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SEARCH PROFILING WITH PARTIAL KNOWLEDGE OF DETERRENCE

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ABSTRACT

Economists studying public policy have generally assumed that the relevant social planner knows how policy affects population behavior. Planners typically do not possess all of this knowledge, so there is reason to consider policy formation with partial knowledge of policy impacts. Here I consider the choice of a profiling policy where decisions to search for evidence of crime may vary with observable covariates of the persons at risk of being searched. To begin I pose a planning problem whose objective is to minimize the utilitarian social cost of crime and search. The consequences of candidate search rules depends on the extent to which search deters crime. Deterrence is expressed through the offense function, which describes how the offense rate of persons with given covariates varies with the search rate applied to these persons. I study the planning problem when the planner has partial knowledge of the offense function. To demonstrate general ideas, I suppose that the planner observes the offense rates of a study population whose search rule has previously been chosen. He knows that the offense rate weakly decreases as the search rate increases, but he does not know the magnitude of the deterrent effect of search. In this setting, I first show how the planner can eliminate dominated search rules and then how he can use the minimax or minimax-regret criterion to choose an undominated search rule.

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1. Introduction

Economists engaged in normative study of public policy generally assume that the relevant social planner knows how policy affects population behavior. For example, economists studying optimal income taxation assume that the planner knows how the tax schedule affects labor supply (e.g., Mirrlees, 1971). Those studying optimal criminal justice systems assume that the planner knows how policing and sanctions affect offense rates (e.g., Polinsky and Shavell, 2000).

In these and other policy domains, social planners may not possess the knowledge that economists assume them to have. Fundamental identification problems and practical problems of statistical inference make it difficult to learn how policy affects behavior. Hence, there is reason to consider policy formation when a planner has only partial knowledge of policy impacts.

I have previously studied an abstract planning problem when policy impacts are partially identified (Manski, 2000; 2002; 2005a, Chapter 2) or are observed in finite samples (Manski, 2004; 2005a, Chapter 3). Here, I examine a specific aspect of law enforcement that has recently been the subject of debate. This is the choice of a *profiling* policy wherein decisions to search for evidence of crime may vary with observable covariates of the persons at risk of being searched. Policies that make search rates vary with personal attributes are variously defended as essential to effective law enforcement and denounced as unfair to classes of persons subjected to relatively high search rates. Variation of search rates by race has been particularly controversial; see, for example, Knowles, Persico, and Todd (2001), Persico (2002), and Dominitz (2003). Whereas recent research on profiling has sought to define and detect racial discrimination, my concern is to understand how a social planner might reasonably choose a profiling policy.

To begin, I pose in Section 2 a planning problem whose objective is to minimize the utilitarian social cost of crime and search. I suppose that search is costly per se, and search that reveals a crime entails costs for punishment of offenders. Search is beneficial to the extent that it deters or prevents crime. Deterrence is expressed through the *offense function*, which describes how the offense rate of persons with given covariates varies with the search rate applied to these persons. Prevention occurs when search prevents an offense from causing social harm. Drawing on Manski (2005b), I use the analytically simple cases of no deterrence and linear deterrence to illustrate how the optimal search rate may depend on the cost parameters and offense function.

Section 3 examines the planning problem when the planner has only partial knowledge of the offense function and, hence, is unable to determine what policy is optimal. To demonstrate general ideas, I consider in depth a specific informational setting that may sometimes be realistic. I suppose that the planner observes the offense rates of a study population whose search rule has previously been chosen. He knows (or finds it credible to assume) that the study population and the population of interest have the same offense function. He also knows that search weakly deters crime; that is, the offense rate weakly decreases as the search rate increases. However, the planner does not know the magnitude of the deterrent effect of search.

In this setting, I first show how the planner can eliminate dominated search rules, ones which are inferior whatever the actual offense function may be. Broadly speaking, low (high) search rates are dominated when the cost of search is low (high); Lemmas 1 and 2 make this precise. I then show how the planner can use the minimax or minimax-regret criterion to choose an undominated search rule. Both criteria are reasonable and tractable, but they yield different policies; Lemmas 3 and 4 derive their explicit forms. The proofs of all lemmas are given in an Appendix.

Sections 2 and 3 consider *ex ante* search, which apprehends offenders before their offenses cause social harm. Section 4 performs parallel analysis for *ex post* search, which apprehends offenders after completion of their offenses. Whereas ex ante search both deters and prevents crime, ex post search only deters. The two types of search have different implications for profiling policy. Lemmas 5 through 8 formalize the analysis of ex post search.

The concluding Section 5 discusses some variations on the planning problems examined in Sections 2 through 4. Formal study of these variations would require fresh analysis. However, the general idea that a planner with partial knowledge of deterrence can eliminate dominated search rules and select a minimax or minimax-regret rule remains applicable.

Although detection of discrimination is not my direct concern, the analysis in this paper has implications for that inferential problem. The models studied in Knowles, Persico, and Todd (2001) and in Persico (2002) imply that, in the absence of discrimination, optimal profiling must equalize the offense rates of persons with different covariates, provided that such persons are searched at all. The present model differs from theirs, and it does not produce their conclusion. Perhaps the most important difference is in the objective functions assumed for the agencies that make profiling policy. They assumed that police on the street aim to maximize the probability of successful searches minus the cost of performing searches. I assume that a planner wants to minimize a social cost function with three components: (a) the harm caused by completed offenses, (b) the cost of punishing offenders who are apprehended, and (c) the cost of performing searches.

2. Optimal Ex-Ante Profiling

2.1. Basic Concepts

I suppose that there exists a large population of potential offenders—formally, the population is an uncountable probability space (J, Ω, P) with $P(j) = 0, j \in J$. Each member of this population decides whether or not to commit an offense, taking into account the chance that he will be searched. Let $t \in [0, 1]$ denote the probability with which a person is searched. Let $y_j(t) = 1$ if person j chooses to commit an offense when the search probability is t, with $y_j(t) = 0$ otherwise.

The planning problem is to choose the probabilities with which persons are searched. Let person j have observable *fixed* covariates $x_j \in X$, with X being the space of possible covariate values. It is important to my analysis that the planner use only fixed covariates to determine search rates. If search rates vary with *malleable* covariates, persons may choose to manipulate their covariates so as to lower the probability of search. I permit no such manipulation of covariates. See Brock (2005) and Persico and Todd (2005) for analyses of some profiling problems in which persons are able to manipulate their covariates.

