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# SIMULTANEITY OF ISSUES AND ADDITIVITY IN BARGAINING

BY HANS PETERS[1]

Simultaneous bargaining over more issues by two bargainers is treated by taking sums of bargaining games and requiring bargaining solutions to satisfy certain (super-) additivity axioms. A (new) characterization of a family of so-called proportional solutions is given with the aid of three axioms: (partial) superadditivity, homogeneity, and weak Pareto optimality. Requiring, besides individual rationality and Pareto continuity, the axioms of restricted additivity, scale transformation invariance, and Pareto optimality, yields an alternative characterization of a family of solutions consisting of all nonsymmetric extensions of Nash's solution. Also these solutions exhibit a (limited) proportionality property. Further, the relation with the Super-Additive solution of Perles and Maschler is discussed, and also the link with Myerson's results on proportional and utilitarian solutions.

## 1. INTRODUCTION

SUPPOSE, TWO PARTIES are facing several (separate) bargaining situations, on (possibly quite) different issues. Handling these situations one by one may yield both parties only small profits. Bargaining, however, over these issues simultaneously, may yield both parties larger total profits, thus reflecting more properly their perhaps strong interests in some of these issues. The following simple example illustrates this.

EXAMPLE 1.1: Mr. $X$ and his wife each have a ticket for a magnificent movie, but, unfortunately, these tickets are not valid for the same show. Now, for each of the two shows for which one of the tickets is valid, there are three alternatives: (a) the ticket-holder watches the movie leaving his/her partner at home, which gives him/her 6 units of utility and his/her partner $-2$ units: (b) they both stay at home, but with the ticket-holder grudging the whole evening: 0 utility for both; (c) they both stay at home and play some card-game: 0 utility for the ticket-holder and 1 unit of utility for the partner. If we suppose for a moment that these utilities are additive, then Mr. $X$ as well as his wife do very well by each one using his/her ticket and receiving a net utility of 4.

In this paper, we will follow the axiomatic approach to the bargaining problem as initiated by Nash [**12**]. We restrict our attention to two-person bargaining problems. Simultaneous bargaining over more issues will be reflected in our model by taking appropriate sums of bargaining games, and its possible advantages for the players by additivity axioms for bargaining solutions.

Formally, a (two-person) *bargaining game* $S$ is a proper subset of the plane satisfying:

(1.1) $S$ is closed, convex and $\sup\{x_i;\, x\in S\}\in\mathbb{R}$ for all $i\in\{1,2\}$;

(1.2) $0(=0,0))\in S$ and $x>0$ for some $x\in S$;

[1] The author would like to express his gratitude to the referees for their valuable comments and suggestions. The proofs of Theorem 3.2 and Lemma 4.6 have been considerably improved by their suggestions.

(1.3) $S$ is comprehensive, i.e. for all $x \in S$ and $y \in \mathbb{R}^2$, if $y \leq x$ then $y \in S$.

Let $B$ denote the family of all bargaining games. When interpreting an $S \in B$, one must think of the following game situation. Two players (bargainers) may cooperate and agree on a feasible outcome $x$ in $S$, giving utility $x_i$ to player $i = 1, 2$, or they may fail to cooperate, in which case the game ends in the disagreement outcome 0. So for any $S \in B$, the disagreement outcome is fixed at 0 (which allows us to omit the usual axiom of translation invariance for bargaining solutions, which are defined below). Closedness of $S$ is required for mathematical convenience; convexity stems from allowing lotteries in an underlying bargaining situation. Further, it is assumed that $S$ is bounded from above, but not from below, since we allow free disposal of utility. The requirement $x > 0$ for some $x \in S$ serves to give each player an incentive to cooperate. Not all of the restrictions in (1.1)–(1.3) are necessary for all of our results, but assuming them simplifies matters and, moreover, none of them goes against intuition.

A (two-person) *bargaining solution* is a map $\phi$: $B \to \mathbb{R}^2$ assigning to each $S \in B$ an outcome $\phi(S) \in S$ and such that Axiom 0 holds:

AXIOM 0: *$\phi(S)$ depends only on (the shape of) $S$.*

Axiom 0 states explicitly that $\phi$ does not depend on an underlying bargaining situation (i.e., a set of lotteries and a pair of utility functions mapping these into the plane). By most authors, this is implicitly assumed or taken for granted (however, cf. Shapley [**16**]). We will explicitly use Axiom 0 in the next section.

Before introducing some further axioms for bargaining solutions, we need a few definitions and notations. A *scale transformation* $a = (a_1, a_2)$ is a vector in $\mathbb{R}^2_{++} := \{x \in \mathbb{R}^2; x > 0\}$. For $a \in \mathbb{R}^2_{++}, x \in \mathbb{R}^2$, $S \in B$, $ax := (a_1x_1, a_2x_2)$ and $aS := \{ax; x \in S\}$. For $\alpha \in \mathbb{R}$, $\alpha > 0$, $\alpha S := (\alpha, \alpha)S$. For $S, T \in B$, $S + T := \{x + y; x \in S, y \in T\}$. (Note that $aS, S + T \in B$.) For $S \in B$, $P(S) := \{x \in S$; for all $y \in S$, if $y \geq x$, then $y = x\}$ denotes the *Pareto optimal subset of $S$*, and $W(S) := \{x \in S$; for all $y \in \mathbb{R}^2$, if $y > x$, then $y \notin S\}$ denotes the *weakly Pareto optimal subset of $S$*.

Let $\phi$: $B \to \mathbb{R}^2$ be a bargaining solution. The following axioms will play an important role.

AXIOM 1 (*Individual Rationality*, IR): *$\phi(S) \geq 0$ for all $S \in B$.*

AXIOM 2 (*Pareto Optimality*, PO): *$\phi(S) \in P(S)$ for all $S \in B$.*

AXIOM 3 (*Weak Pareto Optimality*, WPO): *$\phi(S) \in W(S)$ for all $S \in B$.*

AXIOM 4 (*Scale Transformation Invariance*, STI): *$\phi(aS) = a\phi(S)$ for all $S \in B$, $a \in \mathbb{R}^2_{++}$.*

AXIOM 5 (*Homogeneity*, HOM): *$\phi(\alpha S) = \alpha\phi(S)$ for all $S \in B$, $\alpha \in \mathbb{R}$, $\alpha > 0$.*

AXIOM 6 (*Super-Additivity*, SA): *$\phi(S+T) \geqslant \phi(S)+\phi(T)$ for all $S, T \in B$.*

AXIOM 7 (*Partial Super-Additivity*, PSA): *$\phi(S+T) \geqslant \phi(S)$, $\phi(S+T) \geqslant \phi(T)$ for all $S, T \in B$.*

AXIOM 8 (*Symmetry*, SYM): *$\phi_1(S) = \phi_2(S)$ for all $S \in B$ such that $S = \{(x_2, x_1); x = (x_1, x_2) \in S\}$.*

AXIOM 9 (*Pareto Continuity*, PCO): *$\phi$ is continuous on $(B, \pi)$ where $\pi$ is the metric on $B$ defined by $\pi(S, T) = d_H(P(S), P(T))$ and $d_H$ is the Hausdorff metric.*

Note that Axiom 3 is implied by Axiom 2, 5 by 4, 7 by 6 and 1. (The last implication is the reason why we use the expression *partial* SA rather than *weak* SA: PSA is not implied by SA alone.) Axiom 6, super-additivity, was first formulated by Perles and Maschler [**13**]. Note also that the continuity axiom PCO is weaker than the continuity axiom mostly used (see, e.g., Jansen and Tijs [**5**]). As far as needed, we will discuss all these axioms in due place.

