MULTIPLE-SOLUTION INDETERMINACIES
IN MONETARY POLICY ANALYSIS

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ABSTRACT

This paper discusses four current topics in monetary policy analysis, each of which hinges on the possibility of multiple solutions in rational expectations (RE) models. In three of these cases--involving inflation forecast targeting, the zero-lower bound deflation trap, and the fiscal theory of the price level--analysis based on E-stability and adaptive learnability of the solutions suggests that only one of them is a viable equilibrium candidate. Thus the dangers alleged to prevail, in these cases, are not ones with which actual policymakers need to be concerned. In the case of the Taylor principle, by contrast, policy behavior that violates the principle is genuinely undesirable, since all of the RE equilibria fail to be learnable.

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1. Introduction

A sizeable fraction of recent research on monetary policy has been concerned with issues relating to analytical "indeterminacies"—i.e., multiple solutions in rational expectations (RE) models. Prominent topics for which this type of indeterminacy is central to the analysis include (i) the "Taylor principle" [Clarida, Gali, and Gertler (1999, 2000), King (2000), Woodford (2001)]; (ii) inflation forecast targeting [Woodford (1994, 2002), Bernanke and Woodford (1996), Kerr and King (1996), Carlstrom and Fuerst (2001), Bullard and Mitra (2002)]; (iii) the zero-lower-bound deflation trap [Benhabib, Schmitt-Grohe, and Uribe (2001, 2002), Alstadheim and Henderson (2002)], and (iv) the fiscal theory of the price level [Woodford (1994, 1995, 2002), Sims (1994), Cochrane (1998), Kocherlakota and Phelan (1999)]. The papers just mentioned, moreover, include only a sample of leading items, not an exhaustive listing. Most of the literature features sophisticated RE analysis conducted within dynamic models that reflect optimizing behavior by individual agents and incorporate Taylor-style policy rules.

A few papers have suggested that some of the particular indeterminacy arguments are misleading or irrelevant; these include Buiter (1999) and McCallum (1999b, 2001a, 2001b). For the most part, however, there has been little dissent from the position that these indeterminacies present a genuine problem for monetary policy makers. The purpose of the present paper, by contrast, is to argue that conclusions based on multiple-solution indeterminacy findings are of dubious merit rather generally. In each of the mentioned cases, that is, there is at most one RE solution that should be regarded as plausible, the others reflecting theoretical curiosities that are not of relevance for actual economies. As it happens, the plausible solution in most or all of the cases studied is the

minimum-state-variable (MSV) solution defined in McCallum (1983, 1999b), which is unique by construction in linear models. The principle basis of the argument developed here depends, however, not on any alleged "fundamental" or "bubble-free" nature of the MSV solution, but on the E-stability and adaptive learnability of this solution as defined and explored in important recent publications by Evans and Honkapohja (1999, 2001).

The outline of the paper is as follows. In Section 2, two preliminary issues of a partly terminological but also substantive nature are taken up, so as to avoid ambiguity or confusion later in the discussion. Next, Section 3 provides a brief summary of the E-stability/ least-squares learnability approach and includes a brief argument for its importance. Then in Sections 4-7 the four topics mentioned above are considered in turn, with each presented in the simplest possible setting. Finally, a short concluding section is provided.

2. Preliminaries

There are two partly terminological issues that should be confronted at the outset, so as to avoid ambiguities based on different implicit definitions. The first of these is the nature of the MSV solution. Throughout, I will be using that term to designate the solution yielded by the procedure of McCallum (1983, 1999b), which is designed to be unique by construction. This terminological usage agrees with that of Evans (1986, 1989) and Evans and Honkapohja (E&H) (1992) but differs from that employed in the latter's more recent publications (E&H, 1999, p. 496; 2001, p. 194). Either terminology could be used, of course, but the one adopted here is more appropriate and convenient for the issues at hand.

To clarify the distinction, consider first the univariate model

(1) $y_t = \alpha + aE_t y_{t+1} + cy_{t-1} + u_t$,

where $a \neq 0$ and $u_t = \rho u_{t-1} + \varepsilon_t$ with $|\rho| < 1$ and ε_t being white noise. Then the usual listing

of relevant state variables (i.e., determinants of y_t) would include just y_{t-1} and u_t (plus a

constant term), so the MSV solution will be of the form

(2) $y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 u_t$,

which includes no extraneous state variables and in which ϕ_0, ϕ_1, and ϕ_2 are restricted to

be real. This implies that $E_t y_{t+1} = \phi_0 + \phi_1(\phi_0 + \phi_1 y_{t-1} + \phi_2 u_t) + \phi_2 \rho u_t$ and substitution into

(1) requires that the ϕ_j coefficients must satisfy the following conditions:

(3a) $\phi_0 = \alpha + a\phi_0 + a\phi_1\phi_0$

(3b) $\phi_1 = a\,\phi_1^2 + c$

(3c) $\phi_2 = a\phi_1\phi_2 + a\rho\phi_2 + 1$.

Clearly, the second of these conditions yields two potential values for ϕ_1, namely,

$[1 \pm \sqrt{1-4ac}\,]/2a$. These represent two different functions of a and c, say $\phi_1^{(+)}$ and $\phi_1^{(-)}$,

which therefore define two different RE solutions. In what sense, then, is there a unique

MSV solution? By definition, it is the one that includes no extraneous state variable for

any relevant value of a and c. In other words, extraneous state variables are excluded for

all values of a and c in broad open sets that include a = 0 and c = 0. Thus the MSV

solution will be the one provided by use of the $\phi_1^{(-)}$ root, because it is the one that implies

$\phi_1 = 0$ in cases in which c = 0.[1] (Use of $\phi_1^{(+)}$ would give $\phi_1 = 1/a$ in these cases, even

though y_{t-1} does not appear in the model .) Then with ϕ_1 uniquely determined, the values

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[1] In the very unlikely case that $\phi_1^{(+)}$ and $\phi_1^{(-)}$ both equal 0 at c = 0, then the one with a continuous first
derivative would be designated as ϕ_1 for the MSV solution.

of ϕ_0 and ϕ_2 are given unambiguously by (3a) and (3c). Evans (1986, 1989) termed this the MSV solution—indeed, he coined the term—whereas E&H (1999, 2001) refer to both of the solutions of form (2) as MSV solutions (with non-MSV solutions also including terms such as y_{t-2} and u_{t-1}, as well as "sunspot" variables unrelated to the model).

The same type of procedure applies in multivariate linear models. Suppose that the model includes a m×1 vector y_t of endogenous variables, as in

(4) $y_t = A\ E_t y_{t+1} + C\ y_{t-1} + u_t,$ $A \neq 0$

where $u_t = Ru_{t-1} + \varepsilon_t$ includes exogenous variables and shocks, with R a stable m×m matrix and ε_t a white noise vector. (Constant terms are absent for expositional purposes.) Then the MSV solution will be of the form

(5) $y_t = \Omega\ y_{t-1} + \Gamma\ u_t.$

There are many Ω matrices that will satisfy the quadratic matrix equation analogous to (3b), which is $\Omega = A\Omega^2 + C$, but the MSV value is the one that equals 0 when $C = 0$. In most cases it will coincide with the one whose m eigenvalues are the smallest (in modulus). For additional discussion, see McCallum (1999b).

