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THE DELAYING EFFECT OF FINANCING CONSTRAINTS ON INVESTMENT

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ABSTRACT

We develop a simple model in which a firm considers a number of investment projects. Because of limited financial resources, the firm can undertake at most one project. In line with the literature on real options we stress features like irreversibility, uncertainty and the possibility of postponing the investment decision and show under which conditions limited availability of funds tends to increase the value of waiting.

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I. INTRODUCTION

The lack of unlimited financial means influences our decision making significantly, in all levels of society. Often firms or consumers cannot buy everything they would like, due to the presence of financial constraints. Limited availability of financial resources may force a consumer to decide whether to buy a new car or to replace the kitchen instead of doing both.

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But besides choosing between buying the car or the kitchen at this moment, the consumer also has the possibility of waiting before making an investment. It is quite possible that the existing car will break down next year and he or she will have to buy a new car then. The presence of uncertainty will lead to an incentive for postponement of the investment.

Examples in other fields are also numerous. In this paper we will focus on a firm's investment decision. In our analysis an investment is irreversible. Once the firm has invested in a certain project, it will not be possible to recover the initial expenses and use these to start an alternative project later, or restart the same project at better terms later. Furthermore, the future value of the various projects is surrounded with uncertainty and there is the option for the firm to postpone its investment decision. The real options approach predicts that the incentive to postpone the investment increases with the amount of uncertainty surrounding the future valuation of the project. Hence the higher the uncertainty, the higher the probability the firm will suspend investing.[1]

We will make a distinction between a firm that does not face any binding financial constraints and can invest in all projects that are considered profitable, and a firm that can only undertake at most one project due to limited availability of funds and therefore has to choose how best to spend its money. Hence, a crucial aspect of our model is that there are competing projects engaged in a race for limited funds. This limitation implies that only one project can be implemented ultimately. This scenario is in line with the literature that stresses the role of capital market imperfections due to which some firms may face credit rationing (Hubbard, 1998). This financial obstacle to investment decisions in combination with assumptions standard in the literature on irreversible investment under uncertainty has received very little attention so far.[2] To consider this last case we adopt an approach that is similar to Stulz (1982) and Johnson (1987) who derive the price of options on the maximum or minimum of several financial assets. This paper contributes a model that provides an alternative interpretation to this framework, including the possibility of comparing the investment decision of firms with and without limited financial resources. We are able to derive new comparative static results in the case of N projects and we have a different assumption regarding the distribution of returns to the various projects.[3]

The paper proceeds as follows. In Section II we briefly present the case where a firm decides upon the timing of investing its resources in one

[1] These assumptions are standard in the real options approach advanced by Dixit and Pindyck (1994).

[2] Scarramozino (1997) deals with irreversible investment and finance constraints in the context of incremental investment and estimates a Q-model.

[3] In particular, we assume that such returns have a uniform distribution whereas Stulz (1982) and Johnson (1987) assume returns to follow a Geometric Brownian Motion.

project. In Section III we introduce the presence of a second project and allow for the limited availability of financial resources. Next we discuss the more general case of N projects in Section IV. Finally Section V concludes.

II. ONE INVESTMENT PROJECT

In this section we present a stylized model that captures the main elements of the recent literature on investment under uncertainty (see Dixit and Pindyck, 1994). Consider a firm that has an investment opportunity called project A. If the firm decides to invest in period 1 the net return of the investment is equal to V_A^1. If i indicates a project and t denotes period t then V_i^t should be interpreted as the value minus the sunk cost of investing in project i at time t.[4] The net present value rule suggests making an investment as soon as $V_A^1 > 0$. However, the recent literature on investment argues that the value of V_A^1 should be compared with the expected net present value the investment yields if the firm delays its decision. Suppose that the firm also has the option to start project A in period 2.[5] If the firm invests in period 2 the net value of the project is equal to V_A^2. If the firm invests in the first period, it is impossible to recover the initial sunk cost of investment and to restart the project in period 2. This precludes the firm from setting up the project at better terms in period 2 and from acquiring the potentially larger value V_A^2. To capture the notion that future realizations are uncertain, we presume that V_A^2 is a random variable which is uniformly distributed on the interval $[\mu-\sigma, \mu+\sigma]$. Both μ and σ are strictly positive. The parameter μ denotes the expected value of V_A^2. The variance of V_A^2 equals σ^2. Therefore, a higher σ implies a higher degree of uncertainty surrounding the future benefits of the project.