I assume that it is legal to search differentially among persons with different values of x—if not, then redefine x to be those fixed covariates that the planner can observe and legally use. The planner can a priori distinguish persons with different observed covariates, but cannot distinguish among persons with the same covariates. Hence, a feasible search rule is a function $z: X \rightarrow [0, 1]$ that assigns one search rate z(x) to all persons with the same value of x, but possibly different search rates to persons with different covariates. I assume that offenders are always apprehended through search but are not apprehended otherwise. The analysis here and in Section 3 assumes that search is ex ante.

Let $p(t, x) \equiv P[y(t) = 1|x]$ be the offense function, giving the fraction of persons with covariates x who commit an offense when their search rate is t. Under search rule z, the offense rate among persons with covariates x is $p[z(x), x] = P\{y[z(x)] = 1|x\}$. I need not specify a particular model of offense behavior, but one may find it helpful to envision a threshold-crossing model in which $y_j(t) = 1$ if $t < \tau_j$ and $y_j(t) = 0$ if $t > \tau_j$, where τ_j is a person-specific threshold. Then $P(t < \tau|x) \le p(t, x) \le P(t \le \tau|x)$.

I suppose that the planner wants to minimize a social cost function with three additive components. These are (a) the harm caused by completed offenses, (b) the cost of punishing offenders who are apprehended, and (c) the cost of performing searches. I assume for simplicity that decisions to commit offenses are statistically independent of the harm that these offenses would cause. Then the social cost of search rule z is

$$(1) \quad S(z) = \int a(x) \cdot p[z(x), x] \cdot [1 - z(x)] dP(x) + \int b(x) \cdot p[z(x), x] \cdot z(x) dP(x) + \int c(x) \cdot z(x) dP(x).$$

Consider the first term on the right hand side. For each $x \in X$, $p[z(x), x]$ is the probability that a person with covariates x commits an offense and $1 - z(x)$ is the probability that such a person is not searched; hence, the product $p[z(x), x]\cdot[1 - z(x)]$ is the probability that a person with covariates x commits an offense that causes social harm. The positive constant $a(x)$ is the mean magnitude of the harm caused by an offense. Integrating across the covariate distribution $P(x)$ yields the aggregate social cost due to harm caused by completed offenses.

Next consider the second term. The product $p[z(x), x]\cdot z(x)$ is the probability that a person with covariates x commits an offense but is apprehended. The constant $b(x)$ is the mean net social cost of punishing the offender. I say “net” social cost because punishing an offender may have multiple cost components, not all of which need be positive. Positive social costs may be incurred for the prosecution of offenses, the incapacitation of convicted offenders, and the deleterious effects of punishment on offenders themselves. Negative social costs may arise to the extent that society views retribution for offenses as a social good. Again, integrating across $P(x)$ gives the aggregate social cost of punishing apprehended offenders.

The third term gives the aggregate cost of performing searches. The positive constant $c(x)$ is the mean cost of performing a search on a person with covariates x. The integral $\int c(x)\cdot z(x)dP(x)$ is the aggregate cost of performing searches.

The planner wants to solve the problem $\min_{z \in Z} S(z)$, where Z is the space of feasible search rules. This minimization problem is separable in x. For each $x \in X$, the optimal search rate for persons with covariates x is

$$(2) \quad z^*(x) \equiv \underset{t \in [0, 1]}{\operatorname{argmin}} \; a(x)\cdot p(t, x)\cdot(1 - t) + b(x)\cdot p(t, x)\cdot t + c(x)\cdot t.$$

Inspection of (2) shows that the planning problem gives multiple reasons for profiling. The cost parameters a(x), b(x), or c(x) may vary with x. So may the offense function p(·, x).

The analysis in this paper assumes that $a(x) > b(x) \geq 0$ and that $c(x) \geq 0$. The assumption that $b(x) \geq 0$ asserts that society does not value retribution so highly as to make punishment a net social good. The assumption that $a(x) > b(x)$ asserts that punishment is less costly to society than the harm caused by completed offenses. These assumptions differ considerably from those of Knowles, Persico, and Todd (2001) and of Persico (2002). Translating their model of police-initiated profiling into my notation for the social planning problem, they assumed that $a(x) = 0$ and $b(x) < 0$.

Social cost functions of the form S(·) can express a variety of social objectives through appropriate choice of the cost parameters. However, a law enforcement agency contemplating application of the findings of this paper should be aware that S(·) is utilitarian in character and, hence, does not express any social preference for cross-group equity in profiling. Discussions of racial profiling often verbally convey a preference for equity, but they typically do not formalize what this may mean. See Dominitz (2003) and Durlauf (2005) for efforts to clarify the alternative forms that a preference for equity might take.

2.2. Illustration: No Deterrence and Linear Deterrence

Manski (2005b) uses the analytically simple cases of no deterrence and linear deterrence to illustrate how the optimal search rate may depend on the cost parameters and offense function. One might think that search should focus on the most crime-prone segments of the population. However, I show that this is not necessarily the case if persons differ in the extent to which search deters crime. It may be optimal to search a less crime-prone group and not to search a more crime-prone group if members of the former group are deterrable and those in the latter group are not.

Let $\rho(x) \equiv p(0, x)$ denote the offense rate for persons with covariates x when their search rate is zero. Search does not deter if $p(t, x) = \rho(x)$, $t \in [0, 1]$ and deters linearly if $p(t, x) = \rho(x)\cdot(1 - t)$, $t \in [0, 1]$. In terms of the threshold-crossing model, linear deterrence means that a fraction of $1 - \rho(x)$ of persons with covariates x have negative thresholds and, hence, do not commit an offense even when the search rate is zero. The remaining fraction $\rho(x)$ have thresholds distributed uniformly on the interval [0, 1].

No Deterrence

Let $p(t, x) = \rho(x)$, $t \in [0, 1]$. Then the optimal search rate for persons with covariates x is

$$(3)\quad z^*(x) = \underset{t \in [0,1]}{\operatorname{argmin}}\ a(x)\cdot\rho(x)\cdot(1 - t) + b(x)\cdot\rho(x)\cdot t + c(x)\cdot t$$

$$= 0 \text{ if } c(x) > [a(x) - b(x)]\cdot\rho(x),$$

$$= 1 \text{ if } c(x) < [a(x) - b(x)]\cdot\rho(x).$$

Thus, the optimal search rate is either zero or one. Recall that $a(x) > b(x)$ by assumption. Hence, holding the cost parameters fixed, the optimal search rate is zero if the offense rate $\rho(x)$ is sufficiently small and is one otherwise.

Linear Deterrence

Let $p(t, x) = \rho(x)\cdot(1 - t)$. Then the optimal search rate is

$$(4)\quad z^*(x) = \underset{t \in [0,1]}{\operatorname{argmin}}\ a(x)\cdot\rho(x)\cdot(1 - t)^2 + b(x)\cdot\rho(x)\cdot t(1 - t) + c(x)\cdot t.$$

The cost parameters being non-negative, the quadratic first term of the social cost function is minimized at $t = 1$, the quadratic second term at $t = 0$ and $t = 1$, and the linear third term at $t = 0$. The optimal search rate must resolve this tension.

