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ADJUSTMENT COSTS, DURABLES, AND AGGREGATE CONSUMPTION

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Abstract

Previous tests of the permanent income hypothesis (PIH) have focused on either nondurables or durables expenditures in isolation. This paper studies consumer purchases of nondurables and durables as the outcome of a single optimization problem. It is shown that the presence of adjustment costs of changing durables stocks may substantially affect the time series properties of both components of expenditure under the PIH. However, econometric tests based on this model do not contradict earlier rejections of the PIH in aggregate quarterly data.

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1. Introduction

The validity of Friedman's (1957) permanent-income hypothesis (PIH) as a description of aggregate consumer behavior is an important and long-debated issue in macroeconomics. Recently this hypothesis, sharpened by the implications of rational expectations theory, has come under intensive re-examination. Articles by Hall (1978), Sargent (1978), and Blinder (1981) have exploited restrictions generated by the PIH-cum-rational expectations to test the hypothesis in aggregate time series data; summarizing this evidence, Flavin (1981) concluded that the PIH should be rejected. Using disaggregated data, Hall and Mishkin (1982) and Hayashi (1982b) have also found reason to question the PIH.

A difficulty with all of these papers is that none does a very satisfactory job with the durables expenditure component of consumption. This is a potentially important problem: On the one hand, because of lagged stock adjustment and accelerator effects, an inadequate treatment of durables expenditure might lead to an incorrect rejection of the PIH. (This is true even if the durables component is excluded from the data, if durables and nondurables are nonseparable in consumer utility functions.) On the other hand, if significant imperfections in consumer credit markets (the most likely source of failure of the PIH) do exist, then we would expect this to be reflected most strongly in the pattern of durables purchases (Darby (1972), Mishkin (1976)). Insufficient attention to durables in this case eliminates a good opportunity to distinguish the PIH from the alternatives.

At least two papers have tried to remedy the relative neglect of durables. A note by Mankiw (forthcoming) argued that, under the PIH, expenditures on durables should follow a mixed autoregressive-moving average process; his finding that aggregate durables expenditure is AR(1) is interpreted as evidence against the PIH. (This conclusion seems premature, as the usual alternatives to the PIH appear even less likely to generate the time series behavior which Mankiw found.)

The other paper which concentrates on durables is my own (Bernanke (forthcoming)). I looked at data on income and automobile expenditure for 1400 families over four years. My estimates suggested that consumer behavior in the sample could be well described by the PIH.

As the earlier work tended to exclude durables, however, the papers by Mankiw and by me considered only durables expenditure in isolation. A natural step is to consider the joint behavior of durables and nondurables spending. The present paper does two things: First, it presents a formal dynamic model of the consumer's decision problem under uncertainty. The model assumes that utility is a nonseparable function of nondurables consumption and of the services from durables, and that changing durables stocks involves costs of adjustment. This analysis, which appears to be somewhat more general than those previously used, can be used to derive exact, closed-form decision rules suitable for use in estimation. These rules suggest that the presence of adjustment costs may have a substantial effect on the pattern of nondurable as well as durables purchases.

Second, the paper uses the formal model to test the PIH in aggregate time series data. Under the econometrically tractable assumption of constant interest rates, the PIH is strongly rejected by the data. With variable interest rates, it appears that this conclusion is somewhat moderated but not reversed. Overall, Flavin's earlier findings are supported.

2. The consumer's optimization problem

This section presents an analysis of optimal consumer choice in a dynamic stochastic environment. The model has the following features:

1) The consumer derives utility from both nondurables and the services of durables. Nondurables and durables may enter the utility function nonseparably.

2) There are adjustment costs (which take the form of foregone leisure) associated with changing durables stocks.

3) Utility and adjustment costs are assumed to be quadratic in form.

4) The consumer's income is a general stochastic process.

5) The real interest rate, the relative price of durables, and the rate of stock depreciation follow known, but possibly time-dependent, sequences.

The integration of durables and nondurables purchasing decisions is the most important novelty of the model. Also, previous analyses have typically not allowed time-dependency of interest rates, prices, and depreciation. (These variables are assumed nonstochastic to avoid certain technical problems; however, most of the analysis below also applies in the general stochastic case.) The principal restriction of the model is the assumption that utility and adjustment costs are quadratic. This assumption is required for there to be any hope of deriving closed-form decision rules. It should be noted, however, that all previous formulations of the PIH under uncertainty are also based, either explicitly or implicitly, on a quadratic specification of utility.[1]

To begin, it is assumed that the utility enjoyed by a representative consumer during a given period t is described by

$$(2.1) \qquad U(c_t, K_t, K_{t+1}) = -\tfrac{1}{2}(\bar{c}-c_t)^2 - \tfrac{a}{2}(\bar{K}-K_t)^2 - m(c-c_t)(\bar{K}-K_t) - \tfrac{d}{2}(K_{t+1}-K_t)^2$$

c_t is the quantity of nondurable goods and services consumed during t. K_t and K_{t+1} are the stocks of consumer durables held at the beginning of periods t and t+1, respectively. $\bar{c}$, $\bar{K}$, a, m, and d are parameters. As noted, the expression (2.1) is quadratic in form.

