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MEASUREMENT ERROR AND THE FLOW OF FUNDS ACCOUNTS: ESTIMATES OF HOUSEHOLD ASSET DEMAND EQUATIONS

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ABSTRACT

In the household sector of the Flow of Funds Accounts, the difference between net acquisition of financial assets and net financial savings is equal to a statistical discrepancy which is often quite large relative to the reported changes in asset holdings. This means that the budget restrictions emphasized in the Brainard-Tobin approach to specifying asset demand equations are not satisfied by the data commonly used to estimate such equations. The view adopted in this paper is that the statistical discrepancy should be thought of as resulting from measurement error in the Flow of Funds data. By imposing a structure on the measurement error, a consistent estimator is developed and used to estimate asset demand equations for the household sector. The demand equations are similar in specification to those used by others so that the results allow a direct assessment of the effects of alternative treatments of the statistical discrepancy. The empirical results suggest that qualitative conclusions about the effects of financial flows and interest rates on asset demands are not affected by the way the statistical discrepancy is treated. Quantitative conclusions are, however, affected.

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1. Introduction

It is now more than a dozen years since Brainard and Tobin published their "Pitfalls" paper warning that financial models must be specified so as to be consistent with the underlying budget constraints and wealth identities faced by the agents in the model. An agent's wealth and the specification of n-1 asset demand equations completely determines the agent's asset behavior in an n asset model. Brainard and Tobin [1963] drew attention to two important implications of this fact. First, given wealth, no explanatory variable can appear in only one asset demand equation. This follows since any variable which affects the demand for one asset can only cause a reallocation of the portfolio, since the portfolio size is given by wealth, and must therefore affect the demand for at least one other asset. In particular, the sum across all n equations of the partial derivatives in each demand equation of an explanatory variable (other than wealth) must equal zero. From this restriction the conclusion has been drawn that, in the absence of strong a priori information, the same set of explanatory variables should appear in each equation describing an agent's asset holdings. Second, Brainard and Tobin noted that the basic stock adjustment formulation, in which actual asset holdings change in response to a gap between desired and actual holdings, needs to be generalized in order to be consistent with the restrictions implied by the budget identity. Again, the approach followed was to assume that the change in holdings of any one asset would depend upon the difference for every asset between desired and actual asset holdings.

The budget constraint faced by the economic agent or sector being modelled has thus been at the heart of the empirical financial models which have followed Brainard and Tobin. Quite commonly,[1] this budget constraint has been stated in flow terms: net acquisition of assets minus net acquisition of liabilities equals the change in wealth. In this form, the Brainard and Tobin framework has been used in conjunction with data from the Federal Reserve Board's Flow of Funds Accounts to estimate asset demand equations for various sectors of the economy.

One unfortunate aspect of the Flow of Funds data is that, for most sectors, they fail to satisfy the budget restrictions emphasized in the Brainard-Tobin approach.[2] For example, net acquisition of financial assets minus liabilities (excluding capital gains) for the household sector averaged 10.8 billion per quarter for the 1956:1 to 1978:4 period. This should equal net financial savings (basically income minus taxes and consumer expenditures). However, net financial saving averaged only 7.5 billion per quarter over this same period. The difference between savings and net acquisition of assets minus liabilities, the sector statistical discrepancy in the Flow of Funds Accounts, averaged -3.2 billion per quarter, almost half the size of the total net flow of financial savings. Because the household sector

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[1]Examples of empirical work in the Brainard-Tobin tradition will be discussed in the next section.

[2]In one sense it is fortunate that the Flow of Funds data are not forced to satisfy the budget identities; the resulting discrepancies can alert us to measurement problems which would otherwise be hidden.

is treated as the residual sector in the Flow of Funds Accounts, the statistical discrepancy is generally largest for this sector.

Since one of the attractions of the Brainard-Tobin framework is the set of cross-equation coefficient restrictions imposed by the budget identity, it is somewhat inconvenient that the data themselves fail to satisfy the underlying accounting identities. This failure forces anyone using the Flow of Funds data to make a decision as to how the statistical discrepancy should be treated. Usually whatever decision has been made has been accompanied with little discussion of either the reasons or the consequences. It will be argued in this paper that past treatments of the statistical discrepancy have resulted in the use of estimators which are biased and inconsistent.

The approach adopted in this paper is that the statistical discrepancy should be thought of as resulting from measurement error in the Flow of Funds data. This approach is then used to analyze the implications of alternative treatments of the discrepancy in previous applied work. This will be done in section 2. In section 3, further assumptions which may be appropriate for the household sector are used to impose more structure on the measurement error process. This allows a consistent estimator to be developed that has the additional advantage that, when all asset demand equations contain the same set of variables, the coefficient estimates obtained by estimating each equation separately satisfy the restrictions implied by the budget identity.

Once a consistent estimator is developed, it remains to be seen whether or not a proper treatment of the statistical discrepancy leads to important changes in the parameter estimates obtained in a model of household asset holdings. To assess then the empirical importance of accounting in a consistent way for the measurement error in the data, a model of household asset holdings is estimated using quarterly data for the 1956:1 to 1978:4 period. The model itself is specified in section 4 and the data used in this study are discussed. In section 5, alternative estimates of the model are compared. The paper's conclusions are summarized in section 6.

2. Previous Treatments of the Discrepancy

In order to evaluate the implications of previous treatments of the statistical discrepancy in the household sector accounts, it is necessary to specify a framework of analysis within which alternative approaches can be considered. The original Brainard and Tobin paper and most of the empirical models of household financial behavior using the Flow of Funds Accounts make the basic assumption that the household decision making process can be decomposed into two steps. In the first step, households decide upon levels of consumption and investment in physical assets. The result of this first step is to produce a savings flow which must be allocated to the acquisition of financial assets. The second step in the decision process is to then choose desired levels of financial assets and liabilities.[3] In this stage, the net change in financial asset holdings is treated as exogenously determined (in the first stage).

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[3] The joint determination of consumption, physical investment and financial investment is considered by Walsh [1976], Purvis [1978], and Backus and Purvis [1980].

Suppose y_t^* is the net change in financial asset holdings during period t. Let s_{it}^* be the net acquisition of asset i, i=1,...,k (liabilities are proceeded by a minus sign). The budget restriction is that net acquisitions sum to the net change in financial assets:

$$\sum_1^k s_{it}^* = y_t^* . \tag{2.1}$$

The asterisk denotes that this relationship must hold between the true, not necessarily observable, values of the variables.

Let s_{it} and y_t be the observed values, obtained from the Flow of Funds Accounts, of s_{it}^* and y_t^* respectively. Let d_t be the reported statistical discrepancy. The observed data then satisfy

$$\sum_1^k s_{it} = y_t - d_t \tag{2.2}$$

instead of (2.1). It would seem natural to assume that d_t arises because of measurement error contained in the individual s_{it} and in y_t. Suppose then that we can write

$$s_{it} = s_{it}^* + u_{it}, \quad i=1,\ldots,k \tag{2.3a}$$

$$y_t = y_t^* + v_t \tag{2.3b}$$

where u_{it} and v_t are random measurement errors. Taking all variables to be written as deviations from their sample means, we will assume that $u_t' = (u_{1t},\ldots,u_{kt})$ and v_t have mean zero and covariance matrix given by

$$E\begin{bmatrix} u_t \\ v_t \end{bmatrix} [u_t' \; v_t] = \begin{bmatrix} \Sigma_u & \sigma_{uv} \\ \sigma_{uv}' & \sigma_{vv} \end{bmatrix} = \Omega \tag{2.4}$$

Assume also that $[u_t' \; v_t]$ is asymptotically uncorrelated with $[s_t^{*\prime} \; y_t^*]$ where $s_t^{*\prime} = [s_{1t}^*,\ldots,s_{kt}^*]$. These are the assumptions of a standard errors in variables model.