With $a(x) > b(x)$ and $\rho(x) > 0$, the social cost function has a unique minimum at the value of t that solves the first order condition

$$(5)\quad 0 = 2a(x)\cdot\rho(x)\cdot t - 2a(x)\cdot\rho(x) + b(x)\cdot\rho(x) - 2b(x)\cdot\rho(x)\cdot t + c(x)$$
$$= 2[a(x) - b(x)]\cdot\rho(x)\cdot t - [2a(x) - b(x)]\cdot\rho(x) + c(x).$$

Solving (5), the minimum is at

$$(6)\quad t^*(x) = \frac{[2a(x) - b(x)]\cdot\rho(x) - c(x)}{2[a(x) - b(x)]\cdot\rho(x)} .$$

Hence, the optimal search rate is

$$(7)\quad \begin{aligned} z^*(x) &= 0 && \text{if } t^* < 0, \\ &= t^*(x) && \text{if } 0 \le t^* \le 1, \\ &= 1 && \text{if } t^* > 1. \end{aligned}$$

Holding the cost parameters fixed, the optimal search rate $z^*(x)$ increases with the ceiling offense rate $\rho(x)$, and the offense rate implied by optimal search decreases with $\rho(x)$. The optimal search rate is zero if $\rho(x) \le c(x)/[2a(x) - b(x)]$, making the offense rate equal $\rho(x)$. It is optimal to search all persons with covariates x if $\rho(x) \ge c(x)/b(x)$, making the offense rate equal zero. In intermediate cases it is optimal to search with probability $t^*(x)$, which increases with $\rho(x)$. This makes the offense rate equal $[c(x) - b(x)\cdot\rho(x)]/\{2[a(x) - b(x)]\}$, which decreases with $\rho(x)$.

Ceteris paribus, the optimal search rate decreases as the cost parameter c(x) rises. However, $z^*(x)$ varies nonmonotonically with a(x) and b(x). The more complex relationship of optimal search to these cost parameters stems from the fact that the social cost term $b(x)\cdot\rho(x)\cdot t(1 - t)$ is not monotone in t. This term takes the value 0 when t = 0, rises to $b(x)\cdot\rho(x)/4$ as t increases to ½, and then falls back to 0 as t further increases to 1.

Profiling and Deterrence

To appreciate how deterrence affects profiling policy, it is illuminating to compare the optimal search rates for two groups of persons who are identical in all respects except that search deters one group linearly and does not deter the other. Let ξ and x be distinct elements of X and suppose that persons with these covariate values have identical values of the cost parameters; say $a(\xi) = a(x) = a^*$, $b(\xi) = b(x) = b^*$, and $c(\xi) = c(x) = c^*$. However, the two groups of persons respond differently to search, with $p(t, \xi) = \rho(\xi)$ for all values of t and $p(t, x) = \rho(x)\cdot(1 - t)$.

Suppose that $(a^* - b^*)\rho(\xi) < c^* < (2a^* - b^*)\rho(x)$. Then (3) and (7) show that the optimal search rate for persons with covariates x is positive but the optimal rate for those with covariates ξ is zero. This can occur even if $\rho(\xi) > \rho(x)$, in which case persons with covariates x are strictly less prone to criminality than are those with covariates ξ. Thus, it may be optimal to search a less crime-prone group and not to search a more crime-prone group if members of the former group are deterrable and those in the latter group are not.

3. Partial Knowledge of Deterrence

3.1. Empirical Evidence and Credible Assumptions

Solution of the planning problem of Section 2 requires essentially complete knowledge of the offense function $p(\cdot, \cdot)$. However, this knowledge generally is unavailable in practice. Empirical evidence and credible assumptions may restrict the form of the offense function, but they rarely if ever pin it down fully.

To demonstrate how a planner with partial knowledge of deterrence may choose a profiling policy, I consider decision making in a particular informational setting. I suppose that the planner observes the offense rates of a study population whose search rule has previously been chosen. He thinks it credible to assume that the study population and the population of interest have the same offense function. He also thinks it credible to assume that the offense rate weakly decreases as the search rate increases. However, he does not know anything about the magnitude of the deterrent effect of search.

Let r(x) denote the search rate applied to persons with covariates x in the study population and let q(x) denote the realized offense rate of these persons. Formally, I maintain

Assumption 1 (Study Population): The planner observes $[r(x), q(x)], x \in X$. The planner knows that $q(x) = p[r(x), x], x \in X$.

Assumption 2(Search Weakly Deters Crime): The planner knows that, for each $x \in X$, p(t, x) is weakly decreasing in t.

Taken together, these assumptions imply that

$$(8)\quad t \le r(x) \;\Rightarrow\; p(t, x) \ge q(x),$$

$$t \ge r(x) \;\Rightarrow\; p(t, x) \le q(x).$$

Thus, the planner knows that $p(\cdot, x)$ is weakly decreasing and satisfies (8).

Assumptions 1 and 2 may be more realistic than the traditional economic assumption that the planner knows the offense function. A planner often observes the search and offense rates of a study population.

In particular, he may observe the past search and offense rates of his own jurisdiction or of another jurisdiction with a similar population. In such cases, it may be reasonable to suppose that the study population and the population of interest have (at least approximately) the same offense function.

It usually is reasonable to think that search weakly deters crime. In particular, this holds under the threshold-crossing model cited in Section 2, where $P(t < \tau | x) \le p(t, x) \le P(t \le \tau | x)$. Assumption 2 can hold even if members of the population do not correctly perceive the search rates that they face; it suffices that their perceptions vary monotonically with actual search rates. Assumption 2 is a specific instance of the general idea of *monotone treatment response* developed in Manski (1997).

Although I conjecture that the informational setting posed here may be realistic, I am aware of no research that describes how law enforcement agencies actually perceive the deterrent effect of search. What is clear is that social scientists have usually found it difficult to learn how policing and sanctions affect offense rates. For example, Blumstein, Cohen, and Nagin (1978) explains why it is so hard to establish the deterrent effect of capital punishment on murder. National Research Council (2001) summarizes the very limited information available on the effectiveness of criminal sanctions in deterring drug supply and use.

3.2. Dominated Search Rules

How should a planner use Assumptions 1 and 2 to choose a profiling policy? Although there is no one "correct" answer to this question, a planner clearly should not choose a search rule that is inferior whatever the actual offense function may be.

