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THE EXCESS SMOOTHNESS OF CONSUMPTION: IDENTIFICATION AND INTERPRETATION

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ABSTRACT

The paper investigates the implications of the omitted information problem — that is, the econometric problem which arises because an econometrician cannot explicitly include the complete set of variables potentially used by agents — in the context of the "excess smoothness" phenomenon posed by Deaton [1987]. The paper shows that an econometrician who fails to take into account the effects of omitted information will incorrectly conclude that an empirical finding of excess smoothness of consumption implies that the income process is nonstationary. By contrast, with a more thorough understanding of the omitted information problem, the finding of excess smoothness of consumption is easily explained with two assumptions: a) the consumption data is generated by the excess sensitivity alternative hypothesis, in which consumption is a weighted average of current income and permanent income, and b) agents are forecasting on the basis of a larger information set than the econometrician. Further, excess smoothness is revealed to be consistent with a wide range of stationary income processes as well as nonstationary income processes. Thus the common presumption that the excess smoothness phenomenon is linked in an essential way to the stationarity or nonstationarity of the income process evaporates when omitted information is taken into consideration.

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In a provocative paper, Deaton [1987] pursues a line of reasoning which has subsequently been dubbed the "Deaton paradox": If one abandons the conventional view that income is a stationary process around a deterministic trend for the newer view, promoted by Nelson and Plosser [1982] and Campbell and Mankiw [1987a], that income is nonstationary, then the conventional characterization of permanent income as a "smoothed" version of current income is fundamentally undermined. If the intuition is based on a univariate model of the income process, as in Deaton's original paper, the logic of the argument is that the income series can be reasonably modeled as positively autocorrelated in first differences (or growth rates), implying that a \$1 innovation in current income induces a revision in permanent income of more than \$1. If the variance of revisions in permanent income exceeds the variance of income forecast errors, as this view suggests, then the consumption data should, under the permanent income hypothesis, follow a random walk with innovation variance larger than the innovation variance of the income series. Empirically, however, the variance of the first difference of consumption, Δc_t, is smaller than, or approximately equal to, the variance of innovations in income. Thus Deaton characterizes the permanent income hypothesis as failing in a way which makes consumption "too smooth", a view which turns inside-out the conventional notion that a failure of the permanent income hypothesis causes consumption to be more variable, or less smooth, than predicted by the hypothesis.

While the intuition based on the univariate model of income has considerable appeal, the intuition cannot be made rigorous within the confines of a univariate time series model of income. It has long been recognized (see

Flavin [1981]) that the residuals of a univariate income process, while correlated with revisions in permanent income, will not be proportional to the contemporaneous revisions in permanent income if variables other than lagged income are useful in predicting future income. In response to Deaton [1987], West [1988] showed that if income is generated by a multivariate process, the associated revisions in permanent income will generally have smaller variance than the error-ridden series of permanent income revisions calculated from a univariate income model. In other words, if agents forecast income on the basis of variables other than lagged income, the variance of revisions in permanent income cannot be identified on the basis of a univariate time series model of income.

Because an econometrician attempting to estimate rational expectations models will be unable to explicitly include all of the informational variables available to agents, either because of the absence of data or because of degrees of freedom constraints, a whole class of empirical rational expectations models will be characterized by the property that the information set used by the econometrician contains only a subset of the information set used by agents. This discrepancy between the econometrician's and the agent's information sets — which has variously been described as the "omitted information" issue (from the point of view of the econometrician) or the "superior information" issue (from the point of view of the agent) — was first raised by Shiller [1972] in the context of the expectations hypothesis of the term structure.[1]

For the reasons indicated above, it seems futile to respond to the omitted information problem by simply adding more and more variables to the information set in the econometric model. Instead, several authors, including Hansen and Sargent [1981], West [1988], and Campbell and Deaton [1988], have used projection arguments to finesse the omitted information problem. These projection arguments exploit the property that the optimal behavior of agents, as postulated by the null hypothesis, will cause an observable endogenous variable to encapsulate the information used by agents but unobserved by the econometrician. Under this projection argument, the econometrician need not observe all the variables in the agent's information set individually, since the agent's behavior reveals the appropriate summary measure of the omitted variables in an endogenous "signaling" variable. In the context of the term structure of interest rates, the crucial signaling variable would be the long rate; in a stock market model, the signaling variable would be the stock price; in the permanent income consumption model, the signal would be the consumption series.

Virtually all of the discussion of the use of this projection argument to finesse the omitted information problem has been conducted by first taking as a premise the validity of the null hypothesis under consideration. The result that the omitted information problem can be completely and simply eliminated when the null hypothesis holds is extremely useful for constructing statistical tests of the null hypothesis. That is, the resulting statistical tests are valid even though the econometrician did not explicitly include in the specification all of the variables potentially used by agents for forecasting.

While the projection arguments employed by Hansen and Sargent [1981], West [1988], and Campbell and Deaton [1988] are indeed robust to the exclusion of relevant forecasting variables from the econometrician's information set, they are not robust to arbitrary departures from the null hypothesis. That is, if the null hypothesis — for example, the expectations hypothesis of the term structure, or the permanent income hypothesis — fails in an arbitrary way, the signaling variable (the long rate in the term structure model, or the consumption series in the permanent income model) will not fulfill its crucial role of fully encapsulating all information available to agents, with the consequence that the "omitted information" problem remains a problem.

Focusing on the permanent income application, consider the following alternatives to the permanent income hypothesis:

1) Agents spend 95%, or some other fixed fraction, of their current income each month.

2) Agents save a fixed dollar amount of income each month as a reserve against contingencies.

3) Agents are "completely Keynesian", with a marginal propensity to consume out of current income of unity.

Under any of these three alternative hypotheses, the data series on consumption (or, equivalently, the series on saving) would have no information content which would improve on a univariate income process in predicting future income. If the consumption data are generated by any of these alternative hypotheses, the projection arguments employed by West [1988] and Campbell and Deaton [1988] will not successfully finesse the omitted information problem.

This paper pursues the following main objectives:

1) Explanation of excess smoothness

A simple explanation of the Deaton paradox, i.e. the apparent "excess smoothness" of consumption, is proposed. The explanation of excess smoothness entails two elements: stipulation of a specific alternative structural model of consumption, and consideration of the effects of omitted information under the alternative hypothesis. Since the alternative model of consumption invoked in the explanation of the Deaton paradox is the "excess sensitivity" model used in Flavin [1981], the discussion automatically generates an explanation of the relationship between the concept of "excess smoothness" and the concept of "excess sensitivity." This section of the paper also shows that the phenomenon of excess smoothness is not as closely linked to the presence of a unit root in the income process as is commonly believed.

2) Analysis of omitted information under alternatives to null hypothesis

As mentioned above, virtually all of the previous discussion of the use of projection arguments to solve the omitted information problem has been premised on the assumption that the null hypothesis holds. This paper extends previous results on identification of rational expectations models subject to the omitted information problem by asking whether, and under what conditions, the projection arguments will fully avoid the omitted information problem when the data is generated by some model other than the null hypothesis. While this discussion is conducted in the context of the consumption model, the results to some extent carry over to other applications. In the context of the permanent income application, the analysis shows that if the consumption data are generated by the "excess sensitivity" model used in Flavin [1981] (within which the permanent income model is nested), the basic projection argument successfully eliminates the omitted information problem. The discussion also confirms and amplifies a point made in Hansen and Sargent [1981] that the introduction of a stochastic disturbance into the model — i.e. the introduction of a transitory consumption term, or a preference shock — fundamentally undermines the usefulness of the projection argument in avoiding the omitted information problem.

3) Critique of West [1988] and Campbell and Deaton [1988]

The paper offers a critique of the methodology and conclusions of the recent papers by West [1988] and Campbell and Deaton [1988]. Both of these papers recognize that Deaton's [1987] original finding of excess smoothness, based on a univariate model of the income process, was potentially a spurious finding created by the effects of omitted information. While West [1988] and Campbell and Deaton [1988] use somewhat different applications of the projection argument, their analysis is similar in the sense that both papers first establish that if the null hypothesis is assumed to hold, a projection argument can be employed to completely avoid the omitted information problem. Both papers then use the projection argument to calculate estimates of the variance of revisions in permanent income, var(Δy_t^P), and conclude that even after having taken the omitted information issue into account, the results confirm Deaton's original empirical finding that consumption is "too smooth". I argue that while the result provided by West [1988] and Campbell and Deaton [1988] that var(Δy_t^P) can be identified under the null hypothesis may be of interest for some purposes (such as construction of a statistical test of the null hypothesis), it is not sufficient for establishing excess smoothness, since excess smoothness could only be generated by some type of departure from the null hypothesis. In my view, both the West and the Campbell and Deaton papers ultimately are unsuccessful in establishing that consumption is "too smooth" relative to permanent income; to establish excess smoothness, they would need to provide at least one alternative hypothesis under which var(Δy_t^P) is identified. Neither of the previous analyses provides this; the papers by West and by Campbell and Deaton only establish that var(Δy_t^P) can be identified if the permanent income hypothesis is true.

4) <u>Empirical evidence on the extent of the omitted information problem</u>

Having identified a fortuitous case in which the omitted information problem can be finessed even though the null hypothesis fails to hold, the analysis naturally raises the empirical question of whether the assumptions embodied in the fortuitous case are consistent with the data. The empirical section of the paper provides a statistical test of the assumptions embodied in the fortuitous case, and finds that these assumptions are violated by the data. Even after establishing that the omitted information problem cannot be completely avoided, one would like to know whether a model suffering from misspecification due to omitted information generates inferences which are a) nevertheless fairly accurate, or are b) grossly inaccurate. To this end, the empirical section offers some limited evidence on the magnitude of the discrepancy between the econometrician's estimate of a parameter based on a low dimension VAR which is subject to the omitted information problem and the true parameter based on the full information set.

Section 1: The omitted information problem

The analysis is conducted by working through the effects of omitted information in the context of a particular specification of the time series process generating income. Some of the points made by the paper — in particular, the basic point that under the exact excess sensitivity hypothesis, $var(\Delta y_t^P)$ is still identified — can easily be generalized beyond the assumed specification of the income process. While the quantitative results will, or course, be dependent on the assumed specification for the income process, I have chosen to work in the context of the example rather than seek maximum generality. As an expositional strategy, I believe that the simplicity and concreteness of the illustrative example helps clarify some of the issues, as well as permitting the analytical solution of a wider range of results.

The exogenous specification of the time series process on labor income is assumed to be:

(1) $$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + x_{t-1} + \epsilon_{1t}$$
$$x_t = \epsilon_{2t}$$

In this example, the variable x_{t-1} represents a composite of all the information available to agents but not directly observed by the econometrician. To take the simplest specification, x_t is assumed uncorrelated with lagged labor income (y_{t-i}, $i=1,2,3\ldots$), its own lagged values (x_{t-i}, $i=1,2,3\ldots$), and ϵ_{1t}. Thus x_t is correlated with y_{t+1}, but uncorrelated with y_t. Since the variable x_{t-1} is assumed to represent the composite of all information available to agents aside from lagged labor income, the disturbance ϵ_{1t} is the true income forecasting error perceived by agents in period t; that is:

$$\epsilon_{1t} = y_t - E(y_t | I_{t-1}) \tag{2}$$

where I_t represents the complete information set available to agents. Given the assumptions on x_t, an econometrician estimating a univariate income autoregression would obtain consistent estimates of ρ_1 and ρ_2, but the disturbance, or forecast error, would be an estimate not of ϵ_{1t}, but instead of the sum $x_{t-1} + \epsilon_{1t}$.

As originally stated by Deaton, the proposition that consumption is too smooth compared to permanent income is closely tied to the proposition that income is nonstationary. Equation (1) was chosen as the exogenous specification of labor income because it encompasses both the stationary and nonstationary views of income. For the "conventional" view that income is a stationary process around an exponential trend, interpret y_t as labor income expressed in deviations from trend and assume that the largest root of the process is strictly less than one. For the Nelson and Plosser [1982] and Campbell and Mankiw [1987a] view that income is nonstationary, interpret y_t as labor income (not detrended) and assume that the autoregressive process has a unit root.

The permanent income model used by Campbell and Deaton [1988] and West [1988] follows the basic simplifying assumptions used in Flavin [1981]. These assumptions include: a) labor income, y_t, is exogenously determined, and b) the real rate of return, denoted r, is constant.

