Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

Journal of Macroeconomics 27 (2005) 69–86

Journal of MACROECONOMICS

www.elsevier.com/locate/econbase

Entrepreneurs, moral hazard, and endogenous growth ☆

J. Philipp Reiß [a,*], Lutz Weinert [b]

[a] *Faculty of Economics and Management, Otto-von-Guericke-University of Magdeburg, P.O. Box 41 20, 39016 Magdeburg, Germany*

[b] *BearingPoint GmbH and University of Magdeburg, Magdeburg, Germany*

Received 8 January 2002; accepted 20 October 2003
Available online 1 January 2005

Abstract

We analyze an endogenous growth model with agents differing in their endowments. Poor entrepreneurs with limited liability need to borrow in financial markets to participate in aggregate output production. We show that the first-best solution can either be achieved by decentralized financial contracting or by employing a project-specific subsidy policy.

If additional capital market imperfections are introduced into the model, a negative link between inequality and growth emerges. Then, the impact of inequality on growth increases for a higher degree of frictions.

JEL classification: D31; G14; O16; O41
Keywords: Endogenous growth; Inequality; Moral hazard; Limited liability; Capital market imperfections

☆ An earlier version of this paper was presented at the 2001 Annual Meeting of the Royal Economic Society.
* Corresponding author. Tel.: +49 0391 67 18 324; fax: +49 0391 67 11 136.
E-mail address: reiss@ww.uni-magdeburg.de (J.P. Reiß).

0164-0704/$ - see front matter
doi:10.1016/j.jmacro.2003.10.001

Nomenclature

θ	index of individuals
$\varepsilon_{\theta,t}$	random human capital endowment share
γ	start-up cost of an entrepreneurial project
λ	wealth level of the poorest entrepreneur
λ^{sdc}	wealth level of the poorest entrepreneur with a standard debt contract
μ	index of the poorest borrowing entrepreneur exerting e^*
π^*	expected project payoff with e^*
π_i	project payoff if outcome i realizes
$\pi^{\text{sdc}}(e)$	expected project payoff with a standard debt contract
ρ	riskless rate of interest
$\hat{\rho}$	borrowers' net repayment rate
$\omega_{\theta,t}$	human capital endowment of individual θ in cohort t
Π	random return of an entrepreneurial project
$c(e)$	effort cost function
b_θ	amount of external finance
$d_{\theta,t}$	consumption of individual θ
e	entrepreneurial effort
$F(\varepsilon)$	economy's cdf of capital endowment shares
g	growth rate of aggregate output
i	index of project payoff states
k_θ	individual θ's endowment with physical capital ($\omega_\theta = k_\theta$)
n	number of possible project payoff realizations
$p_i(e)$	probability of project outcome i
s	subsidy ensuring e^*
t_i	repayment to the lender in payoff state i
y_t	aggregate output in period t
A_t	stock of knowledge available at the beginning of period t
R	expected repayment generated by any transfer system T
R^{FBmax}	largest repayment generated by a first-best contract
$R^{\max}$	largest repayment generated by any contract (possibly nonmonotonic)
T	any feasible repayment scheme
U_t^θ	utility of individual θ in cohort t

1. Introduction

The purpose of this paper is to analyze the conditions that lead to optimal economic growth in an agency-model of financial contracting. In this model, a risk-neutral entrepreneur chooses an unobservable level of effort. He may employ the investment funds of a risk-neutral investor while both, entrepreneur and investor, are constrained by limited liability. We show that the first-best solution can either be achieved by decentralized financial contracting or by employing a project-specific subsidy policy.

Recent empirical research, based on cross-country-regression analysis, has identified a negative relation between inequality and growth. Prominent examples include Persson and Tabellini (1994) and Aghion et al. (1999). In response to this finding, models have been constructed which predict lower growth rates as inequality becomes more severe. For surveys of the recent theoretical literature, see Barro (2000), Aghion et al. (1999) and Benabou (1996). There exists a variety of approaches which encompass political turmoils as well as voting behavior as possible transmission channels. Another strand of the literature examines the role of credit-market imperfections due to moral hazard in the inequality-growth context (e.g. Aghion and Bolton, 1997; Piketty, 1997). These contributions build on an incentive argument whereby inequality worsens entrepreneurial *incentives* which in turn depresses the economy's growth rate as emphasized by Aghion et al. (1999). [1] Entrepreneurial investment projects in these models are very specific in that project returns follow a binomial distribution.

The recent contribution of Forbes (2000) to the empirical literature challenges the supposed negative link between income inequality and growth. Given this observation, our model demonstrates that the existence of credit market imperfections due to limited liability in a model of endogenous growth is not necessarily inconsistent with these results.

In contrast to the cited theoretical literature, we assume a general class of investment projects where revenue is discretely distributed. We find that the outcome of decentralized financial contracting can be Pareto-efficient. [2] Thereby credit-market frictions due to moral hazard are overcome and inequality does not affect entrepreneurial incentives and growth anymore. If the Pareto-efficient effort level is not implementable by decentralized contracting, a project-specific subsidy policy can be employed which retains the first-best solution.

The rationale for our result is as follows. Poor entrepreneurs are residual claimants of their project and its effort incentives depend on implementable contracts. The multiple state distribution of payoffs allows for the design of contracts which do not distort the entrepreneur's effort decision. With a two-state payoff distribution, this requires repayments to coincide in both states due to offsetting marginal probabilities. Since in the lower profit state the repayment exceeds the payoff, limited liability prevents the coincidence of transfers. Therefore, the repayment in the low state is always smaller than in the high state, additional effort increases expected repayment, and the borrower's effort choice is suboptimal. In contrast, a richer payoff distribution allows to offset the effect of unequal transfers by the possibility to condition repayments on additional states such that the marginal expected repayment can be

[1] In the model of Aghion et al. (1999), the elimination of inequality is the only way to increase the growth rate to its optimal level. This is achieved by permanent redistribution.

