NBER WORKING PAPER SERIES

TWO NOTES ON EXCHANGE RATE RULES AND ON THE REAL VALUE OF EXTERNAL DEBT

Rudiger Dornbusch

Working Paper No. 567

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

October 1980

Financial support was provided by a grant from the National Science Foundation. The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #567
October, 1980

Two Notes on Exchange Rate Rules and on the Real Value of External Debt

ABSTRACT

This report presents two unrelated, short papers on exchange rate rules and on the real value of the external debt. The paper on exchange rate policy uses the Taylor model of overlapping, longterm wage contracts to ask whether accommodating or PPP oriented exchange rate policies tend to stabilize output. In earlier work I had shown that exchange rate indexing, while destabilizing prices enhances the stability of output. The result is qualified here by showing that the exchange rate not only affects aggregate demand directly but operates also, through the cost of imported intermediates, on the price level. It is shown that unless monetary policy is sufficiently accommodating this latter effect may dominate with the consequence that increased exchange rate indexation reduces output stability.

The paper on the real value of external debt poses the question how to integrate external debt holdings in the traditional framework used to evaluate the effects on real income of changes in world prices. It is shown that integrating debt service liabilities in a comprehensive income measure makes real disposable income equal to the value of output less the real value of real interest payments on the external debt. Furthermore, with the CPI being the appropriate deflator for foreign debt, a rise in export prices raises income in proportion to exports while a rise in import prices lowers real income in proportion to imports. The proper accounting of debt in a comprehensive income framework, noting the intertemporal budget constraints, thus restores the conventional treatment of the income effects of price changes.

Rudiger Dornbusch
Department of Economics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

(617) 253-3648

PPP EXCHANGE RATE RULES AND MACROECONOMIC STABILITY

Taylor (1979) developed a model of longterm overlapping wage contracts where, under rational expectations, disturbances have persistent effects on output and prices because relative and absolute wages exhibit shortterm stickiness. This stickiness arises because labor, in setting the money wage, is not only concerned with employment but also with the relative wage position compared to ongoing and anticipated future contracts. This tendency for relative wage fixity is reinforced if the government pursues an accommodating monetary policy that dispenses with the need for wage-price discipline. In Dornbusch (1979,1980) the Taylor model was extended to the open economy to demonstrate that purchasing power parity (PPP) oriented exchange rate rules operate in the same direction as accommodating monetary policy. They stabilize output at the cost of increased instability of prices.

The present note takes up once more the question of exchange rate rules in the context of the Taylor model. We show here that the exchange rate rule affects the output-price level stability trade-off through two seperate channels. On one hand a PPP oriented exchange rate rule tends to maintain the real exchange rate constant ,thereby stabilizing demand. The other channel, introduced here, considers the effect of exchange rate rules through the supply side on the price level. The exchange rate affects costs and prices through the domestic currency cost of imported intermediate goods. To the extent that wage disturbances that tend to raise prices are accompanied by exchange depreciation, because of a PPP rule, the impact of wage movements on prices is amplified. Thus the second channel introduces the possibility for exchange rate policy to dampen or amplify the impact of wage disturbances on prices and output.

This second channel, operating through the cost side of the economy, introduces the possibility that increased exchange rate accommodation lowers rather than raises the persistence of disturbances while raising the impact of unanticipated wage disturbances on prices and output. These results are of interest in view of the widely shared belief that a PPP oriented exchange rate policy is good policy advice for small countries. We now pursue these issues in the context of the extended Taylor model.

1. The Model

The model is stated in terms of the deviations of output and prices from their given trend. All variables are expressed in logs. The level of output, y, is demand determined with demand depending on the real money supply, m-p, and on the real exchange rate, e-p:

$$(1) \qquad y = a(m-p) + b(e-p) \qquad a, b > 0$$

A rise in the real money stock raises demand as does a depreciation of the real exchange rate or a rise in e-p. A real depreciation raises demand, given the elasticity condition which is assumed to be satisfied with b positive, because it shifts demand from foreign goods to home output.[1]

Monetary and exchange rate policies are specified by rules which the authorities follow in setting nominal money and the nominal exchange rate:

$$(2) \qquad m = \alpha p \qquad e = \beta p \qquad 0 \leq \alpha, \beta \leq 1$$

The coefficients α and β measure the extent to which monetary and exchange rate policies are accommodating. Unit values imply that output is maintained constant because real money and the real exchange rate are held constant. Values of α and β that are less than unity imply less than full accommodation so that a rise in prices lowers real balances and appreciates the real exchange rate thereby inducing a fall in demand and output. The extent to which a rise in prices leads to a contraction in demand and output is summarized by the parameter θ :

$$(3) \qquad y = -\theta p \qquad \theta \equiv a(1-\alpha) + b(1-\beta)$$

---

[1]Foreign prices are assumed given and any repercussion effects from abroad are neglected.

