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DEVELOPMENT AND TESTING OF A DYNAMIC DEMAND THEORY

BY

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> 'Thus I may be utterly convinced of the truth of a statement; certain of the evidence of my perception, overwhelmed by the intensity of my experience: every doubt may seem to me absurd. But does this afford the slightest reason for science to accept my statement? Can any statement be justified by the fact that K.R.P. is utterly convinced of its truth? The answer is, "No"; any other answer would be incompatible with the idea of scientific objectivity.'
>
> K. R. Popper (1959), p. 46

1 INTRODUCTION

The objective of this study is to formulate a refutable theory that endeavours to explain expenditure on certain groups of goods in a number of countries and to test this theory against reality.

Although it is not usual, in Section 2 we give a concise formulation of our scientificophilosophical point of departure. This makes it possible to direct the set-up of this study towards and to interpret it in accordance with this principle. An impression is given of the concept 'refutable theory', on the basis of K. R. Popper's philosophy of science. At the same time some properties of a refutable theory are mentioned. Finally, a number of arguments for the choice of the refutability starting point are discussed.

In Section 3 a general dynamic theory of utility maximization is developed, in which both stockpiling and habit-forming may play a role. The derived demand behaviour proves to have the usual properties. These properties are not refutable, since no specified utility function is assumed. Next a specialization of the general theory is developed which, after a number of supplementary assumptions, leads to the dynamic demand model of Houthakker and Taylor. We go deeper into the properties of this model. The theories discussed relate in

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principle to an individual consumer. It is concluded that the model to be used in this study cannot be based on consistent aggregation, and it is indicated how the aggregation problems are obviated. The assumption of additivity comes up for discussion and we examine the conditions on which the ultimately formulated theories are refutable.

The estimating and testing procedures are discussed in Section 4. Despite a number of unsolved problems, we accept the estimation method. Acceptable testing procedures are available for a number of properties of the system of dynamic expenditure equations to be estimated. We also consider the aprioristic criteria for appraising the result of observation. It is then concluded that the operationalized specific demand theories are refutable.

With the aid of data for Sweden and Switzerland we test in Section 5 the extent to which the system of dynamic demand equations possesses the expected properties. It proves that the empirical results lead to rejection of the specific demand theory with and without the assumption of additivity for the two countries. By way of conclusion a number of considerations follow on the strength of these findings.

2 THE SCIENTIFICOPHILOSOPHICAL POINT OF DEPARTURE

2.1 *The Concept 'Refutable Theory'*

The structure of a theory may be described as a system in which a number of regularities are derived from a number of assumed regularities via analytically correct reasonings, *i.e.* reasonings conceived in conformity with the rules of logic and mathematics. The basic form of a regularity is the strictly universal conditional statement. Popper finds a logical asymmetry in strictly universal conditional statements: observations, however numerous, *cannot* verify a law.[1] A law relates to a set with a number of elements that is infinite or in any case too great for observation. Consequently, it cannot be investigated whether a law is true, *i.e.* is in accordance with the facts.[2] However, the occurrence of an event excluded by a law contradicts that law. Such a regularity is therefore not verifiable, but it is falsifiable (refutable).[3]

A theory, Popper argues, is part of empirical[4] science if it is falsifiable, *i.e.* if it has empirical content. The empirical content of a theory is determined by and

1 See Popper (1959), p. 41.

2 Popper uses Tarski's objectivistic concept of truth. See Popper (1968), pp. 223 *et seq.* and Tarski (1943–4), pp. 341 *et seq.*

3 Popper (1959), p. 41.

4 The explicit introduction of the predicate empirical leaves scope for the subsuming of logic, mathematics, mathematical economics *etc.* under the formal sciences.

identical with the set of basic statements that refute the theory[5]: the potential falsifiers.

Basic statements are propositions with the aid of which theories can be tested.[6] Acceptance of a potential falsifier logically implies acceptance of a refutation of a law. A basic statement posits the existence of observable facts within a spatio-temporally demarcated area[7] and, according to Popper, is therefore intersubjectively testable by observation.[8] However, the question arises as to what should happen if the investigators who together make up the scientific forum,[9] do not arrive at intersubjective agreement on a result of observation and thus on acceptance or rejection of a basic statement. Popper's answer begins by saying that every basic statement uses terms of a universal nature, *i.e.* with a lawlike intension.[9] With the aid of the laws bound up with these universal terms new basic statements can be formulated in their turn, on which a decision can again be made by observation.[10] Intersubjective agreement on the result of observation may imply acceptance of the original basic statement. This procedure is repeatable and has no end in principle. However, Popper argues, the investigators can stop at basic statements on which they reach intersubjective agreement.[11] Basic statements can therefore be tested with new basic statements. However, intersubjective acceptance of a potential falsifier logically implies acceptance of a refutation of the theory concerned.

It was posited above that a theory is part of empirical science if it is refutable. However, probability statements are not only not verifiable but also not falsifiable.[12] Nevertheless, they can be made refutable by a number of aprioristic statistical decisions that lead to criteria for appraising a result of observation. These decisions relate for instance to a level of significance and a probability function. On the strength of compliance or non compliance with the criteria it can be concluded whether to reject or provisionally accept a probability hypothesis. These aprioristic statistical criteria are reconcilable with and necessary for a meaningful acceptance of the refutability starting point for empirical science.

2.2 *Some Properties of a Refutable Theory*

The principal property of a refutable theory is that the set of refuting basic statements is *not empty*. From this principal property the following characteris-

5 Popper (1959), p. 86 and Popper (1968), p. 385.
6 Popper (1968), pp. 387 and 388.
7 Popper (1968), p. 386.
8 Popper (1959), p. 192.
9 Popper (1959), pp. 95 and 423 and Popper (1968), p. 387.
10, 11 Popper (1959), p. 104.
12 See Popper (1959), pp. 191 *et seq.*

tics may be derived:

1. refutable laws do not hold good by definition;
2. an empirical law cannot have the logical form of a strictly existential quantifier;[13]
3. an empirical theory contains at least one refutable law;
4. a refutable law is operationally defined.

The first three properties need no further explanation in this context. With regard to the fourth characteristic, the following may be remarked. The conditions of validity of a law may be defined as the conditions on which the law is assumed to be valid. The intension of these conditions should be such for an empirical law that it relates to properties that are intersubjectively observable for the scientific forum, *i.e.* to an observable non-empty set.[14]

This requirement follows logically from the requirement that the set of refuting basic statements may not be empty. Each basic statement posits the existence of intersubjectively observable facts within a demarcated spatio-temporal area. It is true of all these statements of an empirical law that, on the basis of intersubjective agreement, it can be established whether the facts comply with the conditions of validity of the law. If this is not possible, the basic statements are not potential falsifiers and the law has no empirical content. The conditions of validity must therefore have been operationally defined, *i.e.* they must relate to properties intersubjectively observable for the scientific forum.

In addition it must also be so that the properties or relations predicted or explained in a likewise manner are intersubjectively observable. For in a refuting basic statement the negation of those predicted or explained facts is posited. The investigators must be able to decide intersubjectively whether these phenomena occur or not. If this is not possible, the basic statements are once again not potential falsifiers. The predicted or explained properties must be defined operationally.

A law which does not satisfy these requirements of operationality is not falsifiable and not testable and, considered in itself, is not part of empirical science.[15]

13 A strictly existential quantifier may be defined as a disjunction of an infinite number of propositions. For instance: 'There exists at least one non-white swan.'

14 The intension of a concept is the total of its defining properties. The extension is the set of phenomena possessing these properties.

15 It can be seen from several places in Popper's work that he identifies falsifiability and testability with one another. See inter alia Popper (1968), p. 256: 'the refutability or falsifiability of a theoretical system should be taken as a criterion of its demarcation'... 'a system is to be considered as scientific only if its makes assertions which may clash with observations and a system is in fact tested by attempts to produce such clashes, that is to say by attempts to refute it. Thus testability is the same as refutability and can therefore likewise be taken as a criterion of demarcation.' This is in *contrast*

2.3 *Some Arguments for the Choice of the Refutability Starting Point*

After this outline of the Popperian point of departure it may be as well to consider why an economist ought to replace his philosophy of science by Popper's.

An important reason for this is that the philosophy of refutability minimizes the number of propositions accepted in advance; a process of minimization which, as such, must appeal to many economists. The aprioristic judgments that have to be accepted consist only of the Popperian philosophy of refutability and of statistical aprioristic judgments needed for developing empirical theories, such as an unreliability threshold.

A further argument lies in the actual econometric application of theories. It is not unusual in the practice of empirical research to use originally micro-economic theories at macro-economic level. Instances are the estimating of production functions and of elasticities of substitution,[16] but also the estimating of demand relations for the average consumer on the basis of a utility maximization theory.[17] In this context it is no longer easy, in our opinion, to maintain that theories are used which are formulated around Mill's principal laws of human nature. The refutability point of view gives more scope here: any theory, metaphysical or not, is admissible, if only it leads to empirical laws. But at the same time the principle indicates exact limits: the empirical laws have to be refutable. Operational refuting basic statements must be capable of formulation.

An additional reason for the choice of this new philosophy is that it presents the possibility of making the nature of economic science clear. It will then

to what Klant thinks; see J. J. Klant (1972), p. 183 *et seq.* The methodological viewpoint of K. Klappholz and J. Agassi (see K & A (1959), pp. 60 *et seq.*) thus does correspond to Popper's criteria, likewise in contrast to what Klant posits: a non-operational theory is not testable and not falsifiable; it is for the time being metaphysical, but that does not imply that it is meaningless. See also inter alia Popper (1968), pp. 253 *et seq.* The falsifiability of a theory is therefore not only a logical property. See inter alia Popper (1959), p. 192: 'a basic statement must also satisfy a *material* requirement – a requirement concerning the event which, as the basic statement tells us, is occurring at place k. This event must be an "observable" event, that is to say basic statements must be testable, intersubjectively, by "observation".'