[2] This complements the finding of Innes (1990). He analyses a financial contracting problem with moral hazard and limited liability where the returns to an investment project follow a continuous distribution.

reduced to zero. In this case, marginal effort return fully accrues to the entrepreneur implying a Pareto-efficient effort choice.

The existence of repayment schemes inducing a Pareto-efficient allocation requires state-contingent contracts and costless state verification. With costly state verification, Gale and Hellwig (1985) and Townsend (1979) have shown the optimality of the standard debt contract. With standard debt contracts, entrepreneurs must share marginal effort return with investors. Hence, effort is distorted away from the first-best level such that a negative link between inequality and growth prevails. However, the magnitude of the impact of inequality on economic growth is driven by the degree of capital market imperfections, which is consistent with Barro (2000).

2. The static model

In this section, we introduce an inequality-and-growth model with credit market imperfections. In this model, one generation of risk-neutral individuals succeeds the former one until eternity. Upon birth, individuals in the same cohort receive heterogenous human capital endowments. They may be regarded as entrepreneurs since every agent pursues an investment project. The distinctive feature of our model is a richer payoff distribution of investment projects which may allow for first-best growth unlike similar models with extremely rudimentary distribution specifications (see e.g., Aghion and Bolton, 1997; Aghion et al., 1999).

Each individual lives for two subperiods and is endowed with one unit of raw labor as well as some human capital. In particular, any individual θ in cohort t embodies human capital $w_{\theta,t} = \varepsilon_{\theta,t} \cdot A_t$ where A_t denotes the stock of knowledge available at the beginning of period t measured in "efficiency units". The continuous random variable ε is distributed independently and identically over individuals and cohorts with cdf $F(\varepsilon)$ such that $E(\varepsilon) = 1$ and $\varepsilon \geqslant 0$.

The timing of events is summarized in Fig. 1. In the first subperiod, individuals are young and produce a capital good with a 1:1-technology where each effective unit of labor (raw labor refined by individual knowledge) creates one unit of capital. Since raw labor is fixed at unity, individual θ produces $k_\theta = w_\theta$ units of physical capital. Preferences are represented by:

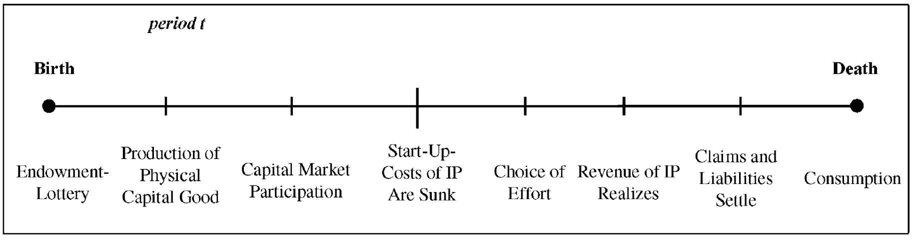

Fig. 1. Timing of events.

$$U_{\theta,t} = d_{\theta,t} - C(e_{\theta,t}, A_t), \tag{1}$$

where d_t denotes consumption at the end of lifetime and $C(\cdot)$ measures nonmonetary costs of effort devoted to an investment project. The opportunity cost of effort reduction is the expected decrease in consumption. Since consumption increases in the stock of knowledge, opportunity costs of effort reduction increase over time and it is natural to assume that effort's costs increase in A_t, too. If e is interpreted as the number of labor hours devoted to the project, the assumption implies that disutility of labor grows as labor productivity increases. This reflects the increasing value of leisure activities as their quality/variety benefits from improved labor productivity. In particular, the effort cost is defined by $C(e_{\theta,t},A_t) = c(e_{\theta,t}) \cdot A_t$. This guarantees a stationary level of first-best effort over time. Total effort costs per efficiency unit, $c(e)$, are supposed to be strictly convex in $e \in [0, \bar{e}]$, i.e. $c',c'' > 0$, $\lim_{e\to 0} c'(e) = 0$ and $\lim_{e\to\bar{e}} c'(e) = \infty$.

In the second subperiod, every individual executes an investment project. In order to become an entrepreneur in period t, an individual needs a fixed amount of capital γA_t which may be thought of as a start-up cost, $\gamma > 0$. Depending on the intensity of effort, revenue Π is a discrete random variable. Let the revenue space of an investment project Π_t be given by $\{A_t \pi_i\}_{i=1}^n$ where $n \geqslant 3$ (for $n = 2$, see Aghion et al., 1999). Without loss of generality, assume outcomes to be in ascending order such that $\pi_i < \pi_j$ for any $i < j$. Due to entrepreneurs' limited liability, we abstract from any subsequent payments, thus $\pi_i \geqslant 0$. The probability of outcome π_i conditional on entrepreneurial effort e is denoted by $p_i(e) \geqslant 0$ which we assume to be twice-differentiable in e. In order to formalize that additional effort is beneficial, we assume that the monotone likelihood ratio property holds, thus $p_i'(e)/p_i(e) > p_j'(e)/p_j(e)$ for all $\pi_i > \pi_j$ (cf. Milgrom, 1981). This implies that $\partial E(\Pi|e)/\partial e > 0$. Moreover, we assume that marginal expected revenue $\partial E(\Pi|e)/\partial e$ is monotonic and differentiable in e and that $\lim_{e\to\bar{e}} \partial E(\Pi|e)/\partial e$ has a finite upper bound. The last assumption implies that expected marginal revenue does not grow to infinity as maximum effort is delivered.

In the following, individuals with sufficient capital to cover start-up costs are *rich.* Individuals who need to borrow are *poor*. Suppose an individual is rich enough to finance her investment project out of her endowment. Then $\gamma A \leqslant k$ and she supplies $k - \gamma A$ to (world) capital markets where the riskless rate of interest is fixed at ρ. Substitution of her budget constraint $d = \pi + (1+\rho)(k - \gamma A)$ into the utility function leads to the expected utility maximization problem of rich entrepreneurs:

$$\max_e \; A \sum_{i=1}^n p_i(e)\pi_i + (1+\rho)(k - \gamma A) - c(e)A. \tag{2}$$

The assumptions about the effort cost function and the investment project guarantee that first-best effort $e^* \in (0, \bar{e})$ is unique and implicitly given by the FOC.

$$\sum_{i=1}^n p_i'(e^*)\pi_i = c'(e^*). \tag{3}$$

Thus, each rich entrepreneur chooses an effort level e^*, such that marginal expected revenue equals marginal effort costs.