We assume it is not possible to postpone the investment decision even after period 2. Therefore, if the firm's management has delayed the investment decision in period 1 it will undertake project A in period 2 if the realization of V_A^2 exceeds zero and otherwise abstain from investing. We assume that $\mu < \sigma$.[6] To decide whether the firm should invest in period 1 it calculates the expected value of the option to invest in the second period:

[4] It is useful to note that we do not model the cost of investing explicitly in our model.

[5] Without losing generality we abstract from discounting.

[6] This assumption implies that the probability that the firm does not implement project A in the second period is strictly positive, because in the worst case scenario $V_A^2 = \mu - \sigma < 0$.

$$F(V_A) = \int_0^{\mu+\sigma} \frac{1}{2\sigma} V_A^2 dV_A^2 = \frac{(\mu+\sigma)^2}{4\sigma} = E > 0. \quad (1)$$

It is optimal to invest in period 1 if $V_A^1 > F(V_A)$. Otherwise the firm will postpone the investment decision until period 2. In line with the real options literature our model indicates that the net present value rule is incorrect (Dixit and Pindyck, 1994). In fact Equation (1) shows that the net present value of the investment in period 1, V_A^1, should be strictly larger than zero to be willing to invest in period 1. Furthermore it is straightforward that the higher the uncertainty as measured by σ, the higher the value of delaying the investment: $\partial E/\partial\sigma > 0$. This result indicates that higher uncertainty tends to depress investment. Various studies provide empirical support for this claim (Guiso and Parigi, 1999; Ghosal and Loungani, 2000). Finally, delaying the investment becomes more likely as the expected value of V_A^2 increases: $\partial E/\partial\mu > 0$.

III. TWO INVESTMENT PROJECTS

The objective of the paper is to provide insights concerning the role of limited availability of funds in determining investment in light of the real options approach. Due to capital market imperfections, firms may find that a shortage of cash constrains investment if the terms at which bank loans can be obtained are unfavourable. This lack of financial means will affect their investment decision.[7]

Suppose that in addition to project A the firm has an alternative investment option called project B. If the firm chooses to invest in project B in period 1 this yields V_B^1. To simplify our analysis, the net present value of project B in the second period is also uniformly distributed on the interval $[\mu - \sigma, \mu + \sigma]$. V_A^2 and V_B^2 are independent. These assumptions affect the generality of our results, but the main argument that the firm has an incentive to learn which project is most profitable holds in this more general setting as well.

Implicitly we assume that the returns of a project implemented in period 1 are insufficient to provide the required funds to start another project in period 2. The best way of thinking about this aspect of the model is that each project's cash flow is spread over many periods. To

[7] Firms may face difficulties in acquiring external financial resources like bank loans or equity because of an information asymmetry between the firm's management and the bank concerning the profitability of investment opportunities. Alternatively the principals (i.e., holders of claims on the firm) cannot perfectly monitor the activities of their agent (i.e., the management team of the firm). See Hubbard (1998) for an excellent review of this literature.

simplify our analysis further, we assume the financial resources of the firm may be used to start an investment project, but the firm is not able to acquire any additional external funds to finance investment. If financial constraints are not binding the firm may choose to invest in both project A and B. The firm should apply the methodology depicted in Section II to both projects separately, since the value of the firm is additive in the values of the two projects. This means that the firm will invest in project A (B, respectively) in the first period if and only if $V_A^1 \geq F(V_A)$, and consider its investment in the second period otherwise. The firm does not have to choose between the two projects, but can evaluate them separately.

