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Testing for multiple regimes in the tail behavior of emerging currency returns

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Abstract

It is by now generally accepted that foreign exchange returns exhibit “heavy tails” as measured by the so-called tail index. However, it is unclear whether the tail behavior remains stationary in the presence of recurrent switches in the exchange rate regime. We therefore test the null hypothesis of tail index constancy by applying a single breaks test “in rounds” which enables the detection of multiple breakpoints. We are able to identify multiple jumps in the tail index of currency returns. Moreover, some breaks coincide with documented shifts in monetary and exchange rate policies.

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1. Introduction

Recent financial meltdowns like the 1997 Asian crisis, the LTCM debacle, the Mexican “Tequila” crisis or the Russian debt crisis have strengthened academic attention into the extremal behavior of financial market returns. Following the seminal work by Mandelbrot (1963) it is now generally accepted that overnight financial market collapses are more likely to occur than back-of-the-envelope calculations using the normal distribution (df) paradigm seem to

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suggest. Stated differently, financial returns exhibit “heavy tails” in contrast to the thin tailed normal df. This “heavy tail” characteristic has been established for most financial asset classes. Empirical studies tried to characterize the thickness of the tails by estimating the so-called tail index α .¹ Loosely speaking this parameter reflects the number of bounded distributional moments that are still finite. The more the probability mass in the tails the lower will be this parameter. For example, for series characterized by $\alpha < 2$, the variance, skewness and kurtosis do no longer exist. As for the normal df, the tail index is infinite because all moments exist and are bounded. Albeit interesting from a purely theoretical perspective, quantifying the tail index is also relevant for financial practitioners (risk managers, financial regulators, etc.) because it is a necessary ingredient in order to calculate quantiles very far into the distributional tail, i.e., quantiles (or Value-at-Risk (VaR) levels) that correspond with a very low tail probability.

From the empirical literature, it emerges that $2 \leq \hat{\alpha} \leq 4$, both across assets, asset types and sample periods. Surprisingly, however, this apparent cross sectional and temporal invariance has barely been the subject of thorough statistical testing.² The issue of whether α is constant across assets or time — especially the latter presumption — is nevertheless crucial for the applicability of extreme value analysis into, e.g., risk management or financial stability assessment. If the amount of probability mass in the tails of the unconditional distribution is shifting through time, the full sample estimates of α and corresponding quantiles probably wrongly reflect the expected frequency of sharp declines in, e.g., portfolio values for certain subperiods of the sample.

The cited empirical literature on the constancy issue mainly focuses on testing for a single known (exogenously selected) breakpoint in α . Recently, however, Quintos et al. (2001) proposed test statistics that allow for the identification of single but unknown (endogenously detected) breakpoints in α . Upon applying a recursive, rolling and sequential testing procedure they were able to detect structural breaks in the tails of three emerging stock markets. Moreover, the detected breakpoints were found to be “meaningful” in the sense that they coincided with periods of financial turmoil or regulatory change. Upon comparing the three procedures in terms of small sample power and estimation accuracy for the structural break date, they found that the proposed recursive procedure performs best. In this paper we extend the Quintos et al. analysis in that we argue that the recursive testing procedure can be used to detect more than one break in the tail behavior. Loosely speaking the approach boils down to applying the recursive test “in rounds”: if a full sample break is detected, the single breaks recursive test is repeated over the two subsamples determined by the initial break date and so forth. This “multistage” implementation of the single breaks test can go on as long as the parameter constancy hypothesis is rejected and the subsamples are not too small.

The testing procedure is applied to Asian and Western exchange rate data because the numerous switches in monetary and exchange rate policies over the recent history make foreign

¹ Numerous studies have made use of semi-parametric estimators derived from extreme value analysis in order to estimate the tail index. Heavy tails in stock markets are most widely documented, see e.g., Jansen and de Vries (1991), Longin (1996), Lux (1996) and Hartmann et al. (2004). Fat tails in foreign exchange returns are investigated, inter alia, by Koedijk et al. (1990, 1992), Hols and de Vries (1991) and Hartmann et al. (2003). Bond extremes have been rather neglected in the empirical literature. de Haan et al. (1994) and Hartmann et al. (2004) constitute two notable exceptions.

² The scant empirical evidence can be subdivided into three categories. A first group of papers investigates structural change in α , see e.g., Phillips and Loretan (1990), Koedijk et al. (1990, 1992), Jansen and de Vries (1991) and Pagan and Schwert (1990). Cross asset tail (in)equalities have been considered by Koedijk et al. (1990) for exchange rates and Jondeau and Rockinger (2003) for stock markets. Finally, asymmetry tests comparing upper and lower stock market tails have been performed by, e.g., Jansen and de Vries (1991) and Lux (1996).

exchange markets the more obvious candidates for exhibiting multiple shifts in tail behavior. Moreover, previous empirical studies already suggested that the tail index may be influenced by the degree of fixity of the forex regime. Koedijk et al. (1990) showed that the installation of the European Monetary System (EMS) did not lead to a lower frequency of extreme currency fluctuations compared to the period of the Snake (1971–1979). However, upon comparing the tails of the nearly fixed currency returns of the Bretton Woods era with the post-Bretton Woods return tails, Koedijk et al. (1992) observed a rise in the tail index.

The rest of the paper is organized as follows. In Section 2 we discuss how a single breaks recursive testing procedure can be generalized toward detecting multiple breaks. The small sample performance of the proposed multiple breaks procedure is assessed in Section 3 by means of a Monte Carlo investigation. Empirical testing results for emerging and developed currency returns are documented in Section 4. Section 5 offers concluding remarks.

2. Single vs. multiple breaks in tail behavior

Consider the six time series of Asian foreign exchange rate returns in Fig. 1. A detailed data description is provided in Appendix A. Obviously, the degree of extreme currency return fluctuation seems to change around the 1997 Asian crisis for most of the currencies. One expects this to be reflected in the value of the tail index over the considered time period. We therefore implement tests of tail shape constancy to the time series in Fig. 1 and, if possible, we relate the detected breakpoints to known changes in monetary or exchange rate regimes.

Let $X_t = \log(S_t/S_{t-1})$ represent log exchange rate returns with S_t standing for the price of the domestic currency per unit of US\$. Thus, a rise in X implies a depreciation (or devaluation). We therefore test for structural change in the upper tail of X as this might reveal that the potential for large drops in currency prices is nonconstant over time.

The nonnormal distribution of exchange rate fluctuations is by now accepted as an empirical stylized fact. Loosely speaking a forex series' empirical distribution function exhibits a "heavy" tail if "extreme" currency fluctuations are expected to strike more often than predicted on the basis of a normal distribution. Formally, the cumulative df $F(x)$ of the return series X exhibits a heavy or *regularly varying* upper tail if

$$\lim_{t \rightarrow \infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}, \quad x > 0, \alpha > 0. \quad (1)$$

The parameter α is called the tail index; it determines the decay of the tail probability if the x is shifted more outward. Clearly, the lower the α the slower the probability decay and the higher the probability mass in the tail of X . The regular variation property implies that all distributional moments higher than α , i.e., $E[X^r]$, $r > \alpha$, are unbounded, signifying the "fat tail property". Popular distributional models like the Student's t , symmetric stable or the Autoregressive Conditional Heteroscedasticity (ARCH) model all exhibit this tail behavior. Their tail behavior is to a first order approximation comparable to the tail of a Pareto distribution $1 - ax^{-\alpha}$ with the same index α . As for the normal distribution, the limit in Eq. (1) renders e^{-x} , i.e., an *exponentially* declining tail. Distributions with tails that exhibit this latter property are classified as thin tailed; but these distributions still possess all moments, and hence do not capture the observed behavior of extreme financial returns.

