Measuring Credit Spread Risk

Incorporating the tails.

Rachel Campbell and Ronald Huisman

Credit risk management has become an increasingly important area of financial risk management, as evidenced by the enormous surge in credit derivatives. The recent global financial crisis, the need for credit protection, and the potential to enhance loan-based credit portfolio yields and the returns on bank capital all have spurred demand for credit derivatives.

A survey by the British Bankers' Association estimated the global credit derivatives market in 1999 to be $586 billion. By 2000, the market had grown to around $893 billion. As of year-end 2001, the market was estimated to have mushroomed to an incredible $1.2 trillion. Forecasts for 2002 estimated a market of over $1.5 trillion.

Accurate assessment of credit risk depends on methods to accurately measure and control potential or expected losses resulting from default. This includes estimation of the credit exposure, the probability of default, and the fraction of market value recoverable at default. Credit spreads, the difference between the risky bond and a risk-free alternative, should therefore reflect the amount of credit risk faced.

Credit spreads change over time for reasons such as varying market conditions, changes in the credit ratings of issuers, or changes in expectations regarding the recovery rate. Traditional quantitative credit risk models assume that expected changes in spreads are normally distributed, but empirical evidence shows that they are more likely to be skewed and fat-tailed. This makes the expected loss distribution for credit portfolios highly skewed and severely fat-tailed.

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Subrahmanyam, Eom, and Uno [1998] show this for Japanese yen swap spreads, and Phoa [1999] provides evidence using Australian dollar swap spread data. Both studies argue that incorporating the apparent fat tails is crucial in order to correctly measure credit risk.

Phoa applies extreme value theory (EVT) to parameterize fat-tailed Fréchet, Weibull, and Gumbel distributions to measure the maximum expected daily widening in swap spreads on the Australian dollar, but the method he uses to assess the amount of tail fatness (the tail index) is known to be biased. He deals with this fact by showing results for two different tail index estimates.

Recent developments in EVT have led to the development of an unbiased tail index estimator that has proven to work successfully in measuring the tail index, and therefore is also able to capture the additional downside risk in value at risk estimates for stocks and exchange rates.[1] We apply this technique to model the tails of the distribution of expected changes in swap spread.

Using data on U.S., U.K., German, and Japanese ten-year swap and government bond rates, we provide evidence of apparent tail fatness in the empirical distributions of the changes. We also show that the approach outperforms the normal distribution in measuring the risk presented by large widenings or tightenings of credit spreads.

CREDIT SPREADS

The expected credit loss (ECL) is measured by the drop in value due to the possibility of default, λ, over a time interval t, and can be expressed simply as the probability of default multiplied by the proportion of the position not recovered:

$$ECL = (1 - f)(\lambda \Delta t)P_t \tag{1}$$

where f is a fraction representing the recovery rate, and P is the price of a risk-free bond at time t.

The credit spread for a given maturity may be written in terms of yields, y, as in Equation (2), where λ is again the probability of default over the same period as the maturity of the risky bond, P^*, and the risk-free bonds:

$$y^* - y = (1 - f)\lambda \tag{2}$$

The credit spread therefore represents the probability of default multiplied by the proportion not recovered. Indeed, using Equation (2), the term structure of default probabilities can be inferred from the term structures of risky and risk-free bonds, in a similar manner to Jarrow and Turnbull [1995].[2]

The term structure of credit spreads (and shocks to credit spreads) is indeed non-trivial. From the credit spread, we can determine much of the risk involved in credit risk. Indeed, it is this factor that is the crucial element in credit risk management. For example, for the next period's estimate of the expected credit loss we can substitute the credit spread for the market's expectation of default and recovery.

Multiplying by the credit exposure (average price is at the 50% confidence level), we get an estimate for the expected credit loss similar to that given in Equation (1), but now in terms of the credit spread:

$$ECL = (y^* - y)P_t^{0.50} \tag{3}$$

If an estimate of the unexpected credit loss (UCL) is required, we multiply the price of the risk-free asset by the worst credit exposure at a chosen confidence interval, c. For risky debt, the credit exposure is the principal, so P_t^c simplifies to the asset's value at risk for a given confidence level. For products like derivatives, it is only when the derivative contract is in the money that potential credit risk arises, so we also need to multiply by the probability of being in the money at time t, denoted by probability m:

$$UCL = m(y^* - y)P_t^c \tag{4}$$

This approach to estimating unexpected credit loss does not take into account the risk associated with changes in the extent of the credit spread, credit spread risk, or changes in the probability of default and the recovery rate. Unless this is incorporated into the worst case credit at risk (CaR) estimate, it is vital that scenario analysis be used to track the sensitivity of the CaR measure to either credit spread risk or changes in default and recovery rates. Changes in the credit spread (credit spread risk) are therefore the risk involved with changes in the extent of the credit spread. This can have implications for worst case scenario analysis of credit risk for fixed-income products, as well as for pricing credit derivative products when the credit spread is a determining factor for the value of the derivative.

