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Exchange Rate Models and Parameter Variation: The Case of the Dollar-Mark Exchange Rate

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1. Introduction

In a recently published paper, Wolff (1987) used varying-parameter estimation techniques, based on recursive application of the Kalman filter, to improve the predictive performance of monetary exchange rate models. Allowing estimated parameters to vary over time was found to enhance the models' forecasting performance for the Dollar-Pound, Dollar-Mark and Dollar-Yen exchange rates. Contrary to earlier results in the literature (Meese and Rogoff, 1983), *ex-post* forecasts for the Dollar-Mark rate compared favorably with those obtained from the naive random walk forecasting rule.

In this paper the Wolff (1987) results are reexamined and extended in two directions. First, test statistics are constructed to test whether the monetary models do *significantly* better than the random walk for the Dollar-Mark case. Second, a true *ex-ante* forecasting experiment is performed in order to test whether the earlier finding for the Dollar-Mark exchange rate remains valid in a context where exchange rate forecasts are generated only on the

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basis of information that was available to economic agents at the time. Such *ex-ante* forecasts based on monetary exchange rate models have not been studied in the literature to date.

2. State-Space Versions of Monetary Exchange Rate Models

Wolff (1987) studied exchange rate models of the following form:

$$s(t) = z'(t)\, c(t) + \varepsilon(t) \tag{1}$$

$$c(t+1) = c(t) + \eta(t). \tag{2}$$

Here $s(t)$ denotes the natural logarithm of the spot exchange rate at time t, $z(t)$ is a vector of explanatory variables, $c(t)$ is a vector of time-varying coefficients and $\varepsilon(t)$ and $\eta(t)$ are uncorrelated white noise disturbance terms. Equation (1) represents a dynamic version of a class of monetary exchange rate models. The vector $z(t)$ contains explanatory variables which are prescribed by different versions of the monetary models. Two variants of the monetary models were studied in Wolff (1987): the Frenkel-Bilson model (due to Frenkel, 1976; and Bilson, 1978) and the Dornbusch-Frankel model (due to Dornbusch, 1976; and Frankel, 1979). For a more precise description of the models, see Wolff (1987). In equation (2) parameter variation in the vector $c(t)$ is allowed for to address possible parameter instability due to a number of factors (on which, see Wolff, 1987). Equations (1) and (2) are in so-called state-space format, so that the Kalman filter can be employed for recursive estimation and prediction.

In Wolff (1987) an *ex-post* forecasting experiment was performed based on equations (1)—(2). On the basis of a monthly dataset it was found that allowing estimated parameters to vary over time enhanced the models' forecasting performance for the Dollar-Mark, Dollar-Yen and Dollar-Pound exchange rates. The forecasting experiment was an *ex-post* experiment in the sense that actual, realized values of the vector of explanatory variables $z(t)$ were used. Contrary to earlier results in the literature (Meese and Rogoff, 1983) *ex-post* forecasts for the Dollar-Mark rate compared favorably with those obtained from the naive random walk forecasting rule for all forecasting horizons that were studied.

3. Significance Tests

In this section significance tests are constructed in order to assess whether the state-space structural models for the Dollar-Mark exchange rate predicted *significantly* better than the random walk forecasting rule. Since the model forecasts are generated for overlapping periods, the prediction errors for a given horizon of n months ($n > 1$) will not be independently distributed. This will require special care in the construction of a suitable test statistic.

The approach that we take to the construction of test statistics is the following. First, we form the time series $e_a(t, n)$ and $e_s(t, n)$ as follows:

$$e_a(t, n) = |w_1(t, n)| - |w_2(t, n)| \quad (3)$$

$$e_s(t, n) = [w_1(t, n)^2] - [w_2(t, n)]^2 \quad (4)$$

for all forecasting horizons n ($n = 1, 3, 6, 12, 24, 36$) and all points in time t ($t = 1, 2, 3, ..., T_n$). In equations (3) and (4) $w_i(t, n)$ is the forecast error resulting from model i's forecast, generated at time t, of the logarithm of the spot exchange rate at time $t + n$. The subscript $i = 1$ refers to the random walk model and $i = 2$ refers to one of the state-space monetary models. T_n is the sample size of the series $[e_a(t, n)]$ or $[e_s(t, n)]$ for a given horizon n. The number $e_a(t, n)$ is the difference in absolute forecast errors between the random walk model and one of the state-space models for a given combination of t and n, and $e_s(t, n)$ is the difference in squared forecast errors.

