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# SANCTIONS

Jonathan Eaton

Maxim Engers

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## ABSTRACT

Sanctions are measures that one party (the sender) takes to influence the actions of another (the target). Sanctions, or the threat of sanctions, have been used, for example, by creditors to get a foreign sovereign to repay debt, or by one government to influence the human rights, trade, or foreign policies of another government. Sanctions can harm the sender as well as the target. The credibility of such sanctions is thus at issue. We examine, in a game-theoretic framework, whether sanctions that harm both parties enable the sender to extract concessions. We find that they can, and that their threat alone can suffice when they are contingent on the target's subsequent behavior. Even when sanctions are not used in equilibrium, however, how much compliance they can extract typically depends upon the costs that they would impose on each party.

Jonathan Eaton
Department of Economics
Rouss Hall
University of Virginia
Charlottesville, VA 22901

Maxim Engers
Department of Economics
Rouss Hall
University of Virginia
Charlottesville, VA 22901

## I. Introduction

Within national boundaries, laws and contracts allow parties to influence each other's actions. A third party, the legal system, can punish those who break laws or breach contracts. In contrast, the interactions of sovereign governments, or of parties under the jurisdictions of different sovereign governments, typically lack third parties to enforce contracts and agreements. Hence, parties to such contracts or agreements must themselves be willing to enforce them if they are to have any effect. Enforcement may then require measures that affect the other party directly, without involving anyone else. Economic sanctions, steps by a government that inflict harm on another country, possibly at a cost to itself, are such measures.

National governments have often used economic sanctions to affect policies of other countries.[1] The United States government, for example, has banned trade with Cuba and South Africa in response to policies of their governments. U.S. trade law calls for trade restrictions against countries found to engage in practices that damage U.S. industry or infringe upon U.S. intellectual property. Finally, collecting repayment from debtor governments may require that creditor countries threaten to curtail financial relations or trade with debtor countries. In each of these circumstances, one national government tries to affect the actions of another by threatening, or by actually taking, measures that are likely to harm both countries.

We examine the potential for sanctions to elicit desired behavior from

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[1]Daoudi and Dajani (1983) and Hufbauer et al. (1985) provide detailed case studies of several historical situations in which national governments have used economic sanctions, successfully and otherwise, to pursue foreign policy objectives.

another party. We consider the interaction over time of two parties, called the <u>sender</u> and the the <u>target</u>.[2] The sender would like to affect the target's actions. It has the power to harm the target, but at a cost to itself.[3]

We consider two types of actions that the sender might wish to affect. One is the target's ongoing choice of some action, such as the target's debt-service payments, trade policies, pollution, or degree of protection of intellectual property. Another is the target's once-and-for-all choice of an irreversible action, such as ceding territory, releasing a hostage, extraditing an accused criminal, or relinquishing power to a new government.

We examine whether sanctions that are costly both to the sender and to the target enable the sender to alter the behavior of the target.[4] When they can, we also consider whether the threat alone of such sanctions is enough, or whether they must actually be used.

The answers depend critically on the dynamics of the interaction between the sender and target. One issue is whether sanctions are contingent on what the target then does, or are purely spiteful in the sense of imposing a cost independent of the target's subsequent actions. Sanctions, for example, might be imposed or renewed only occasionally (as by a legislature), but enforced continuously (as by an executive or judiciary). Legislation could then instruct the executive or judiciary to lift sanctions as soon as the target

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[2]We are using the terminology of Hufbauer et al. (1985).

[3]The relationship between sender and target resembles that between principal and agent in contract theory. See, for example, Ross (1973). Our concern here is not with the nonobservability of the target's (agent's) action, which has been the focus of this literature, but with the sender's (principal's) ability to enforce a contract with the target (agent).

[4]For example, the failure of the grain embargo imposed by the United States against the Soviet Union after its invasion of Afghanistan is commonly attributed to the loss of export revenue it implied for U.S. farmers. See Dauodi and Dajani (1983) or Hufbauer et al. (1985) for a discussion.

performed as specified. The Jackson Amendment linking most favored nation status to free emigration is an example (See Daoudi and Dajani, 1983).

Another issue is whether, when setting its own policy, each party knows the other's current policy. A repeated-game specification implies the contrary. In the context in which economic sanctions are used, however, parties seem to set policies for a period, knowing current policies elsewhere, but also knowing that these policies may change later. Interaction of this sort can be captured by assuming that parties alternate in setting policies.

As a benchmark, we first suppose that the parties do set their policies at the same time. The standard theory of repeated games then applies. As the Folk Theorem implies, if the parties' relationship continues indefinitely then many outcomes can be supported as subgame perfect equilibria. But if stricter equilibrium criteria are applied, the sender has no control over the target: One criterion yielding this result is that the equilibrium be the limit of finite horizon equilibria, what we call a <u>limit equilibrium</u>. Another is that it be <u>Markov perfect</u>. This criterion specifies that each party's strategy depend only on variables that directly affect the parties' current and future payoffs, and not what might affect current and future payoffs only through the response of the other party.[5]

Our paper focuses primarily on limit and on Markov perfect equilibria. These provide a much sharper characterization of outcomes than subgame perfection alone, and we find them to be of intrinsic interest: Many situations may in fact involve only finite interaction, while Markov

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[5]See Maskin and Tirole (1988b) and Farrell and Maskin (1987). Any equilibrium in which responses are payoff relevant is <u>one</u> equilibrium in a specification in which payoff relevance is not imposed <u>a priori</u>. If one party does not respond to payoff irrelevant information then there is no gain to the other of responding to such information.

perfection requires parties to use the subgame perfect equilibrium strategies that are informationally most parsimonious.

In an alternating move framework, if sanctions are noncontingent then it remains a (limit or Markov perfect) equilibrium for the sender to have no power over the target. This is the only limit equilibrium. There are, however, Markov perfect equilibria in which the sender can obtain concessions, but it must actually impose sanctions to do so.

In contrast, contingent sanctions can ensure the sender a degree of control over the target's actions in a limit or Markov perfect equilibrium, and the threat alone of sanctions suffices. If the sender seeks to influence an ongoing policy of the target then, under general conditions, there is a Markov perfect equilibrium in which the level of compliance depends upon the costs of sanctions to both the sender and the target, and on each party's patience. This can also be a limit equilibrium. In the only other possible steady-state Markov perfect equilibrium, which is not a limit equilibrium, the only outcome is for the target to concede to the maximum, i.e., to the point at which conceding more would be worse for it than enduring sanctions and conceding nothing. This can happen if and only if sanctions are not too harmful to the target.

We also find that contingent sanctions can enable the sender to exact a once-and-for-all concession from the target. The sender might or might not actually have to use sanctions.

In summary, the costliness of sanctions to the sender need not render them ineffective, and sanctions can be effective even if, in equilibrium, they are not actually used.[6]

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[6]This finding contrasts with Bulow and Rogoff's (1989, p. 168) result, from a Rubinstein bargaining framework, that, if imposing sanctions on a

## II. The Basic Framework

More formally, we consider the interaction of two parties, the sender and target, each of which controls the level of a particular variable. The target chooses the level $a \in A$, $A \subset \mathbb{R}$, of some activity that affects its own and the sender's utility in opposite directions, while the sender chooses a level $s \in S$, $S \subset \mathbb{R}$, of sanctions that affect both itself and the target adversely. The per period utility of the sender is $u^S(a,s)$, which increases in a, decreases in s, and is continuous in both variables, while the target's utility per period is $u^T(a,s)$, which decreases in a and s, and is continuous in both.[7] Hence the sender most prefers the target to choose the maximum level of a while the target most prefers the minimum level. The per period discount factor is $\delta_S$ for the sender and $\delta_T$ for the target, where $0 \le \delta_i < 1$, $i = S,T$.

With ongoing actions we usually let the target's choice set A be a continuum, and set $A = [0,1]$. We treat irreversible actions as dichotomous, however, and set $A = \{0,1\}$.

Some sanctions, such as the level of a punitive tariff, can be continuously varied over some set S. If so, we set $S = [0,1]$. Other sanctions are more discrete, such as an embargo, boycott or a military attack. Hence we also consider sanctions that are just on or off, and set $S = \{0,1\}$.

The sender's highest possible per period utility level (bliss) is therefore $u^S(1,0)$, achieved when the sender chooses the maximum value of a

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debtor country is costly to creditors, then the "threat to seize shipments is not credible and they will not be paid a peso in a perfect equilibrium."

[7] "Increasing" and "decreasing" are used in the strict sense throughout.

(one) and no sanctions are in place. Bliss for the target is $u^T(0,0)$, attained when a is at its minimum value (zero) and no sanctions are in place. We normalize bliss for each party at one; i.e., we set:

$$u^S(1,0) = u^T(0,0) = 1.$$

The sender's minimum individually rational utility level, the highest per period utility it can achieve given the least advantageous, for the sender, behavior of the target (i.e., setting a = 0), is attained by setting sanctions at zero, yielding $u^S(0,0)$. The target's minimum individually rational utility level, $u^T(0,1)$, occurs when sanctions are at their maximum level (one) and is attained at a = 0. We normalize each party's minimum individually rational utility levels at zero; i.e., we set:

$$u^S(0,0) = u^T(0,1) = 0.$$

The remainder of the paper proceeds as follows: In Sections III, IV, and V the sender seeks to affect the target's ongoing performance, while in Sections VI and VII it tries to make the target take an irreversible action. In Section III the sender and target simultaneously choose s and a each period. In the remaining sections they alternate in choosing. In Sections III and IV sanctions are noncontingent, while elsewhere they are contingent on the target's subsequent performance.