Let Γ denote the set of offense functions that are feasible under Assumptions 1 and 2. For $\gamma \in \Gamma$, let

$$(9) \quad S(z, \gamma) \equiv \int a(x) \cdot \gamma[z(x), x] \cdot [1 - z(x)] dP(x) + \int b(x) \cdot \gamma[z(x), x] \cdot z(x) dP(x) + \int c(x) \cdot z(x) dP(x)$$

be the social cost of search rule z when the offense function is γ. Rule z is *strictly dominated* if and only if there exists another search rule $z' \in Z$ such that $S(z, \gamma) > S(z', \gamma)$ for all $\gamma \in \Gamma$.

The planning problem is separable in x, so it suffices to consider each covariate value separately. Suppressing the symbol x to simplify the notation, let $t \in [0, 1]$ and $s \in [0, 1]$ designate feasible choices for the search rate. Let

$$(10) \quad d(t, s; \gamma) \equiv [a(1 - t) + bt]\gamma(t) + ct - [a(1 - s) + bs]\gamma(s) - cs$$

be the difference in social cost between application of search rates t and s when the offense function is γ. Search rate s is strictly dominated if and only if there exists a t such that $d(t, s; \gamma) < 0$ for all $\gamma \in \Gamma$. Let $D(t, s) \equiv \sup_{\gamma \in \Gamma} d(t, s; \gamma)$. An easily verifiable sufficient condition for strict dominance is that $D(t, s) < 0$.

Lemma 1 evaluates D(t, s). Then Lemma 2 uses the result to determine a set of dominated search rates.

Lemma 1: Let Assumptions 1 and 2 hold. Then

(i) $t < s \le r \Rightarrow D(t, s) = a(1 - t) + bt - aq(1 - s) - bqs + c(t - s)$,

(ii) $t < r < s \Rightarrow D(t, s) = a(1 - t) + bt + c(t - s)$,

(iii) $s \le t < r \Rightarrow D(t, s) = [c - q(a - b)](t - s)$,

(iv) $r \le t < s \Rightarrow D(t, s) = aq(1 - t) + bqt + c(t - s)$,

(v) $r \le s \le t \Rightarrow D(t, s) = c(t - s)$,

(vi) $s < r \le t \Rightarrow D(t, s) = [c - q(a - b)](t - s)$. □

Lemma 2: Let Assumptions 1 and 2 hold. Search rate s is strictly dominated if any of these conditions hold:

(a) Let $c < (a - b)q$. Then s is strictly dominated if $s < r$.

(b) Let $c > (a - b)q$. Then s is strictly dominated if $s > r + [aq(1 - r) + bqr]/c$.

(c) Let $c > a - b$. Then s is strictly dominated if $a(1 - q)/[c - q(a - b)] < s \leq r$ or if $s > \max(r, a/c)$. □

It might have been thought that Assumptions 1 and 2 are too weak to yield interesting dominance findings, but Lemma 2 shows that these assumptions have considerable power. The broad finding is that small (large) values of s are dominated when the search cost c is sufficiently small (large). Parts (a) through (c) give the specifics. Part (a) shows that search rates lower than the rate r observed in the study population are dominated if $c < (a - b)q$. Thus, when the cost parameters satisfy this inequality, the optimal search rate cannot be smaller than the observed rate r. Part (b) shows that search rates sufficiently larger than r are dominated when $c > (a - b)q$. Part (c) shows that some other search rates are dominated if $c > a - b$.

Choosing an Undominated Rule

Elimination of dominated search rules takes one part way toward solution of the planning problem. The literature on decision theory does not provide a consensus prescription for a complete solution, but it does offer various criteria that ensure choice of an undominated alternative. Particularly familiar to economists is Bayesian decision theory. In the present setting, this recommends that the planner place a subjective distribution on $p(\cdot, \cdot)$, say Ψ, and minimize subjective expected social cost. The social cost function (1) is linear in $p(\cdot, \cdot)$, so subjective expected social cost is

$$(11) \quad E_\Psi[S(z)] = \int a(x)\cdot\pi[z(x), x]\cdot[1 - z(x)]dP(x) + \int b(x)\cdot\pi[z(x), x]\cdot z(x)dP(x) + \int c(x)\cdot z(x)dP(x),$$

where $\pi(\cdot, x) \equiv E_\Psi[p(\cdot, x)]$ is the subjective mean of $p(\cdot, x)$. Thus, a Bayesian planner acts as a pseudooptimizer, using the subjective expected offense function π as if it were the actual offense function p.

The Bayesian prescription may be sensible if a planner can substantiate his choice of π, but pseudo-optimization has no special appeal otherwise. A planner who does not want to go the Bayesian route can reasonably apply the minimax or minimax-regret criterion to choose a search rule. These are general principles using whatever partial knowledge of the offense function the planner may have. Each criterion chooses a rule that, in one sense or another, performs uniformly well across all feasible offense functions.

Sections 3.3 and 3.4 describe the minimax and minimax-regret search rules under Assumptions 1 and 2. See Manski (2005a) for exposition of these criteria and for other applications to problems of social planning with partial knowledge of policy impacts.

3.3. Minimax Search

For each candidate search rule $z \in Z$, compute the maximum social cost that can occur across all feasible offense functions; that is, $\max_{\gamma \in \Gamma} S(z, \gamma)$. The minimax criterion selects the search rate that minimizes this maximum social cost. Thus, the minimax search rule solves the problem

$$(12) \quad \min_{z \in Z} \max_{\gamma \in \Gamma} S(z, \gamma).$$

The outer minimization problem is separable in x, so the search rate for persons with covariates x is

$$(13) \quad z^m(x) \equiv \operatorname*{argmin}_{t \in [0,1]} \max_{\gamma \in \Gamma} [a(x)\cdot(1 - t) + b(x)\cdot t]\cdot\gamma(t, x) + c(x)\cdot t.$$

Lemma 3 derives the search rate that solves this problem. I suppress the symbol x to simplify the notation.

Lemma 3: Under Assumptions 1 and 2, the minimax search rate is

$$
\begin{aligned}
(14)\quad z^m &= 0 && \text{if} \quad c \geq a - b \text{ and } a \leq aq(1 - r) + bqr + cr,\\
&= r && \text{if} \quad c \geq a - b \text{ and } a \geq aq(1 - r) + bqr + cr\\
& && \text{or if} \quad (a - b)q \leq c < a - b,\\
&= 1 && \text{if} \quad c \leq (a - b)q. \qquad \square
\end{aligned}
$$

Lemma 3 shows that the minimax search rate can take one of three values: 0, r, or 1. All else equal, the search rate weakly increases with the cost parameter a and decreases with b and c. It weakly increases with the realized offense rate q if $c < a - b$ and decreases with q otherwise.