However, if the firm faces limited availability of funds it has the option of undertaking one project at most. The financial constraint implies that by investing in project A the firm gives up the option to invest in project B. The decision of whether to invest in the first period depends on the expected value of the two investment projects A and B in the second period. Therefore we start solving the firm's decision in the second period:

$$\begin{aligned} \text{invests in A if} \quad & V_A^2 \geq V_B^2 \text{ and } V_A^2 \geq 0, \\ \text{invests in B if} \quad & V_B^2 > V_A^2 \text{ and } V_B^2 \geq 0, \\ \text{does not invest if} \quad & V_A^2 < 0 \text{ and } V_B^2 < 0. \end{aligned} \tag{2}$$

An important feature of our model is that a number of projects are engaged in a race for a limited amount of funds. The limitation is such that only one project can be chosen. In fact we determine the value of the maximum of two random assets. Our approach resembles closely the set-up of Stulz (1982) and Johnson (1987) who derive the price of options on the maximum (and minimum) of several assets. Their approach is more general in the sense that they assume the price of the underlying assets follow a Geometric Brownian Motion, and the expected value and the variance of the rate of return may be different for each asset. Furthermore, the returns of the various assets may be correlated. In contrast we assume that the returns to the projects are identically (i.e., returns of the projects have the same expected value and variance) and independently (and uniformly) distributed. In short, we apply a different assumption regarding the distribution of returns, and take a rather simple structure of the stochastic processes. Though we lose some degree of generality, we gain tractability of the model which allows us to derive some straightforward comparative static results even in the case of N projects, in a way that provides insight and consistency.

We assume that the firm cannot sell the investment option that it did not implement to another firm, because the option results from firm-specific resources or capabilities that cannot be imitated or transferred to other companies (see Barney, 1991). Therefore in the first period the expected value of the two investment projects is given by (see the Appendix):

$$F(V_A, V_B) = \int_0^{\mu+\sigma} \int_{\mu-\sigma}^{V_A} \frac{V_A}{4\sigma^2} dV_B dV_A + \int_0^{\mu+\sigma} \int_{\mu-\sigma}^{V_B} \frac{V_B}{4\sigma^2} dV_A dV_B = \frac{\frac{1}{3}(\mu+\sigma)^2 - \frac{1}{2}(\mu+\sigma)^2(\mu-\sigma)}{2\sigma^2}. \quad (3)$$

Since we assume that $\sigma > \mu$ it can be shown after some straightforward calculations that

$$F(V_A, V_B) = E\left(1\frac{2}{3} - \frac{1}{3}\frac{\mu}{\sigma}\right) > 1\frac{1}{3}E \quad (4)$$

where E is defined in Equation (1). Equation (4) indicates that the presence of two investment opportunities in combination with limited availability of funds raises the critical benchmark at which the firm finds it optimal to invest in the first period by more than 33 percent. This is due to the fact that by investing in either A or B the firm gives up the opportunity to invest in the other project later, which may be undesirable because the unfunded project may yield a favourable outcome in the future.

The results above suggest that the timing of investment by financially constrained and unconstrained firms will differ. Suppose that a population of firms exists in which each firm considers the same investment projects, A and B. Firms that do not face a shortage of cash are more likely to start project A or B in the first period than financially constrained firms, because the critical value E at which these firms are willing to invest is lower than that of the constrained firms.