We now turn to the wage-price process. Prices are determined by wage costs and the costs of imported intermediate goods, the price of the latter being given by the exchange rate:

$$p = (\phi/2)(x+x_{-1}) + (1-\phi)e \tag{4}$$

The price equation reflects the assumption that there are at any time two overlapping or concurrent wage contracts in force. They stipulate respectively x as the current wage in the first year of our contract and x_{-1} as the current wage on a two-year contract entered into last year. Current wage costs are taken as the average of the two contracts. The terms ϕ and $1-\phi$ represent the cost shares of labor and intermediate goods. In combination with the exchange rate rule in (2) we can rewrite the price equation as:

$$p = k(x + x_{-1}) \qquad k \equiv \frac{\phi/2}{1-\beta(1-\phi)} \tag{5}$$

It is apparent from (5) that a more accommodating exchange rate policy-- a value of β more nearly equal to unity-- tends to raise the impact of wage disturbances on prices. Conversely, maintaing a constant nominal exchange rate in the face of wage disturbances tends to dampen their impact on the level of prices, but raises the impact on the real exchange rate. The extent to which the exchange rate can serve a stabilizing role on the supply side depends on th share of intermediate goods in costs, $1-\phi$.[1]

We complete the model by looking at wage formation. The current wage is set by reference to ongoing and anticipated future contracts but depends also on expected employment during the currency of the contract:

$$x = dx_{-1} + (1-d)\tilde{x}_{+1} + \gamma(d\tilde{y} + (1-d)\tilde{y}_{+1}) + u \qquad ;d,1-d,\gamma > 0 \tag{6}$$

where a tilda denotes an expectation and u is a white noise error term. Applying rational expectations, by using (3) and (5) to generate the expectations, and collecting terms yields:

$$c\tilde{x} = d\tilde{x}_{-1} + (1-d)\tilde{x}_{+1} \qquad c \equiv \frac{1+\theta\gamma k}{1-\theta\gamma k} \tag{6'}$$

---

[1] An alternative interpretation takes p to be the price level with $(1-\phi)$ the direct share of importables in the consumer price index.

with a solution, assumed stable: [1]

$$(7) \qquad x = \rho x_{-1} + u \qquad\qquad \rho \equiv \frac{c - (c^2 - 4d(1-d))^{.5}}{2(1-d)}$$

Using (7) in (5) yields the equation for the price level:

$$(8) \qquad p = \rho\, p_{-1} + k(u + u_{-1})$$

and for output:

$$(9) \qquad y = -\theta p = \rho y_{-1} - \theta k(u + u_{-1})$$

We now turn to a discussion of the effects of exchange rate indexation on the stability of output and prices. We remind briefly of the results in Taylor (1979) and Dornbusch (1979,1980). In these models the term k was equal to k=.5. Therefore it is clear from the definitions of c and ρ in (6)' and (7) that increased monetary or exchange rate accommodation would reduce θ and hence raise the inertia or persistence of wages and prices as measured by ρ. With more accommodating policies output is more stable and that stability of output is taken into account in wage settlements and is reflected in a more protracted phasing out o. disturbances. From here we proceed to the effect of increased exchange rate indexation in the extended model that allows for exchange rate effects both on the supply side and the demand side as reflected in a value of $k = (\phi/2)/(1-\beta(1-\phi))$

2. The Effects of Exchange Rate Indexing

In this section we investigate the implications of an increase in exchange rate indexation-- a rise in β -- when the exchange rate plays a role in determining aggregate demand but also exerts direct cost effects. Given the definitions of k, c and ρ we observe that a rise in β affects the extent of persistence as follows:

