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Nicholas Barberis
Ming Huang
Tano Santos

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ABSTRACT

We propose a new framework for pricing assets, derived in part from the traditional consumption-based approach, but which also incorporates two long-standing ideas in psychology: prospect theory, and evidence on how prior outcomes affect risky choice.

Consistent with prospect theory, the investor in our model derives utility not only from consumption levels but also from changes in the value of his financial wealth. He is much more sensitive to reductions in wealth than to increases, the ``loss-aversion" feature of prospect utility. Moreover, consistent with experimental evidence, the utility he receives from gains and losses in wealth depends on his prior investment outcomes; prior gains cushion subsequent losses -- the so-called “house-money" effect -- while prior losses intensify the pain of subsequent shortfalls.

We study asset prices in the presence of agents with preferences of this type, and find that our model reproduces the high mean, volatility, and predictability of stock returns. The key to our results is that the agent's risk-aversion changes over time as a function of his investment performance. This makes prices much more volatile than underlying dividends and together with the investor’s loss-aversion, leads to large equity premia. Our results obtain with reasonable values for all parameters.

Nicholas Barberis
Graduate School of Business
University of Chicago
1101 East 58th St.
Chicago, IL 60637
and NBER
nick.barberis@gsb.uchicago.edu

Ming Huang
Graduate School of Business
Stanford University
Stanford, CA 94305
mhuang@leland.stanford.edu

Tano Santos
Graduate School of Business
University of Chicago
1101 East 58th St.
Chicago, IL 60637
jesus.santos@gsb.uchicago.edu

1 Introduction

For many years now, the standard framework for pricing assets has been the consumption-based approach. Its shortcomings are well-known. With standard utility specifications and parameter values, it does not come close to capturing the stock market's high historical average returns and volatility, nor the striking variation in equity risk premia over time.[1] Over the past decade researchers have used more complicated specifications for utility over consumption in an attempt to approximate the data more closely.[2] However, even the most state-of-the-art approaches typically require controversial values for important parameters such as the investor's risk-aversion over consumption shocks.

In this paper, we argue that the puzzling empirical features of stock returns can be understood by extending the traditional framework in a way that captures two long-standing ideas in psychology: the prospect theory of Kahneman and Tversky (1979) and the evidence of Thaler and Johnson (1990) and others on the influence of prior outcomes.

The psychology literature has for some time now promoted prospect theory as a descriptive model of decision making under risk. This theory has proved helpful in explaining the numerous violations of the expected utility framework documented over the years. Its central feature is that the carriers of value are *changes* in wealth rather than absolute levels or final outcomes. It also emphasizes *loss aversion*, which stipulates that individuals are much more sensitive to reductions in wealth than to increases.

In our paper, we incorporate prospect theory into the standard consumption-based pricing framework: the agent in our model derives utility not only from consumption levels, but also from *changes in the value of his financial wealth* from year to year, in a way motivated by prospect utility; in particular, he is loss-averse over these wealth changes.

Our model specification is also guided by a second strand of research in psychology which finds that prior outcomes influence the way subsequent

[1] See for example, Hansen and Singleton (1983), Mehra and Prescott (1985), and Hansen and Jagannathan (1991).

[2] Recent papers in this line of research include Abel (1990), Campbell and Cochrane (1999), Constantinides (1990), Epstein and Zin (1989, 1991), and Sundaresan (1989). Another strand of the literature emphasizes market incompleteness due to uninsurable income shocks; see for example Heaton and Lucas (1996) and Constantinides and Duffie (1996). Cochrane (1998) and Kocherlakota (1996) provide excellent surveys.

gains and losses in wealth are experienced. Put differently, this work suggests that the utility derived from a specific wealth change is *not* the same in all circumstances. Thaler and Johnson (1990) find that a loss is *less* painful to people when it comes after substantial earlier increases in wealth: those earlier gains "cushion" the subsequent loss, making it more bearable. The analogy in financial markets is that investors may not care much about a stock market dip that follows substantial prior gains because they can still say that they are "up, relative to a year ago", say. Thaler and Johnson (1990) argue that this idea explains another of their findings, namely that people with recent gains act in a less risk-averse manner, taking on bets they would otherwise find unattractive. This result has been labelled the "house money" effect, reflecting gamblers' increased willingness to bet when ahead.

Conversely, there is evidence that after a loss, people tend to shy away from risky bets that they might otherwise take. Thaler and Johnson (1990) argue that this is because losses that come on the heels of other losses are *more* painful to investors than on average. An informal interpretation is that in the aftermath of a painful loss, while the investor is still reeling from the shock, he is particularly sensitive to additional setbacks, increasing his risk-aversion.

We incorporate this evidence on prior outcomes into our model: while our investor cares about yearly gains and losses in the value of his financial wealth, the pain of a loss is not the same in all circumstances but rather depends on his prior investment performance.

By modifying the traditional asset-pricing framework to capture both prospect theory and the effect of prior outcomes, we find that we are able to understand many of the hitherto perplexing features of aggregate data. In particular, starting from an underlying consumption growth process with *low* variance, our model can generate stock returns with a high mean, high volatility and significant predictability, while maintaining a riskless interest rate with low mean and volatility. More importantly, we achieve all this using reasonable values for all the model parameters.

In essence, our story is one of changing risk-aversion. After a run-up in stock prices, our agent is less risk-averse because those gains will cushion any subsequent loss. After a fall in stock prices, he becomes more wary of further losses and hence more risk-averse. This variation in risk-aversion allows returns in our model to be much more volatile than the underlying dividends: an unusually good dividend raises prices, but this price increase also makes the investor less risk-averse, driving prices still higher. We also generate predictability in returns much like that observed in the data: following a significant rise in prices, the investor is less risk-averse and subsequent returns are therefore on average lower.

Our framework also delivers a substantial equity premium without the high risk-aversion over consumption uncertainty that other models inevitably appeal to. The high volatility of returns in our model leads to frequent losses for stocks and those losses cause our loss averse investor considerable discomfort. A high equity premium is therefore required to convince him to hold stocks.

In an early application of prospect theory, Benartzi and Thaler (1995) examine single period portfolio choice for an investor with prospect-type utility. They find that loss aversion makes investors reluctant to invest in stocks, even in the face of a sizeable equity premium. This suggests that bringing prospect theory into a formal pricing model may help us understand the level of average returns. While our work confirms this, we find that loss aversion cannot *by itself* explain the equity premium; incorporating the effect of prior outcomes is a critical ingredient as well. To see this, we also examine a simpler model where prior outcomes are ignored and hence where the utility of gains and losses is the same, regardless of past history. The investor's risk-aversion is then constant over time and stock prices lose an important source of volatility. With less volatile returns and hence less risk, we are no longer able to produce a substantial equity premium. The intuition Benartzi and Thaler (1995) develop in *partial* equilibrium is therefore not immediately transferable to equilibrium pricing models. The additional evidence on the influence of prior outcomes on risky choice is also required.[3]

Our framework offers a distinct alternative to consumption-based models that attempt to understand the empirically observed high mean, high volatility, and significant predictability of equity returns. Campbell and Cochrane (1999) explain these empirical features using an external subsistence level for consumption which generates time-varying risk aversion. Although our model is also based on changing risk aversion, we generate it by introducing loss aversion over wealth fluctuations and incorporating evidence on how investors' loss aversion is affected by their prior returns.

Our treatment of loss aversion is also reminiscent of the study by Epstein and Zin (1990) which offers an explanation for the high equity premium by

[3]Shumway (1997) uses prospect theory to think about the cross-section of asset returns, as opposed to the time series of aggregate returns that we focus on here.

introducing "first order risk aversion" in a recursive utility specification (see also Epstein and Zin (1989, 1991) and Segal and Spivak (1990)). A model with first-order risk aversion is similar to a model of loss aversion that *ignores* the effect of prior outcomes. However, this literature has not yet attempted to address the excess volatility or predictability of equity returns.[4]

Shefrin and Statman (1985) and Odean (1998) have used prospect theory to try to understand the so-called disposition effect, the tendency of investors to sell winning rather than losing stocks. A premise of these papers is that the investor only experiences utility from investment gains or losses when they are *realized* through a sale of stock. This is reasonable for thinking about short-term trading behavior, but is less attractive in the longer-term: an investor is likely to feel the pain of a substantial long-term decrease in a stock's value even before he sells it. Since our focus is understanding low frequency stock market behavior, we do not distinguish between unrealized and realized gains and losses.

Another set of papers, including Barberis, Shleifer, Vishny (1998) and Daniel, Hirshleifer, Subrahmanyam (1998) explain some empirical features of asset returns by assuming that investors exhibit irrationality when making forecasts of quantities such as cashflows. Other papers, including Hong and Stein (1999), suppose that investors are only able to process subsets of available information. In this paper, we take a different approach. While we do modify the investor's preferences to reflect experimental evidence about the sources of utility, the investor remains fully rational and dynamically consistent throughout.[5]

Rather than introduce both prospect theory and the effect of prior outcomes at once, we develop them one at a time. In Section 2, we show how to bring prospect theory into an asset pricing framework, while ignoring the issue of prior outcomes. We investigate the model's ability to explain aggregate data, and find that it is at best an incomplete description of the facts. In Section 3, we argue that the missing ingredient may be the evidence on prior outcomes, and support this claim by extending our model to account for this evidence and presenting a detailed numerical analysis of the equilibrium in this case. Section 4 concludes.

[4]See also Bekaert, Hodrick, and Marshall (1997) for a study of the foreign exchange, equity, and bond markets of the US and Japan using a model based on first-order risk aversion.

[5]See Shleifer (1999) for a recent treatment of irrationality in financial markets.

2 A Preliminary Model with Prospect Utility

Prospect theory was first proposed by Kahneman and Tversky (1979) as a descriptive model of decision making under risk. Its key feature is that the carriers of value are not absolute levels or final outcomes, but rather gains and losses in wealth. These gains and losses are measured relative to a reference point, typically taken to be the status quo. In a financial context, we interpret this as saying that people care not only about consumption levels, but also about *fluctuations in the value of their financial wealth.*[6] In this section, we show how this natural source of utility can be incorporated into asset pricing models.[7]

Our starting point is the traditional consumption-based asset pricing model proposed by Lucas (1978). In this economy, there are a continuum of identical infinitely-lived agents, with a total "mass" of one, and two assets: a riskfree asset in zero net supply, paying a gross interest rate of $R_{f,t}$ between time t and $t+1$; and a risky asset with a total supply of one unit. In the usual way, the risky asset ("stock") is a claim to a stream of perishable output represented by the dividend sequence $\{D_t\}$. We use R_{t+1} to denote its gross return between time t and time $t+1$.

Each agent is endowed with a unit of the risky asset at time 0. In equilibrium, he holds this amount of the risky asset at all times and consumes the dividend stream, so that each period, the dividend D_t equals aggregate consumption $\overline{C}_t$.[8] Dividend growth, or equivalently aggregate consumption growth, follows an i.i.d. lognormal process,

$$\log\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right) = g + \sigma\epsilon_{t+1}, \tag{1}$$

[6] It is important for our story that the investor worry about fluctuations in *financial* rather than total wealth. In the simple model we present below, there is no distinction between the two. In a more general model where total wealth has many components, we would also require some form of *mental accounting,* namely that the investor compartmentalize different types of wealth and worry about fluctuations in each one separately.

[7] Other papers that extend the standard intertemporal consumption utility specification to allow for investment value as a direct source of utility include Bakshi and Chen (1996) and Zou (1994). These two studies assume that wealth can provide investors direct utility in the form of, say, social status, in addition to its implied consumption. However, the framework used in these papers and the economic issues studied are quite different from those here.

[8] We use the notation $\overline{C}_t$ for aggregate per-capita consumption to distinguish it from an individual's consumption, which is simply C_t.

where $\epsilon_{t+1} \sim$ i.i.d. $N(0,1)$. We keep this process simple for two reasons. First, simple though it is, it approximates reality well. Second, it serves to emphasize that one can generate interesting features of asset returns without specifying an intricate model for consumption growth.

