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WHY WAS THERE MANDATORY RETIREMENT?

OR

THE IMPOSSIBILITY OF EFFICIENT BONDING CONTRACTS

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ABSTRACT

Lazear has argued that hours constraints, in general, and mandatory retirement, in particular, form part of an efficient labor market contract designed to increase output by inhibiting worker shirking. Since the contract is efficient, legislative interference is welfare reducing. However, in any case where bonding is costly, the hours constraints will not be chosen optimally. Although it is theoretically possible that bonding is costless, in this case the earnings profile is indeterminate and we should never observe monitoring aimed at reducing shirking. It therefore appears that bonding should be modelled as costly. If so, the role of policy depends on the source of bonding costs, the set of feasible contracts and the policy options which are available to government.

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Lazear (1979, 1981) has argued that hours constraints, in general, and mandatory retirement, in particular, form part of an efficient labor market contract designed to increase output by inhibiting worker shirking. In effect, workers post bonds (perhaps in the form of low initial wages) against malfeasance which are returned to them in the form of higher wages later in the work relation. Workers who are caught shirking are fired and forfeit the bond. Since wages deviate from the value of marginal product, the contract must specify hours as well as wages. If Lazear is correct, since the contract is efficient, legislative interference is welfare reducing. Yet, Congress has recently passed a law making mandatory retirement illegal in most cases.

The Lazear agency model has received considerable attention in the literature for two reasons. First, there is some evidence that wages rise faster than productivity (Medoff and Abraham, 1980), and the Lazear agency model offers an explanation for this finding. Secondly, unlike the shirking based efficiency wage model (Shapiro and Stiglitz, 1984; Bulow and Summers, 1986), the contracts discussed by Lazear are first-best efficient. Researchers who are sceptical about the efficiency wage literature (Carmichael, 1985, 1986) have pointed to the efficiency of the Lazear solution as evidence against the efficiency wage model. Proponents of the efficiency wage model (Akerlof and Katz, 1986) have regarded the possibility of such contracts as a serious challenge.

The major point of this paper is that in any case where bonding is costly, bonding contracts will not be first best efficient. In particular, the hours constraints will not be chosen optimally. Although it is theoretically possible that bonding is costless, the case of costless bonding is not particularly interesting since the earnings profile is indeterminate, and it is no longer possible to make a strong case that wages rise faster than productivity. As a consequence, the bonding/efficiency wage debate appears less significant than its participants appear to believe.

The basic argument is quite simple. If the labor market clears, as it will in a bonding model, a firm which raises the work requirement by an hour must increase compensation by the monetized marginal disutility of an hour's employment. With reasonable restrictions on the relation between disutility of effort on the job and probability of being caught on the one hand, and hours of employment, on the other, it can be shown that it will also have to increase the size of the "bond" that workers are required to post. Since, as argued below, bonding is costly, the cost to the firm of increasing work by an hour exceeds the marginal disutility of an hour's employment to the worker. Consequently, firms will set hours at a level at which the value of marginal product exceeds the marginal disutility to the worker, and contracts will not be first-best efficient.

To complete the sketch of the proof, we need merely establish that bonding is costly. There are two reasons for believing that bonding is costly. The first is theoretical: as Lazear, himself, notes, the wage profile is indeterminate unless bonding is costly. Thus unless bonding is costly, the bonding model has little or no predictive power. The second reason is empirical: Dickens, Katz and Lang (1986) point out that unless bonding is costly at the margin, firms will not engage in any monitoring but instead will deter shirking solely by setting large bonds. Since we observe that firms do engage in monitoring, bonding must be costly at the margin.

It should be noted that the term "costly" is used quite generally. Bonding may be costly for a number of reasons including but not limited to the need to pay outside agents to monitor the contract, differences in discount rates between workers and firms and the potential for firm default.

I show below that, at least for some sources of costly bonding, hours will be set below their optimal level and that mandatory retirement will occur too early. This is consistent with work by Kahn and Lang (1986) and Dickens and Lundberg (1985) which shows that workers wages frequently exceed their marginal value of leisure. The role of policy depends on the source of bonding costs, the set of feasible contracts and the policy options which are available to government.