Using (2.1)-(2.3) we can express the statistical discrepancy in terms of the underlying random measurement errors

$$d_t = y_t - \Sigma s_{it} = v_t - \Sigma u_{it} . \qquad (2.5)$$

With these relationships in mind, we can now consider the treatment accorded to d_t in previous research.

Three classes of studies have made use of the Flow of Funds Accounts for the estimation of empirical models of household asset behavior. The most ambitious have been the attempts to estimate complete portfolio allocation models for all sectors of the economy. Three such projects have been carried out: Bosworth and Duesenberry [1973], Hendershott [1977], and Backus, Brainard, Smith and Tobin [1980]. Next we have models of the asset holding behavior of one sector. For the household sector such studies include Motley [1970], Wachtel [1972], Hendershott and Lemmon [1975], Kopche [1977], Saito [1977], and Backus and Purvis [1980]. The main focus of this paper will be on these first two classes of studies, both of which attempt to estimate equations to explain each s^*_{it} . A final group of papers have studied the demand by households for one particular asset, often as part of a model of the demand for one asset disaggregated by sectors. Papers in this final group include Goldfeld [1973], Friedman [1977, 1980], and Roley [1980a, 1980b].

All of the studies which have estimated complete models of household asset holdings have proceeded along the following lines. Assume that the net acquisition of asset i is a function of the net change in total financial asset holdings and a set of r other variables, x_t , assumed initially to be the same for each i :

$$s^*_{it} = \beta_i y^*_t + \gamma_i x_t + \epsilon_{it}, \qquad i=1,\dots,k \tag{2.6}$$

where ϵ_{it} is a random disturbance term with (y^*_t, x'_t) and ϵ_{it} asymptotically uncorrelated as are ϵ_{it} and $(u'_t\ v_t)$. The budget identity (2.1) implies the following restrictions on the parameters of (2.6):

$$\sum_1^k \beta_i = 1 , \qquad \sum_1^k \gamma_i = 0 . \tag{2.7}$$

If observations on s^*_{it} and y^*_t were available, the set of k equations in (2.6) could each be separately estimated by ordinary least squares producing estimates $\hat{\beta}_i$ and $\hat{\gamma}_i$ with the property that

$$\Sigma\hat{\beta}_i = 1 , \qquad \Sigma\hat{\gamma}_i = 0 .^4 \tag{2.8}$$

If all equations contain the same set of explanatory variables, each equation can be estimated separately despite the cross-equation restrictions on the coefficients implied by (2.7). This result depends upon the inclusion of $y^*_t = \Sigma s^*_{it}$ as one of the explanatory variables.

With the actual observed values of y_t and the s_{it}'s, y_t does not equal the sum of the s_{it}'s. Hence, if each equation were to be estimated separately, the resulting estimators would fail to satisfy the cross-equation restrictions. Theprocedure followed in all but two of the studies cited above[5] has been to use $\Sigma s_{it} = \bar{y}_t$

[4] Letting $z = (y^*x^*)$ be the Tx1+r matrix of T observations on y^* and x^* and s^*_i be the Tx1 vector of observations on s^*_{it} we have $\hat{\delta}_i = (z'z)^{-1}z's^*_i$ where $\hat{\delta}' = (\hat{\beta}_i' \hat{\gamma}_i)$ and $\Sigma\hat{\delta}_i = \Sigma(z'z)^{-1}z's^*_i = (z'z)^{-1}z'y^* = (1\ \ 0)$. See Denton [1978].

[5] Boseworth and Duesenberry [1973] and Goldfeld [1973] are the two exceptions. The procedure used by Bosworth and Dusenberry is discussed below. Goldfeld estimated a basic money demand equation using Flow of Funds data for household money holdings. His work does not fit into the framework of equation (2.6).

as the constraint variable in the asset demand equation in place of y_t. Since when this is done the sum of the dependent variables equals one of the explanatory variables (i.e. $\bar{y}_t$), separate equation by equation estimation yields parameter estimates which satisfy the restrictions imposed by the budget identity.

In other words, equations of the form

$$s_{it} = \beta_i \bar{y}_t + \gamma_i x_t + e_{it}, \quad i=1,\dots,k \tag{2.9}$$

have been estimated. From (2.3a), however,

$$\bar{y}_t = \Sigma s_{it} = y_t^* + \Sigma u_{it} \tag{2.10}$$

from which it follows that the disturbance term in (2.9) is given by

$$e_{it} = \epsilon_{it} + u_{it} - \beta_i \Sigma u_{jt} \tag{2.11}$$

It is clear from equation (2.11) that equation (2.9) suffers from a standard errors in variables problem: $\bar{y}_t = y_i^* + \Sigma u_{it}$ will be correlated with the error term e_{it}. Thus, while it is true that OLSQ applied to (2.9) will have the desirable property that the estimated coefficients will satisfy the appropriate restrictions, OLSQ is a biased and inconsistent estimator. Hence, the estimation procedures used in the studies cited above have been biased and inconsistent.

Unlike the standard errors in variables model, even if $\gamma_i = 0$ so that s_{it}^* is a function only of y_t^*, we are in general unable to determine the direction in which $\hat{\beta}_i$ is biased. The probability limit of the least squares estimator is given by

$$\text{plim}\hat{\beta}_i = \beta_i + (\sum_j \sigma_{ij} - \beta_i \sum_{jk}\sum \sigma_{jk})/\text{plim}\tfrac{1}{T}\Sigma \bar{y}_t^2 \tag{2.12}$$

where σ_{jk} is the jk^{th} element of Σ_u defined in (2.4). $\Sigma\Sigma\sigma_{jk}$ is positive by the positive definiteness of Σ_u, but the sign of $\Sigma\sigma_{ij}$ is indeterminant.

The alternative estimation procedure followed by Bosworth and Duesenberry amounts to using the measured value y_t in the regression equation rather than $\bar{y}_t$. That is, they estimate equations of the form

$$s_{it} = \beta_i y_t + \gamma_i x_t + e'_{it} . \qquad (2.13)$$

In this case, $e'_{it} = \epsilon_{it} + u_{it} - \beta_i v_t$ which is clearly correlated with $y_t = y^*_t + v_t$. Their estimation procedure therefore is also biased and inconsistent.

Despite the measurement errors suggested by the large household sector statistical discrepancy, all the studies of the portfolio behavior of households of which I am aware that have utilized the Flow of Funds data have ignored the problems introduced by such measurement error. Only Hendershott [1977] has attempted to analyze the statistical discrepancies in the Flow of Funds Accounts. He concludes that the most likely explanation for movements in the sectorial discrepancies is that they are due to data errors [pp. 360-361]. Hendershott does not develop the implications of this conclusion for the estimation methods he uses in [1977] as we have done above.

The implications of the statistical discrepancy as measurement error are also important for the approach to financial modelling adopted by Friedman [1977]. In his work, financial flow variables such as y^*_t are viewed as important in determining changes in asset holdings. This approach has also been used by Roley [1980a, 1980b]. We can conclude, however, that the estimation methods used in these papers have produced biased, inconsistent estimates of the impacts of financial flows on household asset holdings.

In the next section, additional structure will be imposed on the measurement error in s_t and $\bar{y}_t$ which will allow a consistent estimation method for equation (2.6) to be developed.

3. Estimation with Measurement Errors[6]

Suppose we are interested in estimating equations explaining the determination of k variables, s_i^*, where $\sum_i s_{it}^* = y_t^*$ and y_t^* is exogenously determined. If we assume that the same set of explanatory variables appears in each of the k equations, as in (2.9), there is no loss of generality if we assume y_t^* is the only explanatory variable. We thus have k equations of the form

$$s_{it}^* = \beta_i y_t^* + \epsilon_{it}, \quad i=1,\ldots,k \tag{3.1}$$

Observations are available on s_{it} and y_t which are related to s_{it}^* and y_t^* by (2.3a) and (2.3b). The following assumptions are made:

A.1: s_t^* is a kx1 vector of random variables given by $s_t^* = \beta y_t^* + \epsilon_t$ where β is a kx1 vector of parameters, ϵ_t a kx1 random variable with $E(\epsilon_t) = 0$, $E(\epsilon_t \epsilon_t') = \Sigma_\epsilon$.

