilibrium prices are solved as

$$P_d^* = \frac{\Theta + B\left(\frac{2}{B} + \frac{C_{f2}}{\mathbf{a}_f}\right)\left(\frac{C_{d1}}{\mathbf{a}_d} - \frac{S_1}{\mathbf{a}_d}\right)}{G\left(\frac{2}{B} + \frac{C_{d2}}{\mathbf{a}_d} - \frac{S_2}{\mathbf{a}_d}\right) + \frac{\theta^2}{B}\left(\frac{1}{B} + \frac{C_{f2}}{\mathbf{a}_f}\right)} \ , \tag{31}$$

$$P_f^* = \frac{D\left(\frac{2}{B} + \frac{C_{d2}}{\mathbf{a}_d} - \frac{S_2}{\mathbf{a}_d}\right) - \theta\left(\frac{1}{B} + \frac{C_{f2}}{\mathbf{a}_f}\right)\left(\frac{A}{B} - \frac{C_{d1}}{\mathbf{a}_d} + \frac{S_1}{\mathbf{a}_d}\right)}{G\left(\frac{2}{B} + \frac{C_{d2}}{\mathbf{a}_d} - \frac{S_2}{\mathbf{a}_d}\right) + \frac{\theta^2}{B}\left(\frac{1}{B} + \frac{C_{f2}}{\mathbf{a}_f}\right)} \ , \tag{32}$$

where

$$\Theta = \left[\left((\theta + B)\left(\frac{1}{B} + \frac{C_{f2}}{\mathbf{a}_f}\right) + 1\right)A + \theta\frac{C_{f1}}{\mathbf{a}_f}\right]\left(\frac{1}{B} + \frac{C_{d2}}{\mathbf{ad}} - \frac{S_2}{\mathbf{a}_d}\right) > 0,$$

$$D = (\theta + B)\left(\frac{1}{B} + \frac{C_{f2}}{\mathbf{a}_f}\right)A + B\frac{C_{f1}}{\mathbf{a}_f} > 0, \tag{33}$$

$$G = \left(B^2 - \theta^2\right)\left(\frac{2}{B} + \frac{C_{f2}}{\mathbf{a}_f}\right) + \frac{\theta^2}{B} > 0. \tag{34}$$

4.2. *Optimal Public Policy*

We now consider the first stage game where the domestic government decides the optimal trade policy.

4.2.1. CASE 1: Linear Tax Scheme

The first case is where the government pre-commit to a public trade policy according to a linear scheme, i.e. $S_1 \neq 0$ and $S_2 = 0$. The social welfare function is differentiated after substituting (31) and (32) into (10). The optimum public trade policy S_1^* becomes

$$S_1^* = -\frac{\mathbf{a}_d \theta^2 \left(BG \left(\frac{A}{B} - \frac{C_{d1}}{\mathbf{a}_d} \right) + \theta D \right) \left(\frac{1}{B} + \frac{C_{f2}}{\mathbf{a}_f} \right)}{BG \left[BG \left(\frac{2}{B} + \frac{C_{d2}}{\mathbf{a}_d} \right) + 2\theta^2 \left(\frac{1}{B} + \frac{C_{f2}}{\mathbf{a}_f} \right) \right]}, \tag{35}$$

The sign of (35) is unambiguously negative ($S_1^* < 0$). This result means that the government should tax domestic exports when the competition is realized through prices. The intuitive explanation of this result is as follows. Unilateral intervention by the domestic government could raise both firms' profits by softening competition instead of shifting profits from the foreign firm towards the domestic one. An export tax reduces the intensity of the domestic firm's competition, consequently both incumbent firms raise their profits. For this reason, when both firms compete by prices they are competitive complements. If one firm raises its price, the another one gains.[8]

In fact, an export tax allows the domestic firm to have a credibility commitment to be less aggressive in the external market and to maintain a relative high price which allows both firms to increase their profits. Notice that this is as much a strategic policy as the export subsidy is. In this case it works to the advantage rather than the disadvantage of the strategic move. Actually, both exporters gain at the expense of the consumers.

In the next part is analyzed the relation between trade policy and changes in the variance of the exchange rate currencies, under each type of policy scheme.

I.- Relation between the linear tax scheme (S_1^*), and changes in the variance of the domestic exchange rate (σ_d^2).

a) Under constant returns:

$$\frac{dS_1^*}{d\sigma_d^2} = -\theta^2 \frac{(2B - \theta) A + B\theta \frac{C_{f1}}{\mathbf{a}_f}}{4B^2 (2B^2 - \theta^2)} \mathbf{a}_{d\sigma_d^2} > 0. \tag{36}$$

Since $\mathbf{a}_{d\sigma_d^2} < 0$, then equation (36) is positive.