The main purpose of this paper is to find bargaining solutions which satisfy (partial) super-additivity. The following example is a translation of the example at the beginning of this section, with the extras of allowing randomization between alternatives and using a bargaining solution. It indicates that it may indeed be advantageous for both players to bargain over more issues simultaneously.

We adopt another notation: for a finite number of vectors $x^1, x^2, \ldots, x^l$ in $\mathbb{R}^2$,

$$S(x^1, x^2, \ldots, x^l) := \{y \in \mathbb{R}^2; y \leqslant x \text{ for some } x \in \operatorname{conv}\{x^1, x^2, \ldots, x^l\}\}.$$

EXAMPLE 1.2. (See Figure 1): Let $\phi: B \to \mathbb{R}^2$ be a bargaining solution satisfying *IR, WPO, SYM*. Then $\phi_i(S((0,1),(6,-2))) + \phi_i(S((1,0),(-2,6))) \leqslant 3$ for $i \in \{1,2\}$, whereas $\phi(S((0,1),(6,-2)) + S((1,0),(-2,6))) = (4,4)$. So it is clearly advantageous for both players to play both games simultaneously.

The organization of the paper is as follows. Section 2 tries to give a foundation to taking sums of bargaining games as a tool for handling simultaneous bargaining over more than one issue, and pays due attention to the super-additive solution of Perles and Maschler [**13**]. In Section 3, a family of super-additive solutions is characterized. This family does not contain the Perles–Maschler solution; it does, however, contain the so-called proportional solutions proposed by Kalai [**6**]. The result of Section 3 might be regarded unsatisfactory in one specific sense: most of the solutions characterized there do not obey Axiom 4, scale transformation invariance. As a possible remedy for this and an alternative, the super-additivity axiom is weakened in Section 4 to an axiom called restricted additivity, and this leads to a new characterization of a family containing the nonsymmetric Nash solutions (Nash [**12**], Harsanyi and Selten [**3**]). The last result is closely related to Aumann [**1**] and Shapley [**16**]. Also in Section 4, the link of the characterizations in Sections 3 and 4 with the paper of Myerson [**11**] will be discussed. Section 5 concludes with a few final remarks.

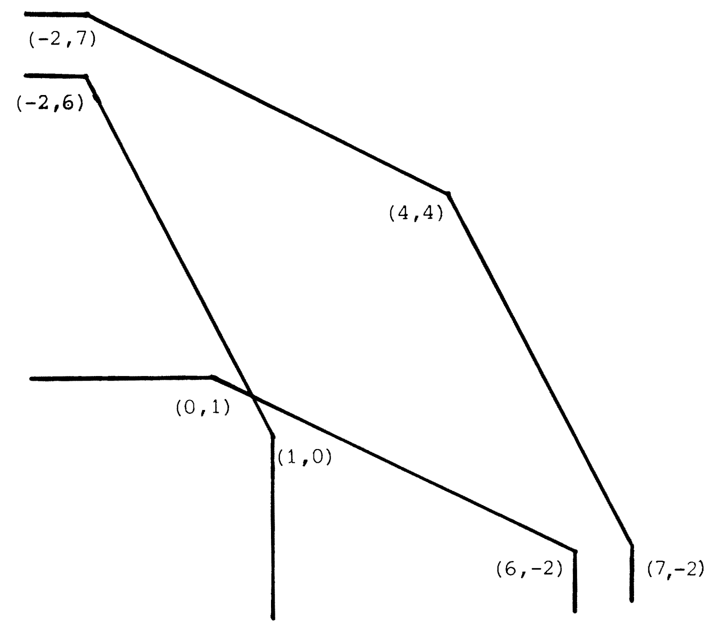


FIGURE 1

## 2. TAKING SUMS OF BARGAINING GAMES—THE SUPER-ADDITIVE SOLUTION OF PERLES AND MASCHLER

Let $P$ denote some *P*rospect space or set of pure (riskless) alternatives, containing a *disagreement alternative* $\bar{p} \in P$, and let $\mathcal{P}$ denote an appropriate mixture set of lotteries on $P$. Suppose there are two bargainers, 1 and 2, with von Neumann-Morgenstern utility functions $u^i$ $(i=1,2)$ defined on $\mathcal{P}$, such that $u^i(\bar{p})=0$. If $P$ is large enough (e.g. $P=\mathbb{R}^2$), then we can view every bargaining game $S$ in $B$ as $S=\{(u^1(l), u^2(l));\ l \in \mathcal{L}\}$ where $\mathcal{L}$ is the mixture set of lotteries corresponding to some subset $L \subset P$ with $\bar{p} \in L$. Suppose now, the two bargainers are faced with two bargaining games $S$ and $T$ in $B$, with $S$ as above and $T=\{(u^1(m), u^2(m));\ m \in \mathcal{M}\}$ where $\mathcal{M}$ is the mixture set of lotteries corresponding to some $M \subset P$ with $\bar{p} \in M$. Simultaneous bargaining over more issues means in this case: bargaining over the product set of lotteries $\mathcal{L} \times \mathcal{M} = \{(l, m);\ l \in \mathcal{L}, m \in \mathcal{M}\}$. Of course, we assume that both bargainers have preferences also on $\mathcal{L} \times \mathcal{M}$, which are represented by utility functions $w^i (i=1,2)$.

In order to use sums of bargaining games as representing simultaneous bargaining, we would like to have $S+T=\{(w^1(l, m), w^2(l, m));\ (l, m) \in \mathcal{L} \times \mathcal{M}\}$ and this is true if we have, for $i=1,2$, $w^i(l, m) = u^i(l) + u^i(m)$ for all $(l, m) \in \mathcal{L} \times \mathcal{M}$. The obvious question then is: when, i.e. under which conditions on the bargainers' underlying preferences, does $w^i$ have this additivity property? This question is answered in detail in Peters [**14**]. There, it is shown that, besides some normalization requirement, two conditions or axioms are necessary and sufficient, namely an axiom of *weak monotonicity* and an axiom of *additive independence*. The latter axiom can also be found in Fishburn [**2**] or Keeney and Raiffa [**8**, p. 231], and requires that player $i$ be indifferent between the lottery which gives him $(l, m)$ and $(l', m')$ both with probability $\frac{1}{2}$, and the lottery which gives him $(l, m')$ and $(l', m)$ both with probability $\frac{1}{2}$, for all $l, l' \in \mathscr{L}$ and $m, m' \in \mathscr{M}$. For further details see Peters [**14**]. In the above argument, we have viewed every bargaining game $S$ as arising from a subset of a large set of alternatives $(P)$, the bargainers' utility functions being the restrictions of their utility functions on the lottery set of this large set. Although we think this is a natural approach, one might still argue that bargaining solutions are supposed to be defined on bargaining games arising from all possible kinds of situations. At this point, Axiom 0 comes into the picture: once the image of a bargaining situation in utility space (i.e., the bargaining game itself) is known, the underlying bargaining situation becomes irrelevant with respect to determination of the solution outcome.