A.2: y_t^* and ϵ_t are asymptotically uncorrelated ($\text{plim}\frac{1}{T}\sum_t y_t^* \epsilon_{it} = 0$ for all i).

A.3: $s_{it} = s_{it}^* + u_{it}$ where u_{it} is a random measurement error. $E(u_{it}) = 0$, $E(u_t u_t') = \Sigma_u$ where $u_t' = (u_{1t},\ldots,u_{kt})$, $\text{plim}\frac{1}{T}\sum_t u_t \epsilon_t' = 0$ and $\text{plim}\frac{1}{T}\sum_t y_t^* u_t = 0$.

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[6] This section draws heavily on sections 3 and 4 of Walsh [1980].

A.4: $y_t = y_t^* + v_t$ where v_t is a random measurement error. $E(v_t) = 0$, $E(v_t^2) = \sigma_{vv}$, $\text{plim}\frac{1}{T}\sum_t v_t \epsilon_t = 0$, and $\text{plim}\frac{1}{T}\sum_t y_t^* v_t = 0$.

A.5: $(\epsilon_t' \; u_t' v_t)$ is distributed independently over time.

A.6: The disturbance term ϵ_t and the measurement errors $(u_t' v_t)$ are jointly normally distributed. $E(u_t v_t) = \sigma_{uv}$.

A.7: y_t^* is normally distributed with mean y^* and variance σ_{yy}^*.

Assumptions (A.1) - (A.4) are standard in measurement error models and will be maintained throughout this paper. The remaining assumptions will be relaxed as the discussion proceeds. Assumptions (A.6) and (A.7) are necessary in order to derive the maximum likelihood estimator of β. It will be shown that the maximum likelihood estimator in this model has an interpretation as an instrumental variable estimator. This result provides a motivation for the instrumental variable estimator that will be developed. Once having served that purpose, assumptions (A.6) and (A.7) can be dropped since the consistency property of the instrumental variable estimator does not require these assumptions. Assumption (A.7) is particularly undesirable, but it is necessary for the derivation of the maximum likelihood estimator. Hsiao [1976] points out the inappropriateness of this assumption for most time series models unless we are dealing with seasonally adjusted, detrended stationary series. Since the empirical papers cited in the previous section do not use detrended data, the emphasis here will be on developing a consistent esatimator which does not require (A.7). To motivate the estimation procedure to be developed, however, we initially assume (A.1) - (A.7).

Consider the following k+1 equations:

$$s_{it} = \beta_i y_t^* + \epsilon_{it} + u_{it} \qquad i=1,\ldots,k \tag{3.2a}$$

$$y_t = y_t^* + v_t \ . \tag{3.2b}$$

In this form, we have k+1 indicators (s_{it}, $i=1,\ldots,k$, and y_t) of the unobservable variable y_t^*. The covariance matrix of the observable indicators is given by

$$E\begin{bmatrix} s_t \\ y_t \end{bmatrix} [s_t' \quad y_t] = \sigma_{yy}^* \begin{bmatrix} \beta\beta' & \beta \\ \beta' & 1 \end{bmatrix} + \begin{bmatrix} \Sigma_\epsilon & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \Sigma_u & \sigma_{uv} \\ \sigma_{uv}' & \sigma_{vv} \end{bmatrix} \tag{3.3}$$

$$= \sigma_{yy}^* \begin{bmatrix} \beta\beta' & \beta \\ \beta' & 1 \end{bmatrix} + \begin{bmatrix} \Sigma_\epsilon & 0 \\ 0 & 0 \end{bmatrix} + \Omega = \Theta \ .$$

Goldberger [1974] discusses models of this type under the assumption that Ω is diagonal. This would be the case if the measurement errors were all independently distributed. He develops maximum likelihood methods of estimation for $k > 2$ (if $k \leq 2$, the system is unidentified). In the present case, Ω is not assumed diagonal so without further restrictions the system is unidentified for all k .

To identify the model, it is necessary to make additional assumptions about the structure of the measurement error covariance matrix Ω . First note that in (3.2a) ϵ_{it} and u_{it} enter only in the form $(\epsilon_{it}+u_{it})$. In (3.3) then, Σ_ϵ and Σ_u enter only in the form $\Sigma_\epsilon + \Sigma_u = \Sigma_\phi$. It will be impossible, therefore, to separately estimate Σ_ϵ and Σ_u. The most we can hope for is to estimate their sum. This implies that assuming Σ_u is diagonal will not reduce the number of parameters we need to estimate and will not help to identify β in (3.3). The sum $\Sigma_\epsilon + \Sigma_u$ cannot be assumed diagonal[7]

[7] Hendershott [1977] makes this assumption.

since the budget constraint implies that the columns of Σ_ϵ sum to zero. Thus, Σ_ϵ cannot be diagonal, and identifying β by imposing restrictions on $\Sigma_\epsilon + \Sigma_u$ does not seem to be a useful approach.

For the household sector of the Flow of Funds Accounts, each u_{it} results from errors in allocating the net acquisition of the ith asset to the various sectors in the economy. Since the household sector is the residual sector in the Accounts, u_{it} will incorporate errors originating in all sectors of the Accounts. The error v_t, on the other hand, arises due to errors in measuring household disposable income and consumption expenditures. The error in y_t originates in the National Income and Product Accounts measurement of household financial savings. It would seem reasonable then to assume that u_t and v_t are asymptotically uncorrelated. We will therefore assume

(A.8) $$\text{plim}\frac{1}{T}\sum_t u_t v_t = 0 .$$

As with most maintained hypotheses, it is unlikely that (A.8) is strictly true. However, it would seem to be a more reasonable assumption upon which to base the estimation of asset demand equations for the household sector than the implicit assumption normally made that the sum of the u_{it}'s is identically zero for all t so that $\sum_i s_{it}$ is an accurate measure of y_t^*.

Under (A.8),

$$\Omega = \begin{pmatrix} \Sigma u & 0 \\ 0 & \sigma_{vv} \end{pmatrix} \qquad (3.4)$$

Letting $P_t' = (s_{1t}, \ldots, s_{kt}, y_t)$ be the vector of observations for the tth period, the likelihood function for a sample of T observations on P_t is given, apart from a constant, by

$$L = |\Theta|^{-\frac{T}{2}} \exp[-\tfrac{1}{2}\sum_t P_t' \Theta^{-1} P_t]$$

$$= |\Theta|^{-\frac{T}{2}} \exp[-\tfrac{1}{2} T \mathrm{tr}(\Theta^{-1} M)] \qquad (3.5)$$

where $M = \frac{1}{T}\sum_t P_t P_t'$ is the matrix of sample variances and covariances among the observable variables. The maximization of L must be carried out subject to two types of constraints. First, we have the relationship between the reduced form parameters in Θ and the structural parameters consisting of β, Σ_ϕ, σ_{vv}, and σ^*_{YY}. This relationship is given by (3.3) and (3.4):

$$\sigma^*_{YY} \begin{bmatrix} \beta\beta' & \beta \\ \beta' & 1 \end{bmatrix} + \begin{bmatrix} \Sigma\phi & 0 \\ 0 & \sigma_{vv} \end{bmatrix} = \Theta . \qquad (3.6)$$

Second, the budget restriction implies that

$$\Sigma\beta_i = 1 . \qquad (3.7)$$

Equation (3.6) expresses the $\frac{1}{2}(k+1)(k+2)$ elements of Θ in terms of σ^*_{YY}, β, σ_{vv}, and the $\frac{1}{2}k(k+1)$ elements of Σ_ϕ. Equation (3.9) implies that β contains only $k-1$ free parameters. The total number of free parameters to be estimated is therefore $1+k-1 + 1 + \frac{1}{2}k(k+1) = \frac{1}{2}(k+1)(k+2)$ so that the model is just identified. This implies that (3.6) and (3.7) place no restrictions on Θ. The maximum likelihood estimator which results from the unconstrained maximization of (3.5) is

$$\hat{\Theta} = M . \qquad (3.8)$$

The maximum likelihood estimators of the structural parameters can be found by solving (3.6) and (3.7) with Θ replaced by $\hat{\Theta}$.