As is well known, the permanent income hypothesis as formulated above is only literally true under strong restrictions on tastes and technology; Christiano, Eichenbaum, and Marshall [1987] detail one set of assumptions under which the permanent income hypothesis holds in general equilibrium. However, relaxing these assumptions about tastes and technology has yet to produce much empirical improvement over the original formulation. Hall's [1978] implementation of the permanent income hypothesis predicted that changes in the level of consumption should be white noise. By contrast, a simple version of the consumption-beta model posits that changes in the log of consumption are a function solely of the expected rate of return. Yet when the latter regression is estimated by instrumental variables, the empirical contribution of the interest rate is effectively zero for macro time series data (Hall [1988]). Further, the consumption-beta version of the Euler equation can be statistically rejected with the same set of variables which indicate rejection of the orthogonality conditions based on the permanent income version of the model. Since the consumption-beta model resembles the permanent income model so closely in terms of its empirical implications and its empirical performance, it seems sensible to retain consistency with the previous authors who have addressed the excess smoothness issue by using the permanent income framework.

Also following Flavin [1981], permanent income is defined as the annuity value of the agent's net worth, where net worth includes the present discounted value of expected future labor income as well as real (non-human) wealth.

(3) $$y_t^p = \left(\frac{r}{1+r}\right)\left[A_t + \sum_{\tau=0}^{\infty} \delta^{\tau} E_t y_{t+\tau}\right]$$

where A_t denotes the agent's real non-human wealth at the end of period t,

$\delta = \frac{1}{1+r}$,

y_t denotes labor income, assumed to be received at the end of period t,

$E_t y_{t+\tau} = E(y_{t+\tau} | I_t)$; that is E_t is shorthand notation for the conditional expectation based on the agent's complete information set I_t.

The evolution of assets is given by:

(4) $$A_{t+1} = (1+r)[A_t + y_t - c_t]$$

where consumption, denoted c_t, is assumed to be paid at the end of the period.

Following Hall [1978], the permanent income hypothesis is specified as:

(5) $$c_t = y_t^p$$

Note that this version of the hypothesis, in which agents consume exactly their permanent income each period, is more restrictive than Friedman's [1957] version of the PIH. Friedman [1957] viewed consumption as containing both a permanent component and a transitory component, and merely assumed that transitory consumption was uncorrelated with permanent consumption. Hall [1978] sharpened the hypothesis by assuming that transitory consumption was identically zero.

If agents consume exactly their permanent income each period, as assumed in equation (5), it's easy to show that permanent income is a martingale, with the implication that

(6) $$\Delta y_t^p = y_t^p - E_{t-1}y_t^p$$

Since the assumption that the real return to wealth is constant rules out unanticipated capital gains, the behavior of the asset stock, A_t, is purely endogenous in this model. While the level of permanent income depends on the level of assets, the change in permanent income over time depends only on the revision in expectations of future labor income because labor income is the only component of total income subject to exogenous shocks:

(7) $$\Delta y_t^p = (\frac{r}{1+r}) \sum_{\tau=0}^{\infty} \delta^\tau (E_t - E_{t-1}) y_{t+\tau}$$

Rewriting the exogenous specification for labor income (equation (1)) in vector form, we have

(8) $$\begin{bmatrix} y_t \\ y_{t-1} \\ x_t \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_2 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ 0 \\ \epsilon_{2t} \end{bmatrix}$$

Denote the 3 × 3 matrix of autoregressive parameters by A. The expectational revision in period t of the present discounted value of expected future income is given by

$$(9) \qquad \sum_{\tau=0}^{\infty} \delta^{\tau}(E_t - E_{t-1})y_{t+\tau} = [1\ 0\ 0][I-\delta A]^{-1}\begin{bmatrix}\epsilon_{1t}\\ 0\\ \epsilon_{2t}\end{bmatrix}$$

Thus the change in permanent income is equal to:

$$(10) \qquad \Delta y_t^p = (\tfrac{r}{1+r})\,\tfrac{1}{\phi}[\epsilon_{1t} + \delta\epsilon_{2t}]$$

where $\phi = 1-\delta\rho_1 - \delta^2\rho_2$.

Suppose that, failing to observe x_t, the econometrician attempts to infer the variance of Δy_t^p from the estimated parameters of the univariate process on income. Based on the misspecified model,

$$(11) \qquad y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \epsilon_{yt}$$

where $\epsilon_{yt} = \epsilon_{2t-1} + \epsilon_{1t}$,

the econometrician would obtain consistent estimates of ρ_1 and ρ_2 and infer the revision in permanent income to be

$$(12) \qquad \Delta\tilde{y}_t^p = (\tfrac{r}{1+r})\,\tfrac{1}{\phi}(\epsilon_{1t} + \epsilon_{2t-1})$$

where $\Delta\tilde{y}_t^p$ is notation for the inferred revision in permanent income based on the econometrician's incomplete information set, as distinguished from Δy_t^p, which denotes the true revision in permanent income.

To develop the explanation of excess smoothness given in Section 2, we need to consider the effects of omitted information on a) the econometrician's estimate of the variance of revisions in permanent income, $\mathrm{var}(\Delta y_t^p)$, and b) the econometrician's estimate of the contemporaneous covariance of the income forecast error with the revision in permanent income, $\mathrm{cov}(\epsilon_{yt},\Delta\tilde{y}_t^p)$. That is, to what extent does the econometrician's failure to observe x_{t-1} (alias ϵ_{2t-1}) cause $\mathrm{var}(\Delta\tilde{y}_t^p)$ to diverge from the true $\mathrm{var}(\Delta y_t^p)$ and $\mathrm{cov}(\epsilon_{yt},\Delta\tilde{y}_t^p)$ to diverge from the true $\mathrm{cov}(\epsilon_{1t},\Delta y_t^p)$?

The ratio of $\mathrm{var}(\Delta y_t^p)$ to the inferred $\mathrm{var}(\Delta\tilde{y}_t^p)$ based on the incomplete information set is given by:

(13) $$\frac{\mathrm{var}(\Delta y_t^p)}{\mathrm{var}(\Delta \tilde{y}_t^p)} = \frac{\delta^2\sigma_2^2 + \sigma_1^2}{\sigma_2^2 + \sigma_1^2} < 1$$

where $\sigma_1^2 = \mathrm{var}(\epsilon_{1t})$ and $\sigma_2^2 = \mathrm{var}(\epsilon_{2t})$.

The ratio $\mathrm{cov}(\epsilon_{1t},\Delta y_t^p)/\mathrm{cov}(\epsilon_{yt},\Delta \tilde{y}_t^p)$ is given by:

(14) $$\frac{\mathrm{cov}(\epsilon_{1t},\Delta y_t^p)}{\mathrm{cov}(\epsilon_{yt}, \Delta \tilde{y}_t^p)} = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} < 1$$

Note that the exogenous specification of the income process chosen for this analysis illustrates the result demonstrated more generally by West [1988] that the omitted information problem will tend to cause the inferred $\mathrm{var}(\Delta \tilde{y}_t^p)$ to overstate the true $\mathrm{var}(\Delta y_t^p)$. In the context of the example, a similar result holds for the covariance term; omitted information causes the inferred $\mathrm{cov}(\epsilon_{yt},\Delta \tilde{y}_t^p)$ to overstate the true $\mathrm{cov}(\epsilon_{1t},\Delta y_t^p)$.

To provide the intuition for understanding the empirical results in Section 5, the analytical example can be used to generate some results on the extent to which the econometrician's inference on $\mathrm{var}(\Delta \tilde{y}_t^p)$ and $\mathrm{cov}(\epsilon_{yt},\Delta \tilde{y}_t^p)$ differ from the corresponding full information moments. To take an interesting limiting case, let $\sigma_1^2 = \mathrm{var}(\epsilon_{1t}) = 0$. In this limiting case, the agent can forecast his labor income one period ahead without error; however, the econometrician estimating the forecasting model as a univariate income autoregression will not achieve perfect one-period ahead forecastability, since the residual in the income autoregression will reflect ϵ_{2t}. Modeling agents as having sufficient information to forecast without error their labor income one period ahead does not, of course, imply that agents have perfect foresight over the whole path of labor income, since agents do not have perfect foresight with respect to the future realizations of x_t.

For $\sigma_1^2 = 0$, equations (13) and (14) become:

(15) $$\frac{\mathrm{var}(\Delta y_t^p)}{\mathrm{var}(\Delta \tilde{y}_t^p)} = \frac{\delta^2\sigma_2^2 + \sigma_1^2}{\sigma_2^2 + \sigma_1^2} = \delta^2$$

$$(16) \qquad \frac{\mathrm{cov}(\epsilon_{1t}, \Delta y_t^P)}{\mathrm{cov}(\epsilon_{yt}, \Delta \tilde{y}_t^P)} = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} = 0$$

To interpret equation (15), note that for a quarterly interest rate of 2%, $\delta^2 = .96$. Further, from equation (13) it is obvious that the assumption $\sigma_1^2 = 0$ drives the ratio of $\mathrm{var}(\Delta y_t^P)/\mathrm{var}(\Delta \tilde{y}_t^P)$ to its lower bound; for general values of σ_1^2 and σ_2^2, the ratio is bounded between δ^2 and 1. Thus, if the data were generated by the process assumed in equation 1, an econometrician who lacked data on x_t and attempted to estimate $\mathrm{var}(\Delta \tilde{y}_t^P)$ with a univariate income autoregression would overstate the true $\mathrm{var}(\Delta \tilde{y}_t^P)$ by no more than 4%. Despite the omitted information problem, the econometrician's inferred $\mathrm{var}(\Delta \tilde{y}_t^P)$ is a fairly accurate measure of the true $\mathrm{var}(\Delta y_t^P)$, at least in the context of the example.

Inquiry into the robustness of the inferred $\mathrm{cov}(\epsilon_{yt}, \Delta \tilde{y}_t^P)$ with respect to omitted information yields exactly the opposite conclusion. For the limiting case characterized by $\sigma_1^2 = 0$, the true $\mathrm{cov}(\epsilon_{1t}, \Delta y_t^P)$ would be zero, while the econometrician would obtain, as a consequence of omitted information, an inferred $\mathrm{cov}(\epsilon_{yt}, \Delta \tilde{y}_t^P) = \sigma_2^2$. Thus in contrast to the robustness of $\mathrm{var}(\Delta \tilde{y}_t^P)$ with respect to omitted information, the econometrician's inferred $\mathrm{cov}(\epsilon_{yt}, \Delta \tilde{y}_t^P)$ may be a very misleading measure of the true $\mathrm{cov}(\epsilon_{1t}, \Delta y_t^P)$.

To understand the robustness, with respect to omitted information, of $\mathrm{var}(\Delta \tilde{y}_t^P)$ as an estimate of $\mathrm{var}(\Delta y_t^P)$, as well as the lack of robustness of $\mathrm{cov}(\epsilon_{yt}, \Delta \tilde{y}_t^P)$ as an estimate of $\mathrm{cov}(\epsilon_{1t}, \Delta y_t^P)$, compare the expectational revisions of future labor income as perceived by agents to those perceived by the econometrician. The impact of ϵ_{1t} on the path of future income is exactly the same for the agent and the econometrician. The discrepancy between the agent's and the econometrician's forecasts arises because the agent observes ϵ_{2t} contemporaneously and incorporates the impact of ϵ_{2t} into the forecasted path of labor income, $E(y_{t+s}|I_t)$, in period t. Because the econometrician does not observe ϵ_{2t} (alias x_t) in period t, the impact of ϵ_{2t} is not incorporated into the econometrician's forecasts for future income until period t+1. As the variance of ϵ_{1t} becomes smaller, relative to ϵ_{2t}, the fact that Δy_t^p depends on current ϵ_{2t} while $\Delta\tilde{y}_t^p$ depends on ϵ_{2t-1} implies that the correlation between the true series Δy_t^p and the inferred series $\Delta\tilde{y}_t^p$ could be very small. Nevertheless, because ϵ_{2t} and ϵ_{2t-1} have the same variance, the moment $\mathrm{var}(\Delta\tilde{y}_t^p)$ would approximate $\mathrm{var}(\Delta y_t^p)$ very closely.