The second preliminary issue to be discussed involves the contention of McCallum (1986, 1999a, 2001b) that it is important to distinguish between two different types of indeterminacy, which may be referred to as <u>nominal indeterminacy</u> and <u>multiple solutions</u>. The term "indeterminacy" first became prominent in monetary economics from a series of writings by Patinkin (1949, 1965) about an alleged logical inconsistency in classical monetary theory. Some of Patinkin's conclusions were disputed by Gurley and Shaw (1960) and the resulting controversy was reviewed in an influential survey article by Johnson (1962). In all of this earlier literature, it must be noted, the

phenonmenon under discussion was "price level indeterminacy" such that the models in question fail to determine the value of any nominal variable, including the money supply. That type of failure occurs basically because of postulated policy behavior that is entirely devoid of any nominal anchor—i.e., there is no concern by the central bank for nominal variables.[2] Since rational private households and firms care only about real variables, according to standard neoclassical analysis, the absence of any "money illusion" by them and by the central bank must imply that no agent (in the model) has any concern for any nominal variable. Thus there is in effect no nominal variable appearing anywhere in the model, so naturally it cannot determine the value of such variables.

The type of indeterminacy under discussion in the current monetary policy literature, with which the present paper is concerned, is very different. Instead of a failure to determine any nominal variable (with no implied problematic behavior for real variables), the recent literature is concerned with a multiplicity of stable equilibria in terms of real variables, typically with an exogenous path specified for some nominal variable.[3] This type of aberrational behavior stems not from the absence of any nominal anchor (a static concept) but from the essentially dynamic fact that various paths of real variables can be consistent with rational expectations under certain conditions. In order to avoid possible semantic confusions, McCallum (1986) proposed that different terms be used for the two types of aberrational behavior—nominal indeterminacy and solution multiplicity, respectively.[4] This proposal has not met with widespread acceptance,

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[2] See Patinkin (1965, p. 309).
[3] It is dynamically stable equilibria that are most relevant, because explosive paths of real variables are often ruled out by transversality conditions that show them to be suboptimal for individual private agents.
[4] The adjective "nominal" was omitted from my original proposal, but seems clearly to be desirable.

although some writers are careful to refer to the second type as involving a "real indeterminacy."

Woodford (2002, Ch. 2, p. 50) has disputed the claim that it is important to distinguish between nominal indeterminacy and multiple solutions (or real indeterminacy). His argument is that a case of multiple solutions (from, e.g., an interest rate policy rule that fails to respect the Taylor principle) is not qualitatively different than the nominal indeterminacy that results (in a flexible price model) because the policy rule refers to no nominal variable at all, "even though in the latter special case it happens that the self-fulfilling expectations have no effect upon expected inflation, interest rates, or real balances" (2002, p. 50)—i.e., upon the model's real variables. I would suggest, however, that this "even though" proviso actually negates the argument being made. Second, Woodford goes on to say that "once we generalize our model to allow for staggered price setting" [i.e., sticky prices] "even a pure interest-rate peg ceases to result in 'nominal indeterminacy' …." But the particular form of sticky prices that Woodford considers is such that the model continues to include nominal variables even when monetary policy supplies no nominal anchor, because private behavior involves a type of dynamic money illusion.[5] If instead one incorporates sticky price adjustments of a type that respects the natural rate hypothesis, as in the P-bar model used by McCallum and Nelson (1999), then nominal indeterminacy will prevail if the monetary authority fails to provide a nominal anchor.[6] Quite generally, nominal indeterminacy occurs if and only if

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[5] Specifically, price adjustments are of the Calvo (1983) type, which does not conform to the natural rate hypothesis—see McCallum and Nelson (1999, pp. 26-27).
[6] The nominal indeterminacy will be basically of the type described in McCallum (1986, pp. 143-9), but with the words "price level" appearing instead of "money stock" in several places. Thus the model will not determine the absolute level of prices but will determine the expected inflation rate. Stochastic properties of the log price level will not be fully determined.

the model includes only real variables whereas multiple-solution RE indeterminacy can occur only in dynamic RE models in which expectations of future endogenous variables appear. This distinction would seem to be of considerable theoretical importance since one concept involves multiple paths of real variables while the other does not, since one is dynamic and the other static, and since they stem from fundamentally different sources.[7] I agree entirely, however, with Woodford's apparent view that nominal indeterminacy is unlikely to be of importance in actual economies, since only a small degree of money illusion—on the part of either private agents or the monetary authority—will rule it out.

3. E-stability and Least Squares Learnability

In this section the object is to provide a short review of the concepts known as E-stability and LS learnability. Evans (1985, 1986), building upon a result of DeCanio (1979), initially developed <u>iterative E-stability</u> as a selection criterion for RE models with multiple solutions. The basic presumption is that individual agents will not be endowed with exact knowledge of the economic system's structure, so it must be considered whether plausible correction mechanisms are convergent. Consider, for example, model (1). The usual "fundamentals" RE solution will be of the form (2), as stated above, but suppose that agents do not initially know the true values of the ϕ_j parameters. If at any date t the agents' prevailing belief is that their values are $\phi_0(n)$, $\phi_1(n)$, and $\phi_2(n)$—where n indexes iterations—so that the <u>perceived law of motion</u> (PLM) is

(6) $y_t = \phi_0(n) + \phi_1(n)y_{t-1} + \phi_2(n)u_t,$

then the implied unbiased expectation of y_{t+1} would be

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[7] Thus the presence of a policy-provided nominal anchor will rule out nominal indeterminacy but not solution multiplicity.

(7) $\phi_0(n) + \phi_1(n)y_t + \phi_2(n)\rho u_t$.

Using this last expression in place of $E_t y_{t+1}$ in (1)—which implies that we have

temporarily abandoned RE—gives

(8) $y_t = \alpha + a[\phi_0(n) + \phi_1(n)y_t + \phi_2(n)\rho u_t] + cy_{t-1} + u_t$.

Thus with rearrangement we have

(9) $y_t = [1-a\phi_1(n)]^{-1} [\alpha + a\phi_0(n) + a\phi_2(n)\rho u_t + cy_{t-1}] + u_t$

as the system's <u>actual law of motion</u> (ALM). Now imagine a sequence of iterations from

the PLM to the ALM. Writing the left-hand side of (9) in the form (6) for iteration n+1

implies that

(10a) $\phi_0(n+1) = [1 - a\phi_1(n)]^{-1}[\alpha + a\phi_0(n)]$

(10b) $\phi_1(n+1) = [1 - a\phi_1(n)]^{-1}c$

(10c) $\phi_2(n+1) = [1 - a\phi_1(n)]^{-1}[a\phi_2(n)\rho + 1]$.

The issue, then, is whether iterations defined by (10) are such that the $\phi_j(n)$ converge to

the ϕ_j values in an expression of form (2) as $n \to \infty$. If they do, then that solution is said

to be iteratively E-stable, and similar investigations can be made for any other RE

solutions. Evans (1986) found that in several prominent and controversial examples the

MSV solution is iteratively E-stable.

On the basis of results by Marcet and Sargent (1989), Evans (1989) and E&H

(1992) switched attention to E-stability without the "iterative" qualification, defined as

follows. Conversion of equations (10) to a continuous form, appropriate as the iteration

interval approaches zero,[8] results in

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[8] There is also a positive speed-of-adjustment coefficient in each of equations (11), but its magnitude is irrelevant for the convergence issue so is usually (as here) set equal to 1. See, e.g., Evans (1989, p. 299).

(11a) $d\phi_0(n)/dn = [1 - a\phi_1(n)]^{-1}[\alpha + a\,\phi_0(n)] - \phi_0(n)$

(11b) $d\phi_1(n)/dn = [1 - a\phi_1(n)]^{-1}c - \phi_1(n)$

(11c) $d\phi_2(n)/dn = [1 - a\phi_1(n)]^{-1}[a\,\phi_2(n)\rho + 1] - \phi_2(n)$.