We study the occurrence of multiple breaks in α by means of Hill's (1975) estimator because it is by far the most widely used tail index estimator. Let $X_{1,n} \leq X_{2,n} \leq \dots \leq X_{n,n}$ stand for the

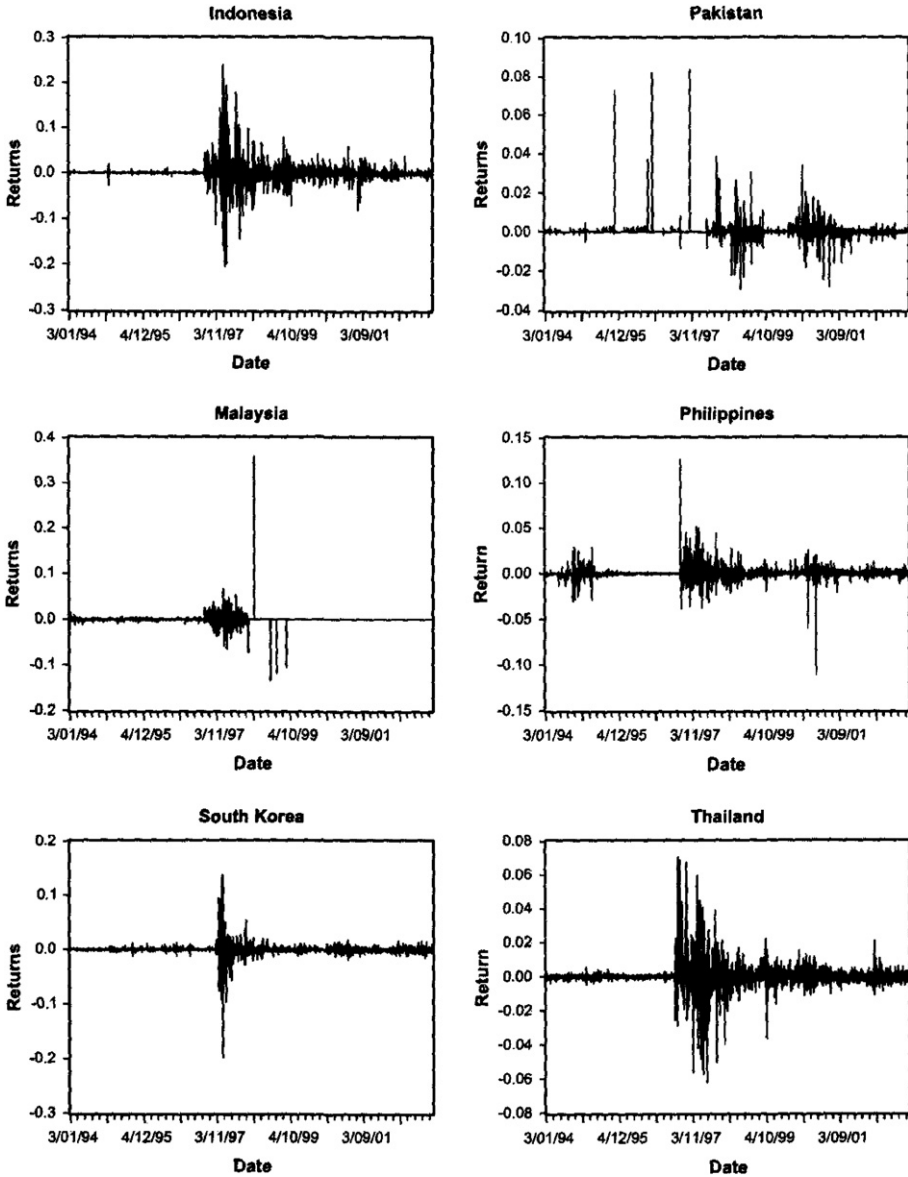


Fig. 1. Daily Asian currency returns (4/1/1994–25/6/2003).

ascending order statistics of a return series X with sample size n . Then the Hill statistic boils down to:

$$\hat{\alpha}_n = \left(\frac{1}{m} \sum_{j=0}^{m-1} \ln \left(\frac{X_{n-j,n}}{X_{n-m,n}} \right) \right)^{-1}, \tag{2}$$

with m the number of highest order statistics used in estimation. Further details are provided in, e.g., Jansen and de Vries (1991) and the recent monograph by Embrechts et al. (1997). Hall (1982) showed for $m/n \rightarrow 0$ as $m, n \rightarrow \infty$ that the statistic $\sqrt{m}((\hat{\alpha}(m)/\alpha) - 1)$ is asymptotically standard normally distributed.

A long standing issue in extreme value analysis constitutes the choice of m . Goldie and Smith (1987) show that one picks m such that it is in a range that minimizes the asymptotic mean-squared error. Consequently, minimizing the sample mean-squared error is the appropriate selection criterion in finite samples. More statistically involved procedures have been proposed for small (Huisman et al., 2001) and large samples (Danielsson et al., 2001). For sake of simplicity, we select the 10% upper order extremes for estimating the tail index, i.e., $m = 0.1n$.

Quintos et al. (2001) propose a recursive, rolling and sequential procedure for detecting single unknown breaks in α . Let t denote the endpoint of a subsample of size $w_t < n$. The recursive estimator uses subsamples $[1; t] \subset [1; n]$ and boils down to:

$$\hat{\alpha}_t = \left(\frac{1}{m_t} \sum_{j=0}^{m_t-1} \ln \left(\frac{X_{t-j,t}}{X_{t-m_t,t}} \right) \right)^{-1}, \tag{3}$$

with $m_t = 0.1t$. The rolling estimator assumes a fixed subsample size $w^* < n$ and estimates the tail index by rolling over the subsample, i.e., the subsample is shifted through the full sample by eliminating past observations and adding future observations such that the subsample size stays constant at w^* .

$$\hat{\alpha}_t^* = \left(\frac{1}{m_{w^*}} \sum_{j=0}^{m_{w^*}-1} \ln \left(\frac{X_{w^*-j,w^*}}{X_{w^*-m_{w^*},w^*}} \right) \right)^{-1}, \tag{4}$$

where $m_{w^*} = 0.1w^*$. Notice that the number of upper order extremes is increasing in the subsample size for the recursive estimator but is constant for the rolling estimator. Finally, the sequential estimator (denoted by $\hat{\alpha}_{2t}$) is identical to the recursive estimator in (3) but calculated in reverse calendar time, i.e., using the more recent observations first.