Now, consider a poor individual which needs to borrow $b = \gamma A - k$ in order to start a business. Let t_i denote the transfer that the entrepreneur has to pay back to the investor if outcome π_i realizes. Due to the entrepreneur's limited liability constraint transfers are bounded by realized revenue, i.e. $0 \leqslant t_i \leqslant \pi_i$. For any repayment scheme $T := \{t_i\}_{i=1}^n$, the expected repayment to the investor is $E(B|e) = A\sum_i p_i(e)t_i$. The expected repayment per efficiency unit is denoted by $R(e) := E(B|e)/A$. The discussion of the lender's participation constraint $E(B|e) \geqslant (1 + \rho)b$ is delegated to Section 3.2. For the moment, assume it to be satisfied. The maximization problem of a borrower choosing effort level $\tilde{e}$ is given by:

$$\max_e A \sum_{i=1}^n p_i(e)(\pi_i - t_i) - c(e)A, \tag{4}$$

leading to FOC,

$$\sum_{i=1}^n p_i'(\tilde{e})\pi_i = c'(\tilde{e}) + \sum_{i=1}^n p_i'(\tilde{e})t_i. \tag{5}$$

Obviously, this FOC differs from (3) in the last term on the RHS which is the expected marginal repayment to the investor. This term is always nonnegative. To see this, assume it to be negative. Then, a new transfer system may be designed that yields the same repayment but induces the first-best solution e^*. [3] Since the lender is indifferent between both transfer schemes, but the entrepreneur has higher residual claims net of disutility if executing e^*, he chooses the first-best repayment contract.

Since we assume the objective function—given the repayment structure—to be strictly concave, a necessary and sufficient condition for a borrower to choose first-best effort is that he receives all the benefits of his marginal effort, i.e. the second term on the RHS vanishes at $e = e^*$:

$$\partial E(B|e^*)/\partial e = 0. \tag{6}$$

Therefore, an additional marginal effort unit must not increase the expected repayment on the loan. Otherwise the entrepreneur chooses an effort level that is too low compared to first-best, i.e. $\tilde{e} < e^*$.

3. First-best contracts

In this section, we derive the conditions for the existence of a repayment contract which satisfies (6) as well as limited liability and is acceptable to the lender. As a result, Pareto-efficient production can be implemented. If the incentive and the repayment constraint cannot be fulfilled, any feasible repayment contract creates a wedge between the marginal revenue of the last effort unit and the entrepreneur's share. This

[3] For a formal proof of this claim, see Lemma 10 in Appendix A.

gap distorts the effort choice and results in suboptimal production as in similar inequality-growth models, cf. Aghion and Bolton (1997) and Aghion et al. (1999). The obstacle to efficient production in this case is the entrepreneur's limited liability.

3.1. The incentive constraint

Since effort is assumed to influence the project outcome in a productive way, additional effort necessarily shifts probability mass from lower profit states to higher ones. Therefore, the introduction of state-dependent transfers in high profit states decreases the entrepreneurial expected return to additional effort and, analogously, transfers in low profit states increase it. The magnitudes of these repayment-scheme-based effects are determined by the size of marginal probabilities and transfers. If transfers are designed such that these effects on marginal effort return exactly balance, the expected marginal repayment to the lender reduces to zero and the effort decision remains undistorted.

The following proposition provides a necessary and sufficient condition for the existence of repayment contracts which solve the incentive problem (6) and highlights the weakness of requirements for the existence of first-best contracts. It suffices that the payoff distribution has at least two strictly positive payoffs with marginal probabilities differing in sign. This directly implies the nonexistence of first-best contracts for two-state-payoff distributions with $\pi_1 = 0$, since repayment occurs only in the higher profit state leading to a positive marginal expected repayment. If the payoff in the lowest of both states is positive, each first-best contract is characterized by transfer coincidence, $t_1 = t_2$, and the limited liability assumption reduces to an unbinding constraint which is usually ruled out. In all other cases, the binding limited liability constraint, $t_1 < t_2$, implies a strictly positive marginal repayment. In contrast, additional payoff states allow for the simultaneity of a binding limited liability constraint and the existence of first-best contracts, since the balance of transfers to eliminate marginal expected repayment can involve more than two payoff states.

Definition 1. A repayment contract $\{t_i\}_{i=1}^{n}$ is trivial if $t_i = 0\,\forall i$.

Proposition 2. *If and only if there exist at least two strictly positive outcomes π_i, π_j, such that $\operatorname{sign}(p_i'(e^*)) = \operatorname{sign}(-p_j'(e^*))$ and $p_i' \neq 0$, then there exist non-trivial repayment contracts $\{t_i\}_{i=1}^{n}$ which solve (6).*

Proof. (I) Necessity: Given that at least one $p_i' \neq 0$, the only possibility not to have two strictly positive outcomes with opposite signs in their corresponding probability derivatives is having $\pi_1 = 0$ with $\operatorname{sign}(p_1'(e^*)) = \operatorname{sign}(-p_k'(e^*))\ \forall k > 1 \wedge p_k' \neq 0$. From $\pi_1 = 0$ it follows that $t_1 = 0$. Therefore (6) reduces to $\sum_{i=2}^{n} p_i'(e^*)t_i = 0$. Since all p_i' in this sum have the same sign (or are zero) and the same is true for all these transfers, there exists no non-trivial repayment contract which can solve (6) in this case. This implies the necessity of the proposition. (II) Sufficiency: Choose any two strictly positive outcomes π_i, π_j, such that $\text{sign}(p_i'(e^*)) = \text{sign}(-p_j'(e^*))$ and $p_i' \neq 0$ and set all transfers $t_k = 0\ \forall k \neq i,j$. Then (6) is given by $p_i'(e^*)t_i + p_j'(e^*)t_j = 0 \iff -p_i'(e^*)/p_j'(e^*) = t_j/t_i$. Since both probability derivatives are opposite in sign and finite, the LHS is a positive constant. Obviously there are infinitely many possibilities to choose transfers t_i, t_j such that $0 < t_i < \pi_i$, $0 < t_j < \pi_j$, and the equation holds. This proves the proposition's sufficiency. □

3.2. The repayment constraint

Our first proposition establishes the existence of repayment contracts that fulfil the incentive constraint if the payoff structure features at least two strictly positive outcomes. The expected value of the resulting repayment has an upper bound. This upper bound is determined by the distribution of payoffs because of the entrepreneur's limited liability. Notice that payoffs π_i, transfers t_i, and the maximum repayment $R^{\text{FB max}}$ are measured per efficiency unit, i.e. consumption units per unit of knowledge, for notational ease.

