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TIED WAGE-HOURS OFFERS AND  
THE ENDOGENEITY OF WAGES

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ABSTRACT

In the standard model of labor supply, each worker is a price taker, where the relevant price is an hourly wage rate which is fixed in the short run, and which does not depend upon the number of hours supplied. With this basic assumption, the wage can be regarded as exogenous for the purpose of estimating a labor supply function. This paper proposes and implements a pair of tests for the exogeneity of wages in a longitudinal labor supply model, and for the particular failure of exogeneity associated with jobs that offer wage-hour packages.

The first test is very simple--it amounts to a test of whether hours Granger-cause wages at the individual level. The second test involves a simultaneous estimation of labor supply and wage offer equations. Both tests indicate that the offered wage is related to hours worked, though the offer locus is, for this sample, very flat. The principal conclusion is that labor supply equations cannot properly be estimated in isolation from the process generating wages, even when long time series are available on a sample of individuals.

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I. Introduction

In the standard model of labor supply, each worker is a price taker. The relevant price for each individual is an hourly wage rate which is fixed in the short run, and which does not depend upon the number of hours supplied. With this basic assumption, the wage can be regarded as exogenous for the purpose of estimating a labor supply function, and the process by which wages are determined need not be specified. This paper proposes and implements a test for the exogeneity of wages, and for the particular failure of exogeneity associated with jobs that offer wage-hour packages.

With cross-sectional data, wage exogeneity is not a plausible assumption and is seldom invoked as such. In the long run, individuals may influence their own earning power via investment in human capital. Thus the wage in a cross-section labor supply equation is likely to be correlated with the stochastic term due to unobserved tastes and abilities which have helped to determine the wage, as well as current labor supply. A longitudinal analysis, in which wages and hours are observed for the same sample over a number of time periods, can avoid the resulting simultaneity bias by eliminating individual constants in the labor supply equation. As long as wages are exogenous to the individual in the short run, we can proceed to estimate labor supply functions using the "within" variation in wages and hours.

This requirement may not be met if the wage is part of a 'package deal' associated with a particular job. Most jobs appear to permit very limited

variation in hours worked at the discretion of the employee (though absenteeism may generate some short-run flexibility). This suggests that fixed hours per week may be part of a package which includes the wage rate and other working conditions, but labor supply analysis need not change as long as there is no systematic relationship between hours and wages. If a continuum of hours are offered by firms, each worker simply chooses a job which imposes desired hours. However, if market equilibrium generates a locus of wage/hours combinations representing different jobs, each individual must optimize subject to this constraint, and the wage will be endogenous. At present we have little evidence regarding the existence, or the shape, of such a locus. Previous studies have relied upon cross-section wage equations, in which the simultaneity of wages and hours is likely to be a problem if preferences are heterogeneous.<sup>1</sup>

The manner in which such a choice set may be established will not be analyzed here. Suffice it to say that employers may wish to make tied offers of hourly wage and work week if production or other costs vary with shift length or with days worked. Such cost differences could arise from fixed costs of shift changeover, hiring and training, or from a non-constant marginal product as an individual worker puts in more hours per week.<sup>2</sup> The wage will be endogenous whenever the slope of the wage-hours offer locus is non-zero.

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<sup>1</sup> H. Rosen (1976) estimated a sizable positive relationship between wages and hours for married women, while Abowd and Ashenfelter (1981) found that wages are negatively related to expected hours per year for stably employed white men. Ehrenberg and Schumann (1981) conclude that, on average, workers do not receive higher straight-time wages for mandatory overtime.

<sup>2</sup> Market equilibria in which jobs have multiple characteristics are analyzed by S. Rosen (1974) and by Lewis (1969), who suggested that a "market equalizing wage curve" is likely to be associated with variation in hours worked. Also see Perlman for a survey of work on the standard 40-hour week.

This paper devises a pair of tests for a non-horizontal wage-hours offer locus using monthly longitudinal data on a sample of married men. The first test is very simple -- it amounts to a test of whether hours Granger-cause wages at the individual level. The second test involves a simultaneous estimation of labor supply and wage offer equations, with all variables expressed as deviations from individual means of a three-year time series. Both tests provide evidence for the existence of tied wage-hours offers, though the second indicates that the offer locus is, for this sample, very flat.