If we let M_{xz} be the sample covariance between x and z , then setting the left side of (3.6) equal to $\hat{\Theta} = M$ yields

$$\hat{\sigma}^*_{yy} \hat{\beta}_i = M_{s_i y} \qquad (3.9)$$

where ^ denotes an estimated value. Summing both sides over i and using (3.7),

$$\hat{\sigma}^*_{yy} \sum_i \hat{\beta}_i = \hat{\sigma}^*_{yy} = \sum_i M_{s_i y} , \qquad (3.10)$$

Combining (3.9) and (3.10), the maximum likelihood estimator of β_i is given by

$$\hat{\beta}_i = M_{s_i y} / \sum_i M_{s_i y} \qquad (3.11)$$

$$= \sum_t s_{it} y_t / \sum_i \sum_t s_{it} y_t .$$

Reversing the order of summation in the denominator of (3.11) and recalling that $y_t = \sum_i s_{it}$, we can rewrite the formula for $\hat{\beta}_i$ as

$$\hat{\beta}_i = \sum_t s_{it} y_t / \sum_t \bar{y}_t y_t . \qquad (3.12)$$

The maximum likelihood estimator of β_i is equal to the instrumental variable estimator of β_i in the regression of s_{it} on $\bar{y}_t$ with y_t used as the instrumental variable.

When s^*_{it} and y^*_t are replaced by s_{it} and $\bar{y}_t$ in (3.1), the error term, given by (2.11), becomes $\epsilon_{it} + u_{it} - \sum_i u_{it}$. By (A.8) this error term is asymptotically uncorrelated with v_t . By (A.2) - (A.4), it is also asymptotically uncorrelated with y^*_t . Therefore, since $\text{plim}\frac{1}{T}\sum \bar{y}_t y_t = \sigma^*_{yy}$, $y_t = y^*_t + v_t$ qualifies as an instrumental variable in the regression of s_{it} on $\bar{y}_t$.

Assumption (A.7) is unlikely to hold, but it is unnecessary for the consistency of the estimator given by (3.12). This gives us a simple, consistent estimator for a model of household asset behavior which has the advantage that the estimation of each equation separately will, when the same variables appear in each equation, yield coefficient estimates which satisfy the restrictions implied by the budget constraint. Summing (3.12) over i shows that

$$\sum_i \hat{\beta}_i = \sum_{it}\sum s_{it} y_t / \sum_t \bar{y}_t y_t = 1 \qquad (3.13)$$

Using $\bar{y}_t$ as the basic explanatory variable insures that the adding-up restrictions are satisfied; using y_t as an instrumental variable insures the consistency of the estimator.

While y_t^* is unobservable, we do have two measures of it, $\bar{y}_t$ and y_t. By using only $\bar{y}_t$, the informational content of y_t is ignored. Using both measures of y_t^* enables the parameters of the model to be consistently estimated.

If each equation contains, in addition to y_t^*, a set of other explanatory variables x_{it} which may differ across equations, the basic relationship among the observable s_{it} and $\bar{y}_t$ is

$$s_{it} = \beta_i \bar{y}_t + \gamma_i' x_{it} + \epsilon_{it} + u_{it} - \beta_i \sum_j u_{jt} \qquad (3.14)$$

$$= \beta_i \bar{y}_t + \gamma_i' x_{it} + e_{it}$$

where $e_{it} = \epsilon_{it} + u_{it} - \beta_i \sum_j u_{jt}$. The budget restriction implies that

$$\sum_i e_{it} = \Sigma \epsilon_{it} + \Sigma u_{it} - \Sigma \beta_i \Sigma u_{jt} \qquad (3.15)$$

$$= 0 + \Sigma u_{it} - \Sigma u_{jt} = 0$$

so that the covariance matrix of e_t, Σ_e, is singular. Letting z^j denote the $(k-1)x1$ vector obtained by dropping the jth element of the $kx1$ vector z,

$$s_t^j = \beta^j \bar{y}_t + \Gamma' x_t + e_t^j \qquad (3.16)$$

where x_t is a vector of all explanatory variables appearing in any equation and Γ' is a conformal matrix of coefficients. Each row of Γ' has zeros corresponding to the elements of x_t not appearing in that equation. In (3.15) we have a system of k-1 equations which could be estimated by the method of Zellner [1962] with two modifications. First, y_t should be used as an instrumental variable for $\bar{y}_t$, and second, if the deleted jth equation did not include all the elements of x_t, there remain cross equation restrictions on the k-1 rows of Γ'. For a more complete discussion of this case, see Walsh [1980].

We have so far assumed that all disturbance and measurement error terms are serially independent. Since this is unlikely to be the case in any model using time series data, it is necessary to consider the additional estimation problems which arise when the error terms are not all serially independent. In (3.14), the error term e_{it} could be serially correlated due to the presence of serial correlation in either ϵ_{it} or any of the u_{jt}'s. However, Brainard and Tobin's "Pitfalls" methodology applies to the specification of the properties of the error terms as well as to the systematic part of each equation; the budget constraint imposes restrictions on the serial correlation properties of $e_t' = (e_{1t}, \ldots, e_{kt})$ since, from (3.15), $\Sigma e_{it} = 0$, and Σ_e is singular. Berndt and Savin [1975] consider maximum likelihood estimation in a model with a singular covariance matrix and autoregressive errors. Their method can be easily modified to be used with the instrumental variable estimator proposed here.

Assume that we can write e_t as an autoregressive process

$$e_t = B(L)e_{t-1} + \psi_t \tag{3.17}$$

where $B(L)$ is a kxk matrix of polynomials in the lag operator L, and ψ_t is a serially independent error term. The matrix B(L) is a function of the time series properties of ϵ_t and u_t since e_t is a composite disturbance term (see Pagan [1973]). Equation (3.15) implies that $\Sigma\psi_{it} = 0$ and each column of B(L) sums to the same constant.

In going from (3.14) to (3.16), the jth equation was eliminated so that the resulting system of equations would have a nonsingular covariance matrix. In order to go from (3.17) to an expression for e_t^j, the method of Berndt and Savin [1975] can be used. Letting $b_{ij}(L)$ be the ijth element of B(L) we can write

$$\begin{aligned} e_t^j &= \begin{bmatrix} b_{11}(L) - b_{1j}(L) & \ldots & b_{1k}(L) - b_{1j}(L) \\ \vdots & & \\ b_{k1}(L) - b_{kj}(L) & \ldots & b_{kk}(L) - b_{kj}(L) \end{bmatrix} e_{t-1}^j + \psi_t^j \\ &= C(L)e_{t-1}^j + \psi_t^j \end{aligned} \tag{3.18}$$

where $C(L)$ is a $(k-1)x(k-1)$ matrix of polynomials in L. Each column of C(L) sums to zero.

Substituting (3.18) into (3.16)

$$s_t^j = \beta^j \bar{y}_t + \Gamma' x_t + C(L)e_{t-1}^j + \psi_t^j \,. \tag{3.19}$$

The unknown elements of β^j, Γ', and C(L) can be estimated by the methods discussed in Fair [1972]. In the actual estimation reported in section 5 below, e_t was assumed to follow a first order autoregressive process. The residuals from an initial consistent estimation of (3.16) were used to estimate equations of the form

$$e_{it} = \sum_{m \neq j} C_{im} e_{mt-1} + \psi_{it} . \qquad (3.20)$$

From the estimated results, certain elements of $C(L) = C$ were assumed to be zero. The remaining elements along with β^j and Γ' were simultaneously estimated by the joint estimation of the k-1 equations in

$$s_t^j = \beta^j \bar{y}_t + \Gamma' x_t + C(s_{t-1}^j - \beta^j \bar{y}_{t-1} - \Gamma' x_{t-1}) + \psi_t^j \qquad (3.21)$$

subject to the implied cross-equation restrictions. For example, if $k=3$, $j=3$ and $c_{11} = c_{21} = 0$, (3.21) would become

$$s_{1t}^3 = \beta_1^3 \bar{y}_t + \gamma_1' x_{1t} + c_{12} s_{2t-1}^3 - c_{12} \beta_2^3 \bar{y}_{t-1} - c_{12} \gamma_2' x_{2t-1} + \psi_{1t-1}^3$$

$$s_{2t}^3 = \beta_2^3 \bar{y}_t + \quad + \gamma_2' x_{2t} + c_{22} s_{2t-1}^3 - c_{22} \beta_2^3 \bar{y}_{t-1} - c_{22} \gamma_2' x_{2t-1} + \psi_{2t-1}^3$$

so that β_2^3 and γ_2' appear in both equations.