## III. Exacting Ongoing Performance: Simultaneous Moves

Consider a situation in which the target chooses some level of action each period, while the sender simultaneously decides what level of sanctions to impose. Say that sanctions are noncontingent.

Any one-shot play of this game has, as a unique Nash equilibrium in dominant strategies, the sender setting $s = 0$ and the target setting $a = 0$. That is, the option of imposing sanctions fails to give the sender any ability to affect the target's choice.

This outcome remains the only Nash equilibrium outcome if the game is finitely repeated. However, if it is repeated an indefinite number of periods, then the relationship between the sender and target is an infinitely-repeated game, for which there are many other subgame perfect equilibrium outcomes.

Figures 1a and 1b depict the set of possible per period utility levels of the sender (on the horizontal axis) and target (on the vertical axis). Points on the northeast frontier of this set represent Pareto-efficient outcomes.

Figure 1a is drawn under the assumption that $u^T(1,0) > 0$, i.e., that setting $a = 1$ and suffering no penalty yields a per period utility above the minimum individually rational utility level (normalized at zero): Sanctions inflict so much harm on the target that the target prefers to perform at any feasible level and avoid sanctions rather than to suffer the penalty. Here sanctions have <u>overkill</u> capacity.

Figure 1b is drawn under the opposite assumption: The target prefers to suffer sanctions at their worst rather than to perform at the maximum feasible level. Here sanctions have <u>limited</u> capacity. In this case we define $\tilde{a}$ as the action level at which the target's per period utility, with sanctions at zero, is at the minimum individually rational level, i.e., $u^T(\tilde{a},0) = 0$.

The Folk Theorem (Fudenberg and Maskin, 1986) assures that, for $\delta_S = \delta_T$ sufficiently close to 1, there exist subgame perfect equilibria sustaining any feasible outcome that strictly Pareto dominates the minimum individually rational payoff pair. In the diagrams these outcomes correspond to all feasible points in the northeast quadrant.

The outcome s = a = 0 (no performance and no penalization) is of course still sustainable, but there are many other possibilities as well. For the case illustrated in Figure 1a, any Pareto-efficient outcome is sustainable, including the outcome s = 0 and a = 1 (bliss for the sender), but inefficient outcomes in which some penalization occurs are also sustainable.

In Figure 1b the target's individual rationality constraint admits as steady states only outcomes in which the level of compliance is less than $\bar{a}$. In this case the cost of the penalty to the target limits the extent of compliance that the sender can exact in steady state.

Note that, in the limit as $\delta_S$ approaches one, the cost to the sender of imposing sanctions does not affect the set of efficient sustainable outcomes. The sender finds it worth incurring any finite cost for a finite number of periods in order to extract a higher level of compliance in perpetuity.[8]

The case in which sanctions are contingent or actions or sanctions (or

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[8]For discount factors $\delta_S = \delta_T$ sufficiently close to one and for N sufficiently large, the following "trigger" strategies provide one way of supporting compliance at an action level a* as a subgame perfect outcome: Consider first the following rule R for the sender: Set s = 1 if the target has set a < a* in any of the previous N periods. The strategy for the sender is to set s = 1 if a < a* in any of the previous N periods and if the sender has always adhered to R previously, and to set s = 0 otherwise. The strategy for the target is to set a = a* if the sender has always adhered to R previously and a = 0 otherwise. For the overkill case (illustrated in Figure 1a) a* can lie anywhere in [0,1]. For the case in which sanctions have limited capacity (illustrated in Figure 1b), a* cannot exceed $\bar{a}$.

both) are dichotomous can be handled similarly.[9] [10]

In conclusion, a wide range of possible (efficient and inefficient) outcomes can be supported as subgame perfect equilibria in a repeated game with costly sanctions. However, the only limit or Markov perfect equilibrium repeats the outcome of a one-shot game, with actions and sanctions at zero. Hence, the sender can extract a performance level above zero only if the parties expect to interact indefinitely and condition their decisions on past decisions that no longer affect current or future payoffs.[11]

## IV. Exacting Ongoing Performance: Alternating Moves with Noncontingent Sanctions

So far we have examined the efficacy of sanctions in a repeated game in which the parties choose simultaneously each period, without having observed the other's current choice. Perhaps a more realistic assumption is that Party 1 sets its choice for a period of time (which may be very short), having observed Party 2's previous choice, which remains in effect for the moment. Having observed 1's choice, which itself remains in effect for the moment, 2 may subsequently respond by making a different choice. After 2 responds, 1

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[9]If sanctions are contingent, utilities at outcomes (0,0), (0,1) and (1,0) are unaffected, as is the efficient frontier (along which $s = 0$). Hence the set of efficient sustainable outcomes is unaffected.

[10]If both are dichotomous then only the four extreme points in Figure 1a or 1b are attainable in the one-shot game. Average or expected utility pairs in the convex hull of these points are attained by generating the four outcomes with various possible frequencies: either as the realization of a random process with a given distribution (i.e. by "correlated strategies") or as a periodic (but deterministic) function of time. In an average or expected utility sense, then, the Folk Theorem applies as above.

[11]Farrell and Maskin's (1987) requirement that equilibria be "weakly renegotiation-proof" reduces the set of possible subgame perfect outcomes to the shaded regions in Figures 1a and 1b. See Appendix A for an explanation.

can make another choice, knowing how 2 responded to its previous choice, and so on. Each time it chooses, each party takes the entire sequence of future choices into account in deciding its best current choice.

*An Alternating Move Framework*

This type of interaction can be studied by assuming that the parties choose only in alternate periods, with only one party choosing in each period. The outcome in any period is determined by one party's current choice and the other party's choice the previous period.[12]

Applying this framework to our situation means that the sender chooses a level of sanctions having observed the target's current performance level, but knowing that the target can change this level before the sender can reset the level of sanctions. Similarly, the target decides its action having observed the current severity of sanctions, but realizing that the sender can reset the level of sanctions before the target can respond.

With the additional requirement that strategies be payoff relevant, each party's strategy can be specified as a reaction function of its rival's current choice. Hence, we specify the sender's strategy in setting the level of sanctions s as a function $R^S(a)$ of the target's current action level a and the target's strategy in setting a as a function $R^T(s)$ of s. If mixed strategies are used then $R^S(a)$ and $R^T(s)$ are random variables. Where there is no ambiguity we use $R^S(a)$ and $R^T(s)$ to denote the support of these variables' distributions.

Markov perfection implies that the maximum discounted present value of

[12]Cyert and de Groot (1970), Maskin and Tirole (1987,1988a,1988b) Gertner (1986), Davies (1987) and Eaton and Engers (1989,1990) have analyzed duopolistic competition in a similar alternating-move framework. The Rubinstein (1982) bargaining model also posits an alternating-move framework.

current and future payoffs to the sender at the time it sets the sanction level $s$ depends only on the target's current action. Given $R^T$, this value, denoted $V^S(a)$, can be obtained recursively by dynamic programming:

$$V^S(a) = \sup_s E\{u^S(a,s) + \delta_S u^S[R^T(s),s] + \delta_S^2 V^S[R^T(s)]\},$$

where E is the expectations operator. Equivalently, for the target, the maximum discounted present value of current and future payoffs at the time it sets the action level a depends only on the sender's current sanction level. Given $R^S$, this value, denoted $V^T(s)$, is similarly obtained:

$$V^T(s) = \sup_a E\{u^T(a,s) + \delta_T u^T[a,R^S(a)] + \delta_T^2 V^T[R^S(a)]\}.$$

A <u>Markov perfect equilibrium</u> is a pair of reaction functions $R^S$ and $R^T$ such that, given $R^T$, for each a in A, $R^S(a)$ attains $V^S(a)$ and, given $R^S$, for each s in S, $R^T(s)$ attains $V^T(s)$.

Henceforth, we use the unqualified term "equilibrium" to mean "Markov perfect equilibrium" and refer to limit equilibria explicitly.

*<u>The Impossibility of Exacting Performance Efficiently</u>*

It is still true that, when sanctions are noncontingent, the pair of strategies $R^S(a) = 0$ for all a and $R^T(s) = 0$ for all s is an equilibrium, with the outcome $s = a = 0$ every period: If the sender will under no circumstances impose sanctions in the future then the target has no reason to set $a > 0$ even if sanctions were for some reason currently in place. Similarly, if the target will under no circumstances set $a > 0$ in the future then sanctions impose only costs and no benefits to the sender.

Furthermore, while this is the only limit equilibrium, there can be other equilibria. But any outcome that these support will not be efficient: If the target (at least occasionally) sets $a > 0$, then the sender must (at least occasionally) set $s > 0$. Hence the only equilibrium outcome in which sanctions are never actually imposed is one in which the target never performs. This result follows from:

Proposition 1: If $R^S[R^T(0)] = 0$ then $R^T(s) = 0$ for all $s$.

The proof of this result, and all our remaining ones, are in Appendix B.