Definition 3. Let $R^{\text{FB max}}$ denote the maximum expected repayment from a first-best transfer system. It is the solution to the following problem:

$$R^{\text{FB max}} \equiv \max_{\{t_i\}} \sum p_i(e^*)t_i,$$
$$\text{subject to:} \quad \sum p_i'(e^*)t_i = 0 \quad \text{and} \quad 0 \leqslant t_i \leqslant \pi_i.$$

In a decentralized market economy with poor entrepreneurs, production efficiency requires $R^{\text{FB max}} \geqslant \gamma(1+\rho)$ for any borrower. Otherwise, no contract exists which gives the lender at least his outside option and induces the entrepreneur to deliver first-best effort. If $R^{\text{FB max}}$ is not sufficiently large to cover the repayment required by the lender, the difference is given by $s = (1+\rho)(\gamma A - \omega) - AR^{\text{FB max}}$. If production efficiency is to be achieved, this calls for a project-specific subsidy policy in the sense that the borrowers receives $s/(1+\rho)$.

4. The dynamics

Aggregate output is the sum of returns to all investment projects. Since there is a unit mass of individuals, the economy's capital endowment is equal to $E(w_{\theta,t}) = A_t$. If all investment projects, each requiring γA as a sunk capital cost, are undertaken, aggregate costs of entrepreneurship amount to γA. Hence the economy is a lender in world financial markets if $\gamma > 1$ and a borrower if $\gamma < 1$. If $\gamma = 1$ the economy is endowed with exactly that amount of physical capital which is needed to provide every individual with the project's start-up cost. Since this assumption has no impact on the incentive argument for redistribution, we set γ to unity. Let $\pi(\varepsilon)$ denote the expected return of a project executed by an individual with capital endowment ε per efficiency unit, then aggregate output is given by:

$$y_t = A_t \int_0^\infty \pi(\varepsilon)\mathrm{d}F(\varepsilon).$$

As shown in the preceding section, depending on the specification of investment projects, there may be an endowment level which is too low to allow for a decentralized first-best outcome. Denote the wealth level per efficiency unit of the poorest individual exerting first-best effort by μ, then aggregate output can be reexpressed as:

$$y_t = A_t\left\{\int_0^\mu \pi(\varepsilon)\mathrm{d}F(\varepsilon) + [1 - F(\mu)]\pi^*\right\},$$

where first-best revenue equals π^* and the integral is strictly less than $F(\mu)\pi^*$. Note that μ equals zero if an individual borrowing the total start-up cost delivers first-best effort and otherwise $\mu = \gamma - R^{\mathrm{FB\,max}}/(1+\rho)$.

Following the literature, we assume for the evolution of the stock of knowledge that $A_t = y_{t-1}$. Since the growth rate of aggregate output, g_t, is approximately $\ln y_t/y_{t-1}$, we obtain:

$$g_t = \ln\left\{\int_0^\mu \pi(\varepsilon)\mathrm{d}F(\varepsilon) + [1 - F(\mu)]\pi^*\right\}. \tag{7}$$

Now we are ready to analyze the effects of inequality: If investment projects are such that every individual delivers first-best effort, μ equals 0 and the economy's growth rate is given by:

$$g_t^* = \ln \pi^*.$$

If some entrepreneurs cannot commit to first-best effort, μ is positive and the economy's actual growth rate is lower than g_t^*. Notice that the transmission channel of inequality to lower growth requires investment projects to generate a positive μ which is not necessarily the case as has been demonstrated in the preceding section. Otherwise this link vanishes. If, however, $\mu > 0$ a project-specific subsidy-policy implements optimal growth.

5. Credit-market frictions

In this section, we demonstrate how credit-market frictions serve as a catalyst for the effect of inequality on growth. As inequality increases, a higher degree of credit-market imperfections leads to a stronger negative effect on growth. Although Barro (2000) suggests that the magnitude of inequality's impact on growth is conditioned on credit-market imperfections, a direct theoretical foundation seems not yet available. Rather, inequality-growth models with imperfect credit markets pay attention to a fixed set of credit-market barriers (see e.g. Aghion and Bolton, 1997; Banerjee and Newman, 1993; Galor and Zeira, 1993; Piketty, 1997).

In our model, credit-market frictions can affect aggregate output and its growth rate through either credit rationing or effort distortions. For a given income distribution, the development of the financial system increases aggregate output's growth rate which is not uncontroversial as Levine (1997) argues. Our theoretical result complements the empirical findings in Rajan and Zingales (1998) and these surveyed in Levine (1997).