3.3. Minimax-Regret Search

For each feasible offense function $\gamma \in \Gamma$, let $S^*(\gamma) \equiv \min_{z \in Z} S(z, \gamma)$ denote the smallest social cost achievable when the offense function is γ. The regret of search rule z in state of nature γ is $S(z, \gamma) - S^*(\gamma)$. Thus, regret measures the difference between the social cost delivered by rule z and that delivered by the best possible rule. The minimax-regret criterion selects the search rule that minimizes maximum regret across all states of nature. Thus, the minimax-regret search rule solves

$$(15)\quad \min_{z \in Z} \ \sup_{\gamma \in \Gamma} S(z, \gamma) - S^*(\gamma).$$

The outer minimization problem is separable in x, so the search rate for persons with covariates x is

(16) $z^{mr}(x)$

$$= \underset{t \in [0,1]}{\text{argmin}} \; \sup_{\gamma \in \Gamma} \Big\{ [a(x)\cdot(1-t) + b(x)\cdot t]\cdot\gamma(t, x) + c(x)\cdot t \; - \min_{s \in [0,1]} \; [a(x)\cdot(1-s) + b(x)\cdot s]\cdot\gamma(s, x) + c(x)\cdot s] \Big\}$$

$$= \underset{t \in [0,1]}{\text{argmin}} \; \sup_{\gamma \in \Gamma,\, s \in [0,1]} \; [a(x)\cdot(1-t) + b(x)\cdot t]\cdot\gamma(t, x) + c(x)\cdot t \; - \; [a(x)\cdot(1-s) + b(x)\cdot s]\cdot\gamma(s, x) - c(x)\cdot s$$

$$= \underset{t \in [0,1]}{\text{argmin}} \; \max_{s \in [0,1]} \; D(t, s; x),$$

where D(t, s; x) was defined in Section 3.2, with the notation x suppressed.

Derivation of an explicit expression for the minimax-regret search rate is not intrinsically difficult, but it is laborious to catalog how the rate depends on the cost parameters and the evidence observed in the study population. Lemma 4 performs the derivation for the special case where $b = 0$ and $c < aq$. In what follows, the notation x is again suppressed.

Lemma 4: Let $b = 0$ and $c < aq$. Under Assumptions 1 and 2, the minimax-regret search rate is

(17) $z^{mr} = (aq + cr)/(aq + c)$. □

Lemma 4 shows that, when $b = 0$ and $c < aq$, the minimax-regret search rate can take any value between r and 1, depending on the values of aq and c. In particular, the rate is r if $aq = 0$ and is 1 if $c = 0$. In contrast, the minimax search rate always equals 1 for these values of (a, b, c, q).

4. Ex Post Search

When search is ex post, the social cost function is given not by S(z) but rather by

(18) $S'(z) = \int a(x)\cdot p[z(x), x]dP(x) + \int b(x)\cdot p[z(x), x]\cdot z(x)dP(x) + \int c(x)\cdot z(x)dP(x).$

The difference between $S'(z)$ and $S(z)$ is that the probability of an offense causing social harm is now $p[z(x), x]$, whereas earlier it was $p[z(x), x]\cdot[1 - z(x)]$. This section succinctly repeats the analysis of Section 3, using $S(\cdot)'$ rather than $S(\cdot)$ as the social cost function.

4.1. Dominated Search Rules

As in Section 3, the planning problem is separable in x, so it suffices to consider each covariate value separately. Suppressing the symbol x to simplify the notation, let $t \in [0, 1]$ and $s \in [0, 1]$ designate feasible choices for the search rate. Let

(19) $d'(t, s; \gamma) \equiv (a + bt)\gamma(t) + ct - (a + bs)\gamma(s) - cs$

be the difference in social cost between application of search rates t and s when the offense function is γ. Let $D'(t, s) \equiv \sup_{\gamma \in \Gamma} d'(t, s; \gamma)$. A sufficient condition for search rate s to be strictly dominated is that there exists a t such that $D'(t, s) < 0$.

Lemma 5 evaluates $D'(t, s)$. Lemma 6 uses the result to determine dominated search rates.

Lemma 5: Let S' be the social cost function. Let Assumptions 1 and 2 hold. Then

(i) $t < s \leq r \Rightarrow D'(t, s) = a + bt - aq - bqs + c(t - s)$,

(ii) $t < r < s \Rightarrow D'(t, s) = a + bt + c(t - s)$,

(iii) $s \leq t < r \Rightarrow D'(t, s) = (b + c)(t - s)$,

(iv) $r \leq t < s \Rightarrow D'(t, s) = aq + bqt + c(t - s)$,

(v) $r \leq s \leq t \Rightarrow D'(t, s) = (bq + c)(t - s)$,

(vi) $s < r \leq t \Rightarrow D'(t, s) = (bq + c)(t - s)$. □

Lemma 6: Let S′ be the social cost function. Let Assumptions 1 and 2 hold. Search rate s is strictly dominated if any of these conditions hold: (a) $a(1 - q)/(c + bq) < s \leq r$; (b) $s > \max(r, a/c)$; (c) $s > r + (aq + bqr)/c$. □

4.2. Minimax Search

Lemma 7 repeats the derivation of Lemma 3. Whereas the minimax search rate for ex ante search could take the values {0, r, 1}, we find that it now can take only the values {0, r}. Moreover, the present minimax search rate is zero whenever the earlier minimax search rate is zero.

Lemma 7: Let S′ be the social cost function. Under Assumptions 1 and 2, the minimax search rate is

(20) $z^{m'} = 0$ if $a \leq aq + bqr + cr$,

$= r$ if $a \geq aq + bqr + cr$. □

4.3. Minimax-Regret Search

Lemma 8 repeats the derivation of Lemma 4. Whereas the minimax-regret search rate for ex ante search in the setting of Lemma 4 was a unique value between r and one, we find that in the corresponding setting of Lemma 8 it equals zero in some cases and takes any value between r and one in others.

Lemma 8: Let S′ be the social cost function. Let b = 0 and c < aq. Under Assumptions 1 and 2, the minimax-regret search rate is

$$(21)\quad z^{mr\prime} = 0 \qquad \text{if} \quad a \leq aq + cr,$$

$$= \text{all } t \geq r \qquad \text{if} \quad a > aq + cr. \qquad \square$$

5. Conclusion

This paper has shown how a planner who does not know the deterrent effect of search can use data from a study population to choose a reasonable search profiling policy. I say a "reasonable" rather than "optimal" policy because our planner does not have enough information to solve the optimization problem that he would like to solve. What he can do is eliminate dominated policies and use a well-defined criterion to choose among the undominated policies. Minimax and minimax-regret are two such criteria, although they certainly are not the only ones.