* Klant's concept of falsifiability is rather different from Popper's. The latter uses it as demarcation criterion for testable theories of empirical science. In the case of Klant falsifiable theories may be non-testable and testable ones may be non-falsifiable.

* Our italics.

16 See for instance Hildebrand and Liu (1965), Fisher (1969), Ringstad (1971), Solow (1967), Tinbergen (1974), Bowles (1969), Dougherty (1972).

17 See inter alia Barten (1966, 1967, 1969), Barten and Geyskens (1974), Byron (1968, 1970 I, 1970 II), Deaton (1972, 1975), Court (1967), Houthakker and Taylor (1970), Luch (1971), Mattei (1971).

become apparent whether economic science is able to formulate successful theories. Perhaps it can then develop from a 'premature science' with dogmatic trends opposed to one another into a 'mature science' evolving paradigmatically.[18] Or it becomes clear that economics cannot really be practised as an empirical science and is more of a deductive science or an economic philosophy.

3 THE THEORY

3.1 *The General Dynamic Model*

Let us assume that a consumer is aiming at utility maximization and that his utility index for period t, v_t, is a function of the quantity of goods bought in that period, $\mathbf{q}_t$ (an $n \times 1$ vector), of the actual stocks of the same goods maintained at the end of the preceding period, $\mathbf{v}_{t-1}$ (an $n \times 1$ vector) and of the habits which he has formed with regard to the consumption of those goods at the end of the preceding period, $\mathbf{h}_{t-1}$ (an $n \times 1$ vector):

$$v_t = v(\mathbf{q}_t', \mathbf{v}_{t-1}', \mathbf{h}_{t-1}'). \tag{1}$$

Consumption in period t, $\mathbf{c}_t$ (an $n \times 1$ vector) is by definition:

$$\mathbf{c}_t \equiv \mathbf{D}_t\mathbf{q}_t + \mathbf{E}_t\mathbf{v}_{t-1} \tag{2}$$

where $\mathbf{D}_t$ and $\mathbf{E}_t$ are diagonal matrices whose principal diagonal elements indicate which part of the purchase and the stocks is consumed. These elements, $0 < d_{it} < 1$ and $0 < e_{it} < 1$, may be functions of the consumption habits formed with regard to the i-th good.

$$d_{it} = d_i(h_{it-1}) \qquad i = 1, \ldots, n \tag{3}$$

and

$$e_{it} = e_i(h_{it-1}) \qquad i = 1, \ldots, n \tag{4}$$

h_{it-1} is the i-th element of $\mathbf{h}_{t-1}$.

It follows from this that the stock of good i, v_{it}, is a function of q_{it}, v_{it-1} and h_{it-1}.

$$v_{it} = v_i(q_{it}, v_{it-1}, h_{it-1}) \tag{5}$$

18 See Kuhn (1970), in particular pp. 12 and 13.

namely as follows:

$$\mathbf{v}_t = (\mathbf{I} - \mathbf{D}_t)\mathbf{q}_t + (\mathbf{I} - \mathbf{E}_t)\mathbf{v}_{t-1}$$

It may further be assumed that the consumption habits formed at the end of period t are functions of $\mathbf{c}_t$ and $\mathbf{h}_{t-1}$:

$$h_{it} = h_i(\mathbf{c}_t', \mathbf{h}_{t-1}') \text{ for } i = 1, \ldots, n \tag{6}$$

and thus $h_{it} = k_i(\mathbf{q}_t', \mathbf{v}_{t-1}', \mathbf{h}_{t-1}')$ (7)

We assume that the utility function (1) is continuous and has continuous partial derivatives of the 1st and 2nd order and that $\dfrac{\partial v}{\partial q_i} > 0$ for $i = 1, \ldots, n$.

The utility function (1) is suitable for accommodating the plausible aprioristic notions that, *ceteris paribus*, the consumer attaches a lower value to a purchased package according as he has greater stocks and attaches a higher value to the same package according as his consumption habits are higher. The preference ordering that can be derived from the utility function (1) by considering only the purchased quantities ($\mathbf{q}_t$) with given actual stocks and consumption habits can change with every change in stocks and habits. This model therefore allows of changing preferences with regard to the goods purchased.

The limiting condition in the consumer's pursuit of utility maximization is his income, which we assume he spends in its entirety:

$$\mathbf{p}_t'\mathbf{q}_t = \mu_t \tag{8}$$

where μ_t is the income in period t. The utility maximization problem is therefore: maximize v_t under the subsidiary condition $\mathbf{p}_t'\mathbf{q}_t = \mu_t$ for $t = 1, \ldots, m$.

This problem can be investigated with Lagrange's auxiliary function:

$$\Lambda(v_t, \lambda_t) = v_t - \lambda_t(\mathbf{p}_t'\mathbf{q}_t - \mu_t) \text{ for } t = 1, \ldots, m \tag{9}$$

where Λ is the Lagrangian function and λ_t the Lagrangian multiplier. Sufficient conditions for a strict and absolute maximum of v_t in any period under the subsidiary condition laid down are that $\mathrm{d}\Lambda_t = 0$ and $\mathrm{d}^2\Lambda_t < 0$ for $t = 1, \ldots, m$. For the maximization problem in period t, $\mathbf{v}_{t-1}$ and $\mathbf{h}_{t-1}$ are constants. The first condition is satisfied if:

$$\left.\begin{array}{l} \mathbf{u}_t - \lambda_t \mathbf{p}_t = 0 \\ \mathbf{p}_t'\mathbf{q}_t = \mu_t \end{array}\right\} \text{ for } t = 1, \ldots, m \tag{10}$$

where $\mathbf{u}_t$ is the $n \times 1$ vector $\left[\dfrac{\partial v}{\partial q_{it}}\right]$.

The second condition is satisfied if:

$$\mathbf{y}'\mathbf{L}_t^q\mathbf{y} < 0 \text{ subject to}$$

$$\mathbf{p}_t'\mathbf{y} = 0 \text{ for } t = 1, \ldots, m \tag{11}$$

where $\mathbf{L}_t^q$ is the matrix $\left[\dfrac{\partial^2 v}{\partial q_{it}\partial q_{it}}\right]$ and $\mathbf{y}$ is a vector unequal to zero. Conditions (10) and (11) are invariant for monotonic transformations of a specified utility function. The utility concept used is therefore ordinal and decreasing marginal utility is not assumed. Utility functions satisfying (11) are strictly quasi-concave.

System (10) makes it possible with a suitably specified utility function to write $\mathbf{q}_t$ as certain functions of $\mathbf{p}_t$, μ_t, $\mathbf{v}_{t-1}$ and $\mathbf{h}_{t-1}$. This derived system is then regarded as the description of the consumer's demand behaviour with the given utility function. For a general model this conclusion is often reproduced as follows:

$$q_{it} = q_i(\mathbf{p}_t', \mu_t, \mathbf{v}_{t-1}', \mathbf{h}_{t-1}') \text{ for } i = 1, \ldots, n \tag{12}$$

This system (12) is suitable for accommodating a negative effect on the demanded quantity of good i as a result of – *ceteris paribus* – an increase in the stock of that good or of a reduction in the consumption habit.

For the general Hicksian utility maximization model it is possible to derive a number of properties for the demand relations without specifying a certain utility function. This also applies to the dynamic model reproduced above. If we wish to trace the effect of a change in prices, income, stocks and consumption habits on the utility-maximizing package chosen, this can be done by means of the system of total differentials of (10).

If we make this system equal to zero, we can approximately solve from it the changes in the package chosen. The desired theoretical properties can likewise be derived from it. These are:

I $$\mathbf{p}_t'\mathbf{q}_{\mu t} = 1 \tag{13}$$

$$\mathbf{p}_t'\mathbf{S}_t = 0 \tag{14}$$

$$\mathbf{p}_t'\mathbf{Q}_{pt} = -\mathbf{q}_t \tag{15}$$

$$\mathbf{p}_t'\mathbf{Q}_{vt} = 0 \tag{16}$$

$$\mathbf{p}_t'\mathbf{Q}_{ht} = 0 \tag{17}$$

Here $\mathbf{q}_{\mu t}$ is the vector $\left[\dfrac{\partial q_{it}}{\partial \mu_t}\right]$, $\mathbf{Q}_{pt}$ is the matrix $\left[\dfrac{\partial q_{it}}{\partial p_{jt}}\right]$, $\mathbf{Q}_{vt}$ is the matrix

$\left[\frac{\partial q_{it}}{\partial v_{jt-1}}\right]$, $\mathbf{Q}_{ht}$ is the matrix $\left[\frac{\partial q_{it}}{\partial h_{jt-1}}\right]$ and

$$\mathbf{S}_t = \mathbf{Q}_{pt} + \mathbf{q}_{\mu t}\mathbf{q}_t'$$

These properties are the aggregation restrictions of this model.

II $\mathbf{S}_t\mathbf{p}_t = 0$ (18)

$\mathbf{Q}_{pt}\mathbf{p}_t = -\mathbf{q}_{\mu}\cdot\mu_t$ (19)

These are the well-known homogeneity restrictions, which also hold good for this model.

III $\mathbf{S}_t = \mathbf{S}_t'$ (20)

This is the symmetry restriction.

IV $\mathbf{x}'\mathbf{S}_t\mathbf{x} < 0$ for each $\mathbf{x}$ not equal to zero and $\mathbf{x} \neq \beta\mathbf{p}_t$ (21)

Equation (21) is the negativity restriction.

The question arises whether this general theory is empirical on the basis of the refutability criterion. In answering this question we shall confine our attention in this context to the derived system of demand relations (12). We can then formulate the conclusion of the theory with regard to the demand behaviour of the consumer in the form of an existential quantifier: 'There exists at least one specification of (12) with equations (13) to (21) as properties.' A justification of this lies in the fact that (12) is not given in specified form by the general theory. If one succeeds in observing a specification of (12) which, however, lacks the theoretical properties, it is still possible that observation of a different specification does not yield the properties. It follows from this that for the conclusion considered no refutable basic statement can be formulated and therefore that part of the theory is not refutable.[19] For the sake of good order it should be added that this proposition does not imply that conclusions of a different kind cannot derived from this theory.