I. The Basic Model

Let us begin by considering a simple two-period model. In the first period, the worker is employed and receives a wage. Second period payment is conditional on not shirking in the first period. Since second period compensation cannot be made conditional on not shirking, all workers will shirk in the second period. Consequently, the second period takes the form of a pension, P. Assume that the size of the pension (appropriately discounted) which is returned to the worker if he does not shirk depends on the size of the initial bond but that the increase is less than one for one. Under these circumstances, the no shirk condition is given by

(1) $$w - e(h) + P(b) \geq w + (1-q(h))P(b)$$

or

(2) $$P(b) \geq e(h)/q(h)$$

where w is the wage, e is the disutility of effort on the job (value of shirking), h is the number of hours worked, b is the bond and q is the probability of shirking being detected,[1] and $0 \leq P' < 1$. It seems reasonable to assume that $e'>0$ and that $e''\geq 0$, that is that the marginal disutility of effort on the job is always positive and that it is nondecreasing in hours worked. The probability of detection, q, is assumed to follow a Poisson process with constant arrival probability over any period of a given length. As a consequence, $q''<0$.[2]

It is worth noting that under these assumptions, the no shirking constraint can be inverted to give a maximum hours constraint

$$h \leq h^*(P(b)) \tag{3}$$

which says simply that for any pension level, there is a maximum number of hours the worker can be employed without shirking. The no shirking constraint and the maximum hours constraint are identical so that if the no shirking constraint is binding so is the maximum hours constraint. The maximum hours form of the no shirking constraint is useful for the multiperiod model.

The firm maximizes profits, π,

$$\pi = v(h) - w \tag{4}$$

where v is the value of the worker's output. It is assumed that $v'>0$ and that $v''\leq 0$. The firm maximizes (4) subject to the no shirking constraint and the constraint that the expected value of the job to the worker must reach the competitive level. Denote the monetized value of not working at the firm (i.e. the value of leisure or of alternative employment) as s(h). The firm must offer at least the competitive value of employment so that

(5) $$w - e(h) - s(h) - b + P(b) \geq 0$$

where s is the shadow value of time, which may be the value of leisure or the value of alternative employment. The term P(b)-b, can be thought of as the cost of the bond or pension. Presentation is somewhat simplified if equation (5) is rewritten as

(6) $$w - e(h) - s(h) - c(P)$$

where c(P) is the cost of the pension.

A seemingly critical issue is whether the firm can steal the bond. If the firm may, under certain circumstances choose to pocket the bond, then profits will depend on the probability that the firm will steal the bond and the size of the bond. In what follows, it is assumed that for a given size of bond, the firm knows in advance whether it will be more profitable to honor the commitment or simply steal the bond. If firms are heterogeneous with respect to the level of the bond at which it becomes profitable for them to steal rather than honor the commitment, workers will discount the bond by the probability that it will be stolen. P(b) can be thought of as the expected bond repayment. Thus this formulation is quite general. The results are not fundamentally different although somewhat more complicated if the firms does not know in advance whether it will cheat and thus must take account of the possible value of cheating.

Under these assumptions, the firm maximizes profits subject to the no shirking constraint (2) and the minimum job value constraint (5). Substituting the constraints into (4) yields

$$(7) \qquad \pi = v(h) - e(h) - s(h) - c[e(h)/q(h)]$$

Maximizing (7) with respect to hours gives the first order condition

$$(8) \qquad v' - e' - s' = c'(e'/q - eq'/q^2).$$

The restrictions on e'' and q'' are sufficient to ensure that the right hand side of (8) is positive so that hours are chosen so that vmp exceeds the marginal disutility of employment.[3] Whether this departure from the "standard" efficiency condition has any significance for policy depends on whether in some way government can effect thet marginal cost of bonding. If not, the contract is constrained efficient.

It will also be the case that workers will feel constrained to work fewer hours than they desire. To see this note that the profit maximizing firm will hire workers until the point that the wage equals vmp. Since $v'' \leq 0$, the hourly wage ($w/h = v/h$) is at least as great as v' and thus greater than the marginal disutility of employment giver. by $e'+s'$. As a result, workers will desire more hours.

Hours constraints are binding in this model despite the fact that the wage equals the value of marginal product. Thus, contrary to the model of first best efficient contracts developed by Lazear, divergence of VMP and wages is not necessary for the existence of hours constraints.

It is important to note that market clearing is essential to this result. To see this, consider an efficiency wage model designed to parallel the bonding model just developed. The firm offers a pension, P, conditional on the worker not being caught shirking and maximizes profits subject to the no shirking constraint (2). Since (5) is not binding, it sets w equal to zero so that profits are given by

$$(9) \qquad \pi = v(h) - P - c(P) = v(h) - e(h)/q(h) - c[e(h)/q(h)].$$

Maximizing (9) with respect to (h) gives the first order condition

$$(10) \qquad v' = (1+c')(e'/q - eq'/q^2.)$$

There is no determinate relation between the right hand side of (10) and the marginal disutility of employment. Consequently, in an efficiency wage model, the choice of hours may be greater than or less than the first best efficient level. Similarly, if workers view their hourly wage as P/h, it is indeterminate whether workers will want to work more or less than determined by the firm.