5.1. Credit rationing

Here, credit rationing designates the inability to obtain the necessary amount of external finance to cover start-up costs. Loans are denied whenever the largest expected repayment from a feasible contract falls short of the amount required by investors. The entrepreneurial limited liability constraint is essential for the possibility of credit rationing, otherwise it is not even beneficial to execute the project with first-best effort. [4]

Definition 4. Let $R^{\max}$ denote the maximum expected repayment from a repayment scheme. It is the solution to the following problem:

$$R^{\max} \equiv \max_{\{t_i\}} \sum p_i(\tilde{e})t_i,$$
$$\text{subject to:} \quad (5) \quad \text{and} \quad 0 \leqslant t_i \leqslant \pi_i.$$

Limited liability generates credit-rationing if $R^{\max} < (1+\rho)\gamma$ which depends on the particularities of the investment project involved. There are a number of reasons why borrowers may face a repayment rate exceeding principal plus interest. These include all sorts of transaction costs arising from writing and enforcing contracts or accessing external finance as well as imperfect competition within the domestic sector of financial intermediation. Any of these capital-market imperfections leads to an upward bias of the borrowers' net repayment rate. For a stylized incorporation of these imperfections into our model, let $1+\hat{\rho}$ denote the repayment rate faced by borrowing entrepreneurs. The difference $\hat{\rho} - \rho \geqslant 0$ accounts for the severity of the outlined type of imperfections in financial markets. [5]

If the wealth level per efficiency unit of the poorest individual executing a (partially or fully) externally financed project is denoted by $\lambda \in [0, \mu]$, the economy's growth rate (7) can be written as:

$$g_t = \ln\left[\pi^* - F(\lambda)\pi^* - \int_{\lambda}^{\mu} \pi^* - \pi(\varepsilon)\mathrm{d}F(\varepsilon)\right]. \tag{8}$$

[4] The possibility of credit-rationing implies $R^{\max} < (1+\rho)\gamma$. Without limited liability, borrowing entrepreneurs always deliver first-best effort. Due to the absence of an outside option for labor, $R^{\max} = E[\Pi|e=e^*] - C(e^*)$ such that we have $E[\Pi|e=e^*] - C(e^*) < (1+\rho)\gamma$.

[5] Generally, the imperfection premium $\hat{\rho} - \rho$ is a function of the loan's size, however, we ignore this subtlety to keep the exposition simple.

As credit-rationing intensifies, which is captured by a higher imperfection interest premium, the poorest entrepreneur must be richer, hence λ rises and the growth rate declines.

Proposition 5. (a) *There is credit rationing iff* $R^{\max} < (1+\hat{\rho})\gamma$, *then* $\lambda = \gamma - R^{\max}/(1+\hat{\rho})$ *where* $0 < \lambda < \mu$. *Otherwise* $\lambda = 0$. (b) *If there is credit rationing, financial development reducing it increases the economy's growth rate.*

Proof. (a) The proof is trivial, once $\lambda < \mu$ is established. For $\lambda < \mu$, consider the effect of a feasible marginal increase of a transfer in any payoff state j on $R^{\mathrm{FB}\max}$ (provided $\mu > 0$):

$$\left.\frac{\mathrm{d}R^{\mathrm{FB\ max}}}{\mathrm{d}t_j}\right|_{\mathrm{d}t_i=0,\,\forall i\neq j} = p_j(e^*) + \frac{\partial \tilde{e}(T^{\mathrm{FB\ max}})}{\partial t_j}\sum_i p_i'(e^*)t_i.$$

Since the marginal repayment to the investor equals zero by the characterizing property of first-best contracts, the second term cancels and the derivative is strictly positive. Hence, $R^{\max}$ must exceed $R^{\mathrm{FB}\max}$ implying the feasibility of loans larger than the one received by an individual with endowment μ. It follows that entrepreneurs repaying $R^{\max}$ must be poorer than entrepreneurs with wealth μ facing $T^{\mathrm{FB}\max}$ which implies $\lambda < \mu$. (b) Financial development reducing credit rationing means lowering $\hat{\rho}$ and thus reducing λ. Differentiating (8) w.r.t. λ verifies the claim. □

5.2. Costly state verification

We implicitly assumed that state-contingent contracts are costless enforceable. Under the assumption of costly state verification, Townsend (1979) and Gale and Hellwig (1985) derive the empirically important standard debt contract as the optimal contract which is a third-best contract in our model. [6] The restriction of feasible repayment schemes to the class of standard debt contracts, which we interpret as an additional credit-market imperfection, necessarily creates a negative link between inequality and growth in the presence of limited liability constraints. If there are growth distortions due to credit rationing, these are magnified by the introduction of costly state verification.

Definition 6. If entrepreneur and investor agree on a standard debt contract (SDC), the entrepreneur repays a payoff-independent amount of $\tau > 0$ or the full payoff if the project's return falls short of τ, i.e. $t_i = \tau$ if $\pi_i \geqslant \tau$ and $t_i = \pi_i$ otherwise.

[6] We are indebted to an anonymous referee for this observation. First-best contracts induce borrowers to exert e^*. Second-best contracts rule out Pareto-improvements for contracting borrowers requiring a level of external finance below $R^{\mathrm{FB\ max}}/(1+\hat{\rho})$. Usually, these second-best contracts are state-contingent. If only standard debt contracts are feasible, these are Pareto-inferior to state-contingent contracts and, hence, termed third-best contracts.

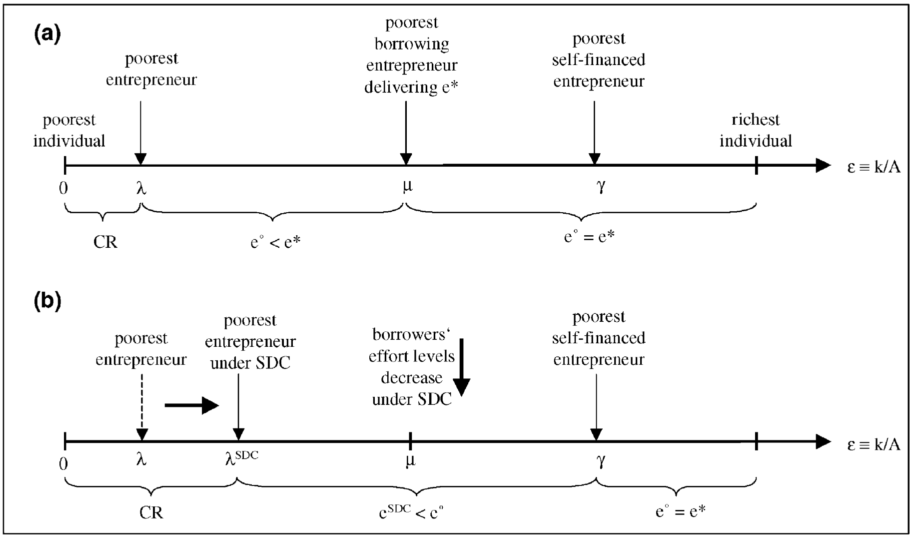

Fig. 2. Effects of standard debt contract restriction.