If the differential equation system (11) is such that $\phi_j(n) \to \phi_j$ for all j, the solution (2) is

E-stable. The crucial feature of this continuous version of the iterative process is that it is

intimately related to an adaptive learning process that is modeled as taking place in real

time.[9] For most models of interest, that is, values of parameters analogous to the ϕ_j in (2)

that are estimated by LS regressions, based on data from periods t−1, t−2,…,1 and used

to form expectations in period t, will converge to the actual values in (2) as time passes if

equations (11) converge to those values and (2) is dynamically stable (non-explosive).

Also, such convergence will not occur if equations (11) do not converge. Thus E-

stability and LS learnability typically go hand in hand. This result, which is discussed

extensively by Evans and Honkapohja (1999, 2001), is useful because it is technically

much easier, in many cases, to establish E-stability than to establish LS learnability. The

latter concept is arguably the more important, in a fundamental sense, as learnability of

some type might be regarded as a necessary condition for the relevance of a RE

equilibrium.

Some analysts have expressed doubts concerning the relevance of the LS

learnability criterion; see, e.g., Buiter and Panigirtzoglu (2002). As a sufficient condition

the criterion is not very convincing—obviously, other learning procedures could be

considered—but as a necessary condition it seems highly attractive. In this regard, note

that the LS learning process assumes that (i) agents are collecting an ever-increasing

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[9] The E-stability process is itself conceived of as taking place in notional time (meta time).

number of observations on all relevant variables while (ii) the structure is remaining unchanged. Furthermore, (iii) the agents are estimating the relevant unknown parameters with an appropriate estimator in (iv) a properly specified model. Thus if a proposed RE solution is not learnable by the process in question, it would seem rather implausible that it could prevail in practice.

4. The Taylor Principle

Let us begin our set of indeterminacy examples by considering the Taylor principle in a simple version of today's near-canonical monetary policy model.[10] The latter consists of an optimizing demand relation, the Calvo price-adjustment scheme, and a monetary policy rule for the one-period interest rate:

$$(12) \quad y_t = E_t y_{t+1} + b_0 + b_1(R_t - E_t \Delta p_{t+1}) + v_t \qquad\qquad b_1 < 0$$

$$(13) \quad \Delta p_t = \beta E_t \Delta p_{t+1} + \alpha y_t \qquad\qquad \alpha > 0$$

$$(14) \quad R_t = r + \Delta p_t + \mu_1(\Delta p_t - \pi^*) + \mu_2 y_t.$$

Here y_t, Δp_t, and R_t represent the output gap, inflation, and the interest rate while v_t is a AR(1) disturbance term, with AR coefficient ρ, incorporating shocks to preferences and the natural-rate (i.e., flexible-price) level of output (treated for simplicity as exogenous).[11] Also, π^* is the central bank's inflation target, $r = -b_0/b_1$ is the average real interest rate, and β is the discount factor for private agents ($0 < \beta < 1$).

In this setting, the Taylor principle is a condition pertaining to policy parameters μ_1 and μ_2 that is said to be necessary for desirable behavior of the inflation rate. The

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[10] Some economists have objected to the term "Taylor principle" on the grounds that the basic idea was recognized earlier by other analysts. But the term has by now been adopted by so many writers that I feel its use is, given Taylor's (1999b) emphasis on the idea, appropriate.

[11] The model is often written with disturbance terms in (13) and/or (14), but they are unnecessary and irrelevant for present purposes. If there is variable government consumption, v_t will also include its expected change. Other fiscal variables are irrelevant under fairly broad, but not universal, conditions—see Section 7 below.

condition is often expressed for $\mu_2 = 0$, in which case it becomes $\mu_1 > 0$. Then the model can be written as (13) plus

(15) $y_t = E_t y_{t+1} + b_0 + b_1[r + (1+\mu_1)\Delta p_t - \mu_1\pi^* - E_t\Delta p_{t+1}] + v_t$

and the MSV solution is of the form

(16a) $\Delta p_t = \phi_{10} + \phi_{11}v_t$

(16b) $y_t = \phi_{20} + \phi_{21}v_t$.

Thus $E_t\Delta p_t = \phi_{10} + \phi_{11}\rho v_t$ with $E_t y_{t+1} = \phi_{20} + \phi_{21}\rho v_t$ and the basic undetermined coefficients procedure implies the following solution:

(17a) $\Delta p_t = \pi^* + \alpha[(1-\rho)(1-\rho\beta) - \alpha b_1(1+\mu_1-\rho)]^{-1}v_t$

(17b) $y_t = (1-\beta)\pi^*/\alpha + (1-\rho\beta)[(1-\rho)(1-\rho\beta) - \alpha b_1(1+\mu_1-\rho)]^{-1}v_t$.

Suppose, however, that one considers non-MSV solutions of the form

(18a) $\Delta p_t = \phi_{10} + \phi_{11}v_t + \phi_{12}\Delta p_{t-1} + \phi_{13}v_{t-1}$

(18b) $y_t = \phi_{20} + \phi_{21}v_t + \phi_{22}\Delta p_{t-1} + \phi_{23}v_{t-1}$.

Then going through the same steps as before one finds that relations (18) admit a multiplicity of RE solutions. One of these is the MSV solution given by (17) but there are others as well. Their dynamic properties clearly depend upon ϕ_{12}, whose value is given by the quadratic equation $\beta\phi_{12}^2 - \phi_{12}(1+\beta-\alpha b_1) + [1-\alpha b_1(1+\mu_1)] = 0$. Given our structural sign restrictions, one root of the latter is invariably greater than 1, but the other root will lie between 0 and 1, yielding a second stable solution, if $\mu_1 < 0$. Thus there is multiple-solution indeterminacy if the Taylor-principle condition does not hold.

Is this result—two stable RE solutions—something to be concerned about? My proposed answer is "not necessarily." Suppose that there were some convincing reason to believe that one of the solutions would prevail and that it is one that will not support

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"sunspot" terms. Then the existence of one or more additional solutions would not pose a practical problem for policy conducted according to the rule (14). Or, to approach the issue in a different fashion, let us ask: what undesirable real-world phenomenon is supposed to result as a consequence of the existence of two stable solutions? Clearly it is not explosive inflation, as some discussions of the failure of the Taylor principle would seem to suggest. Instead, it seems that it is the possibility of sunspot solutions and their associated variability that is of principal concern in situations of indeterminacy.[12] But, as stated above, we must consider whether such a solution is likely to prevail in the case under discussion.

In that regard, there is in my opinion a genuine problem that prevails when $\mu_1 < 0$. It is that neither of the RE solutions is learnable, in the least-squares sense described above. There is therefore no good reason to believe that either of the RE solutions would provide a plausible description of the behavior of inflation and the output gap, even if the simple model provided by (12) and (13) were adequate.

How does one carry out E-stability analysis for the two stable solutions found for the model (13)(15)? Given its simple structure, a starting point might be to write the model in first order form, i.e., as

(19) $x_t = AE_tx_{t+1} + u_t$,

where x_t is the column vector $[\Delta p_t \; y_t]'$ and u_t is the relevant vector of exogenous driving variables, assumed to be first-order autoregressive with stable parameter matrix R.[13] Then one could examine the eigenvalues of A and apply results developed by Evans (1986, 1989) and generalized by E&H (2001, Ch. 10). These indicate that the MSV

[12] Woodford (1986) has shown that under certain conditions there is an if-and-only-if relationship between local indeterminacy (with a continuum of solutions) and existence of sunspot equilibria.
[13] Constant terms are neglected in this discussion for simplicity.

solution is E-stable if and only if both eigenvalues of A have real parts less than 1 and, in addition, all four products of the eigenvalues of A and R have real parts less than 1. Implementation of this procedure is tedious, however, and furthermore does not provide results pertaining to all of the non-MSV solutions.[14] Fortunately, thorough analysis has been provided by the recent papers of Bullard and Mitra (2002) and Honkapohja and Mitra (2001). Together they indicate that, in the setting at hand, learnability of the MSV solution obtains if and only if $\mu_1 > 0$, i.e., if the Taylor principle is respected by the policy rule.[15] In addition Honkapohja and Mitra (2001, p. 19) establish that non-MSV equilibria are not learnable in the model at hand—their Example 1—when the Taylor principle is not satisfied. Honkapohja and Mitra suggest that this result provides an important argument—one that differs from the suggestion of Clarida, Gali, and Gertler (2000)—in support of the idea that it is crucial for interest rate policy rules to satisfy the Taylor principle.