The three tests are constructed using the sequences:

$$Y_n^2(t) = \left(\frac{tm_t}{n} \right) \left(\frac{\hat{\alpha}_t}{\hat{\alpha}_n} - 1 \right)^2, \tag{5}$$

$$V_n^2(t) = \left(\frac{w^*m_{w^*}}{n} \right) \left(\frac{\hat{\alpha}_t^*}{\hat{\alpha}_n} - 1 \right)^2, \tag{6}$$

$$Z_n^2(t) = \left(\frac{tm_t}{n} \right) \left(\frac{\hat{\alpha}_t}{\hat{\alpha}_{2t}} - 1 \right)^2, \tag{7}$$

Expressions (5) and (6) measure the fluctuation in the recursive and rolling values, respectively, of the Hill statistic relative to their full sample counterpart $\hat{\alpha}_n$ whereas the sequential test uses (7) to compare the fluctuations of the recursive with the reverse recursive estimator. The null hypothesis of interest is that the tail index is temporally invariant. More specifically, let α_t be the tail index of the distribution of X_t . For numerical reasons the above tests are evaluated

over a compact subset of $[0; 1]$, i.e., t equals the integer part of nr for $r \in R_\tau = [\tau; 1 - \tau]$ and for small $\tau > 0$.³ The null hypothesis of constancy then takes the form

$$H_0 : \alpha_{[nr]} = \alpha, \quad \forall r \in R_\tau.$$

with the alternative hypothesis $H_A: \alpha_{[nr]} \neq \alpha$ for some $r \in R_\tau$. Conform with [Quandt's \(1960\)](#) seminal work on structural change tests for time series models, the candidate-break date r can be selected such as to maximize the value of the sequences of test statistics in (5)–(7). At this date, the constancy hypothesis is most likely to be violated.

Asymptotic critical values can be easily derived for the sup-values of the three testing procedures. However, the test sequences Y_n^2 , V_n^2 and Z_n^2 need additional scaling to ensure proper convergence behavior when the data are non-i.i.d. Indeed, it is well known that forex returns exhibit nonlinear dependencies like ARCH effects (volatility clustering) which necessitates the scaling. The stronger the volatility clustering, the larger the scaling factor has to be. More details on the ARCH robust procedure in general and how the scaling factor can be consistently estimated are provided in [Quintos et al. \(2001\)](#). We now have to select r for, e.g., the recursive test such that $Y_n^2(t)$ – appropriately scaled – is maximal:

$$Q_{r \in R_\tau} = \sup \widehat{\eta}_t^{-1} Y_n^2(t), \quad (8)$$

where $\widehat{\eta}_t$ is the estimate of the time varying scaling factor. The null of parameter constancy is rejected if the sup-value exceeds the asymptotic critical values, e.g., for the recursive test we reject parameter constancy if $Q > \xi_p$ with $p = 5\%$ or 1% . [Quintos et al.](#) also performed a Monte Carlo investigation in order to compare the small sample performance of the three procedures in terms of power, size and ability to consistently estimate the break date. The three tests are found to exhibit negligible size distortion in small samples. Moreover, the direction of the change in α under the alternative hypothesis seems to play a crucial role in determining the power of the tests in small samples. The recursive and rolling tests satisfactorily detect a decrease in α . On the other hand, the power of the rolling test for detecting an increase in α is far superior to the recursive test. As for the sequential test, the power differs quite a lot depending on the location of the break and the direction of the change in α but is most of the time lower than for the other tests.⁴ Finally, the recursive test is found to estimate the break date with the highest accuracy (lowest small sample bias) in small samples of size $n = 500$ provided the tail changes from thick to thin under the alternative hypothesis.

Notice that the poor small sample performance of the recursive test in detecting upward jumps is only an apparent problem. Indeed, if one lacks prior knowledge on the direction of

³ Sets like R_τ are often implemented in the construction of parameter constancy tests, see e.g., [Hawkins \(1987\)](#) and [Andrews \(1993\)](#). The restricted choice of r implies that $\tau n \leq t \leq (1 - \tau)n$. The lower bound can be justified by the fact that recursive and rolling estimates become too unstable and inefficient when the subsample is very small. As a result the tests would systematically reject the null of parameter constancy for small t , also when there are actually no breaks. On the other hand, the tests will never identify breaks for t equal to or very close to n because the testing values in (5)–(7) are close to zero in that latter case. Thus a computational efficiency argument justifies the upper bound choice for t . In the empirical applications we set the smallest recursive sample size equal to 200, i.e., $\tau = 200/n$.

⁴ The poor performance of the recursive test when $\alpha_1 < \alpha_2$ can be understood by observing that extremal returns occurring in the initial recursive sample will partly remain in the selection of m highest order statistics when the sample size is increased. This initial extremes dominance does not occur for the rolling test since the influence on $\widehat{\alpha}$ of extremal behavior that occurs in the initial sample gradually drops out when the rolling window is shifted through the total sample. For further intuition on the small sample power outcomes we refer to the simulation section in [Quintos et al.](#)

the jump in the tail index, the recursive test can be performed by applying Eqs. (5)–(8) both in calendar time (“forward” recursive test) and by inverting the sample (“backward” or “reverse” recursive test). A decrease of the tail index should then be signaled by the forward test whereas an increase should be detected by the backward test. Because of its superior performance, we opt to work with Quintos’ single breakpoint recursive test in this paper and we apply it into a multiple breaks setup.

The single breakpoint recursive testing approach can now be readily generalized toward detecting multiple breakpoints in α . The proposed procedure implies testing the presence of $b + 1$ breaks *given* that there are b breaks in the following five steps⁵:

- Step 1: Perform the single break recursive test (8) over the full sample in order to test $H_0: b = 0$. If no break is detected (i.e., $Q_{r \in [\tau, 1-\tau]} = \sup \hat{\eta}_t^{-1} Y_n^2(t) < \xi_{5\%}$), the “no breaks” null hypothesis is not rejected and the testing procedure ends here.
- Step 2: If a statistically significant full sample break ($b = 1$) is detected at date t_0 , the sample is partitioned into two subsamples $[\tau n, t_0 - 1]$ and $[t_0 + 1, (1 - \tau)n]$.
- Step 3: The recursive test is repeated on each of the subsamples. If no subsample break is detected (i.e., $Q_{t_1} = \sup Y_{[t_0+1, (1-\tau)n]}^2(t) < \xi_{5\%}$ (respectively, $Q_{t_1} = \sup Y_{[\tau n, t_0-1]}^2(t) < \xi_{5\%}$)), the procedure ends with one single break located at date t_0 .
- Step 4: If a subsample break is detected at a date $t_1 \in [t_0 + 1, (1 - \tau)n]$ (respectively, $t_1 \in [\tau n, t_0 - 1]$), there are multiple breaks $b \geq 2$.
 - Step 4a: The presence of additional breaks might have distorted the initial break date estimate in step 2, as this was determined unconditionally to any other break. We therefore run the recursive test for the sample $[\tau n, t_1]$ (respectively, $[t_1, (1 - \tau)n]$) in order to check whether the break over these subsamples – if any – coincides with t_0 . If both dates are identical, we keep the break at t_0 , otherwise the break with the highest significance level is retained. Steps 2–4a are then performed again, until the breakpoint in step 4a is identical to this obtained in step 2.
- Step 5: The sample partitioning and accompanied testing (steps 2–4a) are repeated until the null hypothesis of constant α can no longer be rejected.