It can be shown that if the expected value of the future returns of the two projects, i.e., μ, increases that:

$$\frac{\partial F(V_A, V_B)}{\partial \mu} = \frac{\mu+\sigma}{2\sigma}\left(\frac{3}{2} - \frac{1}{2}\frac{\mu}{\sigma}\right) > \frac{\mu+\sigma}{2\sigma} > 0. \quad (5)$$

This result implies that the firm's incentive to delay the investment decision increases with a higher expected future return μ. The same holds if the parameter σ measuring the amount of uncertainty surrounding the projects increases:

$$\frac{\partial F(V_A, V_B)}{\partial \sigma} = \frac{\partial E}{\partial \sigma}\left(1\frac{2}{3} - \frac{1}{3}\frac{\mu}{\sigma}\right) + \frac{1}{3}E\frac{\mu}{\sigma^2} > \frac{\partial E}{\partial \sigma}\left(1\frac{1}{3}\right) + \frac{1}{3}E\frac{\mu}{\sigma^2} > 0. \quad (6)$$

Therefore, higher uncertainty tends to increase the incentive to postpone investing.[8]

IV. *N* INVESTMENT PROJECTS

The above findings readily extend to the case where the financially constrained firm has the opportunity to choose one project out of N possibilities. As in the previous section, we assume that the future returns of project i denoted by V_i^2 are identically and independently distributed on the interval $[\mu-\sigma,\ \mu+\sigma]$. In the second period the firm:

$$\begin{aligned}&\text{invests in project } i \text{ if} && V_i^2 \geq V_j^2 \text{ and } V_i^2 \geq 0,\ \text{for } i \neq j, i,j \in \{1,\ldots,N\}\\ &\text{does not invest if} && V_i^2 < 0 \text{ for all } i, i \in \{1,\ldots,N\}.\end{aligned} \tag{7}$$

Using partial integration we show in the Appendix that in the first period the expected value of these N projects equals:

$$\begin{aligned}F(V_1,\ldots,V_N) &= \sum_{i=1}^{N} \int_0^{\mu+\sigma} \int_{\mu-\sigma}^{V_i} \cdots \int_{\mu-\sigma}^{V_i} \frac{V_i}{2^N\sigma^N} dV_i \prod_{i\neq j} dV_j\\ &= (\mu+\sigma) - \frac{2\sigma}{N+1} + \frac{(\sigma-\mu)^{N+1}}{(N+1)(2\sigma)^N}\end{aligned} \tag{8}$$

After some straightforward but tedious calculations performed in the Appendix we find for $N=2$ that the expression in Equation (8) is equivalent to the one presented in Equation (3).

In the Appendix it is shown that if the number of projects increases by one the change in the expected value of the projects equals:

$$\begin{aligned}&F(V_1,\ldots,V_{N+1}) - F(V_1,\ldots,V_N)\\ &= \frac{1}{(N+1)(N+2)}\left(2\sigma - \left(\frac{\sigma-\mu}{2\sigma}\right)^{N+1}((N+3)\sigma + (N+1)\mu)\right).\end{aligned} \tag{9}$$

Our assumptions made previously imply that $0 < 1 - \frac{\mu}{\sigma} < 1$. Furthermore, by using mathematical induction it is verified in the appendix that for all $N \geq 1$ it holds that $0 < \frac{N+3}{2^{N+1}} \leq 1$ and $0 < \frac{N+1}{2^{N+1}} \leq 1$. This implies that the change in the expected value of the projects is positive when the number of projects increases.

[8] This finding corresponds with the results of Stulz (1982). If one assumes in his model that the exercise price is equal to zero, the returns to the assets are independently distributed, then higher uncertainty increases the value of the option and hence the option to wait. This follows from his Equations (19) and (12).

$$F(V_1, \ldots, V_{N+1}) - F(V_1, \ldots, V_N) > \frac{1}{(N+1)(N+2)}(2\sigma - (\sigma + \mu)) > 0. \tag{10}$$

Therefore, the critical value at which the firm is willing to invest increases with the number of projects. Loosely interpreted, financing constraints become tighter as N increases since the firm can only select one project.[9] Therefore, the above result in Equation (10) indicates that tighter financing constraints increase the value of waiting.