Up to this point, our framework is entirely standard. We depart from the usual setup in the way we model investors' preferences. In particular, our agents maximize

$$\mathbf{E}\left[\sum_{t=0}^{\infty}(\rho^t \ \frac{C_t^{1-\gamma}}{1-\gamma} + b_t \ \rho^{t+1} \ v(X_{t+1}))\right]. \tag{2}$$

The first term in this preference specification, utility over consumption C_t, is a standard feature of asset pricing models. Although our framework does not require it, we specialize to power utility, the benchmark case studied in the literature. The parameter ρ is the time discount factor, and $\gamma > 0$ is relative risk aversion over consumption shocks.[9]

The second term is an attempt to capture the idea that the investor cares about fluctuations in his financial wealth. The variable X_{t+1} is the gain or loss the agent experiences on his investments between time t and $t+1$, a positive value indicating a gain and a negative value, a loss; the utility the investor receives from this gain or loss is $v(X_{t+1})$. Finally, b_t is an exogeneous scaling factor that we specify later.

It is important to note that even if the second term were not present, the investor would still care about wealth fluctuations, simply because of what those wealth changes mean for consumption. By including the additional term, we are taking the view that wealth fluctuations generate utility *over and above* the indirect utility that comes through consumption: people feel upset when an asset they have invested in does poorly, and this may be due to more than just the fact that they now need to lower consumption. In our simple setting where the investor worries about fluctuations in *total* wealth, it is conceivable that the asset prices we generate may also result from a more complex utility function defined over consumption alone. We don't interpret (2) this way, but prefer to think of it as capturing concern about wealth fluctuations for their own sake. This distinction becomes much more

[9]For $\gamma = 1$, we replace $C_t^{1-\gamma}/(1-\gamma)$ with $\log(C_t)$.

important in more general settings than the one we have here.[10],[11]

Introducing utility over gains and losses in wealth raises two important issues: (i) how does the investor measure his gains and losses; and (ii) how does utility v depend on those gains and losses? We tackle each of these questions in turn.

Measuring Gains and Losses

We interpret the "gains" and "losses" of prospect theory as referring to changes in the value of the agent's *risky asset* holdings. Kahneman and Tversky (1979) suggest that before people evaluate a gamble, they simplify it as much as possible in what is known as the "editing" phase. An important form of editing is "segregation", where any riskless component of a gamble is stripped away. In the context of our model, the investor separates off the known return on his riskless asset holdings, allowing him to focus directly on his investment in the risky asset.

Next, we need to specify the horizon over which gains and losses are measured. Put differently, how often does the agent seriously evaluate his investment performance? We follow the suggestion of Benartzi and Thaler (1995) that the most natural evaluation period is a year. As they point out, we file taxes once a year and receive our most comprehensive mutual fund reports once a year; moreover, institutional investors scrutinize their money managers' performance most carefully on an annual basis. We discuss the sensitivity of our results to this assumption later in the paper.

Our investor therefore monitors year-to-year fluctuations in the value of his stock portfolio and gets utility from those fluctuations. To fix ideas, suppose that S_t, the time t value of the investor's holdings of the risky asset, is \$100. Imagine that by time $t+1$, this value has gone up to $S_t R_{t+1} = \$120$. The exact way the investor measures this gain depends on the *reference level* to which 120 is compared. Kahneman and Tversky (1979) propose that the primary reference level is the status quo, which in our case is the initial value

[10] For example, consider a more general setup where the investor also receives risky labor income but still cares about fluctuations in his *financial* wealth alone. In this model, every little bit of dividend volatility is priced. In a pure consumption-based model, though, only that part of dividend fluctuations correlated with consumption is priced – a dramatically different conclusion.

[11] It is also worth noting that we do not take an explicitly axiomatic approach to constructing the investor's preferences, but simply posit a parsimonious framework that captures our intuition.

$S_t = \$100$. The gain would then be measured as \$20, or more generally as $X_{t+1} = S_t R_{t+1} - S_t$.

This is essentially our approach, but for one modification which we think is realistic in our context: we take the reference level to be the status quo *scaled up by the riskfree rate*, $S_t R_{f,t}$. In our example, and with a riskfree rate of say 5%, this means a reference level of 105. An end-of-period risky asset value of 106 would then lead the investor to code a gain of 1, while a value of 104 would generate a loss of -1. In general terms, the investor will code a gain or loss of

$$X_{t+1} = S_t R_{t+1} - S_t R_f. \tag{3}$$

The idea here is that in an economy offering a riskless return of 5%, the investor is likely to be disappointed if his stock market investment returns only 4%. The riskless return may not be the investor's only point of comparison, although we suggest that it is the most obvious. Our framework can easily accomodate alternative specifications and our results do not depend in any special way on the particular choice of $S_t R_{f,t}$ as the reference level.

Form of the Prospect Theory Term

The form of v, motivated by the work of Kahneman and Tversky (1979), is

$$v(X_{t+1}) = \begin{cases} X_{t+1} & \text{for} \quad X_{t+1} \geq 0 \\ \lambda(X_{t+1}) & \phantom{\text{for}} \quad X_{t+1} < 0 \end{cases}. \tag{4}$$

This is a piecewise linear function, shown in Figure 1. It is kinked at the origin, where the gain equals zero. The parameter λ helps us capture the important feature of prospect theory known as loss aversion, the tendency of individuals to be more sensitive to reductions in wealth than to increases. Any λ greater than one makes the investor loss averse. When we calibrate our model, we follow Tversky and Kahneman (1992) in setting $\lambda = 2.25$, a figure based on experimental findings.

One attractive feature of this utility specification is that it captures individuals' documented aversion to wealth bets over modest stakes. By contrast, the smooth utility functions typically employed in the literature imply that people are close to risk neutral over modest gambles.[12]

[12]Indeed, when smooth utility functions are calibrated to match individuals' risk aversion over small bets, they lead to absurd predictions for preferences over larger gambles. For example, Rabin (1999) shows that an expected utility maximizer who turns down a 50-50 bet of losing \$100 and gaining \$110 would also turn down a 50-50 bet of losing \$1000 and gaining *any* amount of money!

Our formulation follows the prescription of prospect theory by defining utility over gains and losses, and by introducing loss aversion. Kahneman and Tversky (1979) also propose that v should be mildly concave over gains and convex over losses. This curvature is most relevant when choosing between prospects that involve *only* gains or between prospects that involve *only* losses.[13] For gambles that can lead to both gains and losses – such as the one year investment in stocks that our agent is evaluating – loss aversion at the kink is far more important than the degree of curvature away from the kink. For simplicity then, we make v linear over both gains and losses.[14]

In our framework, the "prospective utility" the investor receives from gains and losses is computed by taking the expected value of v, in other words by weighting the value of gains and losses by their probabilities. As a way of understanding Allais-type violations of the expected utility paradigm, Kahneman and Tversky (1979) suggest weighting the value of gains and losses not with the probabilities themselves but with a nonlinear transformation of those probabilities. Again, for simplicity, we abstract from this feature of prospect theory, and have no reason to believe that our results are sensitive to this simplification.

Given the assumed linearity of the prospect theory function, we can write

$$\begin{aligned} v(X_{t+1}) &= v(S_t R_{t+1} - S_t R_{f,t}) \\ &= S_t\ v(R_{t+1} - R_{f,t}), \end{aligned} \tag{5}$$

which means we can think of gains and losses in terms of *returns* instead of dollar amounts. If we also define

$$\hat{v}(R_{t+1}) = v(R_{t+1} - R_{f,t}), \tag{6}$$

we can finally rewrite the agent's objective function as

$$\mathbf{E}\left[\sum_{t=0}^{\infty}(\rho^t\ \frac{C_t^{1-\gamma}}{1-\gamma} + b_t\ \rho^{t+1}\ S_t\ \hat{v}(R_{t+1}))\right], \tag{7}$$

[13] Indeed, it is by offering subjects gambles over *only* losses or *only* gains that Kahneman and Tversky (1979) deduce the shape of the value function. They propose concavity over gains because subjects prefer a gamble that offers \$2000 w.p. $\frac{1}{4}$, \$4000 w.p. $\frac{1}{4}$, and \$0 w.p. $\frac{1}{2}$ to the mean-preserving spread offering \$6000 w.p. $\frac{1}{4}$ and \$0 otherwise. Preferences switch when the signs are flipped, suggesting convexity over losses.

[14] The reader may rightly be concerned that curvature away from the reference point does assume greater importance when prior outcomes are taken into account, as they are in Section 3. We therefore return to this issue in that section.

where

$$\widehat{v}(R_{t+1}) = \begin{cases} R_{t+1} - R_{f,t} & \text{for} \quad R_{t+1} \geq R_{f,t} \\ \lambda(R_{t+1} - R_{f,t}) & \phantom{\text{for}} \quad R_{t+1} < R_{f,t} \end{cases}. \tag{8}$$

The Scaling Term b_t

We scale the prospect theory term in the utility function to ensure that quantities like the price-dividend ratio and risky asset premium remain stationary even as aggregate wealth increases over time. Without a scaling factor, this will not be the case because the second term of the objective function will come to dominate the first as aggregate wealth grows. One reasonable specification of the scaling term is

$$b_t = b_0 \, \overline{C}_t^{-\gamma}, \tag{9}$$

where $\overline{C}_t$ is the *aggregate* per-capita consumption at time t, and hence exogeneous to the investor. By using an exogeneous variable, we ensure that b_t simply acts as a neutral scaling factor, without affecting the economic intuition of the previous paragraphs.[15]

The parameter b_0 is a positive constant that allows us to control the overall importance of utility from gains and losses in wealth relative to utility from consumption. Setting $b_0 = 0$ reduces our framework to the much studied consumption-based model with power utility.

2.1 Equilibrium Conditions

In order to investigate the model's ability to explain the data, we derive equations that govern equilibrium prices. Our investor chooses (C_t, S_t) for all t to maximize (7), subject to the standard budget constraint. We now show that there is an equilibrium in which the riskfree rate and the stock's price-dividend ratio are both constant and stock returns are i.i.d.

That returns are i.i.d. is a direct consequence of the fact that the price-dividend ratio is constant. To see this, note that the stock return is related to the stock's price-dividend ratio, denoted by $f_t \equiv P_t/D_t$, as follows:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{1 + P_{t+1}/D_{t+1}}{P_t/D_t} \frac{D_{t+1}}{D_t} = \frac{1 + f_{t+1}}{f_t} \frac{D_{t+1}}{D_t}. \tag{10}$$

[15] Using aggregate per-capita wealth as a scaling factor works just as well, and does not affect our basic results.

Given the assumption that the dividend growth is i.i.d. (see (1)), a constant price-dividend ratio $f_t = f$ implies that stock returns are i.i.d.

In equilibrium, and under rational expectations about stock returns and aggregate consumption levels, the agents in our economy must find it optimal to consume the dividend stream and to hold the market supply of zero units of the riskfree asset and one unit of stock at all times.[16] The next proposition characterizes the equilibrium.

Proposition 1 *For the preferences given by (7)-(8), there exists an equilibrium in which the gross riskfree interest rate is constant at*

$$R_f = \rho^{-1} e^{\gamma g - \gamma^2 \sigma^2 / 2}, \tag{11}$$

and the stock's price-dividend ratio, f_t, is constant at f and given by

$$1 = \rho \frac{1+f}{f} \mathbf{E}_t \left[e^{(1-\gamma)(g+\sigma\epsilon_{t+1})} \right] + b_0 \rho \mathbf{E}_t \left[\hat{v} \left(\frac{1+f}{f} e^{g+\sigma\epsilon_{t+1}} \right) \right]. \tag{12}$$

We prove this formally in the Appendix. At a less formal level, our results follow directly from the agent's Euler equations for optimality, derived using standard perturbation arguments:

$$1 = \rho R_f \mathbf{E}_t \left[\left(\frac{\overline{C}_{t+1}}{\overline{C}_t} \right)^{-\gamma} \right], \tag{13}$$

$$1 = \rho \mathbf{E}_t \left[R_{t+1} \left(\frac{\overline{C}_{t+1}}{\overline{C}_t} \right)^{-\gamma} \right] + b_0 \rho \mathbf{E}_t \left[\hat{v}(R_{t+1}) \right]. \tag{14}$$

Readers may find it helpful to compare these equations with those derived from standard asset pricing models with time-additive utility functions. The Euler equation for the riskfree rate is the usual one: consuming a little less today and investing the savings in the riskfree rate does not change the investor's exposure to losses on the risky asset. The Euler equation for the risky asset, however, now contains an additional term. Consuming less today and investing the proceeds in the risky asset *increases* the investor's exposure to risky asset losses.