We impose a minimum size for the subsamples of 500 observations in order to be able to use the asymptotic critical values for Q . Quintos et al. (2001) have shown that the size distortion of the structural breaks test in small samples is still acceptably small for samples of this size. The above five-step procedure can now be performed using both the recursive (forward) and reverse recursive (backward) tests independently. This allows for detecting gradual increases (backward test) as well as gradual decreases (forward test) in α .

3. Simulation experiments

In this section we investigate the ability of both the single recursive test and the five-step multistage recursive approach to consistently determine multiple breaks. We opt for the Student’s t parametric distribution model as Data Generating Process (DGP) in our Monte Carlo investigation. Notice that the t -distribution’s degrees of freedom parameter $\nu = \alpha$. The Student’s

⁵ Notice the conceptual analogy between the Bai and Perron (1998) approach toward detecting multiple breaks in the coefficients of linear regression models and our approach.

t model has become a popular workhorse in the empirical exchange rate literature as an alternative to, e.g., the symmetric stable df because it exhibits fat tails while still allowing for a finite variance if $\nu > 2$.

We distinguish four different simulation scenarios for changes in the degrees of freedom parameter ν :

$$\begin{aligned}
 .DGP_1 &: \text{Student}(10)_{(t=1\dots 1000)} \mid \text{Student}(3)_{(t=1001\dots 2000)} \\
 .DGP_2 &: \text{Student}(10)_{(t=1\dots 1000)} \mid \text{Student}(4)_{(t=1001\dots 2000)} \mid \text{Student}(3)_{(t=2001\dots 3000)} \\
 .DGP_3 &: \text{Student}(10)_{(t=1\dots 1000)} \mid \text{Student}(9)_{(t=1001\dots 2000)} \mid \text{Student}(3)_{(t=2001\dots 3000)} \\
 .DGP_4 &: \text{Student}(10)_{(t=1\dots 1000)} \mid \text{Student}(3)_{(t=1001\dots 2000)} \mid \text{Student}(10)_{(t=2001\dots 3000)}
 \end{aligned}$$

The single break process DGP_1 contains a switch from relatively thin tails ($\alpha = 10$) to relatively thick tails ($\alpha = 3$) and will act as a benchmark for comparison with the Quintos et al. single break results. The remaining processes exhibit two breaks in the tail behavior. Whereas DGP_2 and DGP_3 reflect a gradual decline in α over time the final process DGP_4 assumes a U-shape for the tail index. Each subsample contains 1000 observations. The single break for DGP_1 is chosen exactly in the sample middle whereas the two breaks for the remaining data generating processes divide the total sample in three equal parts, i.e., $r = 1/3$ and $2/3$.

Fig. 2 reports the small sample break distributions for the forward and backward recursive single and multiple breaks test. Each of the simulated dfs is based upon 2000 replications. The histograms only reflect breaks that are statistically significant. This implies, inter alia, that it only makes sense to consider the histograms for tests which exhibit reasonable power against the alternative. Otherwise, the histogram would not be very informative because it would be based on a too small number of statistically significant breaks.

The first row of graphs in Fig. 2 presents the forward and backward recursive tests in the presence of a single break. The left graph reveals that the single break date for DGP_1 is accurately estimated by the forward test as most probability mass seems to be concentrated in the distributional centre. The top right graph shows that applying the backward test to the same DGP leads to a more uniform df of the breaks. However, this histogram is not very informative because it is based on a very small number of significant breaks, i.e., the backward test has negligible power when the tail is switching from thin to thick. The second row of graphs in Fig. 2 deals with the behavior of the single recursive forward test when the tail index is gradually declining ($\alpha_1 > \alpha_2 > \alpha_3$). By construction the single break recursive test can only pick up one of the two breaks. Unsurprisingly, the biggest breaks seem to correspond with the largest probability mass in the second row figures. The third row of histograms illustrates how the single recursive test might be implemented to detect more than one break in case the tail index exhibits a U-shape. We do not need a multiple breaks procedure in this case. The histograms show that the first break is picked up by the recursive forward whereas the second break can be detected by reversing the calendar time, i.e., by running the reverse recursive test. Again this is due to the fact that the recursive test has only high power for detecting tail index declines; increases in the tail index can nevertheless be detected by running the backward version of the test. The final row in Fig. 2 graphically depicts the performance of the breakpoint estimates to capture multiple breaks using the five-step iterative procedure based on the recursive test. Each graph in the final row basically depicts two histograms: the full sample breakpoint df and the subsample breakpoint df conditional upon this full sample one. The bimodality in the two histograms clearly shows that the iterative procedure is able to locate the two breaks.