## II. Labor Supply and Wage Offers for an Individual

Let each individual  $i$  solve in period  $t$  the following maximization problem:

$$\max_H U_i(Y_{it}, H_{it}, X_{it}) \text{ subject to } Y_{it} \leq W_{it} H_{it}$$

where  $Y$  is income,  $W$  is the hourly wage rate, and  $H$  is hours worked.  $X$  is a vector of exogenous variables which affect the supply price of labor, such as age and number of children. If the wage is exogenous, we can specify a labor supply function  $H_{it} = h(a_i, W_{it}, X_{it})$ , where  $a_i$  is an individual effect which is constant over time.<sup>3</sup> Since  $a_i$  is unobservable and is likely to be correlated with  $W_{it}$ , longitudinal data is required to estimate this labor supply function. With a simple linear specification of  $h(\cdot)$ , we can take deviations from individual means, and the fixed effect  $a_i$  will drop out.

To introduce tied wage-hours offers, we assume that the wage faced by person  $i$  in period  $t$  is generated by:

$$(1) \quad W_{it} = d_i + fH_{it} + u_{it}$$

The wage is a linear function of hours worked, a fixed individual effect, and a random shift parameter. Corresponding to the wage offer schedule is a "marginal wage income" schedule with slope  $2f$ , as shown in Figure 1. This shows the effective tradeoff between leisure and income faced by the worker,

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<sup>3</sup> A similar labor supply function would result from an intertemporal problem in which the individual faces a lifetime budget constraint. Additive intertemporal preferences must be assumed, however, since there is no direct dependence of labor supply on wages in other periods, except as they enter through  $a_i$ .

and is analogous to the marginal revenue function of the monopolist.

This is clearly a spot-market specification of hours and wages; multi-period employment contracts introduce a significant complication. In the extreme case where the wage is simply "an installment payment on the firm's long-term obligation to the worker" (Hall, 1980), the current wage need not respond to current hours at all.<sup>4</sup>

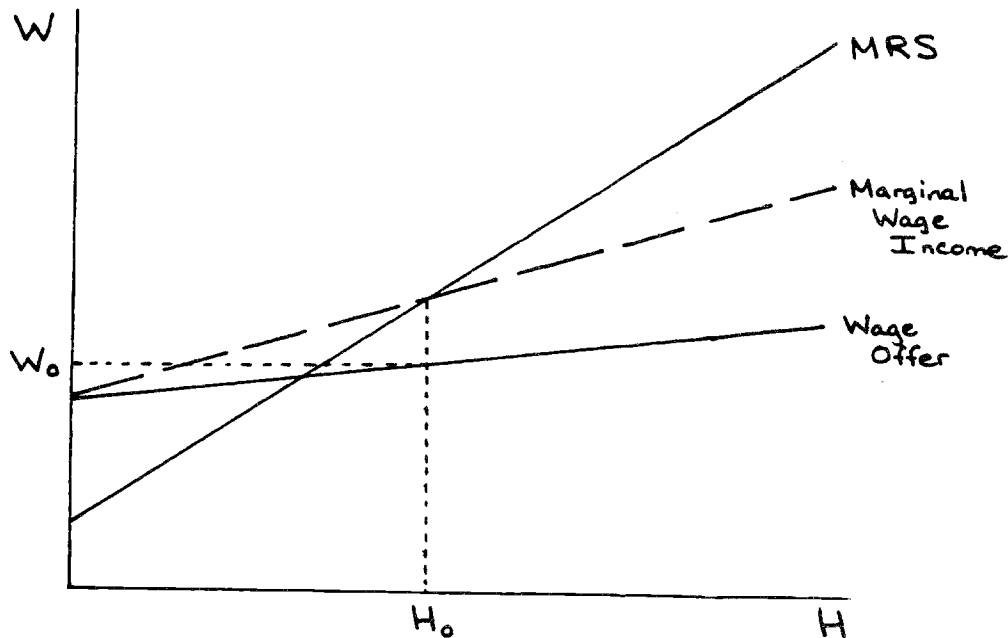


Figure 1

With tied offers, hours worked will be a function of the parameters of the wage-hours offer locus. Assume, for simplicity, that the marginal rate of substitution between income and leisure (MRS in Figure 1) is linear in hours worked. A worker who may choose any wage-hours package from the offer locus will set hours so that marginal wage income is equal to the marginal rate of