I assume that the service flow of durables is proportional to durables stocks; thus the stock measures K_t and K_{t+1} enter utility directly. The third term of the RHS of (2.1) permits the durables and nondurables aggregates to be nonseparable in utility. Positive values of the parameter m imply that durables and nondurables are substitutes.

The last term in the RHS of (2.1) is nonstandard and requires some explanation. In parallel with the literature on capital accumulation by firms[2], this term represents "adjustment costs" associated with net expenditure[3] on durables during period t. The inclusion of some type of adjustment cost is required to motivate lags in stock adjustment. In the case of the consumer, the most likely source of adjustment costs is the time devoted to shopping for (and, possibly, learning how to use) new durables. Assuming that consumers derive utility from leisure time (and dislike shopping), it is appropriate to include these adjustment costs in the utility function, as in (2.1). Not incidentally, having adjustment costs in the utility function (instead of in the budget constraint) and assuming that these costs are quadratic greatly increases the tractability of the problem.

If total utility is taken to be the sum of expected discounted values of present and future period-utilities, then the optimization problem for an infinitely-lived consumer can be written as

$$(2.2) \qquad \max_{\{c_t, K_{t+1}\}} E_0 \sum_{t=0}^{\infty} b^t U(c_t, K_t, K_{t+1})$$

subject to the sequence of budget constraints

$$(2.3) \qquad E_t\{ \sum_{i=t}^{\infty} R_i[c_i + p_i(K_{i+1}-(1-\delta_i)K_i) - y_i]\} = R_t A_t \qquad t = 0,1,2,...$$

where

b = utility discount factor

p_t = the price of durables relative to the price of nondurables in t

δ_t = the rate of physical depreciation of durables in t

y_t = real income in t

A_t = real financial assets at the beginning of t

Initial financial assets A_0 and capital stock K_0 are given. The period-t market discount factor R_t is defined by

$$R_0 = 1 \tag{2.4}$$

$$R_t = \prod_{i=1}^{t} (1+r_{i-1})^{-1} \qquad t=1,2,...$$

with r_t being the prospective real rate of interest between t and t+1. The forcing variables $\{r_t\}$,$\{p_t\}$,$\{\delta_t\}$, and $\{y_t\}$ are time-dependent; $\{y_t\}$ is a stochastic process with a known distribution. The sequence of budget constraints defined in (2.3) requires the consumer to revise his consumption program in each period so as to balance the expected present values of expenditures and receipts; this is a reasonable specification for an infinite-horizon model.

At the beginning of each period t the consumer learns the value of the current income drawing, y_t. Already known are the other forcing variables and the stocks of financial assets and consumer durables left over from the previous period. The consumer's problem is to choose nondurable consumption, c_t, and the amount of net expenditure on durables, $K_{t+1}-K_t$.

The first-order conditions associated with the two decision variables in t are

$$\bar{c} - c_t + m(\bar{K}-K_t) = b(1+r_t)\ E_t[\bar{c} - c_{t+1} + m(\bar{K}-K_{t+1})] \tag{2.5}$$

$$\begin{aligned} &-d(K_{t+1}-K_t) + ba(\bar{K}-K_{t+1}) + bmE_t(\bar{c}-c_{t+1}) \\ &+ bdE_t(K_{t+2}-K_{t+1}) - bE_t\{z_{t+1}[\bar{c}-c_{t+1}+m(\bar{K}-K_{t+1})]\} = 0 \end{aligned} \tag{2.6}$$

where z_{t+1}, the "cost of capital", is given by

(2.7) $$z_{t+1} = r_t p_t + \delta_{t+1} p_{t+1} - (p_{t+1} - p_t)$$

These equations follow from differentiating the objective function with respect to (c_t, K_{t+1}) and using the budget constraint for period t. The interpretation of the equations is as follows: (2.5) says that along the optimal path the consumer must receive the same present utility from an extra unit of nondurable consumption today as from $(1+r_t)$ units tomorrow. The import of (2.6) is that, on the margin, the consumer must be indifferent between the following two strategies: 1) increasing net durables stocks by one unit today, or 2) delaying the net increase in stocks until tomorrow, applying the savings thus achieved towards consumption of nondurables.

(2.5) and (2.6) form a system of stochastic difference equations in c and K. The rest of this section will derive the consumer's optimal program, using these equations and the sequence of budget constraints: Propositions 1 and 2 below will describe a general family of candidate solutions for the system (2.5)-(2.6). Proposition 3 shows how to select the actual solution from the general family. Proposition 4 derives the exact solution for the case where the nonstochastic forcing variables r, p, and δ are constant over time.

<u>Proposition 1.</u> A family of candidate solutions for the utility-maximizing path of nondurables consumption is given by

$$c_t = (\bar{c} + m\bar{K}) - mK_t + R_t G_t / b^t \tag{2.8}$$

where the stochastic process $\{G_t\}$ is an arbitrary martingale.