Inspection of equations (10) and (12) also explains why $\mathrm{cov}(\epsilon_{yt},\Delta\tilde{y}_t^p)$ is not a robust measure of $\mathrm{cov}(\epsilon_{1t},\Delta y_t^p)$. When the econometrician uses ϵ_{yt} as an error-ridden measure of ϵ_{1t} and $\Delta\tilde{y}_t^p$ as an error-ridden measure of Δy_t^p, the consequence of the omitted information problem is to incorporate a common measurement error (ϵ_{2t-1}) into both ϵ_{yt} and $\Delta\tilde{y}_t^p$, artificially increasing the covariance between ϵ_{yt} and $\Delta\tilde{y}_t^p$. Note that if the income model is estimated as a univariate process, the correlation coefficient between ϵ_{yt} and $\Delta\tilde{y}_t^p$ is constrained to equal unity, even if, as in the limiting case in which $\mathrm{var}(\epsilon_{1t})=0$, the correlation coefficient between ϵ_{1t} and Δy_t^p is zero.

Section 2: Explanation of Excess Smoothness

This section of the paper provides a simple explanation of the excess smoothness phenomenon.[2] The essential components of the explanation are: 1) stipulation of a structural alternative hypothesis to the permanent income hypothesis (PIH), and 2) consideration of the effects of the omitted information problem.

Under the PIH, consumption should follow a martingale process, with innovations equal to the innovations in permanent income, hence,

$$\text{(17)}\qquad \mathrm{var}(\epsilon_{ct}) = \mathrm{var}(\Delta c_t) = \mathrm{var}(\Delta y_t^p) = \mathrm{var}(\epsilon_{ypt})$$

where ϵ_{ct} = innovation to a reduced form (i.e., VAR) consumption equation

$\epsilon_{ypt} = y_t^p - E_{t-1}y_t^p$ = innovation to the permanent income series

If the PIH fails to hold, the equality between $\mathrm{var}(\epsilon_{ct})$ and $\mathrm{var}(\Delta c_t)$ also fails to hold. Further, since permanent income, as defined in equation

(3), is an endogenous variable whose evolution is determined in part by consumption behavior, the martingale property (that is, the equality $\mathrm{var}(\Delta y_t^P) = \mathrm{var}(\epsilon_{ypt})$) generally will not hold under alternative consumption hypotheses. In defining the concept of "excess smoothness" one needs to choose between $\mathrm{var}(\epsilon_{ct})$ and $\mathrm{var}(\Delta c_t)$ as a measure of consumption variability, and choose between $\mathrm{var}(\Delta y_t^P)$ and $\mathrm{var}(\epsilon_{ypt})$ as a measure of variability of permanent income. In this paper consumption will be described as "excessively smooth" if

$$\mathrm{var}(\epsilon_{ct}) < \mathrm{var}(\epsilon_{ypt}), \tag{18}$$

which is a definition of excess smoothness consistent with Campbell and Deaton [1988].[3]

The explanation of excess smoothness relies on the specification of a specific alternative hypothesis, rather than thinking of the alternative as the completely general alternative "any behavior other than the behavior predicted by the PIH". The alternative hypothesis is the model used in Flavin [1981] which posits that consumption exhibits excess sensitivity to current income:

$$c_t = \beta y_t^T + y_t^P. \tag{19}$$

In this alternative hypothesis, consumption is assumed to increase by \$1 in response to a \$1 increase in permanent income. In addition, the hypothesis entertains the possibility that consumption will increase by \$$\beta$ in response to a \$1 increase in transitory income, y_t^T. In this Keynesian-type alternative model, β can be thought of as the marginal propensity to consume out of transitory income, where transitory income is defined as the residual, $y_t^T = (y_t + (\frac{r}{1+r})A_t) - y_t^P$. (Remember that y_t denotes labor income, not total income.) Using the definition of permanent income given in equation (3), the excess sensitivity alternative hypothesis can be stated as:

$$c_t = \beta\left[y_t + (\tfrac{r}{1+r})A_t\right] + (1-\beta)\left[(\tfrac{r}{1+r})A_t + (\tfrac{r}{1+r})\sum_{\tau=0}^{\infty} \delta^\tau E_t y_{t+\tau}\right] \tag{20}$$

Taking first differences of equation (20) and rearranging gives:

$$(21)\qquad \Delta c_t = \beta\Delta y_t + (1-\beta)(\frac{r}{1+r})\sum_{\tau=0}^{\infty}\delta^{\tau}(E_t - E_{t-1})y_{t+\tau} + (\frac{r}{1+r})\Delta A_t$$

$$- r(1-\beta)\left[y_{t-1} - (\frac{r}{1+r})\sum_{\tau=0}^{\infty}\delta^{\tau}E_{t-1}y_{t-1+\tau}\right]$$

Using the accounting identity

$$(22)\qquad s_t = (\frac{r}{1+r})A_t + y_t - c_t$$

in conjunction with equation (20), it's easy to show that the excess sensitivity hypothesis, stated in terms of saving rather than consumption, implies[4]

$$(23)\qquad s_t = (1-\beta)\left[y_t - (\frac{r}{1+r})\sum_{\tau=0}^{\infty}\delta^{\tau}E_t y_{t+\tau}\right]$$

Using equation (23), equation (21) can be rewritten as

$$(24)\qquad \Delta c_t = \beta\Delta y_t + (1-\beta)(\frac{r}{1+r})\sum_{\tau=0}^{\infty}\delta^{\tau}(E_t - E_{t-1})y_{t+\tau} + (\frac{r}{1+r})\Delta A_t - rs_{t-1}$$

From equation (4), which describes the evolution of assets, and the definition of saving (22), one can show that ΔA_t and s_{t-1} are related by $\Delta A_t = (1+r)s_{t-1}$, with the implication that the last two terms in equation (24) cancel, leaving

$$(25)\qquad \Delta c_t = \beta\Delta y_t + (1-\beta)(\frac{r}{1+r})\sum_{\tau=0}^{\infty}\delta^{\tau}(E_t - E_{t-1})y_{t+\tau}$$

Note that even though transitory income and permanent income were defined as the transitory and permanent components of total income, inclusive of asset income, in the statement of the excess sensitivity hypothesis (equation (20)), the terms involving asset income cancel out, with the result that the first difference of consumption is a weighted average of the first difference of labor income, Δy_t, and the expectational revision of the annuity value of future labor income. To simplify the notation, note that the expectational revision of the annuity value of future labor income is equal to the first difference of the permanent income series which would be generated under the null hypothesis (equation (7)). Thus the excess sensitivity hypothesis can be expressed as:

(26) $$\Delta c_t = \beta\Delta y_t + (1-\beta)\Delta y_t^p$$

where Δy_t = first difference of labor income, and

$$\Delta y_t^p = (\frac{r}{1+r})\sum_{\tau=0}^{\infty}\delta^{\tau}(E_t - E_{t-1})y_{t+\tau}$$

To avoid confusion, it is important to stress that Δy_t^p in equation (26) and subsequently throughout the paper refers to the first difference of the permanent income series which would be generated under the null hypothesis, not the first difference of the actual permanent income series generated under the alternative hypothesis; the former is a martingale while the latter is not. If notational purity were the only consideration, it would be preferable to state the alternative hypothesis as $\Delta c_t = \beta\Delta y_t + (1-\beta)\epsilon_{ypt}$. However, the simplicity of thinking about the alternative hypothesis as modeling the change in consumption as a weighted average of the change in labor income and the change in permanent income seemed sufficiently valuable to justify a slight abuse of notation. The derivation above (equations (20) through (26)), shows that the intuitively appealing statement of the alternative hypothesis in equation (26) is rigorously grounded in the assumptions of the model, provided that Δy_t^p is understood to denote the first difference in the permanent income series which would be generated under the null hypothesis.

Equation (26) can then be rewritten as:

(27) $$\Delta c_t = \beta\left[E(\Delta y_t|\Omega_{t-1})\right] + \beta\epsilon_{yt} + (1-\beta)\Delta y_t^p$$

where Ω_{t-1} is an information set which is a subset of the agent's complete information set, I_{t-1}, and

ϵ_{yt} = innovation in Δy_t relative to information set Ω_{t-1}

Thus, under the excess sensitivity alternative hypothesis, the disturbance to a reduced form consumption equation would be given by:

(28) $$\epsilon_{ct} = \beta\epsilon_{yt} + (1-\beta)\Delta y_t^p$$

where $\epsilon_{ct} = \Delta c_t - E(\Delta c_t|\Omega_{t-1})$

If "excess smoothness" is defined as in equation (18), excess smoothness will occur if:

(29) $\mathrm{var}(\epsilon_{ct}) = \beta^2\mathrm{var}(\epsilon_{yt}) + 2\beta(1-\beta)\mathrm{cov}(\Delta y_t^p, \epsilon_{yt}) + (1-\beta)^2\mathrm{var}(\Delta y_t^p) < \mathrm{var}(\Delta y_t^p)$

Suppose we view income as a univariate process, as in Deaton [1988]. Then

(30) $$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \epsilon_{yt},$$

implying

(31) $$\Delta y_t^p = \left(\frac{r\delta}{1-\delta\rho_1-\delta^2\rho_2}\right)\epsilon_{yt}$$

which further implies that the income innovation, ϵ_{yt}, and the revision in permanent income, Δy_t^p, are perfectly correlated. Thus for the special case in which income is a univariate process, the excess smoothness condition takes the form:

(32) $$\mathrm{var}(\epsilon_{ct}) = \beta^2\mathrm{var}(\epsilon_{yt}) + 2\beta(1-\beta)\sigma_{\epsilon_{yt}}\sigma_{\Delta y_t^p} + (1-\beta)^2\mathrm{var}(\Delta y_t^p) < \mathrm{var}(\Delta y_t^p)$$

which in turn can be simplified to the condition

(33) $$\mathrm{var}(\epsilon_{ct}) = (\beta\sigma_{\epsilon_{yt}} + (1-\beta)\sigma_{\Delta y_t^p})^2 < \sigma^2_{\Delta y_t^p}$$

The message of equation (33) is as follows: If the consumption data is generated by the excess sensitivity alternative hypothesis, and if labor income is a univariate process (i.e., there is no omitted information problem), an empirical finding of excess smoothness of consumption ($\mathrm{var}(\epsilon_{ct}) < \mathrm{var}(\Delta y_t^p)$) would imply that

(34) $$\sigma_{\epsilon_{yt}} < \sigma_{\Delta y_t^p} ,$$

i.e., a finding of excess smoothness would imply that a $1 innovation to labor income induces a revision in permanent income of more than $1. Thus one way to interpret the apparent excess smoothness of consumption is to view the consumption data as being generated by three assumptions: 1) labor income is a univariate process, 2) as a structural hypothesis, consumption is excessively sensitive to current income, and 3) labor income is nonstationary. Note that if one views income as a univariate process, there is a close relationship between the inequality relating $\text{var}(\epsilon_{ct})$ and $\text{var}(\Delta y_t^p)$ and the stationarity of the income process.

However, the interpretation of an empirical finding of excess smoothness of consumption changes dramatically if we drop the assumption that labor income is a univariate process and allow for omitted information. Returning to equation (29), consider the extreme but nonetheless interesting case in which agents can forecast their labor income one period in advance without error (i.e., the special case in which $\sigma_1^2=0$). In this case the covariance between the revision in permanent income, Δy_t^p, and the forecast error in a univariate income autoregression, ϵ_{yt}, would be zero. For this limiting case, the excess smoothness condition becomes

$$\text{(35)} \qquad \text{var}(\epsilon_{ct}) = \beta^2\text{var}(\epsilon_{yt}) + (1-\beta)^2\text{var}(\Delta y_t^p) \;<\; \text{var}(\Delta y_t^p)$$

where $\epsilon_{ct} = \Delta c_t - E(\Delta c_t | y_t, y_{t-1}, \ldots)$. For values of the excess sensitivity parameter between zero and one, $0<\beta<1$, the weights on $\text{var}(\epsilon_{yt})$ and $\text{var}(\Delta y_t^p)$ will sum to less than unity; for example, for a plausible value of $\beta = .5$, $\beta^2 = (1-\beta)^2 = .25$.[5] Thus if we assume that the consumption data is generated by the excess sensitivity alternative hypothesis and further assume that agents have perfect one-period-ahead forecastability of labor income, excess smoothness of consumption would arise as long as $\text{var}(\epsilon_{yt}) < \left[\frac{2-\beta}{\beta}\right]\text{var}(\Delta y_t^p)$. That is, for $\beta = .5$, the inequality $\text{var}(\epsilon_{ct}) < \text{var}(\Delta y_t^p)$ would obtain as long as $\text{var}(\epsilon_{yt}) < 3\text{var}(\Delta y_t^p)$. Thus while a nonstationary process for labor income would generate excess smoothness, a wide range of stationary processes would also generate excess smoothness.