In the next section the actual model to be estimated using these methods will be specified. The data which are used will also be described.

4. Model Specification and Data

Because the purpose of this paper is to assess the implications for the previous work discussed in section 2 of using an estimation method which attempts to account for the measurement error problem, a fairly standard specification of the household sector's asset demand equations will be made. This will allow for some confidence in relating the conclusions reached here to these earlier studies.

Almost all of the empirical work on household asset holdings has utilized a generalized stock adjustment framework in which

$$A_{it} - A_{it-1} = \sum_j \Theta_{ij}(A^*_{jt} - A_{jt-1}) \qquad (4.1)$$

where A_{it} is the stock of asset i actually held and A^*_{jt} is the desired stock of asset j. It is also common to follow the specification of Brainard and Tobin [1968] and include a term $\beta_i \Delta w_t$ where $\Delta w_t = w_t - w_{t-1}$ is the change in wealth during the period. This last term is included because of the presence of transaction costs; it is cheaper to allocate new financial flows than it is to reallocate existing asset holdings.

In addition to the adjustment equations in (4.1), it is necessary to specify the determinants of desired asset holdings. It is common to assume desired holdings are homogeneous of degree one in wealth and that $\alpha^*_{it} = A^*_{it}/w_t$ is a linear function of interest rates (or their logs) as well as perhaps other variables such as income.

Friedman [1977] has modified this basic framework to allow for interaction between the desired asset allocation for asset i, α^*_{it}, and the effect of financial flows, $\beta_i \Delta w_t$, by assuming $\beta_i = \alpha^*_{it}$. The coefficient of Δw_t changes as interest rate changes affect desired holdings of the ith asset. This generalized adjustment framework has been further developed by Roley [1980a].

The model to be estimated in this paper will be similar to the original Brainard-Tobin model, but the stock adjustment model will not be used to motivate the chosen specification. The stock adjustment model has the desirable property that it allows certain estimated coefficients to be interpreted as speeds of adjustment. Unfortunately, a common problem is that the empirical results usually produce estimated speeds of adjustment which are too slow to be plausible. Thus, while the type of equation derived from (4.1) seems to "work" well empirically, it is not necessarily due to the fact that households adjust their portfolios in the manner assumed by the stock adjustment model.

If we think of a household deciding upon levels of asset holdings during period t, the household's actions will depend upon the state variables describing the resources available to the household at the start of the period and the set of variables, such as interest rates, describing the characteristics of the assets the household is considering holding. In the absence of transaction costs, the household's initial position can be completely characterized by its wealth at the start of the period plus the assumed exogenous financial flow Δw_t.[8] Asset demands would therefore depend upon $w_{t-1} + \Delta w_t = w_t$. With no transaction costs, the composition of w_{t-1} is irrelevant.

When transaction costs are introduced, a dollar in the form of A_{it-1} is not equivalent to a dollar in the form of A_{jt-1} if differential costs are involved in transforming holdings of A_i and A_j into any other asset. Now to describe the initial state of the household, we need to completely specify the composition of w_{t-1}. Each A_{it-1} as well as Δw_t will enter the demand functions for assets in period t. In general then we would have

[8] In the more general models considered by Walsh [1976] and Backus and Purvis [1980], Δw_t is also a decision variable of the household so that the initial position is characterized by w_{t-1} alone.

$$A_{it} = f_i(\Delta w_t, A_{1t-1}, \ldots, A_{2t-1}, x_t) \quad (4.2)$$

where x_t is a vector of additional variables relevant for asset demands.

These relevant variables in x_t are assumed to be a vector of interest rates, scaled by lagged wealth, and personal disposable income. It is assumed that (4.2) can then be approximated by

$$A_{it} = a_{oi} + a_{i1}\Delta w_t + \sum_j b_j A_{jt-1} + c_i PDY_t + \sum_j d_{ij} r_{jt} w_{t-1} + e_{it} \quad (4.3)$$

where e_{it} is a random error term. In order to write this in a form that parallels the flow equations estimated by others and considered in the previous section, subtract A_{it-1} from both sides of (4.3) to yield

$$s_{it} = A_{it} - A_{it-1} = a_{io} + a_{i1}\Delta w_t + \sum_{j \neq i} b_{ij} A_{jt-1} - (1-b_{ii})A_{it-1} + \Sigma c_i PDY_t + \Sigma d_{ij} r_{jt} w_{t-1} + e_{it} \quad (4.4)$$

The budget constraint, $\sum_i s_{it} = \Delta w_t$, implies that the following restrictions on the coefficients must hold:

$$\Sigma a_{io} = \sum_i b_{ij} - 1 = \sum_i c_i = \sum_i d_{ij} = \sum_i e_{it} = 0$$
$$\sum_i a_{i1} = 1 \quad (4.5)$$

In the results reported in the next section, household net acquisition of financial assets is disaggregated into five categories:[9]

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[9]A detailed description of the data is contained in the appendix.

net acquisition of currency and demand deposits (MON), time and savings accounts (TIME), credit market instruments (BOND), nonmarketable assets (NONMKT), and net purchases of equities (NETP). Net acquisition of financial liabilities is disaggregated into two categories: net acquisition of mortgages (MORT) and net acquisition of other liabilities (LIAB). Corresponding to the variable $\bar{y}_t$ in section 3 we have

$$\Delta\bar{w}_t = \bar{y}_t = \sum_{1}^{7} s_{it} = \text{MON} + \text{TIME} + \text{BOND} + \text{NONMKT} + \text{NETP} - \text{LIAB} - \text{MORT} \qquad (4.6)$$

Since asset stocks also appear in (4.4), they were obtained by decumulating from the end of quarter stocks for 1978:4 using the seasonally adjusted quarterly flows (MON, TIME, etc.). This produced stock series which were consistent with the quarterly flow data. Asset stocks are labeled SMON, STIME, etc.

Seven interest rates appear in the asset demand equations: a rate on time and savings accounts (RTIME), the commercial paper rate (RCP), the Aa corporate bond rate (RBOND), the dividend yield for the Standard and Poor's 500 (REQUITY), the rate on business loans as a substitute for a consumer loan rate (RBL), the secondary market yield on FHA insured loans (RMORT), and since asset demands should depend upon real rates of return, the expected rate of inflation (EXPINF). This last variable was estimated as the rate of inflation predicted by a regression of actual inflation on a 10 quarter lag on past inflation, a 10 quarter lag on past rates of growth of the money supply and a 5 quarter lag on past unemployment rates.

Corresponding to y_t in section 3, we define Δw_t as

$$\Delta w_t = \Delta \bar{w}_t + SD_t \tag{4.7}$$

where SD_t is the statistical discrepancy in the Flow of Funds Accounts for the household sector.

5. Empirical Results

The asset demand equations were estimated over the 1956:1 to 1978:4 period. Each equation was initially estimated by OLSQ with $\Delta \bar{w}_t$ as an explanatory variable so the estimated coefficients satisfy the constraints in (4.5). Several of the equations appeared to have serially correlated residuals, so the procedure discussed in section 3 was used. This involved regressing the residuals $\hat{e}^j_t$ on $\hat{e}^j_{t-1}$ where the equation deleted (the jth equation) was the one for net purchases of equities. NETP was very small on average during the sample period with little variance. The initial OLSQ estimates of the NETP equation were imprecise with few coefficients statistically significant. In the remainder of the analysis, the NETP equation was deleted; estimates for this equation can be obtained residually from the budget identity and the estimates of the remaining k-1 (in this case, 6) equations.