Lemma 7. (a) *A standard debt contract always precludes first-best effort.* (b) *Borrowers strictly prefer repayment schemes inducing them to exert more effort* (*without exceeding the first-best level*) *among repayment-neutral contracts.* (c) *Contracts inducing the entrepreneur to exert more effort than the first-best level are never implemented.*

Proof. See the Appendix A. □

In a world with state-contingent repayment schemes, which allow for first-best and second-best contracts, the confinement to SDCs has two effects: The effort level of any borrowing entrepreneur falls and credit rationing amplifies. Effort levels decrease because SDCs inhere larger marginal expected repayments than state-contingent contracts which reduce each borrower's share of additional expected project return to additional effort. Thus, SDCs preclude first-best effort as emphasized by Lemma 7. Credit rationing broadens if the repayment-maximizing contract is non-monotonic which always occurs if the investment project's payoff distribution is not extremely robust to changes in effort. [7] As an unambiguous result of both effects, the growth rate of aggregate output diminishes for a given income distribution. Fig. 2 is drawn for a scenario with credit-rationing (CR) and $\mu > 0$. Part (a) illustrates the

[7] As an extreme, consider a payoff distribution completely independent of effort. Then, the repayment-maximizing contract obviously confiscates the full return to the project in every payoff state which conforms to the definition of a standard debt contract. A weak sufficient condition for the exclusion of the repayment-maximizing contract from the SDC class is $t_n^{\max} < \pi_{n-1}$.

static equilibrium where state-contingent contracts are feasible. Part (b) highlights how this allocation qualitatively changes as the choice of repayment contracts is confined to SDCs.

Let λ^{sdc} denote the wealth level of the poorest individual executing a project under the SDC constraint and expected project returns subject to the SDC restriction by $\pi^{\text{sdc}}(\varepsilon)$. Then, the growth rate of aggregate output is given by:

$$g_t^{\text{sdc}} = \ln\left[\pi^* - F(\lambda)\pi^* - \int_{\lambda}^{\mu} \pi^* - \pi(\varepsilon)\mathrm{d}F(\varepsilon) - \int_{\lambda}^{\lambda^{\text{sdc}}} \pi(\varepsilon)\mathrm{d}F(\varepsilon) - \int_{\lambda^{\text{sdc}}}^{\gamma} \pi(\varepsilon) - \pi^{\text{sdc}}(\varepsilon)\mathrm{d}F(\varepsilon)\right]. \tag{9}$$

The first three terms arise also without costly state verification, see (8). The fourth term accounts for additional credit rationing due to confinement to SDCs and the last integral measures the reduction of aggregate output implied by switching from optimal contracts to SDCs. Since SDCs preclude borrowers from exerting first-best effort (Lemma 7), only entrepreneurs not relying on external finance deliver e^*. The next proposition summarizes these results.

Proposition 8. *The confinement to standard debt contracts induces borrowing entrepreneurs to exert less effort and depresses aggregate output's growth rate. Credit rationing broadens if the project's outcome is not too effort-insensitive, otherwise its size remains unchanged.*

Proof. See Appendix A. □

5.3. Growth effects of inequality

Changes in the wealth distribution $F(\varepsilon)$ never affect growth if there is no asymmetric information or no limited liability constraint. Thus, if the credit market is perfect, there is no relation between inequality and growth. Section 3 demonstrates that credit market imperfections in the sense of the limited liability constraints with moral hazard are not sufficient to create a negative link between wealth distribution and aggregate output's growth rate. For such a relationship, the existence of additional frictions in financial markets such as credit rationing or costly state verification is necessary.

Given costly state verification or credit rationing, growth rates (8) and (9) decline if the densities of endowments at the lower end increase while these at relatively higher endowment levels are non-increasing, hence additional inequality depresses growth. Intuitively, if there is a larger share of poor individuals, a larger number of entrepreneurs relies on higher levels of external finance or is credit rationed, a lower number faces little external finance and there are less self-financed entrepreneurs. In Fig. 2(a), the hypothetical pdf above the support becomes more skewed to the right.

It is easy to see that a shift in the wealth distribution towards more inequality as described above triggers a higher growth reduction if there are more credit market frictions: Suppose costly state verification is absent, then the shift reduces the growth rate due to a larger number of credit rationed individuals. If there is costly state verification instead, growth is reduced by the larger number of entrepreneurs relying on higher levels of external finance. If there is costly state verification from the beginning, a higher degree of credit market frictions is reflected in a larger imperfection premium $\hat{\rho} - \rho$ leading to more credit rationing for a given distribution. If there is a distributional shift towards more inequality, there is an increased chance for entrepreneurs to become credit rationed, if credit market frictions are stronger. Essentially, additional inequality more severely reduces growth if there is a higher degree of credit market imperfections. [8]

6. Conclusion

We have shown that if moral hazard is present in a lender-borrower relationship, the specification of the lender's payoff function is crucial for the contracts that can be implemented. If the investor's outside option does not exceed the maximum possible expected repayment from first-best contracts, then limited liability, and therefore incentives and inequality, are no obstacle to growth. The reason is that the repayment contract does not distort the entrepreneur's effort choice and no static inefficiency arises.

If the lender's outside option happens to exceed maximum repayment, a project-specific subsidy is sufficient to resolve any inefficiencies due to incentive problems caused by inequality. The size of any such subsidy is always lower than the amount of redistribution arising from policies in the Aghion et al. (1999) spirit which provide the entrepreneur with the difference between the project's cost and her endowment. We assume a discrete payoff function for the entrepreneur's project. Innes (1990) analysis suggests that our result should also hold for a continuous payoff schedule.