EXHIBIT 1

Summary Statistics—January 1990–December 1999

Credit Spread	U.S.	U.K.	Germany	Japan
Observations	2610	2610	2610	2610
Average Daily Shift	-5.747E-06	-2.797E-04	1.226E-04	-1.782E-04
Standard Deviation	0.076	0.088	0.062	0.069
Skewness	-0.052	0.015	-0.042	-0.344
Kurtosis	7.224	9.352	7.353	28.723

HISTORICAL CREDIT SPREAD TIGHTENINGS AND WIDENINGS

We provide empirical evidence of the probability distribution of credit spread changes, so that one can more accurately determine worst case scenario analysis for credit risk management, and the pricing and hedging of derivatives products on credit spreads.

To estimate the distribution of shifts in credit spreads for a variety of countries, we employ daily data for the U.S., the U.K., Germany, and Japan from Datastream over the period January 1990 through December 1999. The credit spread prices the additional risk over a base asset such as the Treasury bill rate. We therefore use ten-year government bond yields for each country as the base asset.

The swap rate is commonly taken as a proxy for the AA credit rate, since the swap market is significantly deeper and more liquid than the corporate bond market.[3] The two other factors that also tend to affect the movement of swap spreads are interest rates and liquidity, but the literature generally takes the swap rate for analysis of credit spread risk. We also use the ten-year Datastream swap rate, which is a value-weighted index of the middle yield on U.S. swaps. The swap spread (credit spread) is the swap rate less the yield on the current ten-year government bond.

As a word of caution, it may not always be appropriate to use the Treasury yield as the risk-free rate as Treasuries are more liquid and repo at lower rates. It might therefore be more appropriate to use a swap rate as the risk-free rate. A further limitation is that in using a constant rating series we are not able to reflect spread shifts that result from rating migrations.

The summary statistics for the daily shifts in credit spreads are given in Exhibit 1. We can see that the average daily shift is extremely small with standard deviations ranging from 6.2% for Germany, to 8.8% for the U.K. The distribution of credit spread shifts in Japan is highly skewed, and all country credit markets exhibit significant excess kurtosis.

Deviations from normality will result in a higher than stipulated probability of large movements in credit spreads under the assumption of normally distributed returns. The assumption of Gaussian innovations generates a lower probability of extreme movements, so the assumption of normality is likely to underestimate the credit spread risk of either large tightenings or widenings in credit spreads. The degree of misspecification is of course vital for accurate estimation in risk management for both credit risk and worst case scenario analysis.

The histogram of shifts in swap spreads is given for the U.S. in Exhibit 2 against the probabilities assuming normality. We do indeed observe a greater-than-normal probability of extreme movements in credit spreads, exemplifying the small but looming potential for increases in default risk to have severe implications on the size of credit spread risk.

The prevalence of skewed distributions could also result in an alternative probability for large downward rather than upward shifts in the swap spread, so we look at both tails of the distribution of shifts in swap spreads. A simple approach to modeling the additional tail fatness in distributions is to parameterize the Student-t distribution with degrees of freedom in accordance with the tail estimation as described below. This approach follows the VaR-x approach of Huisman, Koedijk, and Pownall [1998], but instead of value at risk estimation, we focus on quantile estimates. These quantile estimates can then be directly incorporated into scenario analysis for credit at risk analysis, or indirectly for pricing far out-of-the-money credit risk derivatives.