After the above it will be clear that, if this general theory is specialized by a suitable specification of the utility index function, a specified system of demand

19 This conclusion holds good by analogy for the properties of the unspecified system of demand functions that can be derived from the general static Hicksian utility maximization theory. Samuelson calls these properties: 'restrictions upon demand functions and price-quantity data,' going on to say: 'these could be refuted or verified under ideal observational conditions' (see P. Samuelson (1965), pp. 92 *et seq.*). It will be clear that we do not share this opinion.

equations can be derived. This specialized theory is in that case no longer non-falsifiable for the above reason. We shall therefore work out a model of this kind below.

3.2 *The Specific Dynamic Model*

Let us assume that a consumer is aiming at utility maximization, with as cardinal example of his preference ordering the utility function:

$$v_t = \mathbf{c}'\mathbf{q}_t + \mathbf{f}'\mathbf{v}_{t-1} + \mathbf{g}'\mathbf{h}_{t-1} + \tfrac{1}{2}(\mathbf{q}_t'\mathbf{F}\mathbf{q}_t + \mathbf{q}_t'\mathbf{G}\mathbf{v}_{t-1} + \mathbf{v}_{t-1}'\mathbf{G}\mathbf{q}_{t-1} + \\ + \mathbf{v}_{t-1}'\mathbf{H}\mathbf{v}_{t-1} + \mathbf{q}_t'\mathbf{K}\mathbf{h}_{t-1} + \mathbf{h}_{t-1}'\mathbf{K}\mathbf{q}_t + \mathbf{h}_{t-1}'\mathbf{M}\mathbf{h}_{t-1}) \qquad (22)$$

Here **c**, **f** and **g** are vectors of constant coefficients and **F**, **G**, **H**, **K** and **M** matrices of constant coefficients. The limiting condition is again:

$$\mathbf{p}_t'\mathbf{q}_t = \mu_t$$

Conditions (10) are then:

$$\mathbf{c} + \mathbf{N}\mathbf{q}_t + \mathbf{P}\mathbf{v}_{t-1} + \mathbf{R}\mathbf{h}_{t-1} - \lambda_t\mathbf{p}_t = 0 \\ \mathbf{p}_t'\mathbf{q}_t = \mu_t \qquad (23)$$

With this utility function conditions (11) are:

$$\mathbf{y}'\mathbf{N}\mathbf{y} < 0 \text{ subject to} \\ \mathbf{p}_t'\mathbf{y} = 0, \ \mathbf{y} \neq 0 \qquad (24)$$

In (23) **N** is a symmetrical matrix with as *ij*-th element: $\frac{1}{2}(f_{ji} + f_{ij})$, where $[f_{ij}]$ is the matrix **F**. **P** and **R** are likewise symmetrical matrices.

A likewise adequate condition instead of equation (11) is

$$\mathbf{y}'\mathbf{L}_t^q\mathbf{y} < 0, \ \mathbf{y} \neq 0 \qquad (25)$$

Here this is:

$$\mathbf{y}'\mathbf{N}\mathbf{y} < 0, \ \mathbf{y} \neq 0 \qquad (26)$$

Replacement of (11) by (25) implies introduction of a cardinal utility concept, since (25) is invariant only for linear transformations of the utility function. The marginal utility is therefore not only positive but also decreasing.

We further assume that:

$$\mathbf{v}_t = \mathbf{S}\mathbf{s}_t \qquad (27)$$

and

$$\mathbf{h}_t = \mathbf{T}\mathbf{s}_t \tag{28}$$

where **S** and **T** are diagonal matrices with constant coefficients.

The variable $\mathbf{s}_t$ (an $n \times 1$ vector) can be interpreted as Houthakker's 'state variable,'[20] in which both changes in actual stocks and in 'psychological stocks' (consumption habits) find expression. We then obtain for (23):

$$\begin{aligned} &\mathbf{c} + \mathbf{N}\mathbf{q}_t + \mathbf{O}\mathbf{s}_{t-1} - \lambda_t \mathbf{p}_t = 0 \\ &\mathbf{p}_t'\mathbf{q}_t = \mu_t \end{aligned} \tag{29}$$

where $\mathbf{O} \equiv \mathbf{PS} + \mathbf{RT}$.

The utility function[21] corresponding to (29) is:

$$\begin{aligned} \upsilon_t = \mathbf{c}'\mathbf{q}_t + \mathit{2}'\mathbf{s}_{t-1} + \tfrac{1}{2}(\mathbf{q}_t'\mathbf{F}\mathbf{q}_t + \mathbf{q}_t'\mathbf{U}\mathbf{s}_{t-1} + \\ + \mathbf{s}_{t-1}'\mathbf{U}\mathbf{q}_t + \mathbf{s}_{t-1}'\mathbf{V}\mathbf{s}_{t-1}) \end{aligned} \tag{30}$$

The system of specified demand equations that can be solved from (29) is:

$$\begin{aligned} \mathbf{q}_t = (\mathbf{N}^{-1} - \mathbf{N}^{-1}\mathbf{p}_t(\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{p}_t)^{-1}\mathbf{p}_t'\mathbf{N}^{-1})(-\mathbf{c} - \mathbf{O}\mathbf{s}_{t-1}) + \\ + \mathbf{N}^{-1}\mathbf{p}_t(\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{p}_t)^{-1}\mu_t \end{aligned} \tag{31}$$

However, this system is not linear in the demand-determining variables. If we assume that we have values of λ_t available, we can write:

$$\mathbf{q}_t = \mathbf{N}^{-1}(-\mathbf{c} - \mathbf{O}\mathbf{s}_{t-1} + \mathbf{p}_t\lambda_t) \tag{32}$$

Since data on the values of the state variable are not available, a supplementary assumption must be made:

$$\mathbf{c}_t = \mathbf{D}\mathbf{q}_t + \mathbf{E}\mathbf{v}_{t-1} \qquad (33)^{22}$$

where $\mathbf{c}_t$ is an $n \times 1$ vector of the quantity of goods consumed in period t. In contrast to (2) the diagonal matrices of consumption ratios **D** and **E** are not assumed to be dependent on the consumption habits. Changes in these habits

20 See Houthakker and Taylor (1970), pp. 10 *et seq.*

21 The utility function has been used by Mattei (1970) and Houthakker and Taylor (1970).

22 Assumption (33) is a specialization of (2).

thus makes themselves felt only in the composition of the package purchased. It follows from (33) that:

$$\mathbf{v}_t = (\mathbf{I} - \mathbf{D})\mathbf{q}_t + (\mathbf{I} - \mathbf{E})\mathbf{v}_{t-1} \tag{34}$$

and therefore via (27) and (28)

$$\mathbf{h}_t = (\mathbf{TS}^{-1} - \mathbf{TS}^{-1}\mathbf{D})\mathbf{q}_t - (\mathbf{I} - \mathbf{TS}^{-1}\mathbf{ES}^{-1})\mathbf{h}_{t-1} \tag{35}$$

and

$$\mathbf{s}_t = \mathbf{Xq}_t + \mathbf{Ys}_{t-1} \tag{36}$$

where

$$\mathbf{X} \equiv \mathbf{S}^{-1} - \mathbf{S}^{-1}\mathbf{D} \text{ and } \mathbf{Y} \equiv \mathbf{I} - \mathbf{S}^{-1}\mathbf{ES}^{-1}$$

Substitution of (36) in (32) gives:

$$\mathbf{q}_t = \mathbf{N}^{-1}(-\mathbf{c} + \lambda_t\mathbf{p}_t - \mathbf{O}(\mathbf{Xq}_{t-1} + \mathbf{Ys}_{t-1})) \tag{37}$$

Now:

$$\mathbf{s}_{t-1} = \mathbf{O}^{-1}(\lambda_t\mathbf{p}_t - \mathbf{c} - \mathbf{Nq}_t) \tag{38}$$

Thus:

$$\mathbf{q}_t = \mathbf{Aq}_{t-1} + \mathbf{B}\lambda_t\mathbf{p}_t + \mathbf{C}\lambda_{t-1}\mathbf{p}_{t-1} + \mathbf{d} \tag{39}$$

where:

$$\mathbf{A} \equiv \mathbf{N}^{-1}\mathbf{OYO}^{-1}\mathbf{N} - \mathbf{N}^{-1}\mathbf{OX}$$

$$\mathbf{B} \equiv \mathbf{N}^{-1}$$

$$\mathbf{C} \equiv \mathbf{N}^{-1}\mathbf{OYO}^{-1}$$

$$\mathbf{d} \equiv (\mathbf{N}^{-1}\mathbf{OYO}^{-1} - \mathbf{N}^{-1})\mathbf{c}$$

System (39) is linear in $\mathbf{q}_t$ and $\lambda_t \cdot \mathbf{p}_t$.

Assuming that (39) can be observed in this form[23] we shall now investigate which of the theoretical properties derived in the preceding section apply by

23 We assume here that observation yields values of **A**, **B**, **C** and **d** on which the observation procedure imposes no theoretical properties.

definition and therefore are not refutable and thus also not testable. In this specialized model

$$\mathbf{q}_{\mu t} = \mathbf{N}^{-1}\mathbf{p}_t(\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{p}_t)^{-1} \tag{40}$$

$$\mathbf{Q}_{pt} = \lambda_t\mathbf{N}^{-1} - \mathbf{N}^{-1}\mathbf{p}_t(\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{p}_t)^{-1}\mathbf{p}_t'\mathbf{N}^{-1}\lambda_t - \mathbf{q}_{\mu t}\mathbf{q}_t' \tag{41}$$

$$\mathbf{S}_t = \lambda_t\mathbf{N}^{-1} - \mathbf{N}^{-1}\mathbf{p}_t(\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{p}_t)^{-1}\mathbf{p}'\mathbf{N}^{-1}\lambda_t \tag{42}$$

$$\mathbf{Q}_{vt} = \mathbf{N}^{-1}\mathbf{P} - \mathbf{N}^{-1}\mathbf{p}_t(\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{p}_t)^{-1}\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{P} \tag{43}$$

$$\mathbf{Q}_{ht} = \mathbf{N}^{-1}\mathbf{H} - \mathbf{N}^{-1}\mathbf{p}_t(\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{p}_t)^{-1}\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{H} \tag{44}$$

$$\mathbf{Q}_{st} = \mathbf{N}^{-1}\mathbf{O} - \mathbf{N}^{-1}\mathbf{p}_t(\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{p}_t)^{-1}\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{O} \tag{45}$$

In (45) $\mathbf{Q}_{st} = [\partial q_{it}/\partial s_{jt-1}]$.