A law enforcement agency that observes an appropriate study population and that contemplates practical use of the findings reported here should first ask whether a utilitarian social cost function of form S(·) or S′(·) adequately expresses its objective. If so, the agency must decide what values to use for the cost parameters [a(x), b(x), c(x)], $x \in X$. This done, Lemmas 1 through 8 may be applied.

The informational setting posed in Assumptions 1 and 2 seems more realistic than the traditional economic assumption that planners know how policy affects population behavior. However, I can only conjecture that these assumptions reasonably approximate what law enforcement agencies actually know about the deterrent effect of search. Fresh analysis would be required for other informational settings, but the basic message of the paper would remain applicable. A planner can use whatever knowledge he has to eliminate dominated search rules and then use minimax, minimax-regret, or another criterion to choose an undominated rule.

Many structural variations on the planning problem warrant attention. It may be that search only sometimes apprehends offenders, rather than always as assumed here. A planner may be able to implement both ex ante and ex post search rules, rather than one or the other as assumed here. A planner may be able to choose not only a search rule but also the severity with which apprehended offenders are punished.

Offender behavior may differ from the assumptions maintained here. Offense decisions may have endogenous social interactions, each person's decision to commit an offense depending not only on the search rate that he faces but also on the prevalence of offenses within the population. It may be that the personal covariates that planners observe are malleable. When profiling makes search rates vary with malleable covariates, persons may choose to manipulate their covariate values to lower the probability of search.

Researchers contemplating analysis of endogenous social interactions and malleable covariates should be aware that these phenomena substantially complicate the determination of optimal, dominated, minimax, and minimax-regret search rules. Throughout this paper, the planning problem was separable across persons with different covariates, a fact that enormously simplified analysis. Endogenous social interactions and malleable covariates make the planning problem non-separable; hence, much more difficult to study.

Appendix: Proofs of the Lemmas

Lemma 1

(i) $t < s \leq r \Rightarrow \gamma(t) \geq \gamma(s) \geq q$. Maximization over Γ occurs by setting $\gamma(t) = 1$ and $\gamma(s) = q$. This gives $D(t, s) = a(1 - t) + bt + ct - a(1 - s)q - bsq - cs$.

(ii) $t < r < s \Rightarrow \gamma(t) \geq q \geq \gamma(s)$. Maximization over Γ occurs by setting $\gamma(t) = 1$ and $\gamma(s) = 0$. This gives $D(t, s) = a(1 - t) + bt + ct - cs$.

(iii) $s \leq t < r \Rightarrow \gamma(s) \geq \gamma(t) \geq q$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = \delta$ for some $\delta \geq q$. This gives $D(t, s) = [c - \delta(a - b)](t - s)$. Given that $a > b$ and $t \geq s$, the maximum is attained by setting $\delta = q$. Hence, $D(t, s) = [c - q(a - b)](t - s)$.

(iv) $r \leq t < s \Rightarrow q \geq \gamma(t) \geq \gamma(s)$. Maximization over Γ occurs by setting $\gamma(t) = q$ and $\gamma(s) = 0$. This gives $D(t, s) = a(1 - t)q + btq + ct - cs$.

(v) $r \leq s \leq t \Rightarrow q \geq \gamma(s) \geq \gamma(t)$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = \delta$ for some $\delta \leq q$. This gives $D(t, s) = [c - \delta(a - b)](t - s)$. Given that $a > b$ and $t \geq s$, the maximum is attained by setting $\delta = 0$. Hence, $D(t, s) = c(t - s)$.

(vi) $s < r \leq t \Rightarrow \gamma(s) \geq q \geq \gamma(t)$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = q$. This gives $D(t, s) = [c - q(a - b)](t - s)$.

Q. E. D.

Lemma 2

(a) Part (vi) of Lemma 1 showed that if $s < r \leq t$, then $D(t, s) = [c - q\cdot(a - b)]\cdot(t - s)$. Hence, $D(t, s) < 0$ for all such (t, s).

(b) Part (iv) of Lemma 1 showed that if $r \leq t < s$, then

$$D(t, s) = aq(1 - t) + bqt + c(t - s) = aq + [c - q(a - b)]t - cs.$$

Consider the right hand side as a function of t. The function is minimized at $t = r$, giving $D(r, s) = aq + [c - q(a - b)]r - cs$. If $s > r + [aq(1 - r) + bqr]/c$, then $D(r, s) < 0$.

(c) Part (i) of Lemma 1 showed that if $t < s \leq r$, then

$$D(t, s) = a(1 - t) + bt - aq(1 - s) - bqs + c(t - s) = a(1 - q) + [c - (a - b)]t - [c - q(a - b)]s.$$

Consider the right hand side as a function of t. The function is minimized at $t = 0$, giving $D(0, s) = a(1 - q) - [c - q(a - b)]s$. If $s > a(1 - q)/ [c - q(a - b)]$, then $D(0, s) < 0$.

Part (ii) of Lemma 1 showed that if $t < r < s$, then

$$D(t, s) = a(1 - t) + bt + c(t - s) = a + [c - (a - b)]t - cs.$$

Consider the right hand side as a function of t. The function is minimized at $t = 0$, giving $D(0, s) = a - cs$. If $s > a/c$, then $D(0, s) < 0$.

Q. E. D.

Lemma 3

For each value of t, the inner maximization problem in (12) is solved by setting the offense rate to its largest feasible value; that is, $\gamma(t) = 1[t < r] + q \cdot 1[t \geq r]$. Hence,

$$z^m \equiv \underset{t \in [0, 1]}{\text{argmin}} \quad [a(1-t) + bt] \cdot \{1[t < r] + q \cdot 1[t \geq r]\} + ct.$$

To solve this problem, I first consider the two domains $t < r$ and $t \geq r$ separately, and then combine them.

First let $t < r$. In this domain, the minimization problem is $\min_{t<r} \; a(1-t) + bt + ct$. If $c \geq a - b$, the solution is $t = 0$ and the minimax value is a. If $c < a - b$, the criterion function decreases as $t \to r$, with limit value $a(1-r) + br + cr$.

Next let $t \geq r$. In this domain, the minimization problem is $\min_{t \geq r} \; a(1-t)q + bqt + ct$. If $c \geq (a-b)q$, the solution is $t = r$ and the minimax value is $a(1-r)q + bqr + cr$. If $c < (a-b)q$, the solution is $t = 1$ and the minimax value is $bq + c$.

Now combine the two domains. If $c \geq a - b$, the solution on $t \in [0, 1]$ is $t = 0$ if $a \leq a(1-r)q + bqr + cr$ and is $t = r$ if $a \geq a(1-r)q + bqr + cr$. If $(a-b)q \leq c < a - b$, the solution is $t = r$. If $c < (a-b)q$, the solution is $t = 1$.

