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# CONTRACTS AS REFERENCE POINTS 

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#### Abstract

We argue that a contract provides a reference point for a trading relationship: more precisely, for parties' feelings of entitlement. A party's ex post performance depends on whether he gets what he is entitled to relative to outcomes permitted by the contract. A party who is shortchanged shades on performance. A flexible contract allows parties to adjust their outcome to uncertainty, but causes inefficient shading. Our analysis provides a basis for long-term contracts in the absence of noncontractible investments, and elucidates why "employment" contracts, which fix wage in advance and allow the employer to choose the task, can be optimal.

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## 1. Introduction

What is a contract? Why do people write (long-term) contracts? The classical view held by economists and lawyers is that a contract provides parties with a set of rights and obligations, and that these rights and obligations are useful, among other things, to encourage long-term investments. ${ }^{1}$ In this paper we present an alternative, and complementary, view. We argue that a contract provides a reference point for the parties' trading relationship: more precisely for their feelings of entitlement. We develop a model in which a party's ex post performance depends on whether the party gets what he is entitled to relative to the outcomes permitted by the contract. A party who is shortchanged shades on performance, which causes a deadweight loss. One way the parties can reduce this deadweight loss is for them to write an ex ante contract that pins down future outcomes very precisely, and that therefore leaves little room for disagreement and aggrievement. The drawback of such a contract is that it does not allow the parties to adjust the outcome to the state of the world. We study the trade-off between rigidity and flexibility. Our analysis provides a basis for long-term contracts in the absence of noncontractible relationshipspecific investments, and throws light on why simple "employment" contracts are sometimes optimal.

To motivate our work, it is useful to relate it to the literature on incomplete contracts. A typical model in that literature goes as follows. A buyer and seller meet initially. Since the future is hard to anticipate, they write an incomplete contract. As time passes and uncertainty is resolved, the parties can and do renegotiate their contract, in a Coasian fashion, to generate an ex post efficient outcome. However, as a consequence of this renegotiation, each party shares some of the benefits of prior (noncontractible) relationship-specific investments with the other party.

[^0]Recognizing this, each party underinvests ex ante. The literature studies how the allocation of asset ownership and formal control rights can reduce this underinvestment. ${ }^{2}$

While the above literature has generated some useful insights about firm boundaries, it has some shortcomings. ${ }^{3}$ Two that seem particularly important to us are the following. First, the emphasis on noncontractible ex ante investments seems overplayed: although such investments are surely important, it is hard to believe that they are the sole drivers of organizational form. Second, and related, the approach is ill-suited for studying the internal organization of firms, a topic of great interest and importance. The reason is that the Coasian renegotiation perspective suggests that the relevant parties will sit down together ex post and bargain to an efficient outcome using side payments: given this, it is hard to see why authority, hierarchy, delegation, or indeed anything apart from asset ownership matters.

We believe that in order to develop more general and compelling theories of organizational form it is essential to depart from a world in which Coasian renegotiation always leads to ex post efficiency. ${ }^{4}$ The purpose of our paper is to move in this direction. To achieve this goal we depart from the existing literature in two key ways. First, we drop the assumption made in almost all of the literature that ex post trade is perfectly contractible. Instead we suppose that trade is only partially contractible. ${ }^{5}$ Specifically, we distinguish between perfunctory performance and consummate performance, or performance within the letter of the contract and performance within the spirit of the contract. ${ }^{6}$ Perfunctory performance can be

[^1]judicially enforced, while consummate performance cannot. ${ }^{7}$ Second, we introduce some important behavioral elements. We suppose that a party is happy to provide consummate performance if he feels that he is getting what he is entitled to, but will withhold some part of consummate performance if he is shortchanged-we refer to this as "shading." An important assumption we will make (for most of the paper) is that a party's sense of entitlement is determined by the contract he has written. A companion assumption, also significant, is that the contract in question is negotiated under relatively competitive conditions. A final element of the story is that there is no reason why parties' feelings of entitlement should be consistent. In particular, when the contract permits more than one outcome, each party may feel entitled to a different outcome.

These ingredients yield the above-described trade-off between flexibility and rigidity. A flexible contract has the advantage that parties can adjust the outcome to the state of the world, but the disadvantage that any outcome selected will cause at least one party to feel aggrieved and shortchanged, which leads to a loss of surplus from shading. An optimal contract trades off these two effects. Our theory explains not only why parties will write somewhat rigid contracts, but also the nature of the rigidity. The parties are more likely to put restrictions on variables over which there is an extreme conflict of interest, such as price, than on variables over which there is some alignment of interest, such as the nature or characteristics of the good to be traded. Among other things, our model shows why simple employment contracts, which fix price (wage) in advance and allow the employer to choose the task, may be optimal. More generally, the model explains why the wage should vary with the task if some tasks are systematically costlier than others.

[^2]For most of the paper we suppose that parties' feelings of entitlement are controlled entirely by the contract they have written. In reality other influences on entitlements are sometimes important. For example, parties may look to related transactions to determine whether they are being fairly treated. This consideration allows for a rich new set of possibilities; we examine these briefly in Section 5. Although our analysis is preliminary, we show that external measures of entitlement can interfere with an ex ante contract, and that it may therefore be optimal for the parties to postpone contracting, i.e., the optimal ex ante contract may be "no contract."

The paper is organized as follows. Section 2 presents the model and discusses the two key assumptions of partial contractibility and shading. In Section 3 we analyze a case where there is uncertainty about value and cost but not about the type of good to be traded. In Section 4 we consider a second case where there is uncertainty about the nature of the good. Section 5 allows for the possibility of influences on entitlements other than the initial contract. In Section 6 we discuss renegotiation. Finally, Section 7 concludes. The Appendix considers a more general class of contracts than those studied in the text and includes proofs of theorems.

## 2. The Model

We consider a buyer $B$ and a seller $S$ who are engaged in a long-term relationship. The parties meet at date 0 and can trade at date 1 . We assume a perfectly competitive market for buyers and sellers at date 0 , but that competition is much reduced at date 1 : in fact, for the most part we suppose that B and S face bilateral monopoly at date 1 . In other words, there is a "fundamental transformation" in the sense of Williamson (1985).

We do not model why this fundamental transformation occurs. It could be because the parties make relationship-specific investments, but there may be other more prosaic reasons. For example, imagine that B is organizing a wedding for his daughter. S might be a caterer. Six months before the wedding, say, there may be many caterers that B can approach and many weddings that S can cater. But it may be very hard for B or S to find alternative partners a week before the wedding. While there are no very obvious relationship-specific investments here, the fundamental transformation seems realistic, and the model applies.

It would be easy to fit relationship-specific investments explicitly into the analysis, but we would then suppose that these investments are contractible. That is, an important feature of our model is that it does not rely on noncontractible investments.

We make some standard assumptions. Any uncertainty at date 0 is resolved at date 1 . There is symmetric information throughout, and the parties are risk neutral and face no wealth constraints.

We now come to the two assumptions that represent significant departures from the literature. First we suppose that ex post trade is only partially contractible. Specifically, while the broad outlines of ex post trade are contractible, the finer points are not. As noted in the Introduction, we distinguish between perfunctory and consummate performance, or performance within the letter of the contract and performance within the spirit of the contract. Perfunctory performance is enforceable by a court while consummate performance can never be judicially enforced.

For instance, in the wedding example, a judge can determine whether food was provided, but not the quality of the cake or whether the catering staff was friendly to the guests.

Before we describe our second (set of) assumption(s), it is useful to provide a time line; see Figure 1. The parties meet and contract at date 0 . At this stage there may be uncertainty and so the parties typically choose to write a flexible contract that admits several outcomes. At date 1 the uncertainty is resolved and the parties refine the contract, that is, they decide which outcome to pick. After this, trade occurs and the degree of consummate performance is determined. ${ }^{8}$

| $\quad 0$ | 1 |
| :--- | :--- |
| Parties meet and <br> contract. | Contract refined after <br> resolution of uncertainty. <br> Trade occurs and degree of <br> consummate performance <br> determined. |

Figure 1

We now come to our second key departure from the literature. We make a number of assumptions -- some behavioral -- about the determinants of consummate performance. First, we suppose that consummate performance does not cost significantly more than perfunctory performance: either it costs slightly more or it may even cost slightly less, that is, a party may actually enjoy providing consummate performance. Either way, a party is roughly indifferent between providing consummate and perfunctory performance.

Given this approximate indifference we take the view that a party will be willing to provide consummate performance if he is "well treated," but not if he is "badly treated." The

[^3]idea is that either consummate performance is costly but a party is naturally (slightly) altruistic, and is prepared to incur the cost if he is well treated; or that consummate performance is pleasurable, but a party is willing to forego this pleasure to punish a partner who did not treat him generously. ${ }^{9}$

We make the crucial assumption that a party is "well treated" if and only if he receives what he is entitled to; and that the date 0 contract acts as a reference point for entitlements. In fact, for most of the paper we suppose that the contract is the sole reference point for entitlements (but see Section 5). What we mean by this is that a party does not feel entitled to more than the best outcome permitted by the contract. So, for example, if the date 0 contract specifies just one outcome, then each party will feel that he is getting exactly what he is entitled to if that outcome occurs. ${ }^{10}$ We discuss the assumption that the contract acts as a reference point further below. As we shall make clear, this assumption is linked to a companion assumption that the contract is negotiated under relatively competitive conditions. ${ }^{11}$

[^4]Matters become more complicated if the contract specifies more than one outcome. Now we take the view that the parties may no longer agree about what they are entitled to. In particular, if the contract says that either outcome $a$ or outcome $b$ can occur, then one party may feel entitled to a and the other to b . We do not model why these differences in entitlements arise, but we have in mind the kinds of effects described in the self-serving bias literature. ${ }^{12}$ For example, each party may feel that he has taken an action between dates 0 and 1 that has contributed to the trading opportunity available at date 1 , and that he deserves to be rewarded for this. Virtue is in the eye of the beholder and so each party may exaggerate his own contribution.

To capture conflict as simply as possible we suppose that each party feels entitled to the best outcome permitted by the contract. However, our analysis does not depend on such an extreme view of entitlements. ${ }^{13}$ Obviously, opening the black box of entitlements and self-serving biases is an important topic for future research. ${ }^{14}$

The final piece of the story is that getting less than what you are entitled to causes aggrievement and leads to retaliation and "shading." Specifically, for each dollar that a party is shortchanged he shades his performance so that the other party's payoff falls by $\$ \theta$, where $0<\theta$ $\leq 1$. We take $\theta$ to be exogenous - it represents both the desire and opportunity for retaliation $(\theta$ might reflect the probability that a retaliation opportunity arises) - but in future work it would be interesting to endogenize it.

[^5]Shading decisions are made simultaneously by B and S, and are not observable to an outsider. Hence they are not contractible, even at date 1 (recall our assumption that consummate performance can never be enforced judicially). Shading decisions are, however, anticipated by both parties, i.e., the parties have rational expectations. Finally, we suppose that shading is infeasible if the parties do not trade at date 1 (trade can be shaded but no trade cannot be shaded).

It is worth saying a bit more about the nature of shading. There are many ways one trading partner can hurt another. For example, a seller can shade by cutting quality, e.g., in the wedding example, she can stint on some of the ingredients of the wedding cake (more colorfully, she can spit in the soup). Another example is withholding cooperation. The buyer may want the seller to turn up half an hour early. The cost to the seller may be low, and she would normally oblige. But if the seller feels aggrieved she may refuse this request at considerable cost to the buyer. A third example would be "working to rule": the seller abides by the strict terms of the contract and offers no more. ${ }^{15}$

Shading is not confined to a seller. While it is harder to imagine a buyer cutting back on quality, it is easy to think of situations where a buyer refuses to make minor concessions or to cooperate (for example, the buyer may turn down the seller's request to come half an hour later). The buyer can also make life difficult for the seller by quibbling about details of performance. Thus, while the assumption that B and S can shade symmetrically and face the same parameter $\theta$ is strong, we view it as a natural starting point for our analysis. In Section 7 we discuss alternative interpretations and extensions of the model for the case where shading is asymmetric.

[^6]In sum we appeal to three key ideas from the behavioral literature: reference points and entitlements, self-serving biases, and reciprocity and retaliation.

At this point, it is useful to illustrate the model with a simple example. Suppose that B requires one unit of a standard good - a widget - from $S$ at date 1 . Assume that it is known at date 0 that B's value is 100 and S's cost is zero: there is no uncertainty. What is the optimal contract?

The answer found in the standard literature is that, in this setting without noncontractible investments, no ex ante contract is necessary: the parties can wait until date 1 to contract. To review the argument, imagine that the parties do wait until date 1. Assume that Nash bargaining occurs and they divide the surplus $50: 50$, i.e., the price $\mathrm{p}=50$. Of course, a

50 : 50 division may not represent the competitive conditions at date 0 . For simplicity, suppose that there is one buyer and many sellers at date 0 , so that in competitive equilibrium $B$ receives all the surplus. Then $S$ will make a lump-sum payment of 50 to $B$ at date 0 : in effect $S$ pays $B$ up front for the privilege of being able to hold $B$ up once the parties are in a situation of bilateral monopoly.