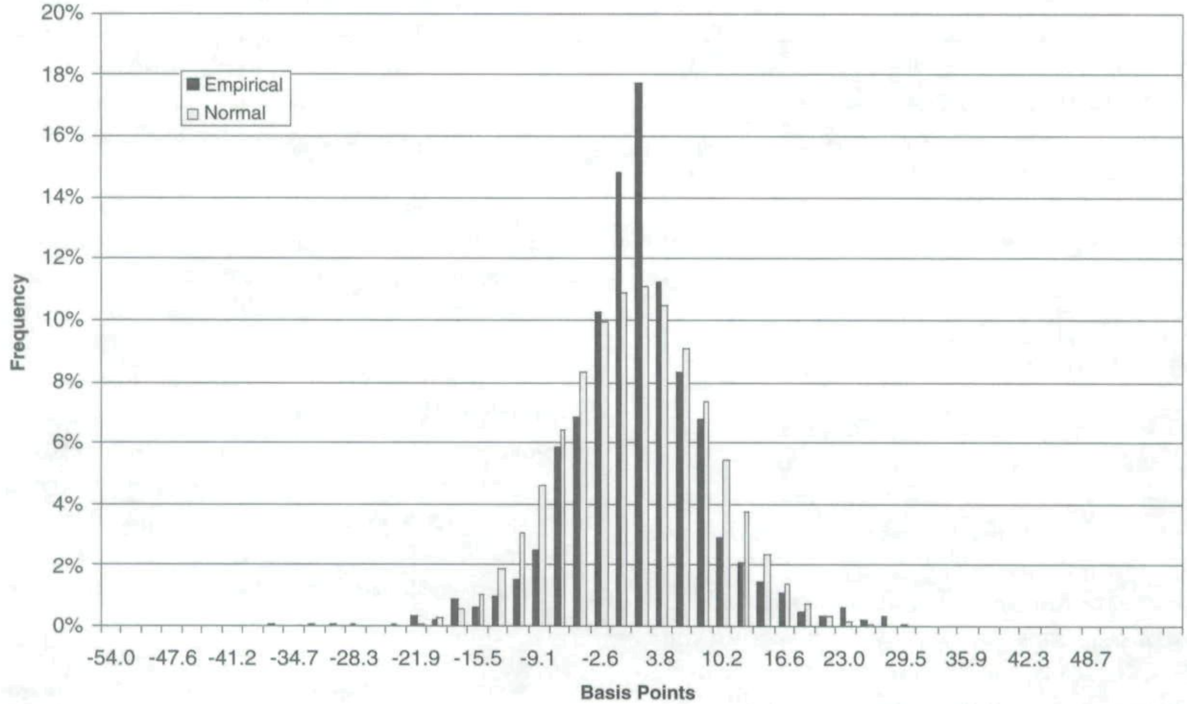
TAIL INDEX ESTIMATION

Recent developments in the application of extreme value theory to risk management enable us to provide a good estimate of the tail index of the distribution of daily movements in credit spreads. Tail index estimation is specification of the degree to which the tail of a distribution exhibits tail fatness; it was first introduced by Hill [1975]. The tail index measures how quickly the distribution's tail approaches zero; the fatter the tail, the slower the speed, and the lower the tail index given.

The tail index has the attractive feature that it is equal to the number of moments of the distribution, and thus can be used to parameterize the Student-t distribution—hence the link to the fatter-tailed Student-t distribution,

EXHIBIT 2

Histogram of Daily Spread Shifts—January 1990–December 1999



which nests the normal distribution as a limiting case. We use a modified version of the Hill estimator, developed by Huisman et al. [2001] to estimate the tail index, modified to account for the bias in the Hill estimator. Specifying k as the number of tail observations, and ordering their absolute values as an increasing function of size, we obtain the tail estimator proposed by Hill.

This is denoted by γ and is the inverse of α:

$$\gamma(k) = \frac{1}{k}\sum_{j=1}^{i} \ln(x_{n-j+1}) - \ln(x_{n-i}) \tag{5}$$

As Phoa [1999] points out, in practical applications of the Hill estimator there is an uncomfortable trade-off between variance and bias. This occurs through the use of fewer observations as we move farther out into the tails of the distribution, so that although the estimate is less biased (reflects more fully the tail of the distribution) the variance of the estimate increases. The bias of the Hill estimator is therefore a function of the sample size used for the estimate, shown in Exhibit 3 for U.S. swap spread data.[4]

Following the methodology of Huisman et al. [2001], we can use a modified version of the Hill estimator to correct for the bias in small samples. A bias-corrected tail index is therefore obtained by observing the bias of the Hill estimator as the number of tail observations increases up to κ, where κ is equal to half of the sample size:

$$\gamma(k) = \beta_0 + \beta_1 k + \varepsilon(k), k = 1, \ldots, \kappa \tag{6}$$

The optimal estimate for the tail index is the intercept β_0. And the α estimate is just the inverse of this estimate. This is the estimate of the tail index that we use to parameterize the Student-t distribution. Recent applications of this approach to estimating market risk have been shown to work well for a variety of financial time series.[5]