Q. E. D.

Lemma 4

Proof: With $b = 0$, the values of $D(t, s)$ derived in Lemma 1 become

$t < s \leq r \Rightarrow D(t, s) = a(1-t) - aq(1-s) + c(t-s),$

$t < r < s \Rightarrow D(t, s) = a(1-t) + c(t-s),$

$s \leq t < r \Rightarrow D(t, s) = (c - aq)(t-s),$

$r \le t < s \Rightarrow D(t, s) = aq(1 - t) + c(t - s)$,

$r \le s \le t \Rightarrow D(t, s) = c(t - s)$,

$s < r \le t \Rightarrow D(t, s) = (c - aq)(t - s)$.

Let $t < r$. I first fix t and maximize D(t, s) over $s \in [0, 1]$. There are three cases to consider:

(i) Given that $c < aq$, $\max_{s: t < s \le r} D(t, s)$ occurs at $s = r$, so $\max_{s: t < s \le r} D(t, s) = a(1 - t) - aq(1 - r) + c(t - r)$.

(ii) $\sup_{s: t < r < s} D(t, s)$ occurs at $s = r$, so $\sup_{s: t < r < s} D(t, s) = a(1 - t) + c(t - r)$.

(iii) $\max_{s \le t < r} D(t, s)$ occurs at $s = t$, so $\max_{s \le t < r} D(t, s) = 0$.

The supremum in case (ii) exceeds the maxima in (i) and (iii). Hence, $\sup_{s \in [0, 1]} D(t, s) = a(1 - t) + ct - cr$.

Minimization over $t < r$ of the expression $a(1 - t) + ct - cr$ yields the minimax-regret search rate within this restricted range of search rates. Given that $c < a$, it follows that $a(1 - t) + ct - cr$ decreases with t. Hence, $\inf_{t < r} \sup_{s \in [0, 1]} D(t, s) = a(1 - r)$.

Now let $t \ge r$. Again, I first fix t and maximize D(t, s) over $s \in [0, 1]$. There are three cases to consider:

(i) $\sup_{s: r \le t < s} D(t, s)$ occurs at $s = t$, so $\sup_{s: r \le t \le s} D(t, s) = aq(1 - t)$.

(ii) $\max_{s: r \le s \le t} D(t, s)$ occurs at $s = r$, so $\max_{s: r \le s \le t} D(t, s) = c(t - r)$.

(iii) $\max_{s: s < r \le t} D(t, s)$ occurs at $s = 0$, so $\max_{s: s < r \le t} D(t, s) = (c - aq)t$.

The supremum in case (iii) is non-positive. Hence, $\sup_{s \in [0, 1]} D(t, s) = \sup[aq(1 - t), c(t - r)]$.

Minimization over $t \ge r$ of $\sup[aq(1 - t), c(t - r)]$ yields the minimax-regret search rate within this restricted range of search rates. The expression $aq(1 - t)$ falls from $aq(1 - r)$ to 0 as t rises from r to 1. The expression $c(t - r)$ rises from 0 to $c(1 - r)$ as t rises from r to 1. Hence, $\sup[aq(1 - t), c(t - r)]$ is minimized when t solves the equation $aq(1 - t) = c(t - r)$; that is, when $t = (aq + cr)/(aq + c)$. Hence, $\min_{t \ge r} \sup_{s \in [0, 1]} D(t, s) = caq(1 - r)/(c + aq)$.

Finally, compare the minimax-regret values over the two ranges $t < r$ and $t \ge r$. The latter value is smaller than the former one. Hence, $(aq + cr)/(aq + c)$ is the overall minimax-regret search rate.

Q. E. D.

Lemma 5

There are six distinct cases:

(i) Let $t < s \le r$. Then $\gamma(t) \ge \gamma(s) \ge q$. Maximization over Γ occurs by setting $\gamma(t) = 1$ and $\gamma(s) = q$. This gives $D'(t, s) = a + bt + ct - aq - bsq - cs$.

(ii) Let $t < r < s$. Then $\gamma(t) \ge q \ge \gamma(s)$. Maximization over Γ occurs by setting $\gamma(t) = 1$ and $\gamma(s) = 0$. This gives $D'(t, s) = a + bt + ct - cs$.

(iii) Let $s \le t < r$. Then $\gamma(s) \ge \gamma(t) \ge q$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = \delta$ for some $\delta \ge q$. This gives $D'(t, s) = (b\delta + c)(t - s)$. Given that $t \ge s$, the maximum is attained by setting $\delta = 1$. This gives $D'(t, s) = (b + c)(t - s)$.

(iv) Let $r \le t < s$. Then $q \ge \gamma(t) \ge \gamma(s)$. Maximization over Γ occurs by setting $\gamma(t) = q$ and $\gamma(s) = 0$. This gives $D'(t, s) = aq + btq + ct - cs$.

(v) Let $r \le s \le t$. Then $q \ge \gamma(s) \ge \gamma(t)$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = \delta$ for some $\delta \le q$. Given that $t \ge s$, the maximum is attained by setting $\delta = q$. This gives $D'(t, s) = (bq + c)(t - s)$.

(vi) Let $s < r \le t$. Then $\gamma(s) \ge q \ge \gamma(t)$. Maximization over Γ occurs by setting $\gamma(t) = \gamma(s) = q$. This gives $D'(t, s) = (bq + c)(t - s)$.

Q. E. D.

Lemma 6

(a) Part (i) of Lemma 5 showed that if $t < s \leq r$, then

$$D'(t, s) = a + bt - aq - bqs + c(t - s) = a(1 - q) + (c + b)t - (c + bq)s.$$

Consider the right hand side as a function of t. The function is minimized at $t = 0$, giving $D'(0, s) = a(1 - q) - (c + bq)s$. If $s > a(1 - q)/ (c + bq)$, then $D'(0, s) < 0$.

(b) Part (ii) of Lemma 5 showed that if $t < r < s$, then

$$D'(t, s) = a + bt + c(t - s) = a + (c + b)t - cs.$$

Consider the right hand side as a function of t. The function is minimized at $t = 0$, giving $D'(0, s) = a - cs$. If $s > a/c$, then $D'(0, s) < 0$.

(c) Part (iv) of Lemma 5 showed that if $r \leq t < s$, then

$$D'(t, s) = aq + bqt + c(t - s) = aq + (c + bq)t - cs.$$

Consider the right hand side as a function of t. The function is minimized at $t = r$, giving $D'(r, s) = aq + cr + bqr - cs$. If $s > r + (aq + bqr)/c$, then $D'(r, s) < 0$.