This "no contract" solution, combined with a lump-sum payment, no longer works in our context. To see why, suppose for the moment that "no contract" means that trade can occur at any price between zero and 100 (we revisit this assumption in Section 5). (Prices above 100 are irrelevant since $B$ will reject the widget and prices below zero are irrelevant since $S$ will refuse to supply.) But this means that when the parties reach date 1 there is much to argue about. ${ }^{16}$ The best contractual outcome possible for B is a zero price and our assumption is that he will feel

[^7]entitled to it; and the best contractual outcome possible for S is a price of 100 and our assumption is that she will feel entitled to it. In spite of these conflicting feelings of entitlement, the parties will settle on some price p between 0 and 100, and trade will occur. However, each party will feel aggrieved and will shade. Since B feels shortchanged by $p$, he shades by $\theta$ p, and since $S$ feels shortchanged by $(100-p)$, she shades by $\theta(100-p)$. Thus the final payoffs are
(2.1) $\mathrm{U}_{\mathrm{B}}=100-\mathrm{p}-\theta(100-\mathrm{p})=(1-\theta)(100-\mathrm{p})$,
(2.2) $\mathrm{U}_{\mathrm{S}}=\mathrm{p}-\theta \mathrm{p}=(1-\theta) \mathrm{p}$,
and total surplus is given by
(2.3) $\mathrm{W}=(1-\theta) 100$.

We see that, independent of p , there is a loss of $100 \theta \cdot{ }^{17}$
What can be done to eliminate this loss? The first point to note is that ex post Coasian renegotiation at date 1 does not do the job. The reason is that shading is not contractible, and thus a contract not to shade is not enforceable. To put it another way, if B offers to pay S more not to shade, then while this will indeed reduce $S$ 's shading since $S$ will feel less aggrieved, it will increase B's shading because B will feel more aggrieved! In fact, it is clear from (2.3) that changes in p do not affect aggregate shading, which is given by $100 \theta$.

[^8]Note that the conclusion that the loss from shading equals $100 \theta$ remains true even if the parties replace renegotiation at date 1 by a mechanism. For example, suppose B and $S$ agree at date 0 that B will make a take-it-or-leave-it offer to S at date 1 . The best offer for B to make is p $=0$. However, $S$ will feel that $B$ could and should have offered $p=100$ since $S$ is entitled to this. Thus S will be aggrieved by 100 , and will shade by $100 \theta$. Hence the loss from shading is again $100 \theta$.

Although these approaches do not work, there is a very simple solution to the shading problem. The parties can write a contract at date 0 that fixes p at some level between 0 and 100 , e.g., if there are many sellers and only one buyer at date 0 , then it would be natural to set $\mathrm{p}=0$. Then there is nothing to argue about at date 1 . Neither party will feel aggrieved or shortchanged since each receives exactly what he or she bargained for and expected. Hence no shading occurs and total surplus equals $100 .{ }^{18}$

As we have argued earlier, a contract that fixes price works because it anchors the parties' expectations and feelings of entitlement: the contract is a reference point. An obvious question to ask is, what changes between dates 0 and 1 ? Why does a date 0 contract that fixes p $=0$ avoid aggrievement by $S$, whereas a date 1 contract that fixes $p=0$ does not? Our view is that the ex ante market plays a crucial role here. Since the date 0 market is more competitive than the date 1 market - for simplicity we have supposed that the date 0 market is perfectly competitive while the date 1 market is perfectly noncompetitive - it provides an external, i.e., objective, measure of what $B$ and $S$ bring to the relationship. $S$ accepts that $p=0$ is a reasonable price for the transaction because there are many other (identical) suppliers at date 0 who are

[^9]prepared to supply at this price. Our assumption is that B and S continue to accept the external measures provided by the date 0 market, now embodied in their contract, once their relationship is underway. ${ }^{19}$ In contrast, if B and S pass up the opportunity to write a contract at date 0 , then by the time date 1 arrives there are no external measures to control entitlements, and the result will be argument, aggrievement, and shading. ${ }^{20}$

The example analyzed in this section is very special because a date 0 contract that fixes price achieves the first-best. The first-best is no longer achievable if either (a) v, $c$ are uncertain; or (b) the nature of the good (the widget) is uncertain. We study case (a) in Section 3 and case (b) in Section 4.

## 3. The Case Where Value and Cost Are Uncertain

In this section we consider the case where $B$ wants one unit of a standard good - a widget - from S at date 1 but there is uncertainty about B's value v and S 's cost c . This uncertainty is resolved at date 1. There is symmetric information throughout, so that $\mathrm{v}, \mathrm{c}$ are observable to both parties. However, v, c are not verifiable, and so state-contingent contracts cannot be written.

We make an important simplifying assumption. We suppose that trade occurs at date 1 if and only if both parties want it, i.e., trade is voluntary. To put it another way, if no trade occurs an outsider (e.g., a judge) cannot tell whether this is because the seller refused to supply the

[^10]widget or the buyer refused to accept it. ${ }^{21}$ As a result, a party cannot be punished for breach of contract. We are confident that the main ideas of this section generalize to the case where specific performance is possible, but the details become more complicated.

Note that the model can also be interpreted as applying to the case where the parties write an "agreement to agree" at date $0 .{ }^{22}$ That is, suppose that the parties intend to use the date 0 agreement as a framework for future negotiation (this corresponds to the refinement process in Figure 1), but for some reason are not yet ready to sign a binding contract. The usual legal presumption is that either party can opt out of such an agreement if future negotiations fail. ${ }^{23}$ Thus the voluntary trade assumption holds.

In this setting the simplest kind of contract consists of a no-trade price $\mathrm{p}_{0}$ and a trade price $\mathrm{p}_{1}$. Given the voluntary trade assumption, trade will occur $(\mathrm{q}=1)$ if and only if

$$
\begin{equation*}
\mathrm{v} \geq \mathrm{p}_{1}-\mathrm{p}_{0} \geq \mathrm{c} \tag{3.1}
\end{equation*}
$$

From (3.1) it is clear that only the difference between $\mathrm{p}_{1}$ and $\mathrm{p}_{0}$ matters, and so, given the existence of lump-sum transfers, we can normalize $p_{0}$ to be zero. ${ }^{24}$

It is worth comparing (3.1) to the first-best trading rule, given by
(3.2) $\mathrm{q}=1 \Leftrightarrow \mathrm{v} \geq \mathrm{c}$.

[^11]We need to deal with one further issue before we proceed. After the uncertainty about v, c is resolved, suppose $\mathrm{v}>\mathrm{c}$ but either $\mathrm{v}<\mathrm{p}_{1}-\mathrm{p}_{0}$ or $\mathrm{c}>\mathrm{p}_{1}-\mathrm{p}_{0}$. At this stage, the parties might want to renegotiate their contract. Renegotiation does not fundamentally change our results and so, for the moment, we ignore it; we return to it in Section 6.

Since we want to allow for contractual flexibility we shall wish to generalize beyond simple contracts. One way to introduce flexibility is to suppose that the contract specifies a notrade price $\mathrm{p}_{0}$ and an interval of trading prices $[\mathrm{p}, \overline{\mathrm{p}}]$. Suppose for simplicity that B chooses the trade price at date 1. Then
(3.3) $\mathrm{q}=1 \Leftrightarrow \exists \mathrm{p} \leq \mathrm{p}_{1} \leq \overline{\mathrm{p}}$ s.t. $\mathrm{v} \geq \mathrm{p}_{1}-\mathrm{p}_{0} \geq \mathrm{c}$.

In other words, trade occurs if and only if B can find a price in the range [p, $\bar{p}]$ so that the parties want to trade ( B will choose the lowest such price). Actually, it is clear that the same trading rule (3.3) holds if $S$ chooses $p_{1}(S$ will choose the highest price in the range $[\mathrm{p}, \overline{\mathrm{p}}]$ that guarantees trade); moreover, the level of shading will be the same given that the parties have the same $\theta$. This feature - that the mechanism for choosing the outcome doesn't matter - is special to the model of this section: it will importantly not hold in the model of Section 4.

It follows from (3.3) that, again, only the difference between $\mathrm{p}_{1}$ and $\mathrm{p}_{0}$ matters, and so we can normalize $\mathrm{p}_{0} \equiv 0$ and rewrite (3.3) as

$$
\begin{align*}
q=1 & \Leftrightarrow \exists \mathrm{p} \leq \mathrm{p} \leq \overline{\mathrm{p}} \text { s.t. } \mathrm{v} \geq \mathrm{p} \geq \mathrm{c}  \tag{3.4}\\
& \Leftrightarrow \mathrm{v} \geq \mathrm{c}, \mathrm{v} \geq \mathrm{p}, \mathrm{c} \leq \overline{\mathrm{p}} .
\end{align*}
$$

More general contracts than $\mathrm{p}_{0} \equiv 0, \mathrm{p} \varepsilon[\mathrm{p}, \overline{\mathrm{p}}]$ are in fact possible. For example, a contract could permit p to lie in some set other than an interval, or it could allow $\mathrm{p}_{0}$ and $\mathrm{p}_{1}$ both to vary. In the Appendix we show that our main ideas extend to the case where a contract consists of an arbitrary set of $\left(p_{0}, p_{1}\right)$ pairs and a mechanism - a game - for choosing among them. In order to carry out this generalization, we need to refine slightly our assumptions about the determinants of a party's aggrievement level.

Given a contract $[\mathrm{p}, \overline{\mathrm{p}}]$ (that is, $\mathrm{p}_{0} \equiv 0, \mathrm{p} \varepsilon[\mathrm{p}, \overline{\mathrm{p}}]$ ), what determines aggrievement? Our hypothesis is that each party feels entitled to the best outcome permitted by the contract. However, each party also recognizes that, given the voluntary trade assumption, he or she cannot hope to obtain more than one hundred percent of the gains from trade. This means that S feels entitled to $p=\operatorname{Min}(v, \bar{p})$ and $B$ feels entitled to $p=\operatorname{Max}(c, p)$. (Another way to think about it is that if S had complete control over the price but had to stick within the contract she would choose $p=\operatorname{Min}(v, \bar{p})$, and if B had complete control over the price but had to stick within the contract he would choose $\mathrm{p}=\operatorname{Max}(\mathrm{c}, \mathrm{p})$.$) Thus aggregate aggrievement equals \{\operatorname{Min}(\mathrm{v}, \overline{\mathrm{p}})-$ Max (c, p) \}. An optimal contract maximizes expected surplus net of shading costs. (Lump-sum transfers are used to reallocate surplus.) Thus an optimal contract solves:

```
(3.5) \(\underset{\underline{p}, \overline{\mathrm{p}}}{\operatorname{Max}} \underset{\mathrm{v} \geq \mathrm{c}}{\int}[\mathrm{v}-\mathrm{c}-\theta\{\operatorname{Min}(\mathrm{v}, \overline{\mathrm{p}})-\operatorname{Max}(\mathrm{c}, \mathrm{p})\}] \mathrm{dF}(\mathrm{v}, \mathrm{c})\),
    \(\mathrm{v} \geq \underline{p}\)
    \(\mathrm{c} \leq \overline{\mathrm{p}}\)
```

where $F$ is the distribution function of $(v, c)$.
The trade-off is clear. A large interval [ $\mathrm{p}, \overline{\mathrm{p}}$ ] makes it more likely that trade will occur if $v \geq c$. (If $p=-\infty, \bar{p}=\infty$, the trading rule becomes the first-best one: $q=1 \Leftrightarrow v \geq c$.) However, it also increases expected shading costs.

We refer to a contract where $\mathrm{p}=\overline{\mathrm{p}}$ as a simple contract, and a contract where $\mathrm{p}<\overline{\mathrm{p}}$ as a non-simple contract. We start off with some cases where the first-best is achievable with a simple contract.

Proposition 3.1. A simple contract achieves the first-best if (i) only $\widetilde{v}$ varies; (ii) only $\widetilde{\mathrm{c}}$ varies; (iii) the smallest element of the support of $\widetilde{\mathrm{v}}$ is at least as great as the largest element of the support of $\widetilde{\mathrm{c}}$.

The proof of Proposition 3.1 is immediate. If only $\widetilde{\mathrm{v}}$ varies, choose a simple contract with $\mathrm{p}=\mathrm{c}$. If only $\widetilde{\mathrm{c}}$ varies, choose a simple contract with $\mathrm{p}=\mathrm{v}$. If (iii) holds, choose a simple contract with p between the smallest v and largest c .

In some cases one needs a non-simple contract to achieve the first-best.

## Example 3.1

Suppose that there are two states of the world. In $\mathrm{s} 1, \mathrm{v}=9, \mathrm{c}=0$. In $\mathrm{s} 2, \mathrm{v}=20, \mathrm{c}=10$. In other words, either v and c are both low or they are both high.

0
s2 20

10

Obviously, one cannot get the first-best with a simple contract since there is no price $p$ that lies both between 0 and 9 and between 10 and 20. However, a contract that specifies an interval of trading prices $[9,10](\mathrm{p}=9, \overline{\mathrm{p}}=10)$, with $B$ choosing the price, does achieve the firstbest. To see why, note that in s1 B will choose $\mathrm{p}=9$ since this is the lowest available price. S will not be aggrieved since, even if S could choose the price, she would not pick a price above 9 given that this would cause $B$ not to trade. In $s 2 B$ picks $p=10$ since this is the lowest price consistent with S being willing to trade. S is again not aggrieved since she couldn't hope for a higher price than 10 given that 10 is the highest available price. Thus, the contract $\mathrm{p}=9, \overline{\mathrm{p}}=10$ achieves trade in both states without any shading.

Note that in this example the optimal contract is unique. Any price range smaller than $[9,10]$ would fail to generate trade in one of the states, and any price range larger than $[9,10]$ would cause aggrievement in at least one of the states. (If $\mathfrak{p}<9$, the parties would argue about where p should be in the range $[\mathrm{p}, 9]$ in s 1 , and if $\overline{\mathrm{p}}>10$, the parties would argue about where p should be in the range $[10, \overline{\mathrm{p}}]$ in s 2 .)

We now turn to an example where the first-best cannot be achieved even with a nonsimple contract.