The important lesson which emerges from this analysis is as follows: If we model the labor income series as a univariate process, as in Deaton [1987], the correlation between the inferred $\Delta\tilde{y}_t^p$ and ϵ_{yt} is artificially constrained to equal unity, with the implication that a nonstationary income process will generate excess smoothness and a stationary income process will generate "insufficient" smoothness. However, in the general case in which the agent's inference of Δy_t^p is based on an information set strictly larger than the econometrician's information set, the correlation between Δy_t^p and ϵ_{yt} may plausibly be much less than unity — in the extreme case of perfect one-period-ahead forecastability of labor income, the correlation is zero. If, due to omitted information, the correlation between Δy_t^p and ϵ_{yt} is small, an empirical finding of excess smoothness of consumption can be interpreted as a result of a) consumption exhibiting excess sensitivity to transitory income ($\beta > 0$), and b) agents forecasting on the basis of a more complete information set than the econometrician. In the general case which allows for omitted information, excess smoothness of consumption is consistent with nonstationary income processes as well as a wide range of stationary income processes.

Section 3: Critique of Campbell and Deaton [1988] and West [1988]

Papers by both West [1988] and Campbell and Deaton [1988] recognized that the original finding of excess smoothness by Deaton [1987] may have been an artifact attributable to the assumption that income was a univariate process. Both papers use a projection argument along the lines of Hansen and Sargent [1981] in an attempt to finesse the omitted information issue; both papers conclude that, even when omitted information is taken into account, Deaton's original finding of excess smoothness of consumption is confirmed. While there are important differences in the way in which the two papers implement the projection argument for dealing with omitted information, both West [1988] and Campbell and Deaton [1988] base their empirical conclusion that consumption to excessively smooth on a discussion which establishes that the upper bound, $\text{var}(\Delta y_t^p)$, can be econometrically identified if the null hypothesis (the PIH) is true.

In this section I argue that establishing that $\text{var}(\Delta y_t^p)$ is identified under the null hypothesis is not germane for establishing an empirical finding of excess — or insufficient — smoothness since excess smoothness could only be generated by some sort of failure of the PIH. An attempt to characterize the way in which consumption departs from the behavior predicted by the permanent income hypothesis in terms of excess smoothness can only succeed if the relevant upper bound, $\mathrm{var}(\Delta y_t^p)$, can be identified under some alternative hypothesis. For an arbitrary alternative hypothesis (e.g., for the list of simple alternative hypotheses mentioned in the introduction) $\mathrm{var}(\Delta y_t^p)$ will not be identified in the presence of omitted information.

Thus, my primary criticism of the analysis in both West [1988] and Campbell and Deaton [1988] is as follows: if their objective is to establish that consumption is "too smooth" relative to permanent income, it is necessary to show that there is at least one alternative hypothesis, consistent with the observation that aggregate consumption violates the orthogonality condition, for which $\mathrm{var}(\Delta y_t^p)$ is identified. Their analysis does not provide this; it only establishes the identifiability of $\mathrm{var}(\Delta y_t^p)$ under the null hypothesis.

This paper supplies the required alternative hypothesis: if consumption is generated by the excess sensitivity model (equation (20)) used in Flavin [1981], $\mathrm{var}(\Delta y_t^p)$ is identified. In the Campbell and Deaton paper, the univariate income model is replaced by a bivariate autoregression of income and saving. In the bivariate autoregression, it is important that the second forecasting variable is the saving series rather than an arbitrary additional variable; Campbell and Deaton use a projection argument to establish that if the permanent income hypothesis holds, the saving series will fully encapsulate all the information contained in the agent's information set but not observed by the econometrician. Hence, if one assumes that the permanent income hypothesis is true, a bivariate autoregression of income and saving will generate valid inferences about the variance of revisions in permanent income. The fortuitous result that generalizing the consumption hypothesis to permit a non-zero marginal propensity to consume out of transitory income does not destroy the identification of var(Δy_t^P) will be recognized as intuitively plausible if one observes from the savings equation (equation (36)) that under the alternative hypothesis of excess sensitivity the saving series is simply a rescaled version of the saving series which would have been generated under the null hypothesis:

$$s_t = (1-\beta)\left[y_t - (\frac{r}{1+r})E_t \sum_{\tau=0}^{\infty} \delta^{\tau} y_{t+\tau}\right]. \tag{36}$$

This property holds for a general process generating labor income and is not specific to the particular income process used as an example. Since allowing the excess sensitivity parameter to differ from zero merely rescales the saving series, the information content of the saving series is not destroyed or diluted by this particular generalization of the hypothesis.

Below, I work through the Campbell and Deaton algorithm for exploiting a bivariate autoregression of income and saving in order to infer var(Δy_t^P). The crucial feature which distinguishes the analysis below from the original analysis presented in Campbell and Deaton is that consumption (and therefore saving) is assumed to be generated by the excess sensitivity alternative hypothesis, while Campbell and Deaton assumed that consumption was generated by the PIH.[6] To save space the remainder of the section concentrates on the analysis of Campbell and Deaton [1988] and does not explicitly work through the argument a second time in the context of West's [1988] formulation of the projection argument.

Using the income process specified in Section 1, the "structural" model for income and saving, under the alternative hypothesis ($0 \le \beta < 1$), is given by:[7]

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + x_{t-1} + \epsilon_{1t} \tag{37a}$$

$$x_t = \epsilon_{2t} \tag{37b}$$

$$(37c) \qquad s_t = (1-\beta)\left[y_t - \left(\frac{r}{1+r}\right)E_t \sum_{\tau=0}^{\infty} \delta^{\tau} y_{t+\tau}\right]$$

For the assumed exogenous process driving labor income, permanent (labor) income can be written as:

$$(38) \qquad \left(\frac{r}{1+r}\right)E_t \sum_{\tau=0}^{\infty} \delta^{\tau} y_{y+\tau} = \left(\frac{r}{1+r}\right)\left[\frac{1}{\phi} y_t + \frac{\delta\rho_2}{\phi} y_{t-1} + \frac{\delta}{\phi} x_t\right]$$

where $\phi = 1 - \delta\rho_1 - \delta^2\rho_2$ as before.

Substituting (38) into (37c), the saving equation becomes:

$$(39) \qquad s_t = (1-\beta)\left[\left(1-\left(\frac{r}{1+r}\right)\frac{1}{\phi}\right)y_t - \left(\frac{r}{1+r}\right)\frac{\delta\rho_2}{\phi} y_{t-1} - \left(\frac{r}{1+r}\right)\frac{\delta}{\phi} x_t\right]$$

In Campbell and Deaton's analysis, the saving variable is exploited to convey the information in the variables not directly observed by the econometrician, in this example, x_t. Thus we need to solve equation (39) for x_t in terms of s_t, y_t, and y_{t-1} and use the resulting equation to eliminate x_{t-1} from the income equation (equation (37a)). With a further substitution to eliminate current y_t from the RHS of the saving equation, the bivariate autoregression of income and saving is:

$$(40) \qquad \begin{bmatrix} y_t \\ s_t \end{bmatrix} = \begin{bmatrix} A(\beta) \end{bmatrix}\begin{bmatrix} y_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ (1-\beta)(\epsilon_{1t} - \Delta y_t^P) \end{bmatrix}$$

$$\text{where} \qquad A(\beta) = \begin{bmatrix} 1 + \dfrac{1-\varphi_1-\varphi_2}{r} & -\left(\dfrac{1}{1-\beta}\right)\dfrac{\phi}{r\delta^2} \\[2ex] (1-\beta)\left[\dfrac{1-\varphi_1-\varphi_2}{r}\right] & 1+r - \dfrac{\phi}{r\delta^2} \end{bmatrix}$$

The notation $A(\beta)$ is used to emphasize the fact that the parameters of the autoregression are functions of the excess sensitivity parameter, β. Note that equation (40) uses the fact that, for the assumed process on labor income, $\Delta y_t^P = \frac{r\delta}{\varphi}(\epsilon_{1t} + \delta\epsilon_{2t})$.

Since income, consumption, and saving are related by an accounting identity, it's easy to map the bivariate autoregression (40) into the corresponding consumption equation. Because the interest rate is assumed constant in this model, and because y_t refers to labor income rather than total income, the income identity (from equations (4) and (22)) implies:

(41) $$\Delta c_t = \Delta y_t - s_t + (1+r)s_{t-1}$$

Applying the income identity (41) to the income and saving equation (40) generates:

(42) $$\Delta c_t = \beta\left[\frac{1-\rho_1-\rho_2}{r}\right]y_{t-1} - \left(\frac{\beta}{1-\beta}\right)\left[\frac{\phi}{r\delta^2}\right]s_{t-1} + \beta\epsilon_{1t} + (1-\beta)\Delta y_t^p$$

The consumption equation implied by the bivariate autoregression of income and saving has the property that under the null hypothesis ($\beta=0$), Δc_t is orthogonal to lagged y_t and lagged s_t. This, of course, is the famous Hall [1978] orthogonality condition for consumption.

Denote the values of the autoregressive parameters under the null hypothesis ($\beta=0$) as A(0). Campbell and Deaton state that under the permanent income hypothesis, the parameters of the bivariate autoregression will satisfy the constraint:[8]

(43) $$r\delta[1\ 0][I - \delta A(0)]^{-1} = [1\ -1]$$

Campbell and Deaton refer to equation (43) as the orthogonality condition because if the income/saving autoregression satisfies equation (43), the implied process for Δc_t will be orthogonal to lagged y_t and lagged s_t. Thus a statistical test of the permanent income hypothesis, that is, a probability statement as to whether the data was or was not generated by the hypothesis, can be obtained by testing the restriction (43) on the autoregressive parameters. Since the parameter restrictions in equation (43) simply translate the exclusion restrictions in a regression of Δc_t on y_{t-1} and s_{t-1} into the equivalent parameter restrictions on the income/saving autoregression, the statistical test of the null hypothesis provided by Campbell and Deaton is equivalent to Hall's original test of the orthogonality condition in all essential respects: both rely on the same set of identifying assumptions (notably the absence of any transitory consumption disturbance, or preference shocks); both are based on estimation of reduced form, rather than structural, parameters; both test the implications of the permanent income hypothesis for the autocovariance structure of aggregate income and consumption.

In addition to the orthogonality condition, Campbell and Deaton specify the "smoothness condition." The smoothness condition compares the actual variance of disturbances to the reduced form consumption equation to the predicted variance of disturbances to the reduced form consumption equation. By the accounting identity, the disturbance to a reduced form consumption equation is just the difference between the reduced form disturbances to income and saving. The predicted variance of reduced form disturbances to consumption, under the null hypothesis, is just the variance of Δy^p_t, as generated by the bivariate autoregression. Thus to check the smoothness condition, Campbell and Deaton compare two quadratic forms in Ω, the 2×2 covariance matrix of the innovations to the income/saving autoregression:

$$[1\ -1]\ \Omega\ [1\ -1]' = r\delta[1\ 0][I - \delta A(0)]^{-1}\ \Omega\ \left[r\delta[1\ 0][I - \delta A(0)]^{-1}\right]' \tag{44}$$

In equation (44) the left hand side represents the actual variance of innovations in consumption and the right hand side represents the predicted variance, i.e., the variance of changes in permanent income. In implementing the smoothness condition, Campbell and Deaton use the estimated covariance matrix from the unrestricted bivariate autoregression as an estimate of Ω, and the unrestricted estimates of the autoregressive parameters, $\hat{A}(\beta)$ to construct the inferred variance of Δy^p_t, i.e., the predicted variance of the consumption disturbances. Thus their conclusion that consumption is too smooth is based on the inequality:

$$[1\ -1]\ \hat{\Omega}\ [1\ -1]' < r\delta[1\ 0][I - \delta\hat{A}(\beta)]^{-1}\ \hat{\Omega}\ [r\delta[1\ 0][I - \delta\hat{A}(\beta)]^{-1}]'. \tag{45}$$

Note that in constructing the two quadratic forms which represent the "actual" variance of reduced form disturbances to consumption (on the left hand side) and the "predicted" variance of reduced form disturbances to consumption, which is also interpreted as an estimate of the variance of Δy_t^p (on the right hand side), both quadratic forms are constructed using exactly the same estimated covariance matrix of reduced form disturbances, $\hat{\Omega}$. Thus the smoothness condition will be satisfied if the orthogonality condition (equation (43)) is satisfied. Further, as long as the data satisfy the orthogonality condition, the smoothness condition will hold by construction for any value of $\hat{\Omega}$. Thus as a statistical test of the permanent income hypothesis, a test based on the smoothness condition is just another, less direct, way of testing the orthogonality condition. Campbell and Deaton recognize and emphasize this point, stating, "the orthogonality condition and the condition for smoothness are identical" (page 19).

