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Characterization of all Individually Monotonic Bargaining Solutions

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Abstract: A description is given of the class of all individually monotonic bargaining solutions by associating with each of these solutions a monotonic curve in the triangle of $\mathbf{R}^2$ with vertices (1, 0), (0, 1) and (1, 1). Also the family of globally individually monotonic bargaining solutions is characterized with the aid of monotonic curves in the unit square of $\mathbf{R}^2$.

1 Introduction

In 1950 *Nash* introduced the two-person bargaining problem. In such a problem two bargainers are involved who can agree upon one of the points in a set S of feasible utility pairs or who can disagree, in which case the payoff is a utility pair d, called the *disagreement point.* The pair (S, d) determines the problem.

In the following we only look at bargaining pairs (S, d) where S is a compact convex subset of $\mathbf{R}^2$, $d \in S$ and such that $s_1 > d_1, s_2 > d_2$ for some $(s_1, s_2) \in S$. The family of these bargaining pairs is denoted by $\underline{B}$. By a solution of the bargaining problem (*bargaining solution*) we mean a map $\phi : \underline{B} \to \mathbf{R}^2$, having the following properties:

(P.1) $\phi(S, d) \geqslant d$ for all $(S, d) \in \underline{B}$ (*Individual Rationality*),

(P.2) $\phi(S, d) \in P(S)$ where $P(S) = \{x \in S; \forall y \in S\ [y \geqslant x \Rightarrow y = x]\}$ is the Pareto boundary of S (*Pareto Optimality*),

(P.3) for each $(S, d) \in \underline{B}$ and each transformation $A : \mathbf{R}^2 \to \mathbf{R}^2$ of the form $A(x_1, x_2) = (a_1 x_1 + b_1, a_2 x_2 + b_2)$ for all $(x_1, x_2) \in \mathbf{R}^2$, where $a_1 > 0$, $a_2 > 0, b_1, b_2 \in \mathbf{R}^2$, we have $\phi(A(S), A(d)) = A(\phi(S, d))$ (*Independence of equivalent utility representations*).

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The bargaining solution $F^{1/2}: \underline{B} \to \mathbf{R}^2$, proposed by *Nash*, is the unique solution ϕ with the following two additional properties:

(P.4) for each $(S, d) \in \underline{B}$ with $d_1 = d_2$ and $S = \{(s_2, s_1) \in \mathbf{R}^2; (s_1, s_2) \in S\}$ we have $\phi_1(S, d) = \phi_2(S, d)$ (*Symmetry*),

(P.5) for all $(S, d), (T, e) \in \underline{B}$ we have $\phi(S, d) = \phi(T, e)$ if $d = e$, $S \subset T$ and $\phi(T, e) \in S$ (*Independence of irrelevant alternatives*).

Maps $F^t: \underline{B} \to \mathbf{R}^2$ $(t \in (0, 1))$, satisfying (P.1), (P.2), (P.3) and (P.5), were considered by *Harsanyi* and *Selten*, and *Kalai*.

Here, $F^t(S, d)$ is the unique point of $\{x \in P(S); x \geqslant d\}$ in which the function $(x_1, x_2) \to (x_1 - d_1)^t (x_2 - d_2)^{1-t}$ attains its maximum.

Also the solutions F^0 and F^1, where for $(S, d) \in \underline{B}$, $F^0(S, d)$ $(F^1(S, d))$ is the lexicographical minimum (lexicographical maximum) of $\{x \in P(S); x \geqslant d\}$, satisfy these axioms. In *de Koster, Peters, Tijs* and *Wakker* it is proved that ϕ satisfies (P.1), (P.2), (P.3) and (P.5) if and only if $\phi \in \{F^t; t \in [0, 1]\}$.

Because of criticism on property (P.5) by many authors, *Kalai* and *Smorodinsky* proposed to look at solutions with the individual monotonicity property, which property is described in the next section. *Kalai* and *Smorodinsky* proved that there is a unique bargaining solution satisfying the symmetry property and the individual monotonicity property. The question arose whether there are more individually monotonic solutions. The purpose of section 2 of this paper is to characterize all these solutions.

In *Kalai* and *Rosenthal* a symmetric bargaining solution was introduced having the property of global individual monotonicity, which property we introduce in section 3. Also in section 3, all solutions having this property, are described.