The foregoing discussion constitutes the main substance of this section, but it may be of interest to illustrate the analysis for an even simpler version of the model. Accordingly, let us retain the demand relation (12) but replace the Calvo price-adjustment equation (13) with the assumption that prices adjust promptly, thereby keeping output continually at its flexible-price level (i.e., keeping $y_t = 0$).[16] Then the system (13) (15) reduces to $0 = b_1[(1+\mu_1)\Delta p_t - \mu_1\pi^* - E_t\Delta p_{t+1}] + v_t$. Clearly, the latter can be written as $\Delta p_t = \mu_1\pi^*(1+\mu_1)^{-1} + (1+\mu_1)^{-1}E_t\Delta p_{t+1} - b_1^{-1}v_t$, which is of form (19). Then the relevant eigenvalue condition for E-stability of the MSV solution is simply $(1+\mu_1)^{-1} < 1$, provided that v_t is white noise or has a positive AR(1) parameter. Thus this

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[14] There might exist "sunspot" solutions in addition to the two solutions of form (18) mentioned above.
[15] See Proposition 2 of Bullard and Mitra (2002).
[16] An equivalent system has been studied by Woodford (2002, Ch. 2).

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special case illustrates more generally the point of this section, that the Taylor principle is of importance because its non-satisfaction leads to a situation in which all RE solutions fail to be learnable.[17]

5. Inflation Forecast Targeting

Next we turn to a second type of multiple-solution indeterminacy, which can arise when the central bank's interest rate policy rule responds to an expected future inflation rate, rather than the actual current inflation rate. This case was introduced by Woodford (1994), and has since been discussed by many analysts including Bernanke and Woodford (1997), Kerr and King (1996), King (2000), Carlstrom and Fuerst (2001), and Bullard and Mitra (2002). Svensson (1997)—who suggests that the relevant form of policy behavior should not be termed inflation forecast targeting but instead "responding to inflation forecasts"—refers to the potential problem as "the Woodford warning."

To illustrate the problem, let us again consider the canonical system (12)-(14), but with $E_t\Delta p_{t+1}$ replacing Δp_t in the policy rule (14). Then with $\mu_2 = 0$ equation (15) becomes

(15') $y_t = E_t y_{t+1} + b_0 + b_1[r + \mu_1 E_t\Delta p_{t+1}] + v_t.$

Again there is a MSV solution of form (16) and also solutions of form (18) based on roots to a quadratic equation for ϕ_{12}. In this case the quadratic is $\beta\phi_{12}^2 - \phi_{12}(1+\beta+\alpha b_1\mu_1) + 1 = 0$. Thus the roots are $\phi_{12} = [\,d \pm \sqrt{d^2 - 4\beta}\,]/2\beta$, where $d = 1+\beta+\alpha b_1\mu_1$. These roots will be imaginary if $0 < \mu_1 < \mu_1^c = [2\beta^{0.5} + 1 + \beta]/(-b_1\alpha)$, in which case the MSV solution will be the only real solution. But if $\mu_1 < 0$ or $\mu_1 > \mu_1^c$, then there will be multiple stable

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solutions.[18]

To assess the E-stability and LS learnability of the MSV and non-MSV solutions we again draw on results of Bullard and Mitra (2002) and Honkapohja and Mitra (2001). Proposition 5 of the former indicates that the MSV solution is E-stable if and only if $\mu_1 > 0$, showing that the Taylor principle is again relevant. For the non-MSV solutions of form (18), moreover, Honkapohja and Mitra (2001, p. 20) find that they are not E-stable for any value of μ_1. Thus the LS learnability criterion lends some support to the suggestion that the MSV solution is the only plausible RE solution even in cases referred to by the Woodford warning.

This support is not complete, for Honkapohja and Mitra (2001, pp. 19-20) find that another form of non-explosive RE solution is E-stable and learnable. Specifically, they find that non-explosive <u>resonant frequency sunspot equilibria</u> can exist and be E-stable in the case under discussion. Indeed, they state that this result "strengthens the worries concerning the indeterminacy problems with forward looking interest rules pointed out" by Bernanke and Woodford (1997).

I would argue, however, that these resonant frequency sunspot equilibria should be viewed as mathematical curiosa, not as plausible paths relevant for economic analysis of actual economies. My argument is based on the nature of the resonant frequency sunspot process. It presumes that there is a finite-state Markov process for an exogenous sunspot variable, with fixed transition probabilities and with specified effects on the system's endogenous variables that result from the different sunspot states. Then for the

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[18] Carlstrom and Fuerst (2001) obtain some quite different results, which they attribute to an altered timing assumption regarding the money or asset balances that provide transaction-facilitating services: start of period versus end of period. Actually, however, the crucial change in their analysis is in the optimizing IS function comparable to (12). That no such change is necessitated by use of start-of-period money balances is shown in various writings, including McCallum (2001a, pp. 20-21).

resonant frequency sunspot solutions to exist, there must be an eigenvalue of the matrix

of state transition probabilities that is exactly equal to the inverse of an eigenvalue of a

matrix such as A in (19) for the model at hand.[19] But if there is no causal connection

between the sunspot disturbance process (i.e., the transition probabilities) and the model's

behavioral parameters, then the requisite condition will hold only on a parameter space of

measure zero. Thus the plausibility of such sunspot equilibria is at least one or two

orders of magnitude smaller than that for sunspots of the more familiar type that permits

their generating process to be any martingale difference process.[20]

In sum, then, I would argue that the Honkapohja and Mitra (2001) analysis

actually provides more support for the view that only MSV equilibria are learnable in the

model at hand—that is, the canonical model with inflation forecast targeting—than for

the view that non-MSV equilibria can be E-stable and learnable.

6. Zero-Lower-Bound Deflation Trap

The third topic to be investigated is prompted by recent papers by Benhabib,

Schmitt-Grohe, and Uribe (2001, 2002), Buiter and Panigirtzoglu (2002), and Alstadheim

and Henderson (2002), among others, which argue that recognition of the existence of a

zero lower bound (ZLB) on nominal interest rates leads to the conclusion that inflation

targeting rules—or ones of the more general Taylor type—are likely to fail. The alleged

reason is that the existence of a ZLB implies that RE solutions to standard optimizing

models with Taylor rules are not globally unique and that one solution, likely to be

attained, involves a deflationary liquidity trap.

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[19] See Honkapohja and Mitra (2001, Proposition 2).
[20] At the time of the conference, both Michael Woodford and George Evans announced that the necessary condition will be less stringent in a nonlinear version of the model. Even so, I would suggest that resonant-frequency sunspots are much less plausible than those of the more familiar type considered in Honkapohja and Mitra's Proposition 3.