## Example 3.2

The example is the same as the previous one except that there is a third state, s 3 , where v is high and c is low.

|  | $\underline{\text { s1 }}$ | $\underline{\text { s2 }}$ | $\underline{\text { s3 }}$ |
| :---: | :---: | :---: | ---: |
| v | 9 | 20 | 20 |
| c | 0 | 10 | 0 |

The first-best cannot be achieved because, in order to ensure trade in s 1 , s 2 , we need $\mathrm{p} \leq 9, \overline{\mathrm{p}} \geq 10$. But such a price range leads to aggrievement and shading in s3.

There are three possible candidates for a second-best optimal contract:
(a) $\mathrm{p}=\overline{\mathrm{p}}=9$.

This contract yields trade in s 1 and s 3 but not in s 2 . Since there is nothing to argue about - the price is fixed at 9 -- there is no shading. Total surplus is given by

$$
\mathrm{W}_{\mathrm{a}}=9 \pi_{1}+20 \pi_{3}
$$

where $\pi_{1}, \pi_{3}$ are the probabilities of $s 1, \mathrm{~s} 3$, respectively.
(b) $\mathrm{p}=\overline{\mathrm{p}}=10$.

This contract yields trade in s 2 and s 3 but not in s1. Since there is nothing to argue about - the price is fixed at $10-$ - there is no shading. Total surplus is given by

$$
\mathrm{W}_{\mathrm{b}}=10 \pi_{2}+20 \pi_{3}
$$

where $\pi_{2}$ is the probability of s 2 .
(c) $\mathrm{p}=9, \overline{\mathrm{p}}=10$.

This contract yields trade in all three states, but there is aggregate aggrievement of 1 in s 3 . Total surplus is given by

$$
\mathrm{W}_{\mathrm{c}}=9 \pi_{1}+10 \pi_{2}+(20-\theta) \pi_{3}
$$

Obviously, which of these contracts is optimal depends on the probabilities $\pi_{1}, \pi_{2}, \pi_{3}$ and $\theta$. Contract (a) is optimal if $\pi_{2}$ is small, contract (b) is optimal if $\pi_{1}$ is small, and contract (c) is optimal if $\pi_{3}$ or $\theta$ is small.

Two observations can be made about this example. First, if third parties are permitted, the first-best can be achieved. Consider a contract that fixes the trade price and makes both B and S pay a large amount to a third party in the absence of trade (i.e., the no-trade price is large and positive for B and large and negative for S ). This leads to trade in all states, and no aggrievement, since the consequences of not trading are dire. However, this arrangement works only because trade is efficient in every state. In a more general example where trade is efficient
in some states but not others, third parties do not guarantee the first-best. In what follows we ignore third parties.

Second, the reader may wonder whether "Maskin mechanisms" could improve matters. ${ }^{25}$ Maskin mechanisms are a way of making observable information verifiable. Note that if the state were verifiable, it would be easy to achieve the first-best. For example, a contract that specifies $p=9$ in $s 1$ and $s 3$, and $p=10$ in $s 2$ would do the job. Call this contract (d).

However, Maskin mechanisms do not work in our situation. A Maskin mechanism is a subtle version of the following. Each party reports the state of the world. If they agree the price is as in contract (d), say. If they disagree something unpleasant happens. The problem is that in s3 S would like B to play the Maskin mechanism as if it were s2 and will be aggrieved by 1 if B refuses to go along with this. On the other hand $B$ will be aggrieved by 1 if $S$ refuses to play the mechanism as if it were s3. Either way aggregate aggrievement in s3 is 1, which yields total surplus equal to $\mathrm{W}_{\mathrm{c}}$, as in contract (c).

It may be useful to talk more generally about state contingency (or, more accurately, lack of state contingency). Why can't agreements between two parties who will both learn the state of the world be made state contingent? In principle, one could imagine $B$ and $S$ having the following conversation at date 0 about contract (c). B could tell $S$ that he will pay 9 at date 1 in all circumstances unless $S$ will not trade at this price; in which case $B$ will raise the price to 10 . B could explain to $S$ that she should not feel aggrieved in $s 3$ when $B$ does not raise the price to 10 since the very fact that S trades in this state shows that B is right about S 's cost: B does not need to raise the price to get S to trade. In other words, B will do exactly what he said he would do, and there is no reason for $S$ to feel angry about it. (To put it a little more formally, a contract

[^12]that makes price a function of an observable but unverifiable state of the world should not cause aggrievement since both parties observe the state and can see whether the other party is sticking to the contract.)

In our opinion the problem with this argument is the following. Between dates 0 and 1 S may undertake some (unmodeled) actions that she feels have contributed to B's payoff. She feels entitled to be rewarded for these actions. (Here we appeal to self-serving biases concerning the magnitude of these actions.) Whatever speech $B$ has made at date 0 , $B$ has the option, if he so chooses, to raise the price to 10 , i.e., this is consistent with the contract. In other words, B can pretend that $S$ 's cost is 10 even if it isn't. If B refuses to recognize $S$ 's contributions and offers only $9, \mathrm{~S}$ will take this to be an ungenerous act and will respond by shading. ${ }^{26}$

This brings to a close our discussion of the case where parties induce flexibility by specifying a price range. It is not clear how common this case is. One reason is that in practice the parties may be able to ensure trade when v and c vary through specific performance. Note, however, that price ranges are observed in the case of agreements to agree. ${ }^{27}$ In any event, we see the model of this section as something of a five-finger exercise. In the next section we consider a model where the uncertainty concerns the nature of the good to be provided. We will show that this model can shed light on the employment relationship.

## 4. The Case Where the Nature of the Good is Uncertain

In this section we consider the case where there is uncertainty about the nature of the good or service B requires from $S$. For example, $S$ might provide secretarial services for $B$, and

[^13]B may not know in advance whether he wants $S$ to type letters or file papers. We will actually use a more colorful example. We will suppose that $B$ is arranging an evening with friends and wants S to perform music. The nature of the music may depend on eventualities that will occur between dates 0 and 1, e.g., who is coming to the evening, what music $S$ is rehearsing for other performances, etc.

To make matters as simple as possible, we will assume that there are two types of music/composers that it might be efficient for S to play: Bach and Shostakovich. In the Appendix we also allow for convex combinations of Bach and Shostakovich, but in the text we will not need to do this. Each composer can take on one of two value-cost combinations, given by ( $\mathrm{v}, \mathrm{c}$ ) and ( $\mathrm{v}-\Delta, \mathrm{c}-\delta$ ), respectively, where $\mathrm{v}>\mathrm{v}-\Delta>\mathrm{c}>\mathrm{c}-\delta$. (Everything is measured in money terms.) In other words a composer can be "high value-high cost" or "low value-low cost." We do not insist on stochastic independence of the two value-cost combinations, but we do impose symmetry, i.e., the probability that Bach is "high value-high cost" and Shostakovich is "low value-low cost" is the same as the probability of the reverse. Thus, there are four states of the world:

|  | $\underline{\mathrm{s} 1\left(\operatorname{Prob} \pi_{1}\right)}$ | $\underline{\mathrm{s} 2\left(\operatorname{Prob}\left(1-\pi_{1}-\pi_{4}\right) / 2\right)}$ | $\underline{\mathrm{s} 3\left(\operatorname{Prob}\left(1-\pi_{1}-\pi_{4}\right) / 2\right)}$ | $\underline{\mathrm{s} 4\left(\operatorname{Prob} \pi_{4}\right)}$ |
| :--- | :---: | :---: | :---: | :---: |
| Bach | $(\mathrm{v}, \mathrm{c})$ | $(\mathrm{v}, \mathrm{c})$ | $(\mathrm{v}-\Delta, \mathrm{c}-\delta)$ | $(\mathrm{v}-\Delta, \mathrm{c}-\delta)$ |
| Shostakovich | $(\mathrm{v}, \mathrm{c})$ | $(\mathrm{v}-\Delta, \mathrm{c}-\delta)$ | $(\mathrm{v}, \mathrm{c})$ | $(\mathrm{v}-\Delta, \mathrm{c}-\delta)$ |

Figure 2

We start with the case $\Delta>\delta$. This implies that the high value-high cost composer yields more surplus than the low value-low cost composer and should be chosen whenever available.

Thus the first-best has any music in states s1 and s4, Bach in s2, and Shostakovich in s3. Expected total surplus is $\mathrm{W}=\mathrm{v}-\mathrm{c}-\pi_{4}(\Delta-\delta)$.

What is the optimal second-best contract given that the state is observable but not verifiable? We continue to assume voluntary trade and set $\mathrm{p}_{0} \equiv 0$. We also focus on contracts that deliver symmetric outcomes, i.e., whatever composer occurs in s 2 , the "mirror image" composer occurs in s3, and the prices are the same in the two states.

It will simplify the presentation to start with the case $\pi_{1}=\pi_{4}=0$, i.e., where s 2 and s3 each occur with probability $1 / 2$. We will see that this case is actually too simple, but it is useful for building up intuition.

There are four natural candidates for an optimal contract: no contract, a contract that fixes price and lets B choose the composer, a contract that fixes price and lets S choose the composer, and a contract that fixes price and composer. We consider these in turn.

## (a) No contract

The parties can always wait until date 1 to contract, by which time they will know whether s2 or s 3 has occurred. They will then bargain over the division of surplus, $\mathrm{v}-\mathrm{c}$. The analysis is similar to that in Section 2: whatever price between c and v is agreed to, the total amount of aggrievement will be $(\mathrm{v}-\mathrm{c})$, and so shading costs will be $\theta(\mathrm{v}-\mathrm{c})$. Hence net surplus is

$$
\mathrm{W}_{\mathrm{a}}=(1-\theta)(\mathrm{v}-\mathrm{c}) .
$$

(b) A contract that fixes the price p such that $\mathrm{c} \leq \mathrm{p} \leq \mathrm{v}-\Delta$ and lets B choose the composer at date 1

Given that price is fixed, B will choose the highest value composer at date 1: Bach in s 2 , Shostakovich in s3. Since $\Delta>\delta$ this is the efficient choice. However, in each state the seller will be aggrieved that B did not choose her favorite composer: Shostakovich in s2, Bach in s3. The seller will be shortchanged by $\delta$, the difference in the costs between the two composers, and so shading costs are $\theta \delta$. Total surplus is

$$
\mathrm{W}_{\mathrm{b}}=\mathrm{v}-\mathrm{c}-\theta \delta .
$$

(c) A contract that fixes the price p such that $\mathrm{c} \leq \mathrm{p} \leq \mathrm{v}-\Delta$ and lets S choose the composer at date 1

Given that price is fixed, S will be inclined to choose the lowest cost composer at date 1 , which is inefficient. B will be aggrieved by $\Delta$, the difference in value between his favorite composer and S's choice, and will shade so that S's payoff falls by $\theta \Delta$. Note that if $\theta \Delta>\delta$, $S$ will be worse off than if she had chosen the high cost composer, and so she will choose the high cost composer after all. On the other hand, if $\theta \Delta<\delta$, S will stick with the low cost choice, and surplus is

$$
\mathrm{W}_{\mathrm{c}}=\mathrm{v}-\Delta-\mathrm{c}+\delta-\theta \Delta .
$$

## (d) A contract that fixes the price p such that $\mathrm{c} \leq \mathrm{p} \leq \mathrm{v}-\Delta$ and fixes the composer

 Suppose the composer is fixed at Bach, say. This yields the efficient choice of composer half the time, and so surplus is$$
\mathrm{W}_{\mathrm{d}}=1 / 2(\mathrm{v}-\mathrm{c})+1 / 2(\mathrm{v}-\Delta-\mathrm{c}+\delta)=\mathrm{v}-\mathrm{c}-1 / 2 \Delta+1 / 2 \delta .
$$

Note that there is no cost of shading in contract (d) since, with price and composer fixed, there is nothing to be aggrieved about.

How do contracts (a) - (d) compare? Since $v-\Delta>c$ and $\Delta>\delta$, it is easy to see that $\mathrm{W}_{\mathrm{b}}$ $>\mathrm{W}_{\mathrm{a}}$. (If $\delta \simeq 0, \mathrm{~W}_{\mathrm{b}} \simeq \mathrm{v}-\mathrm{c}$, i.e., contract (b) achieves approximately the first-best.) Also it is clear that $\mathrm{W}_{\mathrm{b}}>\mathrm{W}_{\mathrm{c}}$ (in contract (c) the composer is less efficient than in (b) and there is more shading). So the choice is between contracts (b) and (d). Algebra tells us that (b) is better if and only if

$$
\begin{equation*}
\Delta>(1+2 \theta) \delta \tag{4.1}
\end{equation*}
$$

The analysis so far is a little misleading. There is a contract that performs better than either (b) or (d), and in fact achieves the first-best. This contract is a variation of (b), in which the constraint $\mathrm{c} \leq \mathrm{p} \leq \mathrm{v}-\Delta$ is dropped. Specifically, consider the contract in which the price $\mathrm{p}=$ v and B chooses the composer. B will make the efficient choice in each state (as in contract (b)), but in addition S will not be aggrieved. The reason is that B exactly breaks even: any composer who is more favorable to $S$ than Bach in s2 and Shostakovich in s3 would cause B to refuse to trade, i.e., such a composer would violate B's individual rationality constraint.