---

[1] To arrive at (6)' we take the expectation across (6) noting that E(u)=0 and $x = \tilde{x}$, $x_{-1} = \tilde{x}_{-1}$. To proceed from (6)' to (7) we define the difference operator: $x = \rho x_{-1}$ and, substituting in (6)' obtain a quadratic in ρ : $c\rho x_{-1} = dx_{-1} + (1-d)\rho^2 x_{-1}$. Solving for ρ yields the expression in (7).

$$(10) \qquad \partial\rho/\partial\beta = \delta\{b\phi - a(1-\alpha)(1-\phi)\} \qquad \delta > 0$$

where δ is a (messy) positive function of the parameters. Equation (10) then implies that increased exchange rate accommodation may raise or lower the persistence of disturbances depending on the relative size of the two channels through which the exchange rate operates.

Suppose the elasticity of demand with respect to the real exchange rate is large and the share of labor costs is large. Under these conditions the term in brackets in (10) is likely to be positive and we have the conventional result, in the spirit of Taylor's analysis, that increased accommodation raises persistence. The other case, by no means unlikely, arises when the exchange rate exerts a substantial effect on the cost side and demand is relatively unresponsive to the real exchange rate. Unless monetary accommodation is complete we then have a case where higher exchange rate indexation will reduce persistence.

This latter case arises when the gain in stability of output achieved by real exchange rate stability is more than offset by the indexation effect on the cost side that tends to reduce demand unless money is fully accommodating. This interpretation becomes apparent when we look at the effect of a wage disturbance on output: $dy/dx = -\theta k$ and ask how that effect varies with the extent of exchange rate accommodation:

$$(11) \qquad \frac{\partial(dy/dx)}{\partial\beta} = \frac{k}{1-\beta(1-\phi)}\{b-(1-\phi)(b+a(1-\alpha))\}$$

Labor, in setting wages, takes into account the output effects of wage settlements. If higher exchange rate indexation implies a larger adverse effect of high wage settlements on output then relative wages will be more flexible and persistence will be reduced. It is interesting to note that (10) and (11) show the interdependence of monetary and exchange rate accommodation once the exchange rate operates also through the supply side. The increased cost indexation following from a higher implies a larger adverse effect on real balances and hence on output the smaller is the extent of monetary accommodation. Conversely, if monetary accommodation is relatively complete the increased cost effects of a high do not have important effects on demand and persistence.

The analysis suggests that monetary and exchange rate policies can, in certain cases, be coordinated to reduce both the impact of price disturbances on output and at the same time reduce the persistence or inertia in wages. If $\phi b-(1-\phi)a$ is negative then is it apparent from (10) that increasing indexation can be accompanied by some increase in monetary accommodation and still not enhance persistence.

An important difference between the Taylor model and our analysis here arises from the impact of changes in exchange rate indexation on the impact of unanticipated disturbances. This is apparent in (8) and (9) where the unanticipated wage movements $(u+u_{-1})$ are multiplied by k which now is not a constant but rather a function of β.[1] From (9), in conjunction with (11), it will be noted that a policy that reduces persistence will also raise the impact of unanticipated disturbances on output. It therefore appears that we cannot discuss the policy trade-off merely in terms of persistence and the coefficient θ but rather should look at the asymptotic variances of output and prices. With prices and output following an ARMA(1,1) process the variances are:

$$(12) \qquad \sigma_p^2 = \frac{2(1+\rho)}{1-\rho^2} k^2 \sigma_u^2 \qquad \sigma_y^2 = \theta^2 \sigma_p^2$$

A numerical exercice, shown in Table 1, reports how the variances behave as a function of the extent of exchange rate indexation. The two cases reported correspond to different signs for the expressions in (10) and (11). The table reveals that independently of the sign in (10) and (11) an increase in exchange rate indexation raises the variability of prices. As regards the variability of output there are two possibilities. With the cost channel of exchange rates dominating the variability of output is increased by a move toward more full indexation. Conversely, if the aggregate demand role of the real exchange rate dominates more accommodation implies a more stable behavior of output.

---

[1]The term u_{-1} is a wage disturbance unanticipated when the contract x_{-1} was set. It represents an unanticipated component of the price level as of period t-1.