For analysis of this topic, consider again the canonical model (12)-(14) but with the flexible price assumption $y_t = 0$ replacing, only for simplicity, the Calvo price adjustment equation (13). Also let $\mu_2 = 0$. Then the model reduces to

(20) $0 = b_0 + b_1[r + (1+\mu_1)\Delta p_t - \mu_1\pi^* - E_t\Delta p_{t+1}] + v_t$

so the MSV solution is of the form

(21) $\Delta p_t = \phi_0 + \phi_1 v_t,$

implying $E_t\Delta p_{t+1} = \phi_0 + \phi_1\rho v_t$. Then substitution into (20) and application of the undetermined coefficient procedure yields the requirement that

(22) $0 = b_0 + b_1[r - \mu_1\pi^* + (1+\mu_1)(\phi_0 + \phi_1 v_t) - (\phi_0 + \phi_1\rho v_t)] + v_t$

holds identically for all realizations of v_t. The latter implies unique values for ϕ_0 and ϕ_1 that, with $r = -b_0/b_1$, yield the MSV solution

(23) $\Delta p_t = \pi^* - [b_1(1 + \mu_1 - \rho)]^{-1}v_t.$

Therefore, since the unconditional expectation $E(v_t) = 0$, it is clear that $E\Delta p_t = \pi^*$, i.e., the long-run average rate of inflation given by the MSV solution is equal to the target value specified by the central bank's policy rule.

There is, however, another solution that satisfies the usual conditions for a RE equilibrium. Consider the solution form

(24) $\Delta p_t = \phi_0 + \phi_1 v_t + \phi_2\Delta p_{t-1} + \phi_3 v_{t-1},$

which implies $E_t\Delta p_{t+1} = \phi_0 + \phi_1\rho v_t + \phi_2(\phi_0 + \phi_1 v_t + \phi_2\Delta p_{t-1} + \phi_3 v_{t-1}) + \phi_3 v_t$. Then the undetermined coefficient conditions are

(25a) $b_1[-\mu_1\pi^* + (1+\mu_1)\phi_0 - \phi_0(1+\phi_2)] = 0$

(25b) $b_1[(1+\mu_1)\phi_1 - \phi_1\rho - \phi_2\phi_1 - \phi_2\phi_3] + 1 = 0$

(25c) $\phi_2^2 = \phi_2(1 + \mu_1)$

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(25d) $b_1[(1+\mu_1)\phi_3 - \phi_2\phi_3] = 0.$

Thus there are two possibilities for ϕ_2, namely, 0 and $1+\mu_1$. If the former is selected we

have the MSV solution as given in (23), but if $\phi_2 = 1+\mu_1$ is designated as relevant, the

solution becomes

(26) $\Delta p_t = -\mu_1\pi^* + (1+\mu_1)\Delta p_{t-1} + \phi_1 v_t + [(1-b_1\rho\phi_1)/(1+\mu_1)]v_{t-1}$

for any ϕ_1. Clearly, with $\mu_1 > 0$ the latter is explosive. Consequently, if the system

"begins" with $\Delta p_{t-1} > \pi^*$ then inflation will increase explosively, and if the startup value

is below π^* then Δp_t will have a <u>tendency</u> to approach $-\infty$, according to (26) and as

illustrated in Figure 1 (which abstracts from the stochastic element provided by v_t) .

But the last statement ignores the existence of a ZLB on the nominal interest rate.

In the flexible price system at hand, the latter translates into a lower bound on $E_t\Delta p_{t+1}$; we

have the restriction $E_t\Delta p_{t+1} \geq -r$. Thus if the system begins with $\Delta p_{t-1} < \pi^*$, inflation

cannot behave as specified by (26). Instead, the alleged outcome is that $\Delta p_t \rightarrow -r$, which

corresponds to $R_t \rightarrow 0$. Crucially, there is no violation of a transversality condition, as

there would be if Δp_t were to approach $-\infty$. So in this case the policy rule (14) fails to

stabilize inflation around its target value, π^*. This is the failure of the Taylor rule

proposed and emphasized in the papers mentioned above.

Again, however, the agenda here is to consider the E-stability of the two solutions

(23) and (26). For both, the analysis for this form of model is provided by Evans (1986,

1989), who shows that the MSV solution (23) is E-stable and learnable, whereas a

solution such as (26) is not E-stable or learnable.[21]

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[21] If one were to believe that the non-MSV solution (26) is relevant, then he would need to consider what
happens if the system begins with $\Delta p_{t-1} > \pi^*$, since there is no transversality condition to rule out $\Delta p_t \rightarrow \infty$.

The foregoing statement applies literally to the model without the ZLB constraint. But the latter does not affect the analysis, which is local in nature, of the MSV solution. Then for the non-MSV solution, we need to replace (20) with the ZLB constraint. This can be done by rewriting (20) so as to pass through the point $(-r, -r)$ and inserting a parameter that controls its slope. Then the ZLB constraint would be imposed by letting the slope approach zero. Thus the analysis would be as before, but with a slope of less than 1.0 at the non-MSV solution, which would imply E-instability.

A more satisfying approach might be to recognize that the lower bound on the nominal interest rate is actually the consequence of a decreasing net marginal benefit, via facilitation of transactions, provided by holdings of money.[22] Then the relevant functional form would be as illustrated in Figure 2. There the MSV solution is at point A and the liquidity trap at point B. For this continuous nonlinear case, the analysis in Chapter 11 of E&H (2001) indicates that the MSV solution is E-stable and the trap solution is not, at least with a small variance for v_t. Accordingly, while a ZLB situation may arise if the target inflation rate is set too low, the indeterminacy-based mechanism described above does not seem plausible.

7. Fiscal Theory of the Price Level

The fourth and final topic to be considered is the fiscal theory of the price level, for which the most important references are Woodford (1994, 1995, 2001, 2002), Sims (1994), Cochrane (1998), and Kocherlakota and Phelan (1999)—with Buiter (1999) and McCallum (1999a, 2001a) as the main critics. The relationship to solution multiplicity is different in this case, as will be seen in the development below. Before undertaking that

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[22] See, e.g., McCallum (2001b).

development, however, it should be emphasized just how drastically unorthodox and counter-traditional the fiscal theory of price level determination is. Specifically, it does not suggest merely that fiscal as well as monetary policy stances are significant for price level behavior; instead it features a leading example in which the price level moves over time in a manner that mimics the path of government bonds and is entirely unlike the path of the money stock. Accordingly, it is not the case that the argument involves fiscal behavior that drives an accommodative monetary authority, as when rapid base money growth is adopted to finance a fiscal deficit. Indeed, it is this drastic aspect of the fiscal theory that has made it a subject of great interest.[23]

What is the policy issue that is posed by the fiscal theory of the price level? The two main suggestions are apparently (i) that the behavior of the price level and other macro variables may be very different than predicted by orthodox monetary analysis and/or (ii) that coordination between monetary and fiscal policy authorities is necessary for satisfactory macroeconomic performance. To consider these suggestions, let us begin with an extremely simple formulation. Specifically, suppose that the (per capita) money demand function for a closed economy is of the textbook form

(27) $m_t - p_t = c_0 + c_1 y_t + c_2 R_t + v_t$ $c_1 > 0, c_2 < 0,$

where m_t, p_t, and y_t are logs of the (base) money stock, price level, and output (income) for period t, while R_t denotes a one-period nominal interest rate. The disturbance v_t is taken for simplicity to be white noise. It is well known that there are rigorous dynamic

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[23] In this regard, an important point is that the type of model typically utilized in the literature's analysis is not of the overlapping generations type, in which the Ricardian equivalence proposition is known to fail— implying that tax changes will affect price level behavior. Instead, the model is basically of the Sidrauski-Brock type, in which Ricardian equivalence results are normally obtained, i.e., results implying that bond-financed tax changes have no effect on the price level or other macroeconomic variables of primary interest. In such a setting, fiscalist positions are truly striking.

general equilibrium models with optimizing agents that will justify (27) as a linear approximation to a combination of implied Euler equations (first-order optimality conditions). Furthermore, assume that the economy is one in which output and the real rate of interest are constant over time so that (27) reduces to

(28) $m_t - p_t = \gamma + \alpha(E_t p_{t+1} - p_t) + v_t$ $\alpha = c_2 < 0,$

which is the familiar Cagan specification for money demand. Also suppose that the quantity of (base) money is kept constant by the central bank, so that

(29) $m_t = m.$

Then (28) and (29) plus rational expectations govern the behavior of p_t for time periods t = 1, 2, …. It is possible that the structure was different prior to period 1.