There is something very fragile about the contract just described. If there is any chance that B's value falls below v it will lead to no trade. In the next proposition we return to the case
where states s1 and s4 have positive probability. The proposition tells us that, as long as $\pi_{4}$ is not too low, fixing p close to v is a bad idea. The proposition also requires $\pi_{1}$ not to be too low in order to rule out subtle contracts that can also do better than (b) and (d). In the Appendix we establish formally:

Proposition 4.1 Assume $\mathrm{v}>\mathrm{v}-\Delta>\mathrm{c}>\mathrm{c}-\delta$, and $\Delta>\delta$. Suppose in addition $\frac{\pi_{4}}{1-\pi_{1}} \geq$ $\frac{\mathrm{m}}{(\mathrm{v}-\mathrm{c}-\Delta+\delta+\mathrm{m})}$, where $\mathrm{m}=\min \{\theta \delta, 1 / 2(\Delta-\delta)\}$, and $\frac{\pi_{1}}{1-\pi_{4}} \geq \frac{\delta}{\Delta+\delta}$. Then the optimal secondbest contract fixes $\mathrm{c} \leq \mathrm{p} \leq \mathrm{v}-\Delta$. In addition, if $\Delta>(1+2 \theta) \delta$, B is given the right to choose the composer, while, if $\Delta<(1+2 \theta) \delta$, the parties fix the composer, at Bach say.

Proposition 4.1 illuminates the different roles played by price and music in the model of this section. Price has no allocative role - its choice is a zero sum game - and so, in order to avoid aggrievement, it is better to fix it in advance. Music does serve an allocative role and so, if $\Delta / \delta$ is large or $\theta$ is small, it makes sense to leave it open. Moreover, if $\Delta / \delta$ is large or $\theta$ is small, B should choose the composer since B will make an efficient choice, and, given that $S$ cares relatively little, aggrievement will be low.

Note that there are two implicit assumptions underlying Proposition 4.1. First, aggregate uncertainty is small ( $\mathrm{v}-\Delta>\mathrm{c}$ ), and so price does not have to vary across the four states in order to ensure that both parties wish to trade, as it did in Section 3. Second, there is no systematic relationship between composer and cost. In contrast, if Shostakovich, say, was on average costlier for S to play than Bach, then it would be optimal to have a higher price for Shostakovich than Bach, in order to reduce S's aggrievement in states where Shostakovich is chosen. In other
words, a generalized version of the model can explain why price should vary with the service S provides if there are systematic differences in costs. ${ }^{28}$

Let's now turn to the case where $\Delta<\delta$, i.e., the low value-low cost composer yields more surplus than the high value-high cost composer. The argument goes through as above except that now it is never optimal for $B$ to choose the composer but it may be optimal for $S$ to choose the composer: Contract (c) may be optimal, but contract (b) is not. We have

Proposition 4.2 Assume $\mathrm{v}>\mathrm{v}-\Delta>\mathrm{c}>\mathrm{c}-\delta$, and $\Delta<\delta$. Suppose in addition $\frac{\pi_{1}}{1-\pi_{4}} \geq \frac{\mathrm{m}^{\prime}}{\left(\mathrm{v}-\mathrm{c}+\mathrm{m}^{\prime}\right)}$, where $\mathrm{m}^{\prime}=\min \{\theta \Delta, 1 / 2(\delta-\Delta)\}$, and $\frac{\pi_{4}}{1-\pi_{1}} \geq \frac{\Delta}{\delta+\Delta}$. Then the optimal second-best contract fixes $\mathrm{c} \leq \mathrm{p} \leq \mathrm{v}-\Delta$. In addition, if $\delta>(1+2 \theta) \Delta$, S is given the right to choose the composer, while, if $\delta<(1+2 \theta) \Delta$, the parties fix the composer, at Bach say.

We believe that Propositions 4.1 and 4.2 can throw light on a classic question: the nature of the employment relationship and the difference between an employee and an independent contractor. In early work, Coase (1937) and Simon (1951) argued that a key feature of the employment relationship is that an employer tells an employee what to do. This view was challenged by Alchian and Demsetz (1972), and the more recent literature has emphasized asset ownership as the distinguishing aspect of these relationships (see Grossman and Hart (1986) and

[^14]Hart and Moore (1990)). ${ }^{29}$ The current model allows us to return to the ideas of Coase and Simon. We interpret the case where B chooses the composer as an employment relationship and the case where S chooses the composer as independent contracting. ${ }^{30}$ That is, if B hires S's musical services for the evening, with the understanding that B will tell S what to do, then S is working for $B$. In contrast, if $B$ engages $S$ to provide an evening of music, with the details of exactly how this is to be done left up to S , then S is an independent contractor. ${ }^{31}$

Propositions 4.1 and 4.2 tell us that when $\theta$ is small, if B cares more about the composer, that is, $\Delta>\delta$, then employment is better (if $\delta \simeq 0$, an employment contract achieves approximately the first-best); while if $S$ cares more about the composer, that is, $\Delta<\delta$, then independent contracting is better (if $\Delta \simeq 0$, independent contracting achieves approximately the first-best). In both cases it is optimal for the parties to fix the price in advance.

While these results are in the spirit of Coase and Simon, they differ from Simon's formal argument in important ways. Simon would also argue that B should choose the composer if B cares more about the composer than S. However, in Simon's model it is not clear why an ex ante contract is needed at all. Since there is no aggrievement (not surprisingly!) and no noncontractible investments, the parties can rely on Coasian bargaining at date 1. Also, a contract that achieves the first-best in Simon's model is one where B has the right to make a take-it-or-leave-it offer to S; i.e., B proposes a price-composer pair, and S can accept or reject it. In other words, in Simon's model there are many optimal contracts (a continuum, in fact), of which the employment contract is just one.

[^15]We saw in Section 2 that this is not true in our model. For example, consider the contract in which B offers a price-composer pair. B will suggest price equal to cost, and there will be aggrievement and shading in all states since $S$ will feel entitled to a higher price. Thus this contract performs strictly worse than the employment contract.

In other words a virtue of our model is that it can explain why the employment contract is uniquely optimal when $\Delta>\delta$ and $\theta$ is small; why independent contracting is uniquely optimal when $\Delta<\delta$ and $\theta$ is small; and why in all the cases considered in this section it makes sense (in the absence of systematic cost differences across composers or tasks) for the parties to fix price ex ante, i.e., to take price off the table.

## 5. External Reference Points

So far we have assumed that a prior contract is the only reference point for the transaction at date 1. In this section we relax this assumption. Our analysis is preliminary and speculative.

It is not difficult to think of situations where parties look outside a contract to determine whether they are being treated fairly. A familiar case is where someone is hired as an employee at a particular wage, and some time later someone else with comparable or even inferior skills is hired by the same employer at a higher wage, perhaps because market conditions have changed. The first person will almost certainly feel unhappy about this even though their wage was determined fairly and competitively at the time. ${ }^{32}$

One way to capture the idea of "external" reference points is as follows. Return to the model of Section 3 where the parties trade a standard good but there is uncertainty about v and c . Suppose that in each state of the world there is a range of "reasonable" prices for the good,

[^16]determined exogenously, and given by [ $\mathrm{p} \min , \mathrm{p} \max$ ]. The interpretation is that this range is based on comparable transactions: trades in other markets at date 1, prices of previous transactions, prices embodied in new contracts written between dates 0 and 1, etc. Any price between p min and p max can be justified to outsiders as being reasonable while other prices cannot.

In order to simplify matters, we will assume that the [ $\mathrm{c}, \mathrm{v}$ ] and [ p min, p max] intervals always intersect; that is, whenever $\mathrm{v} \geq \mathrm{c}, \mathrm{v} \geq \mathrm{p}$ min and $\mathrm{c} \leq \mathrm{p}$ max. This assumption captures the idea that external reference points and internal value and cost are never too far apart.

The [ p min, p max] range plays two roles. First, the range may affect entitlements in the presence of a contract. Suppose that the parties' date 0 contract specifies the range of trading prices $[\mathrm{p}, \overline{\mathrm{p}}]$. We saw in Section 3 that, on the basis of this, $S$ feels entitled to receive Min (v, $\overline{\mathrm{p}}$ ) and B to pay Max ( $\mathrm{c}, \mathrm{p}$ ). We assume that the external reference points [ p min, p max] modify these entitlements only if (i) $\mathrm{p} \min >\operatorname{Min}(v, \bar{p})$ or (ii) $\operatorname{Max}(c, p)>p$ max. In the first case the price $S$ feels entitled to receive is raised to p min; while in the second case the price B feels entitled to pay is lowered to p max.

In other words, $S$ feels entitled to receive more than $\operatorname{Min}(v, \bar{p})$ if (and only if) all external prices lie above $\operatorname{Min}(v, \bar{p})$, i.e., everybody else in the market is receiving more, and $B$ feels entitled to pay less than $\operatorname{Max}(\mathrm{c}, \mathrm{p})$ if (and only if) all external prices lie below Max $(\mathrm{c}, \overline{\mathrm{p}})$, i.e., everybody else in the market is paying less. Note that this formulation gives precedence to an existing contract in the sense that external reference points come into play only when contract prices are far apart from what's going on elsewhere. In future work it would be interesting to explore alternative ways of modeling the interaction between external reference points and prior contracts.

In summary, $S$ feels entitled to receive $\operatorname{Max}(\operatorname{Min}(v, \bar{p}), p$ min) and $B$ to pay $\operatorname{Min}(\operatorname{Max}$ (c, p), p max).

Let's continue with the case where the parties write a date 0 contract $[\mathrm{p}, \overline{\mathrm{p}}]$. The trading rule is given by (3.4) and expected surplus by

$$
\begin{align*}
\mathrm{W}= & \int[\mathrm{v}-\mathrm{c}-\theta\{\operatorname{Max}(\operatorname{Min}(\mathrm{v}, \overline{\mathrm{p}}), \mathrm{p} \min )-\operatorname{Min}(\operatorname{Max}(\mathrm{c}, \underline{\mathrm{p}}), \mathrm{p} \max )\}] \mathrm{dF}(\mathrm{v}, \mathrm{c}, \mathrm{p} \max , \mathrm{p} \min )  \tag{5.1}\\
\quad \mathrm{v} & \geq \mathrm{c} \\
\quad \mathrm{v} & \geq \mathrm{p} \\
& \mathrm{c} \leq \overline{\mathrm{p}}
\end{align*}
$$

where we rewrite the distribution function F to reflect the fact that p min and p max can also depend on the state of the world. Note that (5.1) coincides with (3.5) when $\mathrm{p} \min =-\infty, \mathrm{p}$ max $=$ $\infty$. However, shading costs are higher than before when $\overline{\mathrm{p}}<\mathrm{p}$ min or $\mathrm{p}>\mathrm{p}$ max. Otherwise the tradeoff between flexibility and rigidity is essentially as in Section 3: a large $[\mathrm{p}, \overline{\mathrm{p}}]$ range makes it more likely that trade will occur, but also leads to more shading.

Matters become more interesting if the parties write no date 0 contract. This is where the second role of the external reference range [p min, p max] comes in. Previously we supposed that "no contract" was equivalent to setting $\mathrm{p}=-\infty, \bar{p}=\infty$, i.e., to a very flexible contract. However, we now take the view that when the parties bargain at date 1 in the absence of a date 0 contract, they never consider a price below p min or a price above p max because such prices look unreasonable to outsiders (they might not even be enforced by a court). In other words it is as if the parties had written an initial contract with $\mathrm{p}=\mathrm{p}$ min and $\overline{\mathrm{p}}=\mathrm{p}$ max: the parties bargain in the intersection of the $[\mathrm{c}, \mathrm{v}]$ and $[\mathrm{p} \min , \mathrm{p} \max ]$ ranges, and S feels entitled to Min ( $\mathrm{v}, \mathrm{p} \max$ )
and $B$ to $\operatorname{Max}(\mathrm{c}, \mathrm{p} \min$ ). Given our simplifying assumption that $\mathrm{v} \geq \mathrm{p}$ min and $\mathrm{c} \leq \mathrm{p}$ max whenever $\mathrm{v} \geq \mathrm{c}$, trade will always occur when $\mathrm{v}>\mathrm{c}$, and so expected surplus is given by
(5.2) $\mathrm{W}=\int[\mathrm{v}-\mathrm{c}-\theta\{\operatorname{Min}(\mathrm{v}, \mathrm{p} \max )-\operatorname{Max}(\mathrm{c}, \mathrm{p} \min )\}] \mathrm{dF}(\mathrm{v}, \mathrm{c}, \mathrm{p}$ max, p min $)$. $\mathrm{v} \geq \mathrm{c}$

It is easily seen that (5.2) is different from what is obtained by substituting $\mathrm{p}=-\infty, \overline{\mathrm{p}}=\infty$ in (5.1). To underline the point, with external reference points, "no contract" is not the same as a highly flexible contract.

What determines the choice between writing a date 0 contract and writing "no contract," i.e., leaving things to date 1 ? It is good to write an ex ante contract if p min is small and p max is large. In particular, if $\mathrm{p} \min =-\infty, \mathrm{p} \max =\infty$, one can do at least as well as "no contract" by writing a contract with $\mathrm{p}=-\infty, \overline{\mathrm{p}}=\infty$ (compare (5.1) and (5.2)); and one can usually do better by limiting the $[\mathrm{p}, \overline{\mathrm{p}}]$ range. Another case where it is better to write a date 0 contract is if the range [ p min, p max] does not vary with the state of the world - in this case, one can do at least as well as with no contract by setting $\mathrm{p}=\mathrm{p} \min , \overline{\mathrm{p}}=\mathrm{p}$ max.

What about cases where "no contract" is optimal? The leading one is where the range [p min, p max] is small. Under these conditions trade always occurs when it is efficient (by assumption, a price can be found in the intersection of the [ $\mathrm{p} \min , \mathrm{p}$ max] and $[\mathrm{c}, \mathrm{v}]$ ranges whenever $\mathrm{v} \geq \mathrm{c}$ ), and shading costs are low. For example, if $\mathrm{p} \min =\mathrm{p} \max =\hat{\mathrm{p}}$, "no contract" achieves the first-best.