[8] In the oligopoly theory, an export tax shifts the domestic firm's upward sloping reaction curve and it intersects the foreign firm's reaction function at a higher price levels. The implementation of that optimal domestic policy raises profits of the foreign firm. It does so alleviating oligopolistic rivalry.

In case that the foreign government retaliates against the domestic policy the expected optimal tax policy in both countries would be lower, such as it was pointed out in the Cournot case.

b) Under decreasing returns:

$$\frac{dS_1^*}{d\sigma_d^2} = -\left\{2\left[BG\left(\frac{1}{B}+\frac{C_{d2}}{\mathbf{a}_d}\right)+\theta^2\left(\frac{1}{B}+\frac{C_{f2}}{\mathbf{a}_f}\right)\right]\left(A+\theta\frac{D}{G}+B\frac{C_{d1}}{\mathbf{a}_d}\right) -B^2G\frac{C_{d1}}{\mathbf{a}_d}\frac{C_{d2}}{\mathbf{a}_d}\right\}\theta^2\left(\frac{1}{B}+\frac{C_{f2}}{\mathbf{a}_f}\right)\mathbf{a}_{d\sigma_d^2}/ B\left[BG\left(\frac{2}{B}+\frac{C_{d2}}{\mathbf{a}_d}\right)+2\theta^2\left(\frac{1}{B}+\frac{C_{f2}}{\mathbf{a}_f}\right)\right]^2 > 0. \tag{37}$$

Since $\mathbf{a}_{d\sigma_d^2} < 0$, it can be shown that the right hand side of equation (37) is positive. Thus the domestic government should reduce export tax when σ_d^2 changes (note that $S_1^* < 0$), in both cases.

When σ_d^2 increases, the incumbent domestic firm is softened and it reduces its capacity to compete in the external market. The primary aim of the government is to permit the domestic firm to be more aggressive in the international market, raise its profits, and subsequently the country's welfare itself.

II.- Relation between the linear tax scheme (S_1^*), and changes in the variance of the foreign exchange rate (σ_f^2).

a) Under constant returns:

$$\frac{dS_1^*}{d\sigma_f^2} = \mathbf{a}_d\frac{C_{f1}}{\mathbf{a}_f}\theta^3\left\{2\theta^2+2G\left(\left(B^2-\theta^2\right)\frac{C_{f1}}{\mathbf{a}_f}+B\right)\right\}\frac{\mathbf{a}_{f\sigma_f^2}}{\mathbf{a}_f} /4B^2\left(2B^2-\theta^2\right). \tag{38}$$

The algebraic expression inside the bracket is positive. Since $\mathbf{a}_{f\sigma_f^2} < 0$, therefore the derivative in (38) is negative. If the variance of the foreign currency increases, the domestic government should raise the tax rate.

When σ_f^2 increases, the foreign firm loses its capacity to compete in the international market. This implies that domestic government increases the export tax on the domestic firm so that this firm becomes less aggressive. Hence both firms raise their profits and the society its welfare.

b) Under decreasing returns:

The derivative of the export tax with respect to σ_f^2 is ambiguous.[9] In particular this derivative does not specify in which direction the tax policy should be changed when the foreign currency volatility rises. Then the derivative dW_{b1}/dS_1 shall be examined when σ_f^2 changes. If this derivative increases with σ_f^2 the export tax should decreases. The result is

$$\frac{d\frac{dW_{b1}}{dS_1}}{d\sigma_f^2} = \left\{\theta^2\Omega_2+2\frac{S_1}{\mathbf{a}_d}\frac{C_{f2}}{\mathbf{a}_f}\left[G\left(B\left(B^2-\theta^2\right)\left(\frac{1}{B}+\frac{C_{d2}}{\mathbf{a}_d}\right)+\theta^2\right) +\theta^2\left(B^2-\theta^2\right)\left(\frac{1}{B}+\frac{C_{f2}}{\mathbf{a}_f}\right)\right]\right\}B\frac{\mathbf{a}_{f\sigma_f^2}}{\mathbf{a}_f} /\left[BG\left(\frac{2}{B}+\frac{C_{d2}}{\mathbf{a}_d}\right)+\theta^2\left(\frac{1}{B}+\frac{C_{f2}}{\mathbf{a}_f}\right)\right]^2, \tag{39}$$

[9]The result is shown in Appendix.