<sup>4</sup> Kennan (1983) includes a "wage smoothing" specification in his analysis of aggregate employment-wage movements. Abowd and Card (1983) interpret individual wage-hours variations in the context of a similar model.

substitution. Solving for equilibrium hours as a function of the shift in the offer locus, we have

$$(2) \quad H_{it} = \alpha_i + \left(\frac{1}{\beta-2f}\right)u_{it} + \gamma X_{it} + v_{it}$$

where  $\alpha_i$  incorporates  $d_i$ , as well as differences in preferences and lifetime budget constraints,  $\beta$  is the coefficient on hours in the MRS and all other parameters may also be functions of  $f$ . An individual equilibrium at finite hours requires that the slope of the MRS be greater than the slope of the marginal wage income schedule, or  $\beta > 2f$ .

Taking deviations from the individual means of a time-series on a cross-section, (1) and (2) become

$$(3) \quad h_{it} = \left(\frac{1}{\beta-2f}\right)u_{it} + \gamma x_{it} + v_{it}$$

$$(4) \quad w_{it} = fh_{it} + u_{it}$$

so that  $\alpha_i$  and  $d_i$  disappear.

Substituting in for  $u_{it}$  in (3) gives a final system of (4) and

$$(5) \quad h_{it} = bw_{it} + cx_{it} + e_{it}$$

where  $b = 1/(\beta - f)$ ,  $c = \gamma(\beta - 2f)/(\beta - f)$ , and  $\sigma_e^2 = \sigma_v^2 (\beta - 2f)^2 / (\beta - f)^2$

Equation (5) is the "hours function" in Figure 2, which traces out equilibrium wage-hours packages as the offer locus shifts over time.



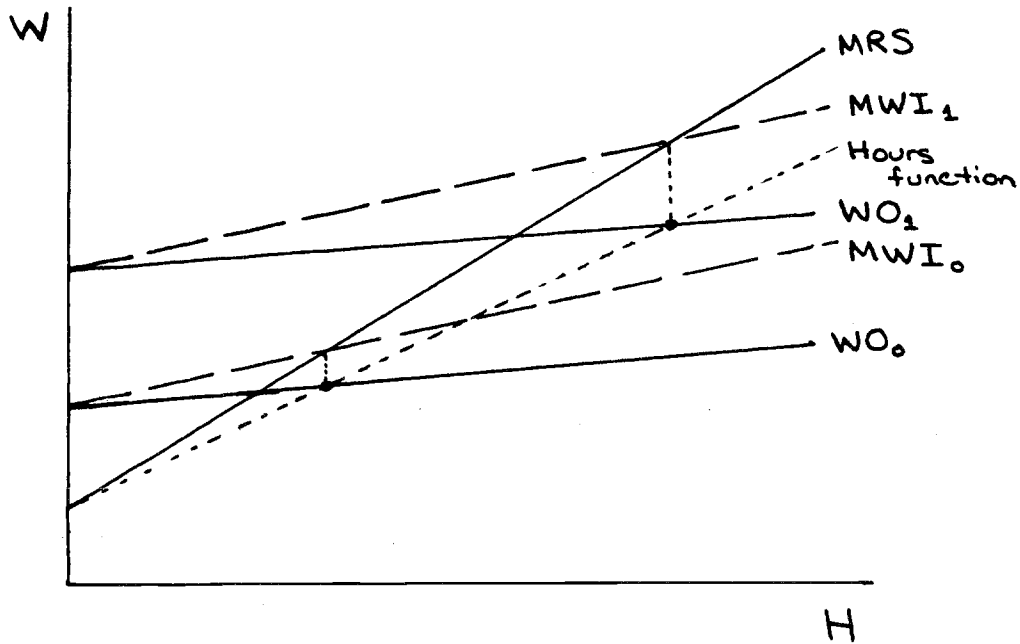


Figure 2

Some obvious generalizations of this hours equation and their implications for the identification of the model will be discussed in section 4. The linearity assumption, however, will be maintained throughout.