The Mean Absolute Error (MAE) of a state-space model's forecast at horizon n is said to be significantly lower (higher) than the MAE of the random walk forecasting rule if the mean of the series $[e_a(t, n);\ t = 1, 2, 3, \ldots, T_n]$ is significantly greater (smaller) than zero. Similarly, the mean square error (MSE) of a state-space model's forecasts at horizon n is said to be significantly lower (higher) than the MSE of the random walk forecast if the mean of the series $[e_s(t, n);\ t = 1, 2, 3, \ldots, T_n]$ is significantly greater (smaller) than zero.

Since the forecasts are overlapping, we use the Hannan efficient estimator (Hannan, 1963) to compute asymptotic t-statistics. Hannan efficient estimation is essentially a procedure for doing Generalized Least Squares (GLS) in the frequency domain, allowing for arbitrary patterns of serial correlation. The construction of the test statistics involves the following steps:

(a) The series $[e_a(t, n);\ t=1, 2, 3, \ldots, T_n]$ or $[e_s(t, n);\ t=1, 2, 3, \ldots, T_n]$ is regressed on a constant term, using Ordinary Least Squares.
(b) The residuals from this regression and the dependent variable are sent to the frequency domain.
(c) A consistent (i. e. *smoothed*) estimate of the spectrum of the residuals is computed.
(d) The dependent variable is Fourier transformed, then filtered [using the estimated spectrum from step (c)] and inverse transformed.
(e) The filtered dependent variable is sent back to the time domain and is regressed on a constant term. The large-sample t-statistic on the constant term in this regression is the test statistic we set out to compute.

The results of this procedure for the Dollar-Mark exchange rate are presented in Table 1 (for MAEs) and Table 2 (for MSEs). Both the Frenkel-Bilson and Dornbusch-Frankel versions of the state-space monetary models are considered. The monthly data that are used are the same as those described in Wolff (1987). The estimation and prediction procedure is also the same. The period over which forecasts are generated ranges from November 1976 through April 1984. As in Meese and Rogoff (1983) and Wolff (1987), natural logarithms of exchange rates are predicted, rather than their simple levels, in order to circumvent any problems arising from Jensen's inequality.

Table 1. *Comparison of Mean Absolute Prediction Errors of State-Space Monetary Models and the Random Walk Forecasting Rule*

Horizon (n) (Months)	MAE (RW)[a]	MAE (FB)[b]	MAE (DF)[c]	Test-Statistic 1[d]	Test-Statistic 2[e]
1	2.52	2.42	2.40	0.62	0.86
3	4.52	4.40	4.29	0.12	0.46
6	6.65	5.94	6.09	0.61	0.64
12	10.58	8.42	8.28	1.52	1.90
24	19.13	12.13	11.50	2.57*	2.70**
36	24.83	13.89	11.55	3.77**	3.75**

[a] MAE (RW) is the MAE of the random walk forecast. All MAEs are approximately in percentage terms.
[b] MAE (FB) is the MAE of the state-space version of the Frenkel-Bilson model.
[c] MAE (DF) is the MAE of the state-space version of the Dornbusch-Frankel model.
[d] Tests whether MAE (FB) is significantly different from MAE (RW).
[e] Tests whether MAE (DF) is significantly different from MAE (RW).
* Denotes statistical significance at the 5% level.
** Denotes statistical significance at the 1% level.

Table 2. *Comparison of Mean Square Prediction Errors of State-Space Monetary Models and the Random Walk Forecasting Rule*

Horizon (n) (Months)	MSE (RW)[a]	MSE (FB)[b]	MSE (DF)[c]	Test-Statistic 1[d]	Test-Statistic 2[e]
1	11.39	10.86	10.77	0.54	0.72
3	30.91	27.68	26.57	0.61	0.97
6	65.69	51.71	49.54	0.90	1.28
12	161.03	109.57	98.76	1.54	2.13
24	431.21	223.31	200.56	2.26*	2.64**
36	708.49	283.95	230.23	3.03**	3.13**

[a] MSE (RW) is the MSE of the random walk forecast. All MSEs are approximately in percent squared.
[b] MSE (FB) is the MSE of the state-space version of the Frenkel-Bilson model.
[c] MSE (DF) is the MSE of the state-space version of the Dornbusch-Frankel model.
[d] Tests whether MSE (FB) is significantly different from MSE (RW).
[e] Tests whether MSE (DF) is significantly different from MSE (RW).
* Denotes statistical significance at the 5% level.
** Denotes statistical significance at the 1% level.