[16]We need to impose rational expectations about aggregate consumption because the agent's utility includes aggregate consumption as a scaling term.

2.2 Numerical Results

With the equilibrium conditions (11) and (12) in hand, we now test the usefulness of our model by checking whether it can match the moments of both consumption growth and asset returns for reasonable values of the parameters. Earlier research suggests that prospect theory may be helpful in explaining at least one perplexing feature of stock returns, namely their high average level. Benartzi and Thaler (1995) analyze the one-period portfolio problem of a loss averse investor. They find that even when confronted with the large historical equity premium, their investor is reluctant to allocate heavily to stocks: the sharp pain experienced when stocks do poorly makes them unattractive. Benartzi and Thaler's partial equilibrium framework does not allow them to address the equity premium puzzle directly; however, their analysis suggests that incorporating prospect theory into a standard equilbrium model may prove fruitful.

Table 1 summarizes our choices of parameter values. For g and σ, the mean and standard deviation of log consumption growth, we follow Ceccheti, Lam, and Mark (1990) who obtain $g = 1.84\%$ and $\sigma = 3.79\%$ from a time series of annual data from 1889 to 1985. These numbers are very similar to those used by Mehra and Prescott (1985) and Constantinides (1990). Campbell and Cochrane (1999) place more emphasis on post-war data which gives $g = 1.89\%$, and $\sigma = 1.22\%$. Since σ varies somewhat by subperiod, we present results for a range of values of σ.

The investor's preference parameters are γ, ρ, λ, and b_0. We set $\gamma = 0.9$, which makes the investor's risk-aversion over consumption shocks close to that of a log utility investor. It is well known that equilibrium models typically require much higher levels of risk-aversion over consumption to match stock return moments. We are therefore making things particularly difficult for ourselves by using a γ as low as 0.9. However, our goal is to explain the data *with reasonable parameter values*; a value of γ that is much higher would be hard to defend as reasonable.

Given the values of g, σ, and γ, we use (11) to choose a rate of time preference ρ that produces a sensibly low level of the riskfree rate. Setting $\rho = 0.98$ brings the riskfree interest rate, $R_f - 1$, close to 3.5%.

The value of λ determines how keenly losses are felt, relative to gains. As mentioned earlier, we follow Tversky and Kahneman (1992) in setting $\lambda = 2.25$.

The final parameter, b_0, determines the relative importance of the prospect utility term in the investor's preferences. We do not have strong priors about what constitutes a reasonable value for b_0. For the time being, we perform the calculations for a range of values of b_0.

Figure 2 presents the implied values of the price-dividend ratio, mean log excess return (equity premium), and riskfree rate for the above parameter values. Each quantity is plotted against σ, the standard deviation of log consumption growth. There are four lines within each graph, each one corresponding to a different level of b_0.

Look first at the results for the equity premium. The solid line corresponds to the case of $b_0 = 0$, where the investor's preferences reduce to the familiar case of power utility over consumption originally analyzed by Mehra and Prescott (1985). The infamous equity premium puzzle is clearly visible in the graph: when $b_0 = 0$, the empirically measured volatility of consumption growth of 3.79% corresponds to a miniscule premium of only 0.06%!

The remarkable finding in this graph is that incorporating loss aversion into the investor's preferences does not lead to an immediate resolution of the equity premium puzzle. Table 2 reports the unconditional moments of returns when $b_0 = 2$. The equity premium corresponding to $\sigma = 3.79\%$ is a mere 0.91%. In fact there is no value of b_0 that can generate the empirical premium of 6% from a consumption growth volatility of 3.79%! As $b_0 \to \infty$, the equity premium we can generate tends to an upper limit, the dotted line in the graph. For a 3.79% consumption growth volatility, the highest possible equity premium is only 1.2%.[17]

This result is surprising in light of Benartzi and Thaler's (1995) suggestion that loss aversion may be helpful in explaining the equity premium. After all, when $b_0 = \infty$, the investor's utility is driven entirely by loss aversion and our framework effectively reduces to a multiperiod version of Benartzi and Thaler's setup. Even for this b_0 however, we can only generate a puny equity premium. Our results therefore point to an unexpected conclusion: loss-aversion *by itself* cannot explain the equity premium.

This conclusion is not sensitive to the length of the investor's evaluation period. It is true that if the investor evaluates his portfolio more frequently than once a year, he is more likely to see losses and hence will be more inclined to charge a higher premium. This effect is not nearly large enough to rescue the model; in any case, one year remains the most natural length

[17]The stock return data in this and other tables is based on the annual returns of the value-weighted NYSE portfolio from 1926 to 1995.

of time between portfolio evaluations.

The difficulty we face in matching the equity premium turns out to be a symptom of a deeper problem that the model of this section shares with all consumption-based models with constant discount rates.[18] It concerns the implications for stock return volatility. Since

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{1 + f_{t+1}}{f_t} \frac{D_{t+1}}{D_t} = \frac{1 + f}{f} e^{g + \sigma \varepsilon_{t+1}}, \tag{15}$$

the volatility of log returns in this model is equal to the volatility of log consumption growth, namely 3.79%. Our model therefore does not come close to matching the empirically observed return standard deviation of 20%! Of course, this is partly due to the fact that models in the Lucas (1978) tradition equate consumption and dividends. However, even using the more generous dividend growth volatility of 11% does not solve this problem satisfactorily.

The unrealistically low stock return volatility generated by the model explains why our conclusion differs from that of Benartzi and Thaler (1995). Even though our investor is loss averse, the losses on the stock are not large enough to scare the investor into demanding a high equity premium as compensation. In their portfolio calculations, Benartzi and Thaler can treat volatility as an exogeneous parameter and set it at its high historical level. In our equilibrium model, we face the more difficult task of generating that volatility endogeneously. Our inability to do so with the current specification also hampers our ability to explain the equity premium.

In Section 3, we draw on recent findings in the psychology literature to modify our model in a simple way, and find that our ability to understand the empirical features of aggregate stock returns is significantly enhanced. In particular, we incorporate evidence on how prior outcomes influence agents' desire to take risk. This leads to a model of changing risk-aversion, and hence time-varying expected returns. We find that this predictability can make stock returns much more volatile than consumption growth, in turn making the equity premium easier to understand.

[18] This category includes papers that use first-order risk-aversion, such as Epstein and Zin (1990). See Campbell (1996) for a discussion of this issue.

3 The Influence of Prior Outcomes

In the model of the previous section, the utility $\widehat{v}(R_{t+1})$ the investor receives from a specific return R_{t+1} is the same, whatever the investor's previous gains or losses. Put differently, the investor evaluates this period's gain or loss with "no memory" of earlier occurences.

A number of recent papers in the psychology literature suggest that prior outcomes do in fact influence the way subsequent gains and losses are experienced, and hence also willingness to take risk. In a pioneering paper, Thaler and Johnson (1990) present evidence on this, using a large sample of Cornell undergraduate and MBA students. Here are some examples of the results they obtained; the percentage of students choosing each option is in parentheses.

1. You have just won $30. Choose between:
(a) A 50% chance to gain $9 and a 50% chance to lose $9 [82]
(b) No further gain or loss [18]

2. You have just lost $30. Choose between:
(a) A 50% chance to gain $9 and a 50% chance to lose $9 [36]
(b) No further gain or loss [64]

These results suggest that prior outcomes influence risky choice. Following a gain, people appear to be more risk-seeking than usual, taking on bets that they would not normally accept. The opposite is true after a loss: the subjects displayed considerable reluctance to accept risky bets.

Thaler and Johnson (1990) offer a natural interpretation of these results. They argue that people's increasing willingness to gamble after a prior gain reflects the fact that a loss is less painful than usual in these circumstances. The earlier gain cushions any subsequent losses, making them easier to bear. In a financial context, an investor does not care much about a loss that comes after substantial gains because he is able to tell himself that he is still "up, relative to a year ago". Thaler and Johnson call this the "house money" effect, because it is reminiscent of the expression "playing with the house money" used to describe gamblers' increased willingness to bet when ahead.

The reason for higher risk aversion following prior losses, according to Thaler and Johnson, is that people are particularly fearful of additional losses when they have just recently experienced a loss. An informal interpretation is that the shock of the initial loss makes people unusually sensitive to further setbacks. Linville and Fisher (1991) offer a more formal way of thinking about this, based on a "renewable resources" model. They argue that people possess limited loss-buffering resources that are used up when coping with a bad outcome. These resources renew over time, but only slowly, which means that people are particularly vulnerable and hence sensitive to losses that occur immediately after other losses.

Thaler and Johnson present further evidence on this point by asking subjects more directly about the discomfort caused by losses in various circumstances. These results clearly support the notion that a loss is less painful after a prior gain and more painful if it comes on the heels of another loss. Typical results are illustrated by the following questions; once again, the percentage of respondents choosing each option is in parentheses.

3.
(a) You lose $9.
(b) You lose $9 after having gained $30
The loss of $9 hurts more in (a) [84] (b) [10] (no difference) [6].

4.
(a) You lose $9
(b) You lose $9 after having lost $30
The loss of $9 hurts more in (a) [22] (b) [75] (no difference) [3].

Linville and Fisher (1991) use a different approach that also points to losses after prior losses being more painful. They ask subjects if they would prefer two unpleasant events to occur close together in time, or far apart. The vast majority of respondents prefer to separate the events, suggesting that having them occur close by would be too overwhelming. Moreover, Linville and Fisher find that when one event is pleasant and the other unpleasant, subjects prefer to have them occur close together in time, the rationale being that the good event cushions the bad one.

One objection to Thaler and Johnson's evidence is that the amounts at stake are too small to elicit serious introspection on the part of subjects. However, similar results were obtained by Gertner (1993) in a clever study involving much larger stakes. He studies the risk-taking behavior of participants in the television game show "Card Sharks," where contestants place bets on whether a card to be drawn at random from a deck will be higher or lower than a card currently showing. He finds that the amount bet is a strongly increasing function of the contestant's winnings up to that point in the show. Once again, this is evidence of more aggressive risk-taking behavior following substantial gains.

The evidence we have presented suggests that in the context of a sequence of gains and losses, people are less risk-averse following prior gains and more risk-averse after prior losses. This may initially appear at odds with Kahneman and Tversky's original value function, which is concave in the region of gains and convex in the region of losses. In particular, the convexity over losses is occasionally interpreted to mean that after a loss, people take on more risk in an attempt to break-even, contrary to our claim.

In fact, the two claims are completely consistent and it is important to understand the distinction between them. An example may be helpful here. Suppose you are spending the day at the horse-races or at a casino. While you may place many individual bets over the course of the day, it is reasonable to suppose that you care primarily about where you stand at the *end* of the day, a natural evaluation point. Put differently, you view your entire day at the casino as a one-shot gamble and apply the prospect value function to your gain or loss at the end of the day.

In this context, the right way to interpret the convex portion of the value function in the domain of losses is the following: it says that if you are nearing the end of the day with losses, you will take on more risk in an attempt to break-even – and this is indeed observed behavior. However, this is not the effect that we are concerned with in our paper. We are interested in what happens *after* the gain or loss at the end of the day is experienced, not before. To continue with our example, suppose that the day ends and that your attempts to break-even have failed, forcing you to acknowledge a painful loss. Our claim, derived from evidence in Thaler and Johnson (1990), is that if you go to the casino again the following day, you will be more risk-averse.

It is important to note that the claim that risk aversion increases after a loss is not derived from prospect theory; indeed it *cannot* be derived from prospect theory. Tversky and Kahneman (1981) themselves emphasize that their theory was developed for elementary, one-shot gambles and that any application to a dynamic context must await further evidence on how people think about sequences of gains and losses. A number of papers, including Thaler and Johnson (1990) and Linville and Fisher (1991) have taken up this challenge, and it is there that we need to look for guidance when implementing a dynamic version of prospect theory.