Our model demonstrates that the existence of credit market imperfections due to limited liability in a model of endogenous growth is not necessarily inconsistent with the new empirical literature which calls the supposed negative link between income inequality and growth into doubt.

The introduction of additional capital market imperfections into the model, creates a negative link between inequality and growth. Then, the impact of inequality on growth is stronger for a higher degree of frictions, which is consistent with Barro (2000).

[8] This assumes that the density of individuals which are not credit rationed under a less imperfect credit market but credit rationed with more imperfections does not decrease with the distributional shift towards inequality. Otherwise there may be less growth reductions with more imperfections.

Acknowledgment

We thank Gerhard Schwödiauer, Dominique Demougin, Jörg Budde, Gangolf Groh, Bertrand Koebel, an anonymous referee, Norbert Gaffke and Axel Lehmann for helpful insights and comments.

Appendix A

Proof of Lemma 7

(a) We show that each interesting SDC generates a strictly positive marginal repayment implying a borrower's effort choice below e^* by FOC (5). Let the expected repayment of any standard debt contract T^{sdc} be given by $R^{\mathrm{sdc}} = \sum_{i=1}^{k-1} p_i\pi_i + \sum_{i=k}^{n} p_i\tau$ where k denotes the lowest payoff state allowing to repay the fixed-payment $\tau \in (\pi_1, \pi_n]$, i.e. $\pi_k = \min\{\pi_i | \pi_i \geqslant \tau, i = 1, \ldots, n\}$, and the effort argument of probabilities is suppressed. Rewriting R^{sdc} and its differentiation leads to

$$\frac{\partial R^{\mathrm{sdc}}}{\partial e} = \pi_1 \sum_{i=1}^{n} p_i' + (\pi_2 - \pi_1) \sum_{i=2}^{n} p_i' + \cdots + (\pi_{k-1} - \pi_{k-2}) \sum_{i=k-1}^{n} p_i' + (\tau - \pi_{k-1}) \sum_{i=k}^{n} p_i'.$$

Due to $\sum_{i=1}^{n} p_i'(e) \equiv 0$ and the fact that negative marginal probabilities correspond only to the lowest payoffs by the payoff distribution's MLRP, the first summation is zero and all remaining summations must be strictly positive due to missing lowest marginal probabilities. Thus, $\partial R^{\mathrm{sdc}}/\partial e > 0$ ruling out first-best effort e^*.

(b) Since borrowers are residual claimants of their projects and expected payoffs net of effort's disutility increase in effort for levels smaller than e^*, borrowers expected utility increases in e for $e < e^*$.

(c) Let any repayment scheme T^0 induce a borrower to exert effort $e^0 > e^*$. It is sufficient to show the existence of a repayment scheme allowing for a Pareto-improvement. By Lemma 10, there exists a repayment contract T^1 which leads to the same expected repayment but induces the borrower to deliver e^*. The investor is indifferent between both contracts, but the borrower is better off with T^1 since $E[\Pi | e] - C(e)$ decreases in e for $e > e^*$. $\square$

Proof of Proposition 8

(I) Effort Distortions: According to Lemma 9 (see below), the expected repayment of any standard debt contract T^{sdc} with $\tau \in [0, \pi_{n-1})$ can be replicated by a nonmonotonic contract T^{nm} inducing the borrower to exert more effort than under T^{sdc} (without exceeding e^*). By Lemma 7b, borrowers strictly prefer the nonmonotonic transfer system T^{nm} to the standard debt contract T^{sdc}. For SDCs with $\tau \in [\pi_{n-1}, \pi_n]$ such dominant nonmonotonic contracts may or may not exist. Given the confinement to SDCs, it follows that all borrowers with $\tau_\theta \in [0, \pi_{n-1})$ exert more effort if nonmonotonic contracts are feasible. If there happens to exist SDCs with

$\tau_\theta \in [\pi_{n-1}, \pi_n]$ which yield a higher expected repayment than any other SDC with $\tau < \pi_{n-1}$, borrowers with these contracts exert not less effort under the unconstrained regime but possibly more effort.

(II) Credit Rationing: In this context, it is trivial that a constrained class of contracts cannot contain a transfer system with a larger expected repayment than $R^{\max}$. By the properties of a repayment-maximizing contract as laid out in Lemma 7, the implied marginal repayment is positive and payoffs in low states, including all states with nonnegative marginal probability, are fully transferred to the lender. If it is possible to increase the transfer in the lowest nonexhausted payoff state j, then the net impact on expected repayment is ambiguous for unspecified payoff structures due to two diametric effects. To see this, consider the total change in expected repayment R in response to a marginal increase of t_j:

$$\left.\frac{\mathrm{d}R}{\mathrm{d}t_j}\right|_{\mathrm{d}t_i=0,\forall i\neq j} = p_j(\tilde{e}) + \frac{\partial \tilde{e}(T)}{\partial t_j}\sum_i p_i'(\tilde{e})t_i.$$

Firstly, expected repayment (cet. par.) directly increases by the probability of payoff state j because of the higher repayment in state j. Secondly, the additional transfer induces the entrepreneur to lower his effort, $\partial\tilde{e}/\partial t_j < 0$ since $p_j' > 0$. This (cet. par.) indirectly reduces the expected repayment from the initial transfer structure. For very effort-insensitive payoff distributions, the direct effect may always outweigh the indirect effect such that the repayment-maximizing contract repays the full project return in every state. As long as $T^{\max}$ leaves the borrower with a positive return only in the highest payoff state such that $t_n^{\max} \in [\pi_{n-1}, \pi_n]$, it is a standard debt contract. If the payoff distribution is slightly more effort-sensitive, $T^{\max}$ is nonmonotonic and the largest feasible loan under SDC must be smaller than in a regime allowing for state-contingent contracts. It follows for sufficiently effort-sensitive payoff structures that the confinement to SDCs may broaden or create credit rationing. Obviously, there is no credit rationing if the largest SDC–repaid-loan covers the amount of external finance needed by the poorest individual to undertake the investment project.