Proof. Noting that $R_{t+1}/R_t = (1+r_t)$ and that $E_t G_{t+1} = G_t$ (by the definition of a martingale), it is straightforward to verify that the stochastic process defined by (2.8) satisfies the first-order condition (2.5).//

<u>Proposition 2.</u> Given a candidate solution for the nondurables process as defined by (2.8), the associated solution for the evolution of durables stocks is

$$K_{t+1} = x_1 K_t + (1-x_1)\bar{K} - (bx_1/d) \sum_{i=0}^{\infty} (x_2)^{-i} E_t(m - z_{t+i+1}) R_{t+i} G_{t+i} / b^{t+i} \tag{2.9}$$

where x_1 and x_2 are scalars to be defined and the assumption is made that

$$a > m^2 \tag{2.10}$$

Proof. Rewrite (2.6) as

$$bE_t K_{t+2} + hK_{t+1} + K_t = (b/d)[(m^2 - a)\bar{K} + E_t(m - z_{t+1}) R_t G_t / b_t] \tag{2.11}$$

where

(2.12) $$h = (-d - ba + bm^2 - bd)/d$$

and we have used (2.8). Following Sargent (1979), p. 198, factor the LHS of (2.11) as $b(1 + hL/b + L^2/b)E_tK_{t+2} = b(1-x_1L)(1-x_2L)E_tK_{t+2}$, where L is the lag operator. Equating powers of L implies

(2.13) $$-h/b = x_1 + x_2$$
$$1/b = x_1x_2$$

Sargent shows that $h < -(1+b)$ guarantees that 1) x_1 and x_2 are real and distinct, and 2) if x_1 is the smaller root, $x_1 < 1 < 1/b < x_2$. A sufficient condition that $h < -(1+b)$ in our problem is assumption (2.10), which is plausible _a priori_ and is satisfied in the data.

By standard methods the stochastic process $\{K_t\}$ that satisfies (2.11) is found to be

(2.14) $$K_{t+1}=x_1K_t-(bx_1/d)\sum_{i=0}^{\infty}(x_2)^{-i}[(m^2-a)\bar{K}+E_t(m-z_{t+i+1})R_{t+i}G_{t+i}/b^{t+i}]$$

Using the fact that

(2.15) $$(1-x_1)(1-x_2)/x_1x_2 = h + b + 1$$

the expression (2.14) may be re-written as (2.9).//

The next proposition specifies the stochastic process $\{G_t\}$.

Proposition 3. The martingale $\{G_t\}$ that completes the definition of the consumer's optimal program is given by

$$(2.16)\qquad G_t = \frac{E_t \sum_{i=0}^{\infty} R_{t+i} y_{t+i} + R_t A_t - \sum_{i=0}^{\infty} R_{t+i} w_0(t,i)}{\sum_{i=0}^{\infty} R_{t+i} w_1(t,i)}$$

(provided the infinite sums exist); where

$$(2.17)\quad w_0(t,i) = [p_{t+i}\ (x_1^{i+1} - (1-\delta_{t+i})x_1^i) - mx_1^i](K_t - \bar{K}) + p_{t+i}\delta_{t+i}\bar{K}$$

$$(2.18)\quad w_1(t,i) = [p_{t+i}\ (x_1 - (1-\delta_{t+i})) - m] \sum_{j=0}^{i-1} q_{t+j} x_1^{i-j-1} + p_{t+i} q_{t+i} + R_{t+i}/b^{t+i}$$

and

$$(2.19)\qquad q_s = -(bx_1/d) \sum_{i=0}^{\infty} (x_2)^{-i}\ (m - z_{s+i+1})\ R_{s+i}/b^{s+i}$$

Proof. The first step is to show that, given that the consumer is following one of the family of programs derived in Propositions 1 and 2, the stochastic process $\{G_t\}$ defined by (2.16) leads to satisfaction of the sequence of budget constraints (2.3).

We look first for an expression for expected expenditure in t+i, $E_t EXP_{t+i}$, defined by

$$(2.20)\qquad E_t(EXP_{t+i}) = E_t(c_{t+i} + p_{t+i}(K_{t+i+1} - (1-\delta_{t+i})K_t)$$

On the <u>presumption</u> (to be verified) that $\{G_t\}$ is a martingale, Propositions 1 and 2 imply

$$E_t(c_{t+i}) = \bar{c} + m\bar{K} - mE_tK_{t+i} + G_t(R_{t+i}/b^{t+i}) \tag{2.21}$$

$$E_t(K_{t+i+1}) = x_1E_tK_{t+i} + (1-x_1)\bar{K} + q_{t+i}G_t \tag{2.22}$$

where G_t is the current realization of $\{G_t\}$ and q_{t+i} is as defined in (2.19). The solution to the difference equation (2.22) (imposing the boundary condition that K_t is a known number) is

$$E_t(K_{t+i+1}) = x_1^{i+1}K_t + \bar{K}(1-x_1^{i+1}) + G_t \sum_{j=0}^{i} q_{t+j}x_1^{i-j} \tag{2.23}$$

(2.23) is valid for integer values of $i \geq -1$, if the convention is adopted that the last term on the RHS of the expression is zero for $i = -1$.

Substituting (2.23) into (2.21) gives an explicit expression for $E_t(c_{t+i})$; substituting the explicit expressions for $E_t(c_{t+i})$, $E_t(K_{t+i})$, and $E_t(K_{t+i+1})$ into (2.20) and collecting terms yields

$$E_t(EXP_{t+i}) = w_0(t,i) + w_1(t,i)G_t \tag{2.24}$$

where $w_0(t,i)$ and $w_1(t,i)$ are known sequences of coefficients defined above.