In this setting, the MSV solution for p_t is of the form

(30) $p_t = \phi_0 + \phi_1 v_t,$

so $E_t p_{t+1} = \phi_0$ and the usual analysis yields

(31) $p_t = m - \gamma - v_t/(1-\alpha).$

Thus, p_t fluctuates randomly around a constant value and if money demand shocks were absent we would have $p_t = m - \gamma$.

But while (31) gives the traditional, "monetarist," bubble-free solutions for this model, there are other expressions as well that satisfy the model with RE. To see this, conjecture a solution of the form

(32) $p_t = \psi_0 + \psi_1 p_{t-1} + \psi_2 v_t + \psi_3 v_{t-1},$

instead of $p_t = \phi_0 + \phi_1 v_t$. Then working through the same type of analysis as before, one finds that the relevant UC conditions are

(33) $0 = \alpha\,\psi_1{}^2 + (1-\alpha)\,\psi_1$

$0 = \alpha\,\psi_1\,\psi_2 + \alpha\,\psi_3 + (1-\alpha)\,\psi_2 + 1$

$0 = \alpha\,\psi_1\,\psi_3 + (1-\alpha)\,\psi_3$

$m = \gamma + \alpha\,\psi_0 + \alpha\,\psi_1\,\psi_0 + (1-\alpha)\,\psi_0.$

By inspection we see that the first of these has two roots $\psi_1{}^{(1)} = 0$ and $\psi_1{}^{(2)} = (\alpha-1)/\alpha$. If

the former is the relevant root, then the same solution as in (31) is obtained. But if $\psi_1{}^{(2)}$

is relevant, then $\psi_3 = -1/\alpha$ and $\psi_0 = (m - \gamma)/\alpha$ while any value of ψ_2 is possible. So an

infinity of solution paths is in this case consistent with the model. Note, moreover, that

$\psi_1{}^{(2)} = (\alpha-1)/\alpha > 1.0$, so most of these solution paths are explosive. One such path is

illustrated in Figure 3, where the random component is suppressed.

There are, however, additional variables and conditions in a fully specified model

of the economy under consideration. In particular, let B_{t+1} denote the (per capita)

quantity of one-period government bonds purchased in t, with each bond purchased at the

price $1/(1+R_t)$ and redeemed in t+1 for one unit of money. Then a full-fledged

optimizing analysis would require that

(34) $\lim_{j\to\infty} E_t\,\beta^j\,(M_{t+j} + B_{t+j})/P_{t+j} = 0,$

i.e., that a transversality condition pertaining to real financial wealth must be satisfied.

Here β is a typical agent's discount factor, $\beta = 1/(1+\rho)$, with $\rho > 0$ so that $0 < \beta < 1$.

(Note that ρ has a different meaning here than in previous sections.)

We are now prepared to describe the fiscalist theory in this setting. With

government bonds recognized, we can write the consolidated government budget

constraint (GBC) in per capita terms as

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(35) $P_t (g_t - tx_t) = M_{t+1} - M_t + (1 + R_t)^{-1} B_{t+1} - B_t,$

where g_t and tx_t are real government purchases and (lump sum) tax collections, respectively. In real terms, this constraint could then be expressed as

(36) $g_t - tx_t = (M_{t+1} - M_t)/P_t + (1 + R_t)^{-1} (P_{t+1}/P_t) b_{t+1} - b_t,$ $t = 1, 2, ...,$

where $b_t = B_t/P_t$. Note the mixed notation being utilized: $b_t = B_t/P_t$ whereas $m_t = \log M_t$ and $p_t = \log P_t$.

Now consider (36) when M_t and thus m_t are constant. Also let the random shock v_t be absent so that P_{t+1} is correctly anticipated in t and suppose that fiscal policy aims for a constant surplus $tx_t - g_t = s > 0$ with $g_t = g$. Then with the real rate of interest on bonds r_t defined by $1 + r_t = (1 + R_t)/(1 + \pi_{t+1})$, where $\pi_{t+1} = (P_{t+1} - P_t)/P_t$, and with $r_t = \rho$, as would be implied by optimizing behavior in the absence of shocks, the government budget constraint becomes

(37) $b_{t+1} = (1 + \rho) b_t + (1 + \rho) (g_t - tx_t)$ $t = 1,2,....$

But since $1 + \rho > 1$, if $g_t - tx_t$ is constant the last equation reveals a strong tendency for b_t to explode as time passes. As t grows without limit, b_t approaches growth at the rate ρ, i.e., behaves like $(1+\rho)^t$. Thus the transversality condition (34) tends to be violated since growth of b_t just offsets the shrinkage of $\beta^t = 1/(1 + \rho)^t$, yielding a limit that is positive.

In fact, in this case there are two paths for b_t that, with $g_t - tx$ constant, will satisfy (37) and also (32)(33)(34) for $t = 1,2,....$ One of these obtains if the value b_1 equals $-(1 + \rho)(g - tx)/\rho$, for then (37) implies that

(38) $b_2 = (1 + \rho) [-(1 + \rho) (g - tx)/\rho] + (1 + \rho) (g - tx)$

    $= (1 + \rho) (g - tx) [-(1+\rho)/\rho + 1] = -(1 + \rho) (g - tx) /\rho$

and that same value prevails in all succeeding periods. Here $b_1 = B_1/P_1$, and B_1 is the

number of nominal government bonds outstanding at the beginning of the initial period,

$t = 1$. Thus if the price level in this first period, P_1, adjusts to equal the value $P_1 =$

$B_1\rho/(1 + \rho)$ (tx – g), then condition (34) as well as (37) will be satisfied. Indeed, this is

what the fiscalist theory predicts: that P_1 adjusts relative to B_1 and tx – g > 0 so as to

satisfy the individual agents' optimality condition (34).

What about the necessary condition for money demand? In this regard, the

fiscalist answer is that although the path just described will not conform to the $p_t = m - \gamma$

solution implied by (31), it can and will satisfy the alternative solution $p_t =$

$[(\alpha - 1)/\alpha]p_{t-1} + (m - \gamma)/\alpha$ for all $t = 2,3,\ldots$. The price level P_1, and thus p_1, is determined

by B_1 and the value of b_1 necessary to satisfy (34), with subsequent P_t and p_t values being

given by (32) with $\psi_1 = (\alpha - 1)/\alpha$. The price level explodes as time passes, despite the

constant value of M_t, but all of the model's equilibrium conditions are satisfied

nevertheless. Since P_t and B_t are growing at the same (explosive) rate, while M_t is

constant, the outcome is rightfully regarded as highly "fiscalist."[24]

But let us consider a second path of b_t that will, with g – tx constant, satisfy the

TC (34) as well as (32), (33), and (37). It is one in which $b_{t+1} = 0$ for all $t = 1, 2 ,\ldots$.

Then, clearly, (34) will be satisfied with $B_{t+1} = 0$ and in that case places no constraint on

P_t values. Thus these are free to obey $p_t = m - \gamma$, as in the special case of (32)-(33) given

by (31). Therefore this solution is the orthodox or monetarist solution.