(III) Growth depression: Trivially, $0 \leqslant \pi^{\mathrm{sdc}}(\varepsilon) \leqslant \pi(\varepsilon)$. The latter inequality is obviously strict for all wealth levels $\varepsilon \in [\mu, \gamma)$ since these allow for nonmonotonic repayment schemes inducing first-best effort. It follows that the last integral in (9) is strictly positive (if not all potential borrowers are credit rationed, i.e. $\lambda^{\mathrm{sdc}} < \gamma$) and thus, the introduction of costly state verification reduces the economy's growth rate from (8). If the SDC restriction generates additional credit rationing, then $\lambda < \lambda^{\mathrm{sdc}}$ and the decrease of the growth rate is reinforced by the second integral in (9). □

Lemma 9. *For any SDC with $\tau < \pi_{n-1}$, there exists a nonmonotonic contract which replicates the expected repayment and induces the entrepreneur to exert a higher level of effort.*

Proof. It is sufficient to show that a nonmonotonic contract exists which induces more effort and a higher repayment than the original SDC. Then, all transfers can be scaled down to generate any lower repayment level as in the proof of Lemma 10. Due to $\tau < \pi_{n-1}$, it is possible to slightly increase transfer t_{n-1} while reducing

t_n at the same time introducing a nonmonotony into the transfer system. In particular, suppose $\mathrm{d}t_n = -p_{n-1}/p_n \mathrm{d}t_{n-1} < 0$ such that a repayment in a higher profit state is reduced and a transfer in a lower profit state enlarged. From (5), the effect of this change in transfers on effort is given by:

$$\left.\frac{\mathrm{d}\tilde{e}}{\mathrm{d}t_{n-1}}\right|_{\mathrm{d}t_n/\mathrm{d}t_{n-1}=p_{n-1}/p_n} = \frac{p_{n-1}(\tilde{e})}{\mathrm{SOC}} \cdot \left[\frac{p'_{n-1}(\tilde{e})}{p_{n-1}(\tilde{e})} - \frac{p'_n(\tilde{e})}{p_n(\tilde{e})}\right] > 0,$$

where $\mathrm{SOC} \equiv \sum p''_i(\pi_i - t_i) - c'' < 0$ and the second factor is negative by definition of MLRP. The redesign's effect on expected repayment is

$$\left.\frac{\mathrm{d}R}{\mathrm{d}t_{n-1}}\right|_{\mathrm{d}t_n/\mathrm{d}t_{n-1}=p_{n-1}/p_n} = \left[\sum p'_i(\tilde{e})t_i\right] \cdot \left.\frac{\mathrm{d}\tilde{e}}{\mathrm{d}t_{n-1}}\right|_{\mathrm{d}t_n/\mathrm{d}t_{n-1}=p_{n-1}/p_n} > 0,$$

where the first factor is the marginal repayment for a SDC which is strictly positive by Lemma 7(a). It follows that the redesign of the repayment scheme increases expected repayment, too. □

Lemma 10. *If any repayment scheme leads a borrower to exert effort $\tilde{e} > e^*$, then there exists a transfer system that replicates expected repayment R° but induces e^*.*

Proof. By (5), $\tilde{e} > e^*$ is equivalent to $\sum p'_i(\tilde{e})t_i < 0$. Therefore, a small transfer increase in some payoff state j such that $p'_j > 0$ is feasible by $\sum p'_i(e) \equiv 0$. Effort falls in response, since $\partial\tilde{e}(T)/\partial t_j = p'_j/\mathrm{SOC} < 0$ where $\mathrm{SOC} \equiv \sum p''_i(\pi_i - t_i) - c'' < 0$. The total effect on expected repayment is strictly positive:

$$\left.\frac{\mathrm{d}R}{\mathrm{d}t_j}\right|_{\mathrm{d}t_i=0,\forall i\neq j} = p_j(\tilde{e}) + \frac{\partial\tilde{e}(T)}{\partial t_j}\sum p'_i(\tilde{e})t_i.$$

Hence, it is possible to increase transfers in states with positive marginal probabilities such that effort equals the first-best level and repayment exceeds R°. By scaling down all transfers proportionately which preserves e^*, the original level of repayment can be replicated. □

References

Aghion, P., Bolton, P., 1997. A theory of trickle-down growth and development. Review of Economic Studies 64, 151–172.

Aghion, P., Caroli, E., García-Peñalosa, C., 1999. Inequality and economic growth: The perspective of the new growth theories. Journal of Economic Literature 37, 1615–1660.

Banerjee, A.V., Newman, A.F., 1993. Occupational choice and the process of development. Journal of Political Economy 101, 274–298.

Barro, R.J., 2000. Inequality and growth in a panel of countries. Journal of Economic Growth 5, 5–32.

Benabou, R., 1996. Inequality and growth. NBER Macroeconomic Annual 11, 11–74.

Forbes, K.J., 2000. A reassessment of the relationship between inequality and growth. American Economic Review 90, 869–887.

Gale, D., Hellwig, M., 1985. Incentive-compatible debt contracts: The one-period problem. Review of Economic Studies 52, 647–664.

Galor, O., Zeira, J., 1993. Income distribution and macroeconomics. Review of Economic Studies 60, 35–52.

Innes, R.T., 1990. Limited liability and incentive contracting with ex ante action choices. Journal of Economic Theory 52, 45–67.

Levine, R., 1997. Financial development and economic growth: Views and agenda. Journal of Economic Literature 35, 688–726.

Milgrom, P.R., 1981. Good news and bad news: Representation theorems and applications. Bell Journal of Economics 13, 369–378.

Persson, T., Tabellini, G., 1994. Is inequality harmful for growth? Theory and evidence. American Economic Review 48, 600–621.

Piketty, T., 1997. The dynamics of the wealth distribution and the interest rate with credit rationing. Review of Economic Studies 64, 173–189.

Rajan, R.G., Zingales, L., 1998. Financial dependence and growth. American Economic Review 88, 559–586.

Townsend, R.M., 1979. Optimal contracts and competitive markets with costly state verification. Journal of Economic Theory 21, 265–293.