Observation of (39) gives the value of **B** and therefore of $\mathbf{N}^{-1}$. Under the given assumptions it is clear that, whatever the value of **B** may be, properties (13) to (19) and $\mathbf{p}_t'\mathbf{Q}_{st} = 0$, the aggregation and homogeneity restrictions, apply by definition and therefore are not testable properties.

Since in empirical research 'total expenditure' is often used as a variable instead of 'income,'[24] the expenditure equation holds good by definition. The loss of the aggregation restriction as a possibly testable property is therefore not a serious matter.

The symmetry restrictions $\mathbf{S}_t = \mathbf{S}_t'$ implies in this model the requirement of symmetry of $\lambda_t\mathbf{N}^{-1} - \mathbf{N}^{-1}\mathbf{p}_t(\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{p}_t)^{-1}\mathbf{p}_t'\mathbf{N}^{-1}\lambda_t$.

This matrix can also be written as:

$$\mathbf{S}_t = \lambda_t\mathbf{N}^{-1} - \mathbf{q}_{\mu t}\cdot\mathbf{q}_{\mu t}'(\mathbf{p}_t'\mathbf{N}^{-1}\mathbf{p}_t)\lambda_t \tag{46}$$

It follows from this that the symmetry of $\mathbf{N}^{-1} \equiv \mathbf{B}$ is a necessary and adequate condition for the symmetry of **S**. Thus the symmetry restriction does not apply by definition in this model, on the basis of the assumption made.

If conditions (11) are assumed for this specific theory, **B** should be such that that negativity restriction is complied with:

$$\mathbf{x}'(\lambda_t\mathbf{B} - \mathbf{B}\mathbf{p}_t(\mathbf{p}_t'\mathbf{B}\mathbf{p}_t)^{-1}\mathbf{p}_t'\mathbf{B}\lambda_t)\mathbf{x} < 0 \tag{47}$$

for all $\mathbf{x} \neq 0$ and $\mathbf{x} \neq \beta\mathbf{p}_t$. But if we assume conditions (25), it is clear that the following must apply as negativity restriction:

$$\mathbf{y}'\mathbf{B}\mathbf{y} < 0, \ \mathbf{y} \neq 0 \tag{48}$$

24 As also in our study.

Summarizing, we may posit that, if it is assumed that the observation procedure yields observation of (39) in the theoretically expected specification,

- the aggregation and homogeneity restrictions apply by definition and therefore are not testable;
- the symmetry restriction and the two negativity restrictions do not apply by definition: an observed **B** must be symmetrical and negative definite or satisfy (47).

The further supplementary assumptions required for calculation of the structural parameters of (39) and the problems with regard to the long-term equilibrium of this model will, if possible, be considered in a following study.

3.3 *The Problems of Aggregation over Goods and Consumers*

The models discussed relate in their most obvious and simple interpretation to the individual consumer demanding elementary goods. Units of an elementary good are perfectly substitutable.

The objective of this study is to explain the expenditure on groups of goods in a national economy. There are consequently two aggregation problems. The theory will have to be amended or extended in the first place for demand relations aggregated over the consumers and in the second place for groups of goods aggregated over goods. At first sight it seems desirable to aggregate consistently, *i.e.* such aggregation that the use of information more detailed than in the aggregation makes no difference to the results of the analysis in question.

However, in our opinion the conclusion seems justified[25] that too many and too restrictive assumptions have to be made for consistent aggregation. For the time being it does not seem possible to base the model to be used in this study on consistent aggregation from a theory relating to the individual consumer with a utility function defined on all elementary goods.

To avoid the problem of aggregation over consumers, we shall follow the tradition of the theory[26] and of empirical research[27] and regard the theory as relating to the representative, average consumer.

25 See Maks and Muysken (1974), pp. II, 15 *et seq.*

26 See for instance Hicks (1956), p. 55: 'To assume that the representative consumer acts like an ideal consumer is a hypothesis worth testing; to assume that an actual person, the Mr. Brown or Mr. Jones who lives around the corner, does in fact act in such a way does not deserve a moment's consideration.'

27 See for instance Houthakker and Taylor (1970), p. 200: 'The theory of the dynamic preference ordering given here is strictly in terms of a single individual, yet we apply it to entire countries. In so doing we ignore the aggregation problem, on which there is a voluminous literature. Rather than add to this inconclusive discussion we simply state as our opinion that of all the errors likely to be

The problem of aggregation over goods may be avoided by introducing groups of goods into the utility function of the representative consumer via quantity indices and leaving it open whether the demanded resultant quantity indices are consistent with those that would be obtained on the basis of a utility function defined over the elementary goods. It is then sensible to posit the theory for the price and quantity index figures used and *not* in general, because of course n groups can be constructed in many ways from the same basic material. But if the posited theory is true for one of these ways, it immediately follows from this that for the remaining partitioning into n groups the theory can hold good only insofar as they are consistent with the true partitioning.

Summarizing, what this amounts to is that we regard the model described in Section 3.2 as relating to the representative consumer of a country. The quadratic utility function (30) describes the preference ordering of the representative consumer, while q_t and s_t are defined respectively as vectors with as elements the quantity indices of the purchases and 'stocks' of the various groups. In the demand equations (39) group price index figures are therefore used.

3.4 *The Additivity Assumption*

A static utility function is additive in all goods (or groups) if the utility function can be written as:

$$v = \psi(\sum_{i=1}^{n} v_i(q_i)) \tag{53}$$

where ψ may be any monotonic transformation of $\sum_{i=1}^{n} v_i(q_i)$. In the case of a cardinal utility[28] concept this definition implies that

$$\frac{\partial^2 v}{\partial q_i \partial q_j} = 0 \text{ for } j, i = 1, \ldots, n \text{ and } j \neq i$$

By analogy we can define this assumption for a dynamic utility function as

$$v = \psi(\sum_{i=1}^{n} (q_{it}, v_{it-1}, h_{it-1})) \tag{54}$$

made in demand analysis, the aggregation error is one of the least troublesome. As evidence we cite our lack of success in finding significant demographic variables (see the first edition of this book) most of which would capture distributional effects.'

28 The additivity assumption is often defined cardinally.

If we assume that (30) is a specialization of

$$\sum_{i=1}^{n} v_i(q_{it}, s_{it-1})$$

the matrices **F**, **U** and **V** must be diagonal. This implies that **N** and **O** are also diagonal. Each monotonic transformation of (30) leaves (29) untouched. For define

$$v_t^* = \varphi(v_t) \tag{55}$$

where φ may be any monotonic transformation of (30).

Conditions (10) are then:

$$\varphi'(v_t)(\mathbf{c} + \mathbf{N}\mathbf{q}_t + \mathbf{O}\mathbf{s}_{t-1}) - \lambda_t^* \mathbf{p}_t = 0 \tag{56}$$

Because $\lambda_t^* = \varphi'(v_t) \cdot \lambda_t$ and $\varphi'(v_t) > 0$ division of (56) by $\varphi'(v_t)$ gives system (29).

System (29) is invariant for monotonic transformations of (30) and therefore also the diagonality of **N** and **O**. It follows from this that matrices **A**, **B** and **C** in (39) are also diagonal.

If we assume, as in Section 4, that (39) is observable, it can then be investigated whether **A**, **B** and **C** are diagonal and thus whether in the model used the additivity assumption, may be made with the groups used. It is clear that this assumption greatly reduces the number of parameters in (39) to be freely estimated. In our specific case the number of parameters to be freely estimated falls from thirty to twelve with a division into three groups. Mattei[29] and Houthakker and Taylor[30] therefore combine in their investigations system (39) with the additivity assumption, but without testing whether additivity is acceptable. We shall try to investigate whether this assumption may be made for the groups which we use.

3.5 *The Conditions for Refutability of the Specific Theory*

First the theory will be discussed *with* the assumption of additivity.

On the basis of the assumption that a system (39) can be observed and that this observation yields values for **A**, **B**, **C** and **d**, the following properties do *not* hold good by definition:

29 See Mattei (1971).

30 See Houthakker and Taylor (1970).

- matrices **A**, **B** and **C** are diagonal; (I)
- all elements on the principal diagonal of **B** are negative; (IIa) or
- matrix **B** satisfies (47). (IIb)

Properties (I) and (IIa) or (IIb) are theoretically expected for observed systems (39). However, the question arises whether systems (39) can be observed. And this brings up the problem of the operationality of this theory. An empirical theory assumes, as stated above, operationality of its conditions of validity and of its conclusions.

To operationalize the conditions of validity of the specific theory we assume that it relates to the demand behaviour of the representative, average consumer with regard to the quantity indices of three kinds of goods, consumer non-durables, consumer durables and services, and likewise relates to the price and quantity indices as calculated on the basis of the method indicated in Section 5.1. Moreover, we confine the assumed validity to the available basic index figure material in Sweden and Switzerland in postwar years.