The absence of tied wage-hours offers corresponds to the restriction  $f=0$ . If hours do not enter the wage equation, then the individual faces a perfectly elastic demand curve for hours worked and (5) is an ordinary labor supply equation. In this case, the parameter  $b$  is equal to  $1/\beta$ , and a consistent estimate can be acquired without estimation of the wage offer equation.

If the restriction  $f = 0$  holds, can we say that wages are exogenous? Employing the concept of weak exogeneity set out by Engle, Hendry, and Richard (1983), wages are weakly exogenous to hours with respect to the parameter  $b$  only if  $b$  can be estimated consistently and efficiently without reference to

the wage equation. Wage exogeneity, in this sense, is more restrictive than the absence of tied offers, since consistency requires that the shocks  $e_{it}$  and  $u_{it}$  be uncorrelated as well as  $f = 0$ .

Estimation of the wage-offer equation (4) depends upon the vector  $x$ . Candidates for exogenous shifters of the labor supply schedule are somewhat limited, since all characteristics which do not vary for an individual over the sample period will have been included in the fixed effect. In most samples, income from non-labor sources and number of children are the only observable possibilities for  $x$ .

In fact, however, the behavior of the errors  $e$  and  $u$  provides an alternative method of identification. Both  $h$  and  $w$  exhibit strong serial correlation, a pattern which can be interpreted as the result of shocks to the individual's supply price, or to their offered wage schedule, which are autocorrelated. As an illustration, first order autocorrelation of the form,

$$e_t = pe_{t-1} + \varepsilon_t$$

$$u_t = qu_{t-1} + \mu_t$$

yields a system,

$$(6) \quad h_{it} = bw_{it} + p(h_{it-1} - bw_{it-1}) + \varepsilon_t$$

$$(7) \quad w_{it} = fh_{it} + q(w_{it-1} - fh_{it-1}) + \mu_t$$

It is assumed, for the remainder of the paper, that  $e$  and  $u$  are not causally related, though the innovations  $\varepsilon$  and  $\mu$  may be correlated.

### III. Do Hours Cause Wages?

Equations (6-7) suggest a very simple and very general test for tied wage-hours offers. The reduced form of the wage equation is,

$$w_{it} = f(bw_{it} + p(h_{it-1} - bw_{it-1}) + \varepsilon_t) + q(w_{it-1} - fh_{it-1}) + \mu_t$$

If  $f=0$ , then the current wage depends only upon past wages and the innovation  $\mu_t$ . If  $f \neq 0$ , then past hours should help to predict the current wage. Thus the presence of tied offers implies that hours cause wages, where causality is of the type described by Granger (1969).

"...We say that  $Y_t$  is causing  $X_t$  if we are better able to predict  $X_t$  using all available [past] information than if the information apart from [past]  $Y_t$  had been used" (p. 428).

The tests reported here are similar to those employed by Sargent (1976). The restriction  $f = 0$  implies that  $E(w_{it}|w_{it-1}) = E(w_{it}|w_{it-1}, h_{it-1})$ , which must hold if hours do not Granger-cause wages. A failure of the necessary condition for non-causality will thus imply the presence of tied offers in this case. This result will generalize to higher order autocorrelation of the demand and supply shocks, so an ordinary least squares regression of wages on lagged hours and lagged wages provides the basis of the first test. Note that the validity of this test does not depend at all upon the specification of the labor supply function.