Satisfaction of the budget constraint in t requires that the present value of expected future expenditures equal the present value of expected future income plus current financial assets; that is, (2.25) $$E_t \sum_{i=0}^{\infty} R_{t+i}EXP_{t+i} = E_t \sum_{i=0}^{\infty} R_{t+i}y_{t+i} + R_tA_t$$

Substitution of (2.24) into (2.25) and solving for G_t yields (2.16); that is, as promised, the value of G_t given by (2.16) insures that the budget constraint is met.

It has been assumed up to this point that the process defined by (2.16) is a martingale. That this is in fact the case can be verified by the following steps: 1) Subtract R_tEXP_t from both sides of (2.25), using the facts that $R_ty_t - R_tEXP_t + R_tA_t = R_{t+1}(1 + r_t)(A_t + y_t - EXP_t) = R_{t+1}A_{t+1}$, to obtain

(2.26) $$E_t \sum_{i=0}^{\infty} R_{t+i+1}EXP_{t+i+1} = E_t \sum_{i=0}^{\infty} R_{t+i+1}y_{t+i+1} + R_{t+1}A_{t+1}$$

2) Verify by direct substitution that

(2.27) $$w_0(t,i) + w_1(t,i)G = w_0(t+1,i-1) + w_1(t+1,i-1)G$$

for $i \geq 1$ and for any number G. 3) Combine (2.26) and (2.27) to get

(2.28) $$G_t = \frac{E_t \sum_{i=0}^{\infty} R_{t+i+1}y_{t+i+1} + R_{t+1}A_{t+1} - \sum_{i=0}^{\infty} R_{t+i+1}w_0(t+1,i)}{\sum_{i=0}^{\infty} R_{t+i+1}w_1(t+1,i)}$$

4) Note that the expression for G_{t+1} differs from (2.28) only by a multiple of a term (the revision, between t and t+1, of the expected present value of income in t+1) that has expectation zero as of t. It follows that $E_tG_{t+1} = G_t$; or, applying induction, that $\{G_t\}$ is a martingale.//

In the econometric work to follow I will emphasize the case in which the nonstochastic forcing variables $\{r_t, p_t, \delta_t\}$ are constant. To prepare for this, and to counterbalance the abstraction of Proposition 3, Proposition 4 presents the explicit calculation of the $\{G_t\}$ sequence for the case of constant forcing variables.

Proposition 4. Suppose that the real interest rate r, the depreciation rate δ, and the relative price of durables p are known to be constants. Without further loss of generality, let p = 1. Then the consumer's optimal program is given by

(2.29) $$c_t = (\bar{c} + m\bar{K}) - mK_t + [b(1+r)]^{-t}G_t$$

(2.30) $$K_{t+1} = x_1 K_t + (1-x_1)\bar{K} + f[b(1+r)]^{-t}G_t$$

(2.31) $$G_t = g_0 + g_1 \sum_{j=0}^{t} [b(1+r)]^j v_j$$

where

(2.32) $$f = -\frac{b(1+r)x_1(m-r-\delta)}{d(1+r-x_1)} > 0, \text{ if } m < r + \delta$$

(2.33) $$v_t = (E_t - E_{t-1}) [\sum_{i=0}^{\infty} (1 + r)^{-i} y_{t+i}]$$

and g_0 and g_1 are constant, time-independent functions of the parameters.

Proof. The stochastic process $\{v_t\}$ is the revision, between t-1 and t, of the expected present value of income as of t. Clearly, $\{v_t\}$ must be serially uncorrelated. Thus the process $\{G_t\}$ defined in (2.31) is a martingale. This fact plus Propositions 1 and 2 imply, after some simplification of the more general expression for K_{t+1}, that (2.29) and (2.30) satisfy the first-order conditions of the optimization problem.

It only remains to be shown that constants g_0 and g_1 can be picked such that the sequence of budget constraints is always satisfied. Since the expression for g_1 is used in the sequel, it will be explicitly stated and derived; the solution approach for g_0 will only be indicated.

g_1 must be chosen so that a shock to the present value of income, v_t, will throw in motion a sequence of expenditures that precisely exhausts, in present value terms, the initial windfall gain or loss v_t. That is, g_1 must be such that

$$\sum_{s=0}^{\infty} dEXP_{t+s}/dv_t = 1 \tag{2.34}$$

is satisfied. To evaluate (2.34) for a given g_1, note that

$$dc_{t+s}/dv_t = -m\ dK_{t+s}/dv_t + [b(1+r)]^{-s}g_1 \tag{2.35}$$

$$dK_{t+s+1}/dv_t = x_1\ dK_{t+s}/dv_t + f[b(1+r)]^{-s}g_1$$

$$s = 0,1,2,\ldots$$

The expressions in (2.35) can be viewed as difference equations. With the boundary condition that $dK_t/dv_t = 0$, the solution to the difference equation for dK_{t+s+1}/dv_t is

$$(2.36)\quad dK_{t+s+1}/dv_t = fg_1 b(1+r)\ ([b(1+r)]^{-(s+1)} - x_1^{(s+1)})/[1-x_1 b(1+r)]$$

(2.36), (2.35), and (2.20) together imply that (2.34) can be written as

$$(2.37)\qquad g_1\left(\frac{b(1+r)^2\theta}{b(1+r)^2-1} + \frac{(1+r)(1+f-\theta)}{1+r-x_1}\right) = 1$$

where

$$(2.38)\qquad \theta = (1+f) + \frac{b(1+r)f(x_1-m+\delta+1)}{1-x_1 b(1+r)}$$

The exact expression for g_1 follows immediately from (2.37).