The regression of $\hat{e}^j_t$ on $\hat{e}^j_{t-1}$ produced an estimate of C. Elements less than one and one-half times their estimated standard errors were assumed to equal zero. The remaining nonzero elements of C were used to transform the six equations for s^j_t into the form of (3.21). Because of the zero elements of C, not all equations now contained the same set of explanatory variables.

Consequently, the equations were estimated jointly using an iterative version of Zellner's method for seemingly unrelated equations. The resulting estimates of the parameters of the asset demand equations and the jointly estimated elements of C are presented in Table 1. In interpreting these results, it should be kept in mind that the explanatory variables are highly correlated. In light of this, it is surprising that 62 of the 106 estimated coefficients exceeded 1.65 times their estimated standard error.

The responses of asset demands to changes in nominal interest rates are reported in rows 2-7 of Table 1. Most of the estimated results conform in sign to a priori expectations, although there are a number of exceptions. Changes in the rate on time deposits and savings accounts (RTIME), for example, do not appear to have a significant effect on households' acquisitions of currency and demand deposits (MON), but MON does respond negatively to the commercial paper rate (RCP) as well as to the corporate bond rate (RBOND). The positive coefficients on the remaining interest rate variables (REQUITY, RBL, and RMORT) are more difficult to interpret. It is worth noting that these results for the six nominal interest rate variables are the opposite of those discovered by Backus and Purvis [1980] who found only the time deposit and savings account rate to be significant. The own rate on money, minus the expected rate of inflation, has a negative but statistically insignificant estimated coefficient. If households respond to real rates of interest, the coefficient on $EXPINF_t \cdot w_{t-1}$ is equal to an own rate response minus the sum of the coefficients on the other interest

Table 1

Estimated Asset Demand Equations, No Correction for Measurement Error, 1956:1 to 1978:4

Explanatory Variables	Dependent Variable						
	MON	TIME	BOND	NONMKT	NETP[1]	LIAB	MORT
$RTIME \cdot w^2_{t-1}$	0.018	0.106*	-0.094*	0.005	-0.033	-0.049*	0.051*
$RCP \cdot w_{t-1}$	-0.072*	-0.015	0.069*	-0.012	0.026	0.012	-0.016
$RBOND \cdot w_{t-1}$	-0.243*	-0.434*	0.592*	-0.070**	0.115	0.037	-0.077**
$REQUITY \cdot w_{t-1}$	0.174*	-0.167*	-0.110	0.028	-0.046	-0.135*	0.014
$RBL \cdot w_{t-1}$	0.038**	0.087*	-0.034	0.053*	0.003	0.073*	0.074*
$RMORT \cdot w_{t-1}$	0.162*	-0.069	-0.004	-0.001	-0.112	0.003	-0.027
$EXPINF \cdot w_{t-1}$	0.012	0.006	-0.002	-0.007	-0.004	0.013	-0.000
PDY	0.471*	0.169	-0.127	-0.123*	-0.141	0.078	0.171*
Δw	0.173*	0.221*	-0.068	0.309*	0.035	-0.168*	-0.162*
$SMON_{t-1}$	-0.408*	0.200*	-0.061	0.134*	0.115	-0.015	-0.005
$STIME_{t-1}$	0.007	-0.050	0.086**	0.028	0.000	0.001	0.070*
$SBOND_{t-1}$	0.003	0.219*	-0.317*	0.077*	-0.013	-0.078*	0.047**
$SNONMKT_{t-1}$	-0.050	0.520*	-0.446*	0.028	0.025	0.161*	-0.071**
$SEQUITY_{t-1}$	0.011*	0.013*	-0.019*	0.006*	-0.003	0.001	0.007*
$SLIAB_{t-1}$	0.168	-0.107	-0.209*	-0.121*	-0.114	-0.288*	-0.095*
$SMORT_{t-1}$	-0.247*	-0.521*	0.626*	0.021	0.084	0.023	-0.060
CONSTANT	0.779	-81.041*	83.445*	-7.066	-8.581	-15.307**	2.843**
ρBOND		0.089*					
ρNONMKT	0.970*	1.098*					0.382*
R^2 (adjusted)	0.5959	0.9344	0.6734	0.9667		0.8994	0.9794
S.E.R.	1.302	1.471	2.065	0.913		1.104	0.640

*t > 2.0

**t > 1.65

[1]This equation is derived from the others using the budget identity.

[2]All interest rate coefficients are multiplied by 100.

rate terms. This combined effect may be close to zero even if MON responds to changes in the expected rate of inflation when other real rates are held constant. From Table 1 the implied response of MON to EXPINF holding real rates constant is negative, but small (-0.067).

Net acquisition of time deposits and savings accounts depends positively on the own rate variable, RTIME, and negatively on the rates on substitutes, RBOND and REQUITY. The coefficient on the commercial paper rate is small and not statistically significant. Net acquisition of credit market instruments (BOND) depends positively on the two own rates, RCP and RBOND, and negatively on RTIME and REQUITY. The coefficient on REQUITY is not statistically significant (t = 1.40), but it is larger in magnitude than the coefficient on RTIME. Backus and Purvis found a significant positive effect of RBL on BOND but that does not show up here, although RBL does appear to affect TIME.

As might be expected, net acquisition of nonmarketables (pension fund reserves, life insurance reserves, savings bonds) is fairly interest inelastic. Only RBOND and RBL have statistically significant coefficients and their values are relatively small. None of the interest rate coefficients are large in the equations for LIAB and MORT either, but several are statistically significant: RTIME, REQUITY, and RBL in the equation for LIAB, RTIME, RBOND, and RBL in the equation for MORT. RBL is estimated to have a positive, rather than the expected negative, effect on LIAB.

These results suggest that asset holdings of the household sector are responsive to changes in interest rates, but the magnitudes involved are small.

Turning to the coefficients on the lagged stock variables, Table 1 shows that in only three of the six estimated equations (MON, BOND, and LIAB) is the coefficient on the own lagged stock negative and statistically significant as would be implied under a stock adjustment interpretation. The estimated coefficients on the lagged stocks differ considerably within each equation which is consistent with asset demands depending upon the household's initial portfolio composition and not just its size (w_{t-1}) as would be the case in the absence of transaction costs.

While income (PDY) is statistically significant only in the MON and NONMKT equations, the financial flow variable $\Delta\bar{w}$ is significant in all but the BOND equation. According to Table 1, 17 cents of a dollar increase in wealth is initially allocated to money holdings and 22 cents to time deposits and savings accounts although the biggest flow (31 cents) is into the nonmarketable category. In addition, total liabilities are reduced by about 35 cents.

Section 2 argued that the system of equations (4.4) should be estimated by using $\Delta w_t = \Delta\bar{w}_t + SD_t$ as an instrumental variable for $\Delta\bar{w}$. Reestimating the basic model using the same correction for autocorrelation as was used in Table 1 and an instrumental variable estimator yielded the coefficient estimates presented in Table 2.