If the objective of the Campbell and Deaton paper were simply to provide a statistical test of the hypothesis, their analysis succeeds on this level; the orthogonality condition (equation (43)) and the smoothness condition (equation (44)) each provide the basis for a statistical test of the permanent income hypothesis. However, I have argued above that each of the two tests proposed by Campbell and Deaton are basically transformations of the original orthogonality condition proposed by Hall. Further, much of the beauty of Hall's original formulation of the orthogonality condition - in terms of the directness of the intuition motivating the test, and its ease of execution - has been lost in the transformations introduced by Campbell and Deaton.

However, Campbell and Deaton interpret their empirical results as going beyond the documentation of a statistical rejection of the permanent income hypothesis to characterize the way in which actual consumption behavior differs from the behavior predicted by the permanent income hypothesis. Concretely, Campbell and Deaton conclude Section 2 of their paper with the following summary of their empirical results:

> In every case, the theoretical innovation variance is larger than the actual innovation variance, and in all but one case, is more than twice as large. Consumption is markedly smoother than it ought to be if the permanent income theory were correct." (page 21)

In my view, Campbell and Deaton's analysis does not succeed in establishing that consumption is "smoother than it ought to be" under the permanent income hypothesis. The projection argument used by Campbell and Deaton in Section 2 to finesse the problem that the econometrician inevitably lacks some of the informational variables used by agents established that if the permanent income hypothesis is true, the bivariate autoregression of income and saving can be exploited to infer the variance of innovations to permanent income as:

$$\text{(46)} \qquad \begin{aligned} \text{var}(\Delta y_t^P) &= r\delta[1\ 0][I - \delta A(0)]^{-1}\ \Omega\ [r\delta[1\ 0][I - \delta A(0)]^{-1}]' \\ &= [1\ -1]\ \Omega\ [1\ -1]' \end{aligned}$$

According to the projection argument used by Campbell and Deaton, it is the optimal consumption behavior of agents (i.e., the validity of the permanent income hypothesis) which endows the saving variable with its crucial property of encapsulating all information available to agents. However, many papers, including their own, have shown that the permanent income hypothesis fails. Since the permanent income hypothesis fails, the one thing that has been established at this point is that consumption, and hence saving, is <u>not</u> optimally responding to new information.

If the permanent income hypothesis fails in an arbitrary way, the saving series will not fulfill its crucial role of fully encapsulating all information available to agents, with the consequence that the variance of revisions in permanent income will not be identified. While $\text{var}(\Delta y_t^P)$ is not identified for an arbitrary departure from the null hypothesis, it's easy to show that $\text{var}(\Delta y_t^P)$ is identified if the data are generated by the excess sensitivity alternative hypothesis; further, Campbell and Deaton's particular algorithm for inferring $\text{var}(\Delta y_t^P)$ works without any modification. To show this, note that

$$\text{(47)} \qquad r\delta[1\ 0][I - \delta A(\beta)]^{-1} = [\,1\ \ \tfrac{-1}{1-\beta}]$$

thus,

$$(48)\qquad r\delta[1\ 0][I-\delta A(\beta)]^{-1}\Omega[r\delta[1\ 0][I-\delta A(\beta)]^{-1}]' = [1\ \tfrac{-1}{1-\beta}]\ \Omega\ [1\ \tfrac{-1}{1-\beta}]'$$
$$= \text{var}\left\{[1\ \tfrac{-1}{1-\beta}]\begin{bmatrix}\epsilon_{1t}\\(1-\beta)(\epsilon_{1t}-\Delta y_t^P)\end{bmatrix}\right\} = \text{var}(\Delta y_t^P)$$

The excess sensitivity hypothesis, as expressed in equation (20), is, to my knowledge, the only context in which one can sensibly test for excess smoothness. Unlike the permanent income hypothesis, the excess sensitivity model is consistent with the violation of the orthogonality restrictions; unlike arbitrary alternative hypotheses, the var(Δy_t^P) is identified under excess sensitivity.

Having established that the Campbell and Deaton method requires, as an identifying assumption, that the exact excess sensitivity model holds, it becomes clear that there is an easier and more intuitive method of estimating var(Δy_t^P). In the exact excess sensitivity model (equation (26)), the disturbance term is simply a rescaled version of the series of revisions in permanent income, Δy_t^P, where the scale factor depends only on the excess sensitivity parameter, β. Thus var(Δy_t^P) can be estimated by first obtaining a consistent estimate of β by instrumental variables or generalized method of moments, then using the estimated parameter, $\hat{\beta}$, to rescale the standard error of estimate of the equation. Obtaining instrumental variables estimates of the excess sensitivity parameter was a primary purpose of Flavin [1981] and [1983], although those papers did not go on to use the estimate of β to generate the corresponding estimates of var(Δy_t^P).[9]

Section 4: The effect of transitory consumption (preference shocks) on identification

The analysis in Section 3 established that if the consumption data is generated by the excess sensitivity hypothesis, as stated in equation (20), var(Δy_t^P) is identified. For emphasis, I will henceforth refer to equation (20) as the "exact excess sensitivity" model, since the hypothesis posits that consumption is an exact linear combination of transitory income and permanent income.[10] The purpose of this section is to explore the consequences for identification of $\mathrm{var}(\Delta y_t^P)$ of generalization of the excess sensitivity model to allow some sort of disturbance. One aspect of the results — the result that the validity of the projection argument depends critically on the absence of a preference shock — was emphasized by Hansen and Sargent [1981]. They state (page 12), "The applicability of both Hall's testing procedures and the statistical model of the present paper depend critically on the consumption function being an exact equation, or equivalently, on 'transitory consumption' being identically zero".

For concreteness, the disturbance will be labelled "transitory consumption". Disturbances which break the exact linear relationship between consumption, transitory income, and permanent income could arise from sources other than transitory consumption — for example, measurement error in consumption, or complicated dynamics induced by adjustment costs. However, I find it more convenient to view the disturbance as purely a transitory consumption term, rather than attempt to treat the disturbance more generally as a composite error reflecting disturbances from several sources.

With the transitory consumption disturbance added, the excess sensitivity hypothesis becomes:

$$c_t = (\frac{r}{1+r})A_t + \beta y_t + (1-\beta)(\frac{r}{1+r})\sum_{\tau=0}^{\infty} \delta^\tau E_t y_{t+\tau} + \theta_t \tag{49}$$

where θ_t = transitory consumption in period t.

Transitory consumption, θ_t, is assumed to be serially uncorrelated, and uncorrelated with current and lagged values of labor income and asset stocks. Using the accounting identity (22), the structural saving equation is now:

$$s_t = (1-\beta)\left[y_t - (\frac{r}{1+r})E_t \sum_{\tau=0}^{\infty} \delta^\tau y_{t+\tau}\right] - \theta_t \tag{50}$$

Persuing the same series of substitutions used to derive the bivariate autoregression under the exact excess sensitivity model, the bivariate autoregression becomes, in the presence of transitory consumption:

$$(51)\quad \begin{bmatrix} y_t \\ s_t \end{bmatrix} = \begin{bmatrix} A(\beta) \end{bmatrix} \begin{bmatrix} y_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} - (\frac{1}{1-\beta})\frac{\phi}{r\delta^2}\theta_{t-1} \\ (1-\beta)(1-\frac{r\delta}{\phi})\epsilon_{1t} - (1-\beta)\frac{r\delta^2}{\phi}\epsilon_{2t} - \theta_t + [1+r - \frac{\phi}{r\delta^2}]\theta_{t-1} \end{bmatrix}$$

where $A(\beta) = \begin{bmatrix} 1 + \frac{1-\rho_1-\rho_2}{r} & -(\frac{1}{1-\beta})\frac{\phi}{r\delta^2} \\ (1-\beta)\left[\frac{1-\rho_1-\rho_2}{r}\right] & 1+r - \frac{\phi}{r\delta^2} \end{bmatrix}$ as before.

Under the exact excess sensitivity model, if labor income is generated as assumed in equation (1), the bivariate autoregression of y_t and s_t is an AR(1) process. When the excess sensitivity model allows for transitory consumption, the vector $[y_t, s_t]'$ becomes an ARMA(1,1) process. The matrix of autoregressive parameters, $A(\beta)$, is the same, with or without transitory consumption.

Denote the vector of reduced form disturbances as $[u_{1t}, u_{2t}]'$:

$$(52)\quad \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} \epsilon_{1t} - (\frac{1}{1-\beta})\frac{\phi}{r\delta^2}\theta_{t-1} \\ (1-\beta)(1-\frac{r\delta}{\phi})\epsilon_{1t} - (1-\beta)\frac{r\delta^2}{\phi}\epsilon_{2t} - \theta_t + [1+r - \frac{\phi}{r\delta^2}]\theta_{t-1} \end{bmatrix}$$

We would like to find the moving average representation of the reduced form disturbances, $[u_{1t}, u_{2t}]'$, of the form:

$$(53)\quad \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} M(L) \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

where $M(L)$ is a (2x2) matrix of moving average polynomials and $[e_{1t}, e_{2t}]'$ is a vector white noise process which represents the innovations in $[y_t, s_t]'$ with respect to the econometrician's limited information set $\{y_t, y_{t-1}, \ldots, s_t, s_{t-1}, \ldots\}$.

For general values of β, ϕ, and r, obtaining an analytical solution for $M(L)$ involves the simultaneous solution of 5 quadratic equations. For any particular set of numerical values for the structural parameters ($\beta, \phi, r, var(\epsilon_{1t}), var(\epsilon_{2t})$, and $var(\theta_t)$), one could solve for the associated moving average matrix numerically. However, I have chosen the alternate route of making one additional simplifying assumption which, while restrictive, substantially simplifies the problem and permits an analytical solution.

The simplifying assumption is that labor income is a random walk ($\rho_1=1, \rho_2=0$). Under this assumption, the persistence measure equals unity, $\frac{r\delta}{\phi}=1$, and the ARMA(1,1) process in equation (51) simplifies to:

$$\begin{bmatrix} y_t \\ s_t \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{\delta(1-\beta)} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} - \frac{1}{\delta(1-\beta)} \theta_{t-1} \\ -\delta(1-\beta)\epsilon_{2t} - \theta_t \end{bmatrix} \tag{54}$$

Defining $\gamma = \delta(1-\beta)$, the reduced form disturbances are:

$$\begin{aligned} u_{1t} &= \epsilon_{1t} - \gamma^{-1}\theta_{t-1} \\ u_{2t} &= -\gamma\epsilon_{2t} - \theta_t \end{aligned} \tag{55}$$

and have a MA representation:

$$\begin{aligned} u_{1t} &= e_{1t} - \alpha e_{2t-1} \\ u_{2t} &= e_{2t} \end{aligned} \tag{56}$$

Equating moments yields the following three equations:

$$var(u_{1t}) = \sigma^2_{e_1} + \alpha^2\sigma^2_{e_2} = \sigma^2_1 + \gamma^{-2}\sigma^2_\theta \tag{57a}$$

$$var(u_{2t}) = \sigma^2_{e_2} = \sigma^2_\theta + \gamma^2\sigma^2_2 \tag{57b}$$

$$cov(u_{1t}, u_{2t-1}) = -\alpha\sigma^2_{e_2} = \gamma^{-1}\sigma^2_\theta \tag{57c}$$

where $\sigma^2_1 = var(\epsilon_{1t})$ and $\sigma^2_2 = var(\epsilon_{2t})$ as before.