The test statistics presented in Tables 1 and 2 indicate that the state-space versions of both models did statistically *significantly* better than the random walk forecasting rule for the longer horizons (two years and up). The improvement was not significant for the shorter horizons. These results should be interpreted with caution, not only because the test statistics pertain to large samples, but also because the overlapping nature of the forecasts reduces the amount of information that is effectively present in our sample, relative to a nonoverlapping sample with an equal number of observations.

4. An Ex-Ante Forecasting Experiment

In the literature to date, only *ex-post* forecasts on the basis of structural exchange rate models have been studied. Because Meese and Rogoff (1983) showed that such *ex-post* forecasts were dominated by simple random walk forecasts, it did not seem fruitful to consider *ex-ante* forecasts (using predicted rather than realized values of the explanatory variables) based on these models.

In the context of varying-parameter versions of the monetary models, however, it is interesting to perform an *ex-ante* forecasting experiment for the Dollar-Mark case in order to test whether the finding that exchange rate forecasts based on state-space versions of the monetary models outperform the random walk forecast, remains valid in a context where exchange rate forecasts are generated only on the basis of information that was available to economic agents at the time.

In order to be able to perform such an *ex-ante* forecasting experiment, univariate time series models of the class advocated by Box and Jenkins (1976) are used to forecast the variables that the monetary models take to be exogenous. The procedure is as follows. Initially the time series models are estimated up through the first forecasting period, November 1976, and predictions of the exogenous variables up to 36 months ahead are generated. The time series models are constructed for differences between domestic and foreign variables rather than for individual variables. The state-space models are also estimated up through November 1976 and exchange rate forecasts are then generated using the predictions from the time series models as expectations of future explanatory variables. Then December 1976 data are added to the sample, the time series model are reestimated and again used for the generation of predictions of the exogenous variables. The parameters of the state-space models are then updated using December 1976 data and exchange rate predictions are formed as before. This recursive process continues until forecasts are formed using April 1984 data. The results are presented in Table 3.

Table 3. *Summary Statistics on the Forecasting Performance of State-Space Monetary Models for the Dollar-Mark Exchange Rate; Explanatory Variables are Predicted on the Basis of Univariate Time Series Models*

Horizon (Months)	ME[a]	MAE	RMSE[b]	U-Statistic[c]	*N*[d]
Frenkel-Bilson Model					
1	0.27	2.48	3.36	0.996	89
3	0.89	4.48	5.45	0.980	87
6	1.66	6.28	7.63	0.942	84
12	3.44	9.54	11.57	0.912	78
24	7.02	17.79	20.65	0.994	66
36	2.10	22.66	27.06	1.016	54
Dornbusch-Frankel Model					
1	0.37	2.46	3.35	0.993	89
3	1.23	4.43	5.45	0.980	87
6	2.38	6.78	8.03	0.990	84
12	4.67	10.33	12.53	0.987	78
24	7.91	20.11	22.98	1.107	66
36	0.91	23.40	27.53	1.034	54

[a] ME denotes the Mean Error of the Forecasts. The ME is approximately in percentage terms.
[b] RMSE denotes the Root Mean Square Error of the Forecasts. The RMSE is approximately in percentage terms.
[c] The U-Statistic is the ratio of the model's RMSE to the RMSE of the random walk forecast.
[d] *N* denotes the number of observations.

When comparing the summary statistics in Table 3 above with those in Tables 4 and 5 in Wolff (1987), it is noted that the *ex-ante* forecasts are on average considerably less accurate than the *ex-post* forecasts: knowledge of the realizations of the explanatory variables of the monetary models is useful when generating forecasts. The *ex-ante* forecasts are especially less accurate than the *ex-post* forecasts for the longer horizons. The U-Statistics in Table 3 are smaller than one in many cases, indicating that the state-space monetary models did better than the random walk even on an *ex-ante* basis (at least for horizons up to one year).

5. Conclusions

In this article a recent finding that *ex-post* exchange rate forecasts on the basis of state-space versions of monetary exchange rate models outperform the simple random walk forecasting rule for the Dollar-Mark exchange rate was reexamined in some detail. Tests statistics were constructed to test whether the models did *significantly* better than the random walk. These statistics indicate that the models did significantly better than the random walk at horizons of 2 years and up, but for the shorter horizons the improvement was not significant.

A true *ex-ante* forecasting experiment was performed to test whether the earlier finding remains valid in a context where exchange rate forecasts are based only on information which was available to economic agents at the time. The results indicate that the state-space monetary models did better than the random walk model, even on an *ex-ante* basis, for horizons up to about one year.

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