TABLE 1

	$\beta = .1$	$\beta = .5$	$\beta = 1$
		Case I	
σ_p^2	.23	.35	.75
σ_y^2	.48	.55	.75
		Case II	
σ_p^2	.23	.38	.96
σ_y^2	.44	.38	.24

Note: CaseI: $a=2, b=.5$ $\phi=\gamma=d=\alpha=.5$; Case II: $a=b=1$ $\phi=\gamma=d=\alpha=.5$; $\sigma_u^2=1$

We noted above the interdependence of monetary and exchange rate policies in determining the effectiveness of increased exchange rate indexation. Equation (11) suggests that with the possibity of an adverse effect of exchange rate accommodation on output stability it becomes important to recognize that these effects can be dampened or averted by a commensurate increase in the extent of monetary accommodation. Of course this stabilizing role of monetary policy comes at the cost of increased instability of prices.

3. Concluding Remarks

PPP oriented exchange rate policies have been widely adopted among developing countries as a way of isolating the foreign trade sector from the vagaries of the macro economy. By and large the profession has accepted such policies as sensible, implicitly placing high cost on the variability of real exchange rates. Such policy advice is questioned here by drawing atrention to the macroeconomic costs of higher exchange rate indexation: increased instability of prices and potentially increased instability of output. The analysis also draws attention to the fact that higher monetary accommodation stabilizes output not only directly, but also by offsetting or dampening the adverse effects of exchange rate indexation working through the cost side of the economy.

The analysis then suggests that the search for optimal stabilisation poloicies should not take the target of a constant real exchange rate as given but rather consider the problem in a broader context that evaluates the relative costs of real instability, price instability and target instability.

References

Dornbusch, R. (1979) "Exchange Rate Policy and Macroeconomic Stability." in J. Williamson (ed.) Exchange Rate Rules: Theory, Lessons and and Prospects for the Crawling Pig. MacMillan, London, 1981.

Dornbusch, R. (1980) Open Economy Macroeconomics, Basic Books, N.Y .

Taylor, J. (1979) " Staggered Wage Setting in a Macro Model ". American Economic Review, Papers and Proceedings, pp.108-118.

CONSUMPTION OPPORTUNITIES AND THE REAL VALUE OF THE EXTERNAL DEBT

Repeated oil price increases have led developing countries to build up large external debts and world inflation has reduced the real value of these debts. Renewed shocks to the terms of trade will require adjustments in trade flows to finance the debt service obligations. In this setting the question of the appropriate deflator for foreign debt has arisen. Suppose import prices rise. As debtors we are better off because of the reduced real value of our debts, but as importers we are clearly worse off. What is the deflator that strikes the balance between these two effects and provides a meaningful measure of the welfare effects of price changes ? Alternative deflators have been proposed. One would be a weighted average of export and import prices reflecting in the weighting the marginal shares of exports and imports in the adjustment process. We argue here for a different measure, namely the consumer expenditure deflator, and show that in combination with the comprehensive income measure this deflator gives a very conventional statement of the welfare effects of price changes.

1. The Concept of Income

The center of the analysis is the proper concept of income. We use the traditional Hicks -Bailey definition of comprehensive income as the value of consumption that can be maintained without impairing future consumption opportunities. Defining as c this value of sustainable consumption or real disposable income it is equal to:

$$c = \frac{Y - i^* ED - ED(\dot{e}-\dot{p})}{P} \qquad (1)$$

where

P = home consumer price index
Y = home nominal value of output
D = external debt in dollars
E = exchange rate, cruzeiros/$
i^* = world nominal rate of interest
$\dot{e},\dot{p}$ = rates of depreciation and home inflation

The definition of real disposable income in (1) recognizes that debt service is a charge against resources and that capital losses, if consumption opportunities are to be maintained, likewise diminish the consumption level afforded by a given level of output.

Equation (1) is readily transformed into an alternative and suggestive form. For that purpose we define the home and foreign real interest rates, $r = i - \dot{p}$ and $r^* = i^* - \dot{p}^*$. We also assume for the present that nominal interest rates, adjusted for expected depreciation are equalized, or $i = i^* + \dot{e}$. These relations are summarized in (2):

(2) $$r = i - \dot{p} \; ; \quad r^* = i^* - \dot{p}^* \quad ; \quad i = i^* + \dot{e}$$

Using these definitions, adding and substracting $\dot{p}^*$ in (1), and rearranging terms uields our expression for sustainable consumption or real disposable income as:

(1)' $$c = Y/P - rd$$

where $d \equiv ED/P$ is the real value of the external debt. Real disposable income thus equals the real value of output less the real value of <u>real</u> interest payments on the external debt.