It is worth noting that this conclusion contrasts sharply with Bulow and Summers (1986) who argue that in an efficiency wage model, firms will prevent workers from being employed part-time. In essence, Bulow and Summers assume that both e'' and q'' equal zero. If this assumption were correct, the firm would set hours until either v' equalled zero or the worker's time endowment was exhausted. It seems likely that in either event, the hours chosen would be quite high and that therefore s(h) would be sufficiently high to make the labor market constraint binding. In effect, it is probable that under the Bulow/Summers assumption, firms would clear the market by setting very high work hours. However, as noted above, $q''=0$ is not a natural assumption. Therefore, it appears that the efficiency wage model does not make any strong predictions about the nature of hours constraints.

II. The multi-period model

When the model is extended to many periods, we require more structure so that it is necessary to assume a particular reason for bonding being costly. In this section I assume that the cost arises from workers having higher discount rates than firms. Bonding will be costly because if the worker is to be equally well off, an increase in the bond will require a repayment to the worker which exceeds the initial value of the bond to the firm. While the precise results may vary depending on the reasons for bonding being costly, the essential result that costly bonding implies the impossibility of first best contracts should be evident from the specific model.

If firms' and workers' discount rates differ, the firm maximizes profits which are given by

$$\text{(11)} \qquad \text{profits} = \Sigma v_t/(1+r)^{t-1} - \Sigma w_t/(1+r)^{t-1} + b - P/(1+r)^T.$$

As noted in the previous section, the no shirking constraint can be inverted to give a maximum hours constraint in each period. The firm maximizes (11) subject to the hours constraint for each period

$$(12) \qquad \Sigma\lambda_t[h_t - h^*_t(\sum_{t+1}^{T}(w_j - e_j(h_j) - s_j(h_j))/(1+i)^{j-t} + P/(1+i)^{T-t+1})\ ].$$

It should be noted that since the constraint requires that h_t be less than h^*_t, assuming λ_t is nonpositive.

Maximization of (11) is also subject to the constraint that over the lifetime the worker receives at least the competitive level of utility

$$(13) \qquad \lambda_0\ [-b + \Sigma(w_t - e_t(h_t) - s_t(h_t))/(1+i)^{t-1} + P/(1+i)^T].$$

The first order conditions with respect to h_t and w_t are given by

$$(14)\ v/(1+r)^{t-1} - \sum_{1}^{t-1}\lambda_j(-e_t' - s_t')h_j{*}'/(1+i)^{t-j} + \lambda_0(-e_t' - s_t')/(1+i)^{t-1} + \lambda_t = 0$$

and

$$(15)\quad -(1+r)^{1-t} - \sum_{1}^{t-1}\lambda_j h_j{*}'/(1+i)^{t-j} + \lambda_0/(1+i)^{t-1} = 0.$$

To prove that the hours constraint is always binding, combine the first order conditions for w_t and w_{t+1} and rearrange terms to get

$$(16) \qquad \lambda_t = r - i/[h_t{*}'(1+r)^t].$$

Provided that i is greater than r so that bonding is costly, the hours constraint will be binding.

To prove that hours will be set below their optimal level, we combine (14) and (15) to obtain

$$(17) \qquad v_t - e_t{}' - s_t{}' = -\lambda_t(1+r)^{t-1}.$$

Since λ_t is negative (see also equation (15), equation (16) implies that hours are set at a level at which the marginal product of labor exceeds the marginal disutility of employment (the disutility of labor plus the marginal value of time) so that hours are sub-optimal.

The fact that hours are set according to the maximum hours constraint does not necessarily imply that workers are constrained to work fewer hours than they desire. If the wage were sufficiently lower than VMP, workers could be constrained to work more than they want even though the level of hours was suboptimal. However, it will be shown in the next section that over the period of the contract, discounted wages exceed VMP. It follows that since, at least in some periods, the wage exceeds VMP and VMP exceeds the marginal disutility of employment, that workers will be constrained to work fewer hours than they wish at least in some periods.

III. Mandatory Retirement

It is also possible to show using this multiperiod model that, if it occurs, mandatory retirement will be set at an age when vmp exceeds the marginal disutility of employment. To do this it is helpful to begin by solving for the hours and wage profiles.

Hours of work each period can be obtained by eliminating λ_t from equations (16) and (17) to obtain

(18) $$(v_t - e_t' - s_t')h_t^{*\prime} = (i-r)/(1+r)$$

which is a function only of h and exogenous variables since by the inverse function rule $h^{*\prime}$ is just $(q/e)'$.