This regression employed 48 months of wage and hours data from a longitudinal sample of low-income married men in the control group of the Denver Income Maintenance Experiment. From the set of 580 who reported

employment information for the duration of the experiment, a subsample was chosen who reported positive hours worked in each month. This left 204 men between the ages of 18 and 58, who earned an average hourly wage below \$4.00 (1972 dollars) but experienced reasonably stable employment. Deviations from individual means were calculated for monthly hours adjusted for seasonal variations, and for the straight-time hourly wage deflated by the Denver CPI and seasonally adjusted. The last 36 months of data were employed to allow for 12 monthly lags.<sup>5</sup>

Lagged hours are highly significant in the wage equations for both the full sample and the subsample of job changers. We can thus reject the exclusion of lagged hours, and reject Granger non-causality of wages by hours. This test provides some evidence that tied wage-hours offers are being faced by the men in this sample. A symmetric test for wages causing hours leads to the expected conclusion that lagged wages do help to predict current hours.<sup>6</sup>

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<sup>5</sup> This lag length was chosen arbitrarily. A four month lag was attempted later, but this did not affect the results. It was hoped that 12 months would be sufficiently long that no omitted lags would be included in the error term. In fact, there is no evidence of autocorrelation in the residuals. Estimation of partial correlation functions (for up to six lags) yields no significant coefficients, and the Q statistic from the sample autocovariances for lags one through six is 6.26, which is  $\chi(6)$  and insignificant.

<sup>6</sup> An alternative test for Granger-causality has been employed by Sims (1972) and others. In this context, it consists of regressing hours on past, current, and future values of the wage, and testing for the significance of future wages. This test was also implemented, using eight past and four future lags. Future wages were significant in the hours equation, confirming the above result that hours Granger-cause wages.

TABLE 1  
F-Tests on Twelve Lagged Months

	<u>F-ratio</u>
Hours in wage equation	2.88
Wages in hours equation	3.50
Critical value for $F_{.99}(12, \infty)$	2.18

It is perhaps worth noting that this causality test would fail if the innovations in the wage-offer and hours equations followed very similar serial correlation patterns. If  $p = q$ , then lagged hours will have a coefficient of zero in the wage equation even if  $f \neq 0$ .

#### IV. Simultaneous Estimation

Since the presence of tied wage-hours offers cannot be dismissed on the basis of a causality test, we now investigate the slope of the offer locus through a simultaneous estimation of the wage and hours equation. In OLS regressions similar to those reported in the previous section, variables which were expected to affect the supply price of labor only and thus identify the wage offer equation, such as non-labor income, did not have a significant impact on hours. Similarly, the unemployment rate in Denver should have been included in the offer equation, but displayed very little variation over the sample period (1972-1974).

However, with first order autocorrelation specified for both  $e$  and  $u$ , the system (6-7) is just identified -- the four reduced form parameters can be translated into the four structural parameters through non-linear restrictions. Unfortunately, the supply and demand equations are symmetric and, as Kennan (1983) notes with respect to a similar structure in an aggregate model, we cannot tell which parameters belong to the wage offer locus and which to the labor supply function, though all are identified. However, we can assume a priori that the slope of the offer locus is less than the slope of the labor supply function, since this is implied by the restriction that the slope of the marginal wage income schedule be less than the slope of the MRS.

In practice, a fourth order autocorrelation scheme is specified for both equations, so that the system is overidentified. Full-information maximum likelihood was used to estimate the parameters of the two equations. This method will generate consistent and efficient estimates if the innovations

$\varepsilon$  and  $\mu$  are normally distributed.<sup>7</sup> Fourth order autocorrelation of the errors  $e$  and  $u$  cannot be rejected as a restriction on a twelfth order process, so only results from the former are reported. The wage data are in dollars per hour and represent the regular straight-time wages only, not overtime payments. Hours are hours per month, adjusted for seasonal variations. All observations are again deviations from individual means, and 36 observations for each individual in the sample are used. The standard deviation of monthly hours is 20.5 hours, and of wages, \$0.51 per hour.

The final results, with standard errors in parentheses, follow.