3.1 Extending the Model to Account for Prior Outcomes

The model of Section 2.1., summarized in (7) and (8), makes the prospect theory term $\widehat{v}$ a function of R_{t+1} alone, implying that investors evaluate current gains and losses with "no memory" of earlier outcomes. The experimental evidence strongly contradicts this. Rather, it suggests that the utility impact of a 10% decline, say, in the value of a stock from \$100 to \$90 depends on the stock's prior returns, on the path by which it arrived at \$100 in the first place.

To capture the influence of prior outcomes, we introduce the concept of a *historical benchmark level* Z_t for the value of the risky asset.[19] We propose that when judging the recent performance of a stock, investors compare S_t, the value of their stock holdings today, to some value Z_t based on the stock's previous price history. Different investors will form this benchmark in different ways. For some investors, it may represent an average of recent stock prices. For others, it may be the specific stock price at salient moments, such as the turn of the year. Whichever way the benchmark level is formed, the difference $S_t - Z_t$, when positive, is the investor's personal measure of how much "he is up" on his investment and conversely, when negative, how much "he is down".

Introducing Z_t is helpful in modelling the influence of prior outcomes on the way subsequent gains and losses are experienced. When $S_t > Z_t$, subsequent losses are less painful because they are cushioned by the investor's prior gains, $S_t - Z_t$. This makes the investor *less* risk-averse than usual. Conversely, when $S_t < Z_t$, the investor is down on his investment. Subsequent losses are more painful, and the investor is *more* risk-averse than usual.

It is clear from this discussion that the way subsequent gains and losses are experienced depends primarily on the *relative* values of S_t and Z_t, rather than their absolute levels. A simple way of capturing the effect of prior outcomes is therefore to write the prospect theory term as $\widehat{v}(R_{t+1}, z_t)$, a function not only of R_{t+1} but also of the ratio $z_t = \frac{Z_t}{S_t}$.[20]

[19] We use the term *benchmark* level to distinguish Z_t from the *reference* level $S_t R_{f,t}$. The reference level determines the size of the gain or loss. The benchmark level Z_t determines the magnitude of the utility received from that gain or loss, in a way that we soon make precise. While we are careful to stick to this terminology, some readers may find it helpful to think of Z_t as a secondary reference level that also affects the investor's decisions.

[20] It is clearly an abuse of notation to use the same notation $\widehat{v}$ now that we have intro-

Case of Prior Gains

We now describe how gains and losses are treated where we take account of prior outcomes. In doing so, we try to be as faithful as possible to the spirit of the experimental evidence. An example may be helpful here; for simplicity, we take the riskfree rate to be 0%, so that $R_{f,t} = 1$.

Suppose that the value of the risky asset has gone up recently, so that its current value of $S_t = \$100$ is higher than the historical benchmark level the investor has in mind, $Z_t = \$90$, say. As discussed above, we can think of \$90 as the value of the stock one year ago, which the investor still remembers. The investor will be less risk-averse than usual since he has built up a reserve of prior gains, measured by $S_t - Z_t = 10$, that will cushion subsequent losses.

Now suppose that over the next year, the value of the stock falls by 20%, from \$100 down to \$80. In Section 2, we measured the pain of this loss as

$$(80 - 100)(2.25) = -45.$$

Since the investor has built up some prior gains, this calculation probably overestimates actual discomfort. We propose a more realistic measure of the pain caused: since the first \$10 drop, from \$100 down to \$90, is completely cushioned by the reserve of prior gains, we penalize it at a rate of only 1, rather than 2.25. The second part of the loss, from \$90 down to \$80 will be *more* painful since all prior gains have already been depleted, and we penalize it at the usual rate of 2.25. The overall disutility of the \$20 loss is then

$$(90 - 100)(1) + (80 - 90)(2.25) = -32.5$$

We can now restate this argument more formally in terms of returns, while still keeping the riskless rate at 0% for now. Suppose that the investor experiences a particularly poor return R_{t+1} after accumulating prior gains. If the investor has no memory of prior outcomes, equation (8) shows that he will code a loss of $R_{t+1} - 1$ and penalize the entire loss at a rate λ. However, if the investor takes his prior gains into account, we suggest that he will separate the loss $R_{t+1} - 1$ into two components: one component will be that part of the loss that is cushioned by the prior gain, and hence is not very painful; and the other component will be that part of the loss that remains once the cushion is depleted, and which is therefore more painful. The worst return that can be entirely cushioned by prior gains is that return which

duced a new argument, but hopefully this will not cause confusion.

brings the stock value down from S_t to its historical benchmark level Z_t, a return of $\frac{Z_t}{S_t} = z_t$. We therefore measure the pain caused by a poor return R_{t+1} by breaking the total loss $R_{t+1} - 1$ into two parts, as

$$R_{t+1} - 1 = (R_{t+1} - z_t) + (z_t - 1) \tag{16}$$

and penalizing each component separately as

$$(\lambda)(R_{t+1} - z_t) + (1)(z_t - 1). \tag{17}$$

In summary, then, we give $\widehat{v}(R_{t+1}, z_t)$ the following form for the case of prior gains, or $z_t \leq 1$:

$$\widehat{v}(R_{t+1}, z_t) = \begin{cases} R_{t+1} - 1 & \text{for } R_{t+1} \geq z_t \\ (z_t - 1) + \lambda(R_{t+1} - z_t) & \phantom{\text{for }} R_{t+1} < z_t \end{cases}$$

For the more relevant case of a nonzero riskless rate $R_{f,t}$, we scale both the reference level S_t and the benchmark level Z_t up by the riskfree rate, so that

$$\widehat{v}(R_{t+1}, z_t) = \begin{cases} R_{t+1} - R_{f,t} & \text{for } R_{t+1} \geq z_t R_{f,t} \\ (z_t R_{f,t} - R_{f,t}) + \lambda(R_{t+1} - z_t R_{f,t}) & \phantom{\text{for }} R_{t+1} < z_t R_{f,t} \end{cases}. \tag{18}$$

Figure 3 illustrates the form of this function for several values of z_t. Low values of z_t represent cases where the investor has built up substantial prior gains. It therefore takes an especially bad return to inflict any substantial pain on the investor.

Case of Prior Losses

The discussion of the last few paragraphs relates to the case when the investor's recent returns were good, enabling him to build up a reserve of prior gains. Suppose now that the value of the risky asset has fallen in recent periods. How are subsequent losses treated in this case? Once again, an example may be helpful. Suppose that the current stock value is $S_t = \$100$, and that the investor's benchmark level is $Z_t = \$110$, higher than \$100 since the stock has been falling. In this example, we take an interest rate of 5%.

Suppose now that over the next year, the value of the stock falls 10%, from \$100 down to \$90. In Section 2, we measured the pain of this loss as

$$(90 - 105)(2.25) = -33.75.$$

However, this is probably too conservative an estimate: experimental evidence suggests any *further* losses that come on the heels of the initial setbacks will be even more painful than on average. We propose the simplest possible modification: we penalize further losses at a rate higher than 2.25.

More formally, then, we define $\hat{v}(R_{t+1}, z_t)$ for the case of prior losses, or $z_t > 1$, as

$$\hat{v}(R_{t+1}, z_t) = \begin{cases} R_{t+1} - R_{f,t} & \text{for} \quad R_{t+1} \geq R_{f,t} \\ \lambda(z_t)(R_{t+1} - R_{f,t}) & \phantom{\text{for}} \quad R_{t+1} < R_{f,t} \end{cases} \tag{19}$$

In other words, the intensity $\lambda(z_t)$ with which losses are penalized depends on the size of earlier losses, measured by z_t. The evidence dictates $\lambda(z_t) > 2.25$. We also make $\lambda(.)$ an *increasing* function of z_t: the larger the prior loss, the higher z_t, and the greater the pain of a further loss. A very simple formulation is

$$\lambda(z_t) = \lambda + k(z_t - 1), \qquad z_t \geq 1. \tag{20}$$

Our results do not depend crucially on the particular functional form used for $\lambda(\cdot)$.

Dynamics of the Benchmark Level

To complete our description of the model, we need to specify how z_t moves over time, or equivalently how the historical benchmark level Z_t reacts to changes in the stock value S_t. The only requirement we impose on Z_t is that it respond *sluggishly* to changes in the value of the risky asset. By this we mean that when the stock price moves up by a lot, the benchmark level also moves up, but by less. Conversely, if the stock price falls sharply, the benchmark level does not adjust downwards by as much.

Sluggishness turns out to be a very intuitive requirement to impose. To see this, recall that the difference $S_t - Z_t$ is the investor's measure of his reserve of prior gains. How should this quantity behave over time? If the return on the stock market is particularly good, investors should feel as though they have increased their reserve of prior gains. Mathematically, this means that the benchmark level Z_t should move up less than the stock price itself, so that the cushion at time $t + 1$, namely $S_{t+1} - Z_{t+1}$, be larger than the cushion at time t, $S_t - Z_t$. Conversely, if the return on market is particularly poor, the investor should feel as though his reserves of prior gains are depleted. For this to happen, Z_t must fall less than S_t.

A simple way of modelling the sluggishness of the benchmark level Z_t is to write the dynamics of z_t as

$$z_{t+1} = z_t \frac{\overline{R}}{R_{t+1}}. \tag{21}$$

where $\overline{R}$ is a fixed parameter. This equation then says that if the return on the risky asset is particularly good, so that $R_{t+1} > \overline{R}$, the state variable $z = \frac{Z}{S}$ falls in value. This is consistent with the benchmark level Z_t behaving sluggishly, rising less than the stock price itself. Conversely, if the return is poor and $R_{t+1} < \overline{R}$, then z goes up. This is consistent with the benchmark level falling less than the stock price.[21]

$\overline{R}$ is not a free parameter in our model, but is determined endogeneously by imposing the requirement that in equilibrium, the mean value of z_t be equal to one. The idea behind this is that *on average* the investor's benchmark level should be the same as the current stock value, so that $z_t = 1$, although of course at any particular moment, the benchmark level may be above or below the current stock value. It turns out that $\overline{R}$ is typically of similar magnitude to the average stock return.

We can generalize (21) slightly to allow for varying degrees of sluggishness in the dynamics of the historical benchmark level. One way to do this is to write

$$z_{t+1} = \eta \left(z_t \frac{\overline{R}}{R_{t+1}} \right) + (1 - \eta)\ (1). \tag{22}$$

When $\eta = 1$, this reduces to (21), which represents a sluggish benchmark level. When $\eta = 0$, it reduces to $z_{t+1} = 1$, which means that the benchmark level Z_t tracks the stock value S_t one-for-one throughout – a very fast moving benchmark level. Note that this is exactly the "preliminary model" considered in Section 2. By varying η between 0 and 1, we alter the sluggishness of the benchmark level – the higher η, the more sluggish Z_t is.