*A Sanctions Cycle*

There are, however, equilibria in which the target does sometimes make concessions and in which the sender does sometimes actually impose sanctions. Say, for example, that sanctions and actions are dichotomous ($S = \{0,1\}$ and $A = \{0,1\}$) and consider the following reactions:

$$R^S(0) = 1; \quad R^S(1) = 0$$

$$R^T(0) = 0; \quad R^T(1) = 1.$$

These generate a cycle of length four periods over which all combinations of actions and sanctions occur: High performance elicits the removal of sanctions that in turn engenders low performance followed by the reimposition of sanctions, etc.

These reactions are an equilibrium if, for instance, $\delta_S^2 \geq 1/2$ and $\delta_T^2 \geq 1/2$ and if preferences are as in (1) below:

*Example 1:*

$$u^S(a,s) = (1+a)(1-s/2)$$
$$u^T(a,s) = (2-a)(1-s/2). \qquad (1)$$

These utility functions have the property that, as a increases, the cost of sanctions decreases to the sender and increases to the target.[13] If a is a transfer of income, for example, since sanctions halve the utilities of both parties, the more the target transfers, the more costly are sanctions to the sender, and the less costly are they to the target.

## V. Exacting Ongoing Performance: Alternating Moves with Contingent Sanctions

So far, we have analyzed a repeated game and an alternating move game in which sanctions are not contingent on the target's level of performance. Two results have emerged: First, in order to raise the target's action level above zero in equilibrium, the sender must (at least on occasion) endure the cost of imposing sanctions. Second, no performance and no sanctions in all periods is always an equilibrium outcome, and it is the only limit equilibrium

[13]For simplicity these utility functions (and their generalizations in Example 2 below) are not normalized. To normalize them, subtract 1 from each. More generally, to normalize any utility function subtract the minimum individually rational utility, and then divide by the difference between utility at bliss and the minimum individually rational utility.

outcome. We shall show, however, that these results do not hold when the sender can impose sanctions that are contingent upon the target's performance.

Suppose that sanctions are dichotomous ($S = \{0,1\}$) while the action is continuous ($A = [0,1]$). In each period in which the sender moves it specifies a threshold t. Sanctions will be experienced in the current period if and only if the current action level is less than t. Similarly, they will be experienced the subsequent period if and only if the target subsequently chooses an action level below t.

The possible decisions of the sender and target can each be classified into two categories: When choosing its action level a, given the sender's current threshold performance level t, the target may either acquiesce by setting a at or above t (thus averting sanctions in the current period), or else balk by choosing a below t (automatically triggering sanctions). Similarly, observing the target's current action level a, the sender may either condone this level by setting t at or below a (thus averting sanctions in the current period) or else spurn it by setting t above a (thus inflicting sanctions in the current period). Matching by the sender is condoning by setting $t = a$, while matching by the target is acquiescing by setting $a = t$.

In this framework, a payoff-relevant strategy for the sender is a reaction function $R^S$ mapping each possible action level to a threshold while a payoff relevant strategy for the target is a reaction function $R^T$ mapping each possible threshold to an action level. To denote that sanctions are in effect if and only if a is below t, let $\sigma(t,a) = 1$ if $a < t$ and $\sigma(t,a) = 0$ if $a \geq t$. As before, dynamic programming gives the value functions $V^S$ and $V^T$ of each party if the reaction function of the other party is specified. Some additional notation will prove useful. The expected discounted utility of the sender, having set a threshold t the previous period, in a period in which the target chooses, is:

$$W^S(t) = E\{u^S[R^T(t),\sigma(t,R^T(t))] + \delta_S V^S[R^T(t)]\}.$$

Similarly, the expected discounted utility of the target, having set an action level a the previous period, in a period in which the sender chooses, is:

$$W^T(a) = E\{u^T[a,\sigma(R^S(a),a)] + \delta_T V^T[R^S(a)]\}.$$

The expected discounted value to the sender of currently choosing an arbitrary threshold t, given the target's action level a, is:

$$Z^S(a,t) = u^S[a,\sigma(t,a)] + \delta_S W^S(t),$$

and the expected discounted value to the target of currently choosing an arbitrary action level a, given the sender's threshold t, is:

$$Z^T(t,a) = u^T[a,\sigma(t,a)] + \delta_T W^T(a).$$

Let $C^S(a)$ denote $\underset{t}{\operatorname{argmax}}\, Z^S(a,t)$ and let $C^T(t)$ denote $\underset{a}{\operatorname{argmax}}\, Z^T(a,t)$. Then $R^S$ and $R^T$ constitute an equilibrium if and only if:

$$\forall a \in [0,1],\ R^S(a) \subset C^S(a) \text{ and } \forall t \in [0,1],\ R^T(t) \subset C^T(t).$$

If so:

$$V^S(a) = \max_t Z^S(a,t) \text{ and } V^T(t) = \max_a Z^T(a,t).$$

$Z^S(a,t)$ is increasing in a; hence $V^S$ is. Similarly, $Z^T(t,a)$ is nonincreasing in t; hence $V^T$ is.

We say that $\dot{a}$ is a steady state of the equilibrium if $\dot{a} = R^T(\dot{a}) = R^S(\dot{a})$. Proposition 2 states that if sanctions have limited capacity then there exists an equilibrium that supports $\bar{a}$, the target's highest individually rational action level, as its unique steady state.

Proposition 2: If $\exists\bar{a} \le 1$ such that $u^T(\bar{a},0) = u^T(0,1)$ then, for $\delta_S$ sufficiently close to one, the following is an equilibrium:

$$R^S(a) = \bar{a} \quad \forall a$$

$$R^T(t) = \begin{matrix} t \text{ for } t \in [0,\hat{a}] \cup (\bar{a}) \\ 0 \text{ for } t \in (\hat{a},\bar{a}) \cup (\bar{a},1], \end{matrix}$$

where $\hat{a}$ satisfies $u^T(\hat{a},0) + \delta_T u^T(\hat{a},1) = 0$.

These reaction functions are depicted in Figure 2. The sender always sets $\bar{a}$ as a threshold, thus spurning all action levels below $\bar{a}$. ($\delta_S$ must be large enough to ensure that condoning at levels below $\hat{a}$ is no better.) At or below $\bar{a}$ the target is indifferent between acquiescing to $\bar{a}$ and spurning to zero, both of which dominate just meeting the threshold above $\hat{a}$. Between $\hat{a}$ and $\bar{a}$ this indifference is resolved by balking to zero but $\bar{a}$ itself is matched. At thresholds above $\bar{a}$ balking to zero is best.

Thus the action level at which the target is at its minimal individually rational utility level, if it is feasible, can be sustained as a steady state. We now show that under fairly general conditions there is just one other equilibrium that supports a steady state. We characterize this equilibrium and show that it supports a band of steady states lying below $\bar{a}$, whose width tends to zero as either party's discount factor tends to one.

We now impose the restrictions (i) that the cost of sanctions to the sender increase and (ii) that the cost of sanctions to the target not increase in the target's action level. That is, $u^S(a,0) - u^S(a,1)$ everywhere increases in a while $u^T(a,0) - u^T(a,1)$ nowhere increases in a.

Under these restrictions we use five lemmata to prove two theorems that ensure the existence of, and completely characterize, equilibrium steady-state outcomes.

Lemma 1 says that there is an action level $\underline{a}$ such that the sender condones all action levels above $\underline{a}$ and spurns all those below $\underline{a}$:

Lemma 1: $\exists \underline{a}$ such that $\forall a < \underline{a}$, $C^S(a) > a$ and $\forall a > \underline{a}$, $C^S(a) \leq a$.

Lemma 2 says that the sender will never spurn to a threshold that it would spurn if the target performed at that level:

Lemma 2: If $b \in C^S(a)$ and $b > a$ then $C^S(b) \leq b$.

Lemma 3 states that, if there is a point n at which the target does not balk (i.e. is "nice"), then the sender, when facing an action level of at least n, never sets a threshold that the target strictly prefers to n:

**Lemma 3:** If $R^T(n) \geq n$, $a \geq n$, and $t \in C^S(a)$ then $V^T(t) \leq V^T(n)$.

Lemma 4 characterizes any point n at which neither balking nor spurning occurs. Raising its action level above n leaves the target worse off as of the following period (i), while lowering the threshold below n makes the sender no better off as of the following period (iii). The target will always match at n (ii), and it is optimal for the sender to match at n (iv). Finally, $V^i(n)$ and $W^i(n)$ have the same values that they would if n were a steady state (v and vi).

Lemma 4: Suppose that $R^S(n) \leq n$ and $R^T(n) \geq n$.

(i) If $a > n$, then $W^T(a) < W^T(n)$.

(ii) If $t \leq n$, then $C^T(t) \leq n$. Thus $R^T(n) = n$.

(iii) If $t < n$, then $W^S(t) \leq W^S(n)$.

(iv) $n \in C^S(n)$.

(v) $V^S(n) = W^S(n) = u^S(n,0)/(1-\delta_S)$.

(vi) $V^T(n) = W^T(n) = u^T(n,0)/(1-\delta_T)$.

Let $\bar{t}$ denote $\min\{\bar{a},1\}$, the largest feasible action level that gives the target at least its minimal individually rational payoff. Let $B = \{t \in [0,1]: C^T(t) \cap [0,t) \neq \phi\}$, i.e., B is the set of all thresholds t at which it is optimal for the target to balk. If B is not empty let $\underline{t} = \inf B$ while if B is empty let $\underline{t} = 1$.