A concrete example may be useful. Consider Example 3.1 of Section 3. Suppose that p $\min =\mathrm{p} \max =\hat{\mathrm{p}}=4.5$ in s 1 and $\mathrm{p} \min =\mathrm{p} \max =\hat{\mathrm{p}}=15$ in s 2. Previously the date 0 contract p
$=9, \overline{\mathrm{p}}=10$ achieved first-best. However, this is no longer true: in s1 B will feel aggrieved that he pays 9 rather than the (external) price 4.5 , and so shades by $4.5 \theta$; and in s 2 S will feel aggrieved that she receives 10 rather than the (external) price 15 , and so shades by $5 \theta$.

In contrast "no contract" does achieve the first-best. Under no contract, at date 1 B and S agree on a price of 4.5 in s 1 and a price of 15 in s 2 - there is nothing to argue about because p $\min =\mathrm{p} \max -$ and there is no aggrievement or shading in either state.

We conclude by mentioning a possible application of the model to explaining contract length. Take our example of a wedding that will occur in six months. In this case the range of reasonable catering prices [ p min, p max] is plausibly quite large (there are many different types of weddings and caterers) and is unlikely to change much over the next six months. According to our analysis this is a situation where it is better to write an ex ante contract rather than "no contract." On the other hand, imagine that the wedding will occur five years from now. There may be a great deal of uncertainty about future catering costs and future market prices for catering services (and the two may be highly correlated) and, to the extent that the latter acts as a reference point for entitlements, fixing a price, or a range of prices, now may create aggrievement with high probability. It may be better to take a wait-and-see attitude and postpone contracting.

Combining the two cases, i.e., considering a situation of two weddings, one in six months and the other in five years, yields the beginnings of a theory of contract length.

The above just skims the surface of what is potentially a very interesting and rich set of issues. We leave the details to future research.

## 6. Renegotiation

We now relax the no-renegotiation assumption. We will in fact consider three views of the renegotiation process. We will see that our results are modified, but not entirely changed, by the possibility of renegotiation. We focus on the models of Sections 3 and 4.

One view of renegotiation is that, after the contract refinement process illustrated in Figure 1 occurs, the parties will always renegotiate to an efficient outcome, and that the parties will rationally anticipate this. How does this affect our analysis?

Start with the model of Section 3. Suppose B and S write a contract consisting of the price range $[\mathrm{p}, \overline{\mathrm{p}}]$. Then after the uncertainty about $\mathrm{v}, \mathrm{c}$ is resolved at date 1 it is possible that $\mathrm{v}>$ c and yet either $\mathrm{v}<\mathrm{p}$ or $\mathrm{c}>\overline{\mathrm{p}}$. In other words, trade is efficient but won't occur under the contract.

According to the first view of renegotiation, B and S will write a new contract. B will feel entitled to $p=c$ and $S$ to $p=v$. Total shading is $\theta(v-c)$, and net surplus is $(1-\theta)(v-c)$. Note that renegotiation does not achieve the first-best whenever $\theta>0$.

This alters the analysis a little. Take Example 3.2. Contract (c) is unchanged since no renegotiation occurs. However, the surplus in contracts (a), (b) rises. Under (a) renegotiation will take place if s2 occurs. Under (b) renegotiation will take place if s1 occurs. Thus total surplus under (a), (b) is now

$$
\begin{aligned}
& \mathrm{W}_{\mathrm{a}}^{\prime}=9 \pi_{1}+10(1-\theta) \pi_{2}+20 \pi_{3}, \\
& \mathrm{~W}_{\mathrm{b}}^{\prime}=9(1-\theta) \pi_{1}+10 \pi_{2}+20 \pi_{3} .
\end{aligned}
$$

Contracts (a) or (b) might now beat contract (c) even if, in the absence of renegotiation, they did not.

How does this view of renegotiation affect the model of Section 4? Suppose $\Delta>\delta$. There were two candidates for an optimal contract, (b) and (d). In contract (b) price is fixed and B chooses the composer. In (d) both price and composer are fixed.

Contract (b) leads to an efficient ex post outcome (apart from shading) and so no renegotiation will occur. However, contract (d) leads to inefficiency in either s2 or s3, and so now renegotiation will occur in one of these states. For example, if the contract specifies Bach and s 3 occurs, S will agree to switch to Shostakovich in return for a side payment. The gains from renegotiation are $(\Delta-\delta)$, and since there will be aggrievement about how these are split, a fraction $\theta$ of them will be lost to shading. Thus surplus under (d) now rises to

$$
\mathrm{W}=1 / 2(\mathrm{v}-\mathrm{c})+1 / 2(\mathrm{v}-\mathrm{c}-\theta(\Delta-\delta))=\mathrm{v}-\mathrm{c}-1 / 2 \theta(\Delta-\delta) .
$$

One problem with this first view of renegotiation is that it makes a strong assumption about the renegotiation process and entitlements within a state. Consider the switch from Bach to Shostakovich in s3 in contract (d). It is supposed that $S$ feels entitled at most to $100 \%$ of the surplus from the switch, i.e., to an increase in price from $p$ to $p+\Delta$; and $B$ feels entitled at most to $100 \%$ of the surplus from the switch, i.e., to an increase in price from p to $\mathrm{p}+\delta$. But in principle $B$ and $S$ could use the renegotiation to demand even more: $S$ could argue that, since price is "on the table," p should move all the way up to v , and B could argue that the price should move all the way down to c. In other words each party could demand $100 \%$ of all the surplus.
(Recall the voluntary trade assumption: each party has the option to quit.) Moreover, they may feel aggrieved if they don't get this.

We now consider a second view of renegotiation that allows for this possibility. It takes a "thin end of the wedge" approach. Suppose that if one party proposes a renegotiation involving a price change, then both parties recognize that any price change is now possible. Assume that the parties will agree to the renegotiation if and only if both parties are made better off. Then contract (d) will be renegotiated in s3 only if

$$
\begin{aligned}
& 1 / 2(v-c)(1-\theta) \geq v-\Delta-p, \\
& 1 / 2(v-c)(1-\theta) \geq p-c+\delta,
\end{aligned}
$$

where we are assuming that the parties split the surplus 50:50. Adding these inequalities yields

$$
(v-c)(1-\theta) \geq v-c-\Delta+\delta,
$$

which will not be satisfied if, say, $\Delta \simeq \delta$. We see that this second view of renegotiation puts some friction into the process: it makes it less likely that the parties will switch from an inefficient composer to an efficient composer.

Note that this second view of renegotiation coincides with the first view for the model of Section 3. The reason is that renegotiation occurs only if the contract leads to no trade, in which case all the surplus is up for grabs anyway.

In our opinion both of the above views of renegotiation are rosy. Each view supposes that the possibility of changing price in one state will not affect parties' feelings of entitlement in
other states. But this is questionable. Return to the model of Section 3. Then, under contract (a), the price is raised to at least 10 in s 2 as a result of renegotiation. Given that price is flexible, why wouldn't $S$ feel entitled to a price change in s3? Of course, if S does think this way, then it is as if the contract specified that the price could be in (at least) the [9,10] range in the first place, and we are back to contract (c).

In our opinion an intellectually more coherent position is that any flexibility in the trading price must be built into the initial contract (assuming an initial contract is written at all - see Section 5). That is, one can set $\mathrm{p}=9$ or $\mathrm{p}=10$ or $\mathrm{p} \varepsilon[9,10]$, but one cannot set $\mathrm{p}=9$ and then change it to $\mathrm{p}=10$.

Moreover, as we have discussed elsewhere, we believe that this position is consistent with legal practice and social custom. ${ }^{33}$ The courts regard contract renegotiations with some suspicion and may overturn them if they believe that opportunism or duress has played a role. (Social attitudes and norms often mirror the law.) To this end, the courts require that renegotiation must be in "good faith," but, since this is difficult to monitor, they will often substitute the requirement that the renegotiation can be justified objectively, e.g., the price increases because the seller is supplying an additional service and her costs have risen. ${ }^{34}$ In the model of Section 3, no extra service is provided, and so there is no objective justification for a price change, say from $\mathrm{p}=9$ to $\mathrm{p}=10 .{ }^{35}$ Similarly, there is no justification for a price change in the model of Section 4 when S switches from Bach to Shostakovich, given that Shostakovich is not objectively (i.e., verifiably) more costly than Bach.

[^17]Given this third view of renegotiation, no renegotiation will occur at all in the models of Sections 3 and 4. Thus this third view of renegotiation is most favorable to our preceding analysis: it survives intact.

## 7. Summary and Conclusions

In this paper we have developed a theory of contracts based on the view that a contract provides a reference point for a parties' trading relationship. The idea is that a contract written early on when an external measure of the parties' contributions to the relationship is provided by competitive markets can continue to govern the parties' feelings of entitlement later when they become locked in to each other. The anchoring of entitlements in turn limits disagreement, aggrievement, and the deadweight losses from shading. We have shown that our theory yields a trade-off between contractual rigidity and flexibility, provides a basis for long-term contracts in the absence of noncontractible investments, and throws light on the nature of the employment relationship. We have also shown that an extension of our theory that allows for external reference points can explain why parties sometimes deliberately write "no contract."

Our theory is based on strong assumptions. Before considering how these might be improved on and relaxed, let us say a little more about why we made them. In principle one could study the trade-off between rigidity and flexibility using more traditional approaches. For example, standard rent-seeking arguments suggest that a flexible contract that "leaves money on the table" will generate inefficiency ex post as the parties fight over the surplus. ${ }^{36}$ Similarly, influence-cost arguments suggest that mechanisms will be costly given that one party will waste resources trying to influence the other party's decision. ${ }^{37}$ However, these theories suffer from

[^18]the following weakness. If, as is usually supposed, ex post trade is perfectly contractible, why can't the parties negotiate around the wasteful rent-seeking and influence activities and move straight to the ex post efficient outcome (assuming symmetric information)?

The main motivation for introducing shading, i.e., for dropping the assumption that ex post trade is contractible, and for making associated behavioral assumptions, is to avoid this conclusion. ${ }^{38}$ In addition our model is tractable and yields intuitive formulae for the costs of flexibility. For example, in Section 4 we saw that letting one party choose the outcome (the composer) will lead to little inefficiency if the other party is approximately indifferent about the choice. It is not clear that more traditional approaches generate such a simple (and we think reasonable) conclusion.

We should make it clear, however, that we are more than happy to consider alternatives in future work. We mention one. Consider a contract that admits two outcomes, $a$ and $b$, and gives the buyer the right to choose between them. If the buyer prefers a and the seller $b$, the buyer may have to spend time persuading the seller of the reasonableness of the choice a in order to ensure consummate performance by the seller. These persuasion costs are a plausible alternative to the shading costs we have focused on. Modeling persuasion costs is not easy but it is an interesting topic for future research.

Let us turn now to how our behavioral assumptions could be refined and relaxed. Our model has a number of "black box" features. We have made strong and somewhat ad hoc assumptions about entitlements, self-serving biases, and shading behavior. We have not derived these from first principles. Opening up the behavioral black box and showing that these

[^19]assumptions are consistent with utility maximizing behavior is an important topic for future research.

We have also made other strong assumptions. One is that shading by $B$ and $S$ is symmetric. As mentioned in the text, it is easier to think of examples of S's shading than of B's shading. If we supposed no B shading, the model as it stands would collapse: the first-best could be achieved by giving $S$ the right to make a take-it-or-leave-it offer to $B$. S would choose the efficient price-output or price-quality combination, thereby extracting all the surplus, and, according to standard subgame perfection arguments, B would accept such an offer. One way to avoid this conclusion is to appeal to the ultimatum game literature that argues that B will not accept S's offer unless S gives B a reasonable fraction of the surplus. ${ }^{39}$ Knowing this, S will allocate some of the surplus to $B$, but since $S$ feels entitled to all the surplus she will be aggrieved and will shade. A second way to avoid the "first-best" outcome is to suppose that B dislikes feeling "taken advantage of" or "exploited" by S, that is, a contract that allows S to make a take-it-or-leave-it offer will cause a psychic loss for B if S chooses not to be generous. For instance, in the wedding example, B might obtain disutility from being gouged ex post by the caterer. This psychic loss is akin to the deadweight loss from shading: the difference is that B internalizes it ("eats it") rather than being able to retaliate and shift it to S . (One could even suppose that $S$ incurs a psychic loss from the fact that $S$ 's offer will be perceived as ungenerous by B.) This approach would bring us closer to that part of the behavioral literature that emphasizes loss aversion. ${ }^{40}$

There are at least two other ways to avoid the "first-best" conclusion when B can't shade. One is to suppose that B must make an ex ante noncontractible investment, so that complete

[^20]hold-up by $S$ is undesirable. A second is to suppose that $S$ is wealth constrained and so cannot compensate B in advance for the $100 \%$ of the surplus that S will obtain ex post.

A further assumption that we have made is that only the outcome and not the process matters for people's feelings of entitlement and well-being. This is strong. Take the employment contract in Section 4, in which the price is fixed and B has the right to choose the composer. If B chooses Bach in state s2, which S doesn't like, S may nonetheless accept and not feel aggrieved about this choice given that $B$ had the right to make it. In other words, $S$ might feel differently about the outcome "Bach" if it is the result of a previously agreed process than if the outcome is arrived at in some other way, e.g., through bargaining.

Generalizing the model to allow for the role of process is highly desirable. Note that we do not think that a well designed process will completely eliminate the costs of flexibility. In the employment example $S$ is likely to accept Bach only if she views B's choice as "reasonable." Convincing S of this may be costly for B; see the above discussion of persuasion costs. Thus our view is that an appropriately constructed model that allows for the role of process will still exhibit an interesting trade-off between contractual flexibility and rigidity.