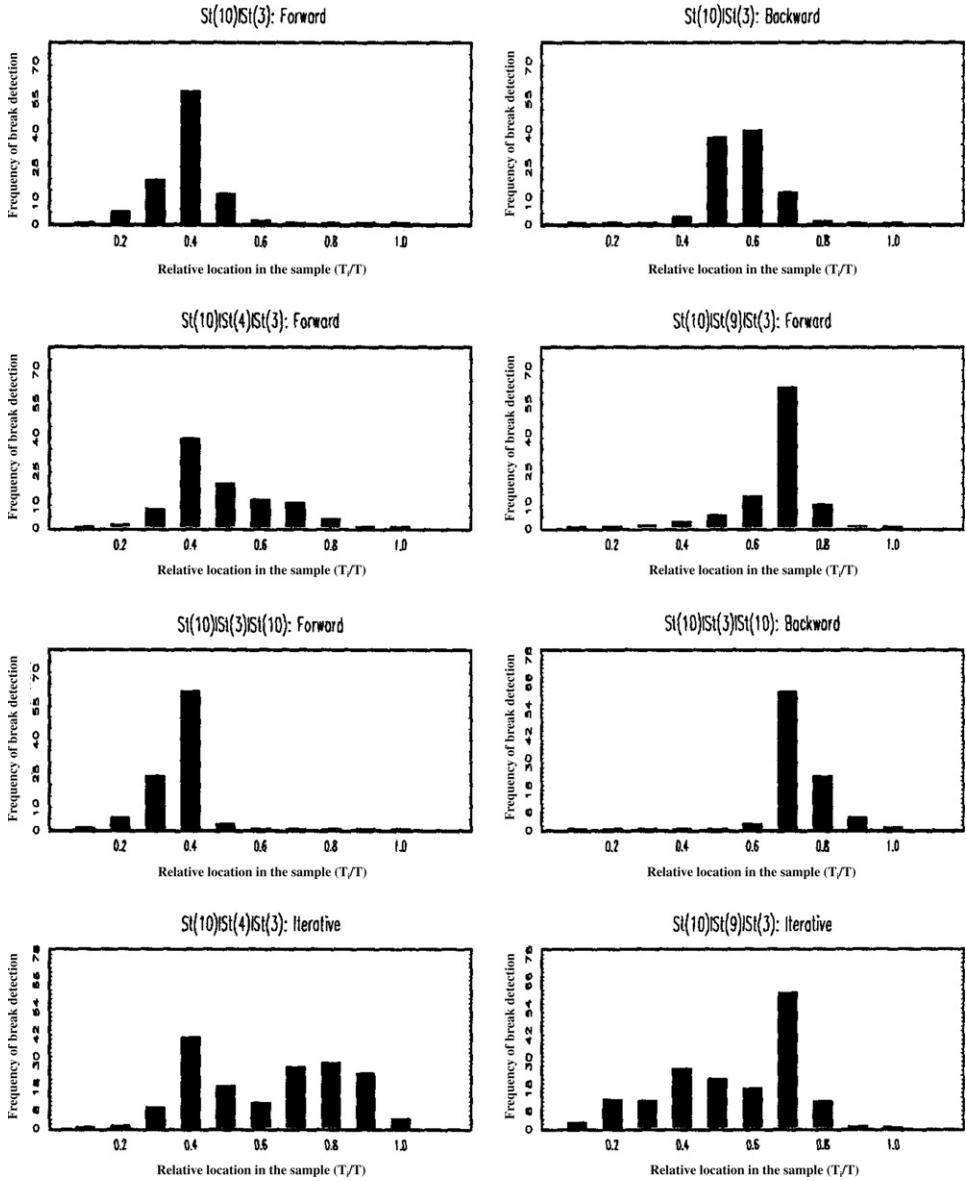


Fig. 2. Simulated distributions of the break dates for different DGPs.

As a complement to the figures, Table 1 reports the sample means and corresponding standard errors for the histograms in Fig. 2.

The simulated break scenarios are reported in the first column whereas the applied testing procedures are mentioned in the second column. The proportion $r = t/n$ reflects the relative position of the break(s) in the sample. The recursive test does well in estimating the single breakpoint (first row). The estimates are only slightly downward biased which is in accordance with the Quintos et al. results who used the symmetric stable df in their

Table 1
Estimates of simulated breakpoints

Simulated break (s) ^a	Used test	Estimated breakpoints (standard error) ^b		
		$r = 1/3$	$r = 1/2$	$r = 2/3$
$(\alpha_1, \alpha_2) = (10; 3)$	1-step rec.	–	0.4977 (0.0938)	–
$(\alpha_1, \alpha_2, \alpha_3) = (10; 4; 3)$	1-step rec.	0.4384 (0.1375)	–	–
$(\alpha_1, \alpha_2, \alpha_3) = (10; 9; 3)$	1-step rec.	–	–	0.6290 (0.0961)
$(\alpha_1, \alpha_2, \alpha_3) = (10; 3; 10)$	1-step rec.	0.3617 (0.1269)	–	–
$(\alpha_1, \alpha_2, \alpha_3) = (10; 3; 10)$	1-step rev. rec.	–	–	0.6625 (0.1010)
$(\alpha_1, \alpha_2, \alpha_3) = (10; 4; 3)$	Multiple rec.	0.4345 (0.1402)	–	0.7293 (0.1179)
$(\alpha_1, \alpha_2, \alpha_3) = (10; 9; 3)$	Multiple rec.	0.3751 (0.1462)	–	0.6278 (0.1007)

^a Estimated breakpoints are reported for the Student's t distribution and for different single and multiple break scenarios.

^b The break estimates are reported for a fixed subsample size of 1000 and for varying locations of the true breakpoints ($r = 1/3, 1/2, 2/3$). The number of Monte Carlo replications is set to 5000 but the estimates are conditioned on the statistically significant breakpoint replications only. The accompanying sampling errors are reported between brackets. Q -tests are calculated starting with a minimum sample size of 200. The number of upper order extremes used in estimating the tail index equals 10% of the total sample size.

simulations. This small “anticipation” effect has already been noticed in a time series framework by Lee and Strazicich (2001). We also investigated the behavior of the single breaks (1-step) recursive test in the presence of multiple (two) breaks (rows 2, 3, 4, 5). By construction the recursive test can only detect one break. The question arises which of the breaks will then be selected. The simulation experiments reveal that the magnitude of the jump is crucial in the breakpoint selection. Indeed, rows 2 and 3 show that the largest jump in α will predominantly be selected. Because the smaller shock is also sometimes selected as single breakpoint, the $r = 1/3$ breakpoint estimate is slightly upward biased (row 2) whereas the $r = 2/3$ breakpoint estimate is slightly downward biased (row 3). On the other hand, the breakpoint bias under the U-shape break scenarios (rows 4 and 5) is much smaller. The smaller bias follows from the low power of the (reverse) recursive test to detect upward (downward) swings. As a consequence, the $r = 2/3$ ($r = 1/3$) break will nearly never be selected as a significant break in the row 4 test (row 5 test). Finally, the two bottom rows report breakpoint estimates for the multiple recursive test. Full sample breaks are estimated in a first round. Just as in rows 2 and 3, the testing procedure will predominantly select the largest jump (0.4345 for $r = 1/3$ in row 6 and 0.6278 for $r = 2/3$ in row 7). The recursive test can now be implemented again for the two subsamples determined by the first round break. The subsample break estimates for the subsamples that actually contain a break are found to be slightly upward biased (0.7293 and 0.3751). Average break estimates for the stationary subsamples are of a spurious nature and are therefore not reported.⁶

⁶ Notice that the probability of detecting statistically significant break estimates for the stationary subsamples (no subsample breaks) is equal to the size of the test (0.05 or 0.01). Thus, an average break estimate can be calculated but will be based on a very small number of statistically significant break estimates which reflect the spurious character of the break outcomes.

4. Identifying emerging currency regimes as multiple breaks

In previous sections we introduced a multistep procedure, centered around the recursive test proposed by Quintos et al. (2001), in order to identify multiple breaks. We also showed that the approach seems to perform well in small to medium-size samples. In this section we apply the technique to a panel of developed currencies and a panel of emerging (Asian) currencies.⁷ We believe that foreign exchange markets are the more obvious candidates for detecting (multiple) breaks in the tail behavior because of the direct link between currency price formation and the exchange rate regime. Moreover, as emerging currencies were more regularly hit by speculative attacks and resulting regime switches than developed currencies in recent times, we expect the latter currency tails to be characterized by a lower number of breaks. We will therefore also consider developed currency breakpoint test results as a benchmark for comparison.

We start the empirical analysis by calculating the recursive Hill estimates that we will use as an input for the stability tests later on. Fig. 3 contains the forward (upper plot) recursive Hill estimates and backward (lower plot) recursive Hill estimates for the tails of six Asian currency return series.

Most striking in the pictures is the nonconstancy of $\hat{\alpha}_t$. For most currencies the forward and backward recursive estimates vary between 1 and 3 (except for the reverse recursive Malaysian estimates near the end of the sample period). Moreover, the forward recursive estimates seem to decline gradually during 1997 before stabilizing later on. From Quintos et al. (2001) and our own simulations we know, however, that the horizontal parts in the forward estimates might hide increases in the tail index because the recursive estimates are dominated by the extremes from the first half of the sample in case of an increase in the tail index later on. The upward sloping backward estimates indeed suggest that the tail index again started to increase in the aftermath of the Asian crisis (from 1998 onwards).