Substituting equation (57b) into (57c), the MA parameter α can be solved for as:

$$\alpha = \frac{-\gamma^{-1}\sigma^2_\theta}{\sigma^2_\theta + \gamma^2\sigma^2_2} \tag{58}$$

Substituting equation (58) into (56) and equating (55) and (56), the innovations with respect to the econometrician's limited information set can be related to the structural disturbances as follows:

$$(59) \qquad e_{1t} = \epsilon_{1t} - \left[\frac{\gamma\sigma_2^2}{\sigma_\theta^2 + \gamma^2\sigma_2^2} \right]\theta_{t-1} + \left[\frac{\sigma_\theta^2}{\sigma_\theta^2 + \gamma^2\sigma_2^2} \right]\epsilon_{2t-1}$$

$$e_{2t} = -\gamma\epsilon_{2t} - \theta_t$$

From equation (59), the variance of forecast errors when forecasting labor income on the basis of the econometrician's information set is:

$$(60) \qquad \sigma_{e_1}^2 = \sigma_1^2 + \left[\frac{\gamma\sigma_2^2}{\sigma_\theta^2 + \gamma^2\sigma_2^2} \right]^2 \sigma_\theta^2 + \left[\frac{\sigma_\theta^2}{\sigma_\theta^2 + \gamma^2\sigma_2^2} \right]^2 \sigma_2^2$$

Obviously, $\sigma_{e_1}^2 > \sigma_1^2$; that is, the variance of the econometrician's forecast error exceeds the variance of the agent's forecast error. From equation (60), it's also easy to show that $\sigma_{e_1}^2 < \sigma_1^2 + \sigma_2^2$; that is, the variance of the econometrician's forecast error is smaller than the forecast error variance of a univariate income model.

Thus while the "structural" time series representation for $[y_t, s_t]'$ in terms of the primitive disturbances is given by equation (54), an econometrician using data only on income and saving will estimate the reduced form time series representation:

$$(61) \qquad \begin{bmatrix} y_t \\ s_t \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{\delta(1-\beta)} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & -\alpha L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

The revision in permanent income in period t as inferred by the econometrician, denoted $\Delta\tilde{y}_t^p$, is:

$$(62) \qquad \Delta\tilde{y}_t^p = r\delta\,[1\ 0][I - \delta A(\beta)]^{-1} \begin{bmatrix} 1 & -\alpha\delta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

where the tilde is used in $\Delta\tilde{y}_t^p$ because the inferred revision in permanent income is not necessarily equal to the true revision in permanent income.

In evaluating equation (62), note that:

$$(63) \qquad r\delta[1\ 0][I - \delta A(\beta)]^{-1} = [\,1\ \ \tfrac{-1}{1-\beta}\,]$$

as before.

Thus the revision in permanent income as inferred by the econometrician will be:

(64) $$\Delta\tilde{y}_t^P = e_{1t} - [\delta\alpha + \frac{1}{1-\beta}]e_{2t}$$

or, equivalently,

(65) $$\Delta\tilde{y}_t^P = \epsilon_{1t} - \left[\frac{\gamma\sigma_2^2}{\sigma_\theta^2 + \gamma^2\sigma_2^2} \right]\theta_{t-1} + \left[\frac{\sigma_\theta^2}{\sigma_\theta^2 + \gamma^2\sigma_2^2} \right]\epsilon_{2t-1} + [\delta\alpha + \frac{1}{1-\beta}][\gamma\epsilon_{2t} + \theta_t]$$

The variance of $\Delta\tilde{y}_t^P$, as calculated from the bivariate ARMA(1,1) on income and saving can be obtained from equations (64), (58), (59), and (60). After simplification, $\text{var}(\Delta\tilde{y}_t^P)$ can be expressed as:

(66) $$\text{var}(\Delta\tilde{y}_t^P) = \sigma_1^2 + \delta^2\sigma_2^2 + (1-\delta^2)\left[\frac{\sigma_\theta^2}{\sigma_\theta^2 + \delta^2(1-\beta)^2\sigma_2^2} \right]\sigma_2^2$$

Under the random walk assumption which was invoked to make the solution of the moving average representation tractable, the persistence measure equals unity ($\frac{r\delta}{\phi} = 1$), with the implication that the "true" revision in permanent income (that is, the revision in permanent income based on the full information set used by agents) has variance:

(67) $$\text{var}(\Delta y_t^P) = \sigma_1^2 + \delta^2\sigma_2^2$$

Several intuitively plausible properties of the inferred $\text{var}(\Delta\tilde{y}_t^P)$ are apparent in equation (66):

1) For $\sigma_\theta^2 > 0$, that is, as long as transitory consumption is not identically zero, the inferred variance of permanent income <u>overstates</u> the true variance of permanent income; $\text{var}(\Delta\tilde{y}_t^P) > \text{var}(\Delta y_t^P)$.

2) Even if consumption is not excessively sensitive to current income ($\beta=0$), the presence of transitory consumption still causes loss of identification of $\text{var}(\Delta y_t^P)$.

3) Holding constant σ_1^2 and σ_2^2, as $\sigma_\theta^2 \to \infty$, $\text{var}(\Delta\tilde{y}_t^P) \to \sigma_1^2 + \sigma_2^2$, which is the variance of inferred revisions in permanent income based on the univariate

income model, $y_t = y_{t-1} + (\epsilon_{2t} + \epsilon_{1t})$. In other words, as transitory consumption increases in importance, the income/saving autoregression does an increasingly poor job of capturing the information available to agents but unobserved by the econometrician.

Section 5: Empirical Results

The discussion in sections 1 through 4 raises several issues which need to be resolved empirically. This section briefly presents empirical evidence on the following three questions:

1) If, in order to achieve identification, we assume that consumption is generated by the exact excess sensitivity model (i.e. we assume that there are no preference shocks), what is the implied estimate of $\mathrm{var}(\Delta y_t^p)$?

2) Is the assumption of no preference shocks violated by the data?

3) Even if we find that the assumption of no preference shocks can be rejected statistically, implying that $\mathrm{var}(\Delta y_t^p)$ is not identified, can we say something about the quantitative importance of the divergence between the econometrician's inference ($\mathrm{var}(\Delta \tilde{y}_t^p)$) and the true $\mathrm{var}(\Delta y_t^p)$, and similarly, the divergence between the inferred $\mathrm{cov}(\epsilon_{yt}, \Delta \tilde{y}_t^p)$ and the true $\mathrm{cov}(\epsilon_{1t}, \Delta y_t^p)$?[11]

Specification tests of the no preference shock assumption

If the exact excess sensitivity hypothesis holds, the one step ahead forecast error in predicting income on the basis of a bivariate autoregression of lagged income and saving cannot be improved upon by adding additional variables to the VAR. Thus the no preference shock assumption can very easily be tested by adding one or more lagged variables to the bivariate income forecasting equation, and conducting an exclusion test for the additional variables. In the context of the assumed process for labor income studied in Section 1, the analytical basis for the exclusion test as a test of the no preference shock assumption is provided in Section 3. However, the assertion that the exact excess sensitivity hypothesis implies that additional (potential) forecasting variables included in the income autoregression will have no explanatory power also holds for a general process for labor income.[12]

The results of the F-tests for the exclusion of additional forecasting variables are reported in Table 1. For comparability with empirical results reported later and with the empirical results in Campbell and Deaton [1988], the income variable was stated in log differences ($\Delta \ln y_t$) and the savings variable as the saving rate (s_t/y_t). For each of the specification tests, the restricted specification consisted of a regression of $\Delta \ln y_t$ on a constant and lagged values of both $\Delta \ln y_t$ and s_t/y_t. The additional variables used in the unrestricted specifications consisted of: the growth rate of the Standard and Poor's Composite Stock Price index ($\Delta \ln S\&P_t$), the growth rate of the Producer Price Index for fuel ($\Delta \ln Pfuel_t$), the first difference of the unemployment rate (Δu_t), and the growth rates of M1 and M2 ($\Delta M1_t$, $\Delta M2_t$, $\Delta M1old_t$, and $\Delta M2old_t$).[13] (For further details on the definition of these variables, see the data appendix.) The variables used throughout the empirical work reported in this section were chosen *a priori* as variables likely to have predictive value for income; they were not chosen as the result of a specification search.

In each of the ten specification tests reported in Table 1, the null hypothesis — that the additional variables have no predictive value when added to a bivariate autoregression of income and savings — can be rejected at the 5% level. In half of the specification tests, it can be rejected at the 1% level as well. The results of the exclusion tests provide uniform and clear-cut evidence that the identifying assumptions underlying the exact excess sensitivity model are inconsistent with the data.

While the additional forecasting variables proved to be statistically significant predictors of future income, it is conceivable that they improve the forecasting ability of the bivariate autoregression by a margin which is quantitatively trivial. As a check on the effect of the inclusion of additional variables on the forecasting performance of the income autoregression, Table 2 reports two goodness-of-fit measures, the $\bar{R}^2$ and the Standard Error of Estimate for a univariate income autoregression, a bivariate autoregression of income and saving, and two VARs containing variables in addition to lagged income and saving. If the exact excess sensitivity hypothesis were true, the bivariate autoregression of income and saving should have a higher $\bar{R}^2$ and lower S.E.E. than a univariate income autoregression, but the two goodness-of-fit measures should not continue to improve as lagged values of additional variables, beyond income and saving, are added to the income equation. However, consistent with the results of the exclusion tests, the goodness-of-fit measures continue to improve as the additional forecasting variables are added to the autoregression. For example, if 4 lags of each variable are included, the $\bar{R}^2$ rises from a value of 16% for the univariate income autoregression to 19% for the bivariate income/saving autoregression, 28% for a 5-variable VAR, and 39% for a 9-variable VAR.

Although $\text{var}(\Delta y_t^p)$ is not identified, strictly speaking, the analytical discussion of the consequences of omitted information suggested the possibility that even when an unresolved omitted information problem precludes the econometrician from observing or inferring the series Δy_t^p, the econometrician's inference of $\text{var}(\Delta\tilde{y}_t^p)$ might nevertheless be a fairly accurate approximation to the true $\text{var}(\Delta y_t^p)$. Intuitively, the econometrician's inference on $\text{var}(\Delta y_t^p)$ turned out to be a fairly robust estimate of the true $\text{var}(\Delta y_t^p)$ because the econometrician's inability to observe parts of the agent's information set primarily meant that a shock to permanent income which agents observed in period t, based on a non-income variable, was not registered by the econometrician until a period or two later when the effect of the shock actually showed up in current income. Thus while the two series — the true Δy_t^p, as perceived by agents based on their complete information set, and the econometrician's inferred $\Delta\tilde{y}_t^p$ based on the incomplete information set — may have modest contemporaneous covariance (and the series Δy_t^p would Granger- cause the inferred series $\Delta \tilde{y}_t^p$) the analytical example used to study the omitted information problem indicated that the two series might nevertheless have essentially the same variance. That is, even though the conditions necessary to establish the identifiability of var(Δy_t^p) are clearly violated by the data, the inference on var(Δy_t^p) based on the bivariate autoregression of income and saving may be a serviceable approximation to the true var(Δy_t^p).

As a check on the robustness of the econometrician's inference on var($\Delta \tilde{y}_t^p$) as an estimate of var(Δy_t^p), a range of VAR's incorporating different information sets was estimated, and the implied series on $\Delta \tilde{y}_t^p$ was calculated for each. Included in the set of VAR's was a univariate autoregression, a bivariate autoregression of income and saving, a 5-variable VAR, and a 9-variable VAR. Table 3 reports the standard deviation of the series on $\Delta \tilde{y}_t^p$ obtained from each VAR. The estimates of the standard devation of $\Delta \tilde{y}_t^p$ are expressed as annualized quarterly growth rates.