The parameter η can also be given an interesting interpretation in terms of the investor's "memory": it measures how far back the investor's mind stretches when recalling past gains and losses. When η is near zero, the benchmark level Z_t is always close behind the value of the stock S_t: prior gains and losses are quickly swallowed up and are not allowed to affect the

[21] The benchmark level dynamics in (21) are one simple way of capturing sluggishness. More generally, we can assume dynamics of the form $z_{t+1} = z_t\ g(R_{t+1}, \overline{R})$, where $g(\cdot, \overline{R})$ is strictly decreasing and equal to 1 at $\overline{R}$.

investor for long. In effect, the investor has a short term memory, recalling only the most recent prior outcomes. When η is closer to one, though, the benchmark level moves sluggishly, allowing past gains and losses to linger and affect the investor for a long time; in other words, the investor has a long memory.[22]

3.2 Equilibrium Conditions

To evaluate our new model, we derive the equations that characterize asset prices in equilibrium. To recap, our investor chooses (C_t, S_t) to maximize

$$\mathbf{E}\left[\sum_{t=0}^{\infty}(\rho^t \; \frac{C_t^{1-\gamma}}{1-\gamma} + b_t \; \rho^{t+1} \; S_t \; \widehat{v}(R_{t+1}, z_t))\right], \tag{23}$$

where for $z_t \leq 1$, $\widehat{v}$ is defined by

$$\widehat{v}(R_{t+1}, z_t) = \left\{ \begin{array}{ll} R_{t+1} - R_{f,t} \\ (z_t R_{f,t} - R_{f,t}) + \lambda(R_{t+1} - z_t R_{f,t}) \end{array} \right. \quad \text{for} \quad \begin{array}{l} R_{t+1} \geq z_t R_{f,t} \\ R_{t+1} < z_t R_{f,t} \end{array}, \tag{24}$$

and for $z_t > 1$,

$$\widehat{v}(R_{t+1}, z_t) = \left\{ \begin{array}{ll} R_{t+1} - R_{f,t} \\ \lambda(z_t)(R_{t+1} - R_{f,t}) \end{array} \right. \quad \text{for} \quad \begin{array}{l} R_{t+1} \geq R_{f,t} \\ R_{t+1} < R_{f,t} \end{array}, \tag{25}$$

with

$$\lambda(z_t) = \lambda + k(z_t - 1). \tag{26}$$

and

$$b_t = b_0 \overline{C}_t^{-\gamma}, \tag{27}$$

and finally,

$$z_{t+1} = \eta \; \left(z_t \frac{\overline{R}}{R_{t+1}}\right) + (1-\eta) \; (1). \tag{28}$$

We propose a one-factor equilibrium in which the Markov state variable, z_t, determines the distribution of all future returns. Specifically, we assume

[22] A simple mathematical argument can be used to show that the "half-life" of the investor's memory is equal to $-\frac{0.693}{\log \eta}$. In other words, after this amount of time, the investor has lost half of his memory. When $\eta = 0.9$, this quantity is 6.6 years and when $\eta = 0.8$, it equals 3.1 years.

that the price-dividend ratio of the stock is a function of the state variable z_t:

$$f_t \equiv P_t/D_t = f(z_t), \tag{29}$$

and then show that there is indeed an equilibrium satisfying this assumption. Applying the above one-factor assumption to equation (10), we show that the distribution of the stock return R_{t+1} is determined by z_t and function $f(\cdot)$ as follows:

$$R_{t+1} = \frac{1 + f(z_{t+1})}{f(z_t)} e^{g+\sigma\epsilon_{t+1}}. \tag{30}$$

In the proposed equilibrium, the riskfree rate is again constant.

The proposition below characterizes the equilibrium.

Proposition 2 *For the preferences given in (23)-(28), there exists an equilibrium in which the gross riskfree interest rate is constant at*

$$R_f = \rho^{-1} e^{\gamma g - \gamma^2 \sigma^2 / 2}, \tag{31}$$

and the stock's price-dividend ratio, as a function of the state variable z_t, satisfies, for all z_t:

$$1 = \rho \mathbf{E}_t \left[\frac{1 + f(z_{t+1})}{f(z_t)} e^{(1-\gamma)(g+\sigma\epsilon_{t+1})} \right] + b_0 \rho \mathbf{E}_t \left[\widehat{v} \left(\frac{1 + f(z_{t+1})}{f(z_t)} e^{g+\sigma\epsilon_{t+1}}, z_t \right) \right]. \tag{32}$$

This proposition is proved in the Appendix. Informally, our results again follow from the agent's Euler equations

$$1 = \rho R_f \mathbf{E}_t \left[\left(\frac{\overline{C}_{t+1}}{\overline{C}_t} \right)^{-\gamma} \right], \tag{33}$$

$$1 = \rho \mathbf{E}_t \left[R_{t+1} \left(\frac{\overline{C}_{t+1}}{\overline{C}_t} \right)^{-\gamma} \right] + b_0 \rho \mathbf{E}_t \left[\widehat{v}(R_{t+1}, z_t) \right]. \tag{34}$$

Once again, the prospect theory term only enters the Euler equation for the risky asset. This enables our model to explain various features of equity returns without simultaneously predicting a high mean and volatility of the riskfree interest rate. This is a point which has frustrated many earlier attempts to understand stock return data by playing with the form of utility over *consumption*. Those earlier models typically use a high risk-aversion γ over consumption shocks to explain the equity premium; unfortunately, that high γ also leads to a strong desire to smooth consumption intertemporally, generating high interest rates. In our model, the contribution of the second term in (34) means that we do not need a high γ to understand the equity premium, allowing us to avoid the riskfree rate puzzle as well.[23]

In Section 3.3, we solve for the price-dividend ratio numerically and use simulated data to show that our model provides a simple way of understanding many of the hitherto puzzling empirical features of aggregate stock returns. In particular, our model is consistent with both a low mean and volatility of consumption growth on the one hand, and a high mean and volatility of stock returns on the other. Moreover, our model generates long-horizon predictability very similar to that observed in empirical studies. Most importantly of all, we obtain these results with reasonable parameter values. For example, our results do not require high levels of risk-aversion over consumption.

It may be helpful to outline the intuition behind these results before moving to the simulations. Return volatility is a good place to start. An important feature of the model of this section is that it allows the volatility of returns and the volatility of consumption growth to be very different. This is critical because the former is high and the latter is low. Indeed, the main failing of the model in Section 2 was that it forced these two quantities to be identical.

To understand what is different in our new model, suppose that there is a positive dividend innovation this period. This will generate a high stock return. However, this high return will also increase the investor's reserves of prior gains, since his benchmark price level goes up more slowly than the stock price itself. This makes the investor less risk-averse, since future losses will be cushioned by the prior gains, which are now larger than before. The investor's lower risk-aversion effectively lowers the rate at which the future dividend stream is discounted, giving stock prices an extra jolt upwards. A similar story holds for a negative dividend innovation. It generates a low stock return, depleting prior gains or increasing prior losses, this time because the investor's benchmark level falls less than the stock price itself. Following

[23]Campbell and Cochrane (1999) is perhaps the only consumption-based model that avoids problems with the riskfree rate. A clever choice of functional form for the habit level over consumption enables them to use precautionary saving to counterbalance the strong desire to smooth consumption intertemporally.

this painful prior loss, the investor is more risk-averse than before, and the increase in risk-aversion pushes prices still lower. The effect of all this is to make returns substantially more volatile than dividend growth.

The fact that returns are now more volatile also leads us to expect a more substantial equity premium from our model. In the framework of Section 2, stock returns were not volatile enough to produce the kind of large losses that scare loss-averse investors into charging a high premium. This is no longer the case.

Long horizon predictability also results naturally in our model. Put simply, since the investor's risk-aversion varies over time depending on his investment performance, expected returns on the risky asset also vary. To understand this in more detail, suppose once again that there is a positive shock to dividends. This generates a high stock return, which in turn lowers the investor's risk-aversion, and pushes the stock price still higher, leading to a higher price-dividend ratio. Since the investor is less risk-averse, subsequent stock returns will be lower on average. Price dividend ratios are therefore inversely related to future returns, in exactly the way that has been documented by numerous studies, including Campbell and Shiller (1988) and Fama and French (1988b).

3.3 Numerical Results

In this section, we present the price-dividend ratio $f(z_t)$ that solves equation (32). The parameter values we use are summarized in Table 1. We set the mean g of log consumption growth equal to 1.84% and the standard deviation to 3.79%. Risk-aversion γ is still 0.9, the time discount rate $\rho = 0.98$, and $b_0 = 2$.

The parameters that are new to the specification of Section 3 are k, η, and $\overline{R}$. We set k, which determines $\lambda(z_t)$ and hence the way the pain of a loss varies with the state variable, equal to 50. To understand what this means, suppose that the state variable z_t is initially equal to 1, and that the stock market then experiences a sharp fall of 10%. From equation (28) with $\eta = 1$, this means that z_t increases by approximately 0.1 to 1.1. From (26), the pain of any additional losses will now penalized at $2.25 + 5 = 7.25$, a more severe penalty.

The variable η controls the sluggishness of the benchmark level. We first present results for $\eta = 1$, the most sluggish case, to see how far our model can stretch in terms of the results it generates. We then present some results for lower η. $\overline{R}$ is not a parameter we have any control over. Rather, it is completely determined by the other parameters *and* the requirement that the equilibrium unconditional mean of z_t be equal to one. Trial and error leads to a value of $\overline{R} = R_f + 4.0\%$ for the case of $\eta = 1$.

Before presenting our results, we briefly describe the way they were obtained. Solving equation (32) is not a trivial task. The complication is due to the fact that z_{t+1} is a function of both ϵ_{t+1} and $f(\cdot)$. In economic terms, our state variable is endogenous: it tracks prior gains and losses, which depend on past returns, themselves endogenous. Equation (32) is therefore self-referential and needs to be solved in conjunction with

$$z_{t+1} = \eta \left(z_t \frac{\overline{R}}{R_{t+1}} \right) + (1 - \eta)\ (1), \tag{35}$$

and

$$R_{t+1} = \frac{1 + f(z_{t+1})}{f(z_t)} e^{g + \sigma \epsilon_{t+1}}. \tag{36}$$

After trying a number of different approaches to this problem, we settled on the following technique. We start out by guessing a solution to (32), $f^{(0)}$ say. We then construct a function $h^{(0)}$ so that $z_{t+1} = h^{(0)}(z_t, \varepsilon_{t+1})$ solves equations (35) and (36) for this $f = f^{(0)}$. This equation determines the distribution of z_{t+1} conditional on z_t.

Given the function $h^{(0)}$, we get an new candidate solution $f^{(1)}$ through the following recursion:

$$\begin{aligned} f^{(i+1)}(z_t) \quad = \quad & \rho \mathbf{E}_t \left[(1 + f^{(i)}(z_{t+1})) e^{(1-\gamma)(g + \sigma \epsilon_{t+1})} \right] \\ & + b_0 \rho f^{(i)}(z_t) \mathbf{E}_t \left[\bar{v} \left(\frac{1 + f^{(i)}(z_{t+1})}{f^{(i)}(z_t)} e^{g + \sigma \epsilon_{t+1}}, z_t \right) \right], \ \forall z_t. \end{aligned} \tag{37}$$

With $f^{(1)}$ in hand, we can calculate a new $h = h^{(1)}$ that solves equation (35) and (36) for $f = f^{(1)}$. This $h^{(1)}$ gives us a new candidate $f = f^{(2)}$ from (37). We continue this process until convergence occurs.

Price-Dividend Ratio

Figure 4 presents the resulting price-dividend ratio as a function of the state variable z_t. It is a decreasing function of z_t. The intuition for this is straightforward: a low value of z_t means that recent returns on the asset have been high, giving the investor a reserve of prior gains. These gains cushion any subsequent losses, making the investor less risk-averse. He therefore discounts future dividends at a lower rate, raising the price-dividend ratio. Conversely, a high value of z_t means that the investor has recently experienced a spate of painful losses; he is now especially sensitive to further losses which makes him more risk-averse and lowers price-dividend ratios.

Distribution of the state variable and of returns

Figure 4 by itelf does not tell us the range of price-dividend ratios we are likely to see in equilibrium. For that, we need to know the distribution of the state variable z_t in equilibrium, and we present it in the top left panel of Figure 5. To obtain it, we use equations (35) and (36) together with the price-dividend ratio graphed in Figure 4 to impute the state variable dynamics $z_{t+1} = h(z_t, \varepsilon_{t+1})$. We then draw a long time series $(\varepsilon_t)_{t=1}^{10,000}$ of 10,000 independent draws from the standard normal distribution and starting with $z_0 = 1$, use h to generate a time series for z_t. Note from the graph that the unconditional mean of z_t is very close to one, and this is no accident. We chose the value of $\overline{R}$ in equation (35) precisely to make the mean of z_t as close to one as possible.

As we generate the time series for z_t period by period, we also compute the returns along the way using equation (36). The top right panel of Figure 5 plots the distribution of returns that we obtain. We now present sample moments computed from these simulated returns; the time series is long enough that sample moments should serve as good approximations to population moments.

Unconditional Means and Volatilities

Our preliminary model is Section 2 utterly failed to reproduce the most basic characteristics of stock returns, their mean and volatility. Using $b_0 = 2$, the equity premium, or mean log excess return, was a paltry 0.91% and volatility only 3.79%. Table 3 shows that the more realistic model we are now using fares much better. The average return on the risky asset in excess of the risk-free rate is now a very substantial 4.1%.

Our success in generating a more sizeable equity premium is largely due to the fact that the current model does a much better job explaining return volatility. Table 3 shows that the equilibrium volatility of returns is now almost 14%. Innovations in consumption growth lead to changes in risk-aversion and hence discount rates, making returns much more volatile than consumption growth. These volatile returns scare the loss-averse investor into charging a much higher equity premium than before.