In anticipation of discussion of the procedure that possibly leads to observation of (39), the following may be stated here. The scientific forum decides on the basis of intersubjective agreement on the observability of a phenomenon and therefore must also decide on the procedure that endeavours to lead to observation of the phenomenon. In the discussion of the observation procedure (in Section 4) for (39) it proves that this system can be observed in stochastic form with the aid of an estimation method. A necessary condition for the specific theory to be empirical is therefore that this estimation method is intersubjectively acceptable. Now if we assume that the observation procedure is acceptable, we can formulate the first operational conclusion of the specific theory. The theory predicts that the demand behaviour of the average consumer can be explained as indicated in (39). This implies the prediction that the observation procedure will lead to observation of (39). This prediction is not automatically refutable. As explained in Section 2.1, for this kind of statement it is necessary to decide a priori on a criterion of acceptance or rejection. These criteria will be stated in Section 4. Suitable choices are the value of the correlation coefficient, the number of significant estimated coefficients and the value of the Durbin-Watson test quantity. If we therefore assume that the estimating procedure is acceptable and that such criteria have been decided on that application of the method does not yield a success by definition, we can formulate the following refuting basic statement: 'The demand behaviour of the average consumer in Switzerland since the Second World War with regard to the groups and indices used does not lead to a satisfactory observation of (39).' The theory thus has empirical content on the basis of the assumptions stated and is refutable.

The specific theory with the assumption of additivity predicts properties (I) and (IIa) or (IIb) for an observed system (39). Now if we assume that the result of observation satisfies the criteria of the estimation procedure, we can try to examine the observed system on these properties. Since (39) is observed in stochastic form, the observed **A**, **B**, **C** and **d** are estimators and their values are subject to a probability distribution. It follows from this that for observation or testing of the theoretically expected properties statistical methods of observation and aprioristic criteria for appraising the result of observation are again required. This will be considered at greater length in Section 4.

The empirical content of a theory is equal to the set of refuting basic statements.[31] For every theoretically expected property for which an observation procedure exists with such criteria that the method is not a success by definition, the number of refuting basic statements of the theory concerned can increase by at least one. According as acceptable observation procedures become available for a greater number of theoretical properties, the empirical content of a theory thus increases. We shall endeavour to test the specific theories with the greatest possible empirical content.

The law derived from the specific theory with the assumption of additivity may be formulated as follows: 'All choice actions by the consumers in Sweden and in Switzerland in the years since the Second World War are such that the demand behaviour of the representative consumer for the three groups stated and the indices to be used

1. results in a satisfactory observation of (39) in stochastic form;
2. has for the observed system (39) property (I);
3. has for system (39) property (IIa) or (IIb).'

As stated above, this law can be called refutable with the assumptions mentioned. If we next assume that there are acceptable observation procedures for properties (I) and/or (IIa) or (IIb) and that it has been decided to use such criteria that application of the method(s) does not lead to success by definition, its empirical content increases. For a second refutable basic statement can be formulated as follows:

'The demand equations (39) successfully observed in Switzerland do not possess the expected property (I).'

Next the specific theory without the assumption of additivity can be dealt with in a briefer compass. On the basis of the assumption that a system (39) can be observed and that this observation yields values for **A**, **B**, **C** and **d**, the following properties do not hold good by definition:

– matrix **B** is symmetrical; (III)

31 See Section 2.1.

– matrix **B** is negative definite; this property implies that all elements on the main diagonal of **B** are negative; (IVa) or:
– matrix **B** satisfies (47). (IIb)

The law to be verified, derived from the specific theory without the assumption of additivity, can be formulated in analogous fashion. The conclusions are then:

1. the result of observation of (39) in stochastic form will satisfy the criteria of the observation procedure;
2. the observed system (39) has property (III);
3. the observed system (39) has property (IVa) or (IIb).

The law can again be called refutable if we assume that there is an acceptable observation procedure for (39) and *a priori* criteria have been laid down. Its empirical content increases if there are acceptable procedures for (III) and/or (IVa) or (IIb) respectively and it has likewise been decided *a priori* to use criteria for appraising the result of observation that do not lead to success by definition.

4 THE OBSERVATION PROCEDURES

4.1 *The Estimation Method*

By estimation method a procedure is meant which is followed to observe the system of equations (39). To find such a procedure use is made of the methods developed in econometrics for estimating equations: after all, this estimating may be interpreted as observing equations.

The system of equations to be estimated (39) contains twice in one term both an unknown parameter and an unknown coefficient. This makes direct estimation of these equations impossible. Houthakker and Taylor have developed a procedure that makes it possible to obtain estimates of both the unknown parameter and the unknown coefficients.[32] However, they confine themselves to describing the estimation procedure, without indicating what value may be attached to the results obtained from the estimating and without going into the limitations of the procedure. It has been attempted below to supply this deficiency to some extent. But first a description of the estimation procedure will be given.

The system of equations to be estimated is:

$$\mathbf{q}_t = \mathbf{A}\mathbf{q}_{t-1} + \mathbf{B}\lambda_t \cdot \mathbf{p}_t + \mathbf{C}\lambda_{t-1} \cdot \mathbf{p}_{t-1} + \mathbf{d} \tag{39}$$

where:

$$\mathbf{p}_t'\mathbf{q}_t = \mu_t \tag{57}$$

32 See Houthakker and Taylor (1970), pp. 201–204. We have used the procedure they describe for estimating the system of equations.

According to (39) and (57):

$$\lambda_t = \frac{\mu_t - \mathbf{p}_t'\mathbf{A}\mathbf{q}_{t-1} - \mathbf{p}_t'\mathbf{C}\lambda_{t-1}\mathbf{p}_{t-1}' - \mathbf{p}_t'\mathbf{d}}{\mathbf{p}_t'\mathbf{B}\mathbf{p}_t} \tag{58}$$

If the values of λ_t $(t = 1, 2, \ldots, n)$ are known, equation (39) is easy to estimate. But these values are not known. Now so as to be able to estimate (39) all the same, Houthakker and Taylor (1970) devised the following procedure.

For each λ_t an arbitrary value, $\hat{\lambda}_t^1$ $(t = 1, 2, \ldots, n)$, is inserted and (39) is estimated for each group of goods. The estimated values of **A**, **B**, **C** and **d** – *viz.* $\hat{\mathbf{A}}^1$, $\hat{\mathbf{B}}$, $\hat{\mathbf{C}}^1$ and $\hat{\mathbf{d}}^1$ – are substituted in (58).[33] One then obtains as estimate of λ_t:

$$\hat{\lambda}_t^2 = \frac{\mu_t - \mathbf{p}_t'\hat{\mathbf{A}}^1\mathbf{q}_{t-1} - \hat{\lambda}_{t-1}^2\mathbf{p}_t'\hat{\mathbf{C}}^1\mathbf{p}_{t-1} - \mathbf{p}_t'\cdot\hat{\mathbf{d}}^1}{\mathbf{p}_t'\hat{\mathbf{B}}^1\mathbf{p}_t} \tag{59}$$

$\hat{\lambda}_t^2$ is substituted in (39); the equation is estimated again and new estimated values are obtained for **A**, **B**, **C** and **d** – $\hat{\mathbf{A}}^2$, $\hat{\mathbf{B}}^2$, $\hat{\mathbf{C}}^2$ and $\hat{\mathbf{d}}^2$. In this way one obtains after l iterations:

$$\hat{\lambda}_t^1 = \frac{\mu_t - \mathbf{p}_t'\hat{\mathbf{A}}^{l-1}\mathbf{q}_{t-1} - \hat{\lambda}_{t-1}^l\mathbf{p}_t'\hat{\mathbf{C}}^{l-1}\mathbf{p}_{t-1} - \mathbf{p}_t'\hat{\mathbf{d}}^{l-1}}{\mathbf{p}_t'\hat{\mathbf{B}}^{l-1}\mathbf{p}_t} \tag{60}$$

as estimated value of λ_t.

The intention of this procedure is to find a collection of lambdas 'such that the estimated values for the individual items of expenditure add up to total expenditure,' (Houthakker and Taylor (1970), p. 201), this being known as the collection of real lambdas.

Now one goes on choosing new values for lambda until the estimated total expenditure does not differ by more than a fraction ε from the actual total expenditure. In mathematical form one gets the requirement:

$$|\mathbf{p}_t'\hat{\mathbf{q}}_t - \mu_t| < \varepsilon\cdot\mu_t \quad \text{for } t = 1, \ldots, m \tag{61}$$

The collection of lambdas corresponding to the estimated total expenditure that satisfies the requirement formulated in equation (61) is accepted as a good estimator of the collection of real lambdas. The values of $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{d}}$ are then seen as good estimators of **A**, **B**, **C** and **d**. In the further consideration of this procedure various problems are encountered.

33 The circumflex above a symbol indicates that it relates to an estimated value. The index above the symbol indicates the number of the iteration. Thus $\hat{\mathbf{A}}^l$ is the estimated value of **A** at the l-th iteration.

The first problem is whether the procedure described above does in fact lead to a collection of lambdas satisfying the above criterion. This will be discussed below.