We will now shortly review the model of Perles and Maschler [**13**] and compare it with our present model. We will describe their results using our own framework. Let $B^0 := \{S \in B;\ x \geq 0 \text{ for all } x \in P(S)\}$. Perles and Maschler prove that there exists a unique solution *PM* on the proper subset $B^0$ of $B$ (the so-called super-additive solution) satisfying the Axioms IR, PO, STI, SA, SYM, PCO. (Perles and Maschler do not need the individual rationality axiom, since they restrict every bargaining game to the positive orthant of the plane.) Dropping SYM gives a two-parameter family of nonsymmetric super-additive solutions. We omit formulas here. Perles and Maschler justify their super-additive solution (or, more specifically, the SA axiom) by the following observation which we copy almost exactly within our own model.

OBSERVATION 2.1: Let $\phi\colon B \to \mathbb{R}^2$ be a solution satisfying SA and HOM. For any game consisting of a lottery on two games $R$ and $S$ in $B$, players who obey $\phi$ will both prefer to reach an agreement before the outcome of the lottery is available.

PROOF: Let $(p, 1-p)$ be the distribution of the lottery, w.l.o.g. $0 < p < 1$. If the players reach an agreement immediately, it must be $\phi(T)$, where $T = pR + (1-p)S$. By HOM and SA,

$$\phi(T) \geq p\phi(R) + (1-p)\phi(S). \tag{2.1}$$

The right-hand side of (2.1) is the expectation of the players from a delayed agreement. *Q.E.D.*

Thus, Observation 2.1 provides a different justification for the SA axiom. Another important difference between the present model and the model of Perles and Maschler is that their solution is restricted to the class $B^0$ where no player has an incentive to commit himself to a feasible outcome which is not individually rational for the other player. Indeed, if one feels that one is actually dealing with noncooperative Nash bargaining games (Perles and Maschler [**13**, p. 167]), then this restriction to $B^0$ is justified. Recall Example 1.2. The outcome of the sumgame, $(4, 4)$, can only be achieved by the sum $(6, -2)+(-2, 6)$. This means that in one game player 1 can commit himself to $(6, -2)$, whereas in the other game player 2 can commit himself to $(-2, 6)$. In a noncooperative setting, such commitments would be impossible: we are stuck in a prisoner's dilemma. Yet in a cooperative setting, where binding agreements are possible, these commitments lead to a net utility profit of 4 for both players. We will assume such a cooperative setting and our main purpose will be to find super-additive solutions defined on $B$.

Perles and Maschler have already indicated that their solution cannot be extended to $B$. This will also follow as a corollary of the results in the next section.

## 3. A FAMILY OF SUPER-ADDITIVE SOLUTIONS

In this section we single out a family of super-additive solutions with the aid of the (weak, odd-numbered) axioms WPO, HOM, and PSA. So we considerably weaken the Perles-Maschler list of axioms in order to avoid an impossibility result. As already remarked before, in Section 1, for individually rational solutions the partial super-additivity axiom follows from super-additivity; it states that in the simultaneous bargaining game, each player should get at least what he can get in each of the composing bargaining games separately. We start with a definition and the main results, and defer discussion to the end of this section.

Definition 3.1: For every $p \in \mathbb{R}^2$ with $p \geq 0$ and $p_1 + p_2 = 1$, the bargaining solution $E^p$: $B \to \mathbb{R}^2$ is defined by

$$\{E^p(S)\} = W(S) \cap \{\alpha p;\ \alpha \in \mathbb{R},\ \alpha > 0\} \quad \text{for all } S \in B.$$

$E^p$ is called the *egalitarian* or *proportional solution with weight vector p.*

For strictly positive weight vectors, these proportional solutions were introduced in Kalai [**6**]. Our main result is the following theorem.

Theorem 3.2: *Let $\phi$: $B \to \mathbb{R}^2$ be a bargaining solution. Then $\phi$ satisfies WPO, HOM, PSA if and only if it is proportional.*

The proof of this theorem will make use of the following three lemmas. In every one of these lemmas, $\phi$ is a bargaining solution satisfying the three axioms of the theorem.

Lemma 3.3: *Let $S \in B$ and $r \in \mathbb{R}^2_{++}$. Then (i) if $r \in int(S)$, then $\phi(S) \geq \phi(S(r))$; (ii) $\phi(S(R)) \geq 0$.*

Proof: (i) Suppose $r \in \text{int}(S)$. Then $\phi(S) \geq \phi(S(r))$, in view of PSA and the fact that $S = S(r) + T$ where $T := \{x - r;\ x \in S\} \in B$. (ii) Suppose $\phi_2(S(r)) < 0$. Then, by HOM, $\phi_2(\frac{1}{2}S(r)) = \frac{1}{2}\phi_2(S(r)) > \phi_2(S(r))$ which contradicts (i). Similarly, the assumption $\phi_1(S(r)) < 0$ leads to a contradiction. Hence $\phi(S(r)) \geq 0$.

*Q.E.D.*

In particular, the following corollary is an immediate consequence of Lemma 3.3.

COROLLARY 3.4: *Every homogeneous and partially super-additive bargaining solution is individually rational.*

PROOF: Follows from the observation that we did not use WPO in the proof of Lemma 3.3. *Q.E.D.*

Henceforth, we may assume that $\phi$ is also individually rational. Let now $L$ be the set $\{p \in \mathbb{R}^2_{++}; p_1 + p_2 = 1\}$.