Lemma 5: B is an interval and $V^T(t)$ is a constant for all $t \in B$.

If B is nonempty we define $V = V^T(t)$ for all $t \in B$. We now state:

Theorem 1: (i) If $\underline{t} \leq \underline{a}$ and there is a steady state then it is at $\underline{a} = \bar{a}$. (ii) If $\underline{a} < \underline{t}$ then each $a \in (\underline{a}, \underline{t})$ is a steady state and the following pairs of inequalities hold with complementary slackness (i.e., at most one in each pair is strict):

$$u^T(\underline{t},0) \geq (1-\delta_T)u^T(\underline{a},1) + \delta_T u^T(\underline{a},0), \qquad 2(a)$$

$$\underline{t} \leq \bar{t}; \qquad 2(b)$$

$$u^S(\underline{a},0) \geq (1-\delta_S)u^S(\underline{a},1) + \delta_S u^S(\underline{t},0), \qquad 3(a)$$

$$\underline{a} \geq 0. \qquad 3(b)$$

Theorem 2: (i) There exists a solution to inequalities (2) and (3), which we denote a* and t*, and there exists an equilibrium whose set of steady states is [a*,t*]. (ii) If, in addition, $u^S$ and $u^T$ are concave in a then a* and t* are unique and the only possible Markov perfect steady-state outcomes are [a*,t*] and, if sanctions have limited capacity, $\bar{a}$.

Figure 3 depicts equilibrium reaction functions that support steady states [a*,t*].

*Example 2:*

To illustrate the theorems, we generalize Example 1 to allow for differences between the cost of sanctions to the sender and to the target and a more general interaction between sanctions and the action level:

$$u^S(a,s) = a - s(F_S + G_S a)$$
$$u^T(a,s) = -a - s(F_T - G_T a), \tag{4}$$

where $a \in [0,1]$ and $s \in (0,1)$. Our restrictions on $u^S$ and $u^T$ require that:

$$1 > G_S > 0,\ 1 > G_T \geq 0,\ F_S > 0,\ \text{and}\ F_T > G_T. \tag{5}$$

In this example, $\tilde{a} = F_T$ if $F_T \leq 1$. (If $F_T > 1$, we are in the overkill case and $\tilde{t} = 1$.)

Conditions (2) and (3) take the following form (where, for convenience, we replace $\underline{t}$ by $w = \underline{t} - \underline{a}$, the width of the band of steady states):

$$G_T\underline{a} + \frac{w}{(1-\delta_T)} \leq F_T \qquad \text{(complementary with } \underline{a} + \underline{w} \leq \tilde{t}\text{)}$$

$$-G_S\underline{a} + \frac{\delta_S w}{(1-\delta_S)} \leq F_S \qquad \text{(complementary with } \underline{a} \geq 0\text{)}.$$

To find the range of steady states we solve:

$$G_T\underline{a} + \frac{w}{(1-\delta_T)} = F_T$$

and

$$-G_S\underline{a} + \frac{\delta_S w}{(1-\delta_S)} = F_S$$

for $w$ and $\underline{a}$ to obtain the solution:

$$w^* = \frac{(1-\delta_T)(G_T F_S + G_S F_T)}{\alpha G_T + G_S}$$

$$a^* = \frac{\alpha F_T - F_S}{\alpha G_T + G_S}.$$

where

$$\alpha = \delta_S(1-\delta_T)/(1-\delta_S)$$

is a measure of the sender's patience relative to the target's.[14] If both parties share a common discount factor $\delta$ then $\alpha = \delta$. Restrictions (5) imply that $w^* > 0$ and, if either $\delta_S$ or $\delta_T$ converges to 1, then $w^* \downarrow 0$, so that the set of steady states converges to a point.

Steady states may lie strictly between zero and one (Case I), $a = 0$ (bliss for the target) may be a steady state (Case II), and $a = 1$ (bliss for the sender) may be a steady state (Case III).

Case I: If $0 < a^* < a^* + w^* < \tilde{t}$, then the range of steady states (other than $\bar{a}$ when sanctions have limited capacity) is $[a^*, a^*+w^*]$, where:

$$a^* = \frac{\alpha F_T - F_S}{\alpha G_T + G_S}$$

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[14]As the time between choices tends to zero, $\alpha$ converges to $\rho_T/\rho_S$, the ratio of the target's continuous discount rate to the sender's. Since $\delta_i = e^{-\rho_i \Delta}$, where $\Delta$ is the interval between choices, the result follows from L'Hôpital's Rule.

which increases in $F_T$ and $\alpha$, and decreases in $F_S$, $G_T$, and $G_S$.[15] Thus lowering the cost of sanctions to the sender or increasing the sender's patience relative to the target's raises the lower bound on steady-state performance levels. Furthermore, as the lowest cost to the target of incurring sanctions, $F_T - G_T$, or $G_T$ rises so does this bound. In this way the sender benefits from being more patient than the target, and from having sanctions that are relatively more painful for the target.

If the parties share a common discount factor that converges to one then the steady states converge to:

$$\frac{F_T - F_S}{G_T + G_S}.$$

If in addition, as in Example 1, $G_S = G_T = G$ and $F_S = F_T - G$ (so that the costs of sanctions are symmetric) then the steady states converge to 1/2. Thus, by the continuity of the expression for $a^*$, if the two parties have nearly equal costs and patience, the outcome is nearly symmetric.

<u>Case II</u>: Zero can be supported as a steady state if and only if $a^* \leq 0$ or, equivalently, $\alpha F_T \leq F_S$. Thus if the target is sufficiently patient relative to the sender, a steady-state performance level of zero can emerge: Holding $\delta_S$ constant below one, as $\delta_T$ approaches one, $\alpha$ approaches 0 so that the inequality is satisfied.

If the parties have the same discount factor converging to one then the condition is that $F_T \leq F_S$: the highest possible cost of sanctions to the

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[15]The result for $\alpha$ is most readily seen by observing that functions of the form $(ax+b)/(cx+d)$ are monotonic in x (by the quotient rule), so we need only compare the expressions when x is zero and when x is infinite.

target is no greater than the lowest possible cost of sanctions to the sender.

Case III: An action level of one can be supported as a steady state if and only if $F_T = 1$, so that $\bar{a} = 1$, or if $F_T \geq 1$ and $a^* + w^* \geq 1$. If the sender is sufficiently patient relative to the target and sanctions have overkill capacity, then the sender can exact a performance level of one: Holding $\delta_T$ constant below one, as $\delta_S$ approaches one, $a^*$ approaches $F_T/G_T > 1$.

If the parties have the same discount factor converging to one then $a^* + w^* \geq 1$ reduces to $F_T - G_T \geq F_S + G_S$: The lowest possible cost of sanctions to the target is no less than their highest possible cost to the sender.

In Example 1, $G_T = G_S = .5$, $F_T = 1$, and $F_S = .5$, so that $\bar{a} = 1$, and a steady state at one can be supported, by Proposition 2. But if $F_T$ is raised slightly then the highest steady state that can be supported is less than one (in fact, it is around one half if discount factors are similar). Thus sanctions with overkill capacity can be less effective than limited sanctions.

To see why, consider behavior just below any maximum sustainable steady state $a^m$. If $\bar{a}$ is the unique steady state, sanctions cannot be too powerful to deter balking at thresholds just below $\bar{a}$. In fact, the target must balk at such thresholds if the sender is to spurn all action levels below $\bar{a}$. However, in any equilibrium that supports a steady state less than $\bar{a}$, sanctions are so powerful that, below $a^m$, acquiescing strictly dominates balking. The target's compliance removes the sender's incentive to spurn action levels just below $a^m$, and such points become steady states as well. There is thus an equilibrium determined by conditions (2) and (3) that supports a band of steady-state action levels below $\bar{a}$.

Example 2 illustrates that, if the sender can commit itself, even for an arbitrarily short while, to sanctions that are contingent on the subsequent behavior of the target, then a considerable degree of compliance can be enforced as part of a Markov perfect equilibrium without sanctions actually being suffered. If utility functions are as in Example 1, for $\delta$ near one, the sender will extract performance that is at least almost half the maximum feasible level.

We have found the general characterization of limit equilibria of an alternating move game with contingent sanctions to be intractable. With utility functions as in Example 2, we have verified that an equilibrium characterized by Theorem 2(i) can be a limit equilibrium.[16] We have also found that the equilibrium of Proposition 2 is not a limit equilibrium.

## VI. Exacting a Single Action: Continuous Sanctions

Suppose now that the sender wants the target to perform an irreversible action, but continue to assume that sanctions are contingent, and that the sender and target alternate in their decisions. Actions are dichotomous, (i.e., $A = \{0,1\}$). Once the target complies, setting $a = 1$, interaction ceases. As long as the target balks, setting $a = 0$, however, the sender can impose sanctions.

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[16]More exactly, we have laboriously verified that in Case I, if the target is the last mover then reaction functions have a similar form to those in the proof of Theorem 2(i). As the horizon lengthens, these converge pointwise to the Markov perfect equilibrium reaction functions. As time moves backward the intervals of matching contract toward the band of steady states supported by the Markov perfect equilibrium. We conjecture that similar results hold for Cases II and III.

When the target complies, its present discounted utility from that point on is normalized at -1 and the sender's is normalized at 1. If the target balks then the next period the sender may impose sanctions at some level s, implying a current utility $u^S(s)$ for itself and $u^T(s)$ for the target, both of which decrease in s. We normalize $u^S(0) = u^T(0) = 0$. Here we allow for a continuum of sanctions, so that $S = [0,1]$.