The recursive and reverse recursive estimates are clearly suggestive of a U-shaped pattern in α over the second half of the 1990s. We were also able to reproduce the U-shape in rolling sub-sample estimates which – as noted earlier – do not suffer from lack of consistency when switching from thick to thin.

It remains to be seen whether the U-shape is statistically significant and not merely a small sample phenomenon. As a second step in the empirical analysis, we therefore applied the single breakpoint forward and backward recursive tests in (8) over the full sample. Fig. 4 reports forward (full lines) and backward (dotted lines) values of $\hat{\eta}_t^{-1} Y_n^2$ in Eq. (5) with the scaling factor $\hat{\eta}_t^{-1}$ reflecting ARCH effects in the forex return series.

Notice that the pictures have a dual vertical scaling because the range of the two series is very different in most cases. The left scale refers to the (full line) recursive testing values whereas the right scale refers to the (dotted line) backward test. Clearly, the forward and backward sup-values correspond with significant breaks as they all lie above the critical values of 1.78 (5%) or 2.56 (1%). Moreover, the forward break dates seem to precede the backward break dates for most currencies. This confirms the U-shape. Finally, the statistical significance of rises

⁷ Testing results for Latin American currencies are available upon request. We decided not to include these results because the bulk of the breakpoints was difficult to interpret economically in terms of regime changes. The erroneous outcomes might be due to frequent currency reforms and resulting data handling. A lot of Latin American countries replaced old currency by new fiat money during the 1990s in order to fight inflation and reduce “monetary overhang”. Data providers typically rescaled backward forex time series by using the conversion rate between old and new currency units. This causes some of the forex series to be basically zero over large parts of the historical sample.

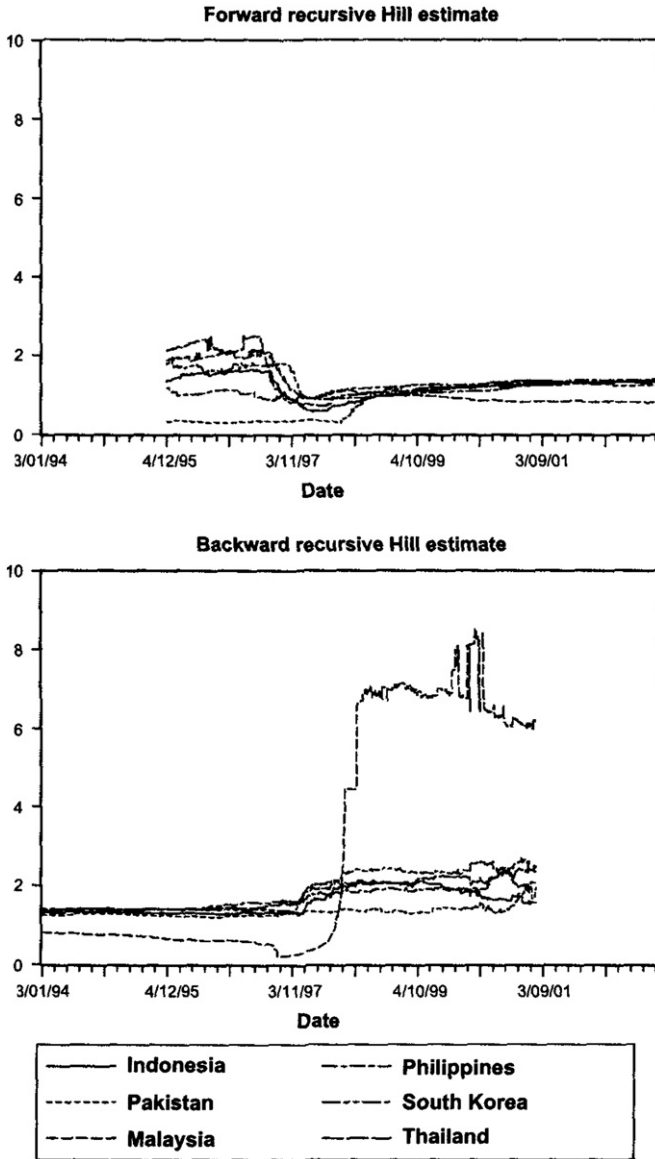


Fig. 3. Recursive (forward) and reverse recursive (backward) Hill estimates for tail indexes of emerging currency returns (3/1/1994–25/6/2003).

in α (backward test) seems higher than the significance of falls (forward test) since the backward testing series usually dominate the forward series in magnitude (except for Thailand and Pakistan).

The corresponding full sample estimates of the break dates are reported in bold in Table 1 with the sup-values between brackets. Reported break dates and sup-values not in bold reflect the subsample breaks estimated using the five-step iterative procedure introduced in Section 2.

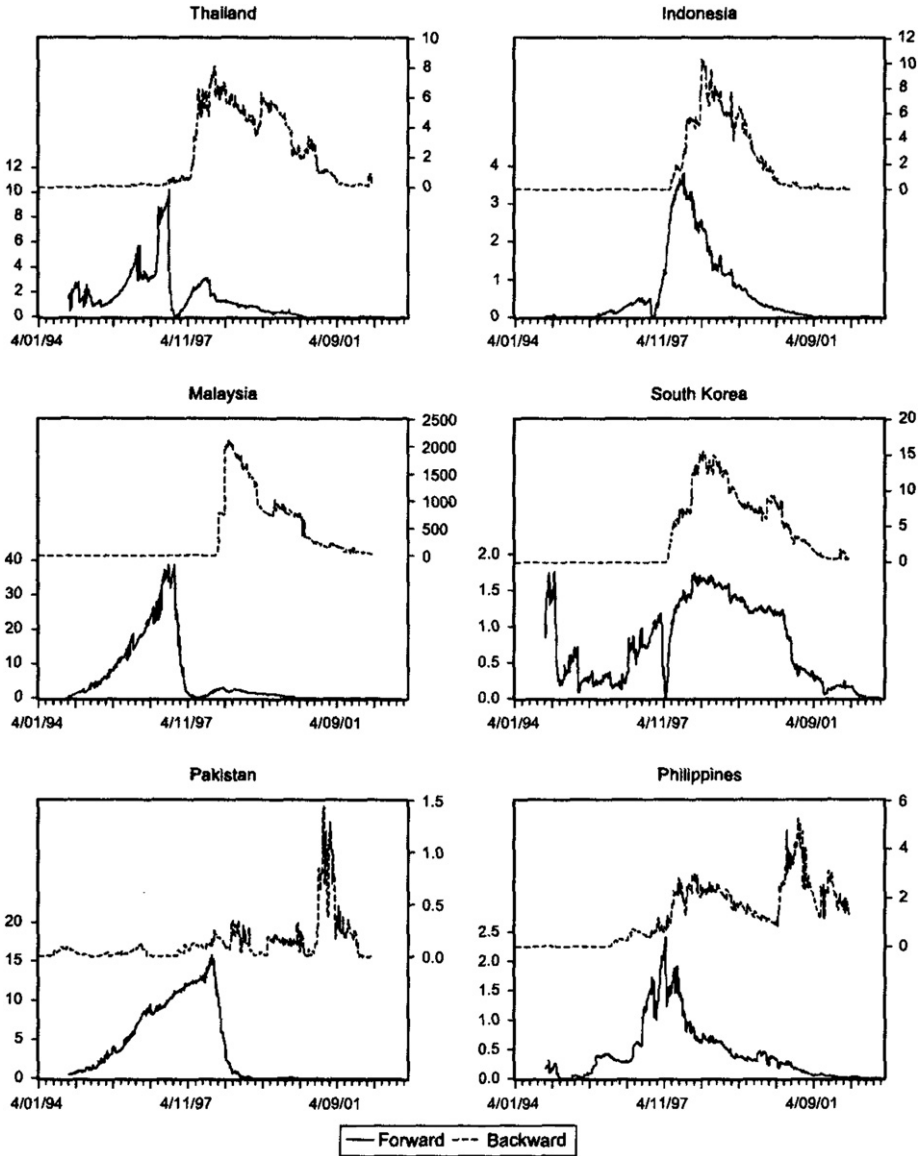


Fig. 4. ARCH-robust recursive (forward) and reverse recursive (backward) endogenous break tests for emerging currency returns (4/1/1994–25/6/2003).