If the number of projects increases to infinity the expected value of the projects becomes:

$$\lim_{N \to \infty} F(V_1, \ldots, V_N) = \mu + \sigma. \tag{11}$$

The result follows from the fact that all random variables V_i^2 are identically and independently distributed on the interval $[\mu - \sigma, \mu + \sigma]$. If the number of projects increases to infinity then with probability one in the second period, the value of one of these projects will be $\mu + \sigma$, the maximum realization possible.

We also find that the value of the investment projects increases with the expected future benefits of the projects:

$$\frac{\partial F(V_1, \ldots, V_N)}{\partial \mu} = 1 - \frac{1}{(N+1)^2} \frac{1}{2^N} \left(1 - \frac{\mu}{\sigma}\right)^N. \tag{12}$$

Since the three terms after the minus sign in the above equation are all larger than zero but smaller than 1, the sign of the derivative is positive, implying that the firm is willing to wait longer if the future prospects of the projects improve. Finally, as we show in the Appendix, higher uncertainty increases the value of the investment projects in the future as well:

$$\frac{\partial F(V_1, \ldots, V_N)}{\partial \sigma} = 1 - \frac{2}{N+1} + \frac{(\sigma - \mu)^N}{(N+1)2^N} \left(\frac{\sigma + N\mu}{\sigma^{N+1}}\right) > 0. \tag{13}$$

Hence, as uncertainty increases it becomes more likely that the firm will wait before investing to see which of the N projects is the most fruitful one.

[9] In fact, the financial constraint does not necessarily become tighter, but the number of alternative projects increases. Thus, the threshold of adoption of a given project increases, as the number of projects increases. This holds true because the probability that a competing project has a more favourable return increases with an increased number of projects.

V. CONCLUSION

We have studied a model in which a firm has a number of potential investment projects. Due to financial constraints it can select only one of these projects. The firm decides to implement one of these projects as the immediate return of this particular project exceeds a certain critical value. This critical value increases if the number of potential projects becomes larger and if uncertainty increases. The reason is that there exists an option value to waiting, because the decision to choose a particular project cannot be reversed. Therefore, waiting allows the firm to learn which project is the best one if future profitability is uncertain. The results of the model hold under the following assumptions. First, the firm can delay its investment decision for one period. Second, the risk characteristics of all projects are identical. Third, the returns to the projects are independently distributed.

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APPENDIX

Derivation of Equation (3)

Since the two terms in $\int_0^{\mu+\sigma}\int_{\mu-\sigma}^{V_A}\frac{V_A}{4\sigma^2}dV_BdV_A+\int_0^{\mu+\sigma}\int_{\mu-\sigma}^{V_B}\frac{V_B}{4\sigma^2}dV_AdV_B$ are equal, we find that $F(V_A,V_B)=2\int_0^{\mu+\sigma}\int_{\mu-\sigma}^{V_A}\frac{V_A}{4\sigma^2}dV_BdV_A=\frac{1}{2\sigma^2}\int_0^{\mu+\sigma} V_A(V_A-\mu+\sigma)dV_A$.
It is straightforward to show that the above integral is equal to the expression in (3).

Derivation of Equation (8)

Since the N terms of $\sum_{i=1}^{N}\int_0^{\mu+\sigma}\int_{\mu-\sigma}^{V_i}\cdots\int_{\mu-\sigma}^{V_i}\frac{V_i}{2^N\sigma^N}\left(\prod_{j\neq i}dV_j\right)dV_i$ are equal, $F(V_1,\ldots,V_N)=\frac{N}{(2\sigma)^N}\int_0^{\mu+\sigma}V_i(V_i-\mu+\sigma)^{N-1}dV_i=$
$\frac{N}{(2\sigma)^N}\left(\frac{(\mu+\sigma)(2\sigma)^N}{N}-\int_0^{\mu+\sigma}\frac{(V_i-\mu+\sigma)^N}{N}dV_i\right)=(\mu+\sigma)-\frac{N}{(2\sigma)^N}\left(\frac{(2\sigma)^{N+1}}{(N+1)N}-\frac{(\sigma-\mu)^{N+1}}{(N+1)N}\right)$.
This is equal to the expression in Equation (8).