It remains to be considered how the GBC (37) can be satisfied with this solution,

i.e., with $B_{t+1} = 0$ for $t = 1, 2, \ldots$ and $tx_t - g > 0$. The explanation is as follows. In a

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[24] There is a serious problem, however, with this solution if B_1 is such that the implied value of P_1 is
smaller than $P^* = Me^{-\gamma}$. In this case the fiscalist equilibrium does not exist because P_t approaches 0,
leading to violation of the transversality condition (34). Also, if tx – g < 0, then a negative price level
would be required for satisfaction of (37) by the assumed value of b_1.

market economy, it is not appropriate to specify fiscal policy as controlling both g_t and tx_t (with an M_t path given) because with such a policy (37) could imply that the number of bonds sold to the private sector is greater than the number demanded. Thus the analysis needs to distinguish between bond supply B^S_{t+1} and bond demand B^D_{t+1}, and policy is appropriately specified in terms of M_t, g_t, and B^S_{t+1} with one relevant equilibrium condition being $B^D_{t+1} \leq B^S_{t+1}$. In the case under consideration, the planned value of $tx_t - g > 0$ reflects B^S_{t+1}, whereas the realized values involve $B_{t+1} = B^D_{t+1} = 0$ and $tx_t - g = 0$. The $tx - g$ values realized are smaller than planned because real revenues from bond sales are larger—zero, rather than the planned negative value, which is $-\rho b_1/(1 + \rho)$. It is not surprising that some such adjustment is needed since the experiment at hand involves monetary (M_t) and fiscal (g_t and B^S_{t+1}) policies that are set independently and exogenously. The monetarist and fiscalist solutions reflect two different ways by which these potentially conflicting policies can be reconciled.

In sum, we end up with two RE solutions that represent two competing hypotheses regarding price level behavior in an economy such as the one under study. The crucial issue, then, is which of the two solutions provides the better guide to reality, i.e., to price level behavior in actual economies? In previous writings (McCallum 1999a, 2001a) I have argued that the traditional equilibrium is the "fundamentals" or "bubble-free" solution provided by the MSV solution concept, whereas the fiscalist solution represents a bubble solution. I have suggested that this is a plausible reason—in addition to existing empirical evidence—for preferring the former, but for many analysts that

argument may not be persuasive.[25] Accordingly, we now consider the E-stability and

learnability properties of the two solutions.

As it happens, that comparison is easy for the case at hand. Write the model as

(39) $p_t = [\alpha/(\alpha-1)] E_t p_{t+1} + (m-\gamma)/(1-\alpha) + [1/(\alpha-1)]v_t$

and note that with $\alpha < 0$, the coefficient on $E_t p_{t+1}$ lies between 0 and 1. Thus the

eigenvalue for A in equation (19) is smaller than 1, and the results described at the end of

Section 4 pertain: the MSV (monetarist) solution is E-stable and learnable, while the non-

MSV (fiscalist) solution is not. Note that although (39) does not explicitly refer to the

fiscal variable B_t, it does not exclude the possibility that the fiscal solution can be

learned, since ψ_1 can adjust toward $(\alpha-1)/\alpha$ and ψ_0 can adjust to the value,

$\log[B_1\rho/(1+\rho)(tx-g)] - [(\alpha-1)/\alpha]p_0$, that makes p_1 equal the fiscalist value described

above.[26]

The foregoing result is for a highly special case; can it be generalized in any way?

With respect to functional form of the money demand equation, the answer is yes.

Ignoring stochastic terms, the linear model that we have used to this point can be

represented graphically as in Figure 3. There the traditional MSV solution is that $p_t = p^*$,

at the intersection point, for each t = 1, 2, …. The fiscalist solution, by contrast, implies

p_t values given by paths such as that of the thin line in Figure 3.   Most of the literature

has, of course, utilized explicit optimizing models that imply an analogous diagram as

shown in Figure 4, where there is a nonlinear P_t to P_{t+1} mapping that has an positive but

increasing slope. Do the results above carry over to such models? Although there are

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[25] For example, Woodford (2001, p. 701) argues that "what constitutes a 'bubble equilibrium' is often in the eye of the beholder…."
[26] This argument can be extended to the case with stochastic disturbances.

some qualifications, the answer is basically "yes." The main point is that E-stability is a local concept, so that conclusions pertaining to the MSV solution in Figure 3 apply to models of the type in Figure 4. Thus the MSV solution is E-stable. Indeed, very recent analysis by E&H (2002b) obtains the same result in an explicitly nonlinear model, and also obtains results as above for the non-MSV solutions.

The foregoing discussion represents our basic application of the paper's argument to the fiscal theory of the price level; it improves upon the argument of McCallum (2001a) significantly by adoption of the learnability criterion. It is clear, however, that the policy specification considered is quite special, so there remains a need to extend the learnability analysis to a broader class of policy regimes. The most well-known specification of such regimes is that of Leeper (1991), in which the monetary authority adjusts a one-period nominal interest rate instrument according to a rule of the form

(40) $\quad R_t = \mu_0 + (1 + \mu_1)\Delta p_t + \theta_t$

while simultaneously the fiscal authority holds $g_t = 0$ and implements a (lump-sum) tax rule of the form

(41) $\quad tx_t = \tau_0 + \tau_1 b_t + \zeta_t.$

Here θ_t and ζ_t are white noise policy shocks. Leeper (1991) classifies monetary policy as "active" if $|(1 + \mu_1)\beta| > 1$ and as "passive" otherwise, and classifies fiscal policy as active if $|\beta^{-1} - \tau_1| > 1$ and passive otherwise.

Very recently, E&H (2002b) have conducted analysis based on policy rules (40) and (41), sensibly focusing on cases in which $1 + \mu_1 > 0$ and $\tau_1 > 0$. Using a linearized version of their extended model, E&H (2002b) find that there are solutions of two types that correspond in several ways to the monetarist and fiscalist solutions discussed above.

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For example, in the monetarist solution the inflation rate depends only upon a constant and θ_t, the current monetary policy shock, while the fiscalist solution has the inflation rate also depending on the previous period's real bond stock. For the most part, the monetarist solution is E-stable and LS learnable when monetary policy is active and fiscal policy passive, whereas the fiscalist solution is E-stable and LS learnable when monetary policy is passive and fiscal policy active. When non-explosive, the fiscalist solution does not imply outcomes inconsistent with beliefs of monetarist economists [e.g., Brunner and Meltzer (1972)] since there is no major discrepancy between the paths of the price level and the money stock.[27] There is also a small region of the policy parameter space that leads to E-stability with explosive solutions, but it seems questionable whether one of these should be regarded as an equilibrium since explosive behavior of the real bond stock would seem to imply failure of a transversality condition that would be necessary for private optimality.

From a practical perspective, emphasis should, I believe, be given to the region in which $(1 + \mu_1)\beta > 1$ and $\rho < \tau_1 < 1 + \rho$, for the following reason. The former condition represents the Taylor principle in the model at hand, while the latter calls for taxes to be levied at a rate that would be large enough to make b_t non-explosive, in the absence of any government revenue from money creation, and yet not so large as to imply that taxes in a single period would reduce government debt by more than the amount outstanding. That is, a positive fraction of outstanding bonds would be retired in each period.[28] Then

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[27] Whether this is a MSV solution remains to be determined.