LEMMA 3.5: *Either (i) $\phi(S(p)) = p$ for some $p \in L$, or (ii) $\phi_2(S(p)) < p_2$ for all $p \in L$, or (iii) $\phi_1(S(p)) < p_1$ for all $p \in L$.*

PROOF: Suppose (i) does not hold. Let $L^1 := \{p \in L; \phi_1(S(p)) < p_1\}$, $L^2 := \{p \in L; \phi_2(S(p)) < p_2\}$, and suppose that $L^1 \neq \emptyset$, $L^2 \neq \emptyset$. Let $p^1 \in L^1$, $p^2 \in L^2$. We show

$$(3.1) \qquad p_1^1 \geq p_1^2.$$

Suppose (3.1) does not hold, i.e. $p_1^1 < p_1^2$ and $p_2^1 > p_2^2$. Let then $q \in \mathbb{R}^2_{++}$ be defined by $q_1 := \frac{1}{2}(p_1^1 + \phi_1(S(p^1)))$, $q_2 := \frac{1}{2}(p_2^2 + \phi_2(S(p^2)))$. Then $q \in \operatorname{int} S(p^1)$, so by Lemma 3.3(i), $\phi_1(S(p^1)) \geq \phi_1(S(q))$. Similarly, $\phi_2(S(p^2)) \geq \phi_2(S(q))$. Altogether we obtain $q > \phi(S(q))$, in contradiction with WPO. So (3.1) must hold.

From (3.1) and our assumption that (i) does not hold, we conclude that there exists $\bar{p} \in L$ such that for all $p \in L$ with $p_1 < \bar{p}_1$ we have $p \in L^2$, and for all $p \in L$ with $p_1 > \bar{p}_1$ we have $p \in L^1$. The proof of the lemma is finished, by contradiction, if we show

$$(3.2) \qquad \phi(S(\bar{p})) = \bar{p}.$$

Suppose (3.2) does not hold, w.l.o.g. suppose $\bar{p} \in L^2$. Let $\alpha > 1$ such that $\alpha\phi_1(S(\bar{p})) + \alpha\phi_2(S(\bar{p})) = 1$, and let $p^* \in L$ be defined by $p^* := \frac{1}{2}(\bar{p} + \alpha\phi(S(\bar{p})))$. Take $\beta \in (0, 1)$ such that for $r := \beta p^*$ we have $r_2 > \phi_2(S(\bar{p}))$, $r_1 < \phi_1(S(\bar{p}))$. Since $p_1^* > \bar{p}_1$, we have $p^* \in L^1$, hence $\phi_2(S(p^*)) = p_2^*$. By HOM, $\phi_2(S(r)) = r_2$. However, $r \in \operatorname{int} S(\bar{p})$ so that $\phi(S(\bar{p})) \geq \phi(S(r))$ by Lemma 3.3(i), in contradiction with $\phi_2(S(r)) = r_2 > \phi_2(S(\bar{p}))$. So (3.2) must hold. *Q.E.D.*

Next, let $p \in L$ with $\phi(S(p)) = p$ if (i) in the above lemma holds, let $p = (0, 1)$ if (iii), and let $p = (1, 0)$ if (ii).

LEMMA 3.6. *For all $S \in B$, $\phi(S) = E^p(S)$.*

PROOF: Let $S \in B$. First suppose $E^p(S) \in P(S)$. If $p > 0$, $\phi(S) \geq \phi(S((1-\varepsilon)E^p(S))) = (1-\varepsilon)E^p(S)$ for $1 > \varepsilon > 0$, by Lemma 3.3(i) and HOM, so we are done by letting $\varepsilon$ go to 0. If $p = (1, 0)$, then take a sequence $r^1, r^2, \ldots$ in $\operatorname{int}(S) \cap \mathbb{R}^2_{++}$ converging to $E^p(S)$. Then again $\phi(S) \geq \phi(S(r^i))$ for each $i = 1, 2, \ldots$, so $\phi_1(S) \geq \phi_1(S(r^i)) = r_1^i$ for each $i = 1, 2, \ldots$, hence $\phi_1(S) \geq E_1^p(S)$. We conclude that $\phi(S) = E^p(S)$. By a similar argument, $\phi(S) = E^p(S)$ if $p = (0, 1)$.

Suppose now that $E^p(S) \notin P(S)$. W.l.o.g. (the other case is similar) there exists $x \in P(S)$ with $x_1 = E_1^p(S)$. Given $\varepsilon > 0$, let $R^\varepsilon \in B$ be defined by $R^\varepsilon := S((\varepsilon, E_2^p(S) - x_2), (0, \varepsilon))$. Let $T^\varepsilon = S + R^\varepsilon$ (cf. Figure 2). Note that $E^p(T^\varepsilon) \in P(T^\varepsilon)$. By the first part of the proof, $\phi(T^\varepsilon) = E^p(T^\varepsilon)$. If $\varepsilon$ goes to 0, $E^p(T^\varepsilon) = \phi(T^\varepsilon)$ converges to $E^p(S)$, and by PSA, $\phi(T^\varepsilon) \geqslant \phi(S)$ for all $\varepsilon$, so $E^p(S) \geqslant \phi(S)$. If $p = (1, 0)$ the proof is finished. If $p > 0$, then also the proof is finished, noting that $\phi(S) \geqslant E^p(S)$ by the argument in the third sentence of the proof.

*Q.E.D.*

An immediate consequence of Lemma 3.6 is that, if $\phi(S(p)) = p$ for some $p \in L$, then this $p$ is unique. The proof of Theorem 3.2 is now straightforward.

PROOF OF THEOREM 3.2: If $\phi$ satisfies the three axioms of the theorem, then $\phi$ is proportional in view of Lemma 3.6 and it is straightforward to verify that a proportional solution satisfies these axioms. *Q.E.D.*

We first remark that the proof of Theorem 3.2 could have been shorter had we added some continuity axiom (e.g. PCO) to our list of axioms. Doing so, however, we would have hidden the fact that such an axiom is not necessary here, whereas it is in the main result of the next section (Theorem 4.9). Note further that, apart from partial superadditivity, we need only two relatively weak axioms (WPO and HOM) to single out the family of proportional solutions in Theorem 3.2. It has turned out (Corollary 3.4) that individual rationality is implied

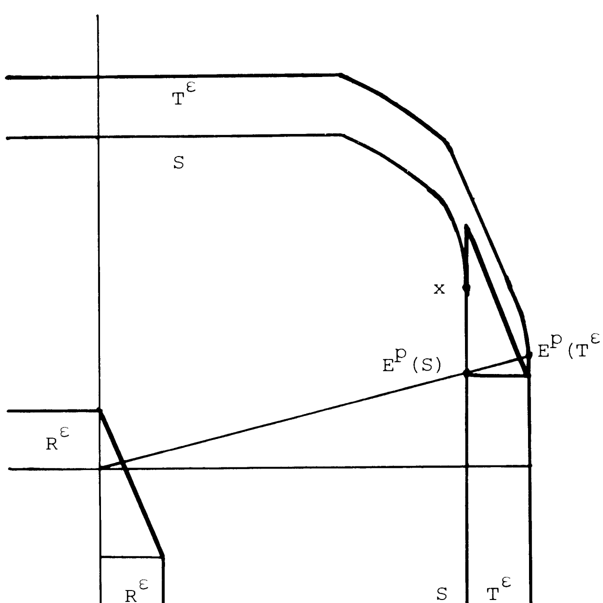


FIGURE 2

by PSA and HOM. (If this had not been the case, we would have required it, since IR is indisputable as an axiom.)