The calculated value of g_1 guarantees, essentially, that if the budget constraint is satisfied in period t-1, it will also be satisfied in t. The parameter g_0 is set in order to satisfy the budget constraint in period 0. The expression for g_0 is messy and its calculation, although straightforward, is tedious; thus I will not report these here. However, the calculation of g_0 and g_1 demonstrates by construction that the program (2.29)-(2.31), already known to be consistent with the first-order conditions, also satisfies the budget constraint in each period.//

The propositions above have fully specified the optimal program of nondurables consumption and durables purchases, both for the general model stated at the beginning of the section and for what will be referred to for brevity as the "constant-interest-rate case". There is one remaining consideration, which is that the transversality condition

$$\lim_{T\to\infty} b^T(K_{T+1}-K_T) = 0 \tag{2.39}$$

be checked. (2.39) requires that net durables expenditures not grow faster than 1/b indefinitely. A sufficient condition for (2.38) to hold is that the stochastic process $\{(m-z_{t+1})R_tG_t/b^t\}$ be of exponential order less than 1/b. For the constant-interest-rate case, this condition boils down to the requirement that $r > 0$.

3. An example

Some insights may be gained by applying these results in a simple example. Suppose that $b(1 + r_t) = 1$, all t; this eliminates time trends from the optimal consumption plans. Also assume $p = 1$ and a constant depreciation rate, as in Proposition 4. Then the decision rule for nondurable consumption becomes

$$c_t = (\bar{c} + m\bar{K}) - mK_t + g_0 + g_1 \sum_{i=0}^{t} v_i \tag{3.1}$$

where, recall, g_0 and g_1 are constants and v_i is the unanticipated revision to total wealth in period i. Alternatively, first-difference (3.1) to get

$$c_t = c_{t-1} - m(K_t - K_{t-1}) + g_1 v_t \tag{3.2}$$

With some manipulation, the decision rule for durables expenditure can in this case be written as

$$K_{t+1} - K_t = (1-x_1)(K_t^* - K_t) \tag{3.3}$$

where K_t^*, the "desired durables stock", is defined by the expression

$$K_t^* = \bar{K} + \frac{(\delta + r - m)}{a - m^2} \left(g_0 + g_1 \sum_{i=0}^{t} v_i\right) \tag{3.4}$$

The definition (3.4) is justified by the fact that K_t^* is the value of K_{t+1} that satisfies the first-order condition (2.6) when there are no adjustment costs ($d = 0$). $\{K_t^*\}$ is a martingale. The response of the desired durables stock to a favorable wealth shock is positive as long as $m < r + \delta$.

Equations (3.2)-(3.4) describe the evolution of the components of consumer spending in this example. The properties of the model in this case can be illustrated by a thought experiment. Suppose that, from an initial steady state, the consumer experiences an unanticipated increase in wealth ($v_t > 0$). This raises the desired durable stock, by (3.4), and stimulates durables expenditure. Because of adjustment costs, however, durables purchases during the period of the shock will not bring the stock to its desired level. Instead, a gradual increase in stocks over the future will be anticipated.

If $m \neq 0$, this gradual adjustment of stocks will affect the pattern of nondurable consumption. Suppose $m > 0$, so that durables and nondurables are substitutes. By (3.2), we see that nondurable consumption will spurt upwards, then decline gradually toward a new steady state. This decline mirrors the increase in durables stocks. Note that nondurable consumption does not follow a random walk, as it would if durables and nondurables were separable in the utility function; instead, the effect of adjustment costs "spills over" from durables to nondurables.

The intuition underlying this example is as follows: The consumer, having received favorable news about his finances, would like to own (say) a better car. However, it takes time to shop for and acquire a new car. In the interim the consumer compensates by visiting expensive restaurants; the binge ends when the new car is purchased. (On the other hand, if $m < 0$ -- the consumer enjoys expensive restaurants more if he can drive to them in a fancy car -- the increase in wealth will cause dining out to be cut back temporarily, in anticipation of the new car.)

It is possible that this model may rationalize Hall's finding that, when c_t is regressed on c_{t-1} and y_{t-1}, the coefficient on y_{t-1} is negative.[4] By (3.2)-(3.4), the covariance of $c_t - c_{t-1}$ with v_{t-1} is equal to $m(1-x_1)(r+\delta-m)g_1 var(v_{t-1})/(a-m^2)$, which is negative when $0 < m < r + \delta$. Presumably, v_{t-1} is functionally related to y_{t-1}, leading to the negative coefficient in Hall's regression.