The expected discounted utility of the sender in any period before the target has complied is then:

$$V^S = \max_s \{u^S(s) + \pi(s)\delta_S + [1-\pi(s)](\delta_S u^S(s) + \delta_S^2 V^S]\},$$

where $\pi(s)$ is the probability of compliance next period given s. The expected discounted utility of the target in any period before it complies is:

$$V^T(s) = \max(-1, u^T(s) + \delta_T E[u^T(s') + \delta_T V^T(s')]),$$

where s is the current sanctions level and s' the (possibly random) level chosen in the subsequent period if the target balks in the current period.

We define three key sanction levels. The first, $s^m$, is such that the target is just indifferent between complying and suffering $s^m$ forever. It is defined by the condition $u^T(s^m)/(1-\delta_T) = -1$. To be minimally effective, sanctions must be at least $s^m$.

The second, $s^b$, is such that the target is just indifferent between complying and suffering $s^b$ currently and never again. It is defined by the condition $u^T(s^b) = -1$. Obviously, if both exist, $s^b > s^m$. To be brutally effective, sanctions must exceed $s^b$.

The third, $s^i$, is such that the <u>sender</u> is just indifferent between imposing $s^i$ currently if it ensures the target's compliance the next period, and never imposing sanctions, ensuring permanent balking. It is defined by the condition $u^S(s^i) + \delta_S = 0$. Sanctions above $s^i$ are <u>incredibly costly</u>.

There are two kinds of equilibria. If $s^i \le s^b$, i.e., if $s^i$ is not brutally effective, then it is an equilibrium for the sender never to impose sanctions and for the target to comply if and only if sanctions are brutal. The outcome is permanent balking with no sanctions. If $s^i < s^m$, i.e., if $s^i$ is not even minimally effective, then this is the only equilibrium outcome.

If, however, $s^i \ge s^m$, i.e., if $s^i$ is minimally effective, then another equilibrium is for the target to comply if and only if sanctions are at least $s^m$ and for the sender to impose sanctions at $s^m$. If the target moves first and sanctions are initially not minimally effective then the outcome is balking in the initial period, followed by the imposition (and suffering) of sanctions at $s^m$ in the next period, with compliance the period after that. If $s^i > s^b$ then this is the only equilibrium outcome. If $s^m \le s^i \le s^b$, i.e., if $s^i$ is minimally, but not brutally effective, then both outcomes are possible.

All the above are also limit equilibria except that there is no multiplicity in this last case: Only permanent balking can be the outcome of a limit equilibrium here.

## VII. Exacting a Single Action: A Single Sanction

If sanctions are dichotomous, so that $S = \{0,1\}$, then outcomes may be much more complicated, and nastier for all: Sanctions may be experienced for more than one period, and the target may delay complying for a while, but not necessarily forever.[17]

In accord with our earlier notation let $u^S(1) = -F_S$ and $u^T(1) = -F_T$.

*Balking Forever: Ineffective or Incredibly Costly Sanctions*

If sanctions are not brutally effective (i.e., $F_T \leq 1$) then one equilibrium is for the target always to balk and for the sender never to impose sanctions. If sanctions are not minimally effective (i.e., $F_T < 1-\delta_T$) then it is the only equilibrium. It is also the only equilibrium if sanctions are incredibly costly (i.e., $F_S > \delta_S$).

However, if sanctions are brutally effective but not incredibly costly then balking forever with no sanctions cannot be an equilibrium outcome. The sender can eventually get the target to comply, but how it does so depends on whether or not sanctions can deter balking.

*Painful Compliance: Nondeterrent Sanctions*

A fourth key characteristic of sanctions is their deterrence effect. If $\delta_T F_T > 1-\delta_T^2$ then sanctions are deterrent: If the target is sure that balking will lead to such sanctions the next period, forcing it to comply the period after that, then it prefers to comply now, even if sanctions are not currently in place. If sanctions are not deterrent or incredibly costly, but are minimally effective then, as with continuous sanctions, an equilibrium is for the target to balk in the absence of sanctions and to comply in their presence, and for the sender always to impose sanctions. The outcome is

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[17]Matsuyama's (1990) analysis of a trade liberalization game between a government and a local firm, where the government wants the firm to become more efficient and the firm wants protection, and Fernandez and Glazer's (1990) analysis of strikes, are similar to the games considered here. In either case, waiting and randomization can occur, and outcomes can be inefficient, even though information is perfect.

initial balking, followed by sanctions, followed by compliance. If, also, sanctions are brutally effective then this is the only equilibrium outcome.

*Mixed Strategy Equilibria*

However, if sanctions are deterrent then the sender will not always impose them: If it did, then the threat of sanctions two periods hence would suffice to enforce compliance the next period, so there is no reason to impose sanctions currently. If sanctions are deterrent then the only possible equilibria in which the target complies involve mixed strategies.

If sanctions are deterrent and not incredibly costly then an equilibrium is for the sender to impose sanctions randomly, and for the target to comply if sanctions are in place, and to randomize between complying and balking in their absence (i.e., mixing at condoning). If, in addition, sanctions are brutally effective, this is the only equilibrium.

The outcome supported by this equilibrium is one in which some delay is expected before the target complies. It may comply without sanctions, or sanctions may be imposed in the period before compliance. Sanctions will not last for more than one period, however.

In another equilibrium in which the sender randomizes, the target mixes between balking and compliance when sanctions are in effect, and always balks in their absence (i.e., mixing at sanctions). This can occur when sanctions are minimally, but not brutally, effective and are not incredibly costly.

Figure 4 illustrates the various possible outcomes as a function of the cost of sanctions to the target $F_T$ and the target's discount factor $\delta_T$, assuming that sanctions are not incredibly costly. Three curves divide the region of possible values into five parts. Above the horizontal line $F_T = 1$

sanctions are brutal while below the diagonal line $F_T = 1-\delta_T$ they are not even minimally effective. Above the curve $F_T = 1/\delta_T - \delta_T$ sanctions are deterrent. The diagram indicates the possible outcomes in each region.

The limit equilibria in the various cases are as follows: Below the brutality boundary only balking forever can occur, while painful compliance remains the equilibrium outcome above the brutality boundary and below the deterrence boundary. Above these two boundaries, the finite-horizon equilibrium is for the sender to impose sanctions every 2i periods from the period of the sender's last possible move, where i satisfies the conditions that $F_S + \delta_S^{2i-3} \geq \delta_S \geq F_S + \delta_S^{2i-1}$, and for the target to comply if sanctions are in effect and otherwise to balk, except just before sanctions are scheduled. The outcome is balking until the period before the first scheduled sanctions, at which time the target complies.

Taking the appropriate limit gives i limit equilibria, indexed by the time of the first scheduled sanctions. These equilibria are not Markov perfect, however: Choices depend upon elapsed time, which is payoff irrelevant. They are also not renegotiation proof: Whenever sanctions are mandated both sender and target prefer to delay them.

## VIII. Conclusion

We have considered the ability of sanctions to exact concessions in a variety of circumstances. A conclusion is that sanctions that are costly for the sender to impose can be credible. A necessary condition to ensure their success in exacting concessions, however, is that the harm caused by the sanctions depend on the target's degree of compliance. Otherwise, zero compliance is always a possible outcome. Sanctions that are purely spiteful (in the sense that the harm that they do does not depend on the target's subsequent behavior) do not ensure compliance.

We have limited ourselves to the interaction of only two parties. We thus ignore the public-goods issues raised by having multiple senders (which have undermined recent attempts to impose sanctions against the People's Republic of China). We also ignore the issues that arise when distinct groups within one country have diverse interests. U.S. farmers, for example, bore the brunt of the U.S. grain embargo against the Soviet Union, while the actions sought by the United States in Japan in recent negotiations under "Super 301" were apparently welcomed by most Japanese consumers.

We have also allowed sanctions to go only one way. If both parties can take actions with external benefits and impose punishments then many more possibilities emerge. Characterizations as tight as those in Theorem 2 are thus unlikely.

Our analysis has focused solely on situations of symmetric information. Even in these, compliance may be delayed, and outcomes can be inefficient in that sanctions may actually have to be used. Informational asymmetries are likely to increase the potential for delay and inefficiency.

Finally, the differences between the simultaneous and alternating move equilibria demonstrate the critical importance of timing. A better understanding of timing would emerge from a model with information lags as well as response lags: Each party learns the other's choice only with delay, and makes its own decision only with further delay. The alternating case describes a situation with long response lags relative to information lags, while the simultaneous case is more descriptive of the opposite. Modern communications technology and political institutions suggest that the first situation better describes the environment in which governments set policy.

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## APPENDIX A: DERIVATION OF RENEGOTIATION-PROOF PAYOFFS IN THE REPEATED SIMULTANEOUS MOVE GAME

Farrell and Maskin's (1987) criterion that a subgame perfect equilibrium be renegotiation-proof requires that the equilibrium have no subgames that Pareto-dominate others. The justification is that, otherwise, when about to embark on a dominated subgame all players would benefit by agreeing to switch strategies to the Pareto-preferred one, and what's to stop them from doing so?