We estimate the tail estimates using the Huisman et al. [2001] alpha estimator for the four countries. The estimates for both tails are given in Exhibit 4. As all the series exhibit excess kurtosis, it is not surprising that the alpha estimates used to parameterize the Student-t distribution generate much fatter-tailed distributions than under normality. We also observe that the alpha estimate for the left tail alone for all the series is slightly smaller than the estimate using both tails and the right tail of the distribution only. This provides evidence of a greater probability attached to credit spread tightenings than to credit spread widenings. This may result from the fact that sharp rises in Treasury yields occur more frequently than sharp drops (see Phoa [1999]).

EXHIBIT 3
Tail Index Estimator

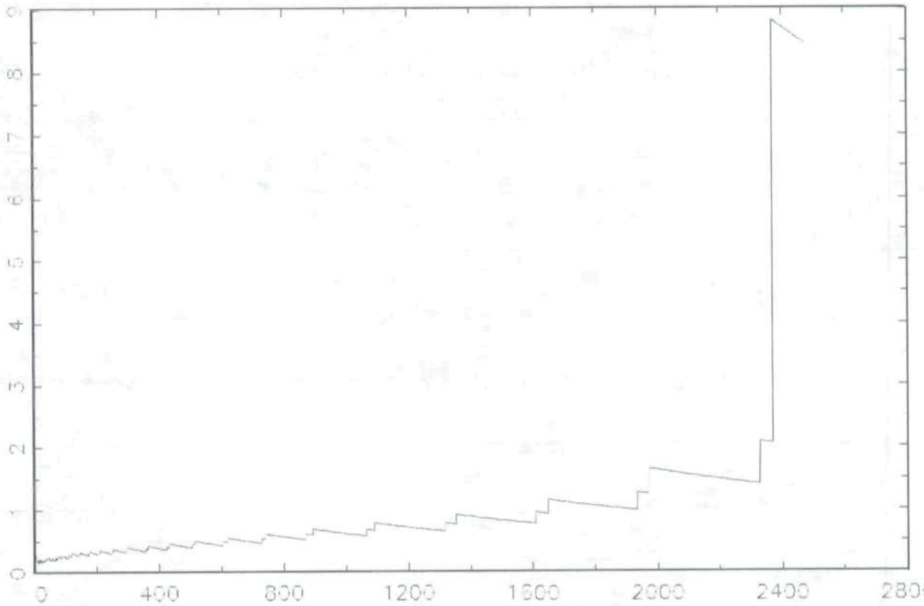


EXHIBIT 4
Alpha Estimates

	U.S.	U.K.	Germany	Japan
Alpha (Both)	**3.848**	**3.423**	**3.550**	**2.939**
Kappa	1305	1305	1305	1305
Alpha (Left)	**3.957**	**3.035**	**2.803**	**2.735**
Kappa	603	618	732	582
Alpha (Right)	**4.506**	**3.835**	**4.230**	**3.561**
Kappa	701	686	572	723

We therefore analyze the quantile estimates for the downward and upward shifts in credit spreads separately, using the tail index estimator for the respective tail. In Exhibit 5 we plot the quantile estimates using the two approaches for quantiles ranging from 7.5% to 92.5% in the right and left tails of the distributions.

In more extreme cases, the assumption of normality severely underestimates the size of the potential shift in the credit spread shift. Indeed this is the case for all the series that we analyze. The results for the quantile estimates for potential daily tightenings and widenings are given in Exhibits 6 and 7.