Given h, it is possible to solve for the wage profile. Since the hours constraint is binding, the no shirking constraint is also binding, and the wage profile can be obtained by backwards induction as in Becker and Stigler (1974) and Akerlof and Katz (1986). In order to stop the worker from shirking in the last period, the pension must be sufficiently large to satisfy the no shirking condition

(19) $$P/(1+i) - e = (1-q)P/(1+i)$$

or

(20) $$P = (1+i)e/q.$$

In every period before the last, the no shirking condition can be solved to give

(21) $$w = e + s + ie/q.$$

In the first period, the wage is indeterminate, but it is possible to solve for the wage minus the bond. The algebraic manipulations in the remainder of this section are somewhat simplified if the first period wage is treated as being determined by (21) and the bond is allowed to adjust to set w_1-b equal to its equilibrium value. Using constraint (13), it can be seen that the bond will be given by

$$(22) \qquad b = (1+i)e/q$$

so that, in effect, workers receive interest on their bond as long as they are working and have the principal returned when they retire. It should be noted that in this model, whether actual upfront bonds are required or whether a lower first period wage is sufficient to clear the market depends on the value of w_1-b and not just on the value of b. This point is discussed in greater detail in Akerlof and Katz (1986).

To determine the mandatory retirement age, we need to consider the effect of increasing the contract by an extra year. The change in profits consists of two parts -- firms gain or lose the difference between the value of marginal product and the wage but gain from delaying repayment of the bond:

$$(23) \qquad \Delta\text{profits} = (v_T - w_T)/(1+r)^T + P/(1+r)^T - P/(1+r)^{T+1}.$$

Substituting from (20) for the pension and from (23) for the wage and rearranging terms gives

$$(24) \qquad \Delta\text{profits} = (1+r)^{-T} \{v-e-s-ie/q + e/q(1 - 1/(1+r))\}.$$

Mandatory retirement occurs when the change in profits equals zero, that is when

$$(25) \qquad v_t - e_t - s_t = (i-r)e_t/(q(1+r))$$

or, in other words when the value of marginal product exceeds the disutility of employment. Thus just as hours are chosen sub-optimally in the bonding model, the mandatory retirement age is chosen sub-optimally.

Moreover, "mandatory" retirement will have to be mandatory. From (23), it is evident that mandatory retirement will only occur when the wage exceeds VMP. Since the wage exceeds VMP and VMP exceeds the disutility of employment, workers would prefer to maintain the employment relation.

A particularly interesting case arises when the e, s and v profiles and thus h* are all independent of age or experience. From (18) it is obvious that, in this case, hours will be set at the same level in each period and that except for the first period, wages will be constant. Thus the wage profile takes the form of an initial low wage or bond, a constant wage over the rest of the employment relation and a pension at retirement. This is in many ways the prototypical bonding model and has all the properties of models in which the value of shirking/disutility of effort is assumed to be exogenous and independent of hours worked.

It is straightforward to show that in this special case, firms need never establish a mandatory retirement age. Firms are indifferent with respect to the length of the contract. To see this note that firms will employ workers up to the point that over the length of the contract the value of marginal product equals compensation or

$$(26) \qquad \Sigma(v-w)/(1+r)^{t-1} + b - P/(1+r)^{T} = 0.$$

Substituting for w, b and P gives the result that the profit maximization condition (26) and the mandatory retirement condition (25) are identical in this model. In other words, extending the contract by one year has no effect on profits in this case, and the firm is indifferent with respect to when the worker retires.

In general, however, the v, e, and s schedules will not be constant over the worker's lifetime. Provided that the disutility of employment rises faster than VMP, the firm will set a mandatory retirement age. As noted above, this retirement age will not be socially optimal. The precise relation between VMP, the wage and the mandatory retirement age depends on the particular reasons for bonding being costly.

IV. Public Policy and Sources of Costly Bonding

Since hours are chosen to maximize profits subject to a constraint on worker well-being, it is not possible to improve on the hours contract by altering hours directly. Similarly, since bonds and pensions are chosen in this manner, government cannot improve welfare by legislating the size of bonds and pensions. However, it is possible that other government policies may improve welfare. In essence, the inefficiency arises because bonds are not sufficiently large. If the cost of bonding could be reduced or the size of bonds increased costlessly by government, welfare would be increased.

Whether such policies exist, depends on the reasons for bonding being costly.

There appear to be three main reasons why bonding might be costly. The first is differences in discount rates, as in sections II and III. The second is possible malfeasance by the firm. The third is the possibility of mistakes.