In the last section we look at the continuity and risk sensitivity of (globally) individually monotonic solutions.

2 Individually Monotonic Solutions

We start with some notations. Let S be a compact convex set in $\mathbf{R}^2$ and $d \in S$. Then $S_d = \{x \in S; x \geqslant d\}$. The *utopia point* of S (*ideal point* of S) is the point $u(S) = (u_1(S), u_2(S))$ with $u_i(S) = \max\{x_i \in \mathbf{R}; x \in S\}$ for $i \in \{1, 2\}$. The *d-ideal point* of S, denoted by $u(S_d)$, is the utopia point of the set S_d. So, if S is the set of attainable utility pairs of a bargaining problem (S, d), then $u_i(S_d)$ is the maximal attainable utility for player i if only utility pairs x are considered with $x \geqslant d$. The smallest comprehensive set containing a set C, is denoted by C^*. Hence, $C^* = \{x \in \mathbf{R}^2; x \leqslant y \text{ for some } y \in C\}$.

Now we introduce the following three partial orders on B:

$(S, d) \subset_1 (T, e)$ if $(S_d)^* \subset (T_e)^*, d = e$ and $u_2(S_d) = u_2(T_e)$,

$(S, d) \subset_2 (T, e)$ if $(S_d)^* \subset (T_e)^*, d = e$ and $u_1(S_d) = u_1(T_e)$,

$(S, d) \subset_{12} (T, e)$ if $(S, d) \subset_i (T, e)$ for $i \in \{1, 2\}$.

Definition

A bargaining solution $\phi : \underline{B} \to \mathbf{R}^2$ is called an *individually monotonic solution* if for all $(S, d), (T, e) \in \underline{B}$ we have:

(IM_1) if $(S, d) \subset_1 (T, e)$, then $\phi_1(S, d) \leqslant \phi_1(T, e)$,

(IM_2) if $(S, d) \subset_2 (T, e)$, then $\phi_2(S, d) \leqslant \phi_2(T, e)$.

Note that, for an individually monotonic solution ϕ, the following properties hold:

(Q.1) $(S, d) \subset_{12} (T, e) \Rightarrow \phi(S, d) \leqslant \phi(T, e)$,

(Q.2) $(S, d) \subset_{12} (T, e), \phi(T, e) \in S \Rightarrow \phi(S, d) = \phi(T, e)$,

(Q.3) $(S, d) \subset_{12} (T, e), \phi(S, d) \in P(T) \Rightarrow \phi(S, d) = \phi(T, e)$.

In (ii) of the next proposition a nice property called *restricted monotonicity* [cf. *Roth*, p. 101], is given. This property proves to be equivalent to the individual monotonicity property. Hence, the proposition says that in case two bargaining pairs have the same disagreement point d and the same d-ideal point, and if the set of feasible utility pairs in the first problem contains that of the second problem, then in the first bargaining problem an individually monotonic solution assigns better utilities to the players than in the second one.

Proposition 1

Let $\phi : \underline{B} \to \mathbf{R}^2$ be a bargaining solution. Then the following two statements are equivalent.

(i) ϕ is an individually monotonic solution.

(ii) For all (S, d) and (T, e) in $\underline{B}$ with $d = e$, $u(S_d) = u(T_e)$ and $S \subset T$, we have $\phi(S, d) \leqslant \phi(T, e)$.

Proof

From $(Q.1)$ it follows immediately that (i) implies (ii). Suppose that (ii) holds. We have to show that (IM_1) and (IM_2) hold. We only prove (IM_2). Take (S, d) and (T, d) in $\underline{B}$ such that $(S_d)^* \subset (T_d)^*$ and $u_1(S_d) = u_1(T_d)$. We have to show that $\phi_2(S, d) \leqslant \phi_2(T, d)$.