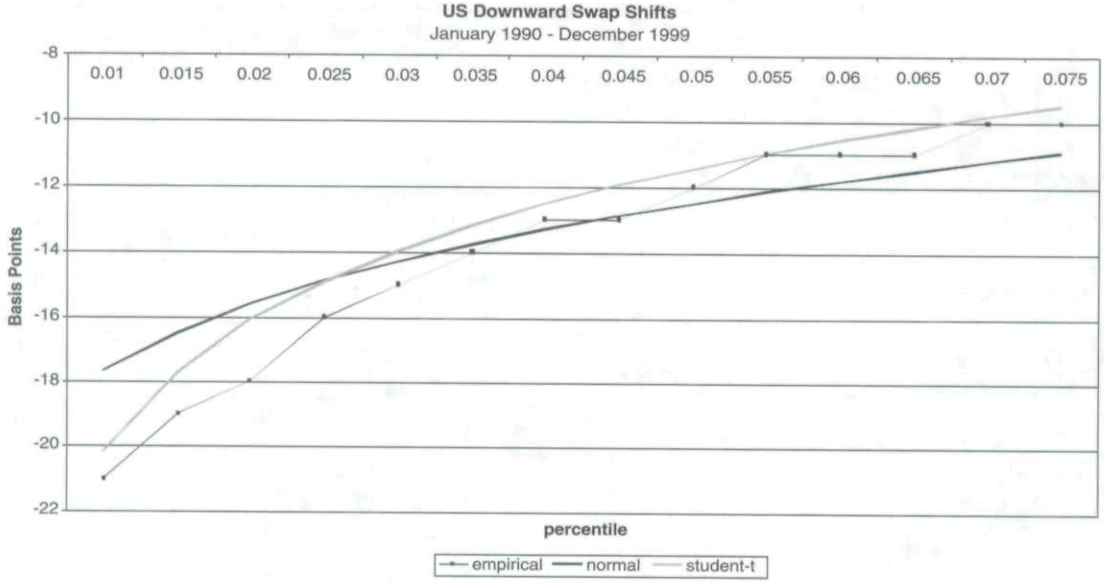
The probability of credit spread tightenings has historically been slightly greater than for similar-sized upward movements, but all the results provide evidence of severe underestimation of the potential changes in large movements of credit spreads. Indeed, the fatter-tailed Student-t distribution parameterized by the alpha tail index estimator provides basis point movements for monthly, yearly, five-, and ten-year events much more in line with those we have observed in recent years. It would therefore appear to be much more prudent to use these higher estimates in risk management techniques and derivatives pricing and hedging strategies incorporating credit spread risk.

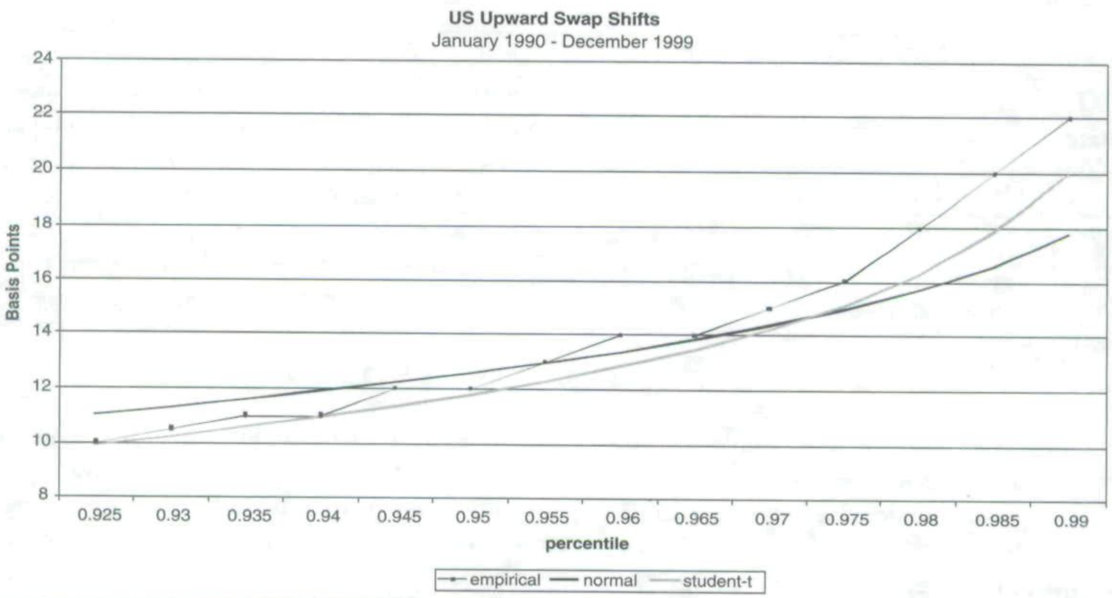
CONCLUSIONS

Estimation of credit spread risk is important not only for pricing and hedging credit derivatives but also for accurate risk management. Small but looming possibilities of default, however, make the expected return distribution for financial products subject to credit risk non-normal. To correctly assess the true probability of large movements in credit widenings and tightenings, we apply techniques developed to incorporate additional downside risk resulting from non-normalities in managing market risk to data on swaps and swap spreads.