[28] It seems peculiar that Leeper (1991) would designate such behavior as representing "passive" fiscal policy, since it represents feedback responses designed to stabilize the stock of bonds outstanding. More generally, Leeper's policy rule specification seems somewhat inappropriately designed for analysis of the issue emphasized by Woodford (1995), namely, whether price level determination is basically monetary or fiscal in nature. To represent the traditional monetary point of view, it would seem preferable to specify

the E&H results indicate that, over this entire region, the monetarist solution is E-stable (and the fiscalist solution is not). This result seems especially important in that it indicates that well behaved (and orthodox) outcomes, as given by the MSV solution, will result when monetary and fiscal policy rules are each sensible on their own terms, with no overt coordination or dependence by either policy authority on the behavior of the other.[29]

8. Concluding Remarks

The foregoing pages have discussed four current topics in monetary policy analysis, each of which hinges in some way on the possibility of multiple solutions that satisfy the usual definition of rational expectations. In two of these cases,[30] analysis of the adaptive learnability of the multiple solutions suggests that only one of them is a viable candidate for a RE equilibrium when account is taken of the need for corrections to occur in response to small departures away from that equilibrium. Thus it is suggested that the dangers alleged to prevail, in these cases, are not ones with which actual policymakers need to be concerned. In the case of the Taylor principle, by contrast, it has been argued that the consequences of policy behavior that violates the principle are genuinely undesirable, since all of the RE equilibria fail to be learnable. Finally, our analysis of the simplest version of the fiscal theory of the price level suggests that the traditional solution is learnable and the fiscalist solution is not. With more general specifications of the policy rules, the outcome is not so clear-cut. It appears doubtful,

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monetary policy in terms of a rule governing responses of a narrow monetary aggregate (e.g., base money) to inflation or price-level departures from target values. This contention does not involve any denial, of course, that most actual central banks employ an interest rate instrument.

[29]The result does not imply, evidently, that there are no parameter regions in which the fiscalist solution (as classified by E&H (2002b)) is learnable.

[30]The forecast targeting and ZLB trap cases.

nevertheless, that any equilibrium, in which price level and money stock behavior are very different, is learnable.

More generally, these examples suggest that learnability, not indeterminacy, should be viewed as the relevant issue for policy-oriented theoretical analysis of monetary policy. More contentiously, it might be argued that RE solution multiplicity should be viewed basically as a mathematical curiosity, stemming from an insufficiently specific definition of rational expectations, rather than as a substantive problem for actual policy makers. Whatever the conclusion drawn, a unified treatment would of course provide a more attractive argument than the foregoing catalog of examples. I hope to be able to produce one in the future.

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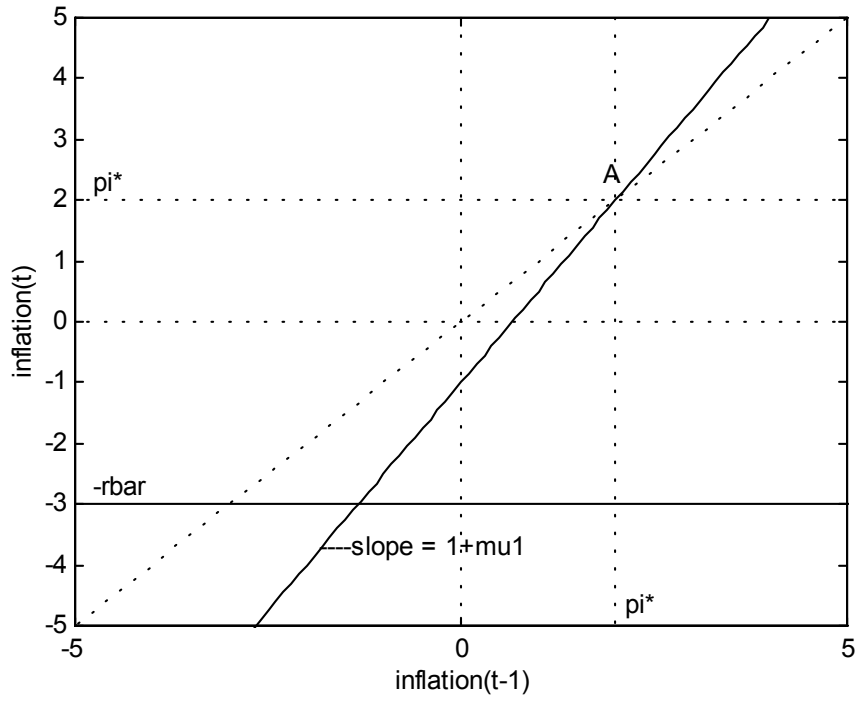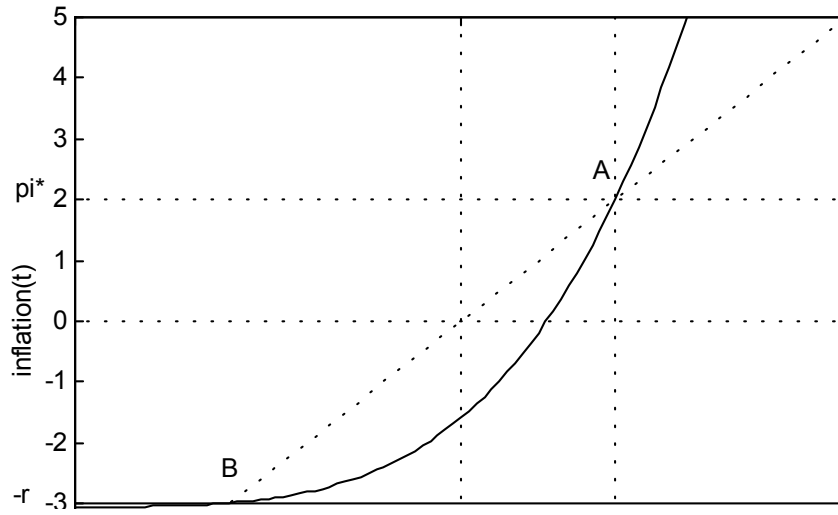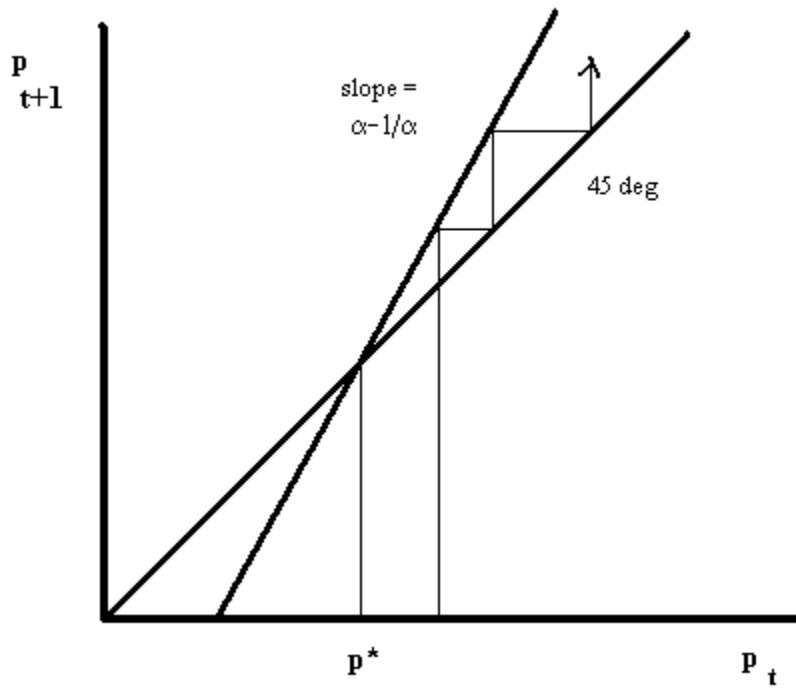
Figure 1

Figure 2

Figure 3

Figure 4