First, let $m_i = \min \{x_i \in \mathbf{R}; x \in T\}$ for $i \in \{1, 2\}$, and let $V = \{x \in (T_d)^*; m_1 \leqslant \leqslant x_1, m_2 \leqslant x_2 \leqslant u_2 (S_d)\}$. Then $S_d \subset V$ and $u(V_d) = u(S_d)$. Hence, by property (ii), $\phi(S_d, d) \leqslant \phi(V, d)$. Furthermore, by applying property (ii) again to (S_d, d) and (S, d), we obtain $\phi(S_d, d) \leqslant \phi(S, d)$ which by (P.2) yields $\phi(S_d, d) = \phi(S, d)$. Hence we have

$$\phi(S, d) \leqslant \phi(V, d). \tag{2.1}$$

Now, let $A : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation with $A(x_1, x_2) = (x_1, d_2 + (u_2(S_d) - d_2)^{-1}(u_2(T_d) - d_2)(x_2 - d_2))$ for all $x \in \mathbf{R}^2$. Then $A(d) = d$, $T \subset A(V)$ and $u((A(V))_d) = u(T_d)$. Hence, by (ii) and (P.3) we have $A(\phi(V, d)) = \phi(A(V), A(d)) = \phi(A(V), d) \geqslant \phi(T, d)$. Hence,

$$\phi_1(V, d) = (A\,\phi(V, d))_1 \geqslant \phi_1(T, d). \tag{2.2}$$

Since $\phi(V, d) \in P(T)$ and $\phi(T, d) \in P(T)$, (2.2) implies

$$\phi_2(V, d) \leqslant \phi_2(T, d). \tag{2.3}$$

Combining (2.1) and (2.3), we obtain $\phi_2(S, d) \leqslant \phi_2(T, d)$. □

In section 1 we have already noted that Kalai and Smorodinsky proved there exists exactly one symmetric, individually monotonic bargaining solution G. The solution G assigns to a bargaining pair (S, d) the unique point of $P(S)$, lying on the line segment with endpoints d and $u(S_d)$. Note that also F^0 and F^1 are individually monotonic.

Our purpose is to find all individually monotonic bargaining solutions. Therefore we look at maps $\lambda : [1, 2] \to \Delta$ where $\Delta = \text{conv}\,\{(1, 0), (0, 1), (1, 1)\}$, with the following property.

(C) For all $s, t \in [1, 2]$ with $s \leqslant t : \lambda(s) \leqslant \lambda(t)$ and $\lambda_1(s) + \lambda_2(s) = s$.

Note that from (C) follows that λ is a continuous map, in the following way. Let $s, t \in [1, 2]$ and $\| x \|_1 = | x_1 | + | x_2 |$ for $x \in \mathbf{R}^2$. Then $\| \lambda(s) - \lambda(t) \|_1 = | \lambda_1(s) - \lambda_1(t) | + | \lambda_2(s) - \lambda_2(t) | = | (\lambda_1(s) + \lambda_2(s)) - (\lambda_1(t) + \lambda_2(t)) | = | s - t |$.

The family of maps satisfying (C) is denoted by Λ and the elements are called *monotonic curves.*

With each $\lambda \in \Lambda$ we now associate a bargaining solution π^λ which is individually monotonic. Let $(S, d) \in \underline{B}$. If $d = (0, 0)$ and $u(S_d) = (1, 1)$, then $\pi^\lambda(S, d)$ is defined as the unique point of $P(S)$ which lies on the curve $\{\lambda(t); t \in [1, 2]\}$. If $d \neq (0, 0)$ or $u(S_d) \neq (1, 1)$, then construct a map $A : \mathbf{R}^2 \to \mathbf{R}^2$ as in (P.3) such that $A(d) = (0, 0)$ and $u((A(S))_0) = (1, 1)$ and put $\pi^\lambda(S, d) := A^{-1}(\pi^\lambda(A(S), A(d)))$, where $\pi^\lambda (A(S), A(d))$ is the unique point of $P(A(S))$ lying on $\{\lambda(t); t \in [1, 2]\}$. We call π^λ the *solution corresponding to the curve* λ. It is then obvious that π^λ satisfies (P.1), (P.2) and (P.3). Furthermore,

Theorem 2

π^λ is an individually monotonic solution.

Proof

In view of Prop. 1 and the definition of π^λ we only have to show that for (S, d) and $(T, e) \in \underline{B}$ with $d = e = (0, 0)$, $u(S_d) = u(T_e) = (1, 1)$ and $S \subset T$, we have $\pi^\lambda (S, d) \leqslant \pi^\lambda (T, e)$.