The downside of our results is that for unexpected events the assumption of normality grossly underestimates credit spread risk in many countries' credit markets. Estimation of swap and credit spread risk for such events is dramatically improved when the severity of the additional downside risk is measured and incorporated into current estimation techniques. These results are crucial not

EXHIBIT 5
Quantile Estimates Using Alternative Parametric Distributions





only for improving credit risk management but also for pricing out-of-the-money credit derivatives.

ENDNOTES

[1]See Huisman et al. [2001], Pownall and Koedijk [1999], and Huisman, Koedijk, and Pownall [1998].

[2]Jarrow and Turnbull [1995] provide a consistent methodology for pricing and hedging derivative securities involving credit risk, assuming no arbitrage and complete markets.

[3]While a confidence level (commonly 95%) is taken for the distribution of the underlying asset, it is not commonly assumed for the distribution of shifts in the credit spread. It is a simple exercise to incorporate this directly into the estimate using a bivariate distribution.

We could have used the Datastream value-weighted index of the middle yield on U.S. corporate bonds index, for example, which includes all maturities and investment-grade credit ratings, but the corporate bond market is still much less liquid, with only weekly data available for the same sample period.

[4]A similar pattern emerges for all the series studied.

[5]See Huisman, Koedijk, and Pownall [1998] for an appli-

EXHIBIT 6

Credit Spread Tightenings—Quantile Estimates

Monthly Event 4.76%	Empirical	Normal	Student-t (a_L)
U.S.	*-12.0*	-12.7	-11.7
U.K.	*-13.0*	-14.7	-12.3
Germany	*-9.0*	-10.4	-8.3
Japan	*-9.0*	-11.5	-9.0
Yearly Event 0.397%	**Empirical**	**Normal**	**Student-t (a_L)**
U.S.	*-29.6*	-20.2	-26.6
U.K.	*-35.0*	-23.4	-32.2
Germany	*-24.7*	-16.5	-22.8
Japan	*-31.9*	-18.3	-25.1
5-Yearly Event 0.079%	**Empirical**	**Normal**	**Student-t (a_L)**
U.S.	*-37.0*	-24.1	-41.4
U.K.	*-57.4*	-27.8	-55.8
Germany	*-32.5*	-19.7	-41.1
Japan	*-62.9*	-21.8	-46.0
10-Yearly Event 0.040%	**Empirical**	**Normal**	**Student-t (a_L)**
U.S.	*-53.4*	-25.6	-49.7
U.K.	*-58.9*	-29.5	-70.4
Germany	*-35.9*	-20.9	-52.7
Japan	*-69.7*	-23.1	-59.4
20-Yearly Event 0.019%	**Empirical**	**Normal**	**Student-t (a_L)**
U.S.	-	-27.0	-59.5
U.K.	-	-31.2	-88.7
Germany	-	-22.1	-67.7
Japan	-	-24.4	-76.7

EXHIBIT 7

Credit Spread Widenings—Quantile Estimates

Monthly Event 4.76%	Empirical	Normal	Student-t (a_R)
U.S.	*12.9*	12.7	12.0
U.K.	*14.0*	14.7	13.5
Germany	*10.0*	10.4	9.7
Japan	*10.0*	11.5	10.3
Yearly Event 0.397%	**Empirical**	**Normal**	**Student-t (a_R)**
U.S.	*26.3*	20.2	25.8
U.K.	*33.5*	23.4	31.0
Germany	*23.6*	16.5	21.3
Japan	*26.3*	18.3	24.6
5-Yearly Event 0.079%	**Empirical**	**Normal**	**Student-t (a_R)**
U.S.	*29.9*	24.1	38.5
U.K.	*43.7*	27.9	48.7
Germany	*29.9*	19.6	32.5
Japan	*38.4*	21.8	39.8
10-Yearly Event 0.040%	**Empirical**	**Normal**	**Student-t (a_R)**
U.S.	*31.9*	25.6	45.4
U.K.	*52.2*	29.6	58.8
Germany	*31.0*	20.9	38.7
Japan	*40.4*	23.2	48.7
20-Yearly Event 0.019%	**Empirical**	**Normal**	**Student-t (a_R)**
U.S.	-	27.0	53.3
U.K.	-	31.2	70.7
Germany	-	22.0	45.8
Japan	-	24.4	59.2

cation to U.S. stocks and bonds, and Pownall and Koedijk [1999] for Asian stock markets, as well as Campbell, Eicholtz, and Huisman [2003] for the U.S. and Dutch real estate markets.

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