By definition:

$$\hat{\lambda}_t^l = \hat{\lambda}_t^{l-1} + \frac{(\mu_t - \mathbf{p}_t'\hat{\mathbf{q}}_t^{l-1}) - (\hat{\lambda}_{t-1}^l - \hat{\lambda}_{t-1}^{l-1})\cdot \mathbf{p}_t'\hat{\mathbf{C}}^{l-1}\mathbf{p}_{t-1}}{\mathbf{p}_t'\hat{\mathbf{B}}^{l-1}\mathbf{p}_t} \tag{62}$$

This can be rewritten as:

$$\hat{\lambda}_t^l = \hat{\lambda}_t^{l-t} + \frac{1}{\dfrac{\partial \mathbf{p}_t'\hat{\mathbf{q}}_t^{l-1}}{\partial \hat{\lambda}_t^{l-1}}} \times$$

$$\times \left\{ (\mu_t - \mathbf{p}_t'\hat{\mathbf{q}}_t^{l-1}) - \frac{\partial \mathbf{p}_t'\hat{\mathbf{q}}^{l-1}}{\partial \hat{\lambda}_{t-1}^{l-1}} (\hat{\lambda}_{t-1}^l - \hat{\lambda}_{t-1}^{l-1}) \right\} \tag{63}$$

Equation (63) indicates that so much is added to the 'old' value of lambda that the 'new' value of lambda would made up the difference between the estimated total expenditure and the actual total expenditure. Allowance is made for this difference in the term

$$(\mu_t - \mathbf{p}_t'\hat{\mathbf{q}}_t^{l-1})$$

of (63). This difference is then corrected by the term

$$\frac{\partial \mathbf{p}_t'\hat{\mathbf{q}}_t^{l-1}}{\partial \hat{\lambda}_{t-1}^{l-1}} (\hat{\lambda}_{t-1}^l - \hat{\lambda}_{t-1}^{l-1})$$

that indicates which part of the difference has already been made up by the change in lambda in the preceding period. The corrected difference is then multiplied by:

$$\frac{1}{\dfrac{\partial \mathbf{p}_t'\hat{\mathbf{q}}_t^{l-1}}{\partial \hat{\lambda}_t^{l-1}}}$$

to ensure that the overall effect of the change in lambda is such that this corrected difference is made up. The reason why the difference is not made up right away, *i.e.* why there are several iterations, is that the estimated values of the coefficients also change with each iteration. The difference will not be made up until these values remain approximately constant upon a new iteration.

This can also be seen if one follows the proposal of Houthakker and Taylor[34] to estimate the following equation instead of equation (39):

$$\mathbf{q}_t = \mathbf{A}\cdot\mathbf{q}_{t-1} + \lambda_t^*\cdot\mathbf{B}\cdot\mathbf{p}_t^* + \lambda_{t-1}^*\cdot\mathbf{C}\cdot\mathbf{p}_{t-1}^* + \mathbf{d} \tag{64}$$

where $\lambda_t^* = \lambda_t\cdot\mu_t$ and $\mathbf{p}_t^* = \mathbf{p}_t/\mu_t$.

In that case the value of lambda must not be estimated in accordance with (60) but in accordance with

$$\lambda_t^{*l} = \frac{1 - \mathbf{p}_t'^*\hat{\mathbf{A}}^{l-1}\mathbf{q}_{t-1} - \lambda_{t-1}^{*l}\mathbf{p}_t'^*\hat{\mathbf{C}}^{l-1}\mathbf{p}_{t-1}^* - \mathbf{p}_t^{*\prime}\mathbf{d}^{l-1}}{\mathbf{p}_t'^*\hat{\mathbf{B}}^{l-1}\mathbf{p}_t^*} \tag{65}$$

Equation (61) can now be rewritten as:

$$|\mathbf{p}_t'^*\cdot\mathbf{q}_t - 1| < \varepsilon \tag{66}$$

This can be converted into:

$$\begin{aligned}|(1-\delta_t) + \mathbf{p}_t'^*(\hat{\mathbf{A}}^l - \delta_t\hat{\mathbf{A}}^{l-1})\mathbf{q}_{t-1} + \mathbf{p}_t'^*(\hat{\mathbf{d}}^l - \delta_t\mathbf{d}^{l-1}) + \\ + \lambda_{t-1}^*\mathbf{p}_t'^*\cdot(\hat{\mathbf{C}}^l - \delta_t\hat{\mathbf{C}}^{l-1})\mathbf{p}_{t-1}^*| < \varepsilon\end{aligned} \tag{67}$$

where:

$$\delta_t = \frac{\mathbf{p}_t'^*\hat{\mathbf{B}}^l\mathbf{p}_t}{\mathbf{p}_t'^*\hat{\mathbf{B}}^{l-1}\mathbf{p}_t^*}$$

It can now be seen that, if the estimated values of the coefficient matrices barely change with successive iterations, (61) will indeed have been satisfied.

A second problem is that when estimating the values of lambda in accordance with equation (65) the value for lambda for each iteration in the first period must be known. It will now be demonstrated that this value must be equal to the value of lambda in the first period at the first iteration.

It can be proved[35] that increasing λ_t^{*1} $(t = 1, 2, \ldots, m)$ by a fraction r results in an increase in the ultimate estimates of λ_t^* $(t = 1, 2, \ldots, m)$ by a fraction r, while the ultimate estimates of **B** and **C** are reduced by a fraction r and the ultimate estimates of **A** and **d** remain unchanged.

34 Houthakker and Taylor (1970), p. 202. The argument adduced for this is that the disturbances than have less of an effect on the estimating procedure. However, we do not see why this is so. Nevertheless, since the procedure is simplified, we follow their proposal.

35 Maks and Muysken (1974), pp. III, 6.

An important conclusion is therefore that as regards λ_t $(t = 1, 2, \ldots, m)$ one estimates only the relation between the elements; no sognificance may be attached to the estimated values of the elements, viewed absolutely!

Since the relation between the lambdas is determined, the values of all lambdas are determined once the value for lambda in the first period is known. The same applies to elements **B** and **C**. This means that the estimated *values* of **B** and **C** correspond to a certain *value* for the lambda in the first period, namely the value imposed by the investigator. Now if one proceeds to determine the value of the lambdas for a following iteration in accordance with equation (65), one utilizes the *values* of **B** and **C**. As value for lambda in the first period one must then again choose the same value as that from the preceding iteration, since only at that value do the estimated *values* of **B** and **C** hold good. This means that with each iteration for lambda in the first period one must choose the same value, *i.e.*:

$$\hat{\lambda}_1^{*l} = \hat{\lambda}_1^{*1} \qquad (l = 2, 3, \ldots, m) \tag{68}$$

A third problem proceeds from the question whether, if one chooses two totally different sets of starting values[36] for lambda, one nevertheless gets two results that give the same relations between λ_t^* $(t = 1, 2, \ldots, m)$ and between the elements of **B** and **C**, and the same values for **A** and **d**. It will be clear that the answer to this question determines whether or not there is any point to the estimation procedure. For if the relation between the elements of **B** and **C** depends on the set of starting values chosen, then perhaps a set of starting values may be found such that **B** is symmetrical; in that case one can never test. However, it proves to be anything but easy to demonstrate the independence of the set of starting values, which is why this will be dealt with in a later study.

A fourth problem is that the whole estimating procedure is based on the intuition that if only one makes the tolerance interval, ε, small enough the estimated coefficient values can hardly differ from the actual coefficient values. This can be illustrated by rewriting the requirement formulated in equation (61) as follows:

$$\begin{aligned} &|\mathbf{p}_t'^*[\hat{\mathbf{A}}^l - \mathbf{A}]\mathbf{q}_{t-1} + \mathbf{p}_t'^*[\hat{\lambda}_t^{*l} \cdot \hat{\mathbf{B}}^l - \lambda_t^*\mathbf{B}]\mathbf{p}_t^* + \\ &\quad + \mathbf{p}_t'^*[\hat{\lambda}_{t-1}^{*l}\mathbf{C}^l - \lambda_{t-1}^*\mathbf{C}]\mathbf{p}_{t-1}^* + \mathbf{p}_t'^*[\hat{\mathbf{d}}^l - \mathbf{d}]| < \varepsilon \qquad t = (1, \ldots, m) \end{aligned} \tag{69}$$

We have not yet succeeded in further supporting the intuition by for instance proof that $\hat{\mathbf{A}}^l$, $\hat{\mathbf{B}}^l$, $\hat{\mathbf{C}}^l$ and $\hat{\mathbf{d}}^l$ converge towards their real values if l becomes

36 By the set of starting values for lambda is meant the set $\lambda_1^1, \lambda_2^1, \ldots$ In this study $\lambda_t^1 = 100$ $(t = 1, 2, \ldots, m)$ is used.

greater. It should, however, be emphasized that the requirement formulated in (69) must hold good for every period.

The investigator is of course free in the choice of ε. Houthakker and Taylor use a tolerance interval of 0.0005; in this study we also use this interval.

A fifth problem is that there is a danger of the results of estimating being obscured by the presence of autocorrelation. The influence of autocorrelation will be investigated for each equation with the aid of the Durbin-Watson test. However, as a result of the small number of degrees of freedom, and also for simplicity's sake, we continue to estimate the equations with the aid of the method of least squares.[37]

It emerges from the above that a satisfactory answer could not already be given to all questions concerning this estimating procedure. Nevertheless, we shall accept and use the method for the time being, partly because it has already been used by others.

This brings us next to the criteria that have to be laid down for appraising the result of observation. In estimating with the aid of the method of least squares one often looks at the value of the correlation coefficient and the standard deviation of the estimated coefficients, finding expression in the t values. At the same time the presence of autocorrelation can be investigated by means of the Durbin-Watson test. However, it is possible to use these test statistics as criteria only if further assumptions are made about the distribution of the disturbances. Since neither theory says anything about the distribution of the disturbances, we assume *a priori* that the disturbances are normally distributed and independent of each other. These assumptions make it possible to utilize the test statistics mentioned above as criteria. In general the aprioristic criteria used for accepting the result of estimating as satisfactory are that the correlation coefficients must be sufficiently high (often a value exceeding 0.8 is meant by this), that an adequate number of coefficient values must be significant ones and that the Durbin-Watson test statistic must lie within the critical values.

When the results of estimating are presented, the values of the above quantities will be given. However, it must not be forgotten in this context that the estimation procedure itself ensures that the value of the correlation coefficient will be very high. Moreover, the significance that may be attached to the Durbin-Watson test statistic must be relativized,[38] because in the set of equations to be estimated one of the explanatory variables is a delayed form of the variable to be explained.

37 Reference may also be made to Mattei (1971), p. 267, where he argues that: 'it is perhaps still better to use direct least squares.'

38 However, this does not directly imply that this quantity is now completely useless; see for instance Christ (1968), p. 527.

As aprioristic criteria we now formulate that

- the correlation coefficients must be greater than 0.80,
- the Durbin-Watson test statistics must lie within the critical value at a significance value of five percent and
- at least one coefficient value corresponding to an explanatory variable must be significant per estimated demand equation at a level of five percent.