Let $s \in [1, 2]$ and $t \in [1, 2]$ be such that $\pi^\lambda (S, d) = \lambda(s)$ and $\pi^\lambda (T, d) = \lambda(t)$. If $s > t$, then in view of $(C): \lambda(s) \geqslant \lambda(t)$ and $\lambda(s) \neq \lambda(t)$, a contradiction with $S \subset T$ and (P.2). So, $s \leqslant t$ and $\pi^\lambda (S, d) = \lambda(s) \leqslant \lambda(t) = \pi^\lambda (T, d)$. □

Note that the Kalai-Smorodinsky solution G corresponds to the curve $\lambda_G \in \Lambda$ with $\lambda_G(t) = (1/2\, t, 1/2\, t)$ for $t \in [1, 2]$. Further, the solutions F^0 and F^1 correspond to λ_0 and λ_1 in Λ where $\lambda_0(t) = (t-1, 1)$ and $\lambda_1(t) = (1, t-1)$ for each $t \in [1, 2]$.

The main result of this section is the following theorem, which states that each individually monotonic solution corresponds to a curve $\lambda \in \Lambda$.

Theorem 3

Let $\phi : \underline{B} \to \mathbf{R}^2$ be an individually monotonic solution. Then there exists $\lambda \in \Lambda$ such that $\phi = \pi^\lambda$.

Proof

For each $t \in [1, 2]$, let $V(t) = \text{conv}\ \{(0, 0), (1, 0), (1, t-1), (t-1, 1), (0, 1)\}$. Define $\lambda : [1, 2] \to \mathbf{R}^2$ by $\lambda(t) = \phi(V(t), 0)$ for all $t \in [1, 2]$. For $1 \leqslant s \leqslant t \leqslant 2$ we have $\lambda(s) = \phi(V(s), 0) \leqslant \phi(V(t), 0) = \lambda(t)$ by (Q.1). Furthermore, for each $t \in [1, 2]$, $\lambda(t) \in P(V(t)) = \text{conv}\ \{(1, t-1), (t-1, 1)\}$, so $\lambda_1(t) + \lambda_2(t) = t$. Hence $\lambda \in \Lambda$. Note that

$$\phi(V(t), 0) = \pi^\lambda (V(t), 0) \text{ for each } t \in [1, 2]. \tag{2.4}$$

We want to prove that $\phi = \pi^\lambda$. In view of (P.3) it is sufficient to show that $\phi(S, 0) = \pi^\lambda (S, 0)$ if $(S, 0) \in \underline{B}$ and $u(S_0) = (1, 1)$. Let $s = \pi_1^\lambda (S, 0) + \pi_2^\lambda (S, 0)$ and let $W = \text{conv}\ \{(0, 0), (1, 0), \pi^\lambda (S, 0), (0, 1)\}$. Then, in view of (2.4),

$$\pi^\lambda (W, 0) = \pi^\lambda (S, 0) = \pi^\lambda (V(s), 0) = \phi(V(s), 0) \in P(W) \cap P(S) \cap P(V(s)). \tag{2.5}$$

In view of (Q.2), (2.5) and $(W, 0) \subset_{12} (V(s), 0)$, we obtain

$$\phi(W, 0) = \phi(V(s), 0). \tag{2.6}$$

In view of (Q.3), (2.5), (2.6) and $(W, 0) \subset_{12} (S, 0)$, we obtain

$$\phi(W, 0) = \phi(S, 0). \tag{2.7}$$

Combining (2.5), (2.6) and (2.7) we conclude that $\phi(S, 0) = \pi^\lambda(S, 0)$. □

3 Globally Individually Monotonic Bargaining Solutions

In the solutions of section 2 the utopia point of S_d played an important role. Now we consider solutions, where the utopia point of S is important. More precisely, we look at bargaining solutions which behave well with respect to the following partial orders on $\underline{B}$

$(S, d) \leqslant_1 (T, e)$ if $S^* \subset T^*$, $d = e$ and $u_2(S) = u_2(T)$,

$(S, d) \leqslant_2 (T, e)$ if $S^* \subset T^*$, $d = e$ and $u_1(S) = u_1(T)$,

$(S, d) \leqslant_{12} (T, e)$ if $(S, d) \leqslant_i (T, e)$ for $i \in \{1, 2\}$.