Finally, it can be easily seen that PSA is implied by SA and IR combined, and that SA is implied by the combination of PSA, WPO, and HOM (since every proportional solution is super-additive). So the following corollary is immediate.

COROLLARY 3.7: *Let $\phi: B \to \mathbb{R}^2$ be a bargaining solution. Then $\phi$ satisfies the axioms WPO, HOM, IR, and SA if and only if it is proportional.*

Recall that a super-additive solution (in the sense of Perles and Maschler [**13**]) is a solution satisfying IR, PO, STI, PCO, SA, SYM.

COROLLARY 3.8: *There does not exist a super-additive solution*: $B \to \mathbb{R}^2$.

PROOF: In view of Corollary 3.7, the only candidate for such a solution would be $E^p$ with $p = (\frac{1}{2}, \frac{1}{2})$, but this solution satisfies neither PO nor STI. *Q.E.D.*

Since von Neumann-Morgenstern utility functions are being assumed, one may find it a drawback for a solution not to satisfy the scale transformation invariance axiom, since this implies that utilities (of different players) are being compared. However, arguments can be given against this objection. For more discussion, we refer to Shapley [**16**], Kalai [**6**], and Myerson [**10**]; and to the final section of the present paper.

There are only two scale transformation invariant proportional solutions: this observation leads to the following corollary immediately.

COROLLARY 3.9: *The only two solutions satisfying WPO, STI, IR, and SA, are the proportional solutions $E^{(1,0)}$ and $E^{(0,1)}$.*

Since these two "tyrannical" solutions are very unlikely to describe an actual bargaining process satisfyingly, we might view Corollary 3.9 as an impossibility result. In the next section, we will considerably weaken the super-additivity axiom to obtain an alternative characterization of a well-known family of solutions, the non-symmetric Nash solutions.

## 4. SOLUTIONS WITH THE RESTRICTED ADDITIVITY AXIOM

In this section, we will describe a family of scale transformation invariant bargaining solutions satisfying the following axiom, where we call an $S \in B$ *smooth at* $x \in S$ if there exists a unique line of support of $S$ at $x$, and where $\phi$ is a bargaining solution.

AXIOM 10 (*Restricted Additivity*, RA): *For all $S$ and $T$ in $B$, if $S$ and $T$ are smooth at $\phi(S)$ and $\phi(T)$ respectively, and $\phi(S)+\phi(T) \in P(S+T)$, then $\phi(S+T) = \phi(S)+\phi(T)$.*

Axiom 10 is a slightly different version of the Conditional Additivity axiom in Aumann [**1**]. We defer most of the discussion and due references to the end of this section, and start with definitions and results.

DEFINITION 4.1: For every $q \in (0,1)$, the bargaining solution $F^q\colon B \to \mathbb{R}^2$ is defined by: for every $S \in B$, $F^q(S)$ (uniquely) maximizes the product $x_1^q x_2^{1-q}$ on $S \cap \mathbb{R}^2_{++}$. The "dictator" solutions $D^1$ and $D^2$ are defined by: for every $S \in B$, $D^i(S)$ is the point in $\{x \in P(S);\ x \geq 0\}$ with maximal $i$th coordinate, for $i = 1, 2$.

The solution $F^{1/2}$ is Nash's solution (Nash [**12**]) and the solutions $F^q(q \in (0,1))$ were derived in Harsanyi and Selten [**3**]. All solutions in Definition 4.1 satisfy the following axiom (cf. Nash [**12**]).

AXIOM 11 (*Independence of Irrelevant Alternatives*, IIA): *For every $S$ and $T$ such that $S \subset T$ and $\phi(T) \in S$, we have $\phi(S) = \phi(T)$.*

Axiom 11 has been amply discussed elsewhere (see Roth [**15**] for discussions and references). In de Koster, Peters, Tijs, and Wakker [**9**] the following result was proved.

THEOREM 4.2: *A bargaining solution $\phi$ satisfies IR, PO, STI, and IIA if and only if it is an element of $\{F^q, D^1, D^2;\ q \in (0,1)\}$.*

Our main result in this section will be that, in Theorem 4.2, IIA can be replaced by restricted additivity and Pareto continuity. First, we have to do some preliminary work. The following lemma gives a geometric characterization of the solutions $F^q(0 < q < 1)$. The proof can be given by using a separating hyperplane theorem and is left to the reader.

LEMMA 4.3: *For every $S \in B$, $F^q(S) = z$ $(\in P(S))$ iff there exists a line of support of $S$ at $z$ with a normal vector $(qz_2, (1-q)z_1)$.*

The following tool will also be of use.

LEMMA 4.4: *Let $S, T \in B$, and $z = x + y \in P(S+T)$ where $x \in S$, $y \in T$. Then we have* (i) $x \in P(S)$, $y \in P(T)$, (ii) *if $l$ is a line of support of $S+T$ at $z$, then there exist lines of support $l'$ and $l''$ of $S$ and $T$ at $x$ and $y$ respectively, such that $l$, $l'$ and $l''$ are parallel,* (iii) *if $S$ and $T$ are smooth at $x$ and $y$ respectively, then $l$, $l'$ and $l''$ in (ii) are unique (and $S+T$ is smooth at $z$).*

PROOF: (i) is straightforward by definition, and (iii) by (ii). To prove (ii), let $l$ be such a line with a normal vector $\lambda$, then $\lambda \geq 0$, and the inner product $\lambda \cdot z = \max\{\lambda \cdot (s+t);\ s \in S,\ t \in T\} = \max\{\lambda \cdot s;\ s \in S\} + \max\{\lambda \cdot t;\ t \in T\}$, hence $\lambda \cdot x = \max\{\lambda \cdot s;\ s \in S\}$ and $\lambda \cdot y = \max\{\lambda \cdot t;\ t \in T\}$, from which (ii) follows immediately. *Q.E.D.*

PROPOSITION 4.5: *Let $\phi \in \{F^q, D^1, D^2;\ q \in (0,1)\}$. Then $\phi$ satisfies the axioms IR, PO, STI, PCO, and RA.*

PROOF: We only prove that every $\phi \in \{F^q, D^1, D^2;\ q \in (0,1)\}$ satisfies the restricted additivity axiom. First, let $\phi = F^q$ for some $q \in (0, 1)$. Let $S, T \in B$ such that $S$ and $T$ are smooth at $x := F^q(S)$ and $y := F^q(T)$ respectively, and $x+y \in P(S+T)$. From Lemma 4.4(iii) it follows that there exists a vector $\lambda \geq 0$ such that $\lambda \cdot x = \max\{\lambda \cdot s;\ s \in S\}$ and $\lambda \cdot y = \max\{\lambda \cdot t;\ t \in T\}$, $\lambda \cdot (x+y) = \max\{\lambda \cdot v;\ v \in S+T\}$. From Lemma 4.3, it follows that $x = \gamma y$ for some $\gamma > 0$, hence $x+y = (1+\gamma)y$. Applying Lemma 4.3 again, it follows that $F^q(S+T) = x+y$.