Table 3 also reports the average value and standard deviation of the price-dividend ratio in our simulations.

Conditional Means and Volatilities

The bottom left panel in Figure 5 plots the conditional expected return as a function of z_t, obtained by numerically integrating the return equation (36) over the conditional distribution of z_{t+1} given by $z_{t+1} = h(z_t, \varepsilon_{t+1})$. The conditional expected return is an increasing function of the state variable. Low values of z_t mean that the investor has accumulated prior gains that will cushion future losses. He is therefore less risk-averse, leading to a lower expected return in equilibrium. The dashed line shows the level of the constant riskfree rate for comparison.

The bottom right panel in Figure 5 graphs the conditional volatility of returns as a function of the state variable. Since much of the return volatility in our model is generated by changing risk-aversion, the conditional volatility in any state depends on how sensitive the investor's risk-aversion in that state is to consumption shocks. The gentle inverted U-shape in the graph reflects the fact that in our specification, risk-aversion is most sensitive near $z_t = 1$, so that conditional volatility is highest near that point. Empirically, volatility has been found to be higher after market crashes than booms, which in our context would mean an upward sloping conditional volatility curve. Of course, we could generate this result by stipulating that investors' risk-aversion is more sensitive in troughs than at peaks. Since we have found no independent evidence of this, we do not impose it in our model.

Autocorrelations

Table 4 presents autocorrelations of log returns and of the price-dividend ratio. As expected, our model produces negatively autocorrelated returns at all lags: high prices lower risk-aversion and lead to lower returns on average. These negative autocorrelations imply long-horizon mean-reversion of the kind documented by Poterba and Summers (1988) and Fama and French (1988a). Moreover, the price-dividend ratio is highly autocorrelated in our model, closely matching its actual behavior.

Long-horizon Predictability

Since the investor's risk-aversion changes over time in our model, expected returns also vary, and hence returns are predictable. To demonstrate this, we use our simulated data to run regressions of cumulative log returns over a k-year horizon on the lagged dividend-price ratio for $k = 1, 2, 3$, and 4,

$$r_{t+1} + r_{t+2} + \cdots + r_{t+k} = \alpha_k + \beta_k(\frac{D_t}{S_t}) + \varepsilon_{k,t}. \tag{38}$$

where r_t is the log return. Table 5 presents the slope coefficients β_k and $R^2(k)$ obtained from our simulated data alongside the empirical values. Note that our simulated results capture the main features of the empirical findings, including an R^2 that increases with the return horizon.

We conclude by presenting some results for different benchmark level dynamics. In particular, we alter the parameter η in (22). The results so far have been for $\eta = 1$, which represents a very sluggish benchmark level. Table 6 shows what happens when we try $\eta = 0.9$ and $\eta = 0.8$. Even before looking at the results, we know that they are unlikely to be as impressive as those for $\eta = 1$. A faster-moving benchmark level makes it harder to accumulate prior gains or losses. The investor's risk-aversion will therefore change more slowly over time, generating lower volatility and hence a lower equity premium. Table 6 shows, though, that the results are still respectable, with even $\eta = 0.8$, generating returns twice as volatile as consumption growth.

Adding the evidence on the effect of prior outcomes has improved our model's ability to make sense of several features of aggregate stock returns. However, even this model is not able to fully match the empirical estimates of the equity premium and volatility. This should not be seen as a weakness of our approach, but on the contrary, as entirely realistic. While we believe that the effects we describe are very relevant for aggregate stock market behavior, we do not insist that they are the only mechanism at work. The literature has produced other possible stories for particular features of the data: the excess volatility of returns, for example, may also also be the result of investors extrapolating earnings trends too far into the future – see Barsky and De Long (1993) and Barberis, Shleifer, Vishny (1998).

Having said this, there is a sense in which the equity premium and volatility numbers that we have generated so far actually represent a very conservative lower bound on what our framework is capable of. This is because we have worked throughout with Lucas' (1978) model, which forces consumption to equal dividends, while in reality they are very different. We adopted this model because it is the simplest possible framework that can illustrate our ideas. The drawback though, is that its restrictive assumption prevents us from demonstrating the full force of our preference specification and from showcasing its full range of predictions.

To see this, imagine a model which does distinguish between dividends and consumption, perhaps by introducing another source of wealth such as labor income. Since stocks are now a claim to volatile dividends rather than to smooth consumption, stock returns will also be more volatile. This much is also true in consumption-based models. However, in those models, separating dividends from consumption has *no effect on the equity premium*: even though stock returns are more volatile, making stocks more risky, they are also less correlated with consumption – because dividends are only weakly correlated with consumption – and this makes stocks less risky. Overall the equity premium is largely unaffected.[24]

In our world, the effect could not be more different. Since our investor worries about wealth fluctuations per se and not simply about the consumption fluctuations they induce, any increase in volatility translates directly into a higher equity premium.

In summary then, separating dividends and consumption should lead to results for volatility and the equity premium that are even more striking than those in the tables. And this is not all. Such a framework would also generate stock returns that are only weakly correlated with consumption growth innovations. The wildly counterfactual perfect correlation typically implied by consumption-based models is a continued source of embarassment for that paradigm.

[24]See Campbell and Cochrane (1999) for an insightful discussion of this point.

4 Conclusion

In this paper, we have proposed a new framework for pricing assets, derived in part from the traditional consumption-based approach, but which also incorporates two long-standing ideas in psychology: the prospect theory of Kahneman and Tversky (1979), and the evidence of Thaler and Johnson (1990) and others on the influence of prior outcomes on risky choice.

Consistent with prospect theory, the investor in our model derives utility not only from consumption levels but also from *changes* in the value of his financial wealth from year to year. He is much more sensitive to reductions in wealth than to increases, the "loss-aversion" feature of prospect utility. Moreover, consistent with experimental evidence, the utility he receives from gains and losses in wealth depends on his prior investment outcomes; prior gains cushion subsequent losses – the so-called "house-money" effect – while prior losses intensify the pain of subsequent shortfalls.

We studied asset prices in the presence of agents with preferences of this type, and found that our model can explain the high mean, volatility, and predictability of stock returns. The key to our results is that the agent's risk-aversion changes over time as a function of his investment performance. This generates time-varying risk premia, which in turn make prices much more volatile than underlying dividends. In combination with the agent's loss-aversion, the high volatility of returns generates large equity premia. Our results obtain with reasonable values for all parameters, including even the investor's risk-aversion over consumption uncertainty.

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6 Appendix: Proof of Propositions

We first prove Proposition 2, since the proof for Proposition 1 follows immediately. We conjecture that in equilibrium, the riskfree gross interest rate is constant at R_f given by (31), and the stock returns have a one-factor Markov structure given by (35), (29), and (30), with $f(\cdot)$ satisfying (32) for all z_t. We then show that, under rational expectations, the representative investor indeed consumes all the dividend and holds the total supply of assets at each time t.

The representative agent's optimization problem is

$$\max_{\{C_t, S_t\}} \mathbf{E}\left[\sum_{t=0}^{\infty}\left[\rho^t \frac{C_t^{1-\gamma}}{1-\gamma} + b_0 \rho^{t+1} \overline{C}_t^{-\gamma} S_t \widehat{v}(R_{t+1}, z_t)\right]\right] \tag{39}$$

subject to the standard budget constraint

$$W_{t+1} = (W_t - C_t) R_f + S_t (R_{t+1} - R_f), \tag{40}$$

where W_t denotes the representative agent's pre-consumption wealth at t.

Let $J(W_t, z_t, t)$ be the value function. It must satisfy, subject to (40), the following Bellman equation:

$$J(W_t, z_t, t) = \max_{(C_t, S_t)} \left[\rho^t \frac{C_t^{1-\gamma}}{1-\gamma} + \mathbf{E}_t\left[b_0 \rho^{t+1} \overline{C}_t^{-\gamma} S_t \widehat{v}(R_{t+1}, z_t) + J(W_{t+1}, z_{t+1}, t+1)\right]\right]. \tag{41}$$

We guess that the value function has the form

$$J(W_t, z_t, t) = \rho^t \left[1 + f(z_t)\right]^{\gamma} \frac{W_t^{1-\gamma}}{1-\gamma}. \tag{42}$$

Let $V(C_t, S_t)$ denote the term inside square brackets in (41):

$$V(C_t, S_t) = \rho^t \frac{C_t^{1-\gamma}}{1-\gamma} + \mathbf{E}_t\left[b_0 \rho^{t+1} \overline{C}_t^{-\gamma} S_t \widehat{v}(R_{t+1}, z_t) + J(W_{t+1}, z_{t+1}, t+1)\right]. \tag{43}$$

Then, under the proposed value function, we can show that the second partial derivative matrix of $V(C_t, S_t)$, at any point (C_t, S_t) with $C_t > 0$ and $S_t > 0$, is negative definite. (This proof is given at the end of this appendix.) Therefore, the necessary and sufficient conditions for a policy of (C_t^*, S_t^*), with $C_t^* > 0$ and $S_t^* > 0$, to maximize $V(C_t, S_t)$ are provided by a pair of Euler equations which are derived by varying C_t and S_t, respectively, around their optimal values of C_t^* and S_t^* and are given below:

$$\rho R_f \mathbf{E}_t \left[[1+f(z_{t+1})]^\gamma \left(\frac{W_{t+1}}{C_t} \right)^{-\gamma} \right] = 1 \tag{44}$$

$$\mathbf{E}_t \left[[1+f(z_{t+1})]^\gamma (R_{t+1} - R_f) \left(\frac{W_{t+1}}{\overline{C}_t} \right)^{-\gamma} \right] + b_0 \mathbf{E}_t \left[\widehat{v}(R_{t+1}, z_t) \right] = 0. \tag{45}$$

We now show that the policy of $C_t^* = \overline{C}_t = D_t$ and $S_t^* = W_t = P_t$, for all t, indeed satisfies the above Euler equations. Under this policy,

$$W_{t+1} = D_{t+1} + P_{t+1} = D_{t+1}(1 + f(z_{t+1})), \tag{46}$$

and the Euler equation (44) becomes

$$\rho R_f \mathbf{E}_t \left[\left(\frac{D_{t+1}}{D_t} \right)^{-\gamma} \right] = 1,$$

which is satisfied by the conjectured riskfree rate in (31). Under the same policy, the Euler equation (45) becomes

$$\mathbf{E}_t \left[(R_{t+1} - R_f) \left(\frac{D_{t+1}}{D_t} \right)^{-\gamma} \right] + b_0 \mathbf{E}_t \left[\widehat{v}(R_{t+1}, z_t) \right] = 0,$$

which is satisfied under the proposed one-factor Markov stock returns in (29)-(30), the conjectured riskfree rate in (31), and the conjectured price-dividend ratio function $f(\cdot)$ that satisfies (32) for all z_t. So under the conjectured bond and stock returns, the representative agent indeed consumes all dividends and holds one unit of the stock. The equilibrium is thus shown to exist.

The proof of Proposition 1 follows exactly as above, except that $z_t \equiv 1$ for all t, $f_t \equiv f$, and the stock returns are i.i.d. over time.