4.2 *The Testing Procedure*

If the observation of the system of demand equations has been found acceptable, it still remains to be seen whether the coefficients of the equation satisfy the properties that have been developed in the preceding chapter. The procedures by which this is examined are called testing procedures.

In the first instance the theoretical properties are (see Section 3.5):

I. the diagonality restriction: matrices $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ are diagonal;

IIa. the negativity restriction: all elements on the principal diagonal of $\hat{\mathbf{B}}$ are negative; or:

IIb. matrix $\hat{\mathbf{B}}$ satisfies equation (47).

If the system of equations proves not to satisfy these properties after an acceptable testing procedure has been developed, the theory with the assumption of additivity is rejected. It can then be investigated whether the system satisfies the properties that have been derived from the theory without the assumption of additivity. These properties are (see Section 3.5):

III. the symmetry restriction: matrix $\mathbf{B}$ is symmetrical;

IVa. the negativity restriction: matrix $\mathbf{B}$ is negative definite; a prerequisite of this is that all elements on the principal diagonal of $\hat{\mathbf{B}}$ are negative; or:

IIb. matrix $\hat{\mathbf{B}}$ satisfies equation (47).

As regards the negativity restriction (IIb), we have not succeeded in finding an acceptable testing procedure: insofar as the theory proceeds from an ordinal utility concept, this theoretical property cannot be observed. However, the negativity restriction (IIa) *can* be observed: if the elements on the principal diagonal of $\mathbf{B}$ are significantly positive, the system does not satisfy this property. As regards the negativity restriction (IVa) it can only be investigated whether one necessary condition has been satisfied that corresponds to property (IIa). Insofar as the theory starts from a cardinal utility concept, this property can therefore be observed on the assumption of additivity and without this assumption it can at most be observed that this property is not present.

For the diagonality restriction (I) and the symmetry restriction (III) Theil[39] has given an acceptable testing procedure. He has developed a test statistic that

39 See Theil (1971), pp. 314 *et seq.*

is F-distributed and tests the null hypothesis that the coefficients of an estimated set of equations satisfy the restrictions imposed.

In this statistic the variance-covariance matrix of the disturbances occurs. This matrix can be approached in two ways. The first approach is the 'conventional' one: one takes the variance-covariance matrix of the disturbances. The second approach assumes that the assumptions on probability distribution must tally with the restrictions to be tested. The disturbances are therefore estimated on the basis that the restrictions laid down have been satisfied, and then the estimated variance-covariance matrix of the disturbances is determined. We do not consider that either of the two approaches automatically enjoys preference; we shall therefore use both approaches of the variance-covariance matrix and in each case estimate the test statistic twice.

So as to be able to appraise the results of testing we therefore establish the following aprioristic criteria: if the test statistic in both cases assumes a value lying below the five percent limit of the F table, the restriction is considered to have been accepted; if this statistic assumes in both cases a value lying above this limit, the restriction is considered to have been rejected; if in the one case a value is obtained lying below the limit and in the other case a value lying above the limit, one of the two approaches will have to be opted for. However, in testing the result always proves to be identical.

In testing the diagonality restriction the null hypothesis is tested that the non-diagonal elements of matrices $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ are nil; in testing the symmetry restriction the null hypothesis is tested that matrix $\hat{\mathbf{B}}$ is symmetrical. It is clear that the fact that with the aid of the estimation procedure only the relation between the elements of **B** and **C** can be determined does not have an effect on the properties of the diagonality for **A**, **B** and **C** and of symmetry for **B**. This therefore does not stand in the way of testing of these properties.

4.3 *Are the Ultimately Formulated Theories Refutable?*

In Section 3.5 it was noted that the ultimately formulated specific demand theories can be called refutable if acceptable methods of estimating and testing are available. A few comments on the acceptability of the estimation method have been made above. In our opinion it is acceptable if, with a normal distribution of the disturbances, the estimators are unbiased, or at any event consistent. A further examination of these properties is therefore necessary. In brief, this amounts to a further analysis of:

- the formal properties of the estimators,
- the sensitivity of the estimators to the collection of starting values,
- the sensitivity of the estimators to the tolerance interval.

Nevertheless we decided to accept the procedure for the time being, partly because it has already been used by others.

In our opinion, for the theoretically expected properties there are acceptable testing procedures available for the diagonality restriction (I) of the specific theory with additivity assumptions and for the symmetry restriction (III) of the specific theory without additivity. As regards the negativity restriction (IIb) of the two theories, we have not succeeded in finding an acceptable testing procedure. However, starting from a cardinal utility concept the negativity restriction (IIa) for the theory with additivity can be acceptably observed. For the theory without additivity it can only be investigated whether one condition necessary for this property has been satisfied or not.

In our opinion both the estimating procedure and a number of methods of testing are therefore acceptable for both theories. Aprioristic criteria have been formulated so as to be able to appraise the result on acceptance or rejection of the property. Moreover, it is clear that the procedures accepted have not been formulated in such a way that they always yield a result satisfying the criteria laid down. The conditions of validity have also been operationalized. Consequently, in our opinion operational refuting basic statements can be formulated for both theories.[40] To our way of thinking they have empirical content and are thus refutable.

5 THE RESULTS OF OBSERVATION FOR SWEDEN AND SWITZERLAND

5.1 *Introduction*

It is clear that on account of the number of degrees of freedom when estimating the system of demand equations (39) one is obliged when specifying these not to make a distinction between too many groups of goods. We use the rule of thumb that the number of explanatory variables in any case may not exceed half the number of data. This means that in most cases a distinction can be made between only three, or at most four, groups of goods. It should be realized that, to give the theory more empirical content, we do not start from the assumption of additivity, but set out to test it. If, in emulation of Mattei and Houthakker and Taylor, we were to proceed *a priori* from this assumption, then of course a distinction can be made between many more groups of goods. However, the empirical content of the theory would then be reduced.

In this study, since the data relate to a large number of groups of goods, we have summarized these data into information relating to three groups of goods,

40 See Section 3.5.

viz. consumer durables, consumer non-durables and services. For the nominal expenditure that is simple: one need only add up the expenditure on those groups of goods being combined to learn the expenditure of the summarizing group. For the price index figure this is not so easy, since it is an index figure. In this study the price index figure for a summarizing group has been calculated in accordance with a method using by Barten.[41]

The quantities in the equations must be interpreted as the real expenditure per capita on a certain group. This expenditure is obtained by dividing the nominal expenditure on that group by its price index figure ($\times \frac{1}{100}$) and by the size of the population.

A problem with which one is confronted is the presence of 'trend breaks' in the time series: the series often display a different nature before and after the Second World War. For this reason we have confined ourselves to time series relating to the period after the Second World War.

In what follows a description will successively be given of the data used for estimating the system of demand equations, of the results of estimation and of the findings from testing of these results.

5.2 *The Data Used*

Below we shall consider in some detail the data have been used for estimating the system of equations. These data relate to Sweden and Switzerland.

5.2.1 Sweden

The data for Sweden have been taken from 'Den privata konsumtionen 1931–1975,' C. J. Dahlman and A. Klevmarken, Stockholm, 1971.

These data concern price index figures (1964 = 100) and expenditure in current prices for nine groups of goods for the period from 1931 to 1968. The data for the groups 'Livsmedel,' 'Drycker och Tabak' and 'Beklädnad'[42] have been summarized in accordance with the method described above into data on the group of consumer non-durables. The data for the groups 'Bostad,' 'Rekreation,' 'Sjukvård och hygien' and 'Ovriga Varor och Tjämster'[43] have likewise been summarized into data on the group of services. The group 'Hushallsutrustning'[44] has remained a separate group: this may be regarded as the group of consumer durables. Of these data, only those relating to the period

41 Barten (1966), pp. 10 *et seq.*

42 Food, tobacco and stimulants, and clothing respectively.

43 Upkeep of the home, cars and travel, recreation, medical care and health articles and services, and other goods and services respectively.

44 Furnishings.

from 1947 to 1968 are used. The population figures have been taken from various issues of the Demographic Yearbook of the United States.

5.2.2 Switzerland[45]

The data for Switzerland were supplied on request by the Bureau Fédéral de Statistique. These data relate to price index figures (1958 = 100) and expenditure in current prices for the period from 1948 to 1969. The data for the groups 'Nährungsmittel,' 'Genussmittel,' 'Bekleidung' and 'Heizung und Beleuchtung'[46] have been summarized in accordance with the method described above into data on the group of consumer non-durables. The data for the groups 'Miete und kleine Unterhaltungskosten,' 'Reinigung,' 'Gesundheidspflege,' 'Verkehrsausgaben,' 'Bildung und Erholung,' 'Versicherung,' 'Dienstbotenlöhne,' 'Ausgaben im Ausland' and 'Verschiedenes'[47] have been summarized into data on the group of services. The group 'Wohnungseinrichtung'[48] has remained a separate group; this may be regarded as the group of consumer durables. The population figures have been taken from various issues of the Demographic Yearbook of the United Nations.

5.3 *The Estimation Results*

5.3.1 General

The results will be given below of estimating the system of demand equations for Sweden and Switzerland. The table in which these data are presented has at all times the following structure:

(1) column – the groups of goods;
(2) column, (**A**) –
(3) column, (**B**) –
(4) column, (**C**) –
(5) column, (**d**) – the estimated value of the elements of matrices **A**, **B**, **C** and **d** that correspond to these groups;
(6) column, (R^2) – the determination coefficient;
(7) column, (S^2) – the sum of the squared estimated disturbances;
(8) column, (DW) – the value of the Durbin-Watson test quantity.[49]

45 These data have also been used by Mattei (1971).
46 Food, stimulants, clothing, and heating and lighting respectively.
47 Rent and costs of minor maintenance, cleaning, health care, expenditure on transport, education and recreation, insurance, servants' wages, expenditure abroad, and miscellaneous respectively.
48 Furnishings.
49 It should be realized that the value of this quantity is not exit because one of the explanatory variables is a delayed form of the variable to be explained.