where

$$\Omega_2 = \left(\frac{1}{B}+\frac{C_{f2}}{\mathbf{a}_f}\right)\left[\frac{C_{f2}}{\mathbf{a}_f}\left(\left(B^2-\theta^2\right)\left(\frac{A}{B}-\frac{C_{d1}}{\mathbf{a}_d}\right)+\frac{\theta\left(B+\theta\right)}{B}A\right)\right. \quad (40)$$
$$\left.+\theta\frac{C_{f1}}{\mathbf{a}_f}\right]+\frac{C_{f2}}{\mathbf{a}_f}\left[G\left(\frac{A}{B}-\frac{C_{d1}}{\mathbf{a}_d}\right)+\frac{\theta}{B}D\right]>0.$$

The numerator in equation (39) is positive, so the overall result is negative. Therefore the government should raise the domestic tax rate in order to soften the intensity of the domestic firms' competition in the external market, and to increase the producers' surplus.

4.2.2. CASE 2: Quadratic Tax Scheme

We now return to the case where the government pre-commits to a quadratic trade policy scheme ($S_1 = 0$, and $S_2 \neq 0$). The optimal trade policy is obtained by setting $dW_2^*/dS_2 = 0$. This gives

$$S_2^* = -\frac{\mathbf{a}_d\theta^2\left(\frac{1}{B}+\frac{C_{f2}}{\mathbf{a}_f}\right)}{BG} < 0\ . \quad (41)$$

That is, the optimal policy once again is to tax the domestic firm. In this case, the tax provides firms with an efficient device which allows them to act as if they were in collusion. So firms gain higher profits, rather than shifting profits towards the domestic firm as it is the usual situation in a Cournot market setting.

I.- Relation between the quadratic tax scheme (S_2^*), and changes in the variance of the domestic exchange rate (σ_d^2).

a) Under constant returns:

$$\frac{dS_2^*}{d\sigma_d^2} = -\frac{\mathbf{a}_{d\sigma_d^2}}{B\left[2\left(\left(\frac{B}{\theta}\right)^2-1\right)+1\right]} > 0. \quad (42)$$

Since $\mathbf{a}_{d\sigma_2^2} < 0$, the derivative is clearly positive. Therefore the domestic government should reduce the tax when σ_d^2 increases.

b) Under decreasing returns:

$$\frac{dS_2^*}{d\sigma_d^2} = -\frac{\theta^2\left(\frac{1}{B}+\frac{C_{f2}}{a_f}\right)\mathbf{a}_{d\sigma_d^2}}{BG} > 0. \quad (43)$$

The derivative of the export tax with respect to σ_d^2 is again positive, and therefore the tax is reduced (note $S_2^* < 0$). These results are similar to those found for the linear export tax scheme. The domestic firm's lower capacity to compete in the external market is caused by a higher level of σ_d^2, which leads the domestic government to cut the export tax rate.

II.- Relation between the quadratic tax scheme (S_2^*), and changes in the variance of the foreign exchange rate currency (σ_f^2).

a) Under constant returns:

$$\frac{dS_2^*}{d\sigma_f^2} = 0. \tag{44}$$

Any change in the variance of the foreign exchange rate does not effect the public trade policy.

b) Under decreasing returns:

$$\frac{dS_2^*}{d\sigma_2^2} = \frac{\theta^4 \mathbf{a}_d}{B^3 G^2} \frac{C_{f2}}{\mathbf{a}_f} \frac{\mathbf{a}_{f\sigma_2^2}}{\mathbf{a}_f} < 0. \tag{45}$$

Under decreasing returns the derivative of the quadratic tax scheme with respect to σ_f^2 is negative. When σ_f^2 increases, the foreign firm is affected negatively and consequently reduces its capacity for competing in the international market. Therefore the domestic government should raise the export tax imposed on the domestic firm.

The theoretical results obtained in this section can be synthesized into the following proposition:

If firms play a Bertrand game, the optimum public policy is to tax exports.

1) *When σ_d^2 increases, then the tax rate decreases under any trade policy scheme.*

2) *When σ_f^2 increases, we have a variety of situations:*

   a) *Under a linear scheme, the optimum export tax increases if firms produce under constant or under decreasing returns.*

   b) *Under a quadratic scheme the optimum export tax should not be changed under constant returns, but this tax should be raised when firms produce under decreasing returns.*

5. Conclusions

The strategic trade policy in a duopoly market setting is analysed as that policy is affected by exchange rate uncertainty. The analysis compares both the linear and the quadratic trade policy schemes. The two schemes produce similar results in terms of social welfare, however the quadratic scheme returns are important for the domestic firm's decision.