Q. E. D.

Lemma 7

The minimax search rate is

$$z^{m'} \equiv \underset{t \in [0,1]}{\text{argmin}} \; \max_{\gamma \in \Gamma} \; (a + bt)\gamma(t) + ct.$$

The inner maximization problem is solved by setting the offense rate to $\gamma(t) = 1[t < r] + q \cdot 1[t \geq r]$. Hence,

$$z^{m'} \equiv \underset{t \in [0,1]}{\text{argmin}} \; (a + bt) \cdot \{1[t < r] + q \cdot 1[t \geq r]\} + ct.$$

To solve the latter problem, I consider the two domains $t < r$ and $t \geq r$ separately and then combine them.

First let $t < r$. The minimization problem is $\min_{t<r} a + bt + ct$. The solution is $t = 0$ and the minimax value is a.

Next let $t \geq r$. The minimization problem is $\min_{t \geq r} aq + bqt + ct$. The solution is $t = r$ and the minimax value is $aq + bqr + cr$.

Hence, the solution over $t \in [0, 1]$ is $t = 0$ if $a \leq aq + bqr + cr$ and is $t = r$ if $a \geq aq + bqr + cr$.

Q. E. D.

Lemma 8

With $b = 0$, the quantity $D'(t, s)$ defined in Lemma 5 becomes

$t < s \leq r \Rightarrow D'(t, s) = a(1 - q) + c(t - s)$,

$t < r < s \Rightarrow D'(t, s) = a + c(t - s)$,

$s \leq t < r \Rightarrow D'(t, s) = c(t - s)$,

$r \leq t < s \Rightarrow D'(t, s) = aq + c(t - s)$,

$r \le s \le t \;\Rightarrow\; D'(t, s) = c(t - s)$,

$s < r \le t \;\Rightarrow\; D'(t, s) = c(t - s)$.

Consider $t < r$. I first fix t and maximize $D'(t, s)$ over $s \in [0, 1]$. There are three cases to consider:

(i) $\sup_{s:\, t < s \le r} D'(t, s)$ occurs at $s = t$ as $s \to t$, so $\sup_{s:\, t < s \le r} D'(t, s) = a(1 - q)$.

(ii) $\sup_{s:\, t < r < s} D'(t, s)$ occurs at $s = r$ as $s \to r$, so $\sup_{s:\, t < r < s} D'(t, s) = a + c(t - r)$.

(iii) $\max_{s \le t < r} D'(t, s)$ occurs at $s = 0$, so $\max_{s \le t < r} D'(t, s) = ct$.

Given that $c < aq$, the supremum in case (ii) exceeds those in cases (i) and (iii). Hence, $\max_{s \in [0, 1]} D'(t, s) = a + c(t - r)$.

Minimization over $t < r$ of the expression $a + c(t - r)$ yields the minimax-regret search rate within this restricted range of search rates. The minimum occurs at $t = 0$, yielding $\min_{t < r} \max_{s \in [0, 1]} D'(t, s) = a - cr$.

Now consider $t \ge r$. Again fix t and maximize $D'(t, s)$ over $s \in [0, 1]$. There are three cases to consider:

(i) $\sup_{s:\, r \le t < s} D'(t, s)$ occurs at $s = t$ as $s \to t$, so $\sup_{s:\, r \le t \le s} D'(t, s) = aq$.

(ii) $\max_{s:\, r \le s \le t} D'(t, s)$ occurs at $s = r$, so $\max_{s:\, r \le s \le t} D'(t, s) = c(t - r)$.

(iii) $\max_{s:\, s < r \le t} D'(t, s)$ occurs at $s = 0$, so $\sup_{s:\, s < r \le t} D'(t, s) = ct$.

Given that $c < aq$, the supremum in case (i) exceeds those in cases (ii) and (iii). Hence, $\sup_{s \in [0, 1]} D'(t, s) = aq$ for all $t \ge r$.

Finally, compare the minimax-regret values over the two ranges $t < r$ and $t \ge r$. If $a - cr \le aq$, then the minimax-regret search rate is $t = 0$. If $a - cr > aq$, then all $t \ge r$ minimize maximum regret.

Q. E. D.

References

Blumstein, A., J. Cohen, and D. Nagin, editors (1978), *Deterrence and Incapacitation: Estimating the Effects of Criminal Sanctions on Crime Rates*, Washington, D.C.: National Academy Press.

Brock, W. (2005), "Profiling Problems with Partially Identified Structure," Department of Economics, University of Wisconsin-Madison.

Dominitz, J. (2003), "How Do the Laws of Probability Constrain Legislative and Judicial Efforts to Stop Racial Profiling?" *American Law and Economics Review*, 5, 412-432.

Durlauf, S. (2005), "Assessing Racial Profiling," Department of Economics, University of Wisconsin-Madison.

Knowles, J., N. Persico, and P. Todd (2001), "Racial Bias in Motor Vehicle Searches: Theory and Evidence," *Journal of Political Economy*, 109, 203-229.

Manski, C. (1997), "Monotone Treatment Response," *Econometrica*, 65, 1311-1334.

Manski, C. (2000), "Identification Problems and Decisions Under Ambiguity: Empirical Analysis of Treatment Response and Normative Analysis of Treatment Choice," *Journal of Econometrics*, 95, 415-442.

Manski, C. (2002), "Treatment Choice Under Ambiguity Induced by Inferential Problems," *Journal of Statistical Planning and Inference*, 105, 67−82.

Manski, C. (2004), "Statistical Treatment Rules for Heterogeneous Populations," *Econometrica*, 72, 1221-1246.

Manski, C. (2005a), *Social Choice with Partial Knowledge of Treatment Response*, Princeton: Princeton University Press.

Manski, C. (2005b), "Optimal Search Profiling with Linear Deterrence," *American Economic Review Papers and Proceedings*, 95, 122-126.

Mirrlees, J. (1971), "An Exploration in the Theory of Optimal Income Taxation," *Review of Economic Studies*, 38, 175-208.

National Research Council (2001), *Informing America's Policy on Illegal Drugs: What We Don't Know Keeps Hurting Us*, C. Manski, J. Pepper, and C. Petrie, editors. Washington, DC: National Academy Press.

Persico, N. (2002), "Racial Profiling, Fairness, and the Effectiveness of Policing," *American Economic Review*, 92, 1472-1497.

Persico, N. and P. Todd (2005), "Using Hit Rates to Test for Racial Bias in Law Enforcement: Vehicle Searches in Wichita," Department of Economics, University of Pennsylvania.

Polinsky, M. and S. Shavell (2000), "The Economic Theory of Public Enforcement of Law," *Journal of Economic Literature*," 38, 45-76.