Comparing the estimates in Tables 1 and 2, fairly sizable changes in the coefficients on $\Delta\bar{w}$ have occurred. The effect of a one dollar change in $\Delta\bar{w}$ on MON has dropped from 17 cents to 12 cents and on TIME from 22 cents to 16 cents. Neither effect is now statistically significant. Since the sum across all equations of

Table 2

Asset Demand Equations, Instrumental Variables Estimation Method[1]

Explanatory Variables	Dependent Variables MON	TIME	BOND	NONMKT	NETP	LIAB	MORT
$RTIME \cdot w_{t-1}$	0.015	0.103*	-0.094*	0.005	-0.032	-0.053*	0.050*
$RCP \cdot w_{t-1}$	-0.077*	-0.002	0.044	-0.003	0.034	0.017	-0.021**
$RBOND \cdot w_{t-1}$	-0.191*	-0.491*	0.675*	-0.103*	0.076	0.011	-0.045
$REQUITY \cdot w_{t-1}$	0.174*	-0.199*	-0.096	0.021	-0.052	-0.167*	0.015
$RBL \cdot w_{t-1}$	0.038**	0.072*	-0.033	0.055*	0.001	0.058*	0.075*
$RMORT \cdot w_{t-1}$	0.149**	-0.058	-0.003	-0.001*	-0.114	0.003	-0.030
$EXPINF \cdot w_{t-1}$	0.013	0.013	-0.009	-0.006	0.004	0.017	-0.002
PDY	0.496*	0.211	-0.060	-0.165*	-0.142	0.160**	0.180*
$\Delta \bar{w}$	0.117	0.159	-0.128	0.363*	0.037	-0.264*	-0.188*
$SMON_{t-1}$	-0.446*	0.254*	-0.170	0.178*	0.152	-0.001	-0.031
$STIME_{t-1}$	0.013	-0.089**	0.106**	0.025	-0.011	-0.033	0.077*
$SBOND_{t-1}$	-0.002	0.209*	-0.349*	0.085*	-0.001	-0.102*	0.044*
$SNONMKT_{t-1}$	-0.105	0.613*	-0.586*	0.077	0.094	0.201*	-0.108*
$SEQUITY_{t-1}$	0.010*	0.013*	-0.021*	0.007*	-0.003	0.000	0.006*
$SLIAB_{t-1}$	0.195*	-0.127**	-0.155**	-0.153*	-0.134	-0.296*	-0.078*
$SMORT_{t-1}$	-0.218*	-0.547*	-0.696*	0.003	0.050	0.025	-0.041
CONSTANT	8.494	-102.482	110.40*	-13.364**	-17.757	-22.991*	8.282
ρ_{bond}		0.066					
ρ_{nonmkt}		0.988*	1.121*				0.403*
R^2 (adjusted)	0.5920	0.9300	0.6505	0.9648		0.8960	0.9801
S.E.R.	1.309	1.520	2.126	0.939		1.122	0.629

[1]See notes to Table 1.

the coefficients on $\Delta\bar{w}$ must equal 1, the fall in the estimated coefficients in the MON and TIME equations must be balanced by changes in other equations. The coefficient on $\Delta\bar{w}$ in the equation for NONMKT has increased from .31 to .36, but the largest change has occurred in the equation for LIAB (.17 in Table 1 versus .26 in Table 2). The estimated coefficient on $\Delta\bar{w}$ in the BOND equation has doubled, but still has a t ratio less than 1.0.

The coefficients on PDY have also changed considerably as a result of the change in estimation methods. The estimated interest rate effects, however, have not changed very much. RCP is no longer statistically significant in the BOND equation whereas it is in the MORT equation now; RBOND is no longer significant in the MORT equation.

Since many of the estimated coefficients in Tables 1 and 2 are small and not statistically significant, a revised version of the asset demand equations which excludes variables from some of the equations was estimated. Since the purpose of this paper is to reestimate consistently this basic type of financial model that has been estimated in the past without regard to measurement error, the OLSQ estimates in Table 1 were used as a guide in respecifying the model. This revised version was then estimated by OLSQ and by the instrumental variable estimator.

In general, a variable was dropped from an equation if the t ratio for its estimated coefficient in that equation was less than 1. Two exceptions were made: the own rate (RMORT) was left in the equation for MORT, and the expected rate of inflation variable was left in all the equations. Since the coefficient on the expected rate of inflation is a combination of the interest rate coefficients if households' respond to real interest rates, dropping some interest rate variables in an equation may greatly affect the estimated coefficient on expected inflation. For this reason it was left in each equation.

Each equation in its modified version was first estimated separately using OLSQ. The calculated residuals from the equations were then analyzed to determine the zero elements of the matrix C in (3.18). The equations were then transformed according to the results of the residuals analysis and the six equations were jointly estimated subject to the cross equation restrictions which result from the autocorrelation transformation. The resulting estimates appear in Table 3.

As might be expected, the major changes in going from Table 1 to Table 3 occur in the coefficients of the lagged stock variables and the autocorrelation structure of the residuals. The coefficient on $\Delta\bar{w}$ has fallen in the equation for NONMKT and risen in the TIME equation. Rather than consider further comparisons of Tables 1 and 3, we can examine the effects of reestimating by instrumental variables the modified specification of Table 3. The resulting estimates are contained in Table 4 while the estimated coefficients on Δw from Tables 1 through Table 4 are brought together for comparison in Table 5.

The estimates in Table 5 show that the estimated coefficient on Δw is most sensitive to the method of estimation in the modified model in the TIME and NONMKT equations. However, even here the absolute differences are relatively small with the instrumental variable estimates implying that an additional 7 cents out of every

Table 3
Modified Model, No Correction for Measurement Error[1]

Explanatory Variables	MON	TIME	BOND	NONMKT	NETP	LIAB	MORT
	Dependent Variables						
$RTIME \cdot w_{t-1}$		0.125*	-0.089*		-0.025	-0.039*	0.050*
$RCP \cdot w_{t-1}$	-0.070*		0.022		0.028		-0.020**
$RBOND \cdot w_{t-1}$	-0.191*	-0.458*	0.552*	-0.143*	0.109		-0.131*
$REQUITY \cdot w_{t-1}$	0.146*	-0.150*	-0.129*		-0.022	-0.155*	
$RBL \cdot w_{t-1}$	0.036*	0.062*		0.047*	0.006	0.078*	0.073*
$RMORT \cdot w_{t-1}$	0.133*			-0.027	-0.114		-0.008
$EXPINF \cdot w_{t-1}$	0.014	-0.020	0.035**	-0.011**	-0.001	0.017**	-0.000
PDY	0.330*			0.003	-0.096		0.237*
$\Delta \bar{w}$	0.157*	0.279*		0.240*	-0.001	-0.155*	-0.170*
$SMON_{t-1}$	-0.358*	0.147*		0.170*	0.041		
$STIME_{t-1}$		-0.018	0.063*		0.007		0.052*
$SBOND_{t-1}$		0.105*	-0.217*	0.119*	0.013	-0.037*	0.057*
$SNONMKT_{t-1}$		0.616*	-0.535*		0.023	0.178*	-0.074*
$SEQUITY_{t-1}$	0.007*	0.000	-0.007**	0.011*	-0.003		0.008*
$SLIAB_{t-1}$	0.150*	-0.169*	-0.157*	-0.064*	-0.051	-0.229*	-0.062*
$SMORT_{t-1}$	-0.206*	-0.519*	0.613*		0.035		-0.077*
CONSTANT	-1.574	-80.234*	75.543*	-12.036*	1.343	-16.603*	-0.355
ρ_{mon}				-0.186*			
ρ_{time}		0.000		-0.051			
ρ_{bond}		-0.035		-0.048			
ρ_{nonmkt}		-1.121*	1.343*	0.139			0.361*
R^2 (adjusted)	0.595	0.941	0.6797	0.9195		0.8970	0.9807
S.E.R.	1.338	2.204	1.578	1.024		1.175	0.661

[1]See notes to Table 1.