Definition

A bargaining solution $\phi : \underline{B} \to \mathbf{R}^2$ is called a *globally individually monotonic (g.i.m.)* solution if for all (S, d) and (T, e) in $\underline{B}$, we have for $i \in \{1, 2\}$:

(GIM$_i$) if $(S, d) \leqslant_i (T, e)$, then $\phi_i(S, d) \leqslant \phi_i(T, e)$.

Many results in section 2, concerning individually monotonic solutions, can be modified to g.i.m. solutions. E.g., we have modifications of (Q.1) – (Q.3) and

Proposition 4

ϕ is a g.i.m. solution iff for all $(S, d), (T, d) \in \underline{B}$ with $u(S) = u(T)$ and $S \subset T$, we have $\phi(S, d) \leqslant \phi(T, e)$.

In *Kalai, Rosenthal* the solution $K : \underline{B} \to \mathbf{R}^2$ was considered, where $K(S, d)$ is the unique Pareto point of S, lying on the line segment with endpoints d and $u(S)$. Obviously, K is a g.i.m. solution. Moreover, by small modifications of the proof in *Kalai, Smorodinsky* one obtains

Theorem 5

K is the unique symmetric g.i.m. bargaining solution.

In this section we want to describe all g.i.m. bargaining solutions.

Let Q be the unit square with vertices (0, 0), (1, 0), (0, 1) and (1, 1). Let Θ be the family of maps $\theta : [0, 2] \to Q$ with the property: (D) For all $s, t \in [0, 2]$, $\theta(s) \leqslant \theta(t)$ if $s \leqslant t$, and $\theta_1(s) + \theta_2(s) = s$.

Note that, just as in section 2 for $\lambda \in \Lambda$, from (D) it follows that $\theta \in \Theta$ is a continuous map.

For $\theta \in \Theta$, let $\psi^\theta : \underline{B} \to \mathbf{R}^2$ be the solution which assigns to an $(S, d) \in \underline{B}$ with $d = (0, 0)$ and $u(S) = (1, 1)$, the unique point of $P(S)$, lying on the curve $\{\theta(t); t \in [0, 2]\}$. Then one easily verifies that ψ^θ is a g.i.m. solution. The following theorem says that all g.i.m. solutions are of this form.

Theorem 6

Let $\phi : \underline{B} \to \mathbf{R}^2$ be a globally individually monotonic bargaining solution. Then there exists a $\theta \in \Theta$ such that $\phi = \psi^\theta$.

Proof

For each $t \in [0, 1]$ let $L(t) = \text{conv}\ \{(-1, 1), (0, t), (t, 0), (1, -1)\}$, and for each $t \in [1, 2]$ let $L(t) = \text{conv}\ \{(-1, 1), (t-1, 1), (1, t-1), (1, -1)\}$. Define $\theta : [0, 2] \to \mathbf{R}^2$ by $\theta(t) = \theta(L(t), 0)$ for each $t \in [0, 2]$. Similarly as in the proof of theorem 3, we obtain $\theta \in \Theta$ and $\phi(L(t), 0) = \psi^\theta(L(t), 0)$ for all $t \in [0, 2]$.

Now take an arbitrary (S, d) with $u(S) = (1, 1)$ and $d = (0, 0)$. We prove that $\theta(S, 0) = \psi^\theta(S, 0)$.

For $i \in \{1, 2\}$, let $m_i = \min\ \{x_i \in \mathbf{R}; x \in S\}$ and $n_i = \min\ \{-1, m_i\}$. Let T be the triangle with vertices $(n_1, 1)$, $(1, n_2)$ and $\psi^\theta(S, 0)$, and let $s = \psi_1^\theta(S, 0) + \psi_2^\theta(S, 0)$. Then

$$\phi(L(s), 0) = \psi^\theta(L(s), 0) = \psi^\theta(S, 0) = \psi^\theta(T, 0) \in P(L(s)) \cap P(S) \cap P(T). \tag{3.1}$$

From (3.1) and $(T, 0) \leqslant_{12} (L(s), 0)$, we obtain

$$\phi(T, 0) = \phi(L(s), 0). \tag{3.2}$$

From (3.1), (3.2) and $(T, 0) \leqslant_{12} (S, 0)$, we obtain

$$\phi(T, 0) = \phi(S, 0). \tag{3.3}$$

Combining (3.1), (3.2) and (3.3) we can conclude that $\phi(S, 0) = \psi^\theta(S, 0)$. □