Since the exclusion tests indicated that the savings variable in the bivariate autoregression was not successfully encapsulating all of the agent's information set, the 5-variable VAR and the 9-variable VAR represent strictly larger effective information sets than either the univariate autoregression or the bivariate autorgression of income and saving. However, despite the result reported earlier that the forecasting performance of the VAR improves significantly as additional variables are included, the associated estimate of var($\Delta \tilde{y}_t^p$) appears to be fairly insensitive to the specification of the information set. I interpret the apparent robustness of the inferred var($\Delta \tilde{y}_t^p$) with respect to the specification of the econometrician's information set as limited empirical evidence in support of the conjecture, made on the basis of the analytical example of the omitted information problem, that the econometrician's inferred var($\Delta \tilde{y}_t^p$) may be a reasonable approximation to the true $\text{var}(\Delta y_t^p)$ even though the econometrician has access to only a limited subset of the agent's information set. The evidence of the robustness of $\text{var}(\Delta\tilde{y}_t^p)$ is described as "limited" because it is always possible that there is a VAR based on a larger or different set of variables for which the implied $\text{var}(\Delta\tilde{y}_t^p)$ would be substantially different from the estimates reported in Table 3.

Table 3 also reports the estimated standard deviation of the consumption innovation, ϵ_{ct}, associated with each VAR. In no case does the addition of additional forecasting variables reverse the finding of excess smoothness; for each of the VARs estimated the standard deviation of $\Delta\tilde{y}_t^p$ exceeds the standard deviation of ϵ_{ct}. In fact, the disparity between $\text{var}(\epsilon_{ct})$ and $\text{var}(\Delta\tilde{y}_t^p)$ seems to widen, if anything, as additional variables are included. This tendency is a result of the apparent robustness of $\text{var}(\Delta\tilde{y}_t^p)$ in combination with the fact that the measure of consumption smoothness ($\text{var}(\epsilon_{ct})$) depends on the information set; as additional variables are added to the VAR, $\text{var}(\epsilon_{ct})$ must either remain constant, or decline. From a different perspective, note that according to the proposed explanation of excess smoothness, ϵ_{ct} is a weighted average of the innovation in the income equation of the VAR (ϵ_{yt}) and Δy_t^p. Since the variance of ϵ_{yt} will decline if additional forecasting variables added to the VAR have predictive value, the reported decline in $\text{var}(\epsilon_{ct})$ is exactly what one should expect if a) consumption is generated by the excess sensitivity model, and b) the saving series fails to fully capture the full information set used by agents, as indicated by the specification tests.

The result stated above — that despite the fact that the econometrician is using only a subset of the information available to agents, he may nevertheless obtain a reasonably accurate inference, $\text{var}(\Delta\tilde{y}_t^p)$, on the true $\text{var}(\Delta y_t^p)$ — should not be misinterpreted as implying the much stronger proposition that the inferred series on $\Delta\tilde{y}_t^p$ is a reasonable proxy for the true series Δy_t^p. To see the fallacy of linking these two propositions, recall that in the analytical discussion of the omitted information problem, the inferred series $\Delta\tilde{y}_t^p$ and the true series Δy_t^p have approximately the same variance but zero contemporaneous correlation for the special case in which agents had perfect one-period-ahead forecastability of income.

One final set of empirical results is required to establish the relationship between a finding of excess smoothness and the stationarity of the income process. Section 2 showed that the relationship between a empirical finding of excess smoothness and the stationarity of the income process depends crucially on the covariance term, $\mathrm{cov}(\Delta y_t^p, \epsilon_{yt})$. If the correlation between Δy_t^p and ϵ_{yt} is small, a finding of excess smoothness is perfectly consistent with a wide range of stationary income processes as well as nonstationary income processes.

Table 4 reports the correlation coefficient between $\Delta\tilde{y}_t^p$ and ϵ_{yt}, for the various VAR models investigated previously. In contrast to the robustness of the estimate of $\mathrm{var}(\Delta\tilde{y}_t^p)$, Table 4 indicates that the correlation coefficient between $\Delta\tilde{y}_t^p$ and ϵ_{yt} keeps dropping as one adds more forecasting variables to the VAR. Based on the 9-variable VAR, the correlation coefficient is around .5 to .66 (depending on the number of lags of each variable included) in contrast to the theoretically constrained correlation of unity implied by the univariate income model. Since even the 9-variable VAR does not include all of the informational variables available to agents, the correlation of the true series Δy_t^p with ϵ_{yt} is presumably even smaller than the .5 to .66 reported in the last row of Table 4. The important conclusion is that while naive estimates of $\mathrm{var}(\Delta\tilde{y}_t^p)$ may provide acceptably accurate approximations to the true $\mathrm{var}(\Delta y_t^p)$, the data do not provide any support for a similar robustness result with respect to $\mathrm{cov}(\Delta\tilde{y}_t^p, \epsilon_{yt})$; using a univariate or bivariate autoregression to make inferences about $\mathrm{cov}(\Delta\tilde{y}_t^p, \epsilon_{yt})$ will tend to overstate the true $\mathrm{cov}(\Delta y_t^p, \epsilon_{yt})$ and the magnitude of the overstatement is probably large.

Conclusions

The omitted information problem — that is, the econometric problem which arises because an econometrician cannot explicitly include in an econometric specification the complete set of variables potentially used by agents — complicates the estimation and interpretation of virtually all empirical rational expectations models. This paper has investigated the implications of the omitted information problem in the context of the empirical phenomenon of the excess smoothness of consumption proposed by Deaton [1987].

Other authors have shown that, if the null hypothesis is true, in many cases the omitted information problem can be completely finessed by using a projection argument. Under this projection argument, the econometrician need not observe the agent's complete information set, since the agent's behavior reveals the appropriate summary measure of the omitted variables in an endogenous signaling variable — in the consumption case, the signaling variable would be consumption, or, equivalently, saving. While the result that the omitted information problem can be completely avoided when the null hypothesis is true is an extremely useful result for some purposes — such as constructing tests of the null hypothesis, or for forecasting when the null hypothesis is not rejected by the data — these results are not immediately useful for characterizing the way in which observed consumption behavior differs from the behavior predicted by the PIH. A substantial part of the analysis, both analytical and empirical, is devoted to studying the implications of the omitted information problem under several different types of departures from the null hypothesis.

In the context of the consumption case, the paper does identify an alternative hypothesis — the exact excess sensitivity hypothesis — for which the signaling variable (i.e., the consumption or saving series) correctly encapsulates the agent's information set, despite the non-optimal behavior of agents. However, this alterative hypothesis is a highly restrictive case; for an arbitrary alternative hypothesis, the omitted information problem cannot be avoided. In particular, the mere addition of a stochastic error term, such as a preference shock, in the behavioral model is sufficient to contaminate the information content of the signaling variable, with the result that the consequences of omitted information are not fully eliminated. Since the effect of the disturbance is to weaken but not fully eradicate the information content of the signal, the application of the projection argument in the presence of a stochastic disturbance in some cases will partially eliminate the consequences of the omitted information.

These additional results on the consequences of omitted information under alternatives to the null hypothesis do not reverse the finding reported earlier by Deaton [1987], West [1988], and Campbell and Deaton [1988], that consumption is "too smooth" in the sense that the variance of reduced form disturbances to consumption is exceeded by the variance of revisions in permanent income. Although the paper tentatively concurs with the conclusion of previous authors that the "excess smoothness" of consumption is a valid characterization of the data, this endorsement is a qualified one and is based on a logically distinct line of reasoning.

While the analytical and empirical investigation of the omitted information problem under various departures from the null hypothesis does not reverse the direction of the inequality, it does dramatically change the economic interpretation of the finding. The paper shows that the naive econometrician who fails to fully take into account the effects of omitted information will incorrectly conclude that the empirical finding of excess smoothness of consumption implies that the income process is nonstationary. By contrast, with a more thorough understanding of the omitted information problem, the finding of excess smoothness of consumption is very easily generated by the conjunction of two assumptions: a) the consumption data is generated by the excess sensitivity alternative hypothesis, in which consumption is a weighted average of current income and permanent income, and b) agents are forecasting on the basis of a larger information set than the econometrician. Further, in the interpretation of the excess smoothness phenomenon which invokes an important role for omitted information, excess smoothness is consistent with a wide range of stationary income processes as well as nonstationary income processes. Thus the presumption which has permeated much of the earlier work on excess smoothness — that the excess smoothness phenomenon was linked in an essential way to the stationarity or non-stationarity of the income process — evaporates when one takes into account the effects of omitted information. The finding that one's perspective on the stylized facts about consumption are not radically altered depending on one's view as to whether income is a stationary or a nonstationary process is a welcome one, since the controversy over the stationarity issue does not appear to be in danger of resolution anytime soon.

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Table 1

F-tests for exclusion of additional variables added to bivariate income autoregression

F-statistic for exclusion of lagged values of:	number of lags of each variable included in the unrestricted specification: 1	2	3	4	5
$\Delta \ln S\&P_t$, Δu_t, $\Delta \ln Pfuel_t$:	$F(3,115) = 4.21$	$F(6,110) = 2.71$	$F(9,105) = 2.23$	$F(12,100) = 2.20$	$F(15,95) = 2.11$
significance level:	.0072	.017	.026	.017	.016
$\Delta \ln S\&P_t$, Δu_t, $\Delta \ln Pfuel_t$, $\Delta M1_t$, $\Delta M2_t$, $\Delta M1old_t$, $\Delta M2old_t$:	$F(7,111) = 2.48$	$F(14,102) = 2.83$	$F(21,93) = 2.15$	$F(28,84) = 2.29$	$F(35,75) = 1.91$
significance level:	.021	.0013	.0066	.0020	.0097

For example, for the entry in the first row, first column, the restricted specification was:

$$\Delta \ln y_t = \alpha + \gamma_1 \Delta \ln y_{t-1} + \gamma_2 s_{t-1}/y_{t-1} + \epsilon_t$$

while the unrestricted specification was:

$$\Delta \ln y_t = \alpha + \gamma_1 \Delta \ln y_{t-1} + \gamma_2 s_{t-1}/y_{t-1} + \gamma_3 \Delta \ln S\&P_{t-1} + \gamma_4 \Delta u_{t-1} + \gamma_5 \Delta \ln Pfuel_{t-1} + \epsilon_t$$

and the F-statistic is for the hypothesis: H_0: $\gamma_3 = \gamma_4 = \gamma_5 = 0$.

As one reads across the row, the number of lagged values of $\Delta \ln y_t$, s_t/y_t, and the additional variables all increase from 1 lagged value to 5 lagged values.

The sample period for all regressions was 1954:4 to 1984:4.

Table 2

Goodness-of-fit of income forecasting equation;
Table reports both the $\bar{R}^2$ and the S.E.E. of the income forecasting equation

variables included in income forecasting equation	number of lags of each variable included in the income forecasting equation:									
	1		2		3		4		5	
	$\bar{R}^2$	S.E.E.	$\bar{R}^2$	S.E.E.	$\bar{R}^2$	S.E.E.	$\bar{R}^2$	S.E.E.	$\bar{R}^2$	S.E.E.
$\Delta \ln y_t$	.17	.785	.17	.788	.17	.788	.16	.790	.19	.775
$\Delta \ln y_t$, s_t/y_t	.21	.766	.20	.770	.20	.772	.19	.777	.26	.743
$\Delta \ln y_t$, s_t/y_t, $\Delta \ln S\&P_t$, Δu_t, $\Delta \ln Pfuel_t$	.27	.737	.27	.738	.27	.737	.28	.731	.36	.693
$\Delta \ln y_t$, s_t/y_t, $\Delta \ln S\&P_t$, Δu_t, $\Delta \ln Pfuel_t$, $\Delta M1_t$, $\Delta M2_t$, $\Delta M1old_t$, $\Delta M2old_t$	.27	.735	.35	.697	.34	.702	.39	.675	.42	.654

Table 2 reports the $\bar{R}^2$, or adjusted R^2, and the S.E.E. of the income forecasting equation for a univariate income autoregression (row 1), a bivariate autoregression of income and saving (row 2), and two VAR's including forecasting variables in addition to lagged income and saving.

Since the income variable is expressed as a growth rate ($\Delta \ln y_t$), the standard error of estimate is expressed in percentage units, i.e., a S.E.E. of .785 indicates that the standard deviation of the forecast errors of $\Delta \ln y_t$ is .785% per quarter.

The sample period for all regressions was 1954:4 to 1984:4.

Table 3

Estimates of the standard deviations of $\Delta\tilde{y}^P_t$ and ϵ_{ct}

$\Delta\tilde{y}^P_t$ = inferred series on revisions in permanent income, based on VAR specified on left.