Derivation that for $N=2$ Equation (8) equals Equation (3).

Starting with Equation (8),

$$F(V_1,V_2)=\frac{1}{2\sigma^2}\left(2\sigma^2\mu+2\sigma^3-\frac{4}{3}\sigma^3+\frac{1}{6}(\mu+\sigma-2\mu)^3\right)$$
$$=\frac{1}{2\sigma^2}\left(2\sigma^2\mu+\frac{2}{3}\sigma^3+\frac{1}{6}\left((\mu+\sigma)^3-6\mu(\mu+\sigma)^2+12\mu^2(\mu+\sigma)-8\mu^3\right)\right)$$
$$=\frac{1}{2\sigma^2}\left(\frac{1}{3}(\mu+\sigma)^3+2\sigma^2\mu+\frac{2}{3}\sigma^3+\frac{1}{6}\left(-(\mu+\sigma)^3-6\mu(\mu+\sigma)^2+12\mu^2(\mu+\sigma)-8\mu^3\right)\right)$$
$$=\frac{1}{2\sigma^2}\left(\frac{1}{3}(\mu+\sigma)^3-\frac{1}{2}\left(\mu^3+\mu^2\sigma-\sigma^2\mu-\sigma^3\right)\right).$$
Equation (3) follows immediately.

Derivation of Equation (9)

$F(V_1,\ldots,V_{N+1})-F(V_1,\ldots,V_N)$
$=\left(\frac{1}{N+1}-\frac{1}{N+2}\right)2\sigma+\left(\frac{\sigma-\mu}{2\sigma}\right)^{N+1}\left(\frac{\sigma-\mu}{N+2}-\frac{2\sigma}{N+1}\right)$
$=\frac{2\sigma}{(N+1)(N+2)}+\left(\frac{\sigma-\mu}{2\sigma}\right)^{N+1}\left(\frac{(N+1)(\sigma-\mu)-2\sigma(N+2)}{(N+1)(N+2)}\right)$. This can be used to derive Equation (9).

Derivation of Equation (10)

We need to show that for all $N>1$, $0<A_N=\frac{N+3}{2^{N+1}}\leq 1$ and $0<B_N=\frac{N+1}{2^{N+1}}\leq 1$. It is sufficient to show that $0<A_N\leq 1$ since $A_N>B_N$. $A_N>0$ is obvious. For $N=1$, $A_N=1$. Suppose that our claim holds for N. $A_{N+1} = \frac{N+3+1}{2^{N+2}} = \frac{A_N}{2} + \frac{1}{2^{N+2}} \leq \frac{1}{2} + \frac{1}{2^{N+2}} \leq 1$. Our assumptions made previously imply $0 < 1 - \frac{\mu}{\sigma} < 1$. Equation (10) follows straightforwardly. *QED*.

Derivation of Equation (13)

$\frac{\partial F(V_1,\ldots,V_N)}{\partial\sigma} = 1 - \frac{2}{N+1} + \frac{1}{(N+1)2^N}\left(\frac{(N+1)(\sigma-\mu)^N\sigma^N-(\sigma-\mu)^{N+1}N\sigma^{N-1}}{\sigma^{2N}}\right)$
$= 1 - \frac{2}{N+1} + \frac{(\sigma-\mu)^N}{(N+1)2^N}\left(\frac{(N+1)\sigma-(\sigma-\mu)N}{\sigma^{N+1}}\right)$. Equation (13) can be obtained by collecting terms. $\frac{\partial F(V_1,\ldots,V_N)}{\partial\sigma}$ is positive, since $1 - \frac{2}{N+1}$ is positive for $N > 1$ and $\left(\frac{(N+1)\sigma-(\sigma-\mu)N}{\sigma^{N+1}}\right) > 0$.