Note that the Kalai-Rosenthal solution K corresponds to the curve θ_K with

$\theta_K(t) = (1/2\, t, 1/2\, t)$ for all $t \in [0, 2]$, while F^1 and F^0 correspond to θ_1 and θ_0 with

$\theta_1(s) = (s, 0)$ for $s \in [0, 1]$ and $\theta_1(s) = (1, s-1)$ for $s \in [1, 2]$,

$\theta_0(s) = (0, s)$ for $s \in [0, 1]$ and $\theta_0(s) = (s-1, 1)$ for $s \in [1, 2]$.

4 Some Remarks

In section 2 and 3 we have characterized all individually monotonic and globally individually monotonic solutions, respectively. It is now easy to derive some other theorems. Most of the proofs of these theorems are straightforward and left to the reader.

(a) First we look at (globally) individually monotonic solutions, which also satisfy the property of independence of irrelevant alternatives.

Theorem 7

(i) The only solutions satisfying (P.5), (IM_1) and (IM_2), are F^0 and F^1.

(ii) The only solutions satisfying (P.5), (GIM_1) and (GIM_2), are F^0 and F^1.

(b) In *Jansen, Tijs* a systematic study of continuity properties of bargaining solutions is made. In the following theorem we characterize all continous (globally) individually monotonic solutions.

Theorem 8

(i) An individually monotonic solution ϕ is continous iff the corresponding curve $\lambda \in \Lambda$ satisfies the following condition:

$\lambda_1(t) < 1$ and $\lambda_2(t) < 1$ for all $t \in [1, 2)$.

(ii) A globally individually monotonic solution ϕ is continuous iff the corresponding curve $\theta \in \Theta$ satisfies the condition:

$\theta_1(t) < 1$ and $\theta_2(t) < 1$ for all $t \in [1, 2)$.

(iii) For a (globally) individually monotonic solution ϕ, at least one of the functions ϕ_1 and ϕ_2 is continuous.

(c) In *Kihlstrom, Roth* and *Schmeidler* and also in *Peters, Tijs* risk sensitivity of bargaining solutions is studied. For our purpose it is sufficient to say that a solution is *risk sensitive* if for each increasing concave transformation $k : \mathbf{R} \to \mathbf{R}$ and each $(S, d) \in \underline{B}$, we have

(RS_1) $\phi_1(K^2(S), K^2(d)) \geqslant \phi_1(S, d)$,

(RS_2) $\phi_2(K^1(S), K^1(d)) \geqslant \phi_2(S, d)$, where $K^1(s) = (k(s_1), s_2)$ and $K^2(s) = (s_1, k(s_2))$ for each $s \in S$, and $K^i(S) = \{K^i(s); s \in S\}$ for $i \in \{1, 2\}$.

Theorem 9

(i) Let ϕ be an individually monotonic bargaining solution. Then ϕ is risk sensitive.

(ii) Let ϕ be a globally individually monotonic bargaining solution. Then ϕ is the risk sensitive.

Proof

We only prove (i). In view of property (P.3), for the proof of (RS_1), it is sufficient to show: for $(S, d) \in \underline{B}$ with $u(S_d) = (1, 1)$ and $d = (0, 0)$, and an increasing concave transformation k with $k(0) = 0$, $k(1) = 1$, we have $\phi_1(K^2(S), 0) \geqslant \phi_1(S, 0)$.

Now, for such k we have $k(x) \geqslant x$ if $x \in [0, 1]$. This implies that

$(S, 0) \subset_{12} (K^2(S), 0)$. Hence, by (Q.1), $\phi_1(S, 0) \leqslant \phi_1(K^2(S), 0)$.

Similarly, one proves (RS_2). □

In *Kihlstrom, Roth* and *Schmeidler* it was already proved that G is risk sensitive and in *Peters, Tijs* that K is risk sensitive.

(d) In *Thomson*, a method of replication of bargaining pairs to n-person bargaining pairs is proposed. This replication method gives rise to an interpretation of the non-symmetry of solutions belonging to a subclass of individually monotonic solutions.

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