To determine what restrictions this requirement imposes on a pure strategy equilibrium, consider the beginning of the subgame of the game that is worst from the target's perspective. Let $v_S$ and $v_T$ denote the average per period payoffs from that period on to the sender and target respectively.

Following Farrell and Maskin (1987) we decompose the pair $(v_S, v_T)$ as:

$$(v_S, v_T) = (1-\delta)(v_S^1, v_T^1) + \delta(v_S^c, v_T^c)$$

where $v_i^1$ is i's first-period expected payoff and $v_i^c$ is the average per-period payoff for the remainder of the subgame, which is itself another subgame. Since $v_T$ is the lowest payoff of any subgame to the target, $v_T \le v_T^c$, and since $v_T$ is a convex combination of $v_T^c$ and $v_T^1$, $v_T^1 \le v_T \le v_T^c$. Since $v_T^c \ge v_T$, the requirement that no subgame Pareto dominate any other implies that $v_S^c \le v_S$. Since $v_S$ is a convex combination of $v_S^c$ and $v_S^1$, $v_S^1 \ge v_S \ge v_S^c$.

However, nothing can be worse for the target than choosing its dominant strategy and finding itself again at the beginning of its worst subgame, i.e.,

$$v_T \ge (1-\delta)u^T(0,s) + \delta v_T$$

or

$$v_T \geq u^T(0,s).$$

The condition that $v_S^1 \geq v_S$ implies that

$$v_S \leq u^S(a,s).$$

These constraints on $v_S$ and $v_T$ restrict the set of possible payoffs to lie within the shaded regions in Figures 1a and 1b. (The parallel exercise performed from the perspective of the sender imposes no restrictions on the set of sustainable subgame perfect payoffs.)

The upper bound $\overline{a}$ on the steady-state performance level implied by the condition that the equilibrium be renegotiation-proof is thus determined by the two conditions:

$$u^S(1,\overline{s}) = u^S(\overline{a},0)$$
$$u^T(\overline{a},0) = u^T(0,\overline{s}). \qquad \text{(A1)}$$

The following is a renegotiation proof equilibrium that, in the limit as $\delta \uparrow 1$, can sustain $\overline{a}$:

Define the following modes:

| | |
|---|---|
| Regular mode: | $a = \hat{a},\ s = 0$ |
| Punishment mode: | $a = 1,\ s = \hat{s}$ |
| Zero mode: | $a = 0,\ s = 0.$ |

The equilibrium strategies call upon the parties to: (i) adhere to punishment mode if, in any of the last N periods, the target has set $a < \hat{a}$ if the game was in regular mode or $a < 1$ if the game was in punishment mode; (ii) adhere to zero mode if the sender has ever set $s < \hat{s}$ in punishment mode; (iii) adhere to regular mode otherwise.

For $\delta$ sufficiently close to 1 and N sufficiently large, the equilibrium is renegotiation-proof as long as

$$u^S(1,\hat{s}) > u^S(\hat{a},0)$$

$$u^T(\hat{a},0) > u^T(0,\hat{s}),$$

which permit steady-state action levels $\hat{a}$ arbitrarily close to $\bar{a}$.

An increase in the cost of sanctions to the target raises $\bar{a}$, while an increase in their cost to the sender lowers $\bar{a}$. Using Example 2 of the text

$$\bar{a} = F_T/(F_S + G_S + F_T).$$

## APPENDIX B: PROOFS

Proof of Proposition 1: Assume that $R^S(a) = 0$ for all $a \in R^T(0)$. $R^T(0)$ must consist of a single point. (If it included points $a_1 < a_2$, since both elicit zero sanctions, and since $u^T$ decreases in a, $a_1$ would be a better response to 0 for the target than $a_2$, so $a_2 \notin R^T(0)$.) So let $\bar{a}$ denote $R^T(0)$. For any $a < \bar{a}$, $R^S(a)$ must exceed 0 with positive probability. (Otherwise a would be a better response to 0 than $\bar{a}$ for the target.)

For all s, $R^T(s) \le \bar{a}$ with probability one. (For any choice $a > \bar{a}$ by the target, its current utility is lower, and the sanctions outcome no more favorable than at $\bar{a}$.)

For the sender, then, setting $s = 0$ is better than setting $s > 0$ at any action level a. (Setting $s = 0$ elicits the highest possible performance level by the target at the least possible cost to the sender.) Hence $R^S(a) = 0$ for all a.

But if the sender does not impose sanctions under any circumstances then the target's only best response can be $R^T(s) = 0$ for all s. □

Proof of Proposition 2: By continuity $\hat{a}$ exists and, because $u^T$ is decreasing in a, it is unique and lies between 0 and $\bar{a}$. Thus $R^T$ is well defined.

Verifying that a pair of reaction functions $R^S$ and $R^T$ constitute an equilibrium requires demonstrating that: (i) if the sender adheres to $R^S$ then $R^T$ is optimal for the target; and (ii) if the target adheres to $R^T$ then $R^S$ is optimal for the sender. With dynamic programming, optimality can be demonstrated by verifying that: (i) if the sender adheres to $R^S$, and the target will adhere to $R^T$ in the future, then $R^T(t)$ is optimal for the target currently at any feasible threshold t; and (ii) if the target adheres to $R^T$, and the sender will adhere to $R^S$ in the future, then $R^S(a)$ is optimal for the sender currently at any feasible action level a.

In principle one must consider all possible actions as alternatives to $R^T(t)$ and all possible thresholds as alternatives to $R^S(a)$. However, since $R^S$ and $R^T$ are piecewise nondecreasing, each term in the expression:

$$Z^S(a,t) = E\{u^S[a,\sigma(t,a)] + \delta_S u^S[R^T(t),\sigma(t,R^T(t))] + \delta_S^2 V^S[R^T(t)]\}$$

is nondecreasing in t except, possibly, at points of discontinuity in one of the three terms, while each term in the expression:

$$Z^T(t,a) = E\{u^T[a,\sigma(t,a)] + \delta_T u^T[a,\sigma(R^S(a),a)] + \delta_T^2 V^T[R^S(a)]\}$$

is decreasing in a except, possibly, at points of discontinuity in one of the three terms. Hence it suffices to check that these points as well as 0 and 1 are not superior to the prescribed actions. Performing this operation is routine, and verifies the result. □

<u>Proof of Lemma 1</u>: We show that if spurning is optimal at some point a then below a only spurning is optimal. If it is optimal for the sender to spurn a then $\exists t > a$, such that:

$$V^S(a) = u^S(a,1) + \delta_S W^S(t) \geq u^S(a,0) + \delta_S W^S(x) \quad \forall x \leq a.$$

Consider the sender's response to any $b < a$. The above inequality and the condition that $u^S(a,0) - u^S(a,1)$ strictly increases in a imply that:

$$Z^S(b,t) = u^S(b,1) + \delta_S W^S(t) > u^S(b,0) + \delta_S W^S(x) \quad \forall x \le b. \quad \square$$

Proof of Lemma 2: Suppose not. Then $\exists c > b$ such that $c \in C^S(b)$. Hence:

$$V^S(a) = u^S(a,1) + \delta_S W^S(b) \ge Z^S(a,c) = u^S(a,1) + \delta_S W^S(c)$$

and:

$$V^S(b) = u^S(b,1) + \delta_S W^S(c) \ge Z^S(b,b) = u^S(b,0) + \delta_S W^S(b).$$

The second inequality implies $W^S(c) > W^S(b)$, which contradicts the first. $\square$

Proof of Lemma 3: For $t \ge n$ this follows by monotonicity. For $t < n$, since

$$u^S(a,0) + \delta_S W^S(t) = Z^S(a,t) = V^S(a) \ge Z^S(a,n) = u^S(a,0) + \delta_S W^S(n),$$

$W^S(t) \ge W^S(n)$ and so

$$E(u^S[R^T(t),0] + \delta_S V^S[R^T(t)]) \ge W^S(t) \ge W^S(n) \ge u^S(n,0) + \delta_S V^S(n),$$

which is only possible if $\exists b \in R^T(t)$ such that $b \ge n$. Thus

$$V^T(t) = Z^T(t,b) = Z^T(n,b) \le V^T(n). \quad \square$$

<u>Proof of Lemma 4</u>:

(i) $W^T(n) \geq u^T(n,0) + \delta_T V^T(n) > u^T(a,0) + \delta_T E(V^T[R^S(a)]) \geq W^T(a)$, by Lemma 3.

(ii) We show that if $a > n$ then $a \notin C^T(t)$. That $R^T(n) = n$ then follows immediately. If $a > n$ then, by (i), $W^T(a) < W^T(n)$, and so

$$Z^T(t,a) = u^T(a,0) + \delta_T W^T(a) < u^T(n,0) + \delta_T W^T(n) = Z^T(t,n).$$

(iii) By (ii), $W^S(t) \leq u^S(n,0) + \delta_S E(V^S[R^T(t)]) \leq u^S(n,0) + \delta_S V^S(n) \leq W^S(n)$.