Finally, we show that the second partial derivatives of $V(C_t, S_t)$ in (43) are negative definite. Let J_W and J_{WW} denote, respectively, the first and second partial derivatives of the value function J with respect to wealth. Then the second partial derivatives of $V(C_t, S_t)$ are given by

$$\begin{aligned} V_{CC}(C_t, S_t) &= -\gamma \rho^t C_t^{-(1+\gamma)} + \mathbf{E}_t \left[R_f^2 J_{WW}(W_{t+1}, z_{t+1}, t+1) \right], & (47) \\ V_{SS}(C_t, S_t) &= \mathbf{E}_t \left[(R_{t+1} - R_f)^2 J_{WW}(W_{t+1}, z_{t+1}, t+1) \right], & (48) \\ V_{CS}(C_t, S_t) &= \mathbf{E}_t \left[-R_f (R_{t+1} - R_f) J_{WW}(W_{t+1}, z_{t+1}, t+1) \right]. & (49) \end{aligned}$$

To show that the second partial derivative matrix of V is negative definite at every strictly positive (C_t, S_t), we need to show that $V_{CC} < 0$ and $V_{CC}V_{SS} - V_{CS}^2 > 0$ everywhere. Under the proposed value function given in (42), $J_{WW} < 0$, so $V_{CC} < 0$ holds. To show that $V_{CC}V_{SS} - V_{CS}^2 > 0$, we can split it into two terms

$$V_{CC}V_{SS} - V_{CS}^2 = A(C_t, S_t) + B(C_t, S_t), \tag{50}$$

where

$$A(C_t, S_t) = -\gamma\rho^t C_t^{-(1+\gamma)}\mathbf{E}_t\left[(R_{t+1} - R_f)^2 J_{WW}(W_{t+1}, z_t + 1, t+1)\right] > 0, \tag{51}$$

and using a shorthand notation for the value function,

$$\begin{aligned} B(C_t, S_t) \;&=\; \mathbf{E}_t\left[R_f^2 J_{WW}(t+1)\right]\mathbf{E}_t\left[(R_{t+1} - R_f)^2 J_{WW}(t+1)\right] \\ &\quad - \left[\mathbf{E}_t\left[-R_f(R_{t+1} - R_f)J_{WW}(t+1)\right]\right]^2. \end{aligned} \tag{52}$$

Define $x \equiv R_f\sqrt{-J_{WW}(t+1)}$ and $y \equiv (R_{t+1} - R_f)\sqrt{-J_{WW}(t+1)}$, and using

$$\mathbf{E}_t(x^2)\mathbf{E}_t(y^2) - \left[\mathbf{E}_t(xy)\right]^2 = \mathbf{E}_t(y^2)\mathbf{E}_t\left[\left(x - \frac{\mathbf{E}_t(xy)}{\mathbf{E}_t(y^2)}y\right)^2\right] > 0,$$

we find that $B(C_t, S_t) > 0$. So $V_{CC}V_{SS} - V_{CS}^2 > 0$, and the second partial derivative matrix of V is indeed negative definite everywhere at $C_t > 0$ and $S_t > 0$.

Table 1: Assumed parameter values for the models in Sections 2 and 3.

Parameter	Section 2	Section 3
g	0.0184	0.0184
σ	0.0379	0.0379
γ	0.9	0.9
ρ	0.98	0.98
λ	2.25	2.25
b_0	(range)	2
k	-	50
η	-	1 (+range)

Table 2: Unconditional moments for returns under the preliminary model. Empirical values for consumption growth come from annual data from 1889-1985; those for stock returns are based on annual NYSE data from 1926-1995.

	Model Value	Empirical Value
Log Consumption growth		
Mean	0.0184	0.0184
Std. Dev.	0.0379	0.0379
Log Excess Stock Return		
Mean	0.0091	0.0603
Std. Dev.	0.0379	0.2002
Sharpe Ratio	0.24	0.3

Table 3: Unconditional moments for returns under the full model. Empirical values for consumption growth come from annual data from 1889-1985; those for stock returns are based on annual NYSE data from 1926-1995.

	Model Value	Empirical Value
Log Consumption growth		
Mean	0.0184	0.0184
Std. Dev.	0.0379	0.0379
Log Excess Stock Return		
Mean	0.041	0.0603
Std. Dev.	0.133	0.2002
Sharpe Ratio	0.31	0.3
Price-Dividend Ratio		
Mean	17.0	25.3
Std. Dev.	3.1	6.7

Table 4: Autocorrelations of log returns and price-dividend ratios in the full model. Empirical values are based on annual NYSE data from 1926-1995.

	Model Value	Empirical Value
Corr(r_t, r_{t-k})		
$k = 1$	-0.10	0.07
$k = 2$	-0.07	-0.17
$k = 3$	-0.06	-0.05
$k = 4$	-0.03	-0.11
$k = 5$	-0.03	-0.04
Corr$((\frac{P}{D})_t, (\frac{P}{D})_{t-k})$		
$k = 1$	0.84	0.70
$k = 2$	0.72	0.50
$k = 3$	0.62	0.45
$k = 4$	0.54	0.43
$k = 5$	0.47	0.40

Table 5: Coefficients and R^2 in regressions of k-year horizon log returns on the lagged dividend-price ratio, $r_{t+1}+r_{t+2}+\cdots+r_{t+k} = \alpha_k+\beta_k(\frac{D_t}{S_t})+\epsilon_{k,t}$. Empirical values are based on annual NYSE data from 1926-1995.

$\beta(k)$, $R^2(k)$	Model Value	Empirical Value
β_1	3.4	4.2
β_2	6.1	8.7
β_3	8.3	12.1
β_4	10.1	15.9
$R^2(1)$	8%	7%
$R^2(2)$	13%	16%
$R^2(3)$	18%	22%
$R^2(4)$	22%	30%

Table 6: Simulation Results for different benchmark level dynamics; lower η means a less sluggish benchmark level.

	$\eta = 1$	$\eta = 0.9$	$\eta = 0.8$
Excess Stock Return			
Mean	0.041	0.030	0.023
Std. Dev.	0.133	0.088	0.066
Sharpe Ratio	0.31	0.34	0.35
Price-Dividend Ratio			
Mean	17.0	20.2	23.7
Std. Dev.	3.1	1.4	0.9
Corr(R_t, R_{t-k})			
$k = 1$	-0.10	-0.13	-0.13
Corr$((\frac{P}{D})_t, (\frac{P}{D})_{t-k})$			
$k = 1$	0.84	0.7	0.62

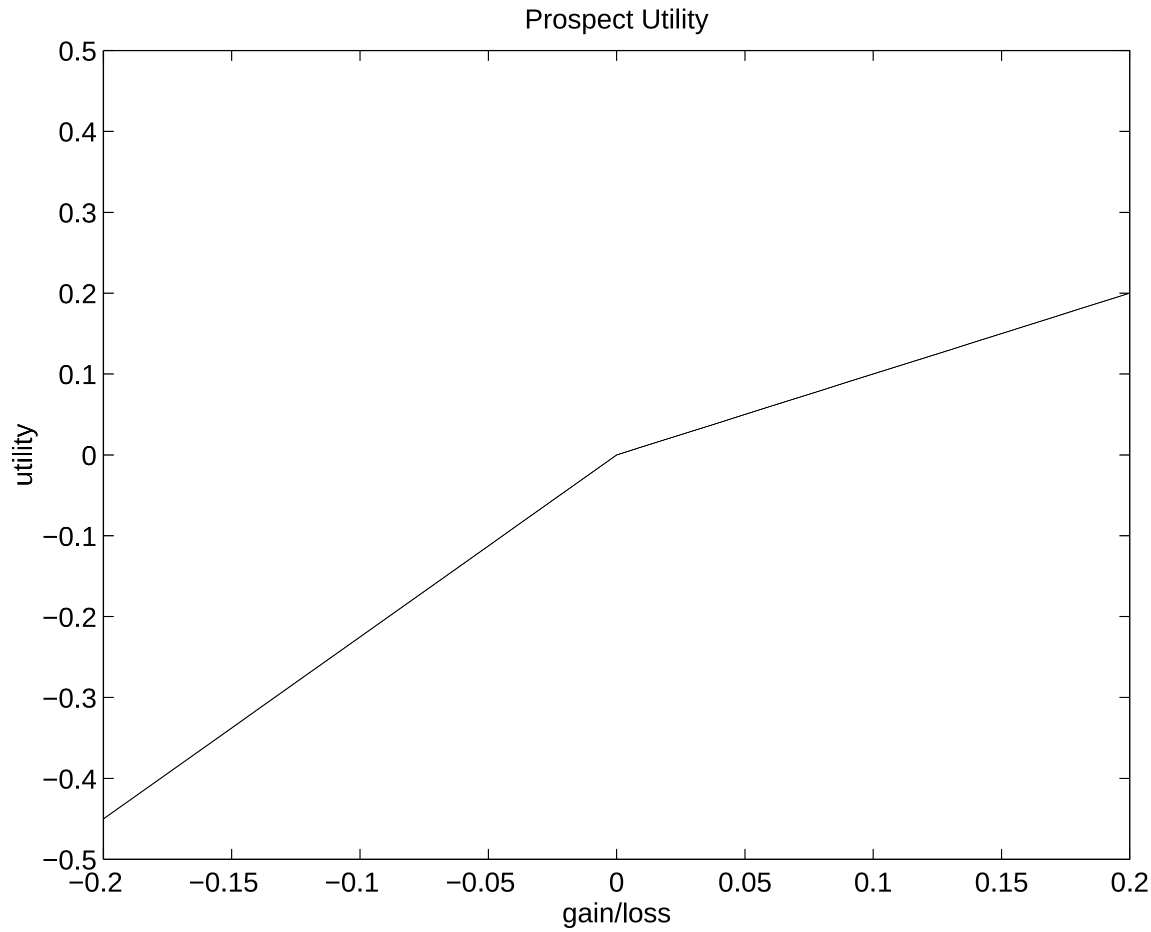

Figure 1. Shape of prospect utility function plotted against the gain/loss, X. Losses are penalized at $\lambda = 2.25$ times the rate the gains are rewarded.

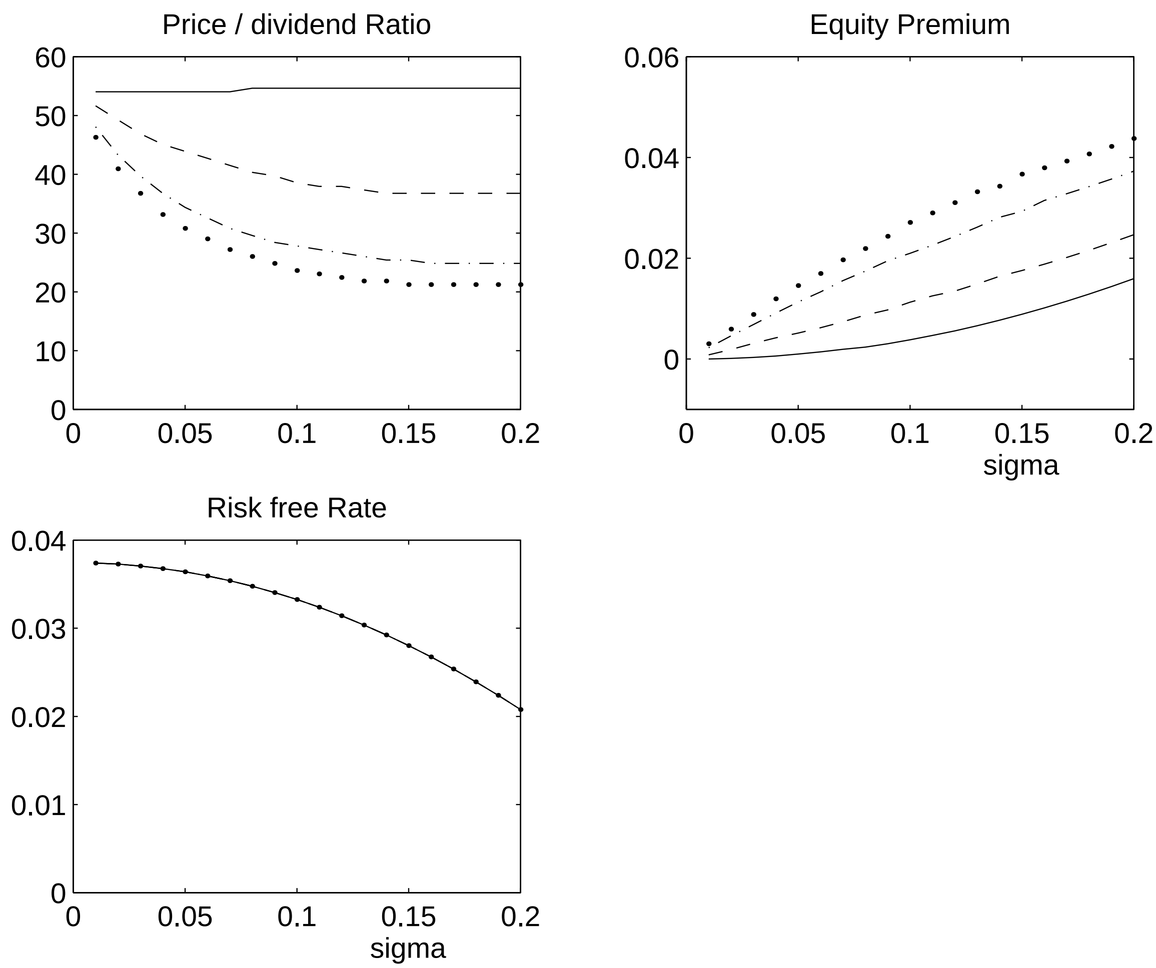

Figure 2. The graph plots the price-dividend ratio, mean log excess return, and risk-free rate as a function of σ, the standard deviation of consumption growth, in an economy where the agent has prospect utility over gains and losses but no memory of prior outcomes. Solid line is for $b_0 = 0$, dashed line for $b_0 = 0.3$, dash-dot line for $b_0 = 2$, and dotted line for $b_0 = \inf$.