Let x and m denote the real value of exports and imports both measured in terms of the consumer price index. We can then write consumption as equal to output, less exports, plus imports:

(3) $$c = Y/P - x + m$$

which in conjunction with (1)' implies that the sustainable current account must be balanced, with a trade surplus equal to the real debt service:

(4) $$x = m + rd$$

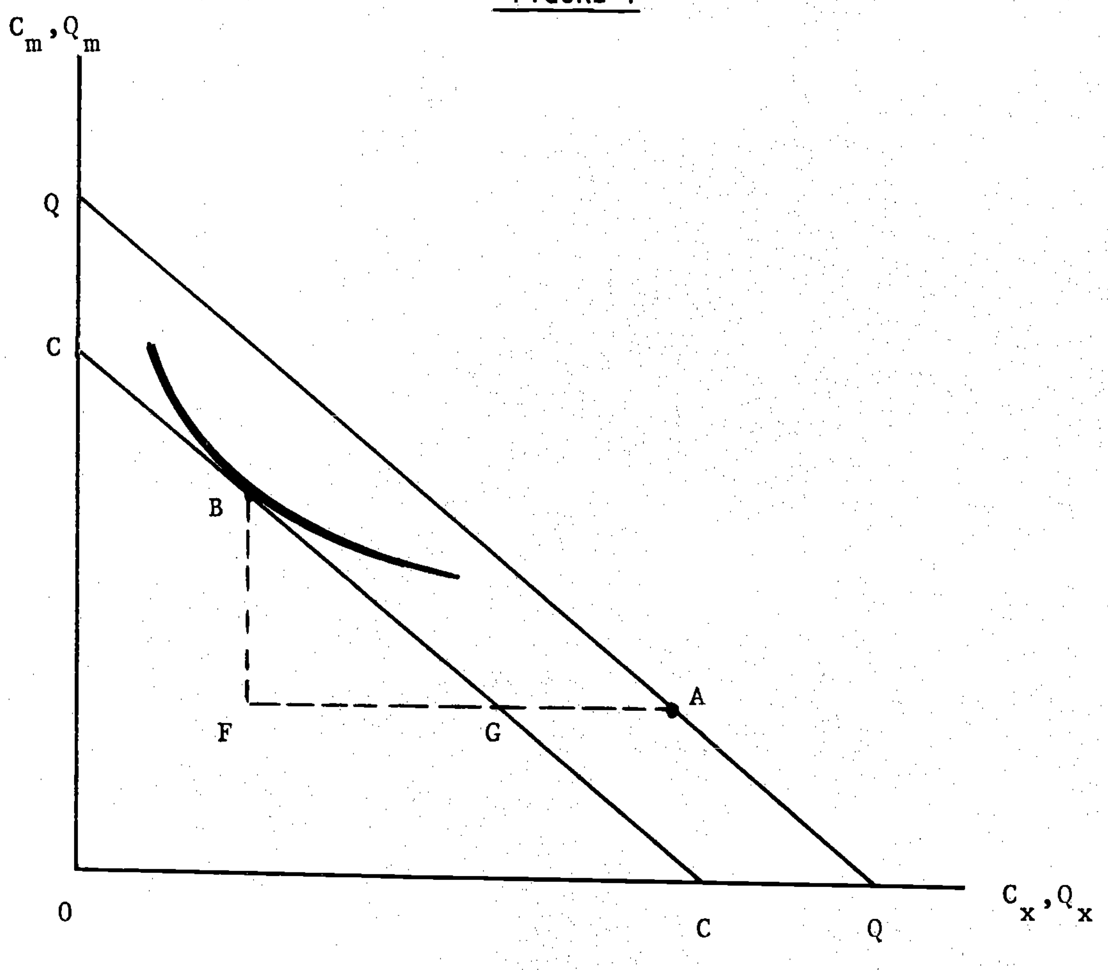
Equations (1)' and (4) describe the <u>average</u> behavior of consumption opportunities and the current account. They are therefore the appropriate benchmark by which to judge the effects of changes in the levels of prices in world trade. We now proceed to that analysis.