Secondly, let $\phi = D^1$, and $S$ and $T$ in $B$ such that $S$ and $T$ are smooth at $D^1(S)$ and $D^1(T)$, respectively, and $D^1(S)+D^1(T) \in P(S+T)$. If $D^1_2(S) = D^1_2(T) = 0$, then $D^1_2(S)+D^1_2(T) = 0$, and so $D^1(S+T) = D^1(S)+D^1(T)$ since $D^1(S)+D^1(T) \in P(S+T)$. Otherwise, in view of Lemma 4.4(iii), the unique lines of support of $S$, $T$ and $S+T$ at $D^1(S)$, $D^1(T)$, and $D^1(S)+D^1(T)$ are the straight lines with equations $x_1 = D^1_1(S)$, $x_1 = D^1_1(T)$, and $x_1 = D^1_1(S)+D^1_1(T)$, respectively. So $D^1(S+T) = D^1(S)+D^1(T)$ since $D^1(S)+D^1(T) \in P(S+T)$.

The third case, $\phi = D^2$, is similar to the second one. *Q.E.D.*

Before proving the converse of the previous proposition, we need two more lemmas.

LEMMA 4.6: *Let $\phi$ be a bargaining solution satisfying IR, PO, and PCO. Let $S \in B$ such that $S$ is smooth everywhere (i.e. at every point of $P(S)$) and such that the line of support of $S$ at $\phi(S)$ has a normal vector with one coordinate equal to 0. Let $z \in P(S)$, $z \neq \phi(S)$. Then there exists an everywhere smooth $S' \in B$ with $S' \subset S$ and $z \in S'$ such that $\phi(S') \neq z$ and such that the line of support of $S'$ at $\phi(S')$ has a strictly positive normal vector.*

PROOF: First note that $\phi(S) = D^1(S)$ or $\phi(S) = D^2(S)$. Assume $\phi(S) = D^2(S)$ (the other case is similar). If $\phi_1(S) = 0$, then an $S'$ as in the lemma can easily be found by cutting off a suitable neighborhood of $\phi(S)$ in $S$ in a smooth way. Suppose now, that $\phi_1(S) > 0$. First, choose $\bar{x} \in P(S)$ with $\phi_2(S) > \bar{x}_2 > z_2$ and such that $\phi_2(T) > z_2$ where $T$ consists of all points of $S$ except those strictly above the straight line through $\bar{x}$ and $\phi(S)$. Such a point $\bar{x}$ exists in view of PCO. We are done if $\phi(T) \neq \phi(S)$ for then we can take, for $S'$, the game $T$ smoothed off at $\phi(S)$ and $\bar{x}$, in view of PCO. Now suppose $\phi(T) = \phi(S)$. For every $\varepsilon$ with $0 \leq \varepsilon \leq \phi_1(S)$, let $S^\varepsilon \in B$ be the game consisting of all points of $S$ except those strictly above the straight line through $\bar{x}$ and the point $(\phi_1(S) - \varepsilon, \phi_2(S))$. Note that $S^0 = T$, so $\phi(S^0) = \phi(T) = \phi(S) = D^2(S) = D^2(S^0)$. Now let $\bar{\varepsilon} := \sup\{\varepsilon \in [0, \phi_1(S)];\ \phi(S^\varepsilon) = D^2(S^\varepsilon)\}$. By PCO, $\phi(S^{\bar{\varepsilon}}) = D^2(S^{\bar{\varepsilon}})$. If $\bar{\varepsilon} = \phi_1(S)$, then we are back in the case of the first paragraph of the proof (where we assumed $\phi_1(S) = 0$). Otherwise, $0 \leq \bar{\varepsilon} < \phi_1(S)$. Then take $\eta$ with $\bar{\varepsilon} < \eta < \phi_1(S)$ small enough such that $(D^2_2(S^\eta) >)\, \phi_2(S^\eta) > z_2$. And take for $S'$ the game $S^\eta$ smoothed off at $D^2(S^\eta)$ and $\bar{x}$. *Q.E.D.*

Let $\Delta \in B$ be defined by $\Delta := S((1,0),(0,1))$.

LEMMA 4.7: *Let $\phi$ be a bargaining solution satisfying IR, PO, STI, PCO, and RA. Let $\mu \in \{F^q, D^1, D^2; q \in (0,1)\}$ be such that $\phi(\Delta) = \mu(\Delta)$. Let $T \in B$ be such that $P(T) \supset conv\{v, w\}$ where $v, w \in \mathbb{R}^2$ satisfy $v_1 + v_2 = w_1 + w_2 = \alpha > 0$, $v_1 > 0$, $w_2 > 0$, $v_2 < 0$, $w_1 < 0$. Then $\phi(T) = \mu(T)$.*

PROOF: (See Figure 3.) By *STI*, $\phi(\delta\Delta) = \mu(\delta\Delta)$ for every $\delta \in (0, \infty)$. Fix $\delta \in (0, \alpha)$. Fix $\varepsilon < \min\{v_1 - \delta, w_2 - d, -v_2, -w_1\}$. Let $D \in B$ be given by the following constraints:

$$\{x \in W(D);\ x_1 \leqslant 0\} = \{(x_1 + \varepsilon, x_2 - \delta - \varepsilon);\ x \in W(T),\ x_1 \leqslant -\varepsilon\},$$

$$\{x \in W(D);\ x \geqslant 0\} = \{x \geqslant 0;\ x_1 + x_2 = \alpha - \delta\},$$

$$\{x \in W(D);\ x_2 \leqslant 0\} = \{(x_1 - \delta - \varepsilon, x_2 + \varepsilon);\ x \in W(T),\ x_2 \leqslant -\varepsilon\}.$$

Let $E \in B$ be given by $E := S((\delta + \varepsilon, -\varepsilon), (-\varepsilon, \delta + \varepsilon))$. Then $E + D = T$. Note that $E$ and $D$ are smooth at every $x \in \{e \in P(E);\ e \geqslant 0\}$ and $y \in \{d \in P(D);\ d \geqslant 0\}$, and that all supporting lines at these points are parallel, with a normal vector $\lambda = (1,1)$. In particular, $x + y \in P(T)$ for every $x \in \{e \in P(e);\ e \geqslant 0\}$, $y \in \{d \in P(D);\ d \geqslant 0\}$. So by PO, IR, and RA, $\phi(T) = \phi(D) + \phi(E)$, hence $\phi_1(E) \leqslant \phi_1(T) \leqslant \phi_1(E) + \alpha - \delta$ and $\phi_2(E) \leqslant \phi_2(T) \leqslant \phi_2(E) + \alpha - \delta$. Letting $\varepsilon$ go to 0 gives, by PCO and the fact that $\phi(\delta\Delta) = \mu(\delta\Delta)$,

$$\mu_1(\delta\Delta) \leqslant \phi_1(T) \leqslant \mu_1(\delta\Delta) + \alpha - \delta,\ \mu_2(\delta\Delta) \leqslant \phi_2(T) \leqslant \mu_2(\delta\Delta) + \alpha - \delta.$$