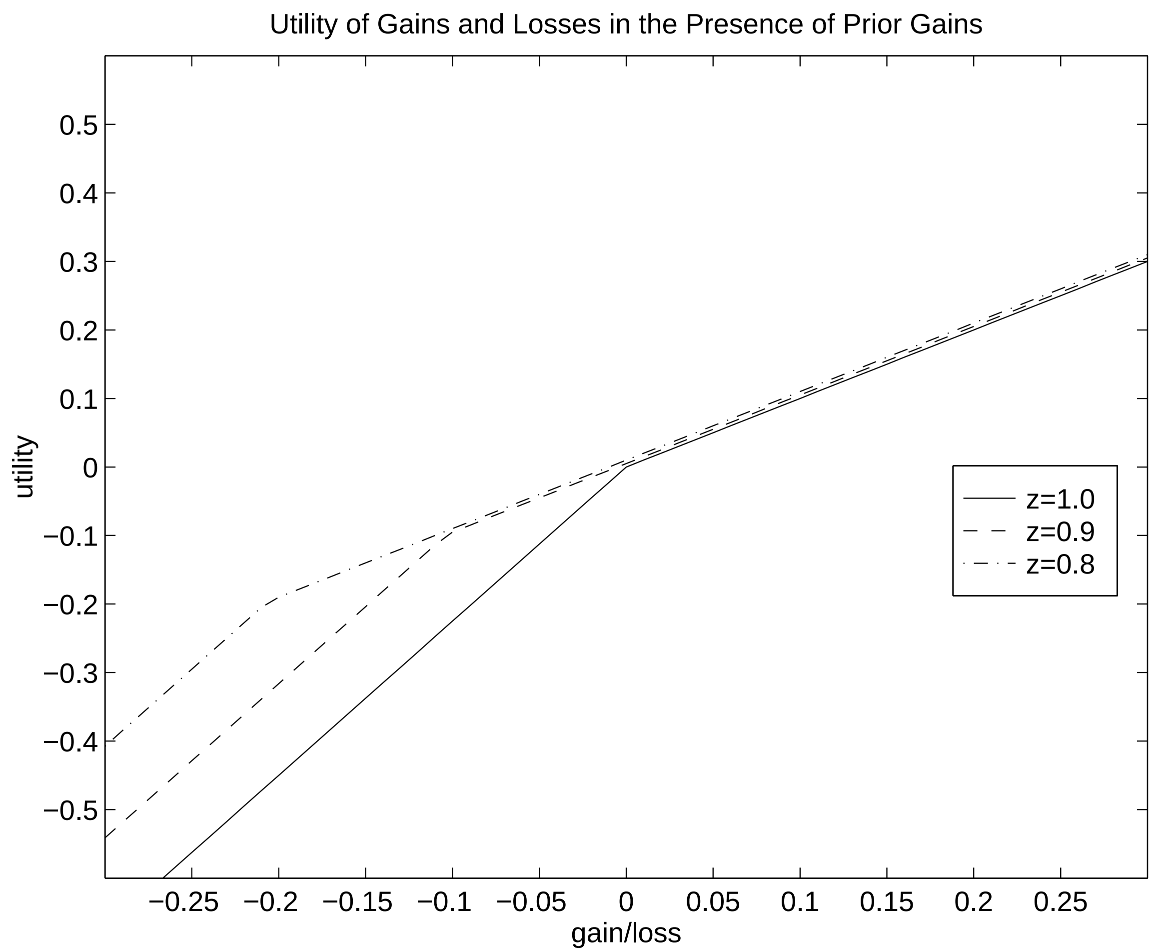

Figure 3. The graph plots the utility from a gain/loss of $R_{t+1} - R_f$ in situations where the investor has had prior gains. The size of the prior gain is measured by the state variable z_t and the riskfree rate is set at 3%.

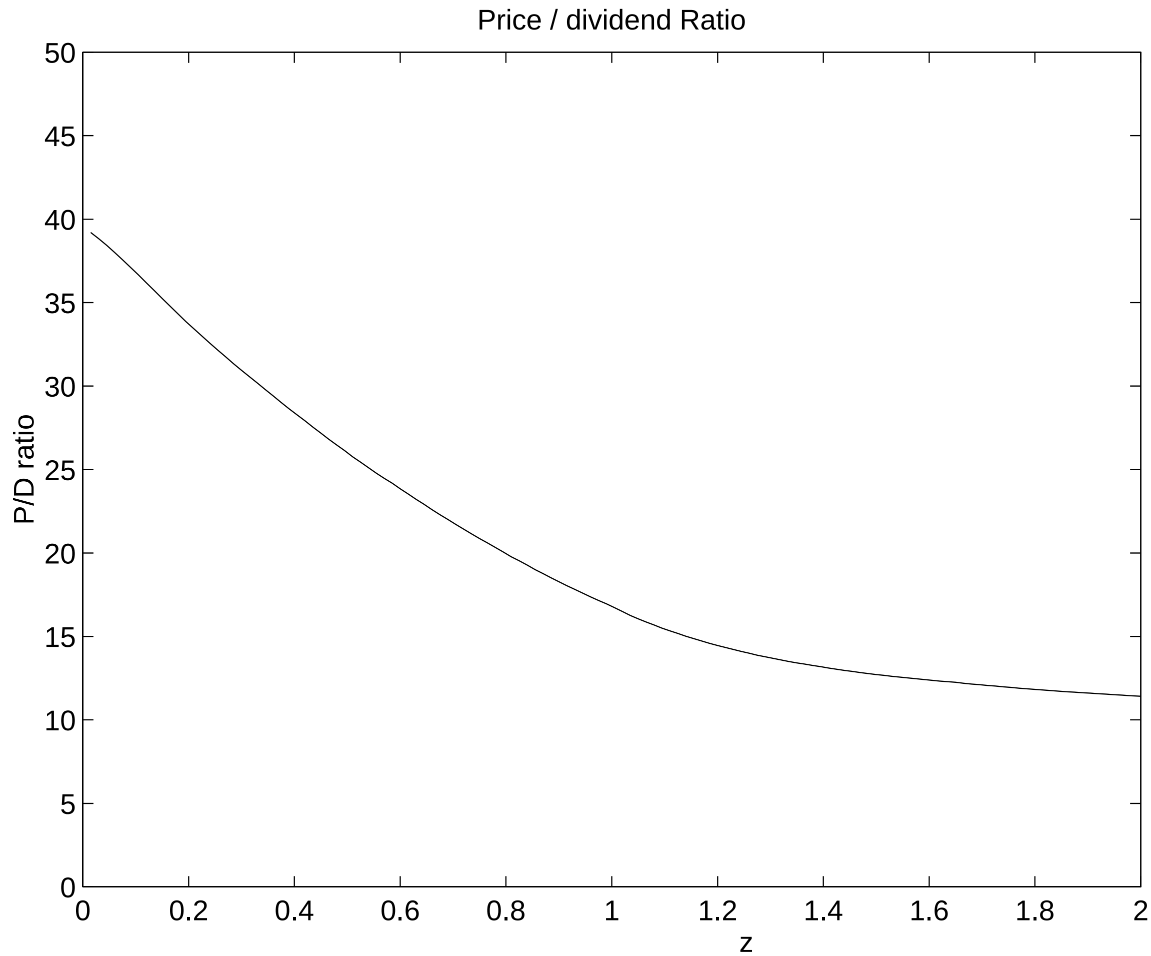

Figure 4. The graph plots the price-dividend ratio against the state variable z in an economy where the agent has prospect utility over gains and losses and where prior outcomes influence risky choice.

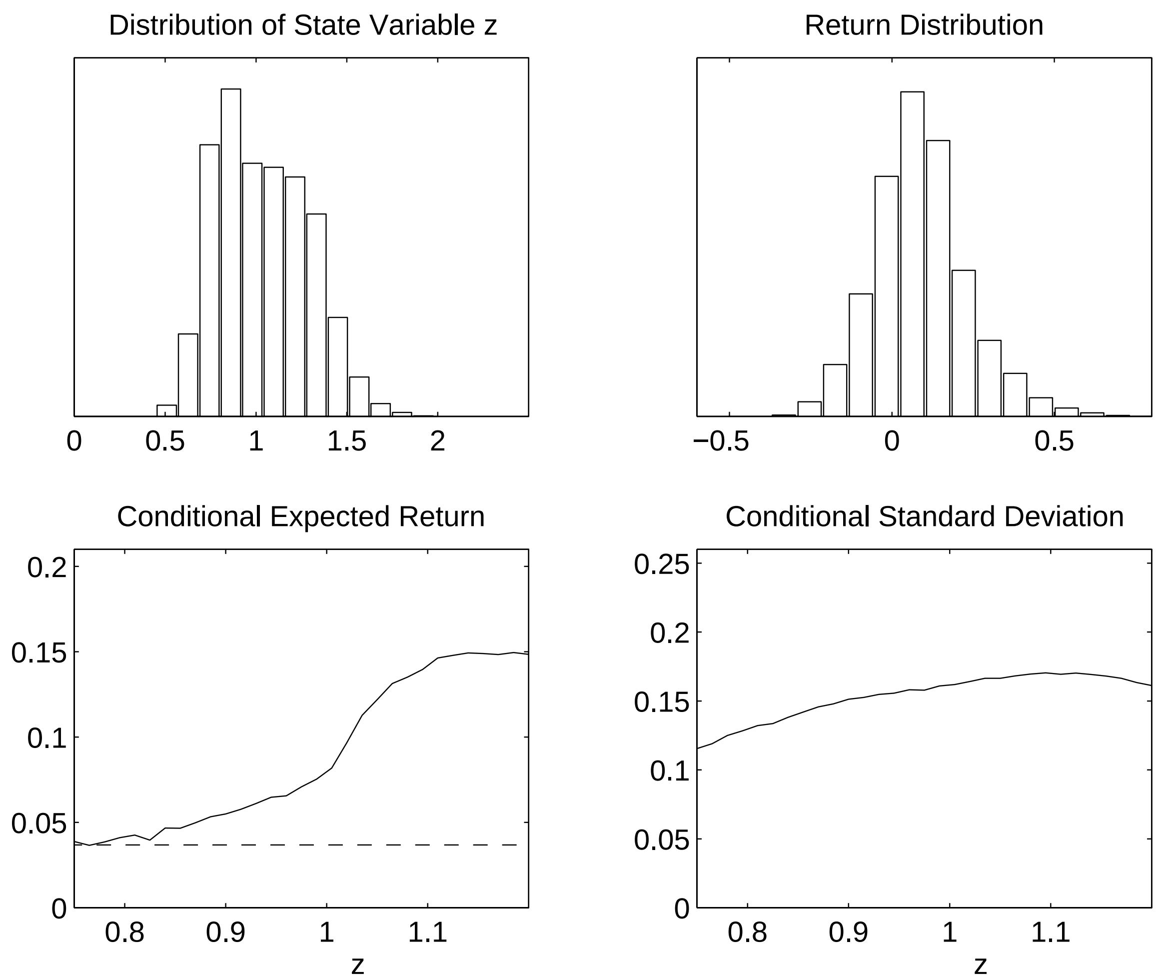

Figure 5. The figure shows the distribution of the state variable z and of stock returns, and plots the conditional expected return and standard deviation of asset returns against the state variable in an economy where the investor has prospect utility over gains and losses and where prior outcomes influence risky choice.