2. Geometry

Before moving to an algebraic statement of the effects of price changes we provide here a geometric interpretation. In Figure 1 we show consumption of importables and exportables (C_m,C_x) as well as production (Q_m,Q_x). Initial production equilibrium is at point A. (We take the case of a fixed production point, although the algebra and the footnote show that we could also look at an equilibrium on the transformation curve). The line QQ through A indicates the value of output at the initial relative prices. From the value of output we subtract the real value of debt service, being rD/P^*_x in terms of exportables and rD/P^*_m in terms of importables. Subtracting these quantities from point A yields points on the consumption frontier CC. Point B is the preferred consumption point with imports BF equal to exports FA less debt service GA, both measured in terms of importables at the prevailing relative price.

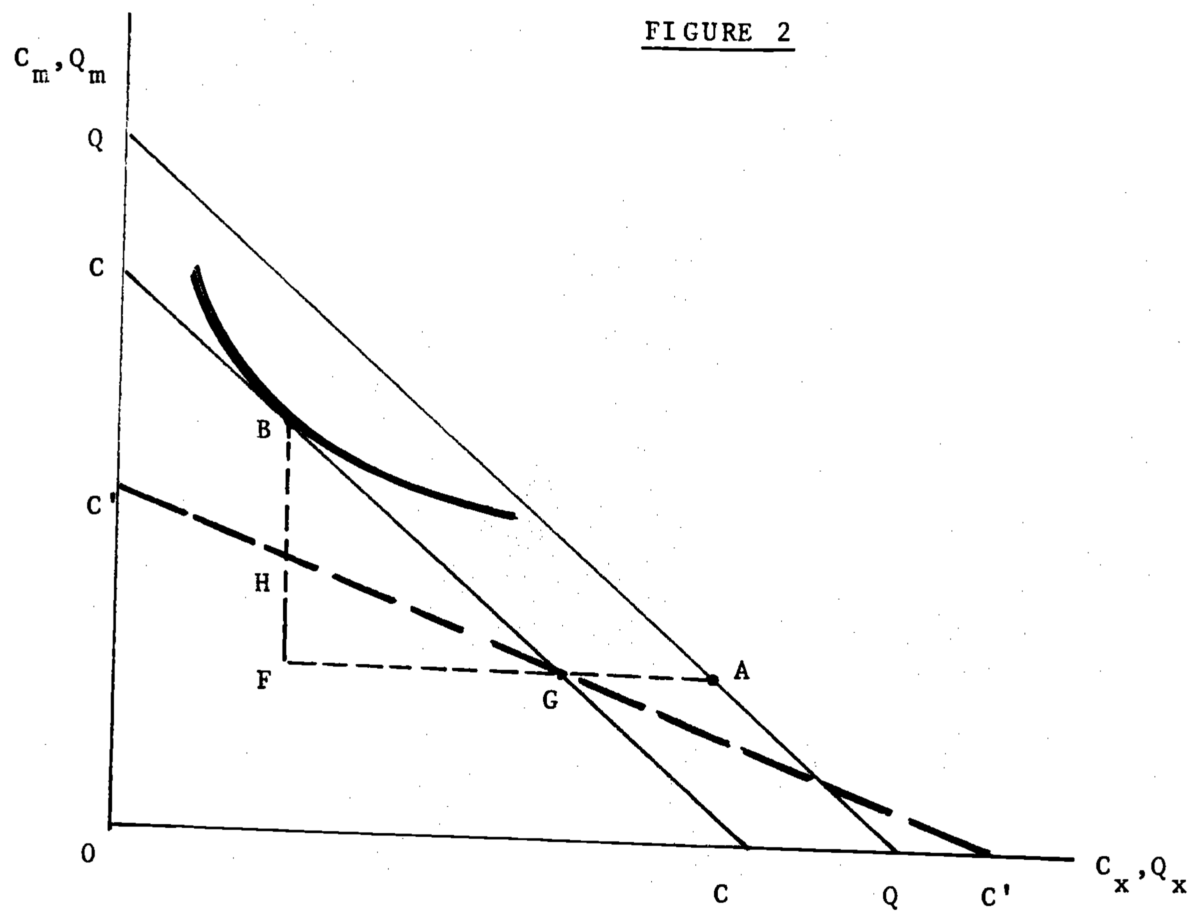
Suppose now a rise in the absolute world price of importables. In Figure 2 we show that the value of production changes as the budget line rotates through point A.[1] To determine what happens to consumption opportuni-