In the Introduction we motivated the paper by pointing out some limitations of existing models of the firm. We believe that the model developed here can help to overcome some of those limitations and can be applied to organizational and contract economics more generally. We have used the model to understand the nature of the employment relationship, and why parties deliberately write "no contract." We believe that there are many other possibilities. We end by mentioning a few of these. First, the model may throw light on the role of the courts in filling in the gaps of incomplete contracts. The idea is that a contractual term provided by the parties may affect entitlements whereas one provided by an outsider - the courts - may not. This
may have efficiency consequences. Second, along related lines, the model may help to explain why parties choose not to index contracts to inflation in order to generate real wage flexibility. If real wages fall because prices rise, this can be blamed on an outsider -- e.g., the government -whereas if nominal wages are reduced by an employer this may generate anger. Finally, the model of Section 4, extended from two to many people, may help us to understand how authority should be allocated, i.e., who, out of a group of individuals, should be boss. We believe that this last application may be a useful step in allowing incomplete contracting ideas to be applied to the very interesting and important topic of the internal organization of firms.

## Appendix A

In this appendix we use the framework of Section 3, first, to refine our assumption about what determines a party's level of aggrievement; and, second, to prove a result giving circumstances where we can restrict attention to contracts in which the no-trade price $\mathrm{p}_{0}$ is zero and one party unilaterally chooses the terms of trade.

To see why a refinement is required, consider a slight variant of the three-state example from Section 3:

|  | $\underline{\mathrm{s} 1}$ | $\underline{\mathrm{~s} 2}$ | $\underline{\mathrm{~s} 3}$ |
| :---: | :---: | :---: | :---: |
| v | 9 | 20 | 10 |
| c | 0 | 10 | 9 |

The only change is that in state $s 3$ the $(\mathrm{v}, \mathrm{c})$ pair equals $(10,9)$ rather than $(20,0)$. All the analysis of contracts (a) - (c) still pertains, except that now they yield expected total surplus of

$$
\begin{aligned}
& \mathrm{W}_{\mathrm{a}}=9 \pi_{1}+\pi_{3}, \\
& \mathrm{~W}_{\mathrm{b}}=10 \pi_{2}+\pi_{3}, \\
& \mathrm{~W}_{\mathrm{c}}=9 \pi_{1}+10 \pi_{2}+(1-\theta) \pi_{3},
\end{aligned}
$$

respectively. As before, in general none of these contracts achieves first-best.
Now consider contract (e), in which $\mathrm{p}_{0} \equiv 0$ and $B$ chooses $\mathrm{p}_{1}$ from a set of three discrete prices, $\left\{8^{1 / 2}, 9^{1 / 2}, 10^{1 / 2}\right\}$. If we think of a contract where $p_{1}$ must lie in an interval $[\mathrm{p}, \overline{\mathrm{p}}]$ as a
"standard" contact (and $\mathrm{p}=\overline{\mathrm{p}}$ as a "simple" contract), then, unlike contracts (a) - (c), contract (e) is "non-standard".

On the face of it, contract (e) achieves first-best because in each state only one $\mathrm{p}_{1}$ out of the three allowable prices delivers trade (remember trade is voluntary, so each party has to be better off than not trading at $\mathrm{p}_{0}=0$ ). Surely, this means that there is no source of aggrievement?

We take the view that this is the wrong answer: contract (e) will generate aggrievement and hence shading. The reason is that in state $s 3$, when $B$ chooses $p_{1}=91 / 2, S$ will feel aggrieved that B didn't choose a $50: 50$ lottery between $p_{1}=91 / 2$ and $p_{1}=101 / 2$, with $B$ committing to trade whatever the outcome of the lottery. The 50:50 odds are such that before the lottery, B would be no worse off than not trading, given that he values the widget at 10 . At a price of $91 / 2$, then, S feels aggrieved by $1 / 2$. Equally, B feels aggrieved by $1 / 2$ too! Even though he has the contractual right to choose $p_{1}$, and chooses $91 / 2$, he would prefer to choose a $50: 50$ lottery between $p_{1}=81 / 2$ and $p_{1}=9^{1 / 2}$, with $S$ committing to trade whatever the outcome: before the lottery, S would be no worse off than not trading, given that the widget costs her 9. In aggregate, shading in state s3 amounts to $(1 / 2+1 / 2) \theta$, which is the same as under contract (c). Worse, in state $s 1$, when B chooses $p_{1}=81 / 2, S$ feels aggrieved that $B$ didn't choose a $50: 50$ lottery between $p_{1}=81 / 2$ and $p_{1}=$ $91 / 2$, with B committing to trade whatever the outcome, so that in this state there is shading. Similarly, there is shading in state s2. All in all, contract (e) is strictly dominated by contract (c).

We are adopting the position that when he or she thinks how aggrieved to feel, a party
(A1) conjures with mixed strategies in the contractual mechanism (and with correlated strategies if the mechanism has simultaneous moves);
(A2) imagines a commitment to trade on the part of the other party, whatever the outcome of the randomization.

However, no-one believes they can force the other party beyond his or her participation constraint. Specifically, no-one thinks they can push the other party's expected payoff below what he or she could get from simply walking away prior to the randomization, refusing to trade.

If the no-trade price $\mathrm{p}_{0}$ varies with the strategies, and cannot simply be normalized to zero, then it is a little less obvious how these participation constraints should be factored into the parties' thinking.

Consider a contract (f) in which B chooses ( $\mathrm{p}_{1}, \mathrm{p}_{0}$ ) from a set of three pairs: $(9,1),\left(9^{1 / 2}, 1 / 2\right)$ or $(10,0)$. If there is no shading, then in state s1 B will choose $\left(p_{1}, p_{0}\right)=(9,1)$, leading to trade at a price of 9 - actually, he is indifferent between this outcome and choosing $\left(p_{1}, p_{0}\right)=(10,0)$, leading to no trade at $p_{0}=0$. In state $s 3 B$ will choose $\left(p_{1}, p_{0}\right)=(91 / 2,1 / 2)$, leading to trade at a price of $91 / 2$ - if he chose $\left(p_{1}, p_{0}\right)=(9,1)$ then $S$ would not trade. In state $s 2$ B will choose $\left(p_{1}, p_{0}\right)=(10,0)$, the only price pair at which $S$ will trade.

Again, on the face of it, if we do not invoke (A1) or (A2), contract (f) looks attractive. It could be argued that, since $B$ is choosing the pair $\left(p_{1}, p_{0}\right)$ he is never aggrieved. And the contract has been cleverly designed so that in state 33 S is aggrieved by only $1 / 2$. She would increase her payoff by $1 / 2$ if, instead of $B$ choosing $\left(p_{1}, p_{0}\right)=(91 / 2,1 / 2)$ leading to trade at $p_{1}=91 / 2, B$ were to choose $\left(p_{1}, p_{0}\right)=(9,1)$ leading to no trade at $p_{0}=1$; equally, $S$ would increase her payoff by $1 / 2$ if B were to choose $\left(p_{1}, p_{0}\right)=(10,0)$ leading to trade at $p_{1}=10$. Arguably, then, shading in state s3 amounts to $1 / 2 \theta$ - half that under contract (c). Likewise, in state $s 1$ there is shading of $1 / 2 \theta$ because S would prefer B to choose $\left(p_{1}, p_{0}\right)=(91 / 2,1 / 2)$ leading to trade at $p_{1}=91 / 2$; and in state $s 2$ there is shading of $\theta$ because $S$ would prefer $B$ to choose $\left(p_{1}, p_{0}\right)=(9,1)$ leading to no trade at $p_{0}=1$. Overall, it therefore could be argued that expected total surplus from contract ( f ) is

$$
(9-1 / 2 \theta) \pi_{1}+(10-\theta) \pi_{2}+(1-1 / 2 \theta) \pi_{3},
$$

which means that if, for example, $\pi_{1}+2 \pi_{2}<\pi_{3}$ and $\theta=1$, contract (f) would dominate contracts (a), (b) and (c).

Once (A1) and, in particular, (A2) are invoked, however, this conclusion changes. To see why, we first need to check what would happen under contract (f) if either party walked away and refused to trade. $B$ would minimize $p_{0}$ by choosing $\left(p_{1}, p_{0}\right)=(10,0)$. So, in reckoning how low they can push the other's payoff, each party thinks in terms of a default no-trade price of $\mathrm{p}_{0}{ }^{*}$ $=0$. Precisely, $\mathrm{p}_{0}{ }^{*}$ is the right-hand side (RHS) of S's participation constraint; and $\left(-\mathrm{p}_{0}{ }^{*}\right)$ is the RHS of B's participation constraint.

Knowing this, we can invoke (A2) to calculate aggrievement levels in each state. In particular, in state $s 3$, when $B$ chooses $\left(p_{1}, p_{0}\right)=\left(9^{1 / 2}, 1 / 2\right), B$ is actually aggrieved by $1 / 2$ because he would prefer to choose $\left(p_{1}, p_{0}\right)=(9,1)$, with $S$ committing to trade - from $(A 2)$, we see that this would not violate $S$ 's participation constraint given $p_{0}{ }^{*}=0 . S$ is also aggrieved by $1 / 2$ because she would prefer $B$ to choose $\left(p_{1}, p_{0}\right)=(10,0)$ and commit to trade. In aggregate, shading in state $s 3$ amounts to $(1 / 2+1 / 2) \theta$ - the same as under contract (c). One can in fact show that the two contracts, (f) and (c), deliver the same expected total surplus.

All this begs the question: assuming (A1) and (A2), can non-zero values of $\mathrm{p}_{0}$ ever help? If not, to find an optimal contract, can we narrow down the class that we need to consider, e.g., restrict attention to contracts in which one party unilaterally chooses the terms of trade?

A general contract C can be viewed as a stochastic mechanism mapping from a pair of messages $\beta$ and $\sigma$, reported by B and S respectively, onto either a pair of (trade, no-trade) prices, $\left(p_{1}, p_{0}\right)$, or onto simply a no-trade price, $p_{0}$. In other words, following the report of messages $\beta$
and $\sigma$, there is an exogenous lottery to determine (i) if trade is allowed or not; (ii) the terms of trade/no-trade. If trade is allowed then it occurs if and only if both parties want it (at price $\mathrm{p}_{1}$ ). Otherwise, there is no trade (at price $\mathrm{p}_{0}$ ). (Remember we are assuming no renegotiation, so if the mechanism specifies no trade then that outcome is final.)

In effect, then, a mechanism allows for probabilistic trade - a surrogate for fractional trade (our widget is assumed to be indivisible).

Under contract C, $\mathrm{p}^{*}$ (the default no-trade price used to determine the RHS's of the parties' participation constraints) is the value of the zero-sum game over the $\mathrm{p}_{0}$ 's specified in the mechanism - under the supposition that one or other party "quits", i.e. always chooses to veto trade, so that the specified $\mathrm{p}_{1}$ 's are irrelevant. Let $\left(\widetilde{\beta}^{*}, \widetilde{\sigma}^{*}\right)$ be the (possibly mixed) equilibrium strategies of this zero-sum message game.

We consider, for each state (v, c), a (possibly mixed strategy) subgame perfect equilibrium of the game induced by contract C . Let $\mathrm{q}(\mathrm{v}, \mathrm{c})$ be the probability of trade in equilibrium, and $p(v, c)$ the expected payment from $B$ to $S$. Since trade is voluntary, $q(v, c)>0$ only if $\mathrm{v} \geq \mathrm{c}$. If there is more than one equilibrium, we pick the one that maximizes $\mathrm{q}(\mathrm{v}, \mathrm{c})$.

No party is worse off than if he or she quit:
$\underline{\text { Lemma } \quad v q(v, c)-p(v, c) \geq-p_{0}{ }^{*}, ~}$

$$
\mathrm{p}(\mathrm{v}, \mathrm{c})-\mathrm{cq}(\mathrm{v}, \mathrm{c}) \geq \mathrm{p}_{0}{ }^{*} .
$$

Proof Consider B. In state (v, c) he could deviate from his equilibrium message-reporting-cum-trading strategy to report $\widetilde{\beta}^{*}$ and always refuse to trade. Were he to do so, his payoff would drop from $\mathrm{vq}(\mathrm{v}, \mathrm{c})-\mathrm{p}(\mathrm{v}, \mathrm{c})$ to, say, $\left(-\hat{p}_{0}\right)$, where $\hat{p}_{0}$ is the ensuing (expected) no-
trade price. But $\hat{p}_{0}$ cannot be any less than $\mathrm{p}_{0}{ }^{*}$, because $\widetilde{\sigma}^{*}$ is a best reply to $\widetilde{\beta}^{*}$ for S in the zero-sum game over the $\mathrm{p}_{0}$ 's. This proves the first inequality in the Lemma. The second follows symmetrically - reversing the roles of B and S .

QED

For future reference, let $H \subseteq[0,1] \times R$ denote the convex hull of $(0,0)$ and all pairs

$$
\left\{\mathrm{q}(\mathrm{v}, \mathrm{c}), \mathrm{p}(\mathrm{v}, \mathrm{c})-\mathrm{p}_{0} *\right\},
$$

where, notice, we are netting prices by subtracting $\mathrm{p}_{0}{ }^{*}$. In the space of quantity $(\mathrm{q})$ and net price ( $\mathrm{p}-\mathrm{p}_{0}{ }^{*}$ ), the set H might look as follows:


Figure 3

Invoking (A1) and (A2), in state (v, c) we define B's [resp. S's] "aspiration level" b(v, c) [resp. $\mathrm{s}(\mathrm{v}, \mathrm{c})$ ] to be his [resp. her] maximum payoff across all correlated message pairs and trading rules subject to the constraint that S [resp. B$]$ gets no less than $\mathrm{p}_{0}{ }^{*}\left[\operatorname{resp.}\left(-\mathrm{p}_{0}{ }^{*}\right)\right]$.

In particular, $B$ and $S$ each imagine that they could jointly precommit to the (mixed) message-cum-trading equilibrium strategies pertaining to some other state, or some convex combination thereof.