Letting $\delta$ go to $\alpha$, gives $\phi(T) = \mu(\alpha\Delta)$, hence $\phi(T) = \mu(T)$ since by definition of $\mu$, $\mu(\alpha\Delta) = \mu(T)$. *Q.E.D.*

PROPOSITION 4.8: *Let $\phi$ be a bargaining solution satisfying IR, PO, STI, PCO, and RA. Let $p := \phi_1(\Delta)$. If $p \neq 0, 1$, then $\phi = F^p$; if $p = 1$, then $\phi = D^1$ and if $p = 0$, then $\phi = D^2$.*

PROOF: (Figure 4.) Let $\mu \in \{F^q, D^1, D^2;\ 0 < q < 1\}$ be the solution such that $\mu_1(\Delta) = \phi_1(\Delta) = p$. Suppose there exists an $S \in B$ such that

(4.1) $\qquad \phi(S) \neq \mu(S)$.

By PCO of $\phi$ and $\mu$, we may suppose that $S$ is smooth everywhere, and by Lemma 4.6, that the line of support of $S$ at $\phi(S)$ has a strictly positive normal vector $\lambda$. By STI, we may further suppose that $\lambda = (1,1)$ and $\phi_1(S) + \phi_2(S) = 1$. Then we have, by Lemma 4.3,

(4.2) $\qquad \phi(S) \neq (p, 1-p)$.

Let $T := S((3,-2),(-2,3))$: then, by Lemma 4.7 and $\mu(T) = (p, 1-p)$, we have

(4.3) $\qquad \phi(T) = (p, 1-p)$.

Further, STI and Lemma 4.7 applied to $S + T$, give

(4.4) $\qquad \phi(S+T) = 2(p, 1-p)$.

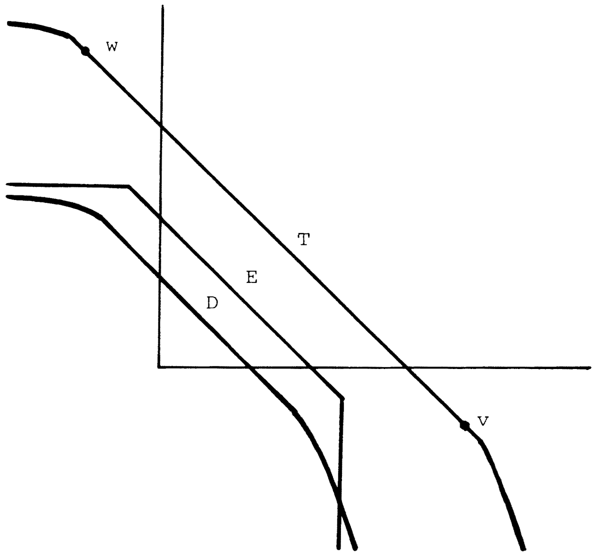


FIGURE 3

On the other hand, since $S$ is smooth at $\phi(S)$, $T$ is smooth at $(p, 1-p)$, and $\phi(S)+(p, 1-p) \in P(S+T)$, we have by RA and (4.3)

(4.5) $\quad \phi(S+T)=\phi(S)+(p, 1-p).$

Combining (4.4) and (4.5) gives $\phi(S)=(p, 1-p)$, in contradiction with (4.2). Hence (4.1) must be false, so $\phi(S)=\mu(S)$ for all $S \in B$. *Q.E.D.*

Propositions 4.5 and 4.8 lead immediately to Theorem 4.9.

THEOREM 4.9: $\{F^q, D^1, D^2; q \in (0,1)\}$ *is the family of all bargaining solutions* $B \to \mathbb{R}^2$ *satisfying IR, PO, STI, PCO, and RA.*

The following example shows that we cannot dispense with the PCO Axiom in Theorem 4.9.

EXAMPLE 4.10: We construct a solution $\phi$: $B \to \mathbb{R}^2$ by first defining it for all games $S$ which satisfy, for $i=1,2$:

(4.6) $\quad \max\{x_i: x \in S, x \geq 0\}=1.$

By applying the appropriate scale transformations, the definition is then extended to $B$, guaranteeing that $\phi$ satisfies STI. So let $S \in B$ such that $S$ satisfies (4.6). We define $\phi(S)$ as follows. If $S$ is smooth at $F^{1/2}(S)$, then $\phi(S) := F^{1/2}(S)$. If $S$ is not smooth at $F^{1/2}(S)$, then also $\phi(S) := F^{1/2}(S)$ except for the case that there exists exactly one other point $x \in P(S) \cap \mathbb{R}^2_+$ such that $S$ is not smooth at $x$; in that case, $\phi(S) := x$.

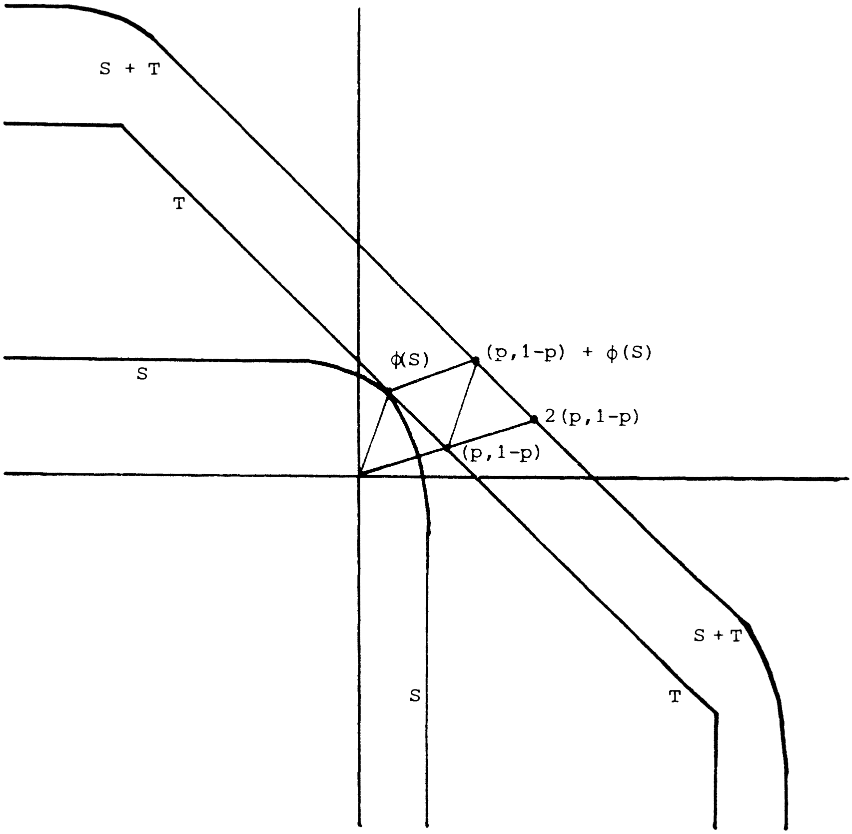


FIGURE 4

It is straightforward to verify that this $\phi$, besides STI, satisfies IR, PO, and RA, but not PCO.