FIGURE 1



ties we have to construct the new budget line C'C'. It is apparent that the value of debt service measured in terms of exportables does not change. Hence point G remains a point of the new budget line C'C'. Through point G we draw a line with the slope given by the new relative price ratio. The value of debt service in terms of importables, of course, is seen to decline as shown by the reduced vertical distance between QQ and C'C'. The new consumption point will lie on the budget line C'C'.

FIGURE 2



Now we can ask how much of an increase in income we would have to give the consumers to allow them to buy the initial consumption basket B at the new prices. Measured in terms of importables the compensating varia-

[1]The argument neglects movements along the transformation curve in response to price changes. For small price changes this effect is second order; for large price changes the geometry and the algebra below overestimate the income effect of price changes. The same applies to the treatment of consumer adjustment below where the Slutzky approximation is exact for small changes.

tion in income is indicated by the vertical distance between C'C' and a parallel through point B,HB. It is readily shown that this compensating variation is equal to:

$$HB = BF - HF = P^*_x(Q_x - rD/P^*_x - C_x)/P^*_m - P^*_x(Q_x - rD/P^*_x - C_x)/P^*_m{}'$$
$$= \bar{m}\,\hat{P}^*_m$$

where $\bar{m}$ denotes the phyical level of imports and P^*_m and $P^*_m{}'$ are the initial and new levels of import prices. Thus the change in real income, measured in terms of importables, is equal to imports times the percentage change in import prices.[1]

As shown in the diagram a rise in import prices reduces welfare, a measure of the reduction in real income being the compensating variation HB. It is apparent that we can use the same technique to show that a rise in export prices raises welfare. (C'C' now rotates through a point on CC vertically below A, becoming steeper). Finally, an equiproportionate change in export and import prices will only reduce the real value of debt service thus shifting CC upward, coming closer to QQ

Having established an intuitive interpretation for our results we now proceed to brief algebraic derivation of the change in real disposable iincome induced by world price changes.

---

[1]With P the price index we can define the change in consumption measured in terms of the index,rather than in terms of importables, as $(P^*_m\bar{m}/P)\,\hat{P}^*_m = m\,\hat{P}^*_m$ where m denotes the real value of imports.

3. The Algebra of Price Changes

To determine the effects of price changes on real disposable income we start with the definitions of the value of output, the consumer price index and import prices, all measured in home currency:

(5) $$Y = P_x Q_x + P_m Q_m \; ; \quad P = P_x^a P_m^{1-a} \quad ; \quad P_m = EP^*$$

Equations (5) reflect the assumption that the home country produces both importables and exportables. In the world market for importables we are price takers, but not necessarily in the export market. The price index is an expenditure weighted index of importable and exportable goods prices.

Using (5) the change in real disposable income is given by:

(6) $$\Delta c = (P_x Q_x / P) \hat{P}_x + (P_m Q_m / P) \hat{P}_m - (Y/P)\hat{P} - rd(\hat{E} - \hat{P})$$

which simplifies to the following expression:[1]

(6)' $$\Delta c = x(\hat{P}_x - \hat{P}_m) + rd \hat{P}^*$$

where x again equals the real value of exports.

Price changes in world trade have two effects on real disposable income. A terms of trade improvement raises real consumption opportunities by the real value of exports times the terms of trade change. This is the conventional income effect of a terms of trade improvement. A rise in the world dollar price level, P*, raises real disposable income by reducing the real value of the debt and hence real debt service. This is represented by the second term in (6)'.

---

[1] From (6) we obtain the simplification by noting that $(Y/P)\hat{P} = a(Y/P)\hat{P} + (1-a)(Y/P)\hat{P}_m$. Next observe that $a(Y/P) = a(c+rd) = ard + P_x C_x / P$ where C_i denotes consumption of commodity i. Further note that the last term in (6) can be written as $rd(\hat{E} - \hat{P}) = rd\,\hat{P}^* + ard(\hat{P}_m - \hat{P}_x)$ and that on average the current account is balanced so that real exports, x, equal real imports, m, plus real interest payments: $x = m + rd$. Using all these substitutions in (6) yields the final form in (6)'.