Hence
(i) $b(v, c) \geq \max \left\{v q-p \mid\left(q, p-p_{0}{ }^{*}\right) \in H\right.$ and $\left.p-c q \geq p_{0}{ }^{*}\right\}$, $q, p$
(ii)

$$
s(v, c) \geq \max _{q, p}\left\{p-c q \mid\left(q, p-p_{0}^{*}\right) \in H \text { and } v q-p \geq-p_{0} *\right\} .
$$

Note that, thanks to the Lemma,

$$
\begin{aligned}
& b(v, c) \geq v q(v, c)-p(v, c), \\
& s(v, c) \geq p(v, c)-c q(v, c),
\end{aligned}
$$

i.e., aspiration levels are at least as high as equilibrium payoffs.

In each state ( $\mathrm{v}, \mathrm{c}$ ), once equilibrium play is over, B shades by reducing S's payoff down
to

$$
p(v, c)-c q(v, c)-\theta\{b(v, c)-[v q(v, c)-p(v, c)]\} .
$$

And S shades by reducing B's payoff down to

$$
v q(v, c)-p(v, c)-\theta\{s(v, c)-[p(v, c)-c q(v, c)]\} .
$$

Hence, in the special case $\theta=1$ (the case considered in the Proposition below), total surplus in this equilibrium equals
(iii) $2(v-c) q(v, c)-b(v, c)-s(v, c)$.

Now consider contract $\hat{C}$, in which $\mathrm{p}_{0} \equiv 0$ and one party (say B - it doesn't matter who) chooses from a set of exogenous lotteries, each corresponding to a different point $(q, p) \in H$ :

$$
\left\{\begin{array}{l}
\text { with probability } \mathrm{q}, \text { trade is allowed at } p_{1}=\frac{p}{q} \\
\text { with probability } 1-\mathrm{q}, \text { trade is not allowed. }
\end{array}\right.
$$

At first sight contract $\hat{C}$ may look a little strange, but that is because it is dealing with probabilistic trade. Note that for $\mathrm{q}=1$ - corresponding to the right-hand edge of the set H in Figure 3 - contract $\hat{C}$ is nothing more than our "standard" contract in which B chooses a trading price $p_{1}$ from an interval $[p, \bar{p}]$. To put this another way, if the upper and lower edges of the set $H$ in Figure 3 were linear rather than piecewise linear, $H$ would correspond to a standard contract.

Proposition Suppose $\theta=1$. Then contract $\hat{C}$ yields at least as much total surplus in each state as does contract C .

Proof Under contact $\hat{C}$, in state (v, c), B's aspiration level is

$$
\hat{b}(v, c)=\max _{q, p}\{v q-p \mid(q, p) \in H \text { and } p-c q \geq 0\}
$$

(iv) $\quad \leq \mathrm{p}_{0} *+\mathrm{b}(\mathrm{v}, \mathrm{c})$
by (i). And S's aspiration level is

$$
\hat{s}(v, c)=\max _{q, p}\{p-c q \mid(q, p) \in H \text { and } v q-p \geq 0\}
$$

$$
\begin{equation*}
\leq-\mathrm{p}_{0}{ }^{*}+\mathrm{s}(\mathrm{v}, \mathrm{c}) \tag{v}
\end{equation*}
$$

by (ii). In each state ( $\mathrm{v}, \mathrm{c}$ ), B chooses the lottery corresponding to a point $(q, p) \in H$ to maximise his net payoff (i.e. net of S's shading), taking into account that $S$ may not be willing to trade at $p_{1}=\frac{p}{q}$ for $\mathrm{q} \neq 0$. That is, given $\theta=1, \mathrm{~B}$ chooses $(q, p) \in H$ to maximise $v q-p-\{\hat{s}(v, c)-[p-c q]\}$ subject to $\mathrm{p}-\mathrm{cq} \geq 0$.

If $\mathrm{v}>\mathrm{c}$, in effect B will maximise the probability of trade, q , subject to $(q, p) \in H$ for some $\mathrm{p} \geq \mathrm{cq}$. Call this maximum $\hat{q}(v, c)$. But from the definition of the set H ,

$$
\left(q(v, c), p(v, c)-p_{0}{ }^{*}\right) \in H ;
$$

and from the second inequality in the Lemma,

$$
p(v, c)-p_{0} * \geq c q(v, c) .
$$

Hence $\hat{q}(v, c)$ is at least $q(v, c)$ whenever $v>c$.

If $\mathrm{v}<\mathrm{c}, \mathrm{B}$ will choose $(\mathrm{q}, \mathrm{p})=(0,0)$; in this case set $\hat{q}(v, c)=0$.

Combining these two cases, we have
(vi) $\quad(v-c)[\hat{q}(v, c)-q(v, c)] \geq 0$ in all states (v, c).

Just as total surplus in state ( $\mathrm{v}, \mathrm{c}$ ) was given by expression (iii) under contract C , so too under contract $\hat{C}$ it is given by
(vii) $\quad 2(v-c) \hat{q}(v, c)-\hat{b}(v, c)-\hat{s}(v, c)$.

But, making use of inequalities (iv) - (vi), we see that the expression in (vii) is no less than that in (iii).

QED

In words, the Proposition states that, without loss of generality, $\mathrm{p}_{0}$ can be normalized to zero and one party (B, say) can be given control over the terms of trade. The subset H of $[0,1] \times R$ is the "design variable". It is this set that the contract specifies, taking any shape (along
the lines of that in Figure 3) - but it must be convex and, for $\mathrm{q}=0$, come to a point at the origin (i.e., when $\mathrm{q}=0, \mathrm{p}=\mathrm{p}_{0}{ }^{*}=0$ ).

Of course, the weakness of this result is that it applies only to the limit case $\theta=1$. However the Proposition is suggestive of other more general results. For example, it may be quite general that (A1) and (A2) are enough to allow us to ignore the possibility of non-zero values of $p_{0}$. The Proposition as it stands may apply if $\theta$ is close enough to 1 . And for lower values of $\theta$, somewhat more complex allocations of control rights over the terms of trade (not merely giving unilateral control to either B or S) may turn out to be optimal. All this awaits further research.

## Appendix B

In this appendix we prove Proposition 4.1.
Think of composers as lying in the [0,1] interval, with $\lambda=0$ corresponding to Bach and $\lambda$ $=1$ corresponding to Shostakovich. In state $s 2$, the value of composer $\lambda$ to $B$ is $v-\lambda \Delta$, and the cost to S is $\mathrm{c}-\lambda \delta$. (That is, $\lambda$ is equivalent to a convex combination of Bach and Shostakovich.) In state $s 3$, the value of $\lambda$ to B is $\mathrm{v}-(1-\lambda) \Delta$, and the cost to S is $\mathrm{c}-(1-\lambda) \delta$.

As a preliminary, we should observe that if no music is played in state $s 4$ then in the other three states the first-best could be achieved using a contract that fixes the price at v and has B choose the composer. In state s2, B would choose $\lambda=0$ (as first-best requires) but there would be no aggrievement on the part of S since at price v no other composer would satisfy B's participation constraint. Likewise in state s3, B would choose $\lambda=1$ and there would be no aggrievement. In state s1, there would be no aggrievement either (B could choose any composer). At this high price B would be unwilling to trade in state s 4 . Overall, expected total surplus would be $\left(1-\pi_{4}\right)(\mathrm{v}-\mathrm{c})$. For small enough $\pi_{4}$, this would be the optimal contract. But we see this as a peculiar case, which we can later confirm is ruled out if $\pi_{4} /\left(1-\pi_{1}\right)$ is above the lower bound in Proposition 4.1. (Incidentally, this is the role of state s4 in the model when $\Delta>$ $\delta$. State s1 plays an analogous role when $\Delta<\delta$.)

From now on, we suppose that music is played in state $s 4$, at price $\mathrm{p}_{4}$, which must lie below B's value, $v-\Delta$. It is straightforward to confirm that in states $s 2, s 3$ and $s 4$ music is also played under an optimal contract. Let the price be $p_{1}$ in state $s 1$. In state $s 2$, suppose composer $\lambda_{2}$ is played at price $\mathrm{p}_{2}$. Since we are restricting attention to symmetric contracts, in state s3 composer $1-\lambda_{2}$ is played, also at price $p_{2}$.

The method we will use to characterize an optimal contract is to include in our mathematical programme only those inequality constraints that are critical. At the end, we will need to confirm that the (many) missing constraints are satisfied. In particular, for now we shall ignore the question of who controls the choice of composer and price.

Let $\mathrm{a}_{2}$ be the total level of aggrievement (B's plus S's) in state s 2 when $\lambda_{2}$ is played at price $\mathrm{p}_{2}$. (By symmetry, $\mathrm{a}_{2}$ is also the total level of aggrievement in state s 3 .) Now S may prefer to switch from composer $\lambda_{2}$ to composer $1-\lambda_{2}$ at the same price $\mathrm{p}_{2}$ (which is admissible in the contract, since that is what occurs in state s3), to reduce her costs from $\mathrm{c}-\lambda_{2} \delta$ to $\mathrm{c}-\left(1-\lambda_{2}\right) \delta-$ unless this switch would violate B's participation constraint, $\mathrm{v}-\left(1-\lambda_{2}\right) \Delta-\mathrm{p}_{2} \geq 0$, in which case the best S could wish for is to switch to composer $\frac{v-p_{2}}{\Delta}$ at price $\mathrm{p}_{2}$ and reduce her costs from c $-\lambda_{2} \delta$ to $c-\frac{v-p_{2}}{\Delta} \delta$. Thus
(i) $\quad \mathrm{a}_{2} \geq \delta \min \left\{1-2 \lambda_{2}, \frac{v-p_{2}}{\Delta}-\lambda_{2}\right\}$.

Let $a_{1}$ be the total level of aggrievement in state $s 1$. Given that $p_{4} \leq v-\Delta$, $a_{1}$ must be at least $p_{2}-v+\Delta$, irrespective of the price $p_{1}$ (if $p_{1}$ does not lie between $v-\Delta$ and $p_{2}$ then $a_{1}$ will be higher still):
(ii) $\quad \mathrm{a}_{1} \geq \mathrm{p}_{2}-\mathrm{v}+\Delta$.

If $a_{4}$ is the total level of aggrievement in state $s 4$, consider the relaxed programme:
Choose $\lambda_{2}, \mathrm{p}_{2}, \mathrm{a}_{1}, \mathrm{a}_{2}$ and $\mathrm{a}_{4}$ to maximize
(iii) $\mathrm{W} \equiv \pi_{1}\left[\mathrm{v}-\mathrm{c}-\theta \mathrm{a}_{1}\right]$

$$
\begin{array}{r}
+\left(1-\pi_{1}-\pi_{4}\right)\left[\mathrm{v}-\mathrm{c}-\lambda_{2}(\Delta-\delta)-\theta \mathrm{a}_{2}\right] \\
+\pi_{4}\left[\mathrm{v}-\mathrm{c}-\Delta+\delta-\theta \mathrm{a}_{4}\right]
\end{array}
$$

subject to (i), (ii) and the constraint that total aggrievement is always nonnegative:
(iv) $a_{1} \geq 0, a_{2} \geq 0, a_{4} \geq 0$.

In a solution to this relaxed programme, the tighter of the lower bound constraints on $a_{1}$ will bind. Likewise for $\mathrm{a}_{2}$. And $\mathrm{a}_{4}=0$.

Now consider the role of $p_{2}$, which affects $W$ in (iii) only via $a_{1}$ and $a_{2}$. Via $a_{1}$, the slope of W w.r.t. $p_{2}$ is $\left(-\pi_{1} \theta\right)$ if $p_{2} \geq v-\Delta$, and is zero otherwise. Via $a_{2}$, the slope of W w.r.t. $p_{2}$ is at most $\left(1-\pi_{1}-\pi_{4}\right) \theta \delta / \Delta$, and is zero if $p_{2}<v-\left(1-\lambda_{2}\right) \Delta$. Hence, from the lower bound on $\pi_{1} /(1-$ $\pi_{4}$ ) in Proposition 4.1, $\mathrm{p}_{2}$ should be as small as possible. However, there is no value in pushing $\mathrm{p}_{2}$ below $\mathrm{v}-\Delta$ since this would not affect W .

Next, consider the role of $\lambda_{2} \in[0,1]$, given $p_{2}=v-\Delta . \quad \lambda_{2}$ affects $W$ only through the middle term in (iii), and via $\mathrm{a}_{2}$. Values of $\lambda_{2}$ above $1 / 2$ are clearly not optimal. If $\Delta>(1+2 \theta) \delta$, $\lambda_{2}$ should be zero; whereas if $\Delta<(1+2 \theta) \delta, \lambda_{2}$ should equal $1 / 2$. These are two conditions that appear in Proposition 4.1.

Where does this leave us? On the one hand, if $\Delta>(1+2 \theta) \delta$ we can implement the above solution to the relaxed programme (viz., $\mathrm{p}_{2}=\mathrm{v}-\Delta, \lambda_{2}=0, \mathrm{a}_{1}=\mathrm{a}_{4}=0$ and $\mathrm{a}_{2}=\delta$ ) using a contract in which the price is fixed at $\mathrm{v}-\Delta$ and B chooses the composer. (Actually, any fixed price between cand $v-\Delta$ would yield the same W.) On the other hand, if $\Delta<(1+2 \theta) \delta$ we can implement the solution (viz. $\mathrm{p}_{2}=\mathrm{v}-\Delta, \lambda_{2}=1 / 2$ and $\mathrm{a}_{1}=\mathrm{a}_{2}=\mathrm{a}_{4}=0$ ) using a contract in which the price is again fixed at $\mathrm{v}-\Delta$, but so too is the composer, at $\lambda_{2}=1 / 2$. (Actually in this latter case, it doesn't matter which composer is fixed; it could instead be Bach, $\lambda_{2}=0$.) The fact that in all cases the solution to the relaxed programme can be implemented using some contract vindicates our earlier decision to omit the other inequality constraints.