ϵ_{ct} = innovations in a reduced form (VAR) consumption equation, based on VAR specified on left.

For each cell, the standard deviation of ϵ_{ct} is on the left, and the standard deviation of $\Delta\tilde{y}^P_t$ is on the right.

variables included in income forecasting equation	number of lags of each variable included in the income forecasting equation: 1	2	3	4	5
	ϵ_{ct} $\Delta\tilde{y}^P_t$	ϵ_{ct} $\Delta\tilde{y}^P_t$	ϵ_{ct} $\Delta\tilde{y}^P_t$	ϵ_{ct} $\Delta\tilde{y}^P_t$	ϵ_{ct} $\Delta\tilde{y}^P_t$
$\Delta\ln y_t$	.0544	.0540	.0592	.0572	.0469
$\Delta\ln y_t$, s_t/y_t	.0334<.0496	.0334<.0519	.0327<.0572	.0320<.0559	.0304<.0481
$\Delta\ln y_t$, s_t/y_t, $\Delta\ln S\&P_t$, Δu_t, $\Delta\ln Pfuel_t$	.0296<.0498	.0293<.0505	.0284<.0567	.0266<.0546	.0254<.0468
$\Delta\ln y_t$, s_t/y_t, $\Delta\ln S\&P_t$, Δu_t, $\Delta\ln Pfuel_t$, $\Delta M1_t$, $\Delta M2_t$, $\Delta M1old_t$, $\Delta M2old_t$	.0278<.0484	.0266<.0474	.0250<.0534	.0235<.0541	.0218<.0546

Sample period for all repressions was 1954:4 to 1984:4.

δ=.9855, which corresponds to an annual interest rate of 6%.

Table 4

Implied estimates of the correlation between ϵ_{yt} and $\Delta\bar{y}_t^p$

ϵ_{yt} = income innovation in VAR specified on left

$\Delta\bar{y}_t^p$ = inferred series on revisions in permanent income, based on VAR specified on left

variables included in income forecasting equation	number of lags of each variable included in the income forecasting equation: 1	2	3	4	5
$\Delta\ln y_t$*	1	1	1	1	1
$\Delta\ln y_t$, s_t/y_t	.772	.800	.863	.925	.993
$\Delta\ln y_t$, s_t/y_t, $\Delta\ln S\&P_t$, Δu_t, $\Delta\ln Pfuel_t$	.672	.619	.707	.738	.838
$\Delta\ln y_t$, s_t/y_t, $\Delta\ln S\&P_t$, Δu_t, $\Delta\ln Pfuel_t$, $\Delta M1_t$, $\Delta M2_t$, $\Delta M1old_t$, $\Delta M2old_t$	.605	.514	.500	.565	.663

Sample period for all repressions was 1954:4 to 1984:4.

δ=.9855, which corresponds to an annual interest rate of 6%.

*Entries in this row must have a theoretical correlation of unity; therefore the theoretical correlation coefficient is reported.

Data Appendix

For conformity with the empirical work in Campbell and Deaton [1988], the labor income and saving data were the same series used in that paper. These series were:

y_t = disposable labor income, seasonally adjusted, real, per capita

yt_t = disposable total income, seasonally adjusted, real, per capita

c_t = total consumption, seasonally adjusted, real, per capita

$s_t = yt_t - c_t$ = saving

s_t/y_t = ratio of saving, as defined above, to disposable labor income

The variables y_t, yt_t, and c_t, which were used in Campbell and Deaton [1988], were originally constructed for use in Blinder and Deaton [1985], and incorporate several adjustments to the standard NIPA concepts. As described by Campbell [1987], these adjustments include:

1) The 1975 tax cut is removed from the disposable income series.

2) Interest payment flows from consumers to business are subtracted from the NIPA disposable income series.

3) Personal non-tax payments to state and local governments are added to both disposable income and consumption, on the grounds that they resemble payment for goods and services more than taxes (e.g., state college tuition, etc.).

4) All series are deflated by total population and by a consumer spending deflator.

5) Disposable total income is broken down into disposable labor income and disposable capital income. In addition to using any allocations of tax payments to capital or labor income which are explicit in the NIPA accounts, proprietor's income and personal income tax payments are attributed to labor and capital income according to their factor shares; social insurance payments are attributed to labor income. For more documentation on the construction of these three variables, the reader is referred to Blinder and Deaton [1985].

Other variables used in the empirical work were constructed as:

$S\&P_t$ = Standard and Poor's Composite Stock Price Index. Quarterly series was compiled using the mid month daily figure in March (for quarter I), June (for quarter II), September (for quarter III), and December (for quarter 4).

$Pfuel_t$ = Producer Price Index: Fuel and Other Related Products and Power, 1967=100. Quarterly series was complied by taking the index value for the middle month of each quarter, i.e., the February value for quarter I, etc.

u_t = Civilian unemployment rate, in per cent, seasonally adjusted.

Quarterly series was complied by taking the monthly figures for March (quarter I), June (quarter II), September (quarter III), and December (quarter IV)

$M1_t$ = M1, in billions of 1982 dollars, seasonally adjusted. Series compiled by taking middle month of each quarter, i.e., February figure for quarter 1, etc. Series contains money stock data for observations 1959:1- 1984:4, contains zeros for observations prior to 1959:1.

$M2_t$ = M2, in billions of 1982 dollars, seasonally adjusted. Series compiled by taking middle month of each quarter, i.e., February figure for quarter 1, etc. Series contains money stock data for observations 1959:1- 1984:4, contains zeros for observations prior to 1959:1.

$M1old_t$ = Total money stock; the sum of Currency and Demand Deposits, in billions of dollars, seasonally adjusted. Series compiled by taking middle month value for each quarter. Series contains money stock data for observations 1953:3 - 1958:4, contains zeros for observations after 1958:4.

$M2old_t$ = $M1old_t$ plus Time Deposits, in billions of dollars, seasonally adjusted. Series compiled by taking middle month value for each quarter. Series contains money stock data for observations 1953:3 - 1958:4, contains zeros for observations after 1958:4.

Footnotes

[1]Shiller [1972] showed that, under the null hypothesis, if the econometrician is forecasting short rates on the basis of an incomplete information set — for example, on the basis of lagged short rates only — the implied restriction between the parameters of the short rate autoregression and the distributed lag of the long rate on lagged short rates was not invalidated by the fact that agents are forecasting on the basis of a larger information set than the econometrician. In fact, the discrepancy between the agent's and econometrician's information sets was a solution rather than a problem, since the discrepancy provided an explanation for the unavoidable finding that a regression of the long rate on lagged short rates does not generate an R^2 of 1.

[2]For an alternative explanation of the excess smoothness phenomenon, see Christiano [1987]. In that paper, Christiano relaxes the constant interest rate assumption and shows that relatively modest interest rate variability may explain a quantitatively important degree of consumption smoothing.

[3] West [1988] defines excess smoothness as arising when $\text{var}(\Delta c_t) < \text{var}(\Delta y^p_t)$. However, whether one chooses $\text{var}(\epsilon_{ct})$ or $\text{var}(\Delta c_t)$ as the measure of consumption variability probably makes little difference in practice; even though the hypothesis that consumption is a random walk can be statistically rejected, the variance of the disturbances to a VAR consumption equation and the variance of the first difference of consumption are not grossly different in magnitude.

[4]In the context of the permanent income hypothesis (i.e., for the special case in which the excess sensitivity parameter, β, is set to zero), Campbell [1987] has referred to equation (23) as the "saving for a rainy day" equation.

[5]For empirical estimates of β in this ballpark, see Flavin [1981], Hayashi [1982], Flavin [1983], or Campbell and Mankiw [1987b].

[6]The analysis in this paper does not appear 100% compatible with the derivations in Campbell and Deaton [1988] because they derive, and work with, a log-linear approximation to the linear permanent income model used in Flavin [1981]. The justification given by Campbell and Deaton for using the log linearization is that a) they perceive a time trend in the variance of innovations to Δy_t, empirically, and b) they favor, on a priori grounds, proportional rather than linear growth. However, in order to state the model in terms of log differences, various approximations need to be made, including not only an assumption that the real interest rate exceeds the mean growth rate of labor income, $r > \mu$, but further, that $\frac{r}{r-\mu} \cong 1$. If, in order to state the consumption model in log linear form it is necessary to approximate the growth rate of income as zero, this seriously undermines the original justification for the log linear model, in my view. For this reason, I have not followed Campbell and Deaton in using their log linearization, and have instead retained the linear version of the model. The choice of linear versus log linear modeling does not affect any of the basic issues.

[7]Under complete Keynesian behavior (β=1), the saving series would be identically zero, $s_t = 0$, with the result that data on saving provides no information concerning variables available to agents but unobserved by the

econometrician. Obviously, $var(\Delta y_t^P)$ will not be identified by a univariate autoregression of income and saving if β=1.

[8]Because Campbell and Deaton work with their log linear approximation to the consumption model, rather than the linear version of the model, their orthogonality condition is actually

$$(43') \qquad [1\ 0][I - \delta A(0)]^{-1} = [\ 1\ -1].$$

Equation (43) is the analog of (43'), simply rederived for consistency with the linear version of the model. That is, if one is working with the log linear model, (43') is the appropriate orthogonality condition, if one is working with the linear model, (43) is the equivalent orthogonality condition.

[9]In Flavin [1981], the alternative hypothesis was considerably more general, in the sense that Δc_t was permitted to respond not only to the current Δy_t but also to seven lagged values of Δy_t. In that paper, the over-parameterization of the alternative hypothesis was required in order to establish an exact correspondence between Hall's [1978] reduced form test of the PIH and the "structural" test of the PIH based on estimation of the excess sensitivity parameter. In the present context, the more parsimonious specification of the alternative hypothesis (equation (26)) is considered, since this is the alternative hypothesis for which the Campbell/Deaton algorithm works without modification.

The parsimonious version of the excess sensitivity hypothesis has also been estimated by instrumental variables by Campbell and Mankiw [1987b]. Campbell and Mankiw [1987b] further differs from Flavin [1981] by considering the analog of equation (26) in growth rates rather than first differences, consistent with their view of income as containing a unit root. Despite the differences in specification, Campbell and Mankiw [1987b] obtain estimates of the excess sensitivity parameter comparable to the estimates in Flavin [1981] and [1983].

[10]I am using the word "exact" in the sense of Hansen and Sargent [1981], "Exact Linear Rational Expectations Models: Specification and Estimation".

[11]Remember that ϵ_{yt} is defined as the innovation in y_t relative to the econometrician's limited information set, Ω_{t-1}, while ϵ_{1t} is defined as the innovation in y_t relative to the agent's complete information set I_{t-1}.

[12]To see this, first review the projection argument used by Campbell and Deaton to establish that the omitted information problem is completely finessed if the PIH is true. If the PIH holds, the optimal behavior of consumption implies that the saving series encapsulates the agent's entire information set; explicitly including the variables of the agent's information set individually would be redundant if saving is already included in the autoregression. Next note that under the exact excess sensitivity hypothesis, saving is just a rescaled version of the saving series which would be generated under the PIH (equation (36)). Thus changing the value of β from zero to a positive value would alter the coefficient on the saving variables in a bivariate autoregression of income and saving, but would not affect the one step ahead forecast of labor income or the income innovation series.

[13]The additional variables are defined as:

$\Delta \ln S\&P_t$ = growth rate of Standard and Poor's Composite Stock Price Index

$\Delta M1_t$ = growth rate of real M1, seasonally adjusted, for 1959:1–1984:4; series contains zeros for observations prior to 1959:1.

$\Delta M2_t$ = growth rate of real M2, seasonally adjusted, for 1959:1–1984:4; series contains zeros for observations prior to 1959:1.

$\Delta M1old_t$ = growth rate of currency and demand deposits, SA, 1953:3–1958:4; series contains zeros for observations after 1958:4.

$\Delta M2old_t$ = growth rate of currency, time deposits, and demand deposits, SA, 1953:3–1958:4; series contains zeros for observations after 1958:4.

$\Delta \ln Pfuel_t$ = growth rate of Producer Price Index: Fuel and Other Related Products and Power

Δu_t = first difference of the civilian unemployment rate, SA

More detail on the definition of the variables is provided in the data appendix.