4. Application to aggregate time series data

This section reports the results of the application of the formal model to aggregate U.S. data. I will concentrate here on what Section 2 called the "constant-interest-rate case", in which not only real interest rates but also the relative price of durables and the depreciation rate are taken to be time-invariant. Making these assumptions greatly simplifies estimation. The effect of allowing interest rates and the other forcing variables to be time-dependent is briefly discussed at the end of this section.

4.1. Data. The data used are quarterly, from the national income and product accounts. The sample period is 1947:I to 1980:II. The end date was chosen because the data were revised as of 1980:III; the unrevised series were used for comparability to earlier work. The basic data were consumption of nondurable goods and services[5], expenditure on durable goods, and disposable income. All the data were per capita, seasonally adjusted, and measured in 1972 dollars. Net durables expenditure and net stock series were constructed using the annual stock data reported in Musgrave (1979). I searched for a constant depreciation rate that matched the gross expenditure data to the endpoints of the annual stock series. The constructed quarterly stock series correlated with the annual data quite well. The implied quarterly rate of depreciation was .0506.

4.2. Use of instrumental variables. In her study of the permanent income hypothesis in aggregate time series data, Flavin used no instrumental variables in estimation; she easily rejected the PIH. In contrast, Hayashi (1982a) employed an instrumental variables technique and found results generally more favorable to the PIH. In light of traditional results on simultaneity bias in the consumption-income relationship, it seemed important to use instruments for income. Accordingly, I began by regressing the disposable income series against a list of instruments.[6] The fitted income seri's was used in place of the original series in all of the subsequent estimation.

An instrumental variables technique was also applied in an attempt to correct for bias arising from the presence of transitory components in expenditure. See section 4.5 below.

4.3 Empirical v series. It was desired to have a "revisions to present value of income stream" series to match the variable v, defined in (2.33). I proceeded as follows: The fitted income series described in 4.2 above was exponentially detrended (to achieve stationarity), then modelled as an AR(8) process with autoregressive coefficients $\rho_1, \rho_2, \ldots, \rho_8$. The residuals of this autoregression may be thought of as estimates of the innovations to current income perceived by consumers during the sample period. Assuming that the statistical model for income was correctly chosen, the revisions series $\{v_t\}$ can be shown to be proportional to the innovations in income, with the factor of proportionality given by

$$\frac{1}{1 - \sum_{i=1}^{8} (1+r)^{-j} \rho_j} \tag{4.1}$$

Formula (4.1) was derived in a similar context by Flavin. The intuition behind this expression is clear: An innovation to current income has a much larger implication for the present discounted value of the consumer's future income when the income process is "persistent"; i.e, the autoregressive coefficients have a sum close to one. Using this approach, and assuming that the quarterly interest rate r was constant and equal to .01, I calculated that a one-dollar innovation to current income during the sample period implied approximately a \$14.84 increase in the total present value of the representative consumer's income stream. (For r=.005, the corresponding estimate was \$16.84; for r=.015, it was \$13.56.) Assuming r=.0125, Flavin found (p. 1000) that the present value of a one-dollar innovation to income was \$17.80.

It should be noted that the method for relating innovations to current and future incomes described here is not the only one possible. For example, a similar approach in which growth rates of income (rather than detrended levels) were fit by an AR(8) process yielded much lower estimates of the long-run implications of current income shocks. My variant of Flavin's method is used as the basis of the results reported here partly to increase comparability with her work and partly because the use of assumptions more favorable to the PIH underscores the robustness of this paper's results.

4.4. Unrestricted estimates. It is straightforward to put the optimal consumer decision rules derived in Proposition 4 into a form suitable for estimation. Quasi-first-differencing (2.29) and (2.30) and using (2.31) gives us

$$(4.2) \qquad c_t = (\bar{c}+m\bar{K})(1-[b(1+r)]^{-1}) + [b(1+r)]^{-1}c_t - mK_t + m[b(1+r)]^{-1}K_{t-1} + g_1v_t$$

$$(4.3) \qquad K_{t+1} = (1-x_1)\bar{K}(1-[b(1+r)]^{-1}) + (x_1+[b(1+r)]^{-1})K_t - (x_1[b(1+r)]^{-1})K_{t-1} + fg_1v_t$$

Equations (4.2) and (4.3) imply a number of nonlinear restrictions across the coefficients of the RHS variables, including the implicit restriction that g_1 is an exact function of the other parameters. Before imposing those restrictions, however, it is instructive to look at unrestricted OLS regressions in the form of (4.2) and (4.3):

$$(4.4) \qquad c_t = \underset{(-0.17)}{-.0014} + \underset{(36.4)}{1.011}\,c_{t-1} - \underset{(-0.80)}{.0448}\,K_t + \underset{(0.80)}{.0439}\,K_{t-1} + \underset{(2.92)}{.0165}\,v_t + \underset{(1.91)}{.0113}\,v_{t-1} + \underset{(0.60)}{.0035}\,v_{t-2}$$

$R^2 = .99906$
D.W. = 1.623

$$(4.5) \qquad K_{t+1} = \underset{(0.89)}{.0010} + \underset{(34.8)}{1.794}\,K_t - \underset{(-15.2)}{0.792}\,K_{t-1} + \underset{(3.23)}{.0180}\,v_t + \underset{(1.34)}{.0077}\,v_{t-1} + \underset{(1.46)}{.0083}\,v_{t-2}$$

$R^2 = .99995$
D.W. = 2.029

t-statistics are in parentheses.