Labor supply:

$$\begin{aligned}
 h_{it} = & 19.3025*w_{it} + 0.4813*e_{it-1} + 0.1029*e_{it-2} + 0.0469*e_{it-3} \\
 & (0.9037) \quad (0.0106) \quad (0.0118) \quad (0.0118) \\
 & + 0.0218*e_{it-4} \\
 & (0.0105)
 \end{aligned}$$

Wage offer:

$$\begin{aligned}
 w_{it} = & 0.0024*h_{it} + 0.5604*u_{it-1} + 0.1341*u_{it-2} + 0.0495*u_{it-3} \\
 & (0.0002) \quad (0.0108) \quad (0.0122) \quad (0.0123) \\
 & + 0.0471*u_{it-4} \\
 & (0.0110)
 \end{aligned}$$

$$\begin{vmatrix} s_{\varepsilon}^2 & s_{\varepsilon\mu} \\ s_{\varepsilon\mu} & s_{\mu}^2 \end{vmatrix} = \begin{vmatrix} 338.998 & -3.308 \\ -3.308 & 0.129 \end{vmatrix} \quad N = 7344$$

<sup>7</sup> See Amemiya (1977). Non-linear two-stage least squares produced very similar results.

The estimated slopes of both the labor supply function and the wage offer locus are positive and significantly different from zero. In particular, the estimate of  $f$  (the coefficient on hours in the wage offer equation) is very precisely estimated, though very small. These results suggest that individuals face, in the short run, a slightly upward-sloping choice locus of wage-hours combinations, even when overtime rates are ignored.

The labor supply response is similar to that found in other longitudinal studies (see, for example, MaCurdy).<sup>8</sup> One dollar per hour yields approximately 4.5 extra hours of work per week, a mean elasticity of about 0.40.<sup>9</sup> The offer curve, however, is extremely flat; the same 4.5 extra hours per week will yield only 4.6 cents per hour, give or take .4 cents. The random components of the supply and demand curves have a contemporary covariance which is significantly negative.

Since the model estimated here is overidentified, it may be useful to consider which of the many possible generalizations will still permit identification of the wage-hours offer locus. The current restrictions placed on the labor supply function are perhaps most difficult to justify. With intertemporal substitution of leisure, a direct dependence of current hours on lagged hours would be introduced and, if expectations regarding lifetime

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<sup>8</sup> Recall that the hours equation is not a proper labor supply function. From equation (5), the coefficient on wages is  $b = \frac{1}{\beta - f}$ , where  $\frac{1}{\beta}$  is the labor supply response to a shift in the wage offer locus. Since the estimated value of  $f$  is so small, however,  $b$  is very close to  $\frac{1}{\beta}$ .

<sup>9</sup> Mean hours per month are 181.6, mean wages \$3.78.



income respond to realized wages, current hours may depend upon lagged innovations in the offer locus as well. With a labor supply function of the form:

$$h_{it} = b_1 w_{it} + b_2 w_{it-1} + b_3 h_{it-1} + e_{it}$$

the parameters  $b_1$ - $b_3$  will not be identified. However, if the simple structure of the wage-offer locus is maintained, its serial correlation structure, as well as the parameter  $f$  which indicated the presence of tied offers, will be identified. The type of test performed in this section, therefore, will be valid for more general specifications of labor supply.

## V. Conclusion

The two tests reported in this paper decisively reject the proposition that wage rates available to an individual in the short run are exogenous with respect to hours worked. Both indicate that the offered wage is related to hours worked -- this is interpreted here as evidence of job "packages" in which wage and hours are tied together. The second test indicates that the two are positively related, even when overtime rates are not included, but that most of the positive covariance between hours and wages is due to supply response, not to tied offers.

The principal conclusion is that labor supply equations cannot properly be estimated in isolation from the process generating wages, even in the ideal situation where long time series are available on a sample of individuals. The negative correlation between innovations in the demand and supply equations and the presence of tied wage-hours offers insures that an estimate of the wage elasticity of labor supply will fail to be unbiased. The quantitative results, particularly the size of the straight-time wage premium for extra hours, should be regarded as representative of this sample only. Both tests, however, can be easily applied to longitudinal data on other types of workers.

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