If bonding costs are due to differences in discount rates, government could improve welfare by taxing older workers and subsidizing younger workers. Since the transfers would not be contingent on performance, they would be consistent with the no shirking constraint. Of course, the worker/firm contract could duplicate the after-tax profile. Instead of having workers post a bond which is not repaid if workers cheat, firms and workers might agree that workers would pay a fine if they were caught shirking. While such contracts might not be legal, firms could instead lend the equivalent of the bond to workers, say "for the purchase of a house," and require repayment regardless of whether the worker remained with the firm. The fact that we do not observe such contracts very often may imply that differences in discount rates are not the source of costly bonding or that these alternative contractual arrangements are equally costly.

If differences in discount rates are important it may be possible to use public policies to affect directly workers' discount rates. For example, if profits are stochastic so that firms may go bankrupt and if firms are risk neutral but workers are risk averse, the value to workers of a one dollar increase in P is less than its cost to the firm so that workers, in effect, have higher discount rates than firms and bonding is costly. It is clear that in this situation some sort of ERISA like policy in which "pensions" are guaranteed can be welfare improving. It is less obvious why the free market could not develop such insurance on its own. Perhaps some combination of moral hazard and adverse selection would provide an explanation.

Similarly, if workers are sometimes misconstrued as shirking, and workers are risk averse, bonding will be costly.[4] In this model, welfare might be increased by some kind of grievance procedure or quasi-judicial review which reduced the probability of a worker being fired. However, again if such improvements were possible. It is difficult to see why they would not be instituted in the agreement between the worker and firm.

The most plausible case for some sort of government intervention arises if there is a possibility of firm malfeasance. If bonding is costly because firms have an incentive to steal the bond, then government may be able to increase welfare by inhibiting firms from malfeasance. In practice, this entails interference with employment at will and may have negative side effects since presumably government would make it more difficult not only to fire workers without cause but also workers who have shirked. Again, it is not impossible that the free market could develop institutions similar to the governmental institutions, but government may have a significant advantage in this area.

Similarly, the desirability of restricting mandatory retirement depends critically on the source of costly bonding. In the model presented in section II and III, simply delaying the mandatory retirement age is not welfare improving. The cost of bonding exceeds the difference between older workers' VMP and their reservation wage. Thus the results of this paper serve to underscore the significance of understanding the reasons that bonding is costly.

The essential message of this paper has been presented using a variety of different models. While the most extensive discussion concerned a model in which workers and firms have different discount rates, it should be clear that the essential point does not depend on the precise formulation. Provided that bonding is costly, it will make sense to use bonding as little as possible so that the no shirking constraint will be binding.

The sub-optimality of the hours and the mandatory retirement age imposed by the contract casts light on the bonding/efficiency wage debate. A major issue between proponents of the two models has been whether the agency problem imposed by shirking results in involuntary unemployment (Carmichael, 1985; Shapiro and Stiglitz, 1984, 1985). Carmichael maintains that the involuntary unemployment which arises in shirking versions of the efficiency wage model depends critically on the assumption that workers cannot post bonds. He argues that even with capital market imperfections, the level of bonds will adjust so that the marginal worker is just indifferent between working and not working.

In the model presented in this paper, it seems reasonable to argue that workers who cannot obtain a job because they have passed mandatory retirement age are involuntarily unemployed although the issue may be more one of semantics or ideology than of substance. They are voluntarily unemployed in the sense that they are unwilling to work at the best offer any employer is willing to make them. On the other hand. their reservation wage is below their value of marginal product. The reason that they cannot reach an agreement with a firm is, of course, that the wage they require plus the cost of bonding is less than their VMP.

In addition, the results suggest that whether or not the agency model leads to involuntary unemployment, it definitely leads to involuntary underemployment. Involuntary underemployment may be as significant an economic phenomenon as involuntary unemployment. Kahn and Lang (1986) estimate that 40 to 45% of hourly workers are involuntarily underemployed. This paper suggests that the agency problem may provide an explanation for the prevalence of such underemployment.

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FOOTNOTES

1. For simplicity it is assumed that the worker either shirks for the entire period or not at all. Formally, this assumption requires that e'' not be too large.

2. Intuitively, while the probability of being caught in any particular hour remains constant, the probability of having already been caught rises as hours worked increases. Consequently, the marginal contribution of the last hours worked to the probability of being caught decreases with hours worked.

3. To see this multiply by q and divide by e to get $e'/e - q'/q$. The first term is at least as great as $1/h$ while the second term is strictly less than $1/h$.

4. This argument is developed formally for the legal system in Polinsky and Shavell (1979).