The comparison of these regression results to equations (4.2) and (4.3) makes the model seem rather promising. The parameter m appears to have a value of about .045 (although the level of significance is low), which would mean that durables and nondurables are substitutes.

The value for m is "large" in the following sense: It seems reasonable to suppose that, empirically, the derivative of durables expenditure with respect to income shocks is positive. For this to be true in the model, it is required that $m < z$, where z is the cost of capital. With r assumed to be .01, the average quarterly value of z is about .07; therefore, a value of m of .045 or so is close to its maximum a priori value.

The regressions also imply a value of x_1 of about .79, or an average stock adjustment rate of about .21 per quarter. This seems plausible.

An anomalous feature of the regressions (4.4) and (4.5) is that both c_t and K_{t+1} depend on lagged as well as current values of the revisions to expected income variable v_t. Consumption is not a random walk, even with the correction for lagged adjustment of durables stocks. Assuming that this is more than misalignment of the data, this finding violates the model and the PIH in their strictest form. However, as a reading of Hall's discussion suggests, this sort of "refutation" of the PIH is not very substantive. We can easily imagine a modified version of the PIH in which there are short lags of perception or action.[7] The behavior of an economy in which the modified PIH governed consumption would be similar to that of an economy in which the strict PIH held.

An interesting refutation of the PIH would be, not that there are lags of perception, action, or data collection, but that there is an excessive short-run response of consumer spending to a perceived change in income or wealth. This would, for example, imply a wider scope for anti-recessionary fiscal policy than there is under the PIH.

Finding out whether "excess sensitivity" is important requires estimation of the fully restricted model.

4.5. Estimation of the restricted model. Before the full model could be applied to the data, one more preliminary issue had to be dealt with; this was the treatment of the trends in the components of consumer spending.

Flavin argued in her paper that a trend in per capita income (due, say, to productivity growth) should induce a trend in per capita consumption, even if individuals tried to maintain level lifetime consumption patterns. This is because old people (whose wealth was accumulated during periods of lower productivity and who therefore have low consumption) are being replaced in the population by younger people (who look forward to higher levels of income and therefore have high consumption.)

While this argument is reasonable, it does not tell us much about the relationship between trends in income and in consumption. For example, the trend in consumption induced by growing income will surely depend on the age distribution in the population; it will also depend on the extent that generations are linked by bequests. At a more disaggregated level, the trend in durables relative to the trend in nondurables will be affected by many factors, such as relative prices and the baskets of durable and nondurable goods which are available. For these reasons, I left exponential trends in nondurable consumption and durables stocks to be estimated jointly with the other parameters.

The full estimated model is given in Table 1. Equations (1) and (2) are the same as (4.2) and (4.3), except that, first, the free trend terms $trend_c$ and $trend_K$ have been introduced and, second, it is assumed that the total response of spending to an income shock is distributed over three quarters. (Longer lags were tried and easily rejected.) It is not required that the distributed lags of durable and nondurable spending have the same shape; however, equation (3) imposes the restriction that the ratio of the total response of durable spending to the total response of nondurable spending be the same as given by the model.

The parameters γ_c and γ_K are defined in equations (4) and (5) to be the difference between the total observed response to an innovation in the present value of income and the response implied by the optimizing model, for nondurable and durable spending respectively. Notice that the parameter g_1 is not freely estimated but is constrained to be a function of other estimated parameters, as in (2.37). The issue of interest is whether the "excess sensitivity" parameters γ_c and γ_K are significantly different from zero.

Estimates of the model which make use of all cross-restrictions are given in Table 2. It was assumed that the real interest rate r was .01 per quarter; changing this assumption affected the estimate of b, but not any other results. Estimation was performed both by nonlinear least squares (Table 2, column 2) and by three-stage nonlinear least squares, using the instruments listed in note 6 (Table 2, column 3). The reason for trying the instrumental variables technique (above and beyond the use of instruments in constructing the fitted income series) was to eliminate transitory components of durable and nondurable spending; these transitory components have the potential of creating "measurement error" bias in the estimates.

However, the estimates generated by nonlinear least squares (especially for m and x_1) appear the more reasonable. This suggests that the potential biases were less important than the efficiency losses arising from the use of instruments.

What light do these estimates shed on the excess sensitivity issue? According to the nonlinear least squares estimates, the actual response of consumers to an income innovation is over three times that predicted by the optimizing model. The hypothesis that both γ_c and γ_K are equal to zero can be rejected with 99% confidence using either estimation approach. This provides strong support for Flavin's earlier finding that the PIH can be rejected in U.S. aggregate quarterly data.

4.6. The variable-interest-rate case. A drawback of the estimates just presented (one that is shared with most previous studies) is the assumption that the real interest rate is constant. Unfortunately, relaxing this assumption introduces many complications. As noted, the analysis of this paper allows time-dependency of interest rates and other exogenous variables, but does not permit them to be stochastic. If we ignore this problem, there are still a number of essentially arbitrary assumptions that have to be made in measuring real interest rates and modelling their time series properties. The results, it turns out, are frequently sensitive to these assumptions.