Proposition 4.1 is proved.
QED

The reader may wonder why the auxiliary condition $\pi_{1} /\left(1-\pi_{4}\right) \geq \delta /(\Delta+\delta)$ is needed in Proposition 4.1. Consider the following variant to a simple employment contract: the basic wage is $\mathrm{v}-\Delta$ in return for performing $\lambda=1 / 2$, but B can pay a supplement $\Delta / 2$ in return for asking $S$ to perform some $\lambda$ in $[0,1]$. In states $s 1$ and $s 4$, $B$ will pay the basic wage only and $\lambda=1 / 2$ will be performed. And in state s2 [resp. s3], B will pay the supplement and ask $S$ to perform $\lambda=0$ [resp. $\lambda=1]$ - note that without factoring in the impact of shading by $\mathrm{S}, \mathrm{B}$ is indifferent about paying the supplement in states $s 2$ and $s 3$, but it can be checked that $S$ shades even more if $B$ does not pay it. In each of states $s 1$, $s 2$ and $s 3$, $S$ would wish that $B$ had paid the supplement and asked her to perform $\lambda=1 / 2$. So she is aggrieved by $\Delta / 2$ in state s 1 , and by $\delta / 2$ in states s 2 and s 3 .
(In state s2, it would be unreasonable for $S$ to wish that $B$ had asked her to perform any $\lambda>1 / 2$ because that would violate B's participation constraint. Similarly, in state s3, any $\lambda<1 / 2$ would be unreasonable.) In state $s 4$, $S$ cannot wish that $B$ pay more than his value, $v-\Delta$, so there is no aggrievement. Overall, then, the contract yields expected total surplus

$$
\pi_{1}[\mathrm{v}-\mathrm{c}-\theta(\Delta / 2)]+\left(1-\pi_{1}-\pi_{4}\right)[\mathrm{v}-\mathrm{c}-\theta(\delta / 2)]+\pi_{4}[\mathrm{v}-\mathrm{c}-\Delta+\delta] .
$$

This can be strictly greater than the expected total surplus yielded by any simple employment contract if the auxiliary condition $\pi_{1} /\left(1-\pi_{4}\right) \geq \delta /(\Delta+\delta)$ is not satisfied. For example, if $v=20$, $\mathrm{c}=10, \Delta=6, \delta=2, \pi_{1}=0.1$ and $\pi_{4}=0.5$, then the above contract yields an expected total surplus $8-(7 \theta / 10)$, whereas the best simple employment contract (viz., B chooses any $\lambda$, and the wage is fixed at $\mathrm{v}-\Delta)$ yields only $8-(4 \theta / 5)$.

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[^0]:    ${ }^{1}$ For up-to-date syntheses of the classical view, see Bolton and Dewatripont (2005) and Shavell (2004).

[^1]:    ${ }^{2}$ See, e.g., Grossman and Hart (1986) and Hart and Moore (1990).
    ${ }^{3}$ For a discussion, see Holmstrom (1999).
    ${ }^{4}$ One obvious possibility is to introduce asymmetric information. To date such an approach has not been very fruitful in the theory of the firm. But see Matouschek (2004).
    ${ }^{5}$ We do not go as far as some of the recent incomplete contracting literature that supposes that ex post trade is not contractible at all (see, e.g., Baker et al. (2006)).
    ${ }^{6}$ The perfunctory and consummate language is taken from Williamson (1975, p. 69).

[^2]:    ${ }^{7}$ For a discussion and examples, see Goldberg and Erickson (1987, p. 388).

[^3]:    ${ }^{8}$ Since a court can determine whether trade took place, any payments that B has promised S conditional on trade must be made: if not, S would sue for breach of contract. In other words, payments are part of perfunctory performance.

[^4]:    ${ }^{9}$ This idea is consistent with the large behavioral economics literature that has examined altruism, reciprocity, and retaliation. For example, in the ultimatum game (see, e.g., Guth et al. (1982)), a suggested split of surplus by the proposer that is seen as "greedy" will often elicit retaliation in the form of rejection by the responder, even though this is costly for the responder. See Camerer and Thaler (1995) for a discussion, and Andreoni et al. (2003) for experimental evidence for the case where the responder can scale back the level of trade rather than rejecting trade entirely. Other important works on reciprocity and retaliation include Akerlof (1982), Akerlof and Yellen (1990), Fehr and Schmidt (1999), Rabin (1993), Fehr et al. (1997), and Bewley (1999); for surveys see Fehr and Gachter (2000) and Sobel (2005). MacLeod (2003) models the role of retaliation in sustaining accurate assessments of worker performance. Direct empirical evidence of retaliation by employees or contractors in response to "bad treatment," in the form of poor performance, negligence, or sabotage, can be found in Lord and Hohenfeld (1979), Giacalone and Greenberg (1997), Greenberg (1990), Krueger and Mas (2004), and Mas (2006).
    ${ }^{10}$ Note that the experimental evidence of Falk et al. (2003) is consistent with the idea that whether a person feels well treated depends not only on the outcome that occurs but also on what other outcomes were available (see also Camerer and Thaler (1995)).
    ${ }^{11}$ The notion of a reference point has played an important role in the recent behavioral economics literature, including that concerned with contractual relationships. Kahneman et al. (1986) provide evidence that for transactions between firms and consumers customers use past prices as a reference point for judging the fairness of a transaction. See also Okun (1981), Falk et al. (2006), Frey (1997, Chapter 2), and Gneezy and Rustichini (2000) for related ideas. Akerlof and Yellen (1990) consider the importance of reference groups in the determination of a fair wage. Benjamin (2006) analyzes the implications of reference points for optimal incentive schemes, and Carmichael and MacLeod (2003) for the hold-up problem. Our paper owes a lot to the above literature, but differs from it in supposing that a contract governing a transaction is a reference point for the transaction itself.

[^5]:    ${ }^{12}$ See, e.g., Hastorf and Cantril (1954), Messick and Sentis (1979), Ross and Sicoly (1979), and Babcock et al. (1995). For discussions, see Babcock and Loewenstein (1997) and Bazerman (1998, pp 94-101).
    ${ }^{13}$ For example, suppose that the parties' feelings of entitlement are stochastic: with some probability the parties' entitlements sum to more than the total surplus available from the contract, and with some probability to less. Then the model goes through as long as parties who receive more than their entitlements do not "over-provide" consummate performance to the point that this makes up for the perfunctory performance that they provide when they receive less than their entitlements. For some experimental evidence consistent with the idea that an increase in performance from receiving a dollar more than one's entitlement is less than the decrease in performance from receiving a dollar less, see Charness and Rabin (2002) and Offerman (2002). Note that conflicting notions of entitlement may also arise because of differences in information about the total surplus available. See Ellingsen and Johannesson (2005).
    ${ }^{14}$ See Rabin (1995) and Benabou and Tirole (2006) for interesting efforts to model self-serving biases.

[^6]:    ${ }^{15}$ For further discussion and examples of seller shading, see Goldberg and Erickson (1987, p. 388), Lord and Hohenfeld (1979), Giacalone and Greenberg (1997), Greenberg (1990), Krueger and Mas (2004), and Mas (2006). One approach to solving the problem of seller shading is for the buyer to withhold some part of his payment until after the transaction is complete, i.e., to offer a bonus or tip to the seller for consummate performance. Such a solution is more likely to work in a repeated relationship than in the one-shot exchange studied here; in a one-shot situation the buyer would have an incentive to claim that the performance wasn't consummate even if it was (another possibility is that the buyer, fearing the seller's wrath, would pay the bonus regardless of the quality of performance). For a more positive view of the effectiveness of bonus schemes, see Scott (2003).

[^7]:    ${ }^{16}$ If B's value of 100 and S's cost of zero were objective (i.e., verifiable), the parties might well agree that the fair outcome is to split the difference and set $\mathrm{p}=50$. We have in mind a more complex situation, where, because value and cost are observable but not verifiable, there is some flexibility in how the parties interpret these variables; this opens the door to conflict. While we do not formalize this notion of flexibility or fuzziness, it would clearly be desirable to do so in the future.

[^8]:    ${ }^{17}$ Note the role of the assumption that $0<\theta \leq 1$. If $\theta>1$, the shading costs are so large that the parties will not trade at all at date 1 in the absence of a date 0 contract. Although we rule out the case $\theta>1$, this does not mean that it is uninteresting.

[^9]:    ${ }^{18}$ Note that we are ignoring "efficiency wage" considerations in our analysis. Regardless of date 0 market conditions, B might well feel that it makes sense to offer $S$ a price in excess of cost in order to encourage better performance (see, e.g., Shapiro and Stiglitz (1984)). However, note that efficiency wage ideas are not inconsistent with our approach. Our view is that, whatever the level of the price, it still makes sense for $B$ and $S$ to fix price in advance in order to avoid argument about the right price later.

[^10]:    ${ }^{19}$ To the extent that the role of the contract is to embody and anchor entitlements, the fact that the contract is legally binding is perhaps of secondary importance. Much of our analysis goes through if the contract is viewed as a nonbinding agreement. See in particular the discussion of agreements to agree in Section 3. Note, however, that legal enforcement may be important in ensuring perfunctory performance, including that B pays S for goods or services received at date 1 .
    ${ }^{20}$ To emphasize this point, suppose that B and S are in a situation of bilateral monopoly already at date 0 , i.e., the date 0 market is no more competitive than the date 1 market. Then, if they agree on $p=50$, say, at date 0 , our view is that each will be aggrieved by 50 and this aggrievement will carry over to date 1 and lead to shading.

[^11]:    ${ }^{21}$ This assumption is taken from Hart and Moore (1988).
    ${ }^{22}$ See, e.g., Corbin (1993, Chapter 2.8) and Farnsworth (1999, pp. 207-222).
    ${ }^{23}$ See, e.g., Corbin (1993, Chapters 2 and 4) and Farnsworth (1999, pp. 207-222).
    ${ }^{24}$ Of course, under the agreement-to-agree interpretation, $\mathrm{p}_{0}$ is necessarily zero.

[^12]:    ${ }^{25}$ See Maskin (1999).

[^13]:    ${ }^{26}$ Of course, even if the contract specifies that $\mathrm{p}=9$ always, as in contract (a), B could in principle unilaterally raise the price to 10 at date 1 to recognize S's efforts. But this would be an act of charity; our assumption is that S does not expect this and is not disappointed when it does not occur.
    ${ }^{27}$ See, e.g., Corbin (1993, Chapter 4.3), Ben-Shahar (2004, pp. 424-425).

[^14]:    ${ }^{28}$ It is worth revisiting our assumptions about aggrievement and shading at this point. Take contract (b), where price is fixed and $B$ chooses the composer. Why is $S$ aggrieved in $s 2$ or $s 3$ when $B$ chooses the high cost composer? In Sections 2 and 3 we offered one justification: S may feel that she has contributed to surplus and should be rewarded with the low cost composer. Here we offer an alternative justification based on subjective views of valuation. Suppose that the state is s2. S can tell herself the following. It's true that B thinks that Bach is worth $\Delta$ more to him than Shostakovich, while the incremental cost to me is only $\delta$, and B therefore feels justified in choosing Bach. However, I, S, think that, even though B doesn't realize this, the true value of Bach is close to v $\Delta$. (Self-serving biases are behind this belief.) Given this, I do not think that B's request is reasonable, and if he insists on Bach, I will respond by shading on performance.

[^15]:    ${ }^{29}$ But see Wernerfelt (1997).
    ${ }^{30}$ This notion of independent contracting differs from Simon's. Simon views independent contracting as corresponding to the case where the price and composer are both fixed.
    ${ }^{31}$ The distinction between an employee and an independent contractor shouldn't be taken too literally. An independent contractor will sometimes do what a buyer tells her to, and an employee sometimes will not. Still, in general terms the notion that an employee is subject to the authority of a boss whereas an independent contractor is not seems valid. For a discussion, see, e.g., Coase (1937, p. 403-404).

[^16]:    ${ }^{32}$ For an interesting discussion of this kind of phenomenon and its implications for labor market practices, see Akerlof and Yellen (1990).

[^17]:    ${ }^{33}$ See Hart and Moore (2004).
    ${ }^{34}$ See Restatement (Second) of Contracts, Section 84(a)(1979); Farnsworth (1999, pp. 276-95); Jolls (1997, pp. 228301); Muris (1981, particularly p. 530); and Shavell (2005).
    ${ }^{35}$ In fact, without some constraint on price changes, a long-term contract would have little meaning. Almost every contract is incomplete in the sense that some ex ante noncontracted-for cooperation is required ex post for the contract to succeed. If each party can demand a large sidepayment for that cooperation that is completely unrelated to costs - you want a glass of water that will cost you $\$ 1,000$ - the initial contract will be vitiated.

[^18]:    ${ }^{36}$ See, e.g., Tullock (1967).
    ${ }^{37}$ See, e.g., Milgrom (1988).

[^19]:    ${ }^{38}$ But see Bajari and Tadelis (2001). Bajari and Tadelis consider a model of the trade-off between flexibility and rigidity, where rigidity, in the form of a fixed price, is good because it encourages efficient cost reduction by the seller S, but bad because it impedes ex post adjustment. Our model has somewhat similar ex post characteristics to Bajari and Tadelis's, but ignores ex ante incentives for cost minimization.

[^20]:    ${ }^{39}$ See, e.g., Camerer and Thaler (1995) and the references in footnote 9.
    ${ }^{40}$ See, e.g., Kahneman and Tversky (1979).