Table 4

Modified Model: Instrumental Variable Estimates[1]

Explanatory Variables	Dependent Variables: MON	TIME	BOND	NONMKT	NETP	LIAB	MORT
$RTIME \cdot w_{t-1}$		0.115*	-0.078			-0.038*	0.054*
$RCP \cdot w_{t-1}$	-0.063*		-0.000				-0.035*
$RBOND \cdot w_{t-1}$	-0.151*	-0.669*	0.761*	-0.122*			-0.050
$REQUITY \cdot w_{t-1}$	0.116*	-0.144*	-0.097			-0.150*	
$RBL \cdot w_{t-1}$	0.032*	0.085*		0.028*		0.074*	0.070*
$RMORT \cdot w_{t-1}$	0.100**			0.015			0.006
$EXPINF \cdot w_{t-1}$	0.015**	0.004	0.008	-0.011**		0.022*	0.001
PDY	0.350*			-0.050			0.198*
$\Delta \bar{w}$	0.115**	0.213*		0.312*		-0.178*	-0.149*
$SMON_{t-1}$	-0.378*	0.067		0.257*			
$STIME_{t-1}$		-0.093*	0.161*				0.082*
$SBOND_{t-1}$		0.207*	-0.240*	0.030		-0.049*	0.035**
$SNONMKT_{t-1}$		0.986*	-0.971*			0.198*	-0.180*
$SEQUITY_{t-1}$	0.007*	0.020*	-0.018*	0.001			0.003**
$SLIAB_{t-1}$	0.113*	-0.460*	0.133**	-0.014		-0.255*	-0.018
$SMORT_{t-1}$	-0.182*	-0.593*	0.702*				-0.027
CONSTANT	-0.235	-128.737*	131.041*	-4.420*		-18.479*	15.719*
ρ_{mon}				-0.222*			
ρ_{time}		0.294*		-0.313*			
ρ_{bond}		-0.104		-0.002			
ρ_{nonmkt}		-1.433*	1.724*	0.223			0.514*
R^2 (adjusted)	0.5802	0.9281	0.6184	0.9024		0.9005	0.9840
S.E.R.	1.362	2.405	1.735	1.127		1.155	0.603

[1] See notes to Table 1.

Table 5
Estimated Coefficients on Δw

Equation	Source: Table 1	Table 2	Table 3	Table 4
MON	0.173 (2.05)	0.117 (1.18)	0.157 (2.67)	0.115 (1.79)
TIME	0.221 (2.17)	0.159 (1.23)	0.279 (4.91)	0.213 (3.19)
BOND	-0.068 (0.50)	-0.128 (0.75)		
NONMKT	0.309 (6.48)	0.363 (6.85)	0.240 (6.43)	0.312 (9.15)
NETP[2]	0.035	0.037	-0.002	0.032
LIAB	-0.168 (2.52)	-0.264 (3.38)	-0.155 (3.52)	-0.178 (3.83)
MORT	-0.162 (3.78)	-0.188 (3.62)	-0.170 (4.85)	-0.149 (3.94)

[1]t ratios in parentheses

[2]calculate from the budget restruction.

dollar change in Δw would end up in nonmarketable assets with 6 cents less in time deposits and savings accounts and 4 cents less in demand deposits and currency. These do not seem like very large magnitudes. In response to a billion dollar increase in Δw, the estimated impact on net acquisition of time deposits and savings accounts during the quarter of the increase would differ by only .2°/o of the average level of total holdings by households and 6°/o of the average size of net acquisitions. The conclusion to be drawn from Table 5 then is that while the sizeable statistical discrepancy for the household sector suggests serious measurement error problems actual coefficient estimates are not dramatically sensitive to the estimation method used.

Within the context of the framework developed in section 2, both of the estimation methods used in this paper would be consistent if $\bar{y}_t$ ($=\Delta\bar{w}_t$) was a true observation on y_t^*. In this case, using $\bar{y}_t$ as the explanatory variable in a least squares regression involves no measurement error problem. As long as v_t is uncorrelated with the measurement error in the individual s_{it}'s (as well as with ϵ_{it}), using y_t as an instrumental variable also produces consistent estimates. However this explanation for the similarity of the coefficients in Table 5 does not seem plausible. Each s_{it}^* for the household sector is measured as a residual and, while the measurement error in s_{it} may be correlated with that in s_{jt}, there is no reason to suppose that the sum of the u_{it}'s over i is identically zero for all t. If Σu_{it} is not identically zero, then $\bar{y}_t$ is not equal to y_t^* for all t and using $\bar{y}_t$ as a regressor leads to inconsistent estimates.

Another possible explanation for the results in Table 5 would be that the measurement errors in $\bar{y}_t$ and y_t are correlated (assumption A.8). In this case neither estimation method is consistent. Without further restricting the model in some way, no consistent estimator exists since the model is unidentified. The failure of assumption A.8 to hold does not in itself imply that similar estimates would be obtained by both estimation methods, but, given the high correlation between $\bar{y}_t$ and y_t (0.963 over the sample period 1956:1 to 1978:4), it is perhaps not surprising.

6. Summary and Conclusions

Previous studies of sectoral financial behavior which have utilized data from the Flow of Funds Accounts have ignored the presence of measurement error. This is potentially a serious problem for the household sector due to the residual nature of this particular sector in the Flow of Funds Accounts. If the statistical discrepancy in the Accounts is interpreted as being the result of measurement error, previous methods used to estimate Brainard-Tobin type models of the financial sector can be shown to be inconsistent.

By imposing additional structure on the measurement error, it was possible to derive a consistent estimator which had the property that, when all equations contain the same set of explanatory variables, single equation estimation produces parameter estimates which satisfy the restrictions implied by the budget identity. Consistency depended upon the assumption that the measurement error in the value of financial savings derived as income minus consumption (from the National Income and Product Accounts) was asymptotically uncorrelated with the measurement error in financial savings derived as net acquisition of financial assets minus liabilities (from the Flow of Funds Accounts).

In order to assess the empirical importance of using a consistent estimator, a model of the household sector's holdings of five asset categories and two categories of liabilities was estimated by two methods. The first involved a straightforward application of Zellner's method for estimating seemingly unrelated equations. The cross equation restrictions implied by the structure of serial correlation in the errors were imposed in the estimation, but no adjustments were made for measurement error. This first estimation method corresponds to the approaches used in previous studies. The second estimation method is an instrumental variables estimation method which was shown to be a consistent estimation procedure despite the measurement errors in the data.

The results for the estimated impact of financial flows on asset demands, summarized in Table 5, revealed some sizable differences between the two estimation procedures when the standard Brainard-Tobin specification was used. In this specification, the same set of explanatory variables appears in each equation. When the equations were modified, dropping variables that appeared to have zero coefficients, the differences produced by the two estimation procedures were much smaller.

This difference in the results obtained in the two versions of the model may be more an indication of the problem of multicollinearity common in models of this type than it is a result of the presence of measurement error. The conclusions to be drawn from the results reported in section 5 are therefore mixed. It is not clear that attempting to correct for measurement error will significantly alter parameter estimates. On the other hand, the estimator developed here is a simple instrumental variables estimator so it would be easy for researchers working on household financial behavior to estimate their models using both the standard method and the one proposed here. If they find little difference in their results, it can only add to the confidence with which their conclusions can be held.

APPENDIX

This appendix contains a description of the data used in this paper.

The following data are from the Flow of Funds Accounts:

NAME	F/F ID CODE
Demand Deposits plus Currency: MON	153020001
Time Deposits and Savings Accounts = TIME	153030005
Credit market instruments	154004005
plus money market shares	634000003
minus savings bonds	313133000
minus mortgages	153065005
equals BOND	
Net purchases of equities	153064005
Life insurance reserves	153050005
plus pension fund reserves	153050005
plus savings bonds	313133000
plus misc. assets	153090005
equals NONMKT	
Net increase in liabilities	154190005
minus security credit assets	153167005
minus home mortgages	153165101
minus other mortgages	153165503
Mortgage Liabilities	153165101
minus mortgage assets	153065005
equals MORT	
Statistical discrepancy	157005005

The rate on time deposits and savings accounts, RTIME, is from the FMP database while all other interest rates are from the Citibank Economic Database and are quarterly averages of monthly data:

NAME	CITIBANK LABEL
Commercial Paper (6 months), RCP	FYCP
Average Yield on Corp. Bonds, RBOND	FYAVG
Dividend price yield for Standard and Poor's Common Stock Composite	FSDXP
Bank rates on short term business loans--RBL	FYST, FY35RR, FY35R
Secondary market yields on FHA mortgages--RMORT	FYFHA

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