TABLE 3.1 – COEFFICIENT VALUES AND OTHER RELEVANT STATISTICS FROM ESTIMATING THE SYSTEM OF DEMAND EQUATIONS FOR SWEDEN, 1947–1968

1	2			3			4			5	6	7	8
	A			**B**			**C**			**d**			
variable explained	(coefficients for quantity $t-1$)			(coefficients for prices t)			(coefficients for prices $t-1$)			constant	R^2	S^2	DW
	N-D.	D.	S.	N-D.	D.	S.	N-D.	D.	S.				
N-D.	.60* (.30)	.73* (.69)	−.09* (.27)	−197477 (39210)	23340* (27925)	23635* (31300)	88992* (59152)	23030* (29717)	−21012* (39250)	2233* (2353)	.992	3952	2.14
D.	.37 (.10)	.27* (.23)	.16* (.09)	−34813 (13410)	6579* (9550)	−24081 (10705)	68552 (20230)	−9662* (10163)	40972 (13423)	−2191 (805)	.994	111	2.49
S.	.32* (.29)	−.16* (.66)	.58 (.26)	−37757* (37899)	1486* (26991)	−143560 (30254)	79582* (57174)	−45734* (28723)	77380 (37937)	1340* (2275)	.999	889	2.15

N-D.: consumer non-durables
D.: consumer durables
S.: services

The standard deviations of the estimators are placed in all cases below the estimated values in parentheses. The estimators that do not prove to differ significantly from zero at a significance level of 5% are designated by an asterisk. It should be realized that it is in no way strange to encounter high determination coefficients since the estimation procedure attends to a good adjustment.

5.3.2 Sweden (1947–1968)

As regards estimation of the system of demand equations for Sweden, a satisfactory result was already obtained after two iterations. The results are presented in Table 3.1 and the values for lambda in Table 3.2.

It is striking that in the explanation of expenditure on consumer non-durables only the coefficient for the prices of that category from the same period is significant. It is also striking that in the explanation of expenditure on services the coefficients for variables that relate to other categories do not significantly differ from zero.

TABLE 3.2 – VALUE OF THE UNKNOWN PARAMETER FOR SWEDEN 1947–1968

Year	$\hat{\lambda}$
1947	.0391
1948	.0361
1949	.0359
1950	.0342
1951	.0299
1952	.0276
1953	.0268
1954	.0256
1955	.0242
1956	.0228
1957	.0218
1958	.0207
1959	.0199
1960	.0187
1961	.0175
1962	.0162
1963	.0150
1964	.0137
1965	.0125
1966	.0119
1967	.0122
1968	.0106

The estimated coefficient values proved to satisfy neither the diagonality restriction nor the symmetry restriction. As regards the diagonality restriction, F values of 2.93 and 28.50 were found from Theil's approach and the 'conventional' approach respectively. The critical value is $F_{18.33} = 1.90$. As regards the symmetry restriction, F values of 4.86 and 26.09 respectively were found. The critical value is $F_{3.33} = 2.89$.

As regards the negativity restriction, it may be noted that none of the elements on the principal diagonal of **B** is significantly positive.

5.3.3 Switzerland (1949–1969)

Here too a result was obtained after three iterations. However, this result was not satisfactory, since only two coefficient values differ significantly from nil.

In anticipation of a following study we then investigated whether the results obtained with a smaller tolerance interval might perhaps display more significant coefficient values. It proved that up to a tolerance interval of 0.00007451 the unsatisfactory result was always obtained after three iterations. However, at a tolerance interval of 0.00007450 no result had yet been obtained after 1600 iterations.

Instead of experimenting still further with the tolerance interval in the ninth decimal – which requires a very great deal of computer time – we investigated, again anticipating a following study, whether a different choice of the set of starting values might not yield better results. The choice fell on that in which the starting value, $\hat{\lambda}^1$, is for each period identical with a constant divided by the real expenditure from the period (originally this was a constant divided by the nominal expenditure). The constant was again put at 100.

At a tolerance interval of 0.00009 a satisfactory result was obtained after 105 iterations. It is presented in Table 3.3. The corresponding lambdas appear in Table 3.4. It will be seen that the majority of the coefficient values are now significant. The significant values do not form a striking pattern.

As in Sweden, the coefficient values proved to satisfy neither the diagonality restriction nor the symmetry restriction. As regards the diagonality restriction, F values were found of 3.33 and 2624.61 by Theil's approach and the 'conventional' approach respectively. The critical value is $F_{18.30} = 1.95$. As regards the symmetry restriction, F values of 9.28 and 332.37 respectively were found. The critical value is $F_{3.30} = 2.92$.

As regards the negativity restriction, it may be noted that none of the elements on the principal diagonal of **B** is significantly positive.

TABLE 3.3 – COEFFICIENT VALUES AND OTHER RELEVANT STATISTICS FROM ESTIMATING THE SYSTEM OF DEMAND EQUATIONS FOR SWITZERLAND, 1949–1969

1	2			3			4			5	6	7	8
	A			**B**			**C**			**d**			
variable explained	(coefficients for quantity $t-1$)			(coefficients for prices t)			(coefficients for prices $t-1$)			constant	R^2	S^2	DW
	N-D.	D.	S.	N-D.	D.	S.	N-D.	D.	S.				
N-D	.56* (.31)	−1.15* (1.37)	.47* (.40)	64887* (38972)	10998* (26611)	−170168 (63348)	64752 (23061)	−135291 (45110)	134966 (45110)	989* (1246)	.999	117	2.54
D.	.43 (.15)	−1.48 (.64)	.55 (.19)	68184 (18324)	23815* (29785)	−123731 (29785)	10795* (10843)	−75967 (21210)	130054 (28175)	−1994 (586)	.997	28	2.67
S.	1.31 (.33)	−4.12 (1.42)	1.69 (.42)	154122 (40616)	33289* (27734)	−272284 (66021)	3718* (24035)	−136034 (47013)	299429 (62452)	−4705 (1298)	.999	127	2.3

N-D.: consumer non-durables
D.: consumer durables
S.: services

TABLE 3.4 – VALUE OF THE UNKNOWN PARAMETER FOR SWITZERLAND 1949–1969

Year	$\hat{\lambda}$
1949	.0291
1950	.0278
1951	.0274
1952	.0268
1953	.0263
1954	.0253
1955	.0244
1956	.0231
1957	.0230
1958	.0230
1959	.0219
1960	.0213
1961	.0196
1962	.0180
1963	.0167
1964	.0155
1965	.0141
1966	.0130
1967	.0119
1968	.0109
1969	.0100

5.4 *Conclusion*

It proved possible to find an acceptable result of observation for Sweden in the 1948–1967 period. As regards the theory with the assumption of additivity, it proved that the diagonality restriction had to be rejected; the negativity restriction (IIa) was not rejected. As regards the theory without the assumption of additivity, it proved that the symmetry restriction must also be rejected. Thus the theory is rejected both with and without the assumption of additivity for Sweden in the 1948–1967 period.

For Switzerland in the 1949–1969 period no satisfactory result can be found by means of the estimating procedure used: hardly any coefficient value differs significantly from nil. However, on the strength of another set of starting values and a different tolerance interval, in anticipation of a following study, an acceptable result of observation was obtained. With regard to the restrictions, precisely the same results were found as in the case of Sweden. Thus the theory, both with and without the assumption of additivity, is also rejected for Switzerland in the 1949–1969 period.

5.5 *Some Considerations with Reference to the Results of Estimation and Testing*

The theories which we used were assumed to hold good for the kinds of goods and indices used. However, an indefinite number of similar theories for different kinds of goods and different kinds of price indices can also be formulated. It cannot be ruled out in advance that testing of one of these theories would give a more satisfactory result. A valuable supplementation of the theory in this respect would be a number of indications that could be used empirically for the kinds of goods and indices to be utilized. As stated in Section 3.3, the notions developed so far from consistent aggregation cannot be used empirically.

In addition, in the specification of the utility function, in the definition of its domain or in the assumptions regarding the 'state variable,' many further variations are conceivable, possibly usable and perhaps more successful in empirical research.

Another possibility, one which precisely our aposterioristic approach may not exclude, is that the representative consumer does not indulge in utility-maximizing behaviour. This assumption implies that the principle of utility maximization will never lead to successful empirical expenditure theories. However, to the extent that this point of departure has been elaborated in operational theories, the results are not exclusively negative.[50] It should nevertheless be borne in mind that the various investigators, in addition to using separate models, do not always utilize the same groups and the same kinds of indices.

The conclusion that we wish to attach to the above is as follows. For further insight into the relevance of the considerations stated here and notably into the question of which kind of indices can best be used and which specifications of the utility function are the most satisfactory, it seems desirable that, firstly, the general utility maximization theory be supplemented by a usable indication of the indices to be used in empirical research and, secondly, that separate theories be tested on the same data material. In a following study we shall, if possible, devote attention to his.

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50 See among others Barten (1966, 1967, 1969), Barten and Geyskens (1974), Byron (1968, 1970 I, 1970 II), Deaton (1975), Court (1967), Houthakker and Taylor (1970), Luch (1971), Mattei (1971).

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Summary

DEVELOPMENT AND TESTING OF A DYNAMIC DEMAND THEORY

In this study a refutable theory is formulated that endeavours to explain expenditure on certain groups of goods in a number of countries. The theory is a general dynamic theory of utility maximization in which both stockpiling and habitforming may play a role. After a number of supplementary assumptions the theory leads to the dynamic demand model of Houthakker and Taylor.

Both the conditions on which the ultimately formulated theory is refutable and the estimation and testing procedures are scrutinized against the background of a concisely formulated scientificophilosophical point of departure. Finally the theory is tested with the aid of data for Sweden and Switzerland. It turns out that the theory has to be rejected.