I experimented with a number of variable-rate models. The following two results were robust and are worth reporting: 1) Permitting real interest rates to vary reduced the measured excess sensitivity of nondurable consumption to income by about half. However, the hypothesis of no excess sensitivity could still be rejected with a high degree of confidence. 2) Durables expenditures exhibit considerable responsiveness to real interest rate changes; but, allowing variable interest rates in the durables equation did not seem to reduce the measured sensitivity of durables spending to income by a significant amount.

More work is needed on the effects of interest rates on consumer spending. At this point, however, it does not seem that dropping the constant interest rate assumption can go more than part way toward explaining the excess sensitivity of consumption to income.

5. Conclusion

Most recent studies of the permanent income hypothesis in aggregate data have found evidence against it. This paper has investigated the robustness of that result in a number of ways:

1) Estimation was based on a model of joint consumer decisions about durable and nondurable purchases, permitting explicit consideration of how accelerator effects or lags in adjustment in durables can "spill over" into decisions about nondurables.

2) Some consideration was given to the variable-interest-rate case.

3) The distinction was made between failure of the PIH due to short lags of perception or action and the more important cause of failure, excessive sensitivity of consumption to income change.

4) Instrumental variables were used to reduce potential simultaneity and measurement error biases.

5) Alternative methods of measuring the relation between innovations to current income and innovations in the present value of

the income stream were tried.

6) Trends in consumption spending and the consumer's discount rate were estimated, rather than imposed _a priori_.

None of these changes overturns the rejection of the PIH in aggregate time series data.

Future research might usefully pursue the variable-interest-case in more detail; it would also be interesting to try to discover why panel data studies seem on the whole more favorable to the PIH than those done in aggregate data. Finally, a research issue is raised by the following fact: Existing studies do not tell us whether the PIH is rejected because consumption has a Keynesian sensitivity to current income; or, because consumers, although forward-looking, have a relatively short horizon. As the policy implications of these two cases are rather different, it would be useful to try to distinguish them.

Notes

1. See, for example, Hall and Mishkin. This statement excludes some analyses which consider only Euler conditions and do not derive explicit decision rules.

2. See Gould (1968), Treadway (1969), Lucas-Prescott (1971).

3. I use net rather than gross expenditure because the resulting decision rules have a slightly neater interpretation. There are no formal problems with having adjustment costs depend on gross expenditure.

4. An analogous finding is in Hall-Mishkin.

5. Flavin excluded the services component because it includes service flows of residential housing. I did not find qualitative differences in the estimates when services were eliminated.

6. The instrument list included a constant, time, time squared, population, and the following variables entering contemporaneously and with a one-quarter lag: real government purchases of goods and services, real defense spending, real exports, the relative price of imports, and beginning-of-period unborrowed reserves.

7. In the investment literature, it is not usually argued that Tobin's q-theory is invalid, even though empirically investment depends on lagged as well as current values of q.

Table 1.

Estimated Model

(1) $$(1 + \text{trend}_c)^{-1} c_t = (\bar{c}+m\bar{K})(1-[b(1+r)]^{-1}) + [b(1+r)]^{-1}c_{t-1} - mK_t + m[b(1+r)]^{-1}K_{t+1} + \beta_{c,0}v_t + \beta_{c,1}v_{t-1} + \beta_{c,2}v_{t-2} + \text{error}$$

(2) $$(1 + \text{trend}_K)^{-1} K_{t+1} = \bar{K}(1-x_1)(1-[b(1+r)]^{-1}) + (x_1+[b(1+r)]^{-1})K_t - (x_1[b(1+r)]^{-1})K_{t-1} + \beta_{K,0}v_t + \beta_{K,1}v_{t-1} + \beta_{K,2}v_{t-2} + \text{error}$$

(3) $$f = (\beta_{K,0} + \beta_{K,1} + \beta_{K,2})/(\beta_{c,0} + \beta_{c,1} + \beta_{c,2})$$

(4) $$\gamma_c = \sum_{i=1}^{3} \beta_{c,i} - g_1(b,r,m,x_1,f)$$

(5) $$\gamma_K = \sum_{i=1}^{3} \beta_{K,i} - fg_1(b,r,m,x_1,f)$$

Table 2.

Results of Estimation

Parameter	NLLSQ estimate	3SLS estimate
$trend_c$	.00639 (3.91)	.00732 (3.11)
$trend_K$	.00204 (3.53)	.00094 (1.16)
b	.990 (733.4)	.990 (616.9)
m	.056 (1.09)	.076 (1.04)
x_1	.786 (15.15)	.903 (11.85)
$\sum_{i=1}^{3} \beta_{c,i}$	.0341 (3.29)	.0347 (3.04)
$\sum_{i=1}^{3} \beta_{K,i}$	.0325 (3.10)	.0206 (1.78)
f	.952 (2.30)	.593 (1.55)
g_1	.01003 (4.47)	.01026 (2.48)
γ_c	.0241 (2.58)	.0245 (2.56)
γ_K	.0229 (2.94)	.0145 (1.74)

r=.01 per quarter
t-statistics are in parentheses

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