Theorem 4.9 is the main result of this section. At first sight, the weakening of super-additivity to restricted addivity may seem somewhat arbitrary, but if we look closer, there is a strong link between the results in this section and those in the previous one. Of course, super-additivity implies restricted additivity. Every proportional solution satisfies IR, PCO, and RA. The solutions $E^{(1,0)}$ and $E^{(0,1)}$ satisfy also WPO and STI. For a solution $\phi$ and $S, T \in B$, say that *RA applies to* $\phi$, $S$ *and* $T$ if $S$ and $T$ are smooth at $\phi(S)$ and $\phi(T)$ respectively, $\phi(S)+\phi(T) \in P(S+T)$ and $\phi(S)+\phi(T)=\phi(S+T)$. Then, as an immediate consequence of Lemmas 4.3 and 4.4, for all $q \in (0,1)$, if RA applies to $F^q$, $S$, and $T$, then $F^q(S)=E^p(S)$, $F^q(T)=E^p(T)$, and $F^q(S+T)=E^p(S+T)$ for some $p>0$. (With a few modifications, also a reversal of this statement holds.) So the restricted additivity axiom entails a kind of restricted proportionality property (not for the dictator solutions $D^1$ and $D^2$, however).

The results in this section are closely related to Aumann's [**1**], where an axiomatic foundation to the so-called nontransferable utility value is given (Shapley [**16**]). Aumann uses a conditional additivity axiom, which is stronger than restricted additivity, in that it does not require smoothness. However, Aumann restricts attention to smooth games, where here we have essentially the Pareto continuity axiom to take care of nonsmoothness. The present paper also covers the nonsymmetric (and nonstrongly individually rational) case, and further, we note that (at least for strongly individually rational solutions) Theorem 4.9 may be extended to characterize solutions for $n$-person games (that is, pure bargaining games, where only one-player coalitions and the all-player coalition are allowed) without difficulty.

The smoothness condition in the definition of RA may be interpreted as "local transferable utility" (Aumann [**1**, p. 14]). It cannot be dispensed with: see the example in Aumann [**1**] or the following one.

EXAMPLE 4.10: Let $\phi = F^{1/2}$ be the (symmetric) Nash solution, take $\Delta$ as before, and $S := S((2, 1))$. Then $\phi(\Delta + S) = (2, 2) \neq (\frac{1}{2}, \frac{1}{2}) + (2, 1) = \phi(\Delta) + \phi(S) \in P(\Delta + S)$. Here $\Delta$ is smooth at $\phi(\Delta)$, but $S$ is not smooth at $\phi(S)$. (See Figure 5.)

We end this section by discussing the relation of the results in Sections 3 and 4 with Myerson's in [**11**]. Myerson considers the effect of timing in social choice, so essentially justifies his approach by Observation 2.1, but unlike Perles and Maschler in [**13**] does not restrict the domain of (in Myerson's case: social choice) problems. His main result (described within our model) reads: if a bargaining solution $\phi$ satisfies SA, WPO, and IIA, then it is either proportional or utilitarian. (By the way, Myerson overlooks here the solutions $E^{(1,0)}$ and $E^{(0,1)}$.) Here $\phi$ is called *utilitarian* if there exists some weight vector $p$ (i.e. $p \geqslant 0$, $p_1 + p_2 = 1$) such that, for every $S \in B$, $\phi(S)$ is a maximizer of $p \cdot x$ where $x \in S$. Apart from the difference in interpretation—simultaneous bargaining over more issues versus timing effect—the main difference between our results and those in [**11**] is the fact that we do not need the IIA axiom as a condition. Parenthetically, note that a utilitarian solution is not completely determined by the definition above. Yet

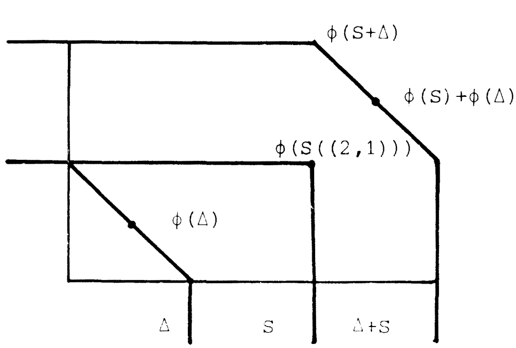


FIGURE 5

the following observation which links the nonsymmetric Nash solutions to the utilitarian solutions, holds: for all $q \in (0, 1)$, if RA applies to $F^q$, $S$, and $T$, then, as before $F^q(S) = E^p(S)$, $F^q(T) = E^p(T)$, and $F^q(S+T) = E^p(S+T)$ for some $p > 0$, and these equalities still hold if we substitute for $E^p$ a utilitarian solution with weight vector $\lambda \|\lambda\|^{-1}$ where $\lambda = (qp_1^{q-1}p_2^{1-q}, (1-q)p_1^q p_2^{-q})$. This observation again follows simply from Lemmas 4.3 and 4.4. It re-establishes the fact that a Nash solution offers a compromise between egalitarian (proportional) and utilitarian principles.

## 5. FINAL REMARKS

We set out, in the present paper, with the problem of simultaneous bargaining over more issues, and have tried to tackle the problem via an axiomatic treatment involving sums of bargaining games and (super-) additivity axioms for bargaining solutions. By these means, we have characterized different families of solutions for the two-person problem, in Sections 3 and 4. The super-additive solution of Perles and Maschler [**13**] vanishes from the scene since, for our purposes, its domain is too restrictive. In Section 3, we have characterized Kalai's (extended) family of proportional solutions, in Section 4 Harsanyi and Selten's (extended) family of nonsymmetric Nash solutions, by means of additivity axioms. A conclusion of Section 3 is that, together with a few standard axioms, the super-additivity axiom only allows "tyranical" solutions if comparisons between the players' utilities are forbidden; in Section 4, we have considered the weaker restricted additivity axiom. One general conclusion from the present results may be that (super)-additivity more or less implies proportionality, i.e. implies to a greater or smaller extent a comparison between the players' utilities.

In the previous section, we indicated one possible extension to $n$-person bargaining games. It is also of interest to look for extensions to the general case of $n$-person games without side payments. Contributions in this area are, besides the already mentioned paper by Aumann [**1**], a paper by Hart [**4**] and a paper by Kalai and Samet [**7**], the former one showing results closely related to the ones obtained by Aumann, the latter one extending, axiomatically, the family of proportional solutions.

*University of Limburg*

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