Bertrand and Cournot models show that the domestic government has a special incentive to introduce a policy action that alters the initial strategic interaction between firms. When firms set quantities in the final-stage game, the optimal policy is to subsidize exports, even if the subsidy itself is merely a transfer. When the uncertainty of the domestic exchange rate increases, the best policy is to reduce the subsidy. The intuition is that an increase in variance effectively makes the domestic firm relatively less efficient and it should therefore command lower subsidies. When the volatility of the foreign exchange rate is high, the export subsidy could be raised if the market size is sufficiently large and the foreign firm's decreasing returns are not sufficiently high.

When firms set prices in the final-stage game, the optimal policy is to tax the exports. Whenever the variance of the domestic exchange rate increases, the domestic firm becomes less aggressive and the government must reduce the tax. The direct effect of this action is to assist the domestic firm vis-à-vis its foreign rival. If firms produce under constant or decreasing returns, the volatility of the domestic exchange rate conducts to reduce the tax rate. In general this policy variation allows the domestic firm to increase its international market share. When the volatility of the foreign exchange rate is high, the export tax should be raised.

Further empirical work must be done to verify the above analytical results. In 1987, Krugman complained about the lack of empirical work on problems concerned with the international market structure and optimal trade policies. Subsequently, a number of empirical works in this field have appeared. The first to evaluate firms' strategic behaviour in imperfect markets was Dixit (1988), who quantifies the competitive relation of American and Japanese automobile firms in the American market. This work gave rise to further studies. In fact, Krishna et al. (1994) revised and extended Dixit's work in the American market, and Smith (1994) applied that approach to some European countries. Following Dixit the demand and production functions in these kinds of models have been re-specified and additional issues such as brand effects have been considered. Many of the more recent empirical works focus exclusively on model calibration to obtain the optimum trade policies including an attempt to apply this theoretical framework to exports of Colombian and Mexican textiles and apparel to the American market (Mesa and Perilla, 2007). Nevertheless, no previous study has considered the uncertainty feature in a theoretical framework such as the one developed here. Additional work is always needed.

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Appendix

The result derived for decreasing returns, when the foreign exchange rate volatility increases, is

$$
\begin{aligned}
\frac{dS_1^*}{d\sigma_2^2} = & \left\{ (B+\theta) \frac{C_{d2}}{\mathbf{a}_d} \left(\frac{1}{B} + \frac{C_{f2}}{\mathbf{a}_f} \right) \left[2\frac{\theta^3}{B^2} A \left(\frac{1}{B} + \frac{C_{d2}}{\mathbf{a}_d} \right) \right. \right. \\
& \left. + G \left(\frac{2}{B} + \frac{C_{d2}}{\mathbf{a}_d} \right) \Omega_7 \right] + 2\frac{\theta^3}{B} \frac{C_{f1}}{\mathbf{a}_f} \left(\frac{1}{B} + \frac{C_{f2}}{\mathbf{a}_f} \right)^2 \frac{2B-\theta^2}{B} \\
& + \theta G \left(\frac{2}{B} + \frac{C_{f2}}{\mathbf{a}_f} \right) \left(\frac{2B^2-\theta^2}{B} - \left(B^2 - \theta^2 \right) \frac{C_{f2}}{\mathbf{a}_f} \right) \\
& \left. + \theta G^2 \frac{C_{f1}}{\mathbf{a}_f} \frac{C_{f2}}{\mathbf{a}_f} \right\} \theta^2 \frac{\mathbf{a}_d}{B} \frac{\mathbf{a}_{f\sigma_2^2}}{\mathbf{a}_f} / G^2 \left[G \left(\frac{2}{B} + \frac{C_{d2}}{\mathbf{a}_d} \right) \right. \\
& \left. + 2\frac{\theta^2}{B} \left(\frac{1}{B} + \frac{C_{f2}}{\mathbf{a}_f} \right) \right]^2 .
\end{aligned} \tag{46}
$$

Remind again that $\mathbf{a}_{f\sigma_2^2} < 0$, and the symbol Ω_7 is defined as

$$
\begin{aligned}
\Omega_7 = & \ \theta \left(\left(B^2 - \theta^2 \right) \left(\frac{2}{B} + \frac{C_{f2}}{\mathbf{a}_f} \right) + \frac{\theta^2}{B} \right) \left(\frac{A}{B} + \frac{B-\theta}{B} \frac{C_{d1}}{\mathbf{a}_d} \right) \\
& - B \left(\left(B^2 - \theta^2 \right) \left(\frac{2}{B} + \frac{C_{f2}}{\mathbf{a}_f} \right) - \frac{\theta \left(2B - \theta \right)}{B} \right) \frac{A}{B}.
\end{aligned}
$$

The relation under decreasing returns is ambiguous. That ambiguity in (46) is solved through the same mathematical analysis explained under similar circumstances in the Cournot setting.