(iv) Since $R^S(n) \leq n$, $\exists t \in C^S(n) \cap [0,n]$. But, by (iii),

$$Z^S(n,n) = u^S(n,0) + \delta_S W^S(n) \geq u^S(n,0) + \delta_S W^S(t) = V^S(n).$$

(v) By (ii) and (iv),

$$V^S(n) = Z^S(n,n) = u^S(n,0) + \delta_S W^S(n) = u^S(n,0)(1+\delta_S) + \delta_S^2 V^S(n).$$

(vi) By (ii),

$$V^T(n) = Z^T(n,n) = u^T(n,0) + \delta_T W^T(n) = u^T(n,0)(1+\delta_T) + \delta_T^2 E(V^T[R^S(n)])$$

$$= u^T(n,0)(1+\delta_T) + \delta_T^2 V^T(n),$$

since $E(V^T[R^S(n)]) = V^T(n)$, by Lemma 3 and because $R^S(n) \leq n$. □

<u>Proof of Lemma 5</u>: If $t_1 < t_2$ then, by monotonicity, $V^T(t_1) \geq V^T(t_2)$. If $t_1 \in B$, $\exists a < t_1$ such that

$$V^T(t_1) = u^T(a,1) + \delta_T W^T(a) = Z^T(t_2,a) \leq V^T(t_2) \leq V^T(t_1).$$

Thus $t_2 \in B$ and $V^T(t_2) = V^T(t_1)$. □

<u>Proof of Theorem 1</u>: (i) We first show that $\underline{t} = 1$ is impossible if $\underline{t} \leq \underline{a}$. Suppose that $\{a_n\}$ converges to $\underline{t} = 1$ from below. Thus the target never balks at $a_n$. By condoning $a_n$ forever the sender obtains $u^S(a_n,0)/(1-\delta_S)$, which converges to $1/(1-\delta_S)$, which exceeds the value of spurning to one, $u^S(a_n,1) + \delta_S/(1-\delta_S) < u^S(1,1) + \delta_S/(1-\delta_S) < 1/(1-\delta_S)$. This contradicts $\underline{a} = 1$. Thus $\underline{t} < 1$, and so B is not empty.

We can thus find a sequence of thresholds $t_n \downarrow \underline{a}$ such that $t_n \in B$ and the sender does not spurn $t_n$. (If $\underline{a} = 1$, for each n choose $t_n = \underline{a}$; if not, choose $t_n > \underline{a}$.) Since $t_n$ is not spurned, the value to the target of acquiescing to $t_n$ is at least $u^T(t_n,0)/(1-\delta_T)$. But since $t_n \in B$, the target is willing to balk. Hence $V = V^T(t_n) \geq u^T(t_n,0)/(1-\delta_T)$, and the limit gives

$$V \geq u^T(\underline{a},0)/(1-\delta_T).$$

If there is a steady state at $\overline{a}$, then, $V^T(\overline{a}) = u^T(\overline{a},0)/(1-\delta_T)$. By Lemma 1, $\overline{a} \geq \underline{a}$, so that $V \leq V^T(\overline{a}) = u^T(\overline{a},0)/(1-\delta_T) \leq u^T(\underline{a},0)/(1-\delta_T) \leq V$ by the above inequality.

Hence $\underline{a}$ is the unique steady state and $V^T(\underline{a}) = V$. By Lemma 2, balking to $a < \underline{a}$ provokes spurning to $t \geq \underline{a} \geq \underline{t}$. Since $t \in B$, balking yields $u^T(a,1)(1+\delta_T) + \delta_T^2 V$, which is maximized at $a = 0$. Hence the optimal balk is to 0, implying that $V = 0$, and hence $u^T(\underline{a},0) = 0$, so that $\underline{a} = \bar{a}$.

(ii) Each point n in the interval $(\underline{a},\underline{t})$ satisfies the assumptions of Lemma 4. Thus $R^T(n) = n$, by (ii), and since $W^S$ is increasing on this interval, by (v), $R^S(n) = n$, so that each point n is a steady state. The inequalities 2(a) and 2(b) follow from the requirement that, at thresholds $t_n \uparrow \underline{t}$, matching is at least as good as balking to levels $a_n \downarrow \underline{a}$ (2(a)) and to zero forever (2(b)). The inequality 3(a) follows from the requirement that, at levels $a_n \downarrow \underline{a}$, spurning to thresholds $t_n \uparrow \underline{t}$ is no better than matching.

Turning to the complementary slackness conditions, we first show that, if $B \neq \phi$, $V = V^T(\underline{t}) = u^T(\underline{t},0)/(1-\delta_T)$. Taking a sequence of thresholds $t_n \uparrow \underline{t}$ shows that $V \leq V^T(\underline{t}) \leq u^T(\underline{t},0)/(1-\delta_T)$, by monotonicity. $\forall t \in B$, $t \geq \underline{t} > \underline{a}$, so

$$V = V^T(t) \geq Z^T(t,t) \geq u^T(t,0)(1+\delta_T) + \delta_T^2 V,$$

and, taking the limit as $t \downarrow \underline{t}$, and combining the previous two inequalities

$$V \geq u^T(\underline{t},0)/(1-\delta_T) \geq V^T(\underline{t}) \geq V,$$

so that $V^T(\underline{t}) = u^T(\underline{t},0)/(1-\delta_T) = V$. Hence, at $\underline{t}$, the target is indifferent between balking and matching. We now show that the target never chooses an action level above $\underline{t}$.

The value of condoning any $a \geq \underline{t}$ is $Z^S(a,t) = u^S(a,0) + \delta_S W^S(t)$. But by Lemma 4(v), $W^S(t)$ is increasing on $(\underline{a},\underline{t})$. Hence $C^S(a) \geq \underline{t}$ so that choosing an action level above $\underline{t}$ yields the target at most $u^T(t,0)(1+\delta_T) + \delta_T^2 V < V$. Thus $C^T(t) \leq \underline{t}$ for all t.

In particular, $C^T(\underline{t}) \leq \underline{t}$, which implies that

$$V^S(\underline{t}) = u^S(\underline{t},0) + \delta_S W^S(\underline{t}) \leq u^S(\underline{t},0)(1+\delta_S) + \delta_S^2 V^S(\underline{t}).$$

By Lemma 4(v), for all $n \in (\underline{a},\underline{t})$, $u^S(n,0)/(1-\delta_S) = V^S(n) \leq V^S(\underline{t})$ by monotonicity. Taking the limit as $n \uparrow \underline{t}$ $V^S(\underline{t}) \geq u^S(\underline{t},0)/(1-\delta_S)$ which, combined with the above inequality, implies that $V^S(\underline{t}) = W^S(\underline{t}) = u^S(\underline{t},0)/(1-\delta_S)$.

We now establish the complementary slackness of 2(a) and 2(b), showing that if $\underline{t} < \overline{t}$ then 2(a) is an equality. It is not optimal for the target to balk to a level above $\underline{a}$, because $W^T(a)$ is decreasing on $(\underline{a},\underline{t})$, by Lemma 4(vi). Balking to below $\underline{a}$ yields at most $\delta_T^2 V$ (since the sender then spurns to a threshold $t \geq \underline{t}$, because $W^S(t)$ is increasing on $(\underline{a},\underline{t})$, by Lemma 4(v)). If $\underline{t} < \overline{t}$ then $V > 0$, so that $\delta_T^2 V < V$. Hence the optimal balk must be to $\underline{a}$ itself.

Since $\underline{a} < \underline{t}$, $R^T(\underline{a}) \geq \underline{a}$ and, since by Lemma 4(vi) $W^T$ is decreasing on $(\underline{a},\underline{t})$, $R^T(\underline{a}) = \underline{a}$. By Lemma 3, $R^S(\underline{a}) \geq \underline{a}$, (because $V^T$ is decreasing on $(\underline{a},\underline{t})$), and so

$$V^T(\underline{a}) = Z^T(\underline{a},\underline{a}) = u^T(\underline{a},0) + \delta_T W^T(\underline{a}) \leq u^T(\underline{a},0)(1+\delta_T) + \delta_T^2 V^T(\underline{a}).$$

Hence $V^T(\underline{a}) \leq u^T(\underline{a},0)/(1-\delta_T)$. By Lemma 4(vi), for all $n \in (\underline{a},\underline{t})$, $u^T(n,0)/(1-\delta_T) = V^T(n) \leq V^T(\underline{a})$ by monotonicity. Taking the limit as $n \downarrow \underline{a}$, $V^T(\underline{a}) \geq u^T(\underline{a},0)/(1-\delta_T)$ which, combined with the above inequality, implies that $V^T(\underline{a}) = W^T(\underline{a}) = u^T(\underline{a},0)/(1-\delta_T)$.

Since, then, at $\underline{t}$, both matching and balking to $\underline{a}$ are optimal:

$$u^T(\underline{t},0)/(1-\delta_T) = V = Z^T(\underline{t},\underline{a}) = u^T(\underline{a},1) + \delta_T W^T(\underline{a}) = u^T(\underline{a},1) + \delta_T u^T(\underline{a},0)/(1-\delta_T),$$

so that 2(a) holds with equality.