Another way of looking at the effects of price changes is to distinguish the role of export and import prices. This is done in (7) where we have substituted for the change in import prices $\hat{P}_m = \hat{E} + \hat{P}^*$ and rearranged terms to obtain:

$$(7) \qquad \Delta c = x(\hat{P}_x - \hat{E}) - m\hat{P}^*$$

In equation (7) the term $\hat{P}_x - \hat{E}$ measures the change in the dollar price of our exportables. The equation shows that a rise in the dollar price of exportables raises real income in proportion to exports. A rise in the dollar price of imports lowers real income in proportion to the level of imports. There is thus a complete symmetry that is assured by the fact that on average the inflation adjusted current account must balance or $x = m + rd$.

4. Extensions

The analysis so far has made two assumptions that are easily relaxed. The first is that depreciation adjusted nominal interest rates are equalized. An alternative is to assume a risk premium μ such that $i = i^* + \dot{e} + \mu$. With this assumption

$$(1)'' \qquad c = Y/P - (r+\mu)d$$

is our definition of real disposable income. The risk premium or "spread" μ reduces the level of real disposable income for a given level of debt. It does not, though, affect the result derived in (7) above.

A second generalisation concerns the role of home goods. Suppose that in addition to exportables and import competing goods the home country produces nontraded goods. This, of course, changes the definitions of nominal income and the price index. It is readily shown that the effects of price changes as derived in (7) remain unaffected. This is so because the home country neither exports nor imports nontraded goods.[1]

5. Concluding Remarks

This note has shown how to integrate the presence of an external debt in the conventional treatment of the income effects of a price change. The analysis has shown that a rise in export prices raises real income in proportion to exports and that a rise in world prices of our imports reduces real income in proportion to imports. The income effects of changes in the real value of the debt service liability are implicit in these results since the difference between exports and imports is equal to the debt service. As noted earlier these measures are only approximate. They are exact for small changes, but provide an overestimate when price changes are large and hence substitution along the transformation curve and along the indifference curve become important. Bearing this qualification in mind our measure provides an upper estimate of the income effect.

From p.7

[1] The value of home output and the price index now are defined as follows:

$$Y = P_x Q_x + P_m Q_m + P_n Q_n \quad \text{and} \quad P = P_x^a P_m^b P_n^{1-a-b}$$

where the subscript n denotes nontraded goods. The definition of sustainable consumption remains $c = Y/P - rd$. A change in prices in world trade yields as an effect the terms developed in (7). This is so since:

$$\Delta c = (P_x Q_x/P)\hat{P}_x + (P_m Q_m/P)\hat{P}_m + (P_n Q_n/P)\hat{P}_n - a(Y/P)\hat{P}_x - b(Y/P)\hat{P}_m -$$
$$-(1-a-b)(Y/P)\hat{P}_n - rd(\hat{E}-\hat{P}) = x\hat{P}_x - m\hat{P}_m - rd(a\hat{P}_x + b\hat{P}_m + (1-a-b)\hat{P}_n - \hat{P}) - rd\hat{E} =$$
$$= x(\hat{P}_x - \hat{E}) - m(\hat{P}_m - \hat{E})$$

where we have used the definition of the expenditure share to note that $C_x P_x/P = a(Y/P - rd)$ and likewise for the other real commodity demands. We also use again the equation for current account balance, $rd = x - m$, and the definition of the price index.

BIBLIOGRAPHICAL NOTE:

For a discussion of the comprehensive income measure used here see:

Hicks, J. Value and Capital, Oxford University Press, 1946.

Bailey, M. National Income and the Price Level, McGrawHill, 1972

Taylor, C.T. and Threadgold, A.R. "Real National Saving and its Sectoral Composition." Bank of England, 1979.

For inflation adjusted balance of payments and budget accounting see:

Freedman, C. "A Note on Interest Payments to Foreigners Under Inflationary Conditions." Canadian Journal of Economics, 1979, pp.291-299.

Siegel, J. "Inflation induced Distortions in Government and Private Statistics:" Review of Economics and Statistics, 1979, pp.83-90.