The “forward” panel results (Panel A) correspond with statistically significant drops in α whereas the “backward” panel outcomes stand for significant rises in the tail index.

Following unsustainable speculative pressures, all considered countries had to abandon their currency pegs against the US\$ during the second half of 1997. Most of these regime changes can be traced back in Table 2. The Central Banks of Thailand, Malaysia and Indonesia announced a managed float on July 2, July 14 and July 11, respectively. For Indonesia, the

Table 2
 ARCH-corrected multiple break date estimates and test statistics: Asian currencies

	Currencies					
	Indonesia	Pakistan	Malaysia	Philippines	South Korea	Thailand
Panel A: Forward breakpoints ^{a,b,c} ($\alpha_1 > \alpha_2$)						
1997	10/7/97 (43.36)**		15/5/97 (38.77)**	24/11/97 (2.41)**		15/5/97 (10.21)**
1998	25/5/98 (3.80)**	20/3/98 (5.80)** 2/7/98 (15.77)**				
1999			5/4/99 (89.21)**			
Panel B: Backward breakpoints ^{a,b,c} ($\alpha_1 < \alpha_2$)						
1996	28/6/96 (4.26)**					
1997			7/2/97 (11.18)**			10/4/97 (11.92)**
1998	6/11/98 (10.33)**		2/12/98 (2118)**	11/8/98 (4.88)**	6/1/98 (6.76)**	22/7/98 (8.15)**
2000					23/11/98 (15.56)**	
2001				3/5/01 (5.24)**		

Values of forward and backward test statistics are reported between brackets. The recursive test calculations start with an initial sample size of 200. The number of upper order extremes used in estimating the tail index equals 10% of the total sample size.

^a Calendar dates of the breaks are in continental (dd/mm/yy) notation.

^b Full sample breaks are in bold and correspond with the global maxima of the sup Q test.

^c * and ** denote statistically significant breakpoints at the 5% and 1% significance levels, respectively; the used asymptotic critical values for Q are equal to 1.78 and 2.54, respectively.

estimated July break nearly perfectly coincides with the date of the regime shift. As for Malaysia and Thailand, the estimated break dates closest to the regime change (May 1997) seem to anticipate the managed float announcement. The Central Bank in the Philippines announced in June 1997 that the Philippine Peso could trade in a wider range; they did, however, not devalue. This half-hearted measure did not lift speculative pressure on the Peso and it continued losing value during the rest of 1997. The June 1997 widening of the bands does not seem to have been picked up by the breakpoint test but the resulting depreciations obviously are (24/11/97). Clearly, the three cited countries were not very successful in curbing the rise in extreme volatility in the immediate aftermath of the exchange rate liberalization. South Korea constitutes a notable exception. The Central Bank in South Korea decided to abandon its defense of the Won in November 1997 which is relatively accurately approximated by the January 1998 break. However, and in contrast to all previously discussed breaks, this latter break date reflects an increase in α , i.e., a reduction in extreme volatility. This might be understood by the fact that the Korean monetary authorities were relatively more successful in curbing increased volatility after they abolished the fixity of the Won. Already in December 1997, they agreed with the IMF on huge bailout package and new regulatory legislation for the financial sector was passed through parliament. Presumably, these measures were perceived as credible and sustainable by financial market participants because they resulted in huge net capital

inflows and a re-stabilization of the Won. The other countries eventually also managed to stabilize the currency but it took them more time. In Indonesia, the cancellation of fuel and food subsidies and the resulting resignation of President Suharto in May 1998 aggravated extreme forex volatility and thus further decreased the tail index α (25/5/98 break). Indonesian authorities were finally able to win back the trust of forex speculators by abolishing the plan for re-introducing a currency board system with a fixed peg in March 1998 and by voting new bankruptcy law (August 1998). (Probably) as a result of these and other measures the Rupiah appreciated with 40.79% during October 1998 (6/11/98 break). In the Philippines, forex turbulence diminished markedly after President Estrada was impeached on the basis of corruption charges and replaced by President Arroyo. The consecutive win in the parliamentary elections of president Arroyo's coalition (May 2001) coincides with the 3/5/01 break. Finally, Malaysia re-established a fixed peg against the US\$ in September 1998 which has proven to be successful since then. This explains the 2/12/98 breakpoint. Pakistan is the only country for which none of the devaluations in the sample period (August 1996, September 1996, October 1996, October 1997) occurs as break in Table 2.⁸

Summarizing, we are able to identify multiple statistically significant breaks and some of them can be linked to changes in monetary regimes or other types of institutional reform. Moreover the forward breaks (drops in α) in the top panel of Table 2 usually precede the backward breaks reported in the bottom panel (rises in α). This further confirms the U-shape already observed in the forward and backward recursive Hill estimates. The fact that we find multiple forward and backward breakpoints indicates that the tail index α did not change all of a sudden but rather gradual. The drops in α are mostly situated during the 1997 Asian crisis era (with an exception for Pakistan) which basically blew away all Asian fixed exchange rate regimes. Also, notice that the lower panel results in Table 2 confirm earlier findings that fully floating systems let exchange rates adjust more smoothly compared to fixing the emerging currency returns against the US\$, see e.g., Koedijk et al. (1990, 1992) for earlier references.

As a benchmark for comparison with the emerging currency panel, we also applied the multiple breaks testing procedure to a set of developed currency returns. Fig. 5 reports the values of $\hat{\eta}_t^{-1} Y_n^2$ for the recursive and reverse recursive test statistic and for five developed currencies against the US\$.

The figure clearly shows that the values of the test statistics do not give rise to significant breaks (except for Japan in the backward test). These results are not too surprising given the lack of explicit regimes in US\$ currency markets.

5. Conclusions

Extreme value techniques gained in popularity in recent years, both in academia and financial practice (risk management, regulation). Taking into account the empirical stylized fact that the returns on foreign exchange exhibit more probability mass in the tails than the normal distribution (heavy tails), extreme value analysis typically focuses upon estimating the so-called tail index α of the return distribution. This parameter is assumed to be constant over time. The question arises whether this assumption is always justified. Recently, Quintos et al.