To establish the complementary slackness of the second pair of inequalities we show that if $\underline{a} > 0$ then 3(a) holds with equality. If the sender faces $a < \underline{a}$, matching yields at least $u^S(a,0)/(1-\delta_S)$. By Lemma 1, below $\underline{a}$ it is optimal to spurn. By Lemma 4(v) $W^S$ is increasing on $(\underline{a},\underline{t})$, so the best spurn is to $\underline{t}$ or above. But, since only balking occurs above $\underline{t}$, the best spurn is to $\underline{t}$ itself. Such a spurn yields

$$u^S(a,1) + \delta_S W^S(\underline{t}) = u^S(a,1) + \delta_S u^S(\underline{t},0)/(1-\delta_S).$$

Thus 3(a) holds with equality. □

<u>Proof of Theorem 2</u>: (i) Let

$$f^T(a) = \max\{t \in [0,\bar{t}]: u^T(t,0) \geq (1-\delta_T)u^T(a,1) + \delta_T u^T(a,0)\},$$

and

$$f^S(a) = \max\{t \in [0,\bar{t}]: u^S(a,0) \geq (1-\delta_S)u^S(a,1) + \delta_S u^S(t,0)\}.$$

Thus, for each a, $f^T(a)$ (respectively $f^S(a)$) gives the t which makes 2(a) (respectively 3(a)) just binding, or $\bar{t}$, if no such t exists. Because $u^S$ and $u^T$ are continuous, so are $f^T$ and $f^S$. Because $u^T$ and $u^S$ are monotonic, $f^T$ and $f^S$ are nondecreasing and $\forall a \in [0,\bar{t}]$, $f^S(a) \geq a$ and $f^T(a) \geq a$.

If $f^T(0) \leq f^S(0)$ then $a^* = 0$, $t^* = f^T(0)$ is a solution to (2) and (3). If $f^T(0) > f^S(0)$ then either $\exists a \in (0,\bar{t}]$ such that $f^T(a) < f^S(a)$, so that there is an interior solution to 2(a) and 3(a) by the intermediate value theorem, or else $\forall a \in (0,\tilde{t}]$, $f^T(a) \geq f^S(a)$, so that $a^* = \min\{a: f^S(a) = \bar{t}\}$ and $t^* = \tilde{t}$ is a solution to (2) and (3).

Given these values of $a^*$ and $t^*$, the following is an equilibrium that supports $\{a^*, t^*\}$ as steady states:

$$R^S(a) = \begin{cases} a & a \in [a^*, t^*] \\ t^* & a < a^* \text{ or } a \geq t^* \end{cases}$$

$$R^T(t) = \begin{cases} t & t \in [0,\hat{a}) \cup [a^*, t^*] \\ a^* & t \in [\hat{a}, a^*) \cup (t^*, 1], \end{cases}$$

where $\hat{a}$ satisfies:

$$u^T(\hat{a},0) + \delta_T u^T(\hat{a},1) = [u^T(a^*,0) - \delta_T^2 u^T(t^*,0)]/(1-\delta_T),$$

if such a value exists, and $\hat{a} = 0$ otherwise.

Checking that this constitutes an equilibrium is routine.

(ii) We rewrite (2) and (3):

$$u^T(\underline{a},0) - u^T(\underline{t},0) \leq (1-\delta_T)u^T[(\underline{a},0) - u^T(\underline{a},1)], \qquad 2'(a)$$

$$\underline{t} \leq \bar{t}; \qquad 2'(b)$$

$$(1-\delta_S)[u^S(\underline{a},0) - u^S(\underline{a},1)] \geq \delta_S[u^S(\underline{t},0) - u^S(\underline{a},0)], \qquad 3'(a)$$

$$\underline{a} \geq 0. \qquad 3'(b)$$

Suppose that there were two distinct solutions $(a_1,t_1)$ and $(a_2,t_2)$. By strict monotonicity of the utility functions, we assume without loss of generality that $a_1 < a_2$ and $t_1 < t_2$. Thus 3'(a) is an equality at $(a_2,t_2)$ and 2'(a) is an equality at $(a_1,t_1)$. Lowering a from $a_2$ to $a_1$ lowers the left-hand side of 3'(a) and, hence, the right-hand side. Since $u^S(a,0)$ is increasing and concave, $t_1-a_1 < t_2-a_2$.

Raising a from $a_1$ to $a_2$ raises neither the right-hand side of 2'(a) nor the left-hand side. Since $u^T(a,0)$ is decreasing and concave this implies that $t_1-a_1 \geq t_2-a_2$. Uniqueness of the solution to (2) and (3) follows from this contradiction. It follows from Theorem 1 that the only possible steady states are $\tilde{a}$ and $[a^*,t^*]$. □

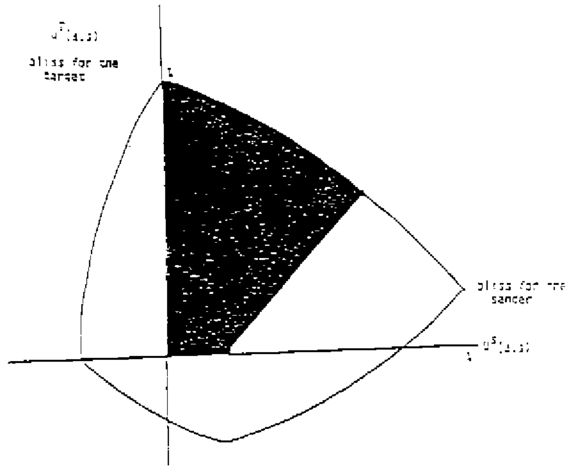



Figure 1a

FEASIBLE OUTCOMES WHEN SANCTIONS HAVE OVERKILL CAPACITY

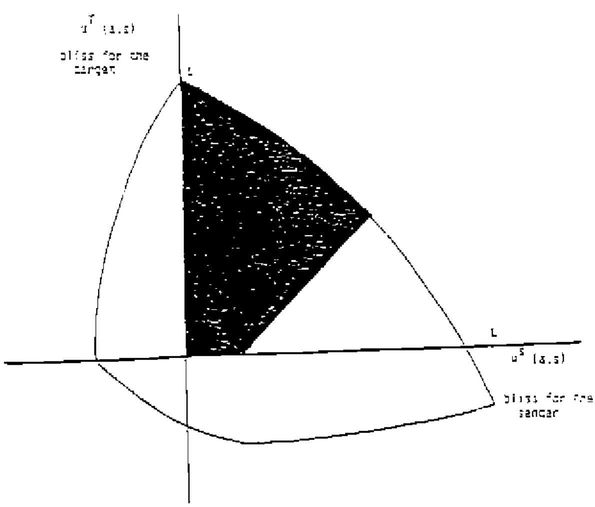



Figure 1b

FEASIBLE OUTCOMES WHEN SANCTIONS HAVE LIMITED CAPACITY

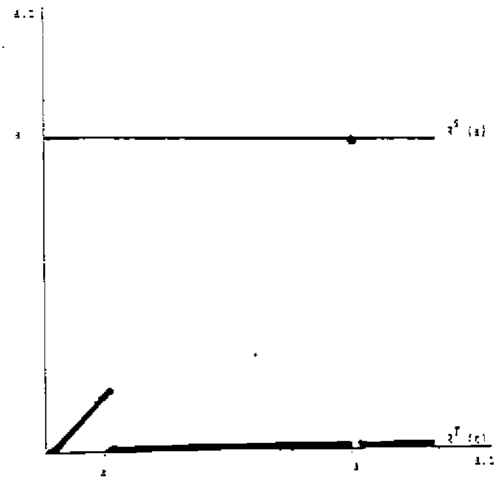


Figure 2

THE EQUILIBRIUM OF PROPOSITION 2

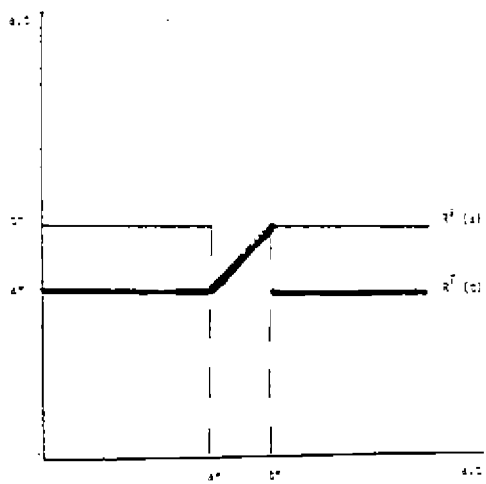
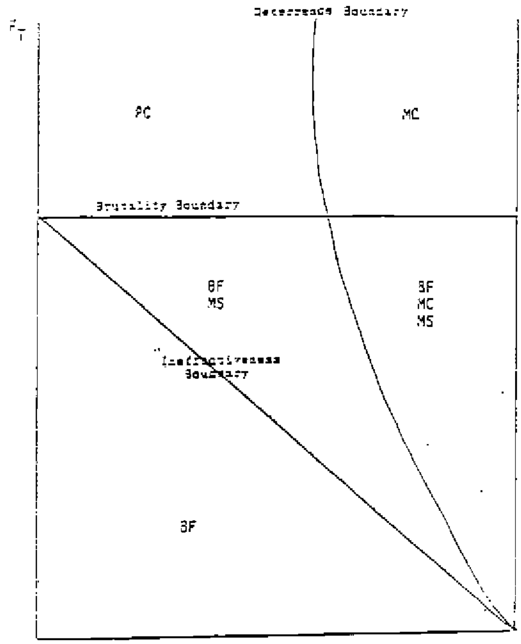


Figure 3

AN EQUILIBRIUM SUPPORTING [a*,b*] AS STEADY STATES




BF: Balking Forever
PC: Painful Compliance
MC: Mixing at Condoning
MS: Mixing at Sanctions

Figure 4

EQUILIBRIUM OUTCOMES WHEN THE SENDER SEEKS AN IRREVERSIBLE ACTION WITH A SINGLE SANCTION