⁸ As concerns the interpretation of the breaks that have been identified for Pakistan, the March 1998 break might be related to the political turmoil raised by the resignation of four government ministers.

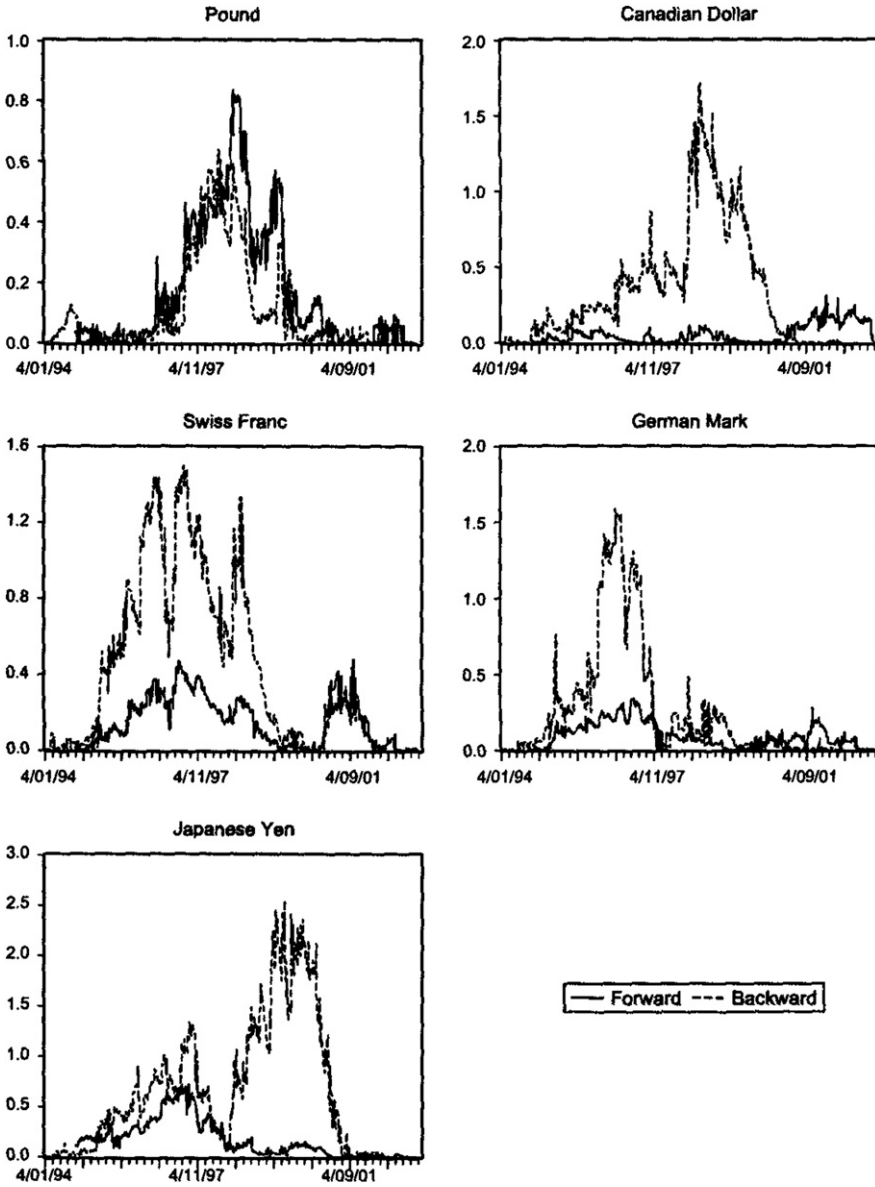


Fig. 5. ARCH-robust recursive (forward) and reverse recursive (backward) endogenous break tests for developed currency returns (4/1/1994–25/6/2003).

(2001) tested whether the tail shape of emerging stock index returns is constant over time by proposing a novel set of endogenous structural breaks tests for α . Using these tests, they found some evidence for structural change in emerging stock market tails indeed.

In this paper we generalized their analysis toward an analysis of *multiple* breaks for *emerging currency* returns. We argued that emerging forex markets are the most obvious candidates for detecting multiple breaks in the tail index α : if the value of the tail index depends on the

exchange rate regime — an often made corroboration in the empirical literature — then one might expect a lot more breakpoints in the tail behavior of emerging currencies compared to developed (industrial) currencies. This is because emerging currency markets have been characterized by nearly endemic switches in currency regimes in recent monetary history.

We then proposed to test for multiple breaks by generalizing the recursive test from the Quintos et al. paper. The approach basically amounts to applying the recursive test both over the full sample and over subsamples. More specifically, we first applied the recursive test to the full sample; if a significant breakpoint was detected then the recursive test was repeated over a partitioning of subsamples. We continued calculating recursive testing values for smaller and smaller subsets of the original sample until we were no longer able to reject the null hypothesis of parameter constancy. A problem with the Quintos et al. recursive approach is that it cannot detect rises in α because the extremes of the initial recursive sample also dominate the tail behavior of recursive α -estimates for larger samples. However, we argued that this problem is more apparent than real because one can perform the test over the “inverse” of the sample, i.e., by reversing the calendar time.

Upon applying the “multistage” version of the recursive and reverse recursive procedure for six Asian currencies we were indeed able to identify multiple breakpoints. Moreover, the recursive test signaled breakpoints that are less recent than its reverse recursive counterpart indicating that the tail index α tends to increase toward the end of the sample period. Initial drops in tail indexes during 1997 might be attributed to speculative attacks and abolishments of exchange rate regimes and the resulting upswings in volatility; whereas the increase in the tail index later on might be linked to widespread attempts in most Asian countries to further liberalize the legal-institutional framework of their financial markets. We also compared the multiple breaks results for emerging markets with testing results for developed currencies and barely found any breakpoints in the tail index of the latter currencies. The smaller amount of regime switches in the developed currency block provides further evidence for the corroboration that there is a relationship between regime switches and structural changes in the tail index. If tail indexes are found to be time varying, extreme value analysis may be less suitable for applications with emerging market data, e.g., calculating Value-at-Risk levels far out in the tails of emerging market currency portfolios.

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Appendix A. Data description and discussion

Data were obtained from Datastream Inc. We downloaded daily nominal bilateral spot rates against the Pound sterling for six emerging Asian currencies (Indonesian Rupiah, Malaysian Ringgit, Thai Baht, Philippine Peso, South Korean Won, Pakistan Rupee) and five developed currencies (Japanese Yen, British Pound, Swiss Franc, Canadian Dollar, German Mark). Notice that we downloaded against the Pound sterling numéraire because this renders the largest cross

section of currencies over the longest possible time span within the Datastream database. US\$ denominated cross rates were then calculated by applying the no triangular arbitrage condition. Thus, a depreciation (appreciation) of the currency against the US\$ corresponds with a rise (fall) in its value. Since January 1999 and the introduction of the Euro, German Mark rates are irrevocably fixed to the Euro, by a conversion rate of 1.9558 DEM/EUR, so that Euro and Mark returns are basically identical, irrespective of the numéraire currency. The sample period for the selected currencies runs from January 3, 1994 until July 25, 2